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Reputation, Career Concerns, and Job Assignments

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Abstract

Does a worker who had a successful career have stronger or weaker incentives to manipulate his reputation than a worker who performed poorly? This paper presents a tractable model that allows us to study career concerns when the strength of a worker's incentives depends on his employment history (the history of his past actions, jobs, and performances). More specifically, the paper incorporates standard job assignments into the main model in Holmstrom's (1999) seminal paper on career concerns. Equilibrium wages, equilibrium job assignments, and the strength of career-concern incentives are the same for all employment histories that lead to the same worker's reputation. (With reputation we refer to beliefs about the worker's future productivity.) We show that, typically, workers with a better reputation have stronger incentives than workers with a worse reputation. Furthermore, we show that when the strength of incentives depends on employment history, (i) a ratchet effect may appear, (ii) in spite of this ratchet effect, incentives may be stronger, and (iii) incentives may be stronger when beliefs about ability are more precise.

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1 Introduction

Fama (1980) suggests that workers are disciplined by opportunities in markets for their services, that is, by their career concerns. These career concerns appear in situations where firms do not know a worker's future productivity, but they learn about it by observing his performance. In general, a worker's compensation is influenced by firms' beliefs about his future productivity. A worker may want to improve his performance in order to make firms believe that he is more productive and thus obtain a better compensation.

Does a worker who had a successful career have stronger or weaker career-concern incentives than a worker who performed poorly? For tractability reasons, previous work sidesteps the relationship between employment history—i.e., the history of a worker's past actions, jobs, and performances—and the strength of career concerns.[1] Earlier research does so by restricting the analysis to environments in which there is only one period with career concerns (and, thus, no relevant employment history), or to multi-period environments in which the strength of career concerns does not depend on employment history.[2] In contrast, we present a multi-period model that allows us to study career concerns when the strength of a worker's incentives to manipulate his future reputation depends on his employment history (with reputation we refer to beliefs about the worker's future productivity).

We study a parsimonious departure from the main model in Holmstrom's (1999) seminal paper on career concerns. In that model, workers can only be employed in one type of job. In contrast, we study an economy with N job types. Each period, a worker can be assigned to any of the N jobs. The output produced by a worker depends on the quality of his labor input (henceforth the quality) and on the job to which he is assigned.

[1] See, for example, Holmstrom (1999), Gibbons and Murphy (1992), Besley and Case (1995), Jeon (1996), Prendergast and Stole (1996), Meyer and Vickers (1997), Dewatripont, Jewitt, and Tirole (1999a, 1999b), Persson and Tabellini (2000), Alesina and Tabellini (2003), De Motta (2003), Ortega (2003), Le Borgne and Lockwood (2004), Eggertsson and Le Borgne (2005), and Shi and Svensson (2006).

[2] For instance, when describing their model, Dewatripont, Jewitt, and Tirole (1999a) explain that "The model is thus a 'two-period' model (effort in period 1, reward in period 2). Its extension to more than two periods is, except in the additive normal case, a complex matter that is left for future research. In a multi-period context, the agent acquires private information about the marginal productivity of her effort and the impact of effort on future wages...." We study a model in which the agent may have private information about the impact of effort on future wages.

Quality depends on the worker's ability, his effort level, and luck. Firms do not observe the worker's ability, effort, and luck; instead they observe his performance. Good current performance by the worker signals that he is capable of good performance in the future. The wage offered to the worker increases if he is expected to perform better. Therefore, the worker exerts effort to improve his current performance.

There are two properties that define a job. One is the fixed amount of output produced by a worker independently of the quality of his labor input. The other is the marginal return to quality. As is standard in the literature on job assignments, we assume that workers who are expected to provide input of higher quality are also expected to be more productive in jobs with higher marginal return to quality.[3] We refer to jobs with higher marginal return to quality as managerial jobs that are at higher levels in a firm's hierarchy.[4]

We show that career concerns are typically stronger for workers with better reputation. A worker's reputation conveys all the payoff-relevant information. That is, for all employment histories that lead to the same reputation at the beginning of a period, equilibrium wages, equilibrium job assignments, and the strength of career-concern incentives are the same. In a competitive labor market, the wage offered to a worker increases with the expected quality of his labor input, and the change in the wage offered with respect to expected quality is increasing in the marginal return to quality of the job to which the worker will be assigned. A worker with a better current reputation is more likely to be assigned to jobs with a higher marginal return to quality. As a result, he expects a larger increase in wages from making firms believe that his labor input will be of better quality. Consequently, he chooses a higher effort level. This finding is consistent with evidence from time-use surveys that shows that workers with more years of education allocate more time for market work (see, for example, Aguiar and Hurst (2007)).[5]

[3]This is the case, for example, in Bernhardt (1995) and Gibbons and Waldman (1999) (see also the references therein). In these studies, workers do not decide their effort level.

[4]For empirical evidence of the "demand for talent at the top," see Cuñat and Guadalupe (2006) and the references therein. Span-of-control models present a theory of why employees with higher ability are assigned to higher levels in hierarchies (see, for example, Rosen, 1982).

[5]Note that one way of exerting more effort on the job is working more hours. Like effort in agency models, some working hours (for example, the time spent thinking about a problem) are not directly observed by the employer. Furthermore, in general, workers with more years of education are more likely to work in managerial jobs, where the marginal return to quality is higher. Thus, these workers could choose to work more hours (or exert more effort) because they expect a larger benefit from making firms believe that the expected quality of

This paper highlights three other lessons from the study of a model of career concerns where the strength of incentives depends on a worker's employment history. First, the paper shows that this dependence may lead to a ratchet effect in implicit incentives: a worker's incentives to exert effort in order to increase his next-period wage may be weakened by the effect of effort on his wages in other future periods. On the one hand, a worker has incentives to exert effort in order to increase his next-period wage: a worker's effort positively affects firms' *current-period learning* about his ability and, thus, exerting effort increases a worker's next-period wage. On the other hand, a worker's effort may negatively affect firms' *next-period learning* about his ability. As a result, exerting effort may decrease a worker's wage after the next period. We show that the latter (ratchet) effect appears because next-period equilibrium effort depends on employment history.

Second, we show that when employment history affects the equilibrium effort level, the effect of effort on next-period wage may be stronger. As a result, in spite of the ratchet effect described above, career concerns may be stronger. Wages reflect expected productivity, which is increasing in the expected effort level. If the equilibrium effort strategy is an increasing function of reputation (as is typically the case), by exerting effort in the current period, a worker not only improves his next-period expected productivity by increasing his expected ability but also increases the effort firms expect him to exert (firms expect the worker to exert the equilibrium effort level). Thus, a worker's incentives to exert effort are strengthened by his desire to make the market believe that he will exert more effort in the future.

Third, the paper shows that a worker's career concerns may be stronger if the prior belief about his ability is more precise. We show that with more precise priors, an improvement in a worker's reputation may imply a greater increase in the next-period effort level firms expect him to exert, which in turn would imply a greater next-period wage increase. As a result, incentives to exert effort may be stronger.

The remainder of this paper is organized as follows. Section 2 presents the framework, Section

their labor input is higher. Of course, other theories could also explain the relationship observed in the data. For instance, if workers with more education are offered a higher market wage, they could choose to sell more observable hours in the labor market. To distinguish between the two theories, one could look at whether workers are paid by the hour and whether employers can observe working hours.

3 discusses results, and Section 4 concludes.

2 Framework

Following Holmstrom (1999), we study a competitive economy with many identical, risk-neutral profit-maximizing firms in the market each period, with labor as the only factor of production. There is a single output good, the price of which is normalized to one. We study a worker's incentives to exert effort in this economy. The number of employment periods is discrete and indexed by $t \in \{1, 2, ..., T\}$.

The amount of output y_t produced by the worker depends on the quality of his labor input (quality) q_t and the job in which he is employed $j_t \in \{1, ..., N\}$. Quality q_t is a stochastic function of ability η_t, and effort $a_t \geq 0$. In particular,

$$q_t = a_t + \eta_t + \varepsilon_t,$$

where ε_t is a normally distributed random variable with expected value 0 and precision h_ε (the variance is $\frac{1}{h_\varepsilon}$). The worker's ability η_t evolves as a random walk. In particular, $\eta_{t+1} = \eta_t + \xi_t$, where ξ_t is assumed to be normally distributed with mean 0 and precision h_ξ.

In any job j, the output produced by the worker would be a linear function of quality. In particular, we make the following assumption.

Assumption 1: If the worker is employed in job j, his output is given by

$$y_t = A^j + \lambda^j \left(a_t + \eta_t + \varepsilon_t\right). \tag{1}$$

Assumption 1 implies that job j can be characterized by two parameters: the amount of output produced independently of quality, A^j, and the marginal return to quality, $\lambda^j > 0$.[6] Without loss of generality, we make the following assumption.

Assumption 2: $\lambda^1 < \lambda^2 < ... < \lambda^N$.

[6]The production function in equation (1) is particularly close to the one studied by Gibbons and Waldman (1999) (who do not study the worker's effort choice). As in previous studies on job assignments, we assume that there is only one type of ability. If $N = 1$, $\lambda^1 = 1$, and $A^1 = 0$, the production function in equation (1) is exactly the one in the main model studied by Holmstrom (1999).

We also assume that no job is fully dominated by the other jobs in the economy. That is, for any job j there is a quality level such that job j is the most productive job for that quality level. This allows us to restrict attention to jobs to which the worker could be assigned in equilibrium.

Neither firms nor the worker know the worker's ability. Before the worker starts his career, firms and the worker share the same belief about his ability. This belief is normally distributed with mean m_1 and precision h_η.

Firms and the worker learn about the worker's ability using Bayesian learning. From this point forward, *belief* refers to *belief about the worker's ability,* unless stated otherwise.

Depending on the precision of the shock that determines the evolution of the worker's ability, h_ξ, the precision of beliefs may be increasing or decreasing with respect to the number of observations of the worker's performances (t) (see Holmstrom, 1999). The following assumption guarantees that the precision of beliefs is constant.

Assumption 3:

$$h_\xi = \frac{h_\eta^2 + h_\eta h_\varepsilon}{h_\varepsilon}. \tag{2}$$

Assumption 3 simplifies the analysis by allowing us to abstract from a tenure effect on the strength of career concerns discussed in previous studies (see, for example, Holmstrom, 1999). We focus on the relationship between employment history and the strength of career concerns, which previous studies left unexplored.

The timing of events within each period is as follows. First, each firm offers a wage to the worker. Second, the worker decides where to work, his employer pays him his wage w_t, and he consumes it.[7] Third, the firm that hired the worker decides in which job he will be employed. Fourth, the worker decides on his effort level. Fifth, the luck component of quality ε_t and the shock that determines the evolution of the worker's ability ξ_t are realized, and output y_t is observed (in period 1, ability η_1 and ε_1 are realized before y_1 is observed).

Ability η_t, the shock that determines the evolution of ability ξ_t, and the luck component of quality ε_t are unobservable. Firms do not observe the worker's effort level a_t (which is of course known by the worker). Job assignments j_t are public information.

[7]As in previous studies of career concerns, this paper does not study the worker's participation decision. It is assumed that the worker always works.

There is a cost to exerting effort, $c(a_t)$, with $c'(a_t) \geq 0$, $c''(a_t) > 0$, and $c'(0) = 0$. The worker's period-t utility equals $w_t - c(a_t)$. Let $\delta \in (0,1)$ denote the firms' and the worker's discount factor.

3 Results

This section is organized as follows. First, we discuss equilibrium learning. Second, we describe the main properties of the equilibrium when the worker can only be employed for three periods. Finally, we discuss whether equilibrium properties change when other employment durations are considered.

3.1 Equilibrium learning

Since beliefs are Gaussian and have a constant precision (because of Assumption 3), their evolution is described by the evolution of their mean. Let m_{ft} and m_{wt} denote the mean of the firms' and the worker's beliefs *at the beginning of period* t (from here on, *at period* t). We refer to a *belief with mean* m as *belief* m. At t, if the firms' and the worker's belief coincide, $m_t = m_{ft} = m_{wt}$ denotes their belief.

Define $s_t \equiv \eta_t + \varepsilon_t$. We refer to s_t as *the signal* of the worker's ability η_t. Bayes' rule implies that the mean of the beliefs at $t+1$ is a weighted sum of the mean at t and the period-t signal. Assumption 3 implies that the precision of the period-$t+1$ beliefs about the signal s_{t+1} is always equal to the precision of the period-t beliefs about the signal s_t. This precision is given by

$$H \equiv \frac{h_\eta h_\varepsilon}{h_\varepsilon + h_\eta}. \tag{3}$$

Since beliefs about the signal are Gaussian and have a constant precision, the evolution of these believes can also be summarized by the evolution of their mean, which is equal to the mean of the beliefs about ability. Assumption 3 implies that the weight of period-t mean beliefs in period-$t+1$ mean beliefs does not depend on t. This weight is given by

$$\mu = \frac{h_\eta}{h_\eta + h_\varepsilon}. \tag{4}$$

Observing output y_t and job j_t allows firms and the worker to compute the signal s_t using their knowledge about the effort exerted by the worker and the production function. The worker knows the effort he exerted. Therefore, he is able to compute the true signal $s_t = \frac{y_t - A^{j_t}}{\lambda^{j_t}} - a_t$. Thus, the worker's belief at $t+1$ is characterized by

$$m_{wt+1} = \mu m_{wt} + (1-\mu)\, s_t.$$

Firms compute the signal using equilibrium effort levels. They are rational and understand the game. In particular, firms know the worker's strategy. They also know the history of outputs $y^{t-1} = (y_1, y_2, ..., y_{t-1})$ and the history of job assignments $j^t = (j_1, j_2, ..., j_t)$. Let $a_{ft}(y^{t-1}, j^t)$ denote the equilibrium period-t effort level computed by firms.[8] The period-t signal computed by firms is given by

$$s_{ft}\left(y^t, j^t\right) \equiv \frac{y_t - A^{j_t}}{\lambda^{j_t}} - a_{ft}\left(y^{t-1}, j^t\right) = s_t + a_t - a_{ft}\left(y^{t-1}, j^t\right). \tag{5}$$

Thus, firms' period-$t+1$ belief is given by

$$m_{ft+1} = \mu m_{ft} + (1-\mu)\left[s_t + a_t - a_{ft}\left(y^{t-1}, j^t\right)\right]. \tag{6}$$

Recall that firms and the worker have the same period-1 belief. Moreover, in any period in which the worker exerts the equilibrium effort level, firms and the worker compute the same signal (see equation (5)). Consequently, the next lemma holds

Lemma 1 *In equilibrium, the firms' and the worker's belief coincide ($m_{ft} = m_{wt}$).*

[8]When the worker decides the effort he will exert in period t, his information is given by $\left(y^{t-1}, j^t, a^{t-1}\right)$ where $a^{t-1} = (a_1, a_2, ..., a_{t-1})$ denotes the history of previous effort levels. For all $t > 1$, let $a_{wt}\left(y^{t-1}, j^t, a^{t-1}\right)$ denote the worker's period-t optimal effort level. Let $a_{w1}(j_1)$ denote the first-period equilibrium effort when the worker is employed in job j_1. Using $\left(y^{t-1}, j^t\right)$ and the history of equilibrium effort levels, firms can compute the period-t equilibrium effort level as

$$a_{ft}\left(y^{t-1}, j^t\right) \equiv a_{wt}\left(y^{t-1}, j^t, a_f^{t-1}\left(y^{t-2}, j^{t-1}\right)\right),$$

where

$$a_f^{t-1}\left(y^{t-2}, j^{t-1}\right) \equiv \left(a_{w1}(j_1), a_{w2}(y_1, j_1, a_{w1}(j_1)), ..., a_{wt-1}\left(y^{t-2}, j^{t-1}, a_f^{t-2}\left(y^{t-3}, j^{t-2}\right)\right)\right)$$

denotes the history of equilibrium effort levels computed using $\left(y^{t-2}, j^{t-1}\right)$.

3.2 A three-working-period version of the model

In this section, we characterize the equilibrium of an economy where the worker can only be employed for three periods, i.e., $t \in \{1, 2, 3\}$. We solve for equilibrium using backward induction.

3.2.1 The last working period

In period 3, there is no future compensation that could be influenced by the worker. Therefore, he always chooses $a_3 = 0$.

The firm that hires the worker assigns him to the job for which his expected productivity is higher. Since $a_3 = 0$, the period-3 quality of the worker's labor input expected by firms is given by the ability expected by firms, m_{f3}. Thus, if a firm assigns the worker to job j, it expects a productivity $A^j + \lambda^j m_{f3}$. Note that the higher the worker's expected ability, the better it is to assign him to a job with a higher marginal return to quality. Thus, we can present the optimal job assignment rule using the following reputation thresholds: $m_3^1 = -\infty$, for all $2 \leq j \leq N$,

$$m_3^j = \frac{A^{j-1} - A^j}{\lambda^j - \lambda^{j-1}}, \tag{7}$$

and $m_3^{N+1} = \infty$. Equation (7) defines m_3^j as the expected quality that makes a firm indifferent between employing the worker in jobs $j-1$ and j, i.e., m_3^j is such that $A^{j-1}+\lambda^{j-1}m_3^j = A^j+\lambda^j m_3^j$. Thus, if $m_{f3} > m_3^j$, a firm would rather employ the worker in job j than in job $j-1$ (recall Assumption 2 states that $\lambda^j > \lambda^{j-1}$).

Since we assumed a competitive labor market, in equilibrium, the wage offered to the worker is equal to his expected productivity, which is computed by firms using the equilibrium job assignment rule. We assume that, when a firm is indifferent between employing the worker in two jobs, it employs the worker in the job with the lowest marginal return to quality λ^j. Following the discussion presented above, the next lemma characterizes equilibrium in the last working period.

Lemma 2 *In period 3, the worker does not exert effort in equilibrium (i.e., $a_{w3}(y^2, j^3, a^2) = 0$ for any history (y^2, j^3, a^2)), he is assigned to job j if and only if $m_3^j < m_{f3} \leq m_3^{j+1}$, and the* *equilibrium wage is given by*

$$\omega_3(m_{f3}) = A^j + \lambda^j m_{f3}, \tag{8}$$

where j is such that $m_3^j < m_{f3} \leq m_3^{j+1}$. Thus, the period-3 wage is the same for all histories (y^2, j^2) that imply the same firms' belief m_{f3}, and is an increasing function of m_{f3}.

3.2.2 The second working period

In the remainder of the paper, the worker's equilibrium effort strategy is characterized using the first-order condition of his maximization problem for a given effort level used by firms to compute the signal.[9] Note that assuming concavity of the worker's maximization problem for a given effort level used by firms to compute the signal assures that, given that effort level, there exists a unique *optimal* effort level. But in order to find the *equilibrium* effort strategy, we have to solve a fixed-point problem: The effort level used by firms to compute the signal has to be equal to the effort level the worker exerts in equilibrium. Concavity does not guarantee that the equilibrium effort strategy exists and is unique. It could be that there is no equilibrium effort level such that if firms use that effort level to compute the signal, the worker chooses that effort level. Also, it could be that there exists more than one effort level such that if firms use that effort level to compute the signal, it is optimal for the worker to choose that effort level.[10]

Let $\phi(s; m, H)$ denote the density function for a signal that is normally distributed with mean m and precision H. For any $j \geq 2$, and any firms' period-two belief m_{f2}, let

$$s_2^j(m_{f2}) \equiv \frac{m_3^j - \mu m_{f2}}{1-\mu} \tag{9}$$

denote the smallest period-2 signal inferred by firms that, in period 3, would make assigning the worker to job j better than assigning him to job $j-1$. That is, $s_2^j(m_{f2})$ is such that

[9] In order to assure global concavity of the worker's problem, it is sufficient to assume enough convexity in $c(a)$. For example, one could find an upper bound for the slope of the marginal benefit curve and assume that the slope of the marginal cost curve is always higher. Another alternative is to assume the standard exponential cost function, $c(a) = a^n$, and to assume that n is high enough. In particular, this makes the worker's problem globally concave in the examples discussed in this paper.

[10] Dewatripont, Jewitt, and Tirole (1999b) present a model with one career-concern period in which there are multiple equilibria. In a multi-period version of their environment, it could be possible to construct equilibria in which the effect of employment histories on the strength of career-concern incentives goes beyond their effect on beliefs.

$\mu m_{f2} + (1-\mu) s_2^j (m_{f2}) = m_3^j$. The next proposition shows that a unique equilibrium effort strategy exists (see Appendix A for the proof).

Proposition 1 *In period 2, for any equilibrium belief m for the firms and the worker, there exists a unique equilibrium effort level $a_2^* (m)$ given by*

$$c'(a_2^* (m)) = \delta (1-\mu) \left[\lambda^1 \int_{-\infty}^{s_2^2(m)} \phi (s; m, H)\, ds + \lambda^2 \int_{s_2^2(m)}^{s_2^3(m)} \phi (s; m, H)\, ds + ... + \lambda^N \int_{s_2^N(m)}^{\infty} \phi (s; m, H)\, ds \right]. \tag{10}$$

Consequently, the equilibrium effort level is the same for all employment histories that imply the same equilibrium belief m.

Equation (10) shows that the period-2 equilibrium effort strategy is such that the marginal cost of exerting effort is equal to the expected marginal benefit of exerting effort. The worker benefits from exerting effort in period 2 because period-2 effort increases the period-3 wage that will be offered to him. As shown in equation (5), the signal computed by firms in period 2 is increasing with respect to the worker's period-2 effort level. Therefore, the worker's period-3 ability expected by firms and thus the period-3 wage are increasing with respect to the period-2 effort level (see equations (6) and (8)). The increase in the period-3 wage implied by an increase in the worker's perceived ability is given by the marginal return to quality λ^j of the job in which firms would employ the worker (see equation (8)). Consequently, when the worker decides his period-2 effort level, he does not know how period-2 effort will affect his period-3 wage, because he does not know to which job he will be assigned in period 3. In the expected marginal benefit of exerting effort (right-hand side of equation (10)), the marginal return to quality of each job j, λ^j, is multiplied by the probability of the worker being assigned to that job next period.

Proposition 1 shows that employment history only affects the strength of a worker's incentives through his reputation. The key insight for understanding why reputation conveys all the payoff-relevant information is as follows. Aside from determining beliefs, histories of outputs and job assignments may only affect the strength of the worker's incentives through the effort level $a_{f2} (y_1, j^1)$ used by firms to compute the period-2 signal $s_{f2} (y_1, j^1)$ (see the worker's maximization problem in Appendix A). In equilibrium, the effort level exerted by the worker is the effort level used by firms to compute the signal. Therefore, the signal computed by firms is the true signal ($s_{f2}(y_1, j^1) = s_2$), which does not depend on the history (y_1, j^1). Thus, the equilibrium effort strategy only depends on the history (y_1, j^1) through beliefs.

Note that proposition 1 shows that the equilibrium effort level does not depend on the job in which the worker is currently employed. This is the case because we assumed that the worker's capability of influencing the signal computed by firms is independent of the job in which he is employed (see equation (5)). Assumption 1 implies that, in any job, the worker can use one unit of effort to substitute one unit of ability.

Reputation and the strength of career concerns. Next, we show that equilibrium effort is increasing with respect to reputation. Recall that, in the expected marginal benefit of exerting effort, the marginal return to quality of each job is multiplied by the probability of the worker being assigned to that job next period. Thus, if the worker believes it is more likely that next period he will be assigned to jobs with a higher marginal return to quality, the expected marginal benefit of exerting effort is higher. The better the worker's reputation, the more likely he will be assigned to such jobs. Consequently, equilibrium effort is increasing with respect to reputation.[11] The following proposition states this result (see Appendix B for the proof).

Proposition 2 *If $N > 1$, the equilibrium effort strategy $a_2^*(m)$ is an increasing function of the worker's reputation m.*

Even though in equilibrium the firms' and the worker's beliefs coincide, the role of each of these beliefs on the worker's decision can be studied separately. In equation (10), the firms' belief determines the minimum signal required for working in a certain job next period, $s_2^j(m)$. As shown in equation (9), this signal is a decreasing function of the worker's reputation. For example, if at the beginning of the period firms believe that the worker is very talented, in the next period he may be assigned to a managerial job even if the current-period signal is low (because the

[11] Milbourn (2003) presents a similar result. He studies a model in which (i) the worker may lose his job, (ii) the worker is more likely to stay on the job if his reputation is better, (iii) the worker's effort has greater influence on the value of the firm when he is more likely to stay on the job, and (iv) the worker's compensation is increasing with respect to the value of the firm. Consequently, incentives to exert effort are stronger when the worker's reputation is better.

firms' next-period belief may still be good enough). That is, if m is higher, $s_2^j(m)$ is lower, and the worker knows that he is more likely to work in a managerial job in the future. Therefore, if the worker's reputation is better, his expected gain from making firms believe that he has higher ability is greater, and his equilibrium effort level is higher. In equation (10), the worker's belief determines the signal density function he uses for evaluating his problem, $\phi(s; m, H)$. Loosely speaking, it determines how likely he thinks it is that a certain signal is realized. If the worker believes he is better and, therefore, he believes a higher signal is more likely, he believes that it is more likely that he will work in a managerial job next period. Consequently, the expected gain from making firms believe that he has higher ability is greater, and he chooses to exert more effort.

Note that if there is only one job in the economy (i.e., if $N = 1$), the equilibrium effort level in equation (10) does not depend on the worker's reputation. Recall that in models of career concerns, a worker exerts effort to improve the quality of labor input he is expected to provide in future periods. If $N = 1$, the worker's compensation is a linear function of the expected quality of his labor input, as in the main model in Holmstrom (1999). Therefore, the expected derivative of the compensation with respect to expected quality does not depend on reputation, and thus the strength of incentives to exert effort does not depend on reputation. In contrast, if $N > 1$, compensation is a piece-wise convex function of expected quality.[12]

Equilibrium wages and job assignments. As in period 3, in period 2, the firm that hires the worker assigns him to the job for which his expected productivity is higher. The period-2 quality of the worker's labor input expected by firms is given by $a_2^*(m_2^j)+m_2^j$. Using this expected quality, we can define reputation thresholds for job assignments, as we did for period 3. As in period 3, the period-2 wage offered to the worker is equal to his expected productivity. The following proposition characterizes the equilibrium job assignment rule and the equilibrium wage function for period 2 (see Appendix C for the proof).

Proposition 3 *There exists a unique set of threshold reputation levels m_2^j such that, in equilib-*

[12] Holmstrom (1999) suggests that expanding his main model to consider multiple jobs would result in a convex compensation function. Rosen (1982) shows that in a span-of-control economy, the compensation function is piece-wise convex.

rium, the worker is assigned to job j in period 2 if and only if $m_2^j < m_{f2} \leq m_2^{j+1}$. The threshold reputations are such that

$$a_2^*(m_2^j) + m_2^j = m_3^j.$$

Therefore, $m_2^j < m_3^j$. The period-2 equilibrium wage is given by

$$\omega_2(m_{f2}) = A^j + \lambda^j \left[a_2^*(m_{f2}) + m_{f2}\right],$$

where j is such that $m_2^j < m_{f2} \leq m_2^{j+1}$. Thus, the period-2 equilibrium wage is the same for all histories that imply the same firms' belief m_{f2}, and is an increasing function of m_{f2}.

Proposition 3 shows that workers with better reputation will work in higher levels in the hierarchies. Thus, even though workers' equilibrium strategies do not depend on their current job assignment, workers in higher levels in the hierarchies exert more effort.

3.2.3 The first working period

In this section, we describe equilibrium in period 1. We show that understanding the worker's interest in influencing the next-period effort level expected by firms is crucial for understanding his incentives.

A ratchet effect. First, we describe how a ratchet effect weakens the worker's incentives to exert effort in period 1. This ratchet effect appears because of a negative effect of period-1 effort on the worker's period-3 wage.[13]

By exerting effort in period 1 the worker improves his period-2 reputation (see equation (6)). On the one hand, the worker benefits from this because it increases the wage he will receive in period 2 (recall that the equilibrium period-2 wage presented in proposition 3 is an increasing function of the worker's reputation).

[13] The ratchet effect in implicit incentives presented in this paper is similar to that in explicit incentives highlighted in previous studies where current performance affects the terms of future explicit incentive contracts (see, for example, Freixas, Guesnerie, and Tirole, 1985, and Meyer and Vickers, 1997). But the mechanism in our paper is different from the one in these studies. Here, the ratchet effect appears through the effect of current effort on future learning and not through the effect of current effort on future contracts.

On the other hand, improving his period-2 reputation may harm the worker because it worsens the period-2 signal of his ability computed by firms and therefore may decrease his period-3 wage. Recall that the signal computed by firms is a decreasing function of the effort they use for computing the signal (see equation (5)). Firms compute the period-2 signal using the equilibrium effort level $a_2^*(m_{f2})$. Thus, since $a_2^*(m_{f2})$ is an increasing function of m_{f2}, the period-2 signal computed by firms is a decreasing function of m_{f2}. If firms compute a lower period-2 signal, this decreases m_{f3} and, therefore, it decreases $w_3(m_{f3})$. This ratchet effect is not present in previous studies of career concerns where equilibrium effort does not depend on employment history and, therefore, a worker does not change the effort firms expect him to exert next-period by changing his next-period reputation.[14]

The remainder of this paper focuses on environments in which the worker wants to exert effort because of career concerns.[15] That is, we focus on environments in which the ratchet effect is not strong enough to prevent the worker from exerting effort. As indicated above, the importance of the ratchet effect depends on the responsiveness of period-2 effort to reputation. A bound to this responsiveness guarantees situations in which the worker wants to exert effort because of career concerns. It is easy to limit this responsiveness. For example, one can assume a low enough value for the precision in the signal distributions.[16]

[14]For instance, Meyer and Vickers (1997) show that a ratchet effect does not appear in their managerial model of career concerns. They also show that a ratchet effect appears in a career-concern model of regulation. But the ratchet effect in this paper is different from the one in their paper. The ratchet effect in their paper does not appear because of the effect of the current action on future learning. In their model of regulation, since a higher perceived ability (or "intrinsic efficiency") translates into a lower regulated price, the agent (the regulated firm) is worse off if he is perceived to have higher ability. This makes their regulation model different from their managerial model in which a worker benefits from improving his reputation (his wage is higher if he is perceived to have higher ability).

[15]We focus on situations in which the worker expects his continuation utility to increase if firms believe his expected ability is higher. The worker's effort at t increases m_{ft+1} (see equation (6)). Consequently, the worker exerts effort if and only if he expects his continuation utility to be increasing with respect to m_{ft+1}.

[16]As explained before, the strength of the incentives to exert effort in period 2 depends on the probability of having a managerial job in period 3. Consequently, the derivative of the equilibrium effort strategy with respect to the worker's reputation can be made small by decreasing the derivative of the probability of having a managerial job with respect to reputation (see Appendix B). This in turn can be achieved by assuming high variance in the signal distributions (a low H). This is illustrated by the examples presented in Figures 1 and 2.

The equilibrium effort level. Let $M(m, s) \equiv \mu m + (1-\mu) s$ denote the mean in the posterior belief when m is the mean in the prior belief and s is the signal used to update the prior. Let

$$r_1(m_{f2}) \equiv \frac{dM(m_{f2}, s + a_2 - a_2^*(m_{f2}))}{dm_{f2}} = \mu - (1-\mu) a_2^{*'}(m_{f2}) \tag{11}$$

denote the derivative of firms' period-3 belief with respect to their period-2 belief for a given period-2 effort level a_2, where $a_t^{*'}(m)$ denotes the derivative of $a_t^*(m)$ with respect to m. For all $j \geq 2$, let

$$s_1^j \equiv \frac{m_2^j - \mu m_1}{1-\mu}$$

denote the smallest period-1 signal inferred by firms that, in period 2, would make assigning the worker to job j better than assigning him to job $j-1$. That is, s_1^j is such that $\mu m_1 + (1-\mu) s_1^j = m_2^j$. The next proposition presents the worker's equilibrium effort level in his first period of employment (see Appendix D for the proof).

Proposition 4 *In the first period of employment, there exists a unique equilibrium effort level a_1^* that satisfies*

$$c'(a_1^*) = P2G + P3G \tag{12}$$

where

$$P2G \equiv \delta(1-\mu)\left[\lambda^1 \int_{-\infty}^{s_1^2} \left[1 + a_2^{*'}(M(m_1, s))\right] \phi(s; m_1, H)\, ds + ... + \lambda^N \int_{s_1^N}^{\infty} \left[1 + a_2^{*'}(M(m_1, s))\right] \phi(s; m_1, H)\, ds\right]$$

and

$$P3G \equiv \delta \int_{-\infty}^{\infty} r_1(M(m_1, s))\, c'(a_2^*(M(m_1, s)))\, \phi(s; m_1, H)\, ds.$$

Career concerns when the worker can affect the effort expected by firms. Equation (12) shows that, as in period 2, in period 1, equilibrium effort is such that the marginal cost of exerting effort is equal to the expected marginal benefit of exerting effort. The worker may benefit from exerting effort in period 1 because period-1 effort may increase the wage that will be offered to him in periods 2 and 3. In the marginal benefit of exerting effort, $P2G$ represents the expected gain from increasing the period-2 wage, and $P3G$ represents the expected gain from increasing the period-3 wage.

$P2G$ shows that when the worker can affect the next-period effort expected by firms, the effect of effort on the next-period wage is stronger. The gain from increasing the next-period wage represented in $P2G$ is similar to the gain from exerting effort in period 2 represented in the right-hand side of equation (10). Exerting effort increases the next-period quality expected by firms and, therefore, it increases the next-period wage. Period-2 effort only increases the next-period expected quality through its direct effect on the next-period expected ability. In contrast, period-1 effort also increases the next-period expected quality through the indirect effect of the next-period expected ability on the next-period equilibrium effort level, which is used by firms to compute next-period expected quality (recall that the period-2 equilibrium effort level is a function of expected ability and the period-3 equilibrium effort level is not). This is why the derivative of the next-period effort level is part of $P2G$ but does not appear on the right-hand-side of equation (10). Note that when the worker can affect the next-period effort expected by firms, since the effect of effort on the next-period wage is stronger, incentives to exert effort because of career concerns may be stronger in spite of the ratchet effect described above.

The expected gain from increasing period-3 wage represented in $P3G$ describes the typical intertemporal tradeoff in dynamic models. In order to influence his period-3 wage, the worker can exert effort in periods 1 and 2. That is, in order to have any given expected period-3 wage, he can choose to exert more effort in period 1 and less effort in period 2, or vice versa. Thus, the worker compares the costs and the *effectiveness* of exerting effort in these two periods.

In $P3G$, r_1 represents the *relative effectiveness* in changing the period-3 wage of exerting effort in period 1 (compared to exerting effort in period 2). Recall that in order to influence his period-3 wage, the worker needs to affect m_{f3}. The worker's period-1 effort affects m_{f2} directly, and affects m_{f3} through m_{f2}. His period-2 effort affects m_{f3} directly. Thus, the relative effectiveness is given by the derivative of $m_{f3} = M\left(m_{f2}, s + a_2 - a_2^*(m_{f2})\right)$ with respect to m_{f2}. If r_1 is lower than one, it implies that a_1 was less effective than a_2 in changing m_{f3}.

In the relative effectiveness, the ratchet effect described above is represented in the second term of the right-hand side of equation (11). As explained before, this effect represents a negative impact of a higher m_{f2} on m_{f3} because of the effect of m_{f2} on the signal computed by firms in period 2. In equation (11), this effect is weighted by $1-\mu$, the weight of the signal in the posterior belief. The first term of the right-hand side of equation (11) represents a positive effect of m_{f2} on m_{f3}. A better prior implies a better posterior. The importance of this effect is represented by the weight of the prior in the posterior, μ.

Career concerns and the degree of uncertainty about ability. In models of career concerns, an increase in uncertainty about ability (a decrease in h_η) brings about an increase in the relative quality of the signal as an indication of ability and, therefore, an increase in the weight of the current-period signal in the next-period belief, $1-\mu$ (see equation (4)). Since the worker affects the signal computed by firms with his effort level, if the signal weight in the next-period belief is higher, the worker has stronger incentives to exert effort (see the marginal benefit of exerting effort in equations (10) and (12)). This insight explains the conventional wisdom in the career-concern literature on the relationship between the degree of uncertainty about ability and the strength of career concerns. For instance, Holmstrom (1999) explains that career-concern incentives "will work more effectively if the ability process is more stochastic." Dewatripont, Jewitt, and Tirole (1999b) explain that "since effort in the career concern model is driven by talent uncertainty, the principal faces a tradeoff between the riskiness of overall performance and effort...." Similarly, Ortega (2003) explains that one can expect career-concern incentives to be stronger for younger workers for whom beliefs about ability are less precise. De Motta (2003) uses this insight to show that when corporate headquarters have more precise information, stand-alone firms provide stronger career-concern incentives than multidivisional firms.

By analyzing a framework in which the strength of career concerns depends on a worker's employment history, we can identify a mechanism that challenges the conventional wisdom in the literature. In the framework presented in this paper, the effect described above is important, however, another effect appears. In particular, in our framework, a positive relationship between equilibrium effort and h_η may arise, as seen in the following example. Suppose there are two jobs in the economy ($N = 2$), $A^1 = A^2 = 0$, $\lambda^1 = 0.5$, $\lambda^2 = 2$, $m_1 = 0$, $c(a) = a^5$, $\delta = 0.9$, and $h_\varepsilon = 1000$. If $h_\eta = 1$, the worker's equilibrium effort level in his first working period is given by $a_1^* = 0.74$. If $h_\eta = 1000$, the worker's equilibrium effort level in his first working period is given by $a_1^* = 0.85$. That is, in the initial working period, incentives are stronger in the economy with less uncertainty about ability. The intuition behind this example is as follows. Suppose h_η is lower. This implies that the distribution of the signal is less precise (H is lower). This in turn implies that equilibrium effort is less responsive to changes in reputation (see the derivative of equilibrium effort with respect to reputation in Appendix B and the examples presented in Figures 1 and 2). Thus, it is more difficult for the worker to increase his next-period wage by making firms believe that he will exert more effort next period. Therefore, incentives to exert effort are weaker.

Equilibrium wages and job assignments. In order to complete the characterization of the three-working-period economy, we present the equilibrium job assignment rule and the equilibrium wage function for the worker's first employment period. As in periods 2 and 3, in period 1, the firm that hires the worker assigns him to the job for which his expected productivity is higher, and the wage offered to the worker is equal to his expected productivity. The following proposition characterizes the equilibrium job assignment rule and the equilibrium wage function for period 1 (the proof of this proposition parallels the proof of proposition 3 presented in Appendix C).

Proposition 5 *There exists a unique set of threshold reputation levels m_1^j such that, in equilibrium, the worker is assigned to job j in period 1 if and only if $m_1^j < m_1 \leq m_1^{j+1}$. The threshold reputations are such that*

$$a_1^* + m_1^j = m_3^j.$$

The equilibrium wage for an inexperienced worker is given by

$$\omega_1 = A^j + \lambda^j \left(a_1^* + m_1\right),$$

where j is such that $m_1^j < m_1 \leq m_1^{j+1}$.

3.3 A T-working-period version of the model

We now discuss whether the insights from the study of the three-working-period economy presented above apply to economies in which the worker can be employed for more than three periods. In the three-working-period version of the model, the relationship between the strength of career concerns and employment history can only be studied in the second employment period (in the first period there is no employment history, and in the last period there are no career concerns). Proposition 2 shows that in a three-working-period model, in the second employment period, career concerns are stronger when the worker's reputation is better. That period is special in that next-period effort does not depend on reputation. We will show that when more than three employment periods are considered, local nonmonotonicities may appear in the equilibrium effort strategy. This implies that the expected quality of a worker's labor input may be decreasing with respect to his reputation because a worker with higher expected ability may be expected to exert less effort. Therefore, since managerial jobs are assigned to workers with higher expected quality, there could be laborers with better reputations than managers.[17]

For the remainder of the paper, we discuss economies with monotone equilibrium assignment rules in which workers with better reputations are assigned to jobs with a higher marginal return to quality.[18] This facilitates the analysis of economies in which the worker can be employed for more than three periods. We will discuss how parameter values can be restricted in order to assure that assignment rules are monotone. We will show that equilibrium job assignment rules may be monotone even in economies that present nonmonotonicities in the equilibrium effort strategies.

3.3.1 Equilibrium.

The next proposition characterizes economies with monotone assignment rules (the derivation of equation (14) follows closely the derivation of equation (12) presented in Appendix D; the

[17]Likewise, Ortega (2003) explains that a firm may want to give more power to workers with lower expected ability when these workers are expected to choose a higher effort level. Ottaviani and Sϕrensen (2001) show that increasing the informational quality of a committee member can decrease the informational value of the committee. Similarly, Ottaviani and Sϕrensen (2006) show that an expert with a better reputation may be less informative in equilibrium than an expert with a worse reputation. They remark that this would make it difficult to endogenously derive a monotonic reputational payoff in a dynamic version of their model with more than two periods. The mechanism through which better reputations weaken incentives in their models is different from the one in our model. Here, concerns about the effect of current actions on future actions expected by the market are crucial. In their two-period models, these concerns are not present.

[18]Similarly, in Martinez (2009), I focus on economies with political career concerns in which the equilibrium reelection rule is monotone.

derivation of equilibrium wages and job assignment rules parallels the derivations for the three-working-period version of the model presented in Appendix C).

Proposition 6 *In an economy in which a worker with higher expected ability is employed at higher levels in the hierarchies, there exists a unique set of threshold reputation levels* m_t^j *such that the worker is assigned to job* j *in period* t *if and only if* $m_t^j < m_t \leq m_t^{j+1}$*. The threshold reputations are such that* $m_t^1 = -\infty$*,* $m_t^{N+1} = \infty$*, and for all* $2 \leq j \leq N$*,*

$$a_t^*(m_t^j) + m_t^j = \frac{A^{j-1} - A^j}{\lambda^j - \lambda^{j-1}}, \tag{13}$$

where $a_T^*(m) = 0$*, and for any* $t < T$*, there exists a unique equilibrium effort strategy* $a_t^*(m)$ *that satisfies*

$$c'(a_t^*(m)) = NPG_t(m) + OPG_t(m), \tag{14}$$

where

$$NPG_t(m) \equiv \delta\left(1-\mu\right)\left[\lambda^1 \int_{-\infty}^{s_t^2(m)} \left[1 + a_{t+1}^{*'}\left(M\left(m,s\right)\right)\right] \phi\left(s;m,H\right) ds + \ldots + \lambda^N \int_{s_t^N(m)}^{\infty} \left[1 + a_{t+1}^{*'}\left(M\left(m,s\right)\right)\right] \phi\left(s;m,H\right) ds\right],$$

$$OPG_t(m) \equiv \delta \int_{-\infty}^{\infty} r_t\left(M\left(m,s\right)\right) c'\left(a_{t+1}^*\left(M\left(m,s\right)\right)\right) \phi\left(s;m,H\right) ds,$$

$$r_t\left(m\right) \equiv \mu - \left(1-\mu\right) a_{t+1}^{*'}\left(m\right),$$

and, for all $j \geq 2$*,*

$$s_t^j(m) \equiv \frac{m_{t+1}^j - \mu m}{1-\mu}.$$

Equilibrium wages are given by

$$\omega_t\left(m_t\right) = A^j + \lambda^j \left[a_t^*\left(m_t\right) + m_t\right], \tag{15}$$

where j *is such that* $m_t^j < m_t \leq m_t^{j+1}$*.*

This proposition shows that, as in the first employment period of the three-working-period version of the model, in any period $t < T$, the marginal gains from exerting effort are the expected increase in the next-period wage ($NPG_t(m)$) and the expected gain from increasing wages in every other future period (represented by the lower marginal cost of exerting effort next period in $OPG_t(m)$). It also shows that for any T, the model can easily be solved backwards as we did for $T = 3$. A unique period-t equilibrium effort strategy can easily be obtained from equation (14) once the unique period-$t+1$ equilibrium effort strategy is known. For example, Figure 1 shows the equilibrium effort strategies for $T-1$, $T-2$, $T-3$, and the limit of the finite-horizon solution when the worker is far enough from retirement for an economy with $N = 2$, $A^1 = A^2 = 0$, $\lambda^1 = 0.5$, $\lambda^2 = 2$, $m_1 = 0$, $c(a) = a^5$, $\delta = 0.9$, and $h_\varepsilon = h_\eta = 1$. For each period, after deriving the equilibrium effort strategy for the period, equilibrium job assignment rules and wages can easily be obtained using equations (13) and (15).

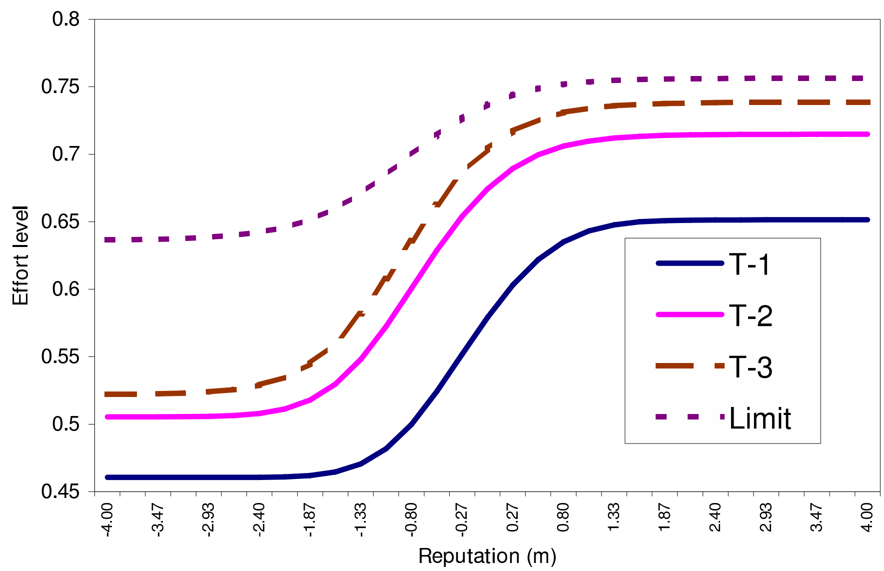

Figure 1: Equilibrium effort strategies ($H = 0.5$)

Recall proposition 1 shows that, when the worker can only be employed for three periods, in the second employment period, employment history only affects the strength of career concerns through its effect on the worker's reputation. Proposition 6 shows that, in any period of the T-working-period version of the model, when employment histories may be more complex, reputation (and t) also conveys all the payoff-relevant information. Beliefs are the relevant state variables. The optimal effort level is the same for all histories (y^{t-1}, j^t, a^{t-1}) that imply the same beliefs m_w and m_f (see the discussion of the optimal off-equilibrium effort strategy in Appendix

D). Equilibrium wages and job assignments are the same for all histories (y^{t-1}, j^{t-1}) that imply the same belief m_f.

The fact that equilibrium strategies can be written as functions of beliefs allows us to increase the assumed maximum length of employment histories without increasing the dimensionality of the state space. Therefore, it facilitates solving the model for any number of employment periods. This insight can also facilitate solving other models of career concerns where employment history affects the strength of incentives. For instance, the insight allows Martinez (2009) to solve an extension of a standard career-concern model of political cycles in which past performances affect the strength of incentives.

3.3.2 Employment history and the strength of career concerns

Next, we describe how the worker's reputation affects the strength of his incentives when he takes into account how his current decision affects the effort level firms expect him to exert. On the one hand, the effect presented in proposition 2 is still important. A worker with a better reputation is more likely to have a managerial job next period. Therefore, he expects a larger increase in his next-period wage if he makes firms believe that he will provide labor input of higher quality. This implies that workers with better reputation could have stronger incentives to exert effort. Figure 1 illustrates how this effect may be dominant and the equilibrium effort strategies may be increasing in reputation.

On the other hand, local nonmonotonicities in the equilibrium effort strategies may appear. Recall that effort affects the next-period wage through the next-period effort level expected by firms. In NPG, this is represented by $a_{t+1}^{*'}(M(m,s))$. As illustrated by Figure 1, $a_{t+1}^{*'}(M(m,s))$ may be decreasing. Thus, a worker with a better reputation may expect his effort will be less effective in making firms believe that he will exert more effort next period. Therefore, he may have weaker incentives to exert effort. For instance, let us consider the example presented in Figure 1 but with $h_\varepsilon = h_\eta = 1000$. Figure 2 shows that in this economy, for reputation levels where $a_{T-1}^{*'}(m)$ is decreasing, $a_{T-2}^{*}(m)$ may be decreasing.

As explained above, nonmonotonicities in the equilibrium effort strategies arise because the

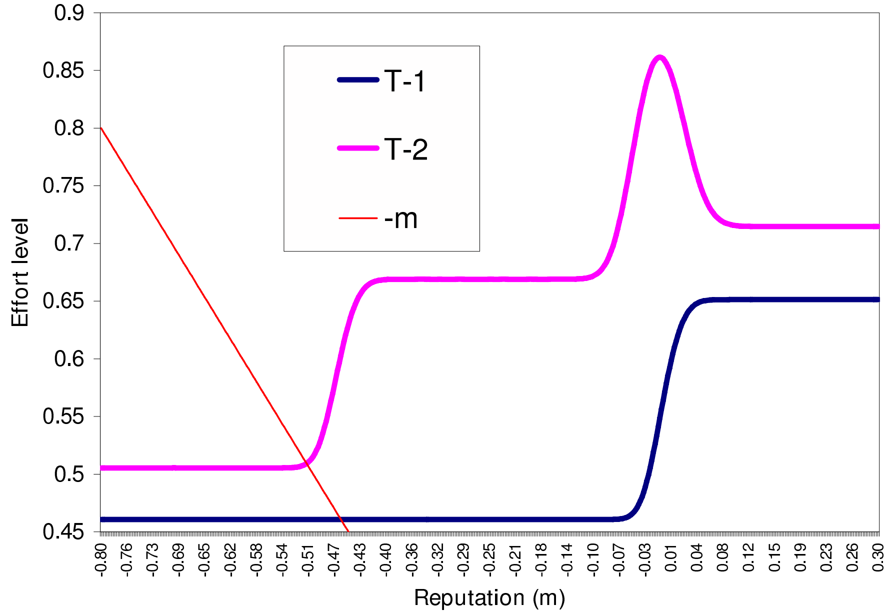

Figure 2: Equilibrium effort strategies ($H = 500$)

derivative of next-period equilibrium effort with respect to reputation may be decreasing. Hence, one could avoid these nonmonotonicities by limiting the responsiveness of this derivative to changes in reputation. Recall that the strength of incentives to exert effort is related to the probability of having a managerial job next period. Thus, both the derivative and the second derivative of this probability can be decreased by decreasing the precision in the distribution of signals, H. Consequently, one could restrict attention to economies with monotone effort strategies by assuming low values of H. This is apparent from comparing the examples presented in Figures 1 and 2—in Martinez (2009) it is shown that, in a political agency model of career concerns, similar restrictions have to be imposed to assure that voters reelect an incumbent policymaker if and only if his reputation is good enough.

Note that, as illustrated in Figure 2, even if the period-t equilibrium effort strategy presents a nonmonotonicity, the period-t equilibrium job assignment rule may be monotone—in the example presented in Figure 2, the worker is assigned to job 2 for all reputations such that the equilibrium effort strategy is above the -45 degree line (see equation (13)). This example illustrates how in economies with nonmonotonic effort strategies, it may be possible to find the equilibrium following proposition 6.

The importance of a higher probability of having a managerial job next period is also illustrated in Figure 2. In general (asides from the nonmonotonic region), equilibrium effort is increasing in reputation. For the more extreme reputation levels, incentives from changing the effort level expected by firms are not important and the equilibrium effort level is determined by the probability of having a managerial job next period.

4 Conclusions

We presented a tractable model of career concerns in which the strength of career concerns depends on a worker's employment history. The strength of career-concern incentives, equilibrium wages, and equilibrium job assignments are the same for all employment histories that lead to the same reputation. This property of the equilibrium facilitates computing the equilibrium for games of any length. We showed that typically, workers with the better reputations exert more effort than workers with the worse reputations. But local nonmonotonicities in the effort-reputation relationship may appear. Furthermore, we showed that when the strength of incentives depends on a worker's employment history, (i) a ratchet effect in implicit incentives may appear, (ii) the effect of current-period effort on next-period wage may be stronger, and (iii) incentives may be stronger if the prior beliefs about ability are more precise.

Appendix A: proof of proposition 1

For any history (y_1, j^1), effort expected by firms $a_{f2}(y_1, j^1)$, and belief m, the worker's period-2 maximization problem can be written as

$$\max_a \left\{ \delta \left[\begin{array}{l} \int_{-\infty}^{s_2^2(m)-a+a_{f2}(y_1,j^1)} \left\{A^1 + \lambda^1 \left[\mu m + (1-\mu)\left(s + a - a_{f2}(y_1, j^1)\right)\right]\right\} \phi(s; m, H)\, ds + \\ \int_{s_2^2(m)-a+a_{f2}(y_1,j^1)}^{s_2^3(m)-a+a_{f2}(y_1,j^1)} \left\{A^2 + \lambda^2 \left[\mu m + (1-\mu)\left(s + a - a_{f2}(y_1, j^1)\right)\right]\right\} \phi(s; m, H)\, ds + ... \\ + \int_{s_2^N(m)-a+a_{f2}(y_1,j^1)}^{\infty} \left\{A^N + \lambda^N \left[\mu m + (1-\mu)\left(s + a - a_{f2}(y_1, j^1)\right)\right]\right\} \phi(s; m, H)\, ds \end{array} \right] - c(a) \right\}.$$

The equilibrium effort level $a_2^*(m)$ satisfies the first-order condition of this problem when firms compute the signal using $a_2^*(m)$ (i.e., evaluated at $a = a_{f2}(y_1, j^1) = a_2^*(m)$). This first-order condition is given by equation (10). The right-hand side in equation (10) does not depend on effort and is positive. Therefore, there exists a unique $a_2^*(m)$ that satisfies equation (10).

Appendix B: proof of proposition 2

In equation (10), the marginal benefit of exerting effort is given by

$$\delta(1-\mu)\left[\lambda^1\int_{-\infty}^{s_2^2(m)}\phi(s;m,H)\,ds+\lambda^2\int_{s_2^2(m)}^{s_2^3(m)}\phi(s;m,H)\,ds+...+\lambda^N\int_{s_2^N(m)}^{\infty}\phi(s;m,H)\,ds\right].$$

The sign of the derivative of this marginal benefit with respect to reputation m is given by the sign of

$$\lambda^1\frac{\partial\int_{-\infty}^{s_2^2(m)}\phi(s;m,H)\,ds}{\partial m}+\lambda^2\frac{\partial\int_{s_2^2(m)}^{s_2^3(m)}\phi(s;m,H)\,ds}{\partial m}+...+\lambda^N\frac{\partial\int_{s_2^N(m)}^{\infty}\phi(s;m,H)\,ds}{\partial m},$$

which is equal to

$$\left(\lambda^1-\lambda^2\right)\phi\left(s_2^2(m);m,H\right)\frac{\partial s_2^2(m)}{\partial m}+\left(\lambda^2-\lambda^3\right)\phi\left(s_2^3(m);m,H\right)\frac{\partial s_2^3(m)}{\partial m}+...+(\lambda^{N-1}-\lambda^N)\phi(s_2^N(m);m,H)\frac{\partial s_2^N(m)}{\partial m}+$$
$$\lambda^1\int_{-\infty}^{s_2^2(m)}\frac{\partial\phi(s;m,H)}{\partial m}ds+\lambda^2\int_{s_2^2(m)}^{s_2^3(m)}\frac{\partial\phi(s;m,H)}{\partial m}ds+...+\lambda^N\int_{s_2^N(m)}^{\infty}\frac{\partial\phi(s;m,H)}{\partial m}ds.$$

Recall that $\lambda^j>\lambda^{j-1}$ (Assumption 2), and $\frac{\partial s_2^j(m)}{\partial m}<0$. Therefore, the first $N-1$ terms in the expression above are positive.

Let x be the job such that $s_2^x(m)\le m<s_2^{x+1}(m)$. We can write the last N terms in the expression above as

$$\lambda^1\int_{-\infty}^{s_2^2(m)}\frac{\partial\phi(s;m,H)}{\partial m}ds+...+\lambda^x\int_{s_2^x(m)}^{m}\frac{\partial\phi(s;m,H)}{\partial m}ds+\lambda^x\int_{m}^{s_2^{x+1}(m)}\frac{\partial\phi(s;m,H)}{\partial m}ds+...+\lambda^N\int_{s_2^N(m)}^{\infty}\frac{\partial\phi(s;m,H)}{\partial m}ds.$$

Recall that $\phi(s;m,H)$ denotes the density function for a normally distributed random variable with mean m. Consequently,

$$-\lambda^x\left[\int_{-\infty}^{s_2^2(m)}\frac{\partial\phi(s;m,H)}{\partial m}ds+...+\int_{s_2^x(m)}^{m}\frac{\partial\phi(s;m,H)}{\partial m}ds\right]=\lambda^x\left[\int_{m}^{s_2^{x+1}(m)}\frac{\partial\phi(s;m,H)}{\partial m}ds+...+\int_{s_2^x(m)}^{m}\frac{\partial\phi(s;m,H)}{\partial m}ds\right]>0$$

Furthermore,

$$-\lambda^x\left[\int_{-\infty}^{s_2^2(m)}\frac{\partial\phi(s;m,H)}{\partial m}ds+...+\int_{s_2^x(m)}^{m}\frac{\partial\phi(s;m,H)}{\partial m}ds\right]\ge-\lambda^1\int_{-\infty}^{s_2^2(m)}\frac{\partial\phi(s;m,H)}{\partial m}ds-...-\lambda^x\int_{s_2^x(m)}^{m}\frac{\partial\phi(s;m,H)}{\partial m}ds,$$

and

$$\lambda^x \left[\int_m^{s_2^{x+1}(m)} \frac{\partial \phi(s; m, H)}{\partial m} ds + ... + \int_{s_2^x(m)}^m \frac{\partial \phi(s; m, H)}{\partial m} ds \right] \leq \lambda^x \int_m^{s_2^{x+1}(m)} \frac{\partial \phi(s; m, H)}{\partial m} ds ... + \lambda^N \int_{s_2^N(m)}^\infty \frac{\partial \phi(s; m, H)}{\partial m} ds.$$

Thus,

$$\lambda^1 \int_{-\infty}^{s_2^2(m)} \frac{\partial \phi(s; m, H)}{\partial m} ds + ... + \lambda^x \int_{s_2^x(m)}^m \frac{\partial \phi(s; m, H)}{\partial m} ds + \lambda^x \int_m^{s_2^{x+1}(m)} \frac{\partial \phi(s; m, H)}{\partial m} ds ... + \lambda^N \int_{s_2^N(m)}^\infty \frac{\partial \phi(s; m, H)}{\partial m} ds \geq 0.$$

Consequently, the marginal benefit of exerting effort is increasing with respect to m. The proposition follows from $c''(a) > 0$.

Appendix C: proof of proposition 3

Expected quality thresholds for job assignments are given by m_3^j. The period-2 quality expected by firms is given by $a_2^*(m_{f2}) + m_{f2}$. Since $a_2^*(m_{f2})$ is increasing with respect to m_{f2}, we know that $a_2^*(m_{f2}) + m_{f2}$ is increasing with respect to m_{f2}. Moreover, we know that for any m_3^j, there is a high enough m_{f2} such that $a_2^*(m_{f2}) + m_{f2} \geq m_3^j$; and there is a low enough m_{f2} such that $a_2^*(m_{f2}) + m_{f2} \leq m_3^j$. Consequently, there is a unique m_{f2} such that $a_2^*(m_{f2}) + m_{f2} = m_3^j$. Considering that the worker is assigned to the position where he is expected to be more productive, and that the equilibrium wage is equal to his expected productivity completes the proof.

Appendix D: proof of proposition 4

For any history (y_1, j_1, a_1) such that the worker's and the firms' beliefs are given by m_{w2} and m_{f2}, respectively, the worker's period-2 maximization problem is given by

$$\max_a \left\{ \delta \left[\begin{array}{l} \int_{-\infty}^{s_2^2(m_{f2})-a+a_2^*(m_{f2})} \left\{ A^1 + \lambda^1 \left[\mu m_{f2} + (1-\mu)\left(s + a - a_2^*(m_{f2})\right) \right] \right\} \phi(s; m_{w2}, H)\, ds \\ + \int_{s_2^2(m_{f2})-a+a_2^*(m_{f2})}^{s_2^3(m_{f2})-a+a_2^*(m_{f2})} \left\{ A^2 + \lambda^2 \left[\mu m_{f2} + (1-\mu)\left(s + a - a_2^*(m_{f2})\right) \right] \right\} \phi(s; m_{w2}, H)\, ds + ... \\ + \int_{s_2^N(m_{f2})-a+a_2^*(m_{f2})}^{\infty} \left\{ A^N + \lambda^N \left[\mu m_{f2} + (1-\mu)\left(s + a - a_2^*(m_{f2})\right) \right] \right\} \phi(s; m_{w2}, H)\, ds \end{array} \right] - c(a) \right\}. \quad (16)$$

Consequently, the optimal period-2 effort level only depends on the worker's employment history through the worker's belief and the firms' belief. Therefore, at the beginning of period 2, the worker's expected lifetime utility only depends on these beliefs. Let

$$W_2\left(m_{w2}, m_{f2}\right) = \omega_2\left(m_{f2}\right) - c(\hat{a}_2^*\left(m_{w2}, m_{f2}\right)) + \delta \int_{-\infty}^{\infty} \omega_3\left(M\left(m_{f2}, s + \hat{a}_2^*\left(m_{w2}, m_{f2}\right) - a_2^*\left(m_{f2}\right)\right)\right) \phi\left(s; m_{w2}, H\right) ds$$

denote this utility, where $\hat{a}_2^*\left(m_{w2}, m_{f2}\right)$ denotes the optimal effort for m_{w2} and m_{f2}. The worker's period-1 maximization problem is given by

$$\max_a \left\{ \delta \left[\begin{array}{l} \int_{-\infty}^{s_1^2 - a + a_1^*} W_2(M\left(m_1, s\right), M\left(m_1, s + a - a_1^*\right))\phi\left(s; m_1, H\right) ds \\ + \int_{s_1^2 - a + a_1^*}^{s_1^3 - a + a_1^*} W_2(M\left(m_1, s\right), M\left(m_1, s + a - a_1^*\right))\phi\left(s; m_1, H\right) ds + ... \\ + \int_{s_1^N - a + a_1^*}^{\infty} W_2(M\left(m_1, s\right), M\left(m_1, s + a - a_1^*\right))\phi\left(s; m_1, H\right) ds \end{array} \right] - c(a) \right\}.$$

The first-order condition of this problem evaluated in equilibrium reads

$$c'\left(a_1^*\right) = \delta\left(1 - \mu\right) \left[\int_{-\infty}^{s_1^2} \left.\frac{\partial W_2}{\partial m_{f2}}\right|_{M(m_1,s),M(m_1,s)} \phi\left(s; m_1, H\right) ds + ... + \int_{s_1^N}^{\infty} \left.\frac{\partial W_2}{\partial m_{f2}}\right|_{M(m_1,s),M(m_1,s)} \phi\left(s; m_1, H\right) ds \right]. \tag{17}$$

For any m such that $m_2^j < m \leq m_2^{j+1}$, let $\lambda_2\left(m\right) \equiv \lambda^j$. By the envelope theorem, for all $m \notin \{m_2^2, m_2^3, ..., m_2^N\}$,

$$\left.\frac{\partial W_2}{\partial m_{f2}}\right|_{m,m} = \lambda_2\left(m\right) \left[a_2^{*\prime}\left(m\right) + 1\right] + \delta r_1\left(m\right) \left[\lambda^1 \int_{-\infty}^{s_2^2(m)} \phi\left(s; m, H\right) ds + ... + \lambda^N \int_{s_2^N(m)}^{\infty} \phi\left(s; m, H\right) ds\right].$$

By equation (10),

$$\left.\frac{\partial W_2}{\partial m_{f2}}\right|_{m,m} = \lambda_2\left(m\right) \left[a_2^{*\prime}\left(m\right) + 1\right] + \frac{r_1\left(m\right)}{1 - \mu} c'\left(a_2^*\left(m\right)\right).$$

Substituting into equation (17) yields equation (12).

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