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Abstract

We explore a political economy model of labor subsidies, extending Meltzer and Richard's median voter model to a dynamic setting. We explore only one source of heterogeneity: initial wealth. As a consequence, given an operative wealth effect, poorer agents work harder, and if the agent with median wealth is poorer than average, a politico-economic equilibrium will feature a subsidy to labor. The dynamic model does not have capital, but it has perfect markets for borrowing and lending. Because tax rates influence interest rates, another channel for redistribution appears, since a decrease in current interest rates favors agents with a negative (below-average) asset position.

By the same token—and as is typically the case in dynamic politico-economic models with rational agents—the setting features time-inconsistency: the median voter would like to commit to not manipulating interest rates in the future. Under commitment, and under the assumption that preferences admit aggregation, we show that labor subsidies subsist only for one period; after that, subsidies are zero. That is, under commitment, the median voter takes advantage of the voting power once and for all. His wealth moves closer to that of the mean (which is zero), but afterwards he refrains voluntarily from further subsidization. Under lack of commitment, which we analyze formally by looking at the Markov-perfect (time-consistent) equilibrium in a game between successive median voters in the same environment. Instead, subsidies persist—they are constant over time—and are more distortionary than under commitment. Moreover, in the situation without commitment, the median voter does not manage to reduce asset inequality, unlike in the commitment case.

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1 Introduction

Consider an economy in which consumers all have the same preferences and income-generating abilities but differ in wealth. Suppose, moreover, that there is collective decision making over taxation and that the politically pivotal group is interested in redistributing resources to itself. If individual-specific taxes or wealth-based taxes are available, such redistribution is easy to accomplish. However, suppose that these redistribution tools are not available and that taxes can only be levied on certain consumption goods; for simplicity, suppose that there are two goods and that one of these goods can be taxed at a proportional rate, whereas the other good cannot be taxed at all. If the proceeds from taxation are rebated in equal per-capita amounts, then there is generally net redistribution from taxation, because rich and poor consumers generally spend different amounts on the good that is taxed. In particular, under a normal-goods assumption, a poor consumer would like a tax on the taxable good, since rich consumers consume more of it and thus pay more in taxes than do poor consumers. Put differently, poor agents would indeed set positive taxes to achieve redistribution, even if taxation is distortionary: a small departure from zero taxes generates only a second-order loss from distortions but a first-order gain from net redistribution.

In the present paper we apply this logic to two specific goods: a typical *consumption* good, on the one hand, and *leisure* on the other. We take labor income to be taxable—i.e., we allow leisure to be subsidized—whereas consumption taxes are not allowed. We thus examine the idea that economies where taxation decisions are made by poorer-than-average consumers will feature *taxation of leisure*: labor subsidies. We formalize the idea using an infinite-horizon dynasty model where wealth differences arise from differences in initial wealth and preferences admit aggregation.

Production uses labor only and the production function is linear in labor; our economy has no physical or other form of real capital accumulation. Because the economy is also closed, this means that in equilibrium average assets must be zero at each point in time; the real interest rate adjusts so that this is the case. The dynamic economy differs from a static one primarily because the interest rate is endogenous. Because of its impact on the total resources available to consume, taxation influences interest rates. In particular, changes in the interest rate are not neutral from the point of view of redistribution: a low interest rate, for example, is good for debtors because it makes it cheaper to roll over loans, whereas for symmetric reasons, a high interest rate is good for creditors. Analysis of the dynamic economy requires a careful examination of how changes in taxes benefit different groups both through direct redistribution and changes in interest rates, aside from the distortions of the labor-leisure choice they lead to.

The possibility of "manipulation" of interest rates also leads to time-consistency problems: politico-economic outcomes differ depending on whether or not there is a commitment device for future policy. The reason is that a tax change today changes aggregate consumption and therefore the interest rate between yesterday and today. Thus, ex post this effect will be ignored if the tax rate is chosen now, but it would have been taken into account had the tax choice been made in advance. The time-inconsistency problem means that in this economy, there must be a difference in outcomes between two voting setups: one with commitment, which we call time-zero voting, and one without, which we refer to as sequential voting.

Our findings are quite striking. In short, we point to (i) rather surprising features of the commitment solution, and (ii) a stark qualitative difference between the solutions with and without commitment. In the case of commitment, there is initial subsidization of labor—unless the median voter is richer than average, in which case there is taxation—and asset inequality shrinks (median wealth, which is below zero to start with, moves toward zero). However, after one period, taxes go to zero and stay there forever, and the asset distribution remains unchanged. This finding, which is reminiscent of the results in Chamley (1986) and Judd (1985)—that optimal capital taxes should be zero in the long run—is stark but it is not a corollary of the Chamley/Judd characterization. It is not entirely general, because it uses the assumption that preferences are in a certain class, however, the choice of preferences is motivated by the fact that they are used in almost all of the applied (quantitative) macroeconomic literature.

We analyze the case without commitment using a Markov-perfect equilibrium notion, as in the politico-economic equilibria studied in Krusell and Ríos-Rull (1999).[1] That is, politico-economic equilibrium taxes are a function of the payoff-relevant state and nothing else. We use median-voter theory and obtain that the state variable in the no-commitment economy is median wealth only. Here it is not easy to provide sufficient conditions for a median-voter theorem. We thus verify single-peakedness numerically for the specific economies we look at. We are not able to prove existence of a Markov-perfect equilibrium globally for our model but, using the first-order condition, we can prove local existence around the point where median assets equal mean assets.[2] One can also see that, if an equilibrium exists globally and is smooth, then it gives higher first-period subsidies than those found under commitment. Thus, a lack of commitment leads to uniformly higher distortions: not only do subsidies remain positive from period two and on, but they are also higher in the first period.

Mathematically, the functional first-order condition is different than in standard optimal control theory problems since we are studying a dynamic game without commitment: the median voter at a point in time cannot bind the hands of future median voters, whether the latter are different individuals or not. Prior analysis focusing on first-order conditions in similar contexts includes the work on individual saving under time-inconsistent preferences, where reminiscent first-order conditions have been derived (so-called "generalized Euler equations"; see Laibson [1997]), and some recent work on dynamic public finance (e.g., see Klein, Krusell, and Ríos-Rull [2005] or Azzimonti, de Francisco and Krusell [2006]). It turns out that the numerical solution of this functional equation is challenging as well; we illustrate this fact in the paper in a final computational section.

Motivation for our work can be found in the earlier political-economy literature. Meltzer and Richard (1981) study a static model of the sort analyzed here but where heterogeneity appears through labor productivity: some agents are more productive, and earn more, than others. A dynamic version of their model without capital would allow asset accumulation on the individual level, and it would allow interest-rate manipulation. However, this manipulation would only have a role to play if agents had different initial asset positions—otherwise, the dynamic Meltzer-Richard model would only be a mechanical repetition of the static one. Thus, what we study here is the most stripped-down version of their economy with asset differences: all the focus is on redistribution due to the wealth differences, and interest-rate manipulation is central. It would seem straightforward to reintroduce productivity differences into the model, which obviously would be a force toward taxation, not subsidization, of labor (assuming, as data suggest, that median labor productivity is below mean labor productivity). The present work also relates to Krusell and Ríos-Rull (1999), who study a dynamic Meltzer-Richard model with capital. In that model, there is a proportional tax on both labor and capital, and the focus is mainly quantitative. In addition, the present paper focuses more on analytical characterization;

[1]See Maskin and Tirole (2001) for a general discussion of the equilibrium concept.

[2]From here on, our results apply to the case of logarithmic utility.

politico-economic equilibria in dynamic models are difficult to characterize, and numerical and analytical work are both important. Furthermore, in any models where interest rates are endogenous and consumers have different asset positions—features that ought to be present in most quantitative political economy models—there is interest-rate manipulation, and the politico-economic aspects of this manipulation therefore should seem to be of general relevance. Finally, our median-voter result under commitment is related to Bassetto and Benhabib (2006), who study a capital taxation economy with a similar preference structure; they use similar reasoning to prove their median-voter result.

The paper is organized as follows. Section 2 describes the basic economic model for exogenous policy. This section demonstrates aggregation and characterizes equilibrium prices (interest rates) and quantities as a function of the sequence of taxes chosen by the government. Section 3 covers the political-economy model where there is commitment to future taxes. In particular, that section demonstrates a median-voter theorem and characterizes the median-voter equilibrium. Section 4 then defines and studies Markov-perfect equilibria when there is no commitment. Section 5 concludes the paper.

2 The economic model

In this section, we describe the economic model for exogenous tax policy: we define sequential competitive equilibria for given sequences of tax rates. We then specialize preferences to admit aggregation and characterize equilibria in this case. This analysis will form the building block for the later analysis of the politically determined policy.

2.1 Labor-leisure choice and wealth heterogeneity

In this infinite horizon economy, time is discrete and there is no uncertainty. Agents value consumption, c, and leisure, l, and preferences are the same for everyone:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t).$$

Agents can trade one period bonds, each promising one unit of consumption next period. The heterogeneity within the population enters only through differences in consumers' initial asset holdings. We will assume that the number of "types" is finite with measure μ_i for type $i \in \{1, 2, \ldots, I\}$. Population size is normalized to one: $\sum_{i=1}^{I} \mu_i = 1$.

Production takes place according to a stationary production function which depends linearly on labor: $Y = N$ (we use capital letters to denote aggregates). Output is used for consumption, and there is no government consumption or investment. We also abstract from capital entirely. The level of asset holdings of individual i is denoted by a_i, which is in zero net supply in equilibrium. Thus, neither storage nor trade with other countries is possible.

There is a constraint on the amount of time for each agent: each consumer has one unit, so that $l_i + n_i = 1$ for all i. We will make assumptions on primitives so that agents' decision problems are strictly concave; hence, all agents of the same type will make the same decisions and we can also write $L_i + N_i = 1$, where L_i and N_i reflect common decisions regarding leisure and labor of all agents of type i. The aggregate labor input is thus $N = \sum_{i=1}^{I} \mu_i N_i$.

2.2 Sequential competitive equilibrium for given tax rates

In a decentralized economy, consumers buy consumption goods at each point in time and sell their labor services to firms under perfect competition. There are two relevant prices determined in equilibrium: the price of one-period bonds, denoted by q, and the wage rate w, both measured in terms of consumption goods in the same period. In addition, we now consider a government that taxes labor income at a proportional rate τ_t in period t and makes equal lump-sum transfers T_t back to all consumers under a balanced budget. Thus, a typical consumer i's budget constraint in period t reads

$$c_{ti} + q_t a_{it+1} = a_{it} + w_t(1 - l_{it})(1 - \tau_t) + T_t.$$

In equilibrium, consumers' holdings of assets have to add up to zero: $\sum_{i=1}^{I} \mu_i A_{it} = 0$, where A_{it} denotes the total holdings of agents of type i. Consumer heterogeneity thus originates in $a_{i0} \neq a_{j0}$ for all $i \neq j$. We define a competitive equilibrium for a given sequence of government policy as follows:

Definition 1 *Given a tax policy $\{\tau_t\}_{t=0}^{\infty}$, a* ***competitive equilibrium*** *is a sequence of prices $\{w_t, q_t\}_{t=0}^{\infty}$ together with a sequence of allocations $\{N_t, \{C_{it}, A_{it+1}, L_{it}\}_{i=1}^{I}, T_t\}_{t=0}^{\infty}$ satisfying the following conditions.*

1. *For all i, $\{C_{it}, A_{it+1}, L_{it}\}_{t=0}^{\infty}$ solves*

$$\max_{\{c_t, a_{t+1}, l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t).$$

subject to

$$c_t + q_t a_{t+1} = a_t + w_t(1 - l_t)(1 - \tau_t) + T_t.$$

for all t, with $a_0 = A_{i0}$. Agents are also subject to a no-Ponzi scheme condition:

$$\lim_{t\to\infty} a_t \Pi_{k=0}^{t-1} q_k = 0.$$

2. *For all t, $w_t = 1$.*

3. *For all t, $N_t = \sum_{i=1}^{I} \mu_i (1 - L_{it})$.*

4. *For all t, $\sum_{i=1}^{I} \mu_i A_{it} = 0$.*

5. *For all t, $T_t = \tau_t N_t$.*

An agent with wealth A_{i0} will choose to accumulate assets and consume leisure so that

$$u_c(c_{it}, 1 - n_{it}) w_t (1 - \tau_t) = u_l(c_{it}, 1 - n_{it})$$

and

$$q_t u_c(c_{it}, 1 - n_{it}) = \beta u_c(c_{it+1}, 1 - n_{it+1}).$$

Together with the budget constraints and a transversality condition, these conditions are sufficient for finding optimal consumer choices. In order to find an equilibrium, we need to aggregate across consumers and find equilibrium bond prices in order to clear asset markets. To accomplish this, we will make additional assumptions on preferences.

2.3 Aggregation

Our additional assumption on utility functions are made in order to deliver aggregation.

Assumption 1: *Let the instantaneous utility be*

$$u(c_t, l_t) = \frac{(c_t^{\alpha} l_t^{1-\alpha})^{1-\sigma}}{1-\sigma}.$$

Under this assumption, the first-order conditions take a particularly simple form. As a result, the competitive equilibrium aggregates and prices are independent of the distribution of asset holdings.[3]

From agent i's leisure choice, we see that all agents will choose the same ratio of consumption to leisure:

$$\frac{l_{it}}{c_{it}} = \frac{1-\alpha}{\alpha(1-\tau_t)}. \tag{1}$$

The same agent chooses asset holdings to satisfy

$$q_t (c_{it}^{\alpha} l_{it}^{1-\alpha})^{1-\sigma} c_{it+1} = \beta (c_{it+1}^{\alpha} l_{it+1}^{1-\alpha})^{1-\sigma} c_{it}.$$

By replacing equation (1) in the condition above we obtain

$$\frac{c_{it+1}}{c_{it}} = \left[\frac{\beta}{q_t \epsilon_t}\right]^{1/\sigma}. \tag{2}$$

where $\epsilon_t = \left[\frac{1-\tau_{t+1}}{1-\tau_t}\right]^{(1-\alpha)(1-\sigma)}$. Thus, the growth rate of consumption is independent of idiosyncratic characteristics. Finally, individual asset holdings must evolve according to

$$a_{it+1} = (a_{it} + (1-l_{it})(1-\tau_{it}) + T_t - c_{it})/q_t. \tag{3}$$

For completeness, we state

Proposition 1: *Under Assumption 1 the competitive equilibrium exhibits aggregation.*

Though the result is well known, we demonstrate it by a guess-and-verify strategy, since the guess plays a role in the further characterization of equilibrium. By our guess, aggregate quantities and prices can be found by solving the maximization problem of a representative agent holding average wealth. We will thus refer to him as the *mean agent*. In other words, his asset holdings are zero, since $\sum_{i=1}^{I} \mu_i A_{it} = 0$. The mean agent would consume goods and leisure according to

$$L_t = \frac{1-\alpha}{1-\alpha\tau_t}$$

and

$$C_t + (1-\tau_t)L_t = 1 - \tau_t + T_t \equiv f_t.$$

Here, we have written resources as the total net-of-tax value of time plus the transfer, which we denote f_t; on the expenditure side, we have expenditures on consumption and leisure.

[3]For a discussion of this result in an economy with capital see, e.g., Krusell and Ríos-Rull (1999) or Azzimonti, de Francisco and Krusell (2006).

In equilibrium $T_t = \tau_t(1 - L_t)$. Therefore,

$$C_t = \alpha f(\tau_t),$$

where $f(\tau_t) = \frac{1-\tau_t}{1-\alpha\tau_t} = f_t$. That is, the function f describes the economically relevant current income of any agent in the economy—since it is based on the total amount of time, not hours actually worked, and all agents have the same time endowment and productivity—as a function of the current tax rate.

The first thing to note is that the consumption and leisure allocations of the mean agent depend only on current taxes; this result really follows since assets of the representative agent must be zero at all points in time (due to aggregation), and thus the representative-agent economy reduces to a static one. So if the tax rates were constant over time, aggregates and prices would also be constant. However, if they are not, there will be changes in the asset distribution over time, as we will see below, even though mean assets remain at zero.

The first-order condition of an agent with average wealth implies

$$\frac{C_{t+1}}{C_t} = \left[\frac{\beta}{q_t \epsilon_t}\right]^{1/\sigma},$$

so the growth rate of consumption is identical to that chosen by an arbitrary agent in this economy. It is immediate, now, that a summing up of leisure, consumption, and asset choices of individual agents, using equations (1)–(2), delivers the expressions for averages given above. Using the definition of consumption of the mean agent, we conclude that the price of bonds in equilibrium must be

$$q_t = \beta \left(\frac{f(\tau_t)}{f(\tau_{t+1})}\right)^{\sigma} \frac{1}{\epsilon_t}.$$

Thus, the interest equals the discount rate if and only if taxes are constant over time.

Aggregation allows us to characterize a competitive equilibrium by stating agents' consumption and leisure decisions as proportional to mean consumption and mean leisure. The proportion is given by their share of after-tax (potential) lifetime income in present value terms. We now proceed to provide the appropriate expressions.

We can insert the equilibrium price of bonds, q_t, into equation (2). The resulting growth rate of consumption depends only on current and next period's taxes:

$$\frac{c_{it+1}}{c_{it}} = \frac{f(\tau_{t+1})}{f(\tau_t)}, \tag{4}$$

with

$$c_{it} = \alpha[f(\tau_t) + a_{it} - q_t a_{it+1}];$$

under the present utility specification, consumption always commands a fraction α of total resources left to spend this period, which are given by the right-hand side of this equation.

Similarly, leisure follows

$$l_{it} = \frac{1-\alpha}{1-\tau_t}[f(\tau_t) + a_{it} - q_t a_{it+1}],$$

The present-value budget constraint of an agent with initial asset holdings a_{i0} is given by

$$\underbrace{\frac{1}{\alpha}\sum_{t=1}^{\infty} c_{it}\Pi_{k=0}^{t-1} q_k + c_{i0}}_{\text{PV consumption}} = \underbrace{f(\tau_0) + a_{i0} + \sum_{t=1}^{\infty}\Pi_{k=0}^{t-1} q_k f(\tau_t)}_{\text{PV income}}$$

We can further simplify this expression and, using equation (4), solve for initial consumption

$$c_{i0} = \alpha f(\tau_0) \left[1 + \frac{a_{i0}}{E(\tau)}\right],$$

where the tax rate without subscript denotes the entire tax sequence, i.e., $\tau \equiv \{\tau_t\}_{t=0}^{\infty}$, and

$$E(\tau) = f(\tau_0) + \sum_{t=1}^{\infty} \Pi_{k=0}^{t-1} q_k f(\tau_t) \tag{5}$$

is the present value of net-of-tax time endowments and transfers received from the government. It is also the lifetime income of the mean agent in present value terms. An arbitrary agent is thus consuming a proportion of mean consumption (recall that the mean agent consumes $C_0 = \alpha f(\tau_0)$). This proportion is given by the share of the given consumer's present-value wealth, including transfers, the net-of-tax time endowment, and initial assets. It is given by

$$\lambda_i(\tau) \equiv \frac{E(\tau) + a_{i0}}{E(\tau)} = 1 + \frac{a_{i0}}{E(\tau)}.$$

After some manipulation of equation (2), we thus find that

$$c_{it} = \underbrace{\alpha f(\tau_t)}_{C_t} \lambda_i(\tau).$$

That is, the level of consumption of an agent with initial wealth a_{i0}, at any point in time, is a proportion of average consumption.

Our equilibrium exists if $\lambda_i(\tau) \geq 0$ for all i, i.e., if all agents can afford positive consumption and leisure, which depends on the given tax sequence. This can be viewed as a restriction on inequality, given a tax sequence, or as a restriction on tax sequences, given an asset distribution. For some asset distributions, however, no tax sequences admit positive consumption, because even the use of infinite subsidies to labor income is not enough to generate positive consumption for the poorest. However, such initial asset distributions do not make sense, since they could never have been chosen in the past.

Summarizing the information above, we can characterize the competitive equilibrium in terms of the allocations chosen by the mean agent. The following proposition summarizes our result.

Proposition 2: *Under Assumption 1 and a given sequence of taxes $\tau = \{\tau_t\}_{t=0}^{\infty}$ satisfying $\lambda_i(\tau) \geq 0$ for all i, there exists a competitive equilibrium, and it is given as follows.*

1. *Transfers are $T_t = \alpha \tau_t f(\tau_t)$.*

2. *The bond price is*

$$q_t = \beta \left(\frac{f(\tau_t)}{f(\tau_{t+1})}\right)^{\sigma} \left[\frac{1 - \tau_t}{1 - \tau_{t+1}}\right]^{(1-\alpha)(1-\sigma)}.$$

3. *Consumption and leisure allocations satisfy*

$$c_{it} = C_t \lambda_i(\tau) \ \ \text{and} \ \ l_{it} = L_t \lambda_i(\tau) \ \ \text{where}$$

$$\lambda_i(\tau) = \left[1 + \frac{a_{i0}}{E(\tau)}\right],$$

with $E(\tau)$, the present value of net-of-tax time endowments plus transfers, is given by equation (5). Per-capita values are

$$C_t = \alpha f(\tau_t)$$

and

$$L_t = \frac{1-\alpha}{1-\alpha\tau_t}.$$

4. *Assets follow*

$$a_{it+1} = (a_{it} - [\lambda_i(\tau) - 1] f(\tau_t)) \frac{1}{q_t}.$$

3 Political economy: time-zero voting with commitment

The equilibrium characterization above implies that any agent who is poorer than average (i.e., with $a_{i0} < 0$) will enjoy less consumption and leisure than the mean agent: $\lambda_i(\tau) < 1$. While this share is constant for a given stream of taxes τ, alternative tax sequences will modify it. It is now clear where the core of the disagreement regarding policy in this economy lies: agents realize that they can modify their share of average consumption and leisure by voting for the appropriate τ. There is a cost associated to this manipulation, and it is given by the distortions introduced in the labor decision. That is, alternative policies have different implications for C_t or L_t, and we know that zero taxes is the best choice for the mean agent, thus leaving $C_t = \alpha f(0) = \alpha$ and $L_t = 1 - \alpha$ as his undistorted, best outcome. If an agent with non-zero initial assets could decide, however, it is clear that a non-zero tax sequence would be chosen, since there is a first-order marginal benefit from departing from zero taxes—the change in the share of resources—and only a second-order loss.

We now need to be specific about the political institution. In this section, we will assume that voting takes place at the beginning of time, and that the policy-setting power has a commitment technology at that point: it can fix a sequence of taxes for all time. We assume that political competition takes place in the form of direct votes over tax sequences, and we assume majority voting.

Definition 2: *A **majority-voting equilibrium (MVE)** is a tax sequence that cannot be beaten in a pairwise voting contest by any other sequence.*

A tax sequence is a high-dimensional object and it is thus far from obvious that this definition has content in our economy—it is well known that the existence of an MVE cannot be guaranteed except under special circumstances. Here, fortunately, the same assumption on preferences that admits aggregation will allow us to show existence of a majority-voting equilibrium: we will provide a median-voter theorem.

We need to derive indirect utility expressions for the different agents, first, which are available in closed form under our assumptions. It is easy to verify that they are given by

$$V_{i0}(\tau) = \lambda_i(\tau)^{1-\sigma} V_0(\tau), \tag{6}$$

where $V_0(\tau)$ is the utility of the agent with average wealth, given by

$$V_0(\tau) = \eta \sum_{t=0}^{\infty} \beta^t \left[\frac{f(\tau_t)}{(1-\tau_t)^{1-\alpha}} \right]^{1-\sigma},$$

with $\eta = \frac{1}{1-\sigma}\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right]^{1-\sigma}$. What is important to note here, from the purposes of existence, is that $\lambda_i(\tau)$ is linear in the only element of heterogeneity in the model: in a_{i0}.

Let τ^* be our candidate majority-voting equilibrium, then it must be the case that this policy is preferred by at least 50% of the population to any other policy. We can show that the equilibrium tax sequence is that preferred by the agent with median wealth, the *median agent*.

3.1 The median-voter theorem

We denote the wealth of the median agent by a_*. Similarly, his preferred tax sequence is denoted τ^*. Thus,

$$\left(1+\frac{a_*}{E(\tau^*)}\right)^{1-\sigma} V_0(\tau^*) \geq \left(1+\frac{a_*}{E(\tau)}\right)^{1-\sigma} V_0(\tau) \tag{7}$$

for all feasible τ, i.e., for all τ such that $\lambda_i(\tau) \geq 0$ for all i.

We now state our median-voter theorem.

Proposition 3: τ^* *is a majority-voting equilibrium.*

The proof is straightforward, but perhaps instructive. For any alternative feasible τ, we know that

$$\left(1+\frac{a_*}{E(\tau^*)}\right)^{1-\sigma} V_0(\tau^*) \geq \left(1+\frac{a_*}{E(\tau)}\right)^{1-\sigma} V_0(\tau).$$

This implies, using simple manipulations, that either

$$a_* \geq \frac{\left(\frac{V_0(\tau)}{V_0(\tau^*)}\right)^{\frac{1}{1-\sigma}} - 1}{\frac{1}{E(\tau^*)} - \frac{1}{E(\tau)}\left(\frac{V_0(\tau)}{V_0(\tau^*)}\right)^{\frac{1}{1-\sigma}}}$$

or

$$a_* \leq \frac{\left(\frac{V_0(\tau)}{V_0(\tau^*)}\right)^{\frac{1}{1-\sigma}} - 1}{\frac{1}{E(\tau^*)} - \frac{1}{E(\tau)}\left(\frac{V_0(\tau)}{V_0(\tau^*)}\right)^{\frac{1}{1-\sigma}}},$$

depending on the signs of the numerators and denominators in these expressions. Therefore, since half of the population has $a_{i0} \geq a_*$ and the other half, $a_{i0} \leq a_*$, working in reverse, it must be that for half of the population, there is agreement with the choice of the median agent: for more than half the population, their a_{i0} is such that

$$\left(1+\frac{a_{i0}}{E(\tau^*)}\right)^{1-\sigma} V_0(\tau^*) \geq \left(1+\frac{a_{i0}}{E(\tau)}\right)^{1-\sigma} V_0(\tau).$$

3.2 Characterization

We now proceed to show that, under the preference specification adopted, subsidies are zero from period 1 and on, whereas they are negative in period 0 if and only if $a_* < 0$.

The key step in the proof is the demonstration that, under our preference specification, $E(\tau)$ can be written as a product of a function only of period-0 taxes, $g(\tau_0)$, and the present-value utility of the mean agent:

Lemma: *$E(\tau) = g(\tau_0)V_0(\tau)$, with $g(\tau_0) \equiv \eta^{-1} f(\tau_0)^{\sigma}(1-\tau_0)^{(1-\alpha)(1-\sigma)}$.*

The proof is contained in Appendix 6.1.

Thus, we conclude that the utility of the median agent can be written as

$$V_{*0}(\tau) = \left[1 + \frac{a_*}{g(\tau_0)V_0(\tau)}\right]^{1-\sigma} V_0(\tau). \tag{8}$$

Now we verify the following.

Proposition 4: *Assume that $\sigma \leq 1$. The politico-economic equilibrium then has the following properties, assuming $a_* < 0$:*

1. *For all $t > 0$, $\tau_t = 0$.*

2. *$\tau_0 < 0$.*

3. *$q_0 > \beta$ and $q_t = \beta$ for all $t > 0$.*

4. *For all i such that $a_{i0} < 0$, $a_{i0} < a_{i1} = a_{i2} = \cdots < 0$ and for all i such that $a_{i0} > 0$, $a_{i0} > a_{i1} = a_{i2} = \cdots > 0$.*

The proof of the first two parts of the proposition is contained in Appendix 6.2. In brief, the proof of the first part involves a straightforward implication of the lemma above, namely, that the first-order condition with respect to τ_t is zero at $\tau_t = 0$, since τ_t for $t > 0$ only appears through $V_0(\tau)$, which we know is maximized at zero taxes; it also involves the verification of a second-order condition. The second-order condition necessitates restricting $-\sigma$ from being too large relative to the median voter's initial debt position; values of σ above one can thus be admitted as well, so long as a_* is above a certain lower bound. The proof of the second part is straightforward. The third and fourth parts of the proposition are immediate implications of the first two parts, together with the earlier characterization of equilibrium prices and quantities for given tax sequences.

The result that taxes are zero in the long run (and even from the second period and on) is surprising; one might think that the redistribution generated by subsidies to labor should not just take place at time zero—that at least a minor amount of redistribution in the future would be better than none. The proposition proves this intuition wrong. Zero taxes in the long run is also reminiscent of the findings of Chamley (1986) and Judd (1985). Here, though it is not an optimal taxation problem, the median agent does have an accumulation problem, and the tax sequence influences the return on savings as well as a period income function $f(\tau_t)$ similar to the Chamley-Judd settings. We do find that savings are undistorted in the long run, but this outcome would result for any constant tax on labor income, whether positive, zero, or negative. The fact that we find that the labor income tax will be exactly zero is, thus, what is different, and surprising. Therefore, there is really no obvious connection between the result here and the Chamley-Judd finding, since zero taxes has a different meaning in the two cases.

The preference specification is key behind the result. In fact, it is possible to extend our result to a utility function that is a power function of a CES function in consumption and leisure (and not just a unitary-elasticity Cobb-Douglas function). However, if the elasticity of substitution is not constant between consumption and leisure, one can demonstrate that the long-run tax is non-zero. Thus, it is the fact that, under homothetic utility specifications, wealth differences do not misalign preferences over tax rates enough that is driving our main finding. There are four kinds of effects when a change of τ_t away from zero is considered. The first two involve the effect on redistribution and the associated distortion to the consumption-leisure choice. That effect would, in isolation, lead the median to choose a negative tax rate. The two other effects involve interest-rate manipulation. A decrease in τ_t will increase labor supply, which will decrease marginal utility of consumption at t, thus raising q_t but lowering q_{t-1}. The former effect is beneficial for the median voter, but the latter is detrimental. Evaluated at $\tau_t = 0$ for all $t > 0$, the net marginal effect of these two changes is negative, and it exactly outweighs the redistributional benefits of lowering the tax rate.[4]

4 Sequential voting without commitment

We now move to the context where there is no commitment technology and taxes for period t are voted on in the beginning of period t (before work decisions have been made). For this environment, we employ the notion of Markov-perfect equilibrium, as used in Krusell and Ríos-Rull (1999). The set of Markov-perfect equilibria is likely large (see the discussion in Krusell and Smith [2003]), and we restrict attention here to Markov-perfect equilibria that would appear as limits of equilibria in the corresponding finite-horizon economies. Thus, we naturally focus on equilibria where equilibrium taxes are a smooth function of the payoff-relevant state variable.

The payoff-relevant state variable in our economy is whatever feature of the asset distribution matters for the political outcome, and no other feature. Again, given taxes, and due to Assumption 1, there is aggregation, and nothing else than mean asset holdings can matter from the economic perspective: in the last period, no other feature matters for outcomes, and thus in the period before the last, no other feature matters either, and so on.[5] What are the politically relevant features of the asset distribution? We will conjecture that there is a median-voter characterization here as well, and we will thus proceed based on this conjecture. In conclusion, the state variable is a_*: the asset holdings of the median agent. From this point on, for simpler notation, we will merely use a to denote median asset holdings.

4.1 Equilibrium definition based on a simple conjecture

We will not define a Markov-perfect equilibrium in full generality, since it consumes space, and such a definition would closely follow the definitions in the literature. Instead we will state our conjecture and define an equilibrium based on it.

We conjecture that taxes in any period are a function of a: $\tau = \Psi(a)$. In other words, Ψ is the fundamental endogenous object we are in search of. Given this conjecture, it should be clear that taxes, and the asset distribution, will be constant from the beginning of time.

[4]In fact, the result that future taxes are zero holds also in finite-horizon settings, perhaps surprisingly. There, one might think that the taxation decision in the last period would lead to a different outcome, since only one interest rate is affected by the last period's tax rate. There, however, the effect of increasing the second-to-last period's q again exactly offsets the gain from redistribution due to a last-period tax cut.

[5]It is conceivable that one can construct a Markov-perfect equilibrium in a political-economy model where other features of the asset distribution matter, even when preferences admit aggregation, but then as a "multiple-equilibrium" phenomenon: other features matter now because they matter in the future. Such equilibria will, however, not be limits of finite-horizon equilibria.

Why should this be true? Suppose that, given any function Ψ, we (rationally) expect taxes not to change. Then, q will equal β, and the asset distribution—in particular, median asset holdings—will not change. As a result, taxes next period will indeed be the same, since they are given by the function Ψ, evaluated at the median asset level. Thus, the conjecture of a constant tax rate is internally consistent. However, could different expectations regarding future taxes be self-fulfilling? Assuming that Ψ is differentiable, one can show that the answer is negative, at least in the neighborhood of $a = 0$.[6]

Proceeding with the guess, we compute the median consumer's utility of any next-period asset level as follows:

$$V(a) \equiv \frac{\left(c(a)^{\alpha} l(a)^{1-\alpha}\right)^{1-\sigma}}{(1-\beta)(1-\sigma)},$$

where

$$c(a) \equiv \left(1 + \frac{a}{e(\Psi(a))}\right) \alpha f(\Psi(a))$$

and

$$l(a) \equiv \left(1 + \frac{a}{e(\Psi(a))}\right) (1-\alpha) \frac{f(\Psi(a))}{1-\Psi(a)},$$

with the function e defining the present-value wealth given the constant tax sequence:

$$e(\Psi(a)) \equiv \frac{1}{1-\beta} f(\Psi(a)).$$

We now consider a one-period deviation from the constant-tax equilibrium, i.e., that taxes in the current period are not necessarily given by $\tau = \Psi(a)$. Now it is useful to define present consumption and leisure

$$\tilde{c}(a, a', \tau) \equiv \left(1 + \frac{a}{\tilde{e}(a, a', \tau)}\right) \alpha f(\tau) \tag{9}$$

and

$$\tilde{l}(a, a', \tau) \equiv \left(1 + \frac{a}{\tilde{e}(a, a', \tau)}\right) (1-\alpha) \frac{f(\tau)}{1-\tau}, \tag{10}$$

with

$$\tilde{e}(a, a', \tau) \equiv f(\tau) + \tilde{q}(a', \tau) e(\Psi(a')) \tag{11}$$

and

$$\tilde{q}(a', \tau) = \beta \left(\frac{f(\tau)}{f(\Psi(a'))}\right)^{\sigma} \left[\frac{1-\tau}{1-\Psi(a')}\right]^{(1-\alpha)(1-\sigma)},$$

the current bond price.

Thus, we can define our Markov-perfect equilibrium as follows:

Definition: *A Markov-perfect equilibrium is a function Ψ solving*

$$\Psi(a) = \arg\max_{\tau} \frac{\left(\tilde{c}(a, a', \tau)^{\alpha} \tilde{l}(a, a', \tau)^{1-\alpha}\right)^{1-\sigma}}{1-\sigma} + \beta V(a'),$$

[6] To do this, one can (i) express asset accumulation of the median agent as a function of current a, current taxes $\Psi(a)$, and future taxes $\Psi(a')$, where a' is next period's median asset holdings, (ii) differentiate the expression, and (iii) evaluate at $a = 0$. This delivers a unique linear solution: $a' = a$.

where a' is given implicitly by the solution to

$$a' = a\left[1 - \frac{1}{\tilde{e}(a,a',\tau)} f(\tau)\right]\frac{1}{\tilde{q}(a',\tau)}.$$

The latter constraint is the asset accumulation equation, slightly simplified, from Proposition 2.

The median-voter theorem holds if the objective above is single-peaked in τ for every a. The shape of the objective is, of course, influenced by Ψ, which itself is endogenous; however, single-peakedness can be verified numerically.

4.2 Analytical findings and comparisons with the commitment solution

In this section, we partially characterize the Markov-perfect equilibrium and compare it to the commitment case. In the next section, we continue our analysis using numerical solution techniques.

From now on, we specialize to the case of $\sigma = 1$ (logarithmic utility). Under this assumption, the present-value wealth from time endowments, net of taxes, and transfers, under a one-period deviation, is independent of asset holdings. Thus equation (11) reduces to

$$\tilde{e}(\tau) = \frac{f(\tau)}{1-\beta},$$

while the price of bonds becomes

$$\tilde{q}(a',\tau) = \beta\frac{f(\tau)}{f(\Psi(a'))}.$$

Consequently, a' is implicitly defined by

$$a' = a\frac{f(\Psi(a'))}{f(\tau)}. \tag{12}$$

Equation (12) determines the saving rule for the median voter, which we denote by $\tilde{h}(a,\tau)$. By replacing the equations above into equations (9) and (10) we obtain expressions for consumption and leisure in the logarithmic case:

$$\tilde{c}(a,a',\tau) = \alpha\left[f(\tau) + a - \tilde{q}(a',\tau)a'\right]$$

and

$$\tilde{l}(a,a',\tau) = \frac{(1-\alpha)}{\alpha}\frac{\tilde{c}(a,a',\tau)}{1-\tau}.$$

Notice that $\tilde{q}(a',\tau)a' = \beta a$, which implies that we can write

$$\tilde{c}(a,\tau) = \alpha\left[f(\tau) + (1-\beta)a\right].$$

The Markov-perfect equilibrium tax function $\Psi(a)$ then solves

$$\max_{a',\tau} \quad \alpha\log\tilde{c}(a,\tau) + (1-\alpha)\log\tilde{l}(a,\tau) + \beta V(a')$$

where

$$V(x) \equiv \frac{1}{1-\beta}\left\{\log \alpha^{2\alpha-1}(1-\alpha)^{1-\alpha} + \log\left[f(\Psi(x)) + x(1-\beta)\right] - (1-\alpha)\log[1-\Psi(x)]\right\},$$

subject to equation (12), i.e., subject to $a' = \tilde{h}(a,\tau)$.[7] It is clear here how the choice for current taxes also influences future outcomes through the effect on future taxes in this model. In particular, $\tau' = \Psi(a') = \Psi(\tilde{h}(a,\tau))$.

Using the assumption that Ψ is differentiable, we can derive a first-order necessary condition. It reads

$$\frac{1}{1-\tau} - \frac{1}{1-\alpha\tau}\frac{1}{1-\tau+a(1-\beta)(1-\alpha\tau)} + \beta\left\{\frac{a}{(1-\tau)[1-\tau+a(1-\beta)(1-\alpha\tau)]}\right. \tag{13}$$

$$\left. - \frac{\alpha\Psi(a')}{(1-\beta)(1-\Psi(a'))(1-\alpha\Psi(a'))}\Psi_a(a')\tilde{h}_\tau(a,\tau)\right\} = 0,$$

where $\tilde{h}_\tau(a,\tau)$ is given by implicit differentiation of equation (12).[8]

The first two terms in equation (13) summarize the static trade-off between the net gain from redistribution resulting from a positive subsidy with the negative effect of the introduced distortion in the leisure-labor decision; it is negative at $\tau = 0$ if and only if the median agent is poorer than the median, $a < 0$. This would be the complete analysis in a Meltzer-Richard (static) version of our economy. The remaining part of the first-order condition—the second and third terms, which are both of second order at $a = 0$—capture how future utility is affected by a change in current taxes. The second term incorporates the effects of an induced change in current prices ($\tilde{q}(a',\tau)$) on future consumption and leisure via changes in bond holdings, assuming that the tax rate next period remains unaffected (that is, keeping $\Psi(a')$ constant). The last term describes the indirect effect on utility through how the current tax rate alters future tax rates.

In a differentiable Markov-perfect equilibrium, condition (13) needs to deliver $\tau = \Psi(a)$: thus Ψ constitutes a fixed point. Moreover, a' will equal a in the equilibrium construction. Therefore, we can state a functional equation in Ψ by imposing these conditions in the equations above, resulting—after some simplifications—in the following:

$$1-\alpha\Psi(a) = \frac{1-\Psi(a)}{1-\Psi(a)+a(1-\beta)(1-\alpha\Psi(a))} - \frac{\alpha\beta\Psi(a)(1-\Psi(a))(1-\alpha\Psi(a))}{(1-\Psi(a))(1-\alpha\Psi(a))+a(1-\alpha)\Psi_a(a)}. \tag{14}$$

This functional equation determines Ψ as a function of a: Ψ has to be such that the equality is met for all values of a. The equation can be labeled a "generalized Euler equation", a term also used in other settings with dynamic decisions made without commitment and a Markov-perfect equilibrium concept, such as the case with savings under time-inconsistent preferences

[7]Here, we use the fact that since consumption can be written as a function of τ and a only, $\tilde{l}(a,\tau) = \frac{(1-\alpha)}{\alpha}\frac{\tilde{c}(a,\tau)}{1-\tau}$.

[8]The derivation of equation (13) involves an intermediate step where we substitute equation (12), where $x = a\frac{f(\Psi(x))}{f(\tau)}$, and use the properties of logarithmic functions to separate the direct effects of τ on $V(x)$ from those resulting from indirect changes through $\Psi(x)$. Using a' as shorthand for $\tilde{h}(a,\tau)$, implicit differentiation of (12) delivers

$$\tilde{h}_\tau(a,\tau) = \frac{a'(1-\alpha\Psi(a')) - a\alpha(1-\Psi(a'))}{(1-\tau)(1-\alpha\Psi(a')) - (\alpha a'(1-\tau) - a(1-\alpha\tau))\Psi_a(a')}.$$

(see, e.g., Laibson, 1997). In these cases, the first order condition also contains the derivative of the unknown policy function. We will discuss the characterization of $\Psi(a)$ in the context of comparing it to the commitment solution.

Under time-zero voting, we can directly apply the insights from Section 3. However, it is useful to take one step back and restate the commitment problem here, because this allows for a convenient parallel to the problem without commitment. The problem can be stated

$$\max_{\{\tau_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\log \left(f(\tau_t) + a_t - \beta \frac{f(\tau_t)}{f(\tau_{t+1})} a_{t+1} \right) - (1-\alpha) \log(1-\tau_t) \right].$$

This problem is unconstrained: one can easily show that the choice for asset holdings delivers

$$a_{t+1} = a_t \frac{f(\tau_{t+1})}{f(\tau_t)},$$

which would have been the constraint representing equilibrium choices (thus, maximization over a larger choice set leads to a choice within the restricted set, so the restriction can be ignored).

Knowing that taxes from period 1 and on are constant, and that this implies that bond holdings are also constant from period 1 on, we obtain, for all a, that taxes in period zero satisfy

$$\frac{1}{1-\tau_0} - \frac{1}{1-\alpha\tau_0} \frac{1}{1-\tau_0 + a(1-\beta)(1-\alpha\tau)} + \beta \frac{a}{(1-\tau_0)[1-\tau_0 + a(1-\beta)(1-\alpha\tau_0)]} = 0, \quad (15)$$

where a now refers to a_0, for ease of comparison with the no-commitment solution. That is, τ_0 denotes the first-period choice for taxes under commitment. Slightly abusing notation, we denote the tax function that solves the equation above by $\tau_0(a)$.

Comparing equations (13) and (15) we see only one difference: the former—the no-commitment case—has an extra term. The reason is that under a period-by-period voting mechanism, the current policymaker does not have a direct control over future taxes: it can only affect them indirectly via the level of a'. If the policy function were constant, i.e., if it did not depend on a, so that $\Psi_a(a) = 0$ in the expression above, then the commitment solution would be time-consistent. However, we know that it cannot be, because the commitment solution does deliver a tax rate that depends on a: simple inspection of equation (15) reveals that $\tau_0(a)$ cannot be independent of a. Thus, in general, $\Psi(a) \neq \tau_0(a)$. We see, however, that for $a = 0$, so long as $\Psi(a)$ is differentiable at 0, both equations (13) and (15) deliver zero taxes: $\Psi(0) = \tau_0(0) = 0$. That is, if the median asset holdings equal the mean, then taxes are zero, no matter how voting takes place. The median voter chooses not to distort labor decisions because he cannot redistribute resources in his favor. Since the median asset holdings do not change (he chooses to hold no debt), taxes in the future will also be zero under commitment. There are no incentives to deviate from this policy once period one arises, and thus the policy is time-consistent.

It is also possible to compare the cases with time-zero voting and sequential voting in the neighborhood of equality of median and mean income ($a = 0$). First, note that initial taxes under commitment allow for a closed-form solution, and that the implied slope of $\tau_0(a)$ can be determined.[9] Thus, it follows in particular that the slope of $\tau_0(a)$ at the origin is given by

$$\left.\frac{d\tau_0}{da_0}\right|_{a_0=0} = \frac{1}{\alpha},$$

i.e., that at least locally around the point where median and mean income coincide, initial taxes are increasing at the rate $1/\alpha > 1$ in median asset holdings. Thus, the higher is the preference weight on consumption relative to leisure (α), the less do subsidies respond to inequality between median and mean income. Intuitively, a low relative value of leisure makes the labor effort differ less across wealth groups, so for each a there is a smaller income discrepancy across groups and, hence, lower benefits from redistribution.

Interestingly, around the point where median wealth equals mean wealth, the local (first-order) sensitivity of first-period taxes to inequality is the same under sequential voting as under time-zero voting, provided that Ψ is twice differentiable at zero. Showing this, i.e., that $\Psi_a(0) = 1/\alpha$, is straightforward: it amounts to differentiating equation (14) with respect to a, then evaluating the resulting expression at $a = 0$, and verifying that $\Psi_a = 1/\alpha$ solves the equation.[10]

Furthermore, it is possible to see that, in the neighborhood of $a = 0$ but for strictly negative values of a, subsidies under the no-commitment case are higher than under commitment, i.e., that

$$\Psi(a) < \tau_0(a).$$

This can be seen as follows. For a small negative level of debt, the third term of equation (13) is negative since (i) the term in brackets is positive, because the median subsidizes when $a < 0$; (ii) $\Psi_a(a) > 0$ at zero; and (iii) $\frac{d\tau'}{d\tau} = \Psi_a(a)\tilde{h}_\tau(a, \tau) < 0$, which can be shown by using the fact that $\Psi(a)$ is increasing in this neighborhood. Given this, at the commitment tax level, the first term is zero while the second one is negative; hence, the equality in equation (13) cannot hold. The median must then adjust τ by increasing the first term. This can only be done by decreasing τ, since the first term is decreasing in τ for small as. These differences will be illustrated numerically in the next section. The reason why subsidies are higher in the first period when there is no commitment comes from the fact that interest-rate manipulation is actually easier when there is no commitment. To see why, from equation (12), a raised subsidy today will make the median asset holding come closer to zero: first, it will lower a'/a since it raises consumption today directly and hence lowers the interest rate; second, since a'/a tends to fall, there is a feedback through the lower implied subsidy rate tomorrow, further lowering the interest rate. Thus, the feedback effect of lower future subsidies coming from a raised subsidy today, which is not present under commitment because future subsidies are always zero in that case, increases the interest-rate effect of a given subsidy change when there is a lack of commitment.

[9] One obtains a second-order polynomial expression for τ_0 for each a, yielding

$$\tau_0^* = \frac{-\alpha[-1 + a_0(\beta - 2)] - \sqrt{\alpha^2[-1 + a_0(\beta - 2)]^2 - 4a_0\alpha(1 + a_0\alpha(1 - \beta))}}{2\alpha(1 + a_0\alpha(1 - \beta))}.$$

[10] The expression involves the product of the second derivative of the policy rule, $\Psi_{aa}(a)$, and the level of assets a. The term cancels out at $a = 0$ under the assumption that $\Psi_{aa}(a) < \infty$.

4.3 A numerical example and a computational challenge

We proceed using the case of logarithmic utility. Using the method described in Krusell, Kuruşçu, and Smith (2002), we solve the model numerically using the parameter values $\alpha = 0.3$ and $\beta = 0.9$.[11] It delivers the following results in a neighborhood of $a = 0$. Figure 1 illustrates

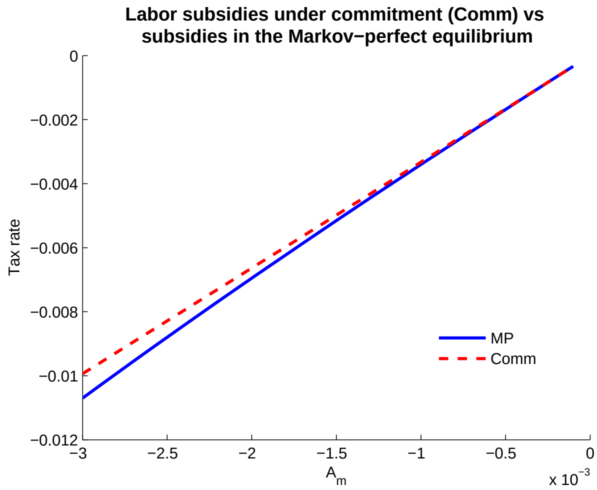

Figure 1: Numerical solution of the model under logarithmic utility

the features derived in the previous section: the commitment and Markov-perfect solutions that coincide at zero asset holdings of the median have the same slope at that point, with the commitment solution giving lower subsidies for strictly negative values of median assets.

It turns out that the model here presents a formidable computational problem. Figure 1 displays the tax function in a very small neighborhood around zero median asset holdings. It is, however, very difficult to extend the results beyond this small neighborhood, and we contend that it is an open problem how to best approximate equilibria in cases like this one. As an illustration, we include numerically computed first-period policy rules (median asset accumulation and the chosen tax rate) for a sequence of economies with an increasing time horizon: we look at horizons of length 1 through 9. These decision rules are solved by discretizing the space of asset holdings, using a very fine grid. Figures 2 and 3 report the results.

As can be seen, first, in Figure 2, assets move toward zero, since the model has a finite horizon; for longer time horizons, the asset rule is closer to the 45-degree line, as expected (in the infinite-horizon Markov-perfect equilibrium, assets are constant over time, i.e., the decision rule is the 45-degree line).

In Figure 3 we see the decision rules for taxes. As expected, the tax function is positively sloped, goes through zero, and is lower, the longer the time horizon: the future benefits of redistribution engineered by raising q in the first period are higher, the longer the future period over which the higher asset holdings are used. However, what is probably more eye-catching in the figure is the fact that wiggles appear in the tax function at longer horizons. We have not been able to find ways of making sure that no wiggles appear. The numerical problems may be related to the fact that one expects the equilibrium tax rate to approach $-\infty$

[11] We have also used a pure grid method, based on the definition of the equilibrium involving value functions. It reproduces these results.

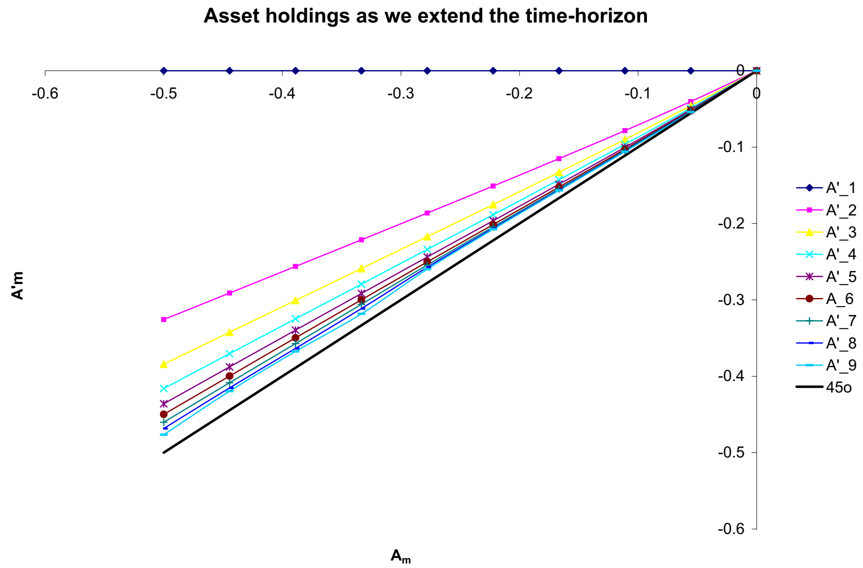

Figure 2: Next-period assets in the first period for a sequence of finite-horizon economies

as median asset holdings approach its lowest feasible value. For the infinite-horizon model, this value, $\underline{a}$, solves $1 + \frac{\underline{a}}{E(-\infty)} = 0$; for logarithmic utility, $\underline{a} = -\frac{1}{\alpha(1-\beta)}$. Numerical instability of Markov-perfect equilibria has been noted before, beginning in Krusell and Smith (2003). The idea there, loosely speaking, is that wiggles/numerical errors, once they appear, tend to reproduce and amplify due to the time-inconsistency inherent in the decision problem. This delivers a non-concave objective function and, eventually, discontinuous behavior.

5 Conclusions

With this paper, we continue the exploration of dynamic macroeconomic models of political economy. We focus on a case where the only source of disagreement on policy is the heterogeneity of initial asset positions: initially well-endowed consumers choose to work less, and a poor median voter thus wants to subsidize leisure. The case is interesting and nontrivial in part because interest rates are endogenous and call for manipulation by the median voter: they are adjusted so as to change his net-present-value wealth position, in effect, by raising the interest rate so that the debt he carries is an easier burden.

The tax sequence chosen by the median voter under commitment involves zero taxes from period 1 and on, presuming that preferences are in the class admitting aggregation. We ensure that a median-voter equilibrium obtains under the same conditions. The tax choice is not time-consistent, however: without commitment, taxes will be constant (and non-zero) over

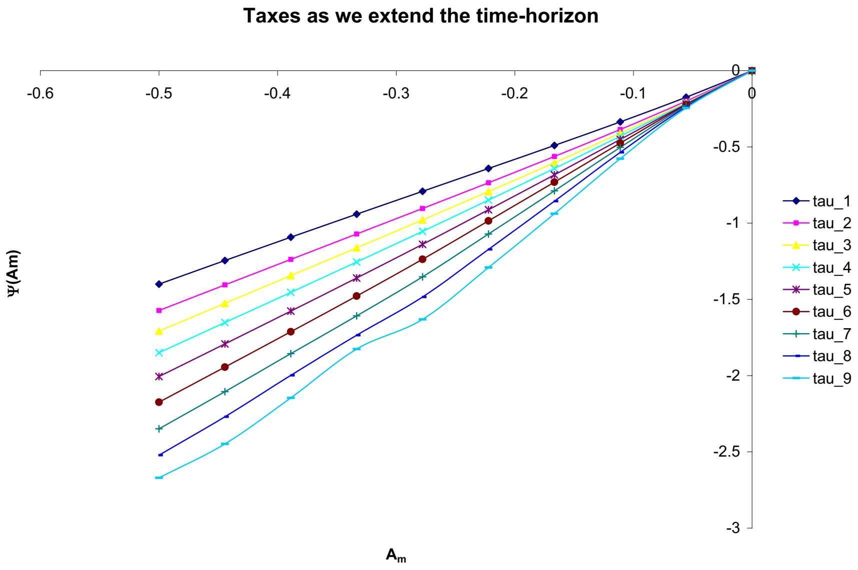

Figure 3: Taxes in the first period for a sequence of finite-horizon economies

time. Asset inequality changes between the first two periods in the case with commitment—inequality is reduced—but under lack of commitment, there is no change in asset inequality, despite the power of the median voter to subsidize labor income.

Open issues remain, particularly regarding the characterization of Markov-perfect equilibria. Even though we show that the present model, like the model of savings under quasi-geometric preferences, reduces to a simple-to-express functional equation—equation (14), which has one unknown function of one variable—it is surprisingly difficult to characterize equilibrium globally, even numerically. It is therefore an outstanding challenge to search for numerical methods aiding the analysis of Markov-perfect equilibria in this and similar kinds of models.

6 Appendix

6.1 Proof of Lemma

We have that

$$E(\tau) = f(\tau_0) + \sum_{t=1}^{\infty} f(\tau_t)\Pi_{k=0}^{t-1} q_k \tag{16}$$

and that

$$q_t = \beta \left(\frac{f(\tau_t)}{f(\tau_{t+1})} \right)^{\sigma} \left[\frac{1-\tau_t}{1-\tau_{t+1}} \right]^{(1-\alpha)(1-\sigma)}.$$

It follows that the discount factor $\Pi_{k=0}^{t-1} q_k$ equals

$$\beta \left(\frac{f(\tau_0)}{f(\tau_1)} \right)^{\sigma} \left[\frac{1-\tau_0}{1-\tau_1} \right]^{(1-\alpha)(1-\sigma)} \times \beta \left(\frac{f(\tau_1)}{f(\tau_2)} \right)^{\sigma} \left[\frac{1-\tau_1}{1-\tau_2} \right]^{(1-\alpha)(1-\sigma)} \times ... \times \beta \left(\frac{f(\tau_{t-1})}{f(\tau_t)} \right)^{\sigma} \left[\frac{1-\tau_{t-1}}{1-\tau_t} \right]^{(1-\alpha)(1-\sigma)}.$$

This can be simplified to

$$\Pi_{k=0}^{t-1} q_k = \beta^t \left(\frac{f(\tau_0)}{f(\tau_t)} \right)^{\sigma} \left[\frac{(1-\tau_0)}{(1-\tau_t)} \right]^{(1-\alpha)(1-\sigma)}.$$

Inserting this expression back into equation(16), we obtain

$$E(\tau) = f(\tau_0) + \sum_{t=1}^{\infty} \beta^t f(\tau_0)^{\sigma} (1-\tau_0)^{(1-\alpha)(1-\sigma)} \left[\frac{f(\tau_t)}{(1-\tau_t)^{(1-\alpha)}} \right]^{1-\sigma},$$

which can be reduced to

$$E(\tau) = f(\tau_0)^{\sigma}(1-\tau_0)^{(1-\alpha)(1-\sigma)} \sum_{t=0}^{\infty} \beta^t \left[\frac{f(\tau_t)}{(1-\tau_t)^{(1-\alpha)}} \right]^{1-\sigma}.$$

Since

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{f(\tau_t)}{(1-\tau_t)^{(1-\alpha)}} \right]^{1-\sigma} = \frac{V_0(\tau)}{\eta},$$

we can write the present value of net income and transfers as a function of the utility of the representative agent, i.e.,

$$E(\tau) = g(\tau_0)V_0(\tau),$$

where $g(\tau_0) = \eta^{-1} f(\tau_0)^{\sigma}(1-\tau_0)^{(1-\alpha)(1-\sigma)}$.

6.2 Proof of Proposition 4

As a first step it is useful to characterize the optimal tax sequence for the representative agent. Taxes must satisfy

$$\frac{\partial V_0}{\partial \tau_t} = \beta^t \eta \left[\frac{f(\tau_t)}{(1-\tau_t)^{(1-\alpha)}} \right]^{-\sigma} \frac{(1-\alpha)\alpha\tau_t}{(1-\tau_t)^{1-\alpha}(1-\alpha\tau_t)^2} = 0 \;\; \forall t.$$

This agent will thus prefer $\tau_t = 0 \forall t$ to any other sequence. Intuitively, this agent obtains no net redistribution (the taxes paid amount to the transfers received) and does not benefit from changing interest rates, since she has no assets. There is no gain from taxing or subsidizing labor, but a second-order loss generated by the distortion it generates on labor decisions. As a result, she finds it optimal to set taxes equal to zero. The median agent on the other hand, can gain from net redistributions and the manipulation of the interest rate sequence.

The indirect time-zero utility function of an agent with wealth a_{i0} can be written as a function of the mean agent's utility and the tax rate at time 0, as we have shown above. That is,

$$V_{i0}(\tau) = \left[1 + \frac{a_{i0}}{g(\tau_0)V_0(\tau)}\right]^{1-\sigma} V_0(\tau).$$

The first-order condition with respect to taxes at time $t \geq 1$ is

$$\frac{\partial V_{i0}}{\partial \tau_t} = \lambda_i(\tau)^{1-\sigma}\left[1 - \frac{(1-\sigma)a_i}{g(\tau_0)V_0(\tau)\lambda_i(\tau)}\right]\frac{\partial V_0(\tau)}{\partial \tau_t} = 0,$$

so the median will choose $\tau_t = 0$ for any $t \geq 1$.

We need to make sure that this is indeed a maximum. The second order condition evaluated at the optimum is

$$\frac{\partial^2 V_{i0}}{\partial \tau_t^2} = \lambda^{1-\sigma}\left[1 - \frac{(1-\sigma)a_i}{g(\tau_0)V_0(\tau)\lambda_i(\tau)}\right]\frac{\partial^2 V_0(\tau)}{\partial \tau_t^2}$$

Then $\frac{\partial^2 V_{i0}}{\partial \tau_t^2} < 0$ if and only if the term in brackets is positive. The latter will hold as long as the agent's initial asset holdings satisfy

$$a_{i0} > -\frac{g(\tau_0)V_0(\tau)}{\sigma}.$$

If $\sigma \leq 1$, this condition is met. In order for the competitive equilibrium allocations to be well defined we need the i's share of income $\lambda_i \geq 0$. But that translates exactly to the condition $a_{i0} > -g(\tau_0)V(\tau)$. Thus, this condition is met whenever $\sigma \leq 1$.

On the other hand, when $\sigma > 1$ the condition is not automatically satisfied. In this case, the second-order conditions might be violated if a_{i0} is too low, which imposes a restriction on the degree of initial inequality for which a zero tax policy will be voted in equilibrium.

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