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Abstract

We study discretionary equilibrium in the Calvo pricing model for a monetary authority that chooses the money supply, producing three main contributions. First, the model delivers a unique private-sector equilibrium for a broad range of parameterizations, in contrast to earlier results for the Taylor pricing model. Second, a generalized Euler equation shows how the monetary authority affects future welfare through its influence on the future state of the economy. Third, we provide exact solutions, including welfare analysis, for the transitional dynamics that occur if the monetary authority loses or gains the ability to commit.

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1 Introduction

Over the last two decades New Keynesian models have become the dominant framework for applied monetary policy analysis. This framework is characterized by optimizing private-sector behavior in the presence of nominal rigidities, typically Calvo (1983) pricing as described by Yun (1996). The fact that some prices are predetermined in these models leads to a time-inconsistency problem for monetary policy, and there is a vast literature studying aspects of discretionary, i.e. time-consistent, policy in New Keynesian models with Calvo pricing. While the typical practice, exemplified by Clarida, Gali and Gertler (1999) and Woodford (2003), has been to work with models approximated around a zero-inflation steady state, a growing literature studies the discretionary policy problem with global methods. This paper contributes to that literature in three ways. First, it shows that for a broad range of parameterizations, the Calvo model delivers a unique private-sector equilibrium when monetary policy is conducted without commitment, in contrast to earlier results for the Taylor model. Second, it derives a generalized Euler equation (GEE), as in Krusell, Kuruscu and Smith (2002) and Klein, Krusell and Rios-Rull (2008), and uses the GEE to decompose the dynamic policy tradeoffs facing a discretionary policymaker. Third, it conducts welfare analysis (without approximation) of the transitional dynamics that occur when a policymaker loses or gains the ability to commit.

The first contribution relates to an existing literature which has identified discretionary policy as a source of multiple equilibria. Private agents make decisions, such as saving or price setting, based on expectations of future policy. Those decisions in turn are transmitted to the future through state variables, creating the potential for a form of complementarity between future policy and expected future policy when policy is chosen under discretion. Viewed from another angle, the fact that policy will react to endogenous state variables can be a source of complementarity among private agents' actions. The link between discretionary policy and multiple equilibria has been especially prominent in the monetary policy literature. In particular, Khan, King and Wolman (2001) and King and Wolman (2004) show that in Taylor-style models with prices set for three and two periods respectively, multiple private-sector equilibria are pervasive under discretion.[1] Calvo and Taylor models are similar in many

[1] Albanesi, Chari and Christiano (2003) show that multiple equilibria arise under discretionary policy in a

ways, yet we find no evidence that discretionary policy generates equilibrium multiplicity in the Calvo model. Although we do not prove the uniqueness of discretionary equilibrium, we show that a policy analogous to the optimal policy in the Taylor model guarantees a unique equilibrium in the Calvo model, despite its greater potential for complementarity than the optimal discretionary policy. We trace the contrasting behavior of the two models to differences in how current pricing decisions affect the distribution of future predetermined prices, and how the future policymaker responds to that distribution.

Uniqueness of private-sector equilibrium opens up the possibility of deriving a GEE, which represents the dynamic tradeoff facing a discretionary policymaker in equilibrium. While the GEE has been extensively studied in fiscal policy applications, and recently extended to the Rotemberg sticky-price model by Leeper, Leith and Liu (2016), to the best of our knowledge it has not previously been derived for the Calvo model.[2] Under discretion, the policy problem is dynamic only to the extent that endogenous state variables affect future welfare, either directly or by shifting the future policymaker's problem. With Calvo price setting, an index of predetermined relative prices is an endogenous state variable. The GEE reveals three distinct channels through which this state variable links current policy to future welfare. First, the state variable affects welfare directly because it acts as an aggregate productivity shifter. Second, the state variable influences the future stance of policy, which affects consumption and leisure. Third, the state variable affects price setting in the future,

model in which a fraction of firms have predetermined prices. Siu (2008) extends King and Wolman's (2004) analysis and Barseghyan and DiCecio (2007) extend Albanesi, Chari and Christiano's (2003) analysis, by incorporating elements of state-dependent pricing and showing that Markov-perfect discretionary equilibrium is unique. Those papers assume that monetary policy is conducted with a money supply instrument. Dotsey and Hornstein (2011) show that with an interest rate instrument there is a unique Markov-perfect discretionary equilibrium in a Taylor model with two-period pricing.

[2] Our paper is closely related to Anderson, Kim and Yun (2010). They study optimal allocations without commitment in the Calvo model. Their approach cannot be used to investigate the possibility of multiple private-sector equilibria for a given policy action, or to derive a GEE. Their solution method, like ours, is based on Chebyshev collocation. While they study a slightly different region of the parameter space, the nature of their solutions is consistent with our findings. Ngo (2014) extends their analysis to a stochastic environment with the zero bound on nominal interest rates, and Leith and Liu (2016) use their approach to compare the Calvo and Rotemberg models.

which has an independent effect on welfare.

When any aspect of public policy suffers from a time-consistency problem, it is important to know the value of commitment. Thus, the third contribution of the paper is to provide exact solutions to the transitional dynamics that occur (i) when an economy that had been operating with optimal policy under commitment unexpectedly finds itself with a policymaker who cannot commit to future policy, and (ii) when an economy that had converged to a discretionary steady state unexpectedly finds itself with a policymaker that can commit to future policy. The first of these transitions is a straightforward analysis of the discretionary equilibrium studied in the first part of the paper. The second transition is more involved, as it requires also solving for the dynamics of optimal policy under commitment. In both cases, we find that the welfare loss (or gain) from the transition is quite close to the steady-state welfare difference between discretion and commitment.

The paper proceeds as follows. The next section contains a description of the Calvo model. Section 3 defines a discretionary equilibrium. Section 4 contains the numerical results for the discretionary equilibrium, emphasizing the issue of multiplicity or lack thereof. Section 5 describes the GEE approach. Section 6 presents the results on transitional dynamics and welfare. Section 7 relates our analysis to the early literature on discretionary monetary policy and concludes. Secondary material is contained in appendixes.

2 The Calvo model

The model is characterized by a representative household that values consumption and dislikes supplying labor, a money demand equation, a competitive labor market, a continuum of monopolistically competitive firms producing differentiated goods, and a monetary authority that chooses the money supply. Each firm faces a constant probability of price adjustment. We assume that the model's exogenous variables are constant, so there is no uncertainty about fundamentals.

2.1 Households

There is a large number of identical, infinitely-lived households. They act as price-takers in labor and product markets, and they own shares in the economy's monopolistically competitive goods-producing firms. Households' preferences over consumption (c_t) and labor input (n_t) are given by

$$\sum_{j=0}^{\infty} \beta^j \left[\ln(c_{t+j}) - \chi \frac{n_{t+j}^{1+\mu}}{1+\mu} \right], \;\; \beta \in (0,1)\,, \; \mu \geq 0, \; \chi > 0,$$

where consumption is taken to be the Dixit-Stiglitz aggregate of a continuum of differentiated goods with elasticity of substitution $\varepsilon > 1$,

$$c_t = \left[\int_0^1 c_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{1}$$

The consumer's flow budget constraint is

$$P_t w_t n_t + R_{t-1} B_{t-1} + \int_0^1 d_t\,(z)\,dz \geq P_t c_t + B_t,$$

where w_t is the real wage, R_t is the one-period gross nominal interest rate, B_t is the quantity of one-period nominal bonds purchased in period t, $d_t\,(z)$ is the dividend paid by firm z, and P_t is the nominal price of a unit of consumption. The aggregator (1) implies the demand functions for each good,

$$c_t\,(z) = \left[\frac{P_t\,(z)}{P_t} \right]^{-\varepsilon} c_t, \tag{2}$$

where $P_t\,(z)$ is the price of good z. The price index is given by

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}}. \tag{3}$$

From the consumer's intratemporal and intertemporal problems we have the efficiency conditions:

$$\begin{aligned} w_t &= \chi c_t n_t^{\mu} \qquad (4) \\ \frac{c_{t+1}}{c_t} &= \beta \left(\frac{R_t}{\pi_{t+1}} \right), \end{aligned}$$

where $\pi_t \equiv P_t / P_{t-1}$ denotes the gross inflation rate between periods $t-1$ and t. We assume there is a money demand equation such that the quantity of money is equal to the nominal value of consumption,

$$M_t = P_t c_t. \tag{5}$$

This constant-velocity money demand equation simplifies the model by abstracting from any distortions arising from money demand, and enables a straightforward comparison with the existing literature (e.g. King and Wolman, 2004). It will be convenient to write the money demand equation normalizing by the lagged price level, which serves as an index of the predetermined nominal prices in period t:

$$m_t \equiv \frac{M_t}{P_{t-1}} = \pi_t c_t. \tag{6}$$

We will refer to m_t as the normalized money supply.

2.2 Firms

Each firm $z \in [0,1]$ produces output $y_t(z)$ using a technology that is linear in labor $n_t(z)$, the only input, with a constant level of productivity that is normalized to unity:

$$y_t(z) = n_t(z).$$

A firm adjusts its price with constant probability $1-\alpha$ each period, as in Calvo (1983).[3] As firms are owned by households, the value of a firm upon adjustment is given by

$$\max_{X_t} \left\{ \sum_{j=0}^{\infty} (\alpha\beta)^j \left(\frac{P_t}{P_{t+j}}\right)\left(\frac{c_t}{c_{t+j}}\right)\left[X_t \left(\frac{X_t}{P_{t+j}}\right)^{-\varepsilon} c_{t+j} - P_{t+j} w_{t+j} \left(\frac{X_t}{P_{t+j}}\right)^{-\varepsilon} c_{t+j}\right]\right\}.$$

The factor α^j is the probability that a price set in period t will remain in effect in period $t+j$. The optimal price is determined by differentiating with respect to X_t. We will denote the profit-maximizing value of X_t by $P_{0,t}$ and we will denote by $p_{0,t}$ the nominal price $P_{0,t}$ normalized again by the previous period's price level, $p_{0,t} \equiv P_{0,t}/P_{t-1}$. Thus, we write the first-order condition as

$$\frac{P_{0,t}}{P_t} = \frac{p_{0,t}}{\pi_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{\sum_{j=0}^{\infty} (\alpha\beta)^j \left(P_{t+j}/P_t\right)^{\varepsilon} w_{t+j}}{\sum_{j=0}^{\infty} (\alpha\beta)^j \left(P_{t+j}/P_t\right)^{\varepsilon-1}}. \tag{7}$$

The real wage is equal to real marginal cost because firm-level productivity is assumed constant and equal to one. With the constant elasticity aggregator (1) a firm's desired markup of price over marginal cost is constant and equal to $\varepsilon/(\varepsilon-1)$. Because the firm cannot adjust its price each period, if the real wage or the inflation rate are not constant then the firm's markup will vary over time. The optimal pricing equation (7) indicates

[3] In Yun's (1996) version of the Calvo model there is price indexation, whereas the version in King and Wolman (1996) has no indexation. We analyze the Calvo model without indexation.

that the firm chooses a constant markup over an appropriately defined weighted average of current and future marginal costs. Note that the economy-wide average markup is simply the inverse of the real wage.

The optimal pricing condition can be written recursively by defining two new variables, S_t and F_t, that are related to the numerator and denominator of (7), respectively:

$$S_t = \pi_t^{\varepsilon} (w_t + \alpha\beta S_{t+1}), \tag{8}$$

$$F_t = \pi_t^{\varepsilon-1} (1 + \alpha\beta F_{t+1}), \tag{9}$$

then,

$$p_{0,t} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{S_t}{F_t}. \tag{10}$$

Because of Calvo pricing, the price index (3) is an infinite sum,

$$P_t = \left[\sum_{j=0}^{\infty} (1-\alpha)\, \alpha^j P_{0,t-j}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{11}$$

but it can be simplified, first writing it recursively and then dividing by the lagged price level:

$$\pi_t = \left[(1-\alpha)\, p_{0,t}^{1-\varepsilon} + \alpha\right]^{\frac{1}{1-\varepsilon}}. \tag{12}$$

Without loss of generality, we have normalized all period-t nominal variables by the period $t-1$ price level.

2.3 Market clearing

Goods market clearing requires that the consumption demand for each individual good is equal to the output of that good:

$$c_t(z) = y_t(z), \tag{13}$$

and labor market clearing requires that the labor input into the production of all goods equal the supply of labor by households:

$$\int_0^1 n_t(z)dz = n_t. \tag{14}$$

The labor market clearing condition is

$$n_t = \sum_{j=0}^{\infty} (1-\alpha)\, \alpha^j n_{j,t}, \tag{15}$$

where $n_{j,t}$ is the labor input employed in period t by a firm that set its price in period $t-j$. Combining this expression with the goods market clearing condition (13), then using the demand curves (2) for each good, and dividing the expression by the consumption aggregator yields

$$\frac{n_t}{c_t} = \sum_{j=0}^{\infty} (1-\alpha)\,\alpha^j \left(\frac{P_{0,t-j}}{P_t}\right)^{-\varepsilon}, \tag{16}$$

which can be written recursively as

$$\Delta_t = \pi_t^{\varepsilon}\left[(1-\alpha)\,p_{0,t}^{-\varepsilon} + \alpha\Delta_{t-1}\right], \tag{17}$$

where

$$\Delta_t \equiv n_t/c_t, \tag{18}$$

and we call Δ_{t-1} the inherited relative price distortion.

2.4 Monetary authority and timing

The monetary authority chooses the money supply, M_t. In a discretionary equilibrium the money supply will be chosen each period to maximize present-value welfare. We assume the sequence of actions within a period is as follows:

1. Predetermined prices ($P_{0,t-j}$, $j>0$) are known at the beginning of the period.
2. The monetary authority chooses the money supply.
3. Firms that adjust in the current period set their prices, and simultaneously all other period-t variables are determined.

Timing assumptions are important in models with staggered price setting. Transposing items 2 and 3 or assuming that firms and the monetary authority act simultaneously would change the nature of the policy problem and the properties of equilibrium.

3 Discretionary equilibrium in the Calvo model

We are interested in studying Markov-perfect equilibrium (MPE) with discretionary monetary policy. In an MPE, outcomes depend only on payoff-relevant state variables; trigger strategies and any role for reputation are ruled out. Hence, it is important to establish what the relevant state variables are. Although there are an infinite number of predetermined nominal prices ($P_{0,t-j}$, $j = 1, 2, ...$), for an MPE a state variable is relevant only if it affects the monetary authority's set of feasible real outcomes. It follows that in an MPE the normalized money supply and all other equilibrium objects are functions of the single state variable Δ_{t-1}. A discretionary policymaker chooses the money supply as a function of the state, taking as given the behavior of future policymakers. In equilibrium, the future policy that is taken as given is also the policy chosen by the current policymaker.

3.1 Equilibrium for arbitrary monetary policy

As a preliminary to studying discretionary equilibrium, it is useful to consider stationary equilibria for arbitrary monetary policy—that is, for arbitrary functions $m = \Gamma(\Delta)$. To describe equilibrium for arbitrary policy we use recursive notation, eliminating time subscripts and using a prime to denote a variable in the next period. The nine variables that need to be determined in equilibrium are S, F, p_0, π, Δ', c, n, w, and m, and the nine equations are the laws of motion for S (8) and for F (9); the optimal pricing condition (10); the price index (12); the law of motion for the relative price distortion (17); the labor supply equation (4); money demand (6); the definition of the relative price distortion (18); and the monetary policy rule $m = \Gamma(\Delta)$.

A stationary equilibrium can be expressed as two functions of the endogenous state variable. The two functions $S(\Delta)$ and $F(\Delta)$ must satisfy the two functional equations

$$S(\Delta) = \pi^{\varepsilon}\left[w + \alpha\beta S(\Delta')\right], \tag{19}$$

$$F(\Delta) = \pi^{\varepsilon-1}\left[1 + \alpha\beta F(\Delta')\right], \tag{20}$$

where the other variables are given recursively by the following functions of Δ:

$$p_0 = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{S(\Delta)}{F(\Delta)}, \tag{21}$$

$$\pi = \left[(1-\alpha)p_0^{1-\varepsilon} + \alpha\right]^{1/(1-\varepsilon)}, \tag{22}$$

$$\Delta' = \pi^{\varepsilon}\left[(1-\alpha)p_0^{-\varepsilon} + \alpha\Delta\right], \tag{23}$$

$$c = \frac{m}{\pi} \tag{24}$$

$$n = \Delta' c, \tag{25}$$

$$w = \chi c n^{\mu}. \tag{26}$$

Given an arbitrary policy of the form $m = \Gamma(\Delta)$, functions $S()$ and $F()$ that satisfy (19)−(26) represent a stationary equilibrium.

3.2 Discretionary equilibrium defined

A discretionary equilibrium is a particular stationary equilibrium with policy given by a mapping from the state to the money supply, $m = \Gamma^*(\Delta)$, in which the following property holds: If the current-period policymaker and current-period private agents take as given that all future periods will be described by a stationary equilibrium associated with $\Gamma^*(\Delta)$, then the current-period monetary authority maximizes welfare by choosing $m = \Gamma^*(\Delta)$ for every Δ.

More formally, a discretionary equilibrium is a policy function $\Gamma^*(\Delta)$ and a value function $v^*(\Delta)$ that satisfy

$$\begin{aligned} v^*(\Delta) &= \max_m \left\{ \ln(c) - \chi \frac{n^{1+\mu}}{1+\mu} + \beta v(\Delta') \right\} \\ \Gamma^*(\Delta) &= \arg\max_m \left\{ \ln(c) - \chi \frac{n^{1+\mu}}{1+\mu} + \beta v(\Delta') \right\} \end{aligned} \tag{27}$$

when $v() = v^*()$. The maximand in (27) can be seen to be a function of m by combining (22)−(26) with optimal pricing by adjusting firms,

$$p_0 = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{\pi^{\varepsilon}\left[w + \alpha\beta S(\Delta')\right]}{\pi^{\varepsilon-1}\left[1 + \alpha\beta F(\Delta')\right]}, \tag{28}$$

where the functions $S()$ and $F()$ satisfy (19) and (20) in the stationary equilibrium associated with $\Gamma^*(\Delta)$. Note the subtle difference between (28) and (19)−(21): in (28), which applies in the current period, we have not imposed a stationary equilibrium. The monetary authority takes as given that the future will be described by a stationary equilibrium. It is an equilibrium outcome, not a constraint, that current-period policy is identical to that which generates the stationary equilibrium in the future.

4 Properties of discretionary equilibrium

We use a projection method to compute numerical solutions for discretionary equilibrium, restricting attention to equilibria that are limits of finite-horizon equilibria. This restriction may further reduce the number of discretionary equilibria (Krusell, Kuruscu and Smith, 2002), and allows us to derive a useful analytical result for the case of a (suboptimal) monetary policy that holds m constant. Computational details are provided in Appendix A. The quarterly baseline calibration is common in the applied monetary policy literature: $\varepsilon = 10$, $\beta = 0.99$, $\alpha = 0.5$, $\mu = 0$, $\chi = 4.5$. Prices remain fixed with probability $\alpha = 0.5$, which means that the expected duration of a price is two quarters. The demand elasticity $\varepsilon = 10$ implies a desired markup of approximately 11 percent. With $\mu = 0$ labor supply is perfectly elastic. Given the values for ε and μ, $\chi = 4.5$ is chosen to target a steady-state level of labor in the flexible-price economy of $n = 0.2$. The baseline calibration is chosen to facilitate comparison with King and Wolman (2004), but many other examples were computed that cover a wide range of structural parameter values.

There are two levels to a complete description of a discretionary equilibrium. First, the equilibrium is characterized by the value function, $v^*(\Delta)$ and the associated monetary policy function, $m = \Gamma^*(\Delta)$, along with the transition function for the state variable and policy functions for the other endogenous variables. Second, for given values of the state variable and the normalized money supply, private-sector equilibrium involves the fixed point of a pricing best-response function. We use the best-response function to study uniqueness.

4.1 Equilibrium functions

Figure 1, Panel A plots the transition function for the state variable as well as the function mapping from the state to the inflation rate in a discretionary equilibrium.[4] The first thing to note is that there is a unique steady-state inflation rate of 5.5 percent annually.[5] Two natural benchmarks against which to compare the steady state of the discretionary equilibrium are the inflation rate with highest steady-state welfare and the inflation rate in the long run

[4] Note that in the model π is a gross quarterly inflation rate, but the figures and the text refer to annualized net inflation rates obtained as $100(\pi^4 - 1)$ percent.

[5] The steady-state inflation rate reaches the range of 9-10 percent in the Calvo model when ε is lower than the baseline value, or when α is larger but not too large. For instance, reducing the demand elasticity to $\varepsilon = 8$, which implies a desired markup of 14 percent, raises the steady-state inflation rate to 9.6 percent. Increasing the probability of no price adjustment to $\alpha = 0.71$ raises the steady-state inflation rate to 9.8 percent, and it declines for even larger values of α. Anderson, Kim and Yun (2010) point out similar relationships between the model's structural parameters and the steady-state inflation rate.

under optimal policy with commitment. Following King and Wolman (1999), we refer to these benchmarks as the golden rule and the modified golden rule respectively. For our baseline parameterization, the golden-rule inflation rate is just barely positive (less than one tenth of a percent) and the modified golden-rule inflation rate is zero. The latter result is parameter-independent; we return to it in Section 6.1.

In addition to showing the steady state, Panel A illustrates the dynamics of the state variable, which exhibit monotonic convergence to the steady state. This means that a policymaker inheriting a relative price distortion that is large relative to steady state finds it optimal to bequeath a smaller relative price distortion to her successor. Together with the monotone downward-sloping equilibrium function for inflation, it follows that the inflation dynamics in the transition from a large relative price distortion (as would be implied by a high inflation rate) involve an initial discrete fall in inflation and a subsequent gradual increase to the steady state.[6]

Panel B of Figure 1 displays the policy variable (m) and welfare (v) as functions of the state variable in the discretionary equilibrium (m is plotted on the left scale and welfare on the right scale).[7] Both functions are downward sloping. Intuition for the welfare function's downward slope is straightforward. By definition, the current relative price distortion represents the inverse of average productivity. But the current relative price distortion is also a summary statistic for the dispersion in relative prices. The higher is the inherited relative price distortion, the higher is the inherited dispersion in relative prices, and through (23) this contributes to a higher dispersion in current relative prices. Higher dispersion in current relative prices in turn reduces current productivity, reducing welfare.

It is less straightforward to understand the downward sloping policy function, $m = \Gamma^* (\Delta)$. At first glance, it seems consistent with the state transition function for m to be decreasing in Δ: if equilibrium involves the relative price distortion declining from a high level, then a large inherited relative price distortion ought to be met with a relatively

[6] Yun's (2005) analysis of the Calvo model with a subsidy to offset the markup distortion displays similar transition dynamics of inflation. But in his model, the steady-state inflation rate under optimal policy is zero, so the transition from a steady state with positive inflation inevitably involves a period of deflation.

[7] In Panel B of Figure 1 we have not converted welfare into more meaningful consumption-equivalent units. We defer a quantitative discussion of welfare to Section 6.

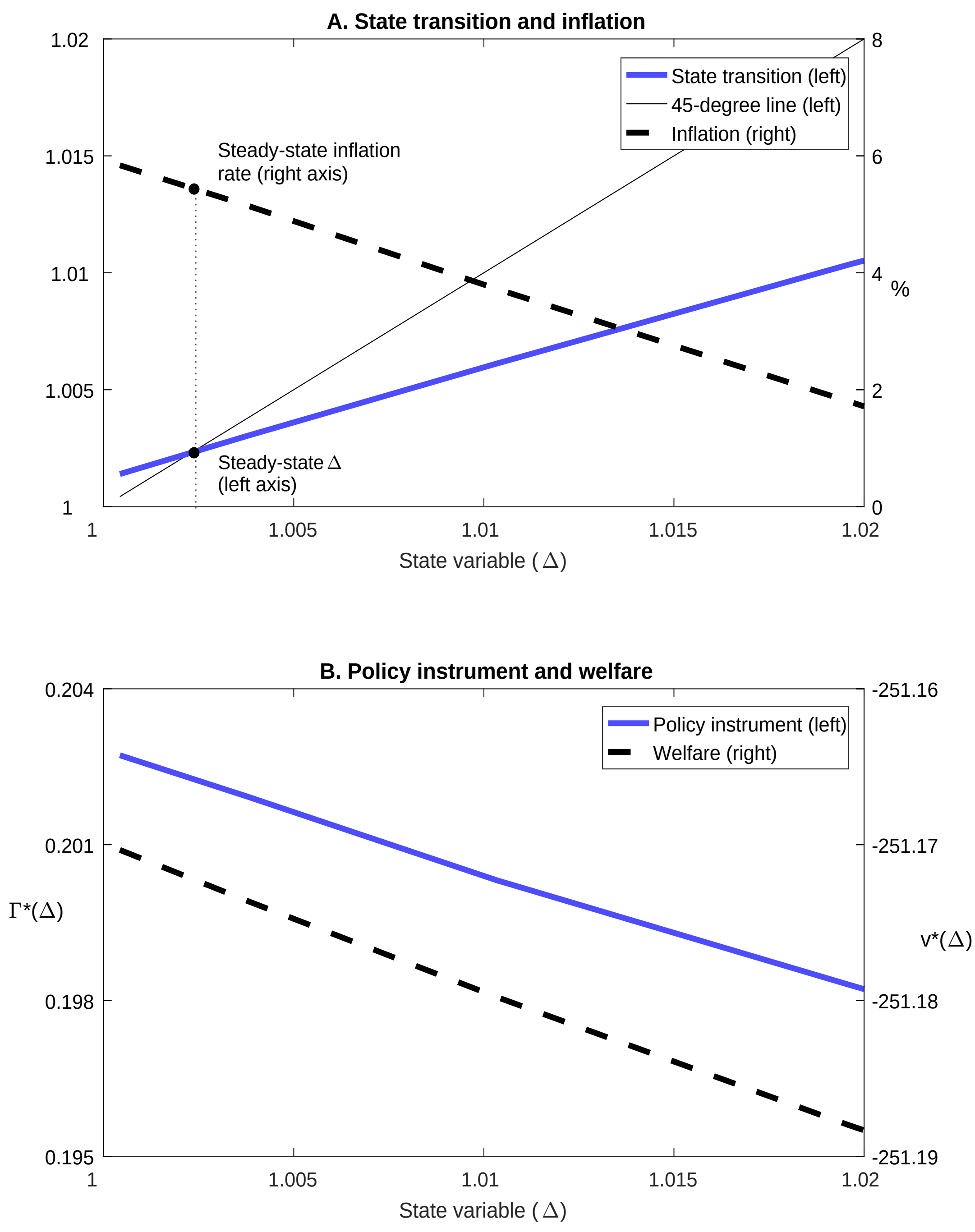

Figure 1: Equilibrium as a function of the state

low normalized money supply, so that newly adjusting firms do not exacerbate the relative price distortion. Looking in more detail, the essential short-run policy trade-off is that the policymaker has an incentive to raise the money supply in order to bring down the markup, but this incentive is checked by the cost of increasing the relative price distortion. It appears that the short-run trade-off shifts in favor of the relative price distortion as the state variable increases. That is, in equilibrium the policymaker chooses lower m at larger values of Δ because the value of the decrease in the markup that would come from holding m fixed at higher Δ is more than offset by welfare costs of a higher relative price distortion.[8]

4.2 Private-sector equilibrium

Our computational approach has led to finding a single discretionary equilibrium. The preceding discussion highlighted some of the properties of the equilibrium for the baseline calibration. Although we have not proved that the equilibrium is unique, in the many other examples described in Appendix A we have found no evidence of multiple equilibria. This is in stark contrast to the Taylor model with two-period price setting, in which King and Wolman (2004) *proved* the existence of multiple discretionary equilibria, which they traced to multiple private-sector equilibria.

To help explain why multiplicity of private-sector equilibrium does not appear in any of our numerical solutions for the Calvo model, we turn to the best-response function for price-setting firms. The best-response function describes an individual firm's optimal price as a function of the price set by other adjusting firms. Figure 2 plots a typical best-response function in a discretionary equilibrium of the Calvo model, using the baseline calibration. It has a unique fixed point, and is concave in a neighborhood of the fixed point. In contrast, the best-response function in the two-period Taylor pricing model is upward sloping, strictly convex and generically has either two fixed points or no fixed points (see King and Wolman, 2004, Figure I).[9]

[8] In the two-period Taylor model, the policymaker also faces a short-run trade-off between reducing the markup and increasing the relative price distortion, but does not face a cost of leaving a higher inherited price distortion to future policymakers. Appendix B provides a description of the Taylor model and a quantitative comparison with the Calvo model.

[9] Our computations have not revealed multiple fixed points in equilibrium. However, we have encountered

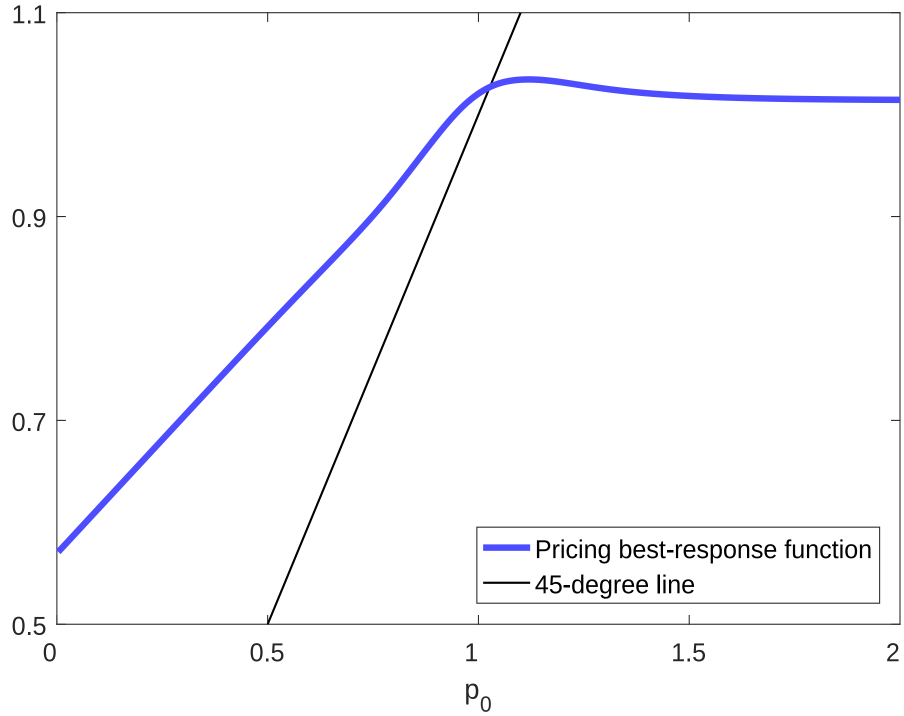

Figure 2: Pricing best-response function: State = 1.004, $m = 0.202$

The starkly different best-response functions in the two models reflect differences in how future monetary policy reacts to the price firms set in the current period. This relationship is linear in the Taylor model: the price set in the current period (P_0) is precisely the index of predetermined nominal prices that normalizes the future money supply, and the normalized money supply is constant in discretionary equilibrium. The relationship is nonlinear in the Calvo model, for two reasons. First, the relationship between P_0 and the future index of predetermined prices (P) is nonlinear. Second, P_0 affects the real state variable to which future policy responds (Δ'). We consider in turn how both these factors weaken the complementarity in price setting.

First, suppose the future policymaker were to set a constant m, raising the nominal money supply in proportion to the index of predetermined prices. In the Taylor model, where such a policy is optimal, the price set by adjusting firms *is* the index of predetermined prices, so the future nominal money supply rises linearly with the price set by adjusting firms. Understanding that this future policy response will occur, and that the price it sets today will also be in effect in the future, an individual firm's best response is to choose a higher price when all other adjusting firms choose a higher price.

In the Calvo model, in contrast, next period's index of predetermined prices comprises an infinite number of lagged prices, of which the price set by adjusting firms today is just one element. Under a constant-m policy, the effect of an increase in prices set today on next period's nominal money supply depends on the effect of such an increase on next period's index of preset prices. That index of preset prices—today's price index—is highly sensitive to low levels of the price set by firms today and relatively insensitive to high levels of the price set by firms today, because goods with higher prices have a lower expenditure share and thus receive a smaller weight in the price index. As the price set by firms goes to infinity, it has no effect on the index of preset prices and no effect on tomorrow's nominal money supply.

Thus, in the Calvo model a constant-m policy would lead to a nominal money supply

instances of multiple fixed points for sub-optimal values of m. In Figure 2 there is a convex region of the best-response function to the left of the fixed point. In the case of multiple fixed points, the convex region of the best-response function intersects the 45-degree line twice, with a third fixed point located on the concave portion.

that is increasing and concave in the price set by adjusting firms. Because a higher future money supply leads firms to set a higher price today, concavity of the future money supply corresponds to decreasing complementarity of the prices set by adjusters. This intuition is confirmed by the following result, which applies to our baseline calibration.

Proposition 1 *Suppose the money supply is always set according to a constant-m policy, regardless of the state, and let $\mu = 0$. Then the Calvo model has a unique private-sector equilibrium.*

Proof. *See Appendix C.* ■

The second reason for weaker complementarity in the Calvo model is that the relationship between the price set by adjusting firms and the future nominal money supply depends on the future state variable. Indeed, the policy maker does not hold m constant, instead lowering it with the state (see Figure 1.B). The response of next period's normalized money supply to the price set by adjusting firms today therefore depends on the relationship between p_0 and Δ'. Combining the market clearing condition (23) with the transformed price index (22) yields

$$\Delta' = \frac{(1-\alpha)p_0^{-\varepsilon} + \alpha\Delta}{\left[\alpha + (1-\alpha)p_0^{1-\varepsilon}\right]^{\varepsilon/(\varepsilon-1)}},$$

which implies that for high (low) values of p_0 the future state is increasing (decreasing) in p_0, holding fixed the current state:

$$\frac{\partial \Delta'}{\partial p_0} = \frac{\varepsilon\alpha\,(1-\alpha)\,p_0^{-\varepsilon-1}}{\left[\alpha + (1-\alpha)\,p_0^{1-\varepsilon}\right]^{1+[\varepsilon/(\varepsilon-1)]}}\left(\Delta p_0 - 1\right). \tag{29}$$

Given that equilibrium m is decreasing in Δ, future m is decreasing in p_0 for high values of p_0 and increasing in p_0 for low values of p_0. That is, a higher price set by adjusting firms—if it is greater than $1/\Delta$—translates into a higher value of the future state, and thus a lower value of the future normalized money supply.[10]

[10]This relationship is reversed at low values of p_0: increases in p_0 reduce the future state, and the policymaker would respond by raising future m. Such low values of p_0 are not relevant for understanding the properties of equilibrium however, because they are associated with suboptimally low values of m. Indeed, if m were low enough that raising m would reduce both the markup and the relative price distortion, there would be no policy trade-off and the policymaker would choose a higher m.

Summarizing the argument: in the Taylor model the normalized money supply is constant in equilibrium, and this results in an increasing convex best-response function with multiple fixed points. In the Calvo model, if policy kept the normalized money supply constant there would be a unique equilibrium: complementarity would be weaker at high p_0 than in the Taylor model, because next period's index of predetermined prices responds only weakly to p_0 at high levels of p_0. Because the normalized money supply is not constant in the Calvo model, the complementarity is weakened even further; m is decreasing in the state, and future m is decreasing in p_0 for high p_0. As both parts of this argument rely on the fact that there are many cohorts of firms with predetermined prices, this feature appears key to explaining why the Calvo model does not have the same tendency toward multiple discretionary equilibria as the Taylor model with two-period pricing.[11]

Although we have not proved uniqueness of equilibrium, our computations have found only one equilibrium in every case, and Proposition 1 gives us confidence that the numerical results do generalize: the constant-m policy, which is key to proving that there are multiple private sector equilibria in the Taylor model, implies a unique private sector equilibrium in the Calvo model. If, as we suppose, MPE is unique, the nature of the equilibrium ought to be invariant to (i) the policy instrument and (ii) whether we use an alternative approach to solving the policy problem, either by solving the GEE or solving the planner's problem as in Anderson, Kim and Yun (2010). For our baseline parameterization we have confirmed that the same steady-state inflation rate obtains whether the policy instrument is the money supply or the nominal interest rate. In addition, we have replicated the steady-state inflation rate of 2.2 percent for Anderson, Kim and Yun's baseline case with $\alpha = 0.75$, $\varepsilon = 11$, and $\mu = 1$, for both interest rate and money supply instruments. Finally, we have computed equilibrium for our benchmark example using the GEE approach, which we discuss in the next section.

[11] This reasoning suggests, however, that a Taylor model with longer duration pricing might not have multiplicity, because the same opportunities to substitute would be present. Khan, King and Wolman (2001) find multiplicity is still present with three-period pricing. Unfortunately, it is computationally infeasible to study discretionary equilibrium in a Taylor model with long-duration pricing.

5 Generalized Euler equation approach

Until this point, we have been careful to allow for the possibility of multiple private-sector equilibria. This has meant eschewing a first-order approach to the policy problem, as we needed to check for uniqueness of private-sector equilibria for all feasible values of m. For the broad range of parameter values that we have studied, however, we have found that private-sector equilibrium is always unique at the optimal choice of m. Therefore, the first-order approach described by Krusell, Kuruscu and Smith (2002) and Klein, Krusell and Rios-Rull (2008, henceforth KKR) is appropriate for our problem, ought to yield equivalent results to those described above, and may provide additional insight into the nature of equilibrium. In this section we follow the approach of KKR for deriving the policymaker's GEE.

5.1 Discretionary equilibrium restated

To derive the GEE, we continue to view the policymaker as choosing the normalized money supply as a function of the state, taking as given the private sector's equilibrium response. We now assume the monetary policy function is differentiable and private-sector equilibrium is unique.

We reformulate the definition of discretionary equilibrium to make it more convenient for deriving the GEE. Some of the functions used in the derivation are known functions of p_0, m and Δ. In particular, the functional forms for consumption, labor input, and the relative price distortion are readily obtained from combining Eqs. (22)$-$(25), so we use the shorthand notation $c = C(p_0, m)$, $n = N(\Delta, p_0, m)$, and $\Delta' = D(\Delta, p_0)$ for these functions. Current utility is given by $u(c, n) = \ln(c) - \chi n^{1+\mu}/(1+\mu)$.

Discretionary equilibrium consists of a value function v, a monetary policy function Γ, and a pricing function h such that for all Δ, $m = \Gamma(\Delta)$ solves[12]

$$\max_{m} \{u(C(p_0, m), N(\Delta, p_0, m)) + \beta v(D(\Delta, p_0))\},$$

$p_0 = h(\Delta)$ satisfies the optimality condition for the price chosen by adjusting firms

$$p_0 [1 + \alpha\beta F(D(\Delta, p_0))] = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \left[(1-\alpha) p_0^{1-\varepsilon} + \alpha\right]^{\frac{1}{1-\varepsilon}} \left[-\frac{u_n}{u_c} + \alpha\beta S(D(\Delta, p_0))\right], \quad (30)$$

[12] We denote the value function and policy function in discretionary equilibrium by v and Γ, dropping for simplicity the asterisk notation used in Section 3.

and $v()$ is given by

$$v(\Delta) \equiv u(C(h(\Delta), \Gamma(\Delta)), N(\Delta, h(\Delta), \Gamma(\Delta))) + \beta v(D(\Delta, h(\Delta))). \tag{31}$$

In (30), $S()$ and $F()$ are the same functions that were defined in (19) and (20). Under our assumptions of differentiability and uniqueness, this description of equilibrium is equivalent to that in Section 3.

5.2 The GEE

We derive a simplified representation of the policymaker's first-order condition by using the envelope condition to eliminate the derivative of the value function, as in KKR. To that end, we first define the firm's "pricing wedge" $\eta(\Delta, m, p_0)$, which is the (out-of-equilibrium) deviation from the optimal price-setting condition:

$$\begin{aligned}\eta(\Delta, m, p_0) &= p_0[1 + \alpha\beta F(D(\Delta, p_0))] \\ &\quad - \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left[(1-\alpha) p_0^{1-\varepsilon} + \alpha\right]^{1/(1-\varepsilon)}\left[-\frac{u_n}{u_c} + \alpha\beta S(D(\Delta, p_0))\right].\end{aligned}$$

Following KKR, given an equilibrium (v, Γ, h) and under some regularity conditions, the implicit function theorem guarantees that there exists a unique function $H(\Delta, m)$, defined on some neighborhood of the steady state, satisfying $\eta(\Delta, m, H(\Delta, m)) \equiv 0$ in that neighborhood. The function H gives the price if the current state is Δ, current money is m, and price-setting firms expect that future money will be determined by the equilibrium policy function Γ. Thus, H describes the private sector's response to a one-time deviation of monetary policy from the equilibrium policy. Continuing as in KKR, using this definition, which implies that $H_m = -\eta_m/\eta_{p_0}$ and $H_\Delta = -\eta_\Delta/\eta_{p_0}$, the first-order condition for the monetary authority is

$$u_c[C_{p_0} H_m + C_m] + u_n[N_{p_0} H_m + N_m] + \beta v'_\Delta D_{p_0} H_m = 0. \tag{32}$$

Note that prime always denotes the next period, never derivative.

The next step is to get an expression for v'_Δ. Begin by differentiating (31) with respect to Δ, replacing $h(\Delta)$ with $H(\Delta, m)$, using the fact that in equilibrium $h(\Delta) = H(\Delta, \Gamma(\Delta))$:

$$\begin{aligned}v_\Delta &= u_c(C_{p_0}(H_\Delta + H_m\Gamma_\Delta) + C_m\Gamma_\Delta) \\ &\quad + u_n(N_\Delta + N_{p_0}(H_\Delta + H_m\Gamma_\Delta) + N_m\Gamma_\Delta) + \beta v'_\Delta(D_\Delta + D_{p_0}(H_\Delta + H_m\Gamma_\Delta)).\end{aligned}$$

From the first-order condition (32) we have

$$\beta v'_\Delta = \frac{-u_c\left[C_{p0}H_m + C_m\right] - u_n\left[N_{p0}H_m + N_m\right]}{D_{p0}H_m}.$$

So the derivative of the value function can be written as

$$v_\Delta = u_n N_\Delta - \frac{\eta_\Delta}{\eta_m}\left(u_c C_m + u_n N_m\right) - \frac{D_\Delta}{D_{p0}}\left[u_c C_{p0} + u_n N_{p0} - \frac{\eta_{p0}}{\eta_m}\left(u_c C_m + u_n N_m\right)\right]. \quad (33)$$

This derivative, the change in equilibrium welfare with respect to a change in the inherited relative price dispersion, consists of three terms. The first term represents the direct effect of a change in the state on current utility. The second term, in parentheses, reflects the fact that even if current pricing behavior does not change, a change in the state variable requires a change in the current money supply in order for the firm's optimality condition to hold. The factor $(-\eta_\Delta/\eta_m)$ represents the change in the money supply with respect to a change in the state along the first-order condition for pricing. Of course, in equilibrium pricing decisions do respond to the state variable; the third term, in brackets, takes care of this effect. When p_0 changes there is a direct effect on utility. There is also an indirect effect because the change in pricing must correspond to a change in the money supply in order for the firms' optimality condition to hold, which explains the factor $-\eta_{p0}/\eta_m$.

Pushing (33) one period forward, we use it to eliminate the value function derivative from the monetary authority's first-order condition (32), therefore writing that first-order condition as a GEE:

$$\Theta + \beta H_m D_{p0}\left[u'_n N'_\Delta - \frac{\eta'_\Delta}{\eta'_m}\left(u'_c C'_m + u'_n N'_m\right) + \frac{D'_\Delta}{D'_{p0}}\frac{\eta'_{p0}}{\eta'_m}\Theta'\right] = 0, \quad (34)$$

where

$$\Theta \equiv u_c C_m + u_n N_m + H_m\left(u_c C_{p0} + u_n N_{p0}\right).$$

The GEE states that in equilibrium, a marginal change in the current money supply leaves welfare unchanged. The variable Θ represents the change in current utility with respect to a change in the current money supply. The term in brackets, v'_Δ from (33), consists of the three effects on future welfare discussed above: the direct effect from a change in the future state variable, the effect of a change in the future money supply associated with the future state, and the effect of a change in the optimal price associated with the future state. The coefficient on future marginal value, $\beta H_m D_{p_0}$ represents discounting and the mapping from a change in current m to a change in the future state.

The GEE highlights the lack of commitment. The optimality condition (34) for the current policymaker incorporates the response of future policy to the endogenous state variable, captured by the terms in C'_m, N'_m, and H'_m of which some are contained in Θ'. This contrasts with the case of commitment, where future policy would not respond to the state variable. The effect of current policy on future welfare directly through the state variable would also be present with commitment, although it would be quantitatively different because the state variable embeds expectations about future policy.

In addition to its analytical value, the GEE can be used as the basis for an alternative approach to computing equilibrium. In a reassuring check on our work above, using the GEE approach we computed an identical steady-state inflation rate of 5.5 percent to that reported in Section 4, although away from steady state the equilibrium differed slightly.[13]

6 Transitions to and from discretion

The relatively high steady-state inflation rate under discretion raises the questions of the cost and benefit, respectively, of losing and gaining the ability to commit. One way to measure these objects is by comparing the steady-state levels of welfare under commitment and discretion. However, both empirical and theoretical considerations suggest that the steady state comparison is inappropriate. Empirically, large changes in the inflation rate rarely occur instantaneously. For example, the famous Volcker disinflation played out over a period of at least three years. Theoretically, we have emphasized the presence of a state variable in the discretionary equilibrium, and commitment induces additional policy inertia, as emphasized by Woodford (2003). Answering these questions, then, means studying the transitional dynamics. Loss of ability to commit involves the transitional dynamics under discretion, starting from the steady state under commitment. Acquisition of ability to commit involves the transitional dynamics under commitment, starting from the steady state under discretion.

[13] In an application to fiscal policy, Azzimonti, Sarte and Soares (2009) also find that different computational approaches produce identical steady states under discretionary policy, but somewhat different dynamics.

Thus, we need to know the steady states and transitional dynamics under both commitment and discretion.

6.1 Optimal allocations with commitment

In considering optimal policy under commitment, we set aside the issue of implementation and solve a social planner's problem. That is, we consider the problem of a planner who can choose current and future prices and quantities, subject to the conditions that characterize optimal behavior by households and firms, and subject to markets clearing. The planner's problem can be written as

$$\max_{\left\{c_t, n_t, w_t, \pi_t, \tilde{S}_t, \tilde{F}_t, p_{0,t}, \Delta_t\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\ln(c_t) - \chi \frac{n_t^{1+\mu}}{1+\mu}\right],$$

subject to the following constraints, for $t = 0, 1, \ldots$:

$$\begin{aligned}
p_{0,t} &= \left(\frac{\varepsilon}{\varepsilon - 1}\right) \pi_t \frac{\tilde{S}_t}{\tilde{F}_t} && (35) \\
\tilde{S}_t &= w_t + \alpha\beta\pi_{t+1}^{\varepsilon} \tilde{S}_{t+1} && (36) \\
\tilde{F}_t &= 1 + \alpha\beta\pi_{t+1}^{\varepsilon-1} \tilde{F}_{t+1}. && (37) \\
\pi_t &= \left[(1-\alpha) p_{0,t}^{1-\varepsilon} + \alpha\right]^{\frac{1}{1-\varepsilon}} && (38) \\
\Delta_t &= \pi_t^{\varepsilon} \left[(1-\alpha) p_{0,t}^{-\varepsilon} + \alpha\Delta_{t-1}\right] && (39) \\
n_t &= \Delta_t c_t && (40) \\
w_t &= \chi c_t n_t^{\mu} && (41)
\end{aligned}$$

These constraints are each familiar from the description of private-sector equilibrium above. However, the optimal pricing condition is transformed slightly, with $\tilde{S}_t \equiv S_t/\pi_t^{\varepsilon}$ and $\widetilde{F}_t \equiv F_t/\pi_t^{\varepsilon-1}$, to reduce the number of state variables in the planning problem. Recall that without commitment, there was a single state variable, Δ_{t-1}. With commitment, the presence of future realizations of variables in the constraints means that there are two additional state variables, the lagged multipliers on the constraints (36) and (37). The first-order conditions for this problem can be simplified to a system of nine nonlinear difference equations in the nine variables $\left\{c_t, \tilde{S}_t, \tilde{F}_t, \pi_t, \Delta_t, \phi_t, \psi_t, \xi_t, \gamma_t\right\}$, where ϕ_t, ψ_t, ξ_t, and γ_t are the Lagrange multipliers on (36), (37), (38), and (39). The nine-equation system is derived in Appendix D.

Solving for the steady state under discretion required us to solve for the functions describing equilibrium dynamics. Under commitment the steady state is simply the time-invariant solution to the nine-equation system implied by the planner's first-order conditions. As shown in Appendix D, the steady-state inflation rate is zero; that is, $\pi = \Delta = p_0 = 1$.[14]

Comparing the steady states under commitment and discretion gives a rough estimate of the cost (benefit) of losing (gaining) the ability to commit to future policies. Using the baseline calibration, both consumption and leisure are slightly higher in the commitment steady state, with zero inflation, than in the discretionary steady state, with 5.5 percent inflation. The welfare difference between the two steady states is equivalent to 0.228 percent of consumption every quarter.[15] In present value terms, the consumption increment represents 5.70 percent of annual consumption. Of course, this calculation is artificial in the sense that the economy cannot simply jump from one steady state to the other. Next we analyze the transitions between steady states.

6.2 Losing the ability to commit

If an optimizing policymaker loses the ability to commit, then the economy behaves according to the transitional dynamics under discretion with an initial condition of $\Delta = 1$, the steady state under commitment. These dynamics can be inferred from Figure 1, but we plot them explicitly in Figure 3. The solid lines display the paths of inflation, the relative price distortion, the markup, and the money growth rate along the transition to the discretionary steady state. Whereas the relative price distortion monotonically increases along the transition, the inflation rate jumps in the initial period (labeled zero) and then smoothly declines to the discretionary steady state. Panel D shows that the money growth rate essentially

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[14] We use the term "steady state" informally in the case of a policymaker with commitment. It is more accurate to refer to this allocation as the limit point in the long run under commitment. A planner who inherited only the state variable $\Delta = 1$ would choose some initial inflation before converging in the long run to zero inflation and $\Delta = 1$; this is the nature of the time-consistency problem.

[15] The welfare calculation involves comparing the discretionary steady state to an allocation on the same indifference curve as the commitment steady state, but with the same wage as the discretionary steady state. The number 0.228 percent represents the parallel rightward shift in the budget constraint (consumption on the horizontal axis).

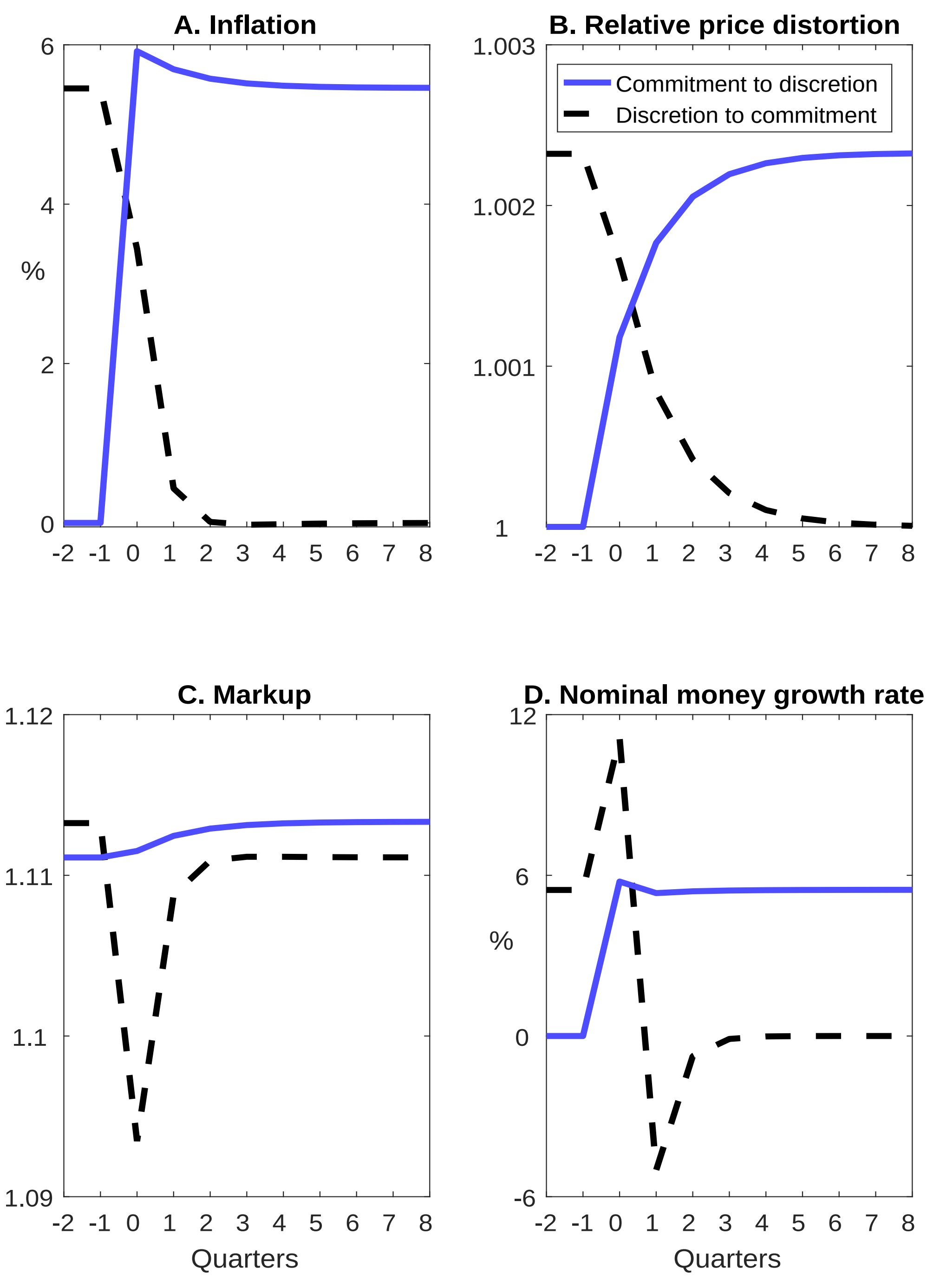

Figure 3: Transitions to and from discretion

mimics the inflation rate. In present value terms, the welfare decline associated with loss of commitment is well-approximated by the steady-state welfare comparison: the representative household would be willing to forego 0.225 percent of consumption each quarter in order to avoid this transition (5.62 percent in present-value terms).

It is notable that the transition to the discretionary steady state does not involve any transitory benefits: along the entire transition both the markup and the relative price distortion are increasing. One might have expected that in the initial period of the transition, the policymaker could effectively exploit preset prices and reduce the markup. It is indeed the case that the markup for non-adjusting firms falls substantially in the initial period. However, this decline is more than offset by an increase in the inflation rate that results from the behavior of adjusting firms. This reasoning uses the identity $P_t/MC_t = (P_t/P_{t-1}) \times (P_{t-1}/MC_t)$. Although the equilibrium path involves the markup rising, it is nonetheless the case that at each point in time the policymaker perceives a tradeoff between reducing the markup and increasing the relative price distortion.

6.3 Gaining the ability to commit

If a policymaker previously operating with discretion gains the ability to commit, the economy behaves according to the dynamics under commitment, beginning in the discretionary steady state and—presumably—ending in the commitment steady state. The dynamics under commitment are represented by the aforementioned nine-variable system of nonlinear difference equations, shown in Appendix D.

To compute the transition path, we conjecture that convergence to the zero-inflation steady state is complete after $T = 40$ quarters. We then have a system of $9 \times T$ equations in the $9 \times T + 5$ variables $\left\{c_t, \tilde{S}_t, \tilde{F}_t, \pi_t, \Delta_t, \phi_t, \psi_t, \xi_t, \gamma_t\right\}_{t=0}^{T-1}$ and $\left\{\Delta_{-1}, \tilde{S}_T, \tilde{F}_T, \pi_T, \gamma_T\right\}$. If we assume there is a unique transition path, then to solve the system of $9 \times T$ equations we need to specify values for the initial condition Δ_{-1} and the terminal conditions $\left\{\tilde{S}_T, \tilde{F}_T, \pi_T, \gamma_T\right\}$. Under the conjecture, the terminal conditions are given by the steady state under commitment (zero inflation). The initial condition is given by the steady-state value of Δ under discretion. From the properties of the Jacobian matrix at the steady state, we know that locally there is a unique stable solution that converges to the steady state. Indeed, we are able to compute a global solution satisfying the conjecture.

The dashed lines in Figure 3 represent the resulting transition path from the steady state with discretion to the steady state with commitment. The transition contains an element of discretionary behavior: in the initial period, since the policy change is unexpected, the policymaker has an incentive to exploit the fixed prices of non-adjusters by increasing the money supply. This was also the case in the transition to discretion, but here the long-run policy involves lower inflation, so the temporary stimulus is not offset by frontloading of larger price increases. Instead, adjusting firms frontload smaller future price increases, more than offsetting any inflationary effects of the temporary monetary stimulus. The welfare benefit of the transition from discretion to commitment is again well approximated by the steady-state welfare comparison: the representative household would require a 0.222 percent increase in consumption each quarter in order to willingly choose the transition from commitment to discretion (5.54 percent in present-value terms).

Initial-period policy under commitment and discretion illustrates the difficulty of exploiting initial conditions. The discretionary monetary authority would like to exploit initial conditions, but in equilibrium even in the short run it is unable to do so because of firms' forward-looking behavior. Conversely, that same forward-looking behavior means that a policymaker who can commit is able to exploit initial conditions (once!) by combining short-run expansionary policy with lower money growth and inflation in the long run.

7 Concluding remarks

The vast literature on time-consistency problems for monetary policy is comprised of two seemingly disparate branches. Much of the profession's intuition is derived from the seminal work by Barro and Gordon (1983), hereafter BG, which in turn built on Kydland and Prescott (1977). They studied reduced-form macroeconomic models in which the frictions giving leverage to monetary policy were not precisely spelled out. In contrast, the staggered pricing models popularized in the last two decades are precise about those frictions. We conclude here by summarizing the paper's three main contributions and then explaining how the analysis relates to BG's early work on time-consistency problems for monetary policy.

The direct motivation for this paper comes from King and Wolman (2004), who showed that in a model with Taylor-style staggered price setting, discretionary policy induces complementarity among firms sufficient to generate multiple private-sector equilibrium. In the Calvo model, which shares many features with the Taylor model but has been much more influential for applied monetary policy analysis (often in its linearized form), we find that the presence of many predetermined variables and a real state variable works against complementarity, and multiple private-sector equilibria do not arise. The state variable measures the dispersion in predetermined relative prices; in contrast, the Taylor model has only one predetermined nominal price and thus no predetermined relative prices.

The Calvo model's combination of a real state variable and unique private-sector equilibrium leads us to an analysis of discretionary equilibrium using the GEE approach. The GEE is a simplified representation of the policymaker's first-order condition, which highlights the various channels through which current policy can affect future welfare. Without commitment, each of these channels works through the state variable; the state variable affects future outcomes directly and it shifts the future policymaker's problem. While the GEE approach has been applied extensively to study fiscal policy, the Calvo application is new to this paper.

Finally, we use the steady-state and dynamic properties of the discretionary equilibrium together with the solution under commitment to study the processes of gaining and losing the ability to commit. The present-value welfare gain or loss is approximately equivalent to 0.2 percent of consumption each period, or a one-time gain or loss of 5.5−5.6 percent of annual consumption. While we compute these welfare numbers using the full transition paths, they differ little from the steady-state welfare comparisons.

At the heart of the time-consistency problem for monetary policy is the notion that a discretionary policymaker takes as given private agents' expectations, but in equilibrium those expectations accurately incorporate the policymaker's optimal behavior. This property holds in modern staggered pricing models such as the Calvo model and in the BG model. In BG, however, the expectations just referred to are current expectations about current policy; dynamics only arise through serial correlation of exogenous shocks. Without other intertemporal links, the policy problem is a static one in BG: treating expectations as fixed, higher inflation is costly in its own right but brings about a beneficial reduction in unemployment. In equilibrium, private expectations are validated, and the policymaker balances the static marginal cost and marginal benefit of additional inflation. In contrast, in staggered pricing models prices set in the past incorporate expectations about current policy. Whereas in BG equilibrium requires that current policy actions be consistent with current-period expectations, in staggered pricing models equilibrium requires that current policy actions be consistent with expectations formed in the past.[16]

The intertemporal nature of price setting also means that unlike BG, staggered pricing models generally contain one or more state variables that can be affected by a policymaker, even under discretion. Thus, the discretionary policymaker does not face a purely static tradeoff between inflation and real activity. That tradeoff is present, but it is complicated by the fact that the current policy action affects tomorrow's state, and thus tomorrow's value function. One of the main points of this paper is that the details of this intertemporal element differ across staggered pricing models, leading to different implications for the nature of equilibrium under discretionary monetary policy.

While staggered pricing models generate a static output-inflation tradeoff superficially similar to the one in BG, these models are explicit about the source of that tradeoff. Because of the forward-looking elements in these models, the details of the policy tradeoff—and whether it even appears in equilibrium—depend critically on the entire path of expected future policy. For example, Section 6 has shown that an unexpected reduction in inflation can be stimulative, if it signals the transition to a permanently lower inflation rate (gaining commitment). Likewise, an unexpected increase in inflation can be contractionary if it signals the transition to a permanently higher inflation rate (losing commitment). Although these transitions may suggest that no output-inflation tradeoff is present, in the discretionary equilibrium the policymaker perceives such a tradeoff: a one-period deviation toward more expansionary policy would raise output and inflation, as it reduced the markup and raised the relative price distortion. The effects on welfare would be offsetting, and thus the policymaker does not deviate.

Of course, the properties of discretionary equilibrium are determined by the specifics of the model. The defining feature of the Calvo model is the assumption that a fraction

[16] With different timing assumptions in staggered pricing models the BG version of expectational consistency would also be required to hold.

of firms are prohibited from adjusting their price. This assumption makes for a relatively tractable framework, undoubtedly the main reason the Calvo model has come to serve as the basis for so much applied work on monetary policy. It has recently become feasible to conduct some forms of policy analysis in models which relax this assumption, allowing firms to adjust their price by incurring a cost. Those models typically have a large number of state variables, currently rendering it infeasible to perform the kind of analysis conducted here (see for example Nakov and Thomas, 2014).[17] Nonetheless, we hope that our work can serve as a useful input for future research on discretionary policy in quantitative state-dependent pricing models.

[17] While Barseghyan and DiCecio (2007) and Siu (2008) study discretionary policy in models with state-dependent pricing, both papers limit the state space by allowing firms to adjust costlessly after one and two periods, respectively.

A Computational details

This appendix describes how numerical solutions for the discretionary equilibrium are computed and how uniqueness of the equilibrium is verified in a large number of examples.

Computing a discretionary equilibrium

The value function and the expressions for $S()$ and $F()$ are approximated with Chebyshev polynomials. This computational method involves selecting a degree of approximation I, and then searching for values of v_i^* and Γ_i^*, for $i = 1 \ldots I$, that solve (27) at the grid points for the state variable Δ_i defined by the Chebyshev nodes. The optimization problem (27) is solved using the following algorithm.

1. *Grids and initial values.* The example of the baseline calibration (i.e., $\alpha = 0.5$, $\beta = 0.99$, $\epsilon = 10$, $\mu = 0$, and $\chi = 4.5$) uses a degree of approximation $I = 12$ on the interval $[1, 1.1]$ for the state variable. As an initial guess for $v()$, $S()$ and $F()$ the discretionary equilibrium for the static model is used, which is the final period of a finite horizon model. To compute the private-sector response to an arbitrary policy, grids $\{m_1, \ldots, m_{I_m}\}$ and $\{p_{0,1}, \ldots, p_{0,I_p}\}$ are specified for the money supply and the optimal price. In the case of the baseline calibration, the grid for m consists of $I_m = 600$ evenly spaced points between 0.01 and 0.25 and the grid for p_0 consists of $I_p = 600$ evenly spaced points between 0 and 2.

2. *Private-sector responses.* For each possible value of Δ and m, compute the private-sector responses by solving (28) as a fixed-point problem. Specifically, compute the right hand side of (28) and call it $\hat{p}_0$, then use linear interpolation to find the fixed points $p_0 = \hat{p}_0$.

3. *Policy function and value function.* On each grid point Δ_i, select the value of m that maximizes the value function. If the value function and policy function that solve the optimization problem are identical to the guess, then they form a discretionary equilibrium. Specifically, iteration j is the final iteration if $||v^{j+1} - v^j||_\infty$ and $||\Gamma^{j+1} - \Gamma^j||_\infty$ are smaller than the tolerance level $1.49 \cdot 10^{-8}$ (the square-root of machine precision). If not, the starting values are updated by pushing out the initial guess one period into

the future, and assuming the one-period-ahead policy and value functions are the ones that solved the optimization problem.

To assess the accuracy of a solution, the difference between the left hand side and the right hand side of (27) is calculated using that solution on a grid of 100,000 points that do not include the Chebyshev nodes. With the baseline calibration, this residual function has a maximum absolute approximation error of order 10^{-6}.

Many other examples were computed that cover a wide range of values for α, ε and μ. These include the values for $\alpha = 0.1$, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.55, 0.6, 0.61, 0.62, 0.63, 0.64, 0.65, 0.66, 0.67, 0.68, 0.69, 0.70, 0.71, 0.72, 0.73, 0.74, 0.75, 0.76, 0.77, 0.78, 0.79, 0.8, 0.85, 0.9, and 0.95, values for $\varepsilon = 6$, 7, 8, 9, and 11, and values for $\mu = 0.25$, 0.5, 0.75, 1, 1.25, 1.5, 1.75, and 2. In addition to these 46 solutions, which consider alternative values of one parameter at a time, solutions were computed for eight combinations of extreme parameter values: $(\alpha, \varepsilon, \mu) = (0.1, 4, 0)$, $(0.1, 4, 2)$, $(0.1, 11, 0)$, $(0.1, 11, 2)$, $(0.9, 4, 0)$, $(0.9, 4, 2)$, $(0.9, 11, 0)$, $(0.9, 11, 2)$.

Uniqueness of the solutions

All the examples described above yield a unique solution. Step 2 in the model solution algorithm allows for the possibility of multiple fixed points at an arbitrary monetary policy at each value of the state. Suppose the money supply that maximizes the value function in iteration $j-1$ induces multiple private-sector responses. Then the inherited relative price dispersion in iteration j is not uniquely determined and neither is the money supply in iteration j. Therefore, if j is the final iteration there are multiple discretionary equilibria. However, this hypothetical sequence of outcomes does not arise in any of our examples, where discretionary equilibrium is always unique. Multiple fixed points were only encountered for sub-optimal values of m, in which case the largest fixed point was arbitrarily selected (the same solutions were found when selecting the smallest fixed point). Specifically, for the alternative calibrations with values of α between 0.63 and 0.69, or with the value $\varepsilon = 6$, the final iteration of the solution algorithm exhibited multiple fixed points for values of m in a range that is positive but smaller than the optimal value of m.

B Comparison with the Taylor model

This appendix describes the two-period Taylor model and provides a quantitative comparison with the Calvo model.

B.1 Model

In the Taylor model each firm sets its price for two periods. The description of the representative household remains unchanged, but the value of a firm upon adjustment is given by

$$\max_{X_t} \left\{ \sum_{j=0}^{1} \beta^j \left(\frac{P_t}{P_{t+1}} \right)^j \left(\frac{c_t}{c_{t+1}} \right)^j \left[X_t \left(\frac{X_t}{P_{t+j}} \right)^{-\varepsilon} c_{t+j} - P_{t+j} w_{t+j} \left(\frac{X_t}{P_{t+j}} \right)^{-\varepsilon} c_{t+j} \right] \right\},$$

and the optimal price satisfies the first-order condition,

$$\frac{P_{0,t}}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \cdot \frac{w_t + \beta \left(P_{t+1}/P_t \right)^{\varepsilon} w_{t+1}}{1 + \beta \left(P_{t+1}/P_t \right)^{\varepsilon - 1}}.$$

This condition indicates the firm chooses a constant markup over a weighted average of current and future marginal costs. Whereas in the Calvo model the index of predetermined prices was given by P_{t-1}, in the Taylor model there is just one predetermined price, $P_{0,t-1}$. Normalizing the optimal price and the price index by $P_{0,t-1}$ and using the definitions $\tilde{p}_{0,t} \equiv P_{0,t}/P_{0,t-1}$ and $p_t \equiv P_t/P_{0,t-1}$, we have

$$\frac{\tilde{p}_{0,t}}{p_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \cdot \frac{w_t + \beta \left(\tilde{p}_{0,t} p_{t+1}/p_t \right)^{\varepsilon} w_{t+1}}{1 + \beta \left(\tilde{p}_{0,t} p_{t+1}/p_t \right)^{\varepsilon - 1}}. \tag{42}$$

Money demand (5) is normalized by the lagged optimal price instead of the lagged price level

$$\tilde{m}_t \equiv \frac{M_t}{P_{0,t-1}} = p_t c_t. \tag{43}$$

We eliminate the predetermined variable from the price index,

$$P_t = \left(\frac{1}{2} P_{0,t}^{1-\varepsilon} + \frac{1}{2} P_{0,t-1}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}},$$

by dividing by the lagged optimal price:

$$p_t = \left(\frac{1}{2} \tilde{p}_{0,t}^{1-\varepsilon} + \frac{1}{2} \right)^{\frac{1}{1-\varepsilon}}. \tag{44}$$

The labor market clearing condition yields

$$\begin{aligned} n_t &= \frac{1}{2}\sum_{j=0}^{1} n_{j,t} \\ \frac{n_t}{c_t} &= \frac{1}{2}p_t^{\varepsilon}\left[(\tilde{p}_{0,t})^{-\varepsilon}+1\right]. \end{aligned} \tag{45}$$

There is one predetermined nominal price ($P_{0,t-1}$), but there are no state variables in the labor market clearing condition. The equations (42)$-$(45) and the labor supply equation (4), together with the behavior of future policymakers, implicitly define the set of feasible values for w_t, c_t, n_t, p_t and $\tilde{p}_{0,t}$ attainable by the current monetary policymaker in the Taylor model. The current policymaker chooses the money supply, or equivalently $\tilde{m}_t$, the money supply normalized by the predetermined price. Unlike the Calvo model, no state variables constrain the monetary authority in an MPE. The lagged optimal price $P_{0,t-1}$ matters for the levels of nominal variables, but is irrelevant for the determination of real allocations.

B.2 Results

Do the seemingly small differences between the Taylor and Calvo models generate similar predictions for inflation under discretion? The remainder of this appendix provides a quantitative comparison of the steady-state inflation rates in the two models. For this comparison, we solve the social planner's problem associated with the Taylor model to circumvent the issue of equilibrium multiplicity. That is, instead of the policymaker choosing the money supply, we assume a planner chooses consumption allocations. The calibration is the same as for the Calvo model: $\varepsilon = 10$, $\beta = 0.99$, $\mu = 0$, $\chi = 4.5$. Since $\alpha = 0.5$ in the Calvo model, the average frequency of price adjustment is two quarters in both models.

Under this calibration, the steady-state inflation rate in the Taylor model is 9.1 percent, exceeding the steady-state inflation rate of 5.5 percent in the Calvo model. In the Taylor model, the policymaker faces only the short-run trade-off between the markup and the relative price distortion. In the Calvo model, when weighing the benefit of a lower markup against the cost of a higher relative price distortion, the policymaker also takes into account the cost to future policymakers of inheriting a higher relative price distortion. By choosing a higher inflation rate today, the policymaker would pass on a higher relative price distortion to the next policymaker, which is an additional cost of high inflation relative to the Taylor model. Therefore, the steady-state inflation rate is lower in the Calvo model.

C Proof of Proposition 1

This appendix presents the proof of Proposition 1. Recall from equation (12) that inflation is the following function of the optimal reset price

$$\pi(p_{0,t}) = \left[(1-\alpha)p_{0,t}^{1-\varepsilon} + \alpha\right]^{\frac{1}{1-\varepsilon}},$$

which is increasing and strictly concave:

$$\begin{aligned}
\pi'(p_{0,t}) &= (1-\alpha)\left[(1-\alpha) + \alpha p_{0,t}^{\varepsilon-1}\right]^{-\frac{\varepsilon}{\varepsilon-1}} = (1-\alpha)\left[\frac{\pi(p_{0,t})}{p_{0,t}}\right]^{\varepsilon} > 0 \qquad (46)\\
\pi''(p_{0,t}) &= -\alpha(1-\alpha)\varepsilon\pi(p_{0,t})^{2\varepsilon-1}p_{0,t}^{3(\varepsilon-1)} < 0, \qquad (47)
\end{aligned}$$

and has a finite limit

$$\lim_{p_{0,t}\to\infty} \pi(p_{0,t}) = \alpha^{\frac{1}{1-\varepsilon}}. \qquad (48)$$

Consistent with the computation of discretionary equilibrium as the stationary limit of the finite-horizon economy, we compute equilibrium with a constant-m policy as the limit of the finite-horizon economy. Let T denote the final period, so $S_{T+1} = F_{T+1} = 0$. Then:

$$\begin{aligned}
S_T &= \left(\frac{\varepsilon}{\varepsilon-1}\right)\pi(p_{0,T})^{\varepsilon-1}\left[\chi m_T + \alpha\beta S_{T+1}\pi(p_{0,T})\right] = \left(\frac{\varepsilon\chi}{\varepsilon-1}\right)\pi(p_{0,T})^{\varepsilon-1}m_T, \qquad (49)\\
F_T &= \pi(p_{0,T})^{\varepsilon-1}\left[1+\alpha\beta F_{T+1}\right] = \pi(p_{0,T})^{\varepsilon-1}, \qquad (50)
\end{aligned}$$

and the pricing best-response function is

$$\hat{p}_{0,T} = \frac{S_T}{F_T} = \left(\frac{\varepsilon\chi}{\varepsilon-1}\right)m_T. \qquad (51)$$

The outcomes $p_{0,T}$, S_T, and F_T do not depend on the state because monetary policy does not depend on the state. Moreover, there can be no complementarity in price setting in period T, because the pricing best-response function (51) of any given firm does not depend on other firms' price decisions.

Note from (49) that $S_T = S_T(m_T, p_{0,T})$ and from (50) that $F_T = F_T(p_{0,T})$. We can now analyze the period $T-1$ pricing best-response function to determine whether there is a unique fixed point. We have:

$$\begin{aligned}
S_{T-1} &= \left(\frac{\varepsilon}{\varepsilon-1}\right)\pi(p_{0,T-1})^{\varepsilon-1}\left[\chi m_{T-1} + \alpha\beta S_T(m_T, p_{0,T})\pi(p_{0,T-1})\right]\\
F_{T-1} &= \pi(p_{0,T-1})^{\varepsilon-1}\left[1+\alpha\beta F_T(p_{0,T})\right],
\end{aligned}$$

so the period $T-1$ best-response function is

$$\hat{p}_{0,T-1} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\left[\frac{\chi m_{T-1} + \alpha\beta S_T(m_T, p_{0,T})\pi(p_{0,T-1})}{1+\alpha\beta F_T(p_{0,T})}\right] \tag{52}$$

The optimal price does not depend on the state because the monetary policy function and the functions S_T and F_T do not depend on the state. To see that the best-response function has a unique fixed point, first write (52) as

$$\hat{p}_{0,T-1} = A_{T-1}(p_{0,T})m_{T-1} + B_{T-1}(m_T, p_{0,T})\pi(p_{0,T-1}),$$

where $A_{T-1}(p_{0,T}) > 0$ and $B_{T-1}(m_T, p_{0,T}) > 0$ because $m_T, p_{0,T} > 0$. It follows from (46)$-$(48) that

$$\begin{aligned}
\frac{\partial \hat{p}_{0,T-1}}{\partial p_{0,T-1}} &= B_{T-1}\pi'(p_{0,T-1}) > 0 \\
\frac{\partial^2 \hat{p}_{0,T-1}}{\partial p_{0,T-1}^2} &= B_{T-1}\pi''(p_{0,T-1}) < 0 \\
\lim_{p_{0,T-1}\to\infty} \hat{p}_{0,T-1} &= \left(\frac{\varepsilon}{\varepsilon-1}\right)\left[\frac{\chi m_{T-1} + \alpha^{\frac{2-\varepsilon}{1-\varepsilon}}\beta S_T(m_T, p_{0,T})}{1+\alpha\beta F_T(p_{0,T})}\right].
\end{aligned}$$

Because the best-response function is always positive and concave and has a finite limit, it has a unique fixed point. Therefore, there exists a unique private-sector equilibrium in period $T-1$.

Write $S_{T-1} = S_{T-1}(m_{T-1}, m_T, p_{0,T-1}, p_{0,T})$ and $F_{T-1} = F_{T-1}(p_{0,T-1}, p_{0,T})$. In period $T-2$ we obtain

$$\begin{aligned}
S_{T-2} &= \left(\frac{\varepsilon}{\varepsilon-1}\right)\pi(p_{0,T-2})^{\varepsilon-1}\left[\chi m_{T-2} + \alpha\beta S_{T-1}(m_{T-1}, m_T, p_{0,T-1}, p_{0,T})\pi(p_{0,T-2})\right] \\
F_{T-2} &= \pi(p_{0,T-2})^{\varepsilon-1}\left[1+\alpha\beta F_{T-1}(p_{0,T-1}, p_{0,T})\right].
\end{aligned}$$

Hence the period $T-2$ best-response function can be written as

$$\hat{p}_{0,T-2} = A_{T-2}(p_{0,T-1}, p_{0,T})m_{T-2} + B_{T-2}(m_{T-1}, m_T, p_{0,T-1}, p_{0,T})\pi(p_{0,T-2}),$$

where $A_{T-2} > 0$ and $B_{T-2} > 0$ because $m_T, m_{T-1}, p_{0,T}, p_{0,T-1} > 0$. By the same arguments as above there is a unique fixed point in period $T-2$.

Repeating the same steps, we can show that for period t,

$$\begin{aligned}
S_t &= \left(\frac{\varepsilon}{\varepsilon-1}\right)\pi(p_{0,t})^{\varepsilon-1}\left[\chi m_t + \alpha\beta S(m_{t+1}, m_{t+2}, \ldots, p_{0,t+1}, p_{0,t+2}, \ldots)\pi(p_{0,t})\right] \\
F_t &= \pi(p_{0,t})^{\varepsilon-1}\left[1+\alpha\beta F(p_{0,t+1}, p_{0,t+2}, \ldots)\right].
\end{aligned}$$

The period-t best-response function can therefore be written as

$$\hat{p}_{0,t} = A_t(p_{0,t+1}, p_{0,t+2}, \ldots)m_t + B_t(m_{t+1}, m_{t+2}, \ldots, p_{0,t+1}, p_{0,t+2}, \ldots)\pi(p_{0,t}),$$

where $A_t > 0$ and $B_t > 0$ because $m_{t+j}, p_{0,t+j} > 0$ for $j = 1, 2, \ldots$. Therefore, by backward induction, there is a unique private-sector equilibrium associated with the arbitrary constant-m policy.

D Derivation of commitment solution

Here we derive the equations characterizing optimal policy with commitment using the following Lagrangian:

$$\begin{aligned}
\mathcal{L} &= \sum_{t=0}^{\infty} \beta^t \left[\ln(c_t) - \chi \frac{n_t^{1+\mu}}{1+\mu}\right] + \sum_{t=0}^{\infty} \beta^t \zeta_t \left[p_{0,t} - \left(\frac{\varepsilon}{\varepsilon - 1}\right) \pi_t \frac{\tilde{S}_t}{\tilde{F}_t}\right] \\
&+ \sum_{t=0}^{\infty} \beta^t \phi_t \left[\tilde{S}_t - \left(w_t + \alpha\beta\pi_{t+1}^{\varepsilon} \tilde{S}_{t+1}\right)\right] + \sum_{t=0}^{\infty} \beta^t \psi_t \left[\tilde{F}_t - \left(1 + \alpha\beta\pi_{t+1}^{\varepsilon-1} \tilde{F}_{t+1}\right)\right] \\
&+ \sum_{t=0}^{\infty} \beta^t \xi_t \left\{\pi_t - \left[(1-\alpha) p_{0,t}^{1-\varepsilon} + \alpha\right]^{\frac{1}{1-\varepsilon}}\right\} + \sum_{t=0}^{\infty} \beta^t \gamma_t \left\{\Delta_t - \pi_t^{\varepsilon} \left[(1-\alpha) p_{0,t}^{-\varepsilon} + \alpha\Delta_{t-1}\right]\right\} \\
&+ \sum_{t=0}^{\infty} \beta^t \Omega_t (n_t - \Delta_t c_t) + \sum_{t=0}^{\infty} \beta^t \theta_t \left[w_t - \chi c_t n_t^{\mu}\right].
\end{aligned}$$

The first order conditions are as follows:

$$p_0: \quad \zeta_t - \xi_t \left[(1-\alpha) p_{0,t}^{1-\varepsilon} + \alpha\right]^{\frac{\varepsilon}{1-\varepsilon}} (1-\alpha) p_{0,t}^{-\varepsilon} + \gamma_t \varepsilon (1-\alpha) \pi_t^{\varepsilon} p_{0,t}^{-\varepsilon-1} = 0 \tag{53}$$

$$\tilde{S}: \quad \phi_t - \phi_{t-1}\alpha\pi_t^{\varepsilon} - \zeta_t \left(\frac{\varepsilon}{\varepsilon-1}\right) \pi_t \frac{1}{\tilde{F}_t} = 0 \tag{54}$$

$$\tilde{F}: \quad \psi_t - \psi_{t-1}\alpha\pi_t^{\varepsilon-1} + \zeta_t \left(\frac{\varepsilon}{\varepsilon-1}\right) \pi_t \frac{\tilde{S}_t}{\tilde{F}_t^2} = 0 \tag{55}$$

$$\begin{aligned}
\pi: \quad & -\phi_{t-1}\varepsilon\pi_t^{\varepsilon-1}\alpha\tilde{S}_t - \psi_{t-1}(\varepsilon-1)\pi_t^{\varepsilon-2}\alpha\tilde{F}_t - \zeta_t \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{\tilde{S}_t}{\tilde{F}_t} \\
& + \xi_t - \gamma_t \varepsilon \pi_t^{\varepsilon-1} \left[(1-\alpha) p_{0,t}^{-\varepsilon} + \alpha\Delta_{t-1}\right] = 0
\end{aligned} \tag{56}$$

$$\Delta: \quad -\Omega_t + \gamma_t - \beta\gamma_{t+1}\alpha\pi_{t+1}^{\varepsilon} = 0 \tag{57}$$

$$c: \quad \frac{1}{c_t} + \theta_t \chi n_t^{\mu} - \Omega_t \frac{n_t}{c_t^2} = 0 \tag{58}$$

$$n: \quad \theta_t \mu \chi c_t n_t^{\mu-1} - \chi n_t^{\mu} + \frac{\Omega_t}{c_t} = 0 \tag{59}$$

$$w: \quad -\theta_t - \phi_t = 0, \tag{60}$$

as well as the constraints (35)−(41) in the body of the text. This is a system of 15 equations, which can be reduced to nine equations as follows. First, eliminate $p_{0,t}$ directly using (35). Second, eliminate θ_t directly using (60). Third, eliminate n_t as $n_t = c_t \Delta_t$ using (40). Fourth, eliminate w_t as $w_t = \chi \Delta_t^{\mu} c_t^{1+\mu}$ using (41). Fifth, eliminate Ω_t using (58) and (40), as

$$\Omega_t = \left(\frac{1}{\Delta_t} + \theta_t \chi c_t^{1+\mu} \Delta_t^{\mu-1} \right).$$

Finally, eliminate ζ_t using (53), noting that we would also substitute out for $p_{0,t}$ using (35):

$$\zeta_t = \xi_t \left[(1-\alpha) p_{0,t}^{1-\varepsilon} + \alpha \right]^{\frac{\varepsilon}{1-\varepsilon}} (1-\alpha) p_{0,t}^{-\varepsilon} - \gamma_t \varepsilon (1-\alpha) \pi_t^{\varepsilon} p_{0,t}^{-\varepsilon-1}.$$

The nine remaining equations are as follows, where the variables $(n_t, w_t, p_{0,t}, \theta_t, \Omega_t, \zeta_t)$ should be understood to be substituted out as described above:

$$\begin{aligned}
\tilde{S}_t &= w_t + \alpha\beta\pi_{t+1}^{\varepsilon} \tilde{S}_{t+1} \\
\tilde{F}_t &= 1 + \alpha\beta\pi_{t+1}^{\varepsilon-1} \tilde{F}_{t+1} \\
\pi_t &= \left[(1-\alpha) p_{0,t}^{1-\varepsilon} + \alpha \right]^{\frac{1}{1-\varepsilon}} \\
\Delta_t &= \pi_t^{\varepsilon} \left[(1-\alpha) p_{0,t}^{-\varepsilon} + \alpha \Delta_{t-1} \right] \\
0 &= \theta_t \mu \chi c_t n_t^{\mu-1} - \chi n_t^{\mu} + \frac{\Omega_t}{c_t} \\
0 &= \phi_t - \phi_{t-1} \alpha \pi_t^{\varepsilon} - \zeta_t \left(\frac{\varepsilon}{\varepsilon-1} \right) \pi_t \frac{1}{\tilde{F}_t} \\
0 &= \psi_t - \psi_{t-1} \alpha \pi_t^{\varepsilon-1} + \zeta_t \left(\frac{\varepsilon}{\varepsilon-1} \right) \pi_t \frac{\tilde{S}_t}{\tilde{F}_t^2} \\
0 &= -\phi_{t-1} \varepsilon \pi_t^{\varepsilon-1} \alpha \tilde{S}_t - \psi_{t-1} (\varepsilon-1) \pi_t^{\varepsilon-2} \alpha \tilde{F}_t - \zeta_t \left(\frac{\varepsilon}{\varepsilon-1} \right) \frac{\tilde{S}_t}{\tilde{F}_t} \\
&\quad + \xi_t - \gamma_t \varepsilon \pi_t^{\varepsilon-1} \left[(1-\alpha) p_{0,t}^{-\varepsilon} + \alpha \Delta_{t-1} \right] \\
0 &= -\Omega_t + \gamma_t - \beta \gamma_{t+1} \alpha \pi_{t+1}^{\varepsilon}
\end{aligned}$$

This is the system of nonlinear difference equations that we must solve to compute the transition path in Section 6.3. It is straightforward to show that zero inflation ($\pi = 1$) solves the steady-state system of equations.

References

[1] Albanesi, S., V.V. Chari, and L.J. Christiano (2003), "Expectation Traps and Monetary Policy," *Review of Economic Studies* 70 (4), 715-741.

[2] Anderson, G., J. Kim and T. Yun (2010), "Using a Projection Method to Analyze Inflation Bias in a Micro-Founded Model," *Journal of Economic Dynamics and Control* 34 (9), 1572-1581.

[3] Azzimonti, M., P.D. Sarte and J. Soares (2006), "Distortionary Taxes and Public Investment when Government Promises Are Not Enforceable," *Journal of Economic Dynamics and Control* 33 (9), 1662-1681.

[4] Barseghyan, L. and R. DiCecio (2007), "Optimal Monetary Policy, Endogenous Sticky Prices, and Multiple Equilibria," *B.E. Journal of Macroeconomics* 7 (1), Article 8.

[5] Barro, R.J. and D. Gordon (1983), "Rules, Discretion and Reputation in a Model of Monetary Policy," *Journal of Monetary Economics* 12 (1), 101-121.

[6] Calvo, G.A. (1983), "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics* 12 (3), 383-398.

[7] Clarida, R., J. Gali and M. Gertler (1999), "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature* 37 (4), 1661-1707.

[8] Dotsey, M. and A. Hornstein (2011), "On the Implementation of Markov-Perfect Monetary Policy," Working Paper 11-29, Federal Reserve Bank of Philadelphia.

[9] Khan, A., R.G. King and A.L. Wolman (2001), "The Pitfalls of Discretionary Monetary Policy," Working Paper 01-16, Federal Reserve Bank of Philadelphia.

[10] King, R.G. and A.L. Wolman (2004), "Monetary Discretion, Pricing Complementarity, and Dynamic Multiple Equilibria," *Quarterly Journal of Economics* 119 (4), 1513-1553.

[11] King, R.G. and A.L. Wolman (1999), "What Should the Monetary Authority Do When Prices Are Sticky?" in *Monetary Policy Rules*, J.B. Taylor ed., University of Chicago Press, 1999, 349-398.

[12] King, R.G. and A.L. Wolman (1996), "Inflation Targeting in a St. Louis Model of the 21st Century," Proceedings, Federal Reserve Bank of St. Louis, 83-107.

[13] Klein, P. P. Krusell and J.-V. Rios-Rull (2008), "Time-Consistent Public Policy," *Review of Economic Studies* 75 (3), 789-808.

[14] Kydland, F.E. and E.C. Prescott (1977), "Rules Rather than Discretion: the Inconsistency of Optimal Plans," *Journal of Political Economy* 85 (3), 473-491.

[15] Krusell, P., B. Kuruscu, and A.A. Smith (2002), "Equilibrium Welfare and Government Policy with Quasi-geometric Discounting," *Journal of Economic Theory* 105 (1), 42-72.

[16] Leeper, E.M., C. Leith and D. Liu. (2016), "Optimal Time-Consistent Monetary, Fiscal and Debt Maturity Policy," Available at SSRN: http://dx.doi.org/10.2139/ssrn.2719457.

[17] Leith and D. Liu. (2016), "The Inflation Bias under Calvo and Rotemberg Pricing," *Journal of Economic Dynamics and Control* 73, 283-297.

[18] Nakov, A. and C. Thomas (2014), "Optimal Monetary Policy with State-Dependent Pricing," *International Journal of Central Banking*, September, 49-94.

[19] Ngo, P.V. (2014), "Optimal Discretionary Monetary Policy in a Micro-Founded Model with a Zero Lower Bound on Nominal Interest Rate," *Journal of Economic Dynamics and Control* 45, 44-65.

[20] Siu, H.E. (2008), "Time Consistent Monetary Policy with Endogenous Price Rigidity," *Journal of Economic Theory* 138 (1), 184-210.

[21] Woodford, M., *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, 2003.

[22] Woodford, M. (2003), "Optimal Interest-Rate Smoothing," *Review of Economic Studies* 70 (4), 861-886.

[23] Yun, T. (2005), "Optimal Monetary Policy with Relative Price Distortions," *American Economic Review* 95 (1), 89-108

[24] Yun, T. (1996), "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics* 37 (2), 345-370.