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Equilibrium Price Dispersion Across and Within Stores*

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Abstract

We develop a search-theoretic model of the product market that generates price dispersion across and within stores. Buyers differ with respect to their ability to shop around, both at different stores and at different times. The fact that some buyers can shop from only one seller while others can shop from multiple sellers causes price dispersion across stores. The fact that the buyers who can shop from multiple sellers are more likely to be able to shop at inconvenient times induces price dispersion within stores. Specifically, it causes sellers to post different prices for the same good at different times in order to discriminate between different types of buyers.

JEL Codes: D43.

Keywords: Search, Price dispersion, Price discrimination, Bargain hunting.

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1 Introduction

It is a well-known fact that the same product is sold at very different prices, even when one restricts attention to sales taking place in the same geographical area and in the same narrow period of time. For instance, Sorensen (2000) finds that the average standard deviation of the price posted by different pharmacies for the same drug in the same town in upstate New York is 22%. In a more systematic study of price dispersion that covers 1.4 million goods in 54 geographical markets within the United States, Kaplan and Menzio (2014b) find that the average standard deviation of the price at which the same product is sold within the same geographical area and the same quarter is 19%. Moreover, it appears that price dispersion is caused by both difference in prices across different stores and difference in prices within each store. For instance, Kaplan and Menzio (2014b) find that on average roughly half of the overall variance of prices for the same good in the same market and in the same quarter is due to the fact that different stores sell the same good at a different price on average, and the other half is due to the fact that the same store sells the same good at different prices in different transactions taking place during the same quarter.

In this paper, we develop a search-theoretic model of price dispersion across and within stores by combining the standard theory of price dispersion of, e.g., Butters (1977) and Burdett and Judd (1983), and the standard theory of intertemporal price discrimination of, e.g., Conslík, Gerstner and Sobel (1984) and Sobel (1984). The resulting model offers a tractable and unified framework to study the extent and shape of price dispersion and its different causes.¹

Specifically, we build a model of the market for an indivisible good. On the demand side, there are buyers who differ with respect to their ability to shop at different stores, as well as with respect to their ability to shop at different times: Some buyers can shop from only one seller while others can shop from multiple sellers, and some buyers can shop only in the daytime while others can shop both in the daytime and in the nighttime. On the

¹Besides intertemporal price discrimination, there are other explanations for why a seller would charge different prices for the same good within the same quarter. First, as in Sheshinski and Weiss (1974), Benabou (1988) or Burdett and Menzio (2013), a seller may change his nominal price during a quarter in order to keep up with the movements in the aggregate price level (more or less frequently depending on inflation and menu costs). Although, this theory of within-store price dispersion seems unlikely to be relevant in a low-inflation environment like the U.S. economy in the 2000s. Second, as suggested by Aguirregabiria (1999), a seller may change his price during a quarter in response to changes in his inventories of the good. Finally, as suggested by Menzio and Trachter (2014), a large seller may change his price over time in order to occasionally price out of the market a fringe of small sellers.

supply side, there are identical sellers and each seller posts a (potentially different) price for the good in the daytime and in the nighttime.

We prove the existence and uniqueness of equilibrium. The equilibrium always features price variation across stores. Moreover, if the buyers who are able to shop at both times are—on average—also able to shop from more stores than the buyers who can only shop in the daytime, the equilibrium features price variation within stores. In particular, the equilibrium is such that some sellers post a strictly lower price in the nighttime than in the daytime. On the other hand, if the buyers who are able to shop at both times of day are—on average—less able to shop at multiple stores, the equilibrium features no price variation within stores. That is, sellers do not vary their price over time.

Intuitively, price dispersion across stores arises because sellers meet some buyers who cannot purchase from any other store and some other buyers who can and—as explained in Butters (1977) and Burdett and Judd (1983)—this heterogeneity induces identical sellers to post different prices for the same good. Price dispersion within stores arises because—if the buyers who are more likely to be able to shop at nighttime are also more likely to be able to shop from multiple stores—a seller can compete more fiercely for these buyers without losing revenues on the other customers by charging a lower price at night than during the day.

The paper’s main contribution is to combine in a unified and simple search-theoretic framework the insights of the literature on price dispersion and the insights of the literature on intertemporal price discrimination. Compared to the classic papers on price dispersion, our model is richer because it introduces a time dimension and characterizing the equilibrium distribution of multidimensional price vectors across stores poses new technical challenges that are first tackled in this paper.² Compared to the classic papers on intertemporal price discrimination, our model is richer as the analysis is carried out in an equilibrium framework that generates price dispersion across stores. Moreover, our model shows that the same element is sufficient to generate both equilibrium price dispersion and intertemporal price discrimination: Heterogeneity across buyers in their ability

²In Butters (1977), Varian (1980), and Burdett and Judd (1983), each seller is indifferent between posting any price on the support of the equilibrium price distribution. Therefore, if sellers choose different prices on different days, these models would generate price dispersion both across and within stores. However, this result would not be robust to the introduction of menu costs (which would discourage sellers from resetting their prices if there is not a strictly positive benefit from doing so) or to the introduction of heterogeneity in the sellers’ cost of production (which would break the seller’s indifference between any price on the support of the equilibrium distribution).

to shop around, at different locations and at different times of the day.³ The empirical evidence in Aguiar and Hurst (2007) and Kaplan and Menzio (2014a, 2014b) suggests that these traits are common to individuals with a relatively low value of time, such as the elderly and the unemployed.

2 Environment

We consider the market for an indivisible good that operates in two periods: daytime and nighttime.⁴ On one side of the market, there is a measure 1 of identical sellers who can produce the good on demand at a constant marginal cost, which we normalize to zero. Each seller simultaneously and independently posts a pair of prices (p_d, p_n) , where $p_d \in [0, u]$ is the price of the good during the day, $p_n \in [0, u]$ is the price of the good during the night, and $u > 0$ is the buyers' valuation of the good. We denote as $G(p_d, p_n)$ the distribution of prices across sellers. Similarly, we denote as F_d the marginal distribution of daytime prices and as F_n the marginal distribution of nighttime prices. Finally, we denote as F_m the marginal distribution of the lowest price of each seller.

On the other side of the market, there is a measure $\theta_x > 0$ of buyers of type x and a measure $\theta_y > 0$ of buyers of type y . The two types of buyers differ with respect to their ability to shop from different sellers, as well as with respect of their ability to shop at different times of the day. In particular, a buyer of type x is in contact with only one seller with probability $\alpha_x \in (0, 1)$ and with multiple (for the sake of simplicity, two) sellers with probability $1 - \alpha_x$. The buyer observes both the morning and afternoon price of the sellers with whom he is contact. However, the buyer is able to shop from these sellers during both the daytime and the nighttime only with probability $1 - \beta_x$. With

³Existing theories of intertemporal price discrimination assume that buyers differ in their valuation of the good. In Conslik, Gerstner, and Sobel, (1984) there are high and low valuation buyers. In Sobel (1984), one type of buyer has a higher valuation and a higher discount factor than the other type. In Albrecht, Postel-Vinay and Vroman (2012), one type of buyer has a higher valuation and consumes the good faster than the other type. Generally, the elements that are needed for intertemporal price discrimination to be profitable and feasible are that: (a) some buyers are willing to pay more for the good than others, and (b) these buyers are also less flexible in the timing of their purchases. In this paper we show that both of these elements follow from one common difference: Some buyers are worse than others at shopping at different stores (which increases their expected willingness to pay) and at different times (which makes them less flexible in the timing of their purchases).

⁴The reader should not interpret day and night literally. The key assumption is that some buyers are flexible with respect to their shopping time and others are not. Therefore, the reader can interpret the nighttime as Monday morning, as a particular day every week, or even as one particular day in any given month. Similarly, the reader can interpret the daytime as every time other than Monday morning or as every day of the week/month except for the one where sales are scheduled.

probability $\beta_x \in (0, 1)$, the buyer is able to shop from the contacted sellers only during the daytime. Similarly, a buyer of type y is in contact with one seller with probability α_y and with multiple (two) sellers with probability $1 - \alpha_y$. A buyer of type y is able to shop from the contacted sellers during both the daytime and the nighttime with probability $1 - \beta_y$ and only during the daytime with probability β_y . Both types of buyers enjoy a utility of $u - p$ if they purchase the good at the price p and a utility of zero if they do not purchase the good. Without loss in generality, we assume that buyers of type x are in contact with fewer sellers than buyers of type y , i.e. we assume $\alpha_x \geq \alpha_y$.

The definition of equilibrium for this model market is standard (see, e.g., Burdett and Judd, 1983 or Head et al 2012).

Definition 1: An equilibrium is a price distribution G such that the seller's profit is maximized everywhere on the support of G .

3 Characterization of equilibrium

In this section, we characterize the equilibrium set. We carry out the analysis in four steps. In Subsection 3.1, we show that we can restrict attention to equilibria in which every seller chooses a price for the good in the night that is non-greater than the price for the good in the day. In Subsection 3.2, we consider an equilibrium in which the profit of a seller attains its maximum for all daytime prices p_d on the support of the marginal distribution F_d , as well as for all nighttime prices p_n on the support of the marginal distribution F_n . We show that—if and only if buyers of type x are in contact with fewer sellers than buyers of type y , and they are less likely to be able to shop in the nighttime—this equilibrium exists and features price dispersion across and within sellers. In Subsection 3.3, we consider an equilibrium in which the profit of a seller attains its maximum for all prices (p_d, p_n) with $p_d = p_n$. We show that—if and only if buyers of type x are in contact with fewer sellers than buyers of type y , and they are more likely to shop in the nighttime—this equilibrium exists and features price dispersion across sellers but not within sellers. Finally, in Subsection 3.4, we rule out other types of equilibria.

3.1 A general property of equilibrium

As a preliminary step, we show that we can restrict attention to equilibria in which sellers post prices (p_d, p_n) such that $p_n \leq p_d$. This is the case because—by assumption—those

buyers who can shop in the nighttime can also shop in the daytime and, hence, a seller posting a higher price in the nighttime than in the daytime enjoys the same profit and exerts the same competition on other seller as if he were to post the same price at both times of day.

To formalize the above argument, consider an equilibrium in which the marginal price distributions are continuous functions⁵ F_d , F_n , and F_m . A seller who posts prices $(p_d, p_n) \in [0, u]^2$ enjoys a profit

$$\begin{aligned} V(p_d, p_n) &= [\mu_{1d} + \mu_{2d}(1 - F_d(p_d))] p_d \\ &\quad + [\mu_{1n} + \mu_{2n}(1 - F_m(\min\{p_d, p_n\}))] \min\{p_d, p_n\}, \end{aligned} \tag{1}$$

where the constants μ_{1d} and μ_{1n} are defined as

$$\begin{aligned} \mu_{1d} &= \theta_x \alpha_x \beta_x + \theta_y \alpha_y \beta_y, \\ \mu_{1n} &= \theta_x \alpha_x (1 - \beta_x) + \theta_y \alpha_y (1 - \beta_y), \end{aligned} \tag{2}$$

and the constants μ_{2d} and μ_{2n} are defined as

$$\begin{aligned} \mu_{2d} &= 2\theta_x (1 - \alpha_x) \beta_x + 2\theta_y (1 - \alpha_y) \beta_y, \\ \mu_{2n} &= 2\theta_x (1 - \alpha_x) (1 - \beta_x) + 2\theta_y (1 - \alpha_y) (1 - \beta_y). \end{aligned} \tag{3}$$

Let us briefly explain (1). The seller meets μ_{1d} buyers who are not in contact with any other seller and who can only shop in the daytime. Each one of these buyers will purchase the good from the seller at the price p_d . The seller meets μ_{1n} buyers who are not in contact with any other seller and who can shop both in the daytime and in the nighttime. Each one of these buyers will purchase the good from the seller at the price $\min\{p_d, p_n\}$. The seller meets μ_{2d} buyers who are in contact with a second seller and who can only shop in the daytime. Each one of these buyers will purchase the good from the seller if p_d is lower than the afternoon price posted by the second seller they contacted, an event that occurs with probability $1 - F_d(p_d)$. Finally, the seller meets μ_{2n} buyers who are in contact with a second seller and can shop both in the daytime and in the nighttime. Each one of these buyers will purchase the good from the seller if $\min\{p_d, p_n\}$ is lower than the lowest price posted by the second seller they meet, an event that occurs with probability $1 - F_m(\min\{p_d, p_n\})$.

⁵The assumption that the distribution functions F_d , F_n , and F_m are continuous is for the sake of exposition only. It is straightforward to generalize Lemma 1 to the case in which these distribution functions have mass points.

A seller posting the prices $(p_d, p_n) \in [0, u]^2$ with $p_n > p_d$ enjoys a profit

$$\begin{aligned} V(p_d, p_n) &= [\mu_{1d} + \mu_{2d}(1 - F_d(p_d))] p_d \\ &+ [\mu_{1n} + \mu_{2n}(1 - F_m(p_d))] p_d. \end{aligned} \tag{4}$$

Notice that the seller's profit does not depend on the nighttime price. Indeed, if $p_n > p_d$, the seller never makes a sale in the nighttime. The customers who can only shop in the daytime will purchase at the price p_d . The customers who can shop both in the daytime and in the nighttime will choose to purchase during the day at the price p_d . Therefore, the seller enjoys the same profit if he were to post the prices (p_d, p_d) rather than (p_d, p_n) .

Now, suppose that there is an equilibrium G in which some sellers post (p_d, p_n) with $p_n > p_d$. Consider an alternative price distribution \hat{G} in which the sellers posting (p_d, p_n) with $p_n > p_d$ change their prices to (p_d, p_d) , while the sellers posting (p_d, p_n) with $p_n \leq p_d$ keep their prices unchanged. Clearly, the marginal price distributions \hat{F}_d and \hat{F}_m associated with \hat{G} are the same as the marginal price distributions F_d and F_m associated with G . Since the prices (p_d, p_n) with $p_n \leq p_d$ maximize the profit of the seller given G , they also maximize the profit of the seller given \hat{G} , as the profit function (1) only depends on the marginals F_d and F_m . Moreover, since the prices (p_d, p_n) with $p_n > p_d$ maximize the profit of the seller given G , the prices (p_d, p_d) maximize the profit of the seller given \hat{G} , as the seller's profit function (1) only depends on the marginal F_d and F_m and, as shown in (4), the seller is indifferent between posting (p_d, p_n) and (p_d, p_d) . Thus, the joint price distribution \hat{G} is an equilibrium and it is—along all relevant dimensions—equivalent to the equilibrium joint price distribution G .

We have therefore established the following Lemma.

Lemma 1. Without loss in generality, we can restrict attention to equilibria G in which every seller posts a price $(p_d, p_n) \in [0, u]^2$ with $p_n \leq p_d$, and the marginal distribution of lowest prices, F_m , is equal to the marginal distribution of night prices, F_n .

3.2 Equilibrium with price dispersion across and within stores

In this section, we look for an equilibrium G in which every seller posts prices $(p_d, p_n) \in [0, u]^2$ with $p_n \leq p_d$ and such that the marginal distribution of daytime prices, F_d , and the marginal distribution of nighttime prices, F_n , are respectively given by

$$F_d(p) = 1 - \frac{\mu_{1d} u - p}{\mu_{2d} p}, \quad \forall p \in [p_{dl}, p_{dh}], \tag{5}$$

and

$$F_n(p) = 1 - \frac{\mu_{1n} u - p}{\mu_{2n} p}, \quad \forall p \in [p_{n\ell}, p_{nh}], \quad (6)$$

where the boundaries of the support of the distributions are

$$\begin{aligned} p_{t\ell} &= \frac{\mu_{1t}}{\mu_{1t} + \mu_{2t}} u, & \text{for } t = \{d, n\} \\ p_{th} &= u, & \text{for } t = \{d, n\}. \end{aligned} \quad (7)$$

Given the marginal price distributions F_d and F_n in (5) and (6), we can identify the region where the profit of the seller attains its maximum. In general, a seller posting prices $(p_d, p_n) \in [0, u]^2$ with $p_n \leq p_d$ attains a profit of

$$\begin{aligned} V(p_d, p_n) &= [\mu_{1d} + \mu_{2d}(1 - F_d(p_d))] p_d \\ &\quad + [\mu_{1n} + \mu_{2n}(1 - F_n(p_n))] p_n. \end{aligned} \quad (8)$$

If the seller posts prices (p_d, p_n) such that $p_d \in [p_{d\ell}, p_{dh}]$, $p_n \in [p_{n\ell}, p_{nh}]$ and $p_n \leq p_d$, his profit is given by

$$V(p_d, p_n) = [\mu_{1d} + \mu_{1n}] u, \quad (9)$$

where (9) follows from (8) and from the expressions for the marginal price distributions F_d and F_n in (5) and (6). Notice that (9) is a constant, i.e., the seller's profit attains the same value for all prices p_d on the support of the marginal distribution F_d and for all prices p_n on the support of the marginal price distribution F_n . Moreover, this profit is equal to the profit that the seller would attain if he were to charge the buyer's reservation price u both in the daytime and in the nighttime and sell only to those buyers who are not in contact with any other seller.

If the seller post prices (p_d, p_n) such that $p_d \in [p_{d\ell}, p_{dh}]$, $p_n \in [0, p_{n\ell})$ and $p_n \leq p_d$, his profit is given by

$$\begin{aligned} V(p_d, p_n) &= \mu_{1d} u + (\mu_{1n} + \mu_{2n}) p_n \\ &< [\mu_{1d} + \mu_{1n}] u, \end{aligned} \quad (10)$$

where the first line on the right-hand side of (10) follows from (8) and the fact that $F_n(p_n) = 0$ for all $p_n \leq p_{n\ell}$, and the second line on the right-hand side of (10) follows from $p_n < p_{n\ell}$ and $p_{n\ell} = u\mu_{1n}/(\mu_{1n} + \mu_{2n})$. Therefore, for any (p_d, p_n) such that $p_d \in [p_{d\ell}, p_{dh}]$, $p_n \in [0, p_{n\ell})$, and $p_n \leq p_d$, the profit of the seller is lower than in (9). This result is intuitive as lowering the price p_n below $p_{n\ell}$ reduces the profit per sale without increasing the probability of making a sale to a night shopper. Similarly, for any (p_d, p_n) such that $p_d \in [0, p_{d\ell})$, $p_n \in [0, p_{nh}]$, and $p_n \leq p_d$, the profit of the seller is lower than in

(9), as lowering the price p_d below p_{dl} reduces the profit per sale without increasing the probability of making a sale to a daytime shopper. Finally, as established in section 3.1, the seller is indifferent between posting the prices (p_d, p_n) with $p_n > p_d$ and the prices (p_d, p_d) .

Taken together, the above observations imply that the seller's profit attains its maximum for all prices (p_d, p_n) such that $p_d \in [p_{dl}, p_{dh}]$, $p_n \in [p_{nl}, p_{nh}]$, and $p_n \leq p_d$, and it attains strictly less than the maximum for all other prices such that $p_n \leq p_d$. Therefore, an equilibrium such that all sellers post a nighttime price lower than the daytime price and where the marginal price distributions F_d and F_n are given as in (5) and (6) exists if and only if we can find a joint price distribution G such that: (a) the support of G lies in the region of prices (p_d, p_n) with $p_d \in [p_{dl}, p_{dh}]$, $p_n \in [p_{nl}, p_{nh}]$, and $p_n \leq p_d$; (b) the joint price distribution G generates the marginals F_d and F_n .

Clearly, a necessary condition for the existence of the desired equilibrium is that the marginal distribution of prices in the daytime first order stochastically dominates the marginal distribution of prices in the nighttime, i.e. $F_d(p) \leq F_n(p)$ for all $p \in [0, u]$. From (5) and (6), it follows that $F_d(p) \leq F_n(p)$ is equivalent to $\mu_{1d}/\mu_{2d} \geq \mu_{1n}/\mu_{2n}$. Moreover, the condition $F_d(p) \leq F_n(p)$ or, equivalently, $\mu_{1d}/\mu_{2d} \geq \mu_{1n}/\mu_{2n}$ is also sufficient for the existence of the desired equilibrium. To see why this is the case, suppose that the distribution of sellers over night prices is the F_n in (6) and a seller who posts p_n in the nighttime posts the price $g(p_n)$ in the daytime, where

$$g(p_n) = \left[\frac{\mu_{1n}}{\mu_{2n}}u + \left(\frac{\mu_{1d}}{\mu_{2d}} - \frac{\mu_{1n}}{\mu_{2n}} \right) p_n \right]^{-1} \frac{\mu_{1d}}{\mu_{2d}} p_n. \quad (11)$$

Given that sellers post $(g(p_n), p_n)$, it is immediate to verify that the marginal distribution of prices in the afternoon is the F_d in (5). Moreover, if $\mu_{1d}/\mu_{2d} \geq \mu_{1n}/\mu_{2n}$, it is easy to verify that the support of the joint price distribution G lies in the region $p_d \in [p_{dl}, u]$, $p_n \in [p_{nl}, u]$, and $p_n \leq p_d$.

Overall, the necessary and sufficient condition for the existence of the desired equilibrium is

$$\frac{\mu_{1d}}{\mu_{2d}} \geq \frac{\mu_{1n}}{\mu_{2n}}. \quad (12)$$

In words, the necessary and sufficient condition (12) states that the ratio of captive buyers—i.e. buyers who are in contact with a particular seller and nobody else—to non-captive buyers—i.e. buyers who are in contact with a particular seller and a second

one—must be greater in the day than at night.

In what follows, we vary the parameters of the model $(\alpha_x, \beta_x, \alpha_y, \beta_y)$ and verify whether condition (12) is satisfied.

Case 1: Buyers of type x are in contact with fewer sellers than buyers of type y and are less likely to shop at night, i.e. $\alpha_x > \alpha_y$ and $\beta_x > \beta_y$. Using (2)-(3) and (5)-(6), it is straightforward to verify that $\alpha_x > \alpha_y$ and $\beta_x > \beta_y$ imply $\mu_{1d}/\mu_{2d} > \mu_{1n}/\mu_{2n}$ and $F_d < F_n$. Since condition (12) is satisfied, there exists a joint price distribution G whose support lies on the required region and that generates the marginals F_d and F_n in (5) and (6). Moreover, since $F_d < F_n$, any such joint price distribution G must be such that a positive measure of sellers posts a strictly lower price at night than during the day. Hence, the equilibrium features price dispersion both across stores—in the sense that the marginal price distributions F_d and F_n are non-degenerate—and within stores—in the sense that a positive measure of sellers sells the good at different prices during different times of the day. As in Butters (1977) and Burdett and Judd (1983), price dispersion across stores emerges because the equilibrium price distribution makes sellers indifferent between posting a high price, enjoying a high profit margin, and selling a small quantity of the good and posting a low price, enjoying a low profit margin, and selling a large quantity of the good. As in Conslik, Gerstner, and Sobel (1984), price dispersion within stores emerges because, when $\alpha_x > \alpha_y$ and $\beta_x > \beta_y$, sellers have the incentive and the opportunity to price discriminate between different types of buyers. Indeed, since the two types of buyers differ in their likelihood to shop at night, sellers face a different composition of buyers in the two times of the day. Moreover, since the type of buyer who is more likely to shop at night is also the type of buyer who is in contact with more sellers, sellers face more competition at night. As a result, sellers find it optimal to post lower prices—in the sense of first order stochastic dominance—at night than during the day.

Case 2: Buyers of type x are in contact with fewer sellers than buyers of type y and they are more likely to shop at night, i.e., $\alpha_x > \alpha_y$ and $\beta_x < \beta_y$. Using (2)-(3) and (5)-(6), one can verify that $\alpha_x > \alpha_y$ and $\beta_x < \beta_y$ imply $\mu_{1d}/\mu_{2d} < \mu_{1n}/\mu_{2n}$. Since condition (12) is violated, there exists no joint price distribution G whose support is on the required region and that generates the marginals F_d and F_n in (5) and (6). Let us explain this result. Since the two types of buyers differ with respect to their ability to shop at night, sellers face a different composition of buyers in the two times of the day. Moreover, since the type of buyer who is more likely to shop at night is in contact with fewer sellers, sellers

face less competition at night. Hence, sellers would like to post higher prices at night than during the day but this would induce. But this is not compatible with equilibrium, as it would induce buyers who can shop at night to visit the sellers during the day.

In between cases 1 and 2, there are two knife-edge cases.

Case 3: Buyers of type x are in contact with fewer sellers than buyers of type y , but are equally likely to shop at night, i.e. $\alpha_x \geq \alpha_y$ and $\beta_x = \beta_y$. Using (2) and (3), it is straightforward to verify that $\alpha_x \geq \alpha_y$ and $\beta_x = \beta_y$ imply that $\mu_{1d}/\mu_{2d} = \mu_{1n}/\mu_{2n}$. In turn, using (5) and (6), it is immediate to verify that $\mu_{1d}/\mu_{2d} = \mu_{1n}/\mu_{2n}$ implies $F_d = F_n$. Since $\mu_{1d}/\mu_{2d} = \mu_{1n}/\mu_{2n}$, we know that there exists a joint price distribution G whose support lies in the required region and that generates the marginals F_d and F_n . Moreover, since $F_d = F_n$ and all sellers must post a nighttime price non-smaller than their daytime price, the only equilibrium G is the one where every seller posts the same price at both times of day. Hence, while the equilibrium features price dispersion across sellers, it does not feature price dispersion within sellers. This result is intuitive. Since the two types of buyers are equally likely to shop at night, sellers face the same composition of buyers and, hence, the same amount of competition in the daytime and in the nighttime. For this reason, the equilibrium marginal price distribution is the same in the two times of day. And since sellers post a lower price at night than during the day, this implies that every individual seller must always post the same price.

Case 4: Buyers of type x are in contact with the same number of sellers as buyers of type y , but they are less likely to shop at night, i.e., $\alpha_x = \alpha_y$ and $\beta_x > \beta_y$. Again, it is straightforward to verify that $\alpha_x = \alpha_y$ and $\beta_x > \beta_y$ imply $\mu_{1d}/\mu_{2d} = \mu_{1n}/\mu_{2n}$ and $F_d = F_n$, which, in turn, implies that every seller posts the same price during the daytime and the nighttime. This result is also intuitive. Since the two types of buyers differ with respect to their ability to shop at night, a seller faces a different composition of buyers in the daytime and in the nighttime. However, since the two types of buyers are in contact with the same number of sellers, this difference in composition does not translate into a difference in competition. As a result, the equilibrium marginal price distribution during the day is the same as during the night, and every individual seller must always post the same price.

The above analysis is summarized in Proposition 1.

Proposition 1. An equilibrium G in which sellers post prices $(p_d, p_n) \in [0, u]^2$ with $p_n \leq p_d$ and such that the marginal price distributions F_d and F_n are as in (5) and (6)

exists if and only if $\alpha_x = \alpha_y$ or $\beta_x \geq \beta_y$. If $\alpha_x > \alpha_y$ and $\beta_x > \beta_y$, the equilibrium features price dispersion across and within sellers. If either $\alpha_x = \alpha_y$ or $\beta_x = \beta_y$, the equilibrium features price dispersion across sellers but not within sellers.

3.3 Equilibrium without within-store price dispersion

In this section, we look for an equilibrium in which the joint price distribution, G , is such that every seller posts the same price for the good during the day and during the night and such that the marginal distribution of daytime and nighttime prices is given by

$$F_d(p) = F_n(p) = 1 - \frac{\mu_{1n} + \mu_{1d}}{\mu_{2n} + \mu_{2d}} \frac{u - p}{p}, \quad \forall p \in [p_\ell, p_h], \quad (13)$$

where the boundaries of the support of the distributions are

$$p_\ell = \frac{\mu_{1n} + \mu_{1d}}{\mu_{1n} + \mu_{1d} + \mu_{2n} + \mu_{2d}} u, \quad p_h = u. \quad (14)$$

First, consider a seller posting the prices (p, p) with $p \in [p_\ell, p_h]$. This seller obtains a profit of

$$\begin{aligned} V(p, p) &= \left[\mu_{1d} + \mu_{1n} + (\mu_{2d} + \mu_{2n}) \frac{\mu_{1d} + \mu_{1n}}{\mu_{2d} + \mu_{2n}} \frac{u - p}{p} \right] p \\ &= [\mu_{1d} + \mu_{1n}] u. \end{aligned} \quad (15)$$

The first line on the right-hand side of (15) follows from (8) and the expression for the marginal price distributions F_d and F_n in (13). The second line on the right-hand side of (15) follows from algebraic manipulation of the first. Notice that the second line on the right-hand side of (15) is a constant. That is, the seller attains the same profit by posting any prices (p, p) on the support of the joint distribution G . Moreover, this profit is equal to the profit that the seller would attain if he were to charge the buyer's reservation price u both in the daytime and in the nighttime and sell only to those buyers who are not in contact with any other seller.

Second, consider a seller posting prices (p_d, p_n) , with $p_d \in [p_\ell, p_h]$, $p_n \in [p_\ell, p_h]$, and $p_n \leq p_d$. This seller obtains a profit of

$$\begin{aligned} V(p_d, p_n) &= \left[\mu_{1d} + \mu_{2d} \frac{\mu_{1d} + \mu_{1n}}{\mu_{2d} + \mu_{2n}} \frac{u - p_d}{p_d} \right] p_d \\ &\quad + \left[\mu_{1n} + \mu_{2n} \frac{\mu_{1d} + \mu_{1n}}{\mu_{2d} + \mu_{2n}} \frac{u - p_n}{p_n} \right] p_n. \end{aligned} \quad (16)$$

Notice that the derivative of the seller's profit with respect to the nighttime price, p_n , is strictly positive if $\mu_{1n}/\mu_{2n} < \mu_{1d}/\mu_{2d}$; it is zero if $\mu_{1n}/\mu_{2n} = \mu_{1d}/\mu_{2d}$; and it is negative if $\mu_{1n}/\mu_{2n} > \mu_{1d}/\mu_{2d}$. Hence, if the seller posts prices (p_d, p_n) with $p_d \in [p_\ell, p_h]$, $p_n \in [p_\ell, p_h]$ and $p_n \leq p_d$, he attains a profit non-greater than (15) if and only if $\mu_{1n}/\mu_{2n} \leq \mu_{1d}/\mu_{2d}$.

Third, consider a seller posting prices (p_d, p_n) , with $p_d \in [p_\ell, p_h]$, $p_n \in [0, p_\ell]$, and $p_n \leq p_d$. This seller's profit is lower than what he could attain by posting the prices (p_d, p_ℓ) , as lowering the price p_n below p_ℓ reduces the profit per sale without increasing the probability of making a sale to a night shopper. Similarly, for any (p_d, p_n) such that $p_d \in [0, p_\ell)$, $p_n \in [0, p_h]$, and $p_n \leq p_d$, the seller's profit is lower than what he could attain by posting the prices (p_ℓ, p_n) , as lowering the price p_d below p_ℓ reduces the profit per sale without the probability of making a sale to a day shopper. Finally, as established in section 3.1, the seller is indifferent between posting the prices (p_d, p_n) with $p_n > p_d$ and the prices (p_d, p_d) .

From the above observations, it follows that the seller's profit is maximized everywhere on the support of the joint price distribution G if and only if

$$\frac{\mu_{1n}}{\mu_{2n}} \leq \frac{\mu_{1d}}{\mu_{2d}}. \quad (17)$$

In words, the necessary and sufficient condition (17) states that the ratio of captive buyers to non-captive buyers must be greater at night than during the day. Notice that condition (17) is the opposite as condition (12) and, hence, for any values of the parameters, there exists either the type of equilibrium studied in Subsection 3.2 or the type of equilibrium studied in this subsection. Moreover, the two types of equilibria coexist only when $\mu_{1n}/\mu_{2n} = \mu_{1d}/\mu_{2d}$, which is a knife-edge configuration of parameters.

In particular, we have the following cases.

Case 1 Buyers of type x are in contact with fewer sellers than buyers of type y and are less likely to shop at night, i.e., $\alpha_x > \alpha_y$ and $\beta_x > \beta_y$. When $\alpha_x > \alpha_y$ and $\beta_x > \beta_y$, condition (17) is violated and, hence, there is no equilibrium in which all sellers post the same price at both times of day, and the marginal price distributions F_d and F_n are given by (13). Intuitively, when $\alpha_x > \alpha_y$ and $\beta_x > \beta_y$, sellers face more competition at night than during the day. For this reason, sellers have an incentive to post lower prices—in the sense of first order stochastic dominance—at night than during the day.

Case 2: Buyers of type x are in contact with fewer sellers than buyers of type y and they

are more likely to shop at night, i.e., $\alpha_x > \alpha_y$ and $\beta_x < \beta_y$. When $\alpha_x > \alpha_y$ and $\beta_x < \beta_y$, condition (17) is satisfied and, hence, there exists an equilibrium in which all sellers post the same price at both times of day, and the marginal price distributions F_d and F_n are given by (13). In this equilibrium, there is price dispersion across stores—in the sense that different sellers post different prices—but no price dispersion within stores—in the sense that every seller posts the same price at all times. Intuitively, when $\alpha_x > \alpha_y$ and $\beta_x < \beta_y$, sellers face more competition during the day than at night. For this reason, sellers want to post a nighttime price as high as possible. However, sellers cannot post a nighttime price higher than the daytime price or, else, buyers who can shop at night will purchase the good during the day. As a result, sellers post a nighttime price equal to the daytime price.

In between cases 1 and 2, there are two knife-edge cases. In these cases, the type of equilibrium that we considered in Subsection 3.2 and the type of equilibrium that we are considering here coexist and coincide.

Case 3: Buyers of type x are in contact with fewer sellers than buyers of type y , but are equally likely to shop at night, i.e., $\alpha_x \geq \alpha_y$ and $\beta_x = \beta_y$. In this case, condition (17) holds with equality. Therefore, there exists an equilibrium in which sellers post the same price during the day and during the night, and the marginal price distributions F_d and F_n are given as in (13). Intuitively, when $\alpha_x \geq \alpha_y$ and $\beta_x = \beta_y$, sellers face the same composition of buyers during the day and during the night and, hence, they have no incentive to vary their price over time. Notice that, when $\alpha_x \geq \alpha_y$ and $\beta_x = \beta_y$, condition (12) holds as well and, hence, there exists also an equilibrium in which the marginal price distributions F_d and F_n are given as in (5) and (6). However, as discussed in the previous subsection, this equilibrium is also such that sellers post the same price in the two periods. Moreover, it is immediate to see that the marginal price distributions F_d and F_n in (5) and (6) are the same as in (13). Hence, the two types of equilibria coexist and are identical.

Case 4: Buyers of type x are in contact with the same number of sellers as buyers of type y , but they are less likely to shop at night, i.e., $\alpha_x = \alpha_y$ and $\beta_x \geq \beta_y$. In this case, condition (17) holds with equality. Therefore, there exists an equilibrium in which sellers post the same price during the day and during the night, and the marginal price distributions F_d and F_n are given as in (13). Intuitively, when $\alpha_x = \alpha_y$ and $\beta_x \geq \beta_y$, sellers face a different composition of buyers during the day and during the night but this

difference in composition does not translate into a difference in competition because both types of buyers are in contact with the same number of sellers. For this reason, sellers have no incentive to vary their price over time. Notice that, also when $\alpha_x = \alpha_y$ and $\beta_x \geq \beta_y$, this equilibrium coexists and coincides with the one studied in Subsection 3.2.

The above analysis is summarized in Proposition 2.

Proposition 2. An equilibrium G in which all sellers post the same price in the morning and in the afternoon and in which the marginal price distributions F_d and F_n are given as in (13) exists if and only if $\alpha_x = \alpha_y$ or $\beta_x \leq \beta_y$.

3.4 Other equilibria

The final step of the analysis is to rule out the existence of any type of equilibrium different from those studied in Subsections 3.2 and 3.3. To this aim, consider an equilibrium distribution of sellers over prices, $G(p_d, p_n)$. Let $F_d(p_d)$ denote the marginal distribution of sellers over daytime prices and as $m_d(p_d)$ the measure of sellers who post a daytime price of p_d , i.e., the mass point associated with the price p_d . Similarly, let $F_n(p_n)$ denote the marginal distribution of sellers over nighttime prices and as $m_n(p_n)$ the measure of sellers who post a morning price of p_n . In light of Lemma 1, we can restrict attention to equilibria in which all sellers post a price $p_n \leq p_d$ and, consequently, such that the marginal distribution of sellers over their lowest price, F_m , is equal to F_n .

In equilibrium, a seller posting prices (p_d, p_n) with $p_n \leq p_d$ attains a profit of

$$V(p_d, p_n) = V_d(p_d) + V_n(p_n), \quad (18)$$

where V_d and V_n are respectively defined as

$$V_d(p_d) = \left[\mu_{1d} + \mu_{2d} \left(1 - F_d(p_d) + \frac{1}{2} m_d(p_d) \right) \right] p_d, \quad (19)$$

and

$$V_n(p_n) = \left[\mu_{1n} + \mu_{2n} \left(1 - F_n(p_n) + \frac{1}{2} m_n(p_n) \right) \right] p_n. \quad (20)$$

In words, $V_d(p_d)$ denotes the seller's profit from daytime trades. In fact, in the daytime, the seller meets μ_{1d} captive buyers and μ_{2d} non-captive buyers. A captive buyer purchases the good from the seller with probability one. A non-captive buyer purchases the good from the seller with probability one if he is in contact with a second seller whose price is strictly

greater than p_d , an event that occurs with probability $1 - F_d(p_d)$ or with probability $1/2$ if he is in contact with a second seller whose price is equal to p_d , an event that occurs with probability $m_d(p_d)$. Similarly, $V_n(p_n)$ denotes the seller's profit from nighttime trades.

Every price pair (p_d, p_n) on the support of the distribution G must maximize the profit $V(p_d, p_n)$ of the seller. We use this property to establish several features of the equilibrium.

Claim 1. The marginal price distributions F_d and F_n have no mass points.

Proof: We begin by proving that F_d has no mass points. On the way to a contradiction, suppose that there exists an equilibrium G in which F_d has a mass point at p_d^* . Consider a seller posting the prices (p_d^*, p_n) with $p_n < p_d^*$. From (18), it follows that this seller can attain a strictly higher profit by posting the prices $(p_d^* - \epsilon, p_n)$ for some $\epsilon > 0$ sufficiently small. Hence, no prices (p_d^*, p_n) with $p_n < p_d^*$ can be on the support of G . Next, consider a seller posting prices (p_d^*, p_d^*) . From (18), it follows that this seller can attain a strictly higher profit by choosing the prices $(p_d^* - \epsilon, p_d^* - \epsilon)$ for some $\epsilon > 0$ sufficiently small. Hence, the prices (p_d^*, p_d^*) cannot be on the support of G . Finally, since G is such that every seller posts a price p_n smaller than p_d , no prices (p_d^*, p_n) with $p_n > p_d^*$ can be on the support of G . We have thus reached a contradiction. The proof that F_n has no mass points is analogous. ■

Claim 2. The marginal price distribution F_d has no gaps and $p_{dh} = u$.

Proof: We first establish that F_d has no gaps. On the way to a contradiction, suppose that F_d has a gap between p_0 and p_1 with $p_1 > p_0$. Since $F_d(p_1) = F_d(p_0)$, a seller posting prices (p_0, p_n) with $p_n \leq p_0$ can attain a strictly higher profit by choosing the prices (p_1, p_n) instead. Hence, the prices (p_0, p_n) with $p_n \leq p_0$ cannot be on the support of G . Similarly, since G is such that every seller posts a price p_n smaller than p_d , no prices (p_0, p_n) with $p_n > p_0$ can be on the support of G . We have thus reached a contradiction. The proof that $p_{dh} = u$ is analogous. ■

Claim 3. Let $p_{n\ell}$ be the lower bound of the support of the marginal price distribution F_n . The profit function $V_n(p_n)$ is weakly increasing in p_n over the interval $[p_{n\ell}, u]$.

Proof: On the way to a contradiction, suppose $V_n(p_n)$ is strictly decreasing over the interval (p_0, p_1) , with $p_{n\ell} \leq p_0 < p_1 \leq u$. If this is the case, $V_n(p_n) < V_n(p_0)$ for all $p_n \in (p_0, p_2)$ where $p_2 > p_1$. Any seller with a daytime price $p_d \geq p_2$, will choose a nighttime price p_n such that $p_n \leq p_d$ and $p_n \notin (p_0, p_2)$. Any seller with a daytime price

$p_d \in (p_0, p_2)$ will choose a nighttime price $p_n \leq p_0$. And any seller with a daytime price $p_d \leq p_0$ will choose a nighttime price smaller than p_d . Therefore, the marginal price distribution F_n has a gap between p_0 and p_2 , i.e., $F_n(p_n) = F_n(p_0)$ for all $p \in (p_0, p_2)$. From (20), it follows that if F_n is constant over the interval (p_0, p_2) , then $V_n(p)$ is strictly increasing over the interval (p_0, p_2) , which contradicts the assumption that $V_n(p)$ is strictly decreasing over the interval (p_0, p_1) . ■

Claim 4. The function $V_n(p_n)$ is either strictly increasing for all $p_n \in [p_{n\ell}, u]$, or it is constant for all $p_n \in [p_{n\ell}, u]$.

Proof: Suppose $V_n(p_n)$ is strictly increasing over some region (p_0, p_1) , where $p_{n\ell} \leq p_0 < p_1 \leq u$. This implies that a seller posting a daytime price $p_d \geq p_1$ chooses a nighttime price $p_n \geq p_1$. A seller posting a daytime price $p_d \in (p_0, p_1)$ chooses a nighttime price $p_n = p_d$. And a seller posting a daytime price $p_d \leq p_0$ must post a nighttime price $p_n \leq p_0$. Therefore, for all $p \in (p_0, p_1)$, the fraction of sellers with a nighttime price smaller than p is equal to the fraction of sellers with a daytime price smaller than p , i.e., $F_n(p) = F_d(p)$ for all $p \in (p_0, p_1)$. Using this fact and $V(p, p) = V(p_1, p_1)$ for all $p \in (p_0, p_1)$, we obtain

$$F_n(p) = F_n(p_1) - \frac{\mu_{1n} + \mu_{1d} + (\mu_{2n} + \mu_{2d})(1 - F_n(p_1))}{\mu_{2n} + \mu_{2d}} \frac{p_1 - p}{p}, \forall p \in (p_0, p_1). \quad (21)$$

Given the expression for F_n in (21), we can compute the derivative of the function $V_n(p_n)$, which is given by

$$V_n'(p_n) = \mu_{1n} - \mu_{2n} \frac{\mu_{1n} + \mu_{1d}}{\mu_{2n} + \mu_{2d}}, \forall p \in (p_0, p_1). \quad (22)$$

The derivative is strictly positive if and only if $\mu_{1n}/\mu_{2n} > \mu_{1d}/\mu_{2d}$. Thus, if $\mu_{1n}/\mu_{2n} \leq \mu_{1d}/\mu_{2d}$, the function $V_n(p_n)$ cannot be strictly increasing over the region (p_0, p_1) and, in light of Claim 3, it must be constant for all $p \in [p_{n\ell}, u]$.

Conversely, suppose $V_n(p_n)$ is constant over some region (p_0, p_1) with $p_0 \geq p_{n\ell}$. In this case, we can prove that $\mu_{1n}/\mu_{2n} \leq \mu_{1d}/\mu_{2d}$. Thus, if $\mu_{1n}/\mu_{2n} > \mu_{1d}/\mu_{2d}$, the function $V_n(p_n)$ cannot be constant over some region (p_0, p_1) and, in light of Claim 3, it must be strictly increasing for all $p \in [p_{n\ell}, u]$. ■

Now, suppose that the equilibrium is such that $V_n(p_n)$ is constant over the interval $[p_{n\ell}, u]$. In this case, it is straightforward to verify that the marginal distribution of nighttime prices, F_n , is given as in (5). Moreover, since $V_n(p_n)$ is constant, the function $V_d(p_d)$ must also be constant over the interval $[p_{d\ell}, u]$. It is also straightforward to verify that this implies that the marginal distribution of daytime prices, F_d , is given as in (6).

Thus, the only equilibrium with a constant $V_n(p_n)$ is the one characterized in Subsection 3.2.

Next, suppose that the equilibrium is such that the function $V_n(p_n)$ is strictly increasing over the interval $[p_{n\ell}, u]$. In this case, every seller posts the same price in the morning and in the afternoon and the marginal price distributions F_d and F_n are identical. In turn, this implies that the marginal price distributions F_d and F_n are given as in (13). Thus, the only equilibrium with a strictly increasing $V_n(p_n)$ is the one characterized in Subsection 3.3.

Thus, we have established the following result.

Proposition 3. Any equilibrium G is such that either: (i) the marginal price distributions F_d and F_n are given as in (6) and (7); or (ii) the marginal price distributions F_d and F_n are given as in (13).

4 Conclusions

We developed a search-theoretic framework that generates equilibrium price dispersion across sellers and within sellers. Price dispersion across sellers is obtained because the buyers are heterogeneous in their ability to shop at different stores. Price dispersion within sellers is obtained when the buyers who are better at shopping at different stores are also better at shopping at less different times and, hence, sellers can discriminate between different types of buyers by varying their price over time. Our model is simpler and its predictions richer than standard models of intertemporal price discrimination. Our model could be estimated by extending the econometric techniques developed by Hong and Shum (2006) and Moraga-Gonzales and Wildenbeest (2009).

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