Working Paper Series

Technology Adoption and Leapfrogging: Racing for Mobile Payments

WP 21-05R

Pengfei Han
Peking University

Zhu Wang
Federal Reserve Bank of Richmond

This paper can be downloaded without charge
from: <http://www.richmondfed.org/publications/>

Richmond • Baltimore • Charlotte

Technology Adoption and Leapfrogging: Racing for Mobile Payments*

Pengfei Han[†] and Zhu Wang[‡]

February 6, 2023

Abstract

Paying with a mobile phone is a cutting-edge innovation that is transforming the global payments landscape. Some developing countries have surprisingly overtaken advanced economies in adopting the mobile payment innovation. We construct a dynamic model with sequential payment innovations to explain this phenomenon, which uncovers how advanced economies' past success in adopting card-payment technology holds them back in the mobile-payment race. Our calibrated model matches the cross-country adoption patterns of card and mobile payments and also explains why advanced and developing countries favor different mobile payment solutions. Based on the model, we conduct quantitative exercises for welfare and policy analyses.

Keywords: Technology Adoption, Payments, FinTech, Financial Development

JEL Classification: E4, G2, O3

*We thank Mark Bills, Yiting Deng, Jeremy Greenwood, Apoorv Gupta, Zhiguo He, Chang-Tai Hsieh, Chad Jones, Boyan Jovanovic, Pete Klenow, Dirk Krueger, Jane Li, Florin Maican, Nitya Nayar, Shumiao Ouyang, Zheng (Michael) Song, Pengfei Wang, Shang-Jin Wei, Kairong Xiao, Wei Xiong, Daniel Yi Xu and participants at various seminars and conferences for helpful comments, and Emily Emick and Brennan Merone for research assistance. The views expressed herein are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Richmond or the Federal Reserve System.

[†]Affiliation: Guanhua School of Management, Peking University, Beijing, China. Email address: pengfeihan@gsm.pku.edu.cn.

[‡]Affiliation: Research Department, Federal Reserve Bank of Richmond, Richmond, VA, USA. Email address: zhu.wang@rich.frb.org.

1 Introduction

The payments system is an essential financial technology infrastructure of the aggregate economy. With the successful launch of general-purpose credit cards in the late 1950s and debit cards in the mid-1980s, the United States has been one of the leading countries in deploying card payment technologies. However, the U.S. is falling behind in adopting the recent mobile-phone-based payment innovation (henceforth, “mobile payment”).

In contrast, Kenya is an early success story for mobile payment adoption. Within four years after being launched in 2007, mobile payment has been adopted by nearly 70% of Kenya’s adult population (Jack and Suri, 2014). While the mobile payment technology in Kenya relies on Short Message Service (SMS), China introduced a mobile payment innovation based on smartphones and Quick Response (QR) codes that has gained explosive growth in usage in recent years. In 2017, a total of 276.8 billion mobile payment transactions were made in China, equivalent to 200 transactions per capita.¹

Figure 1. ADOPTION OF CARD AND MOBILE PAYMENTS (2017)

Figure 1 compares the adoption of card and mobile payments in three countries: Kenya, China, and the U.S. Figures 1A and 1B report the percentage of the adult population (age 15 and above) having a debit card and using a mobile payment service, respectively.² As shown by the figures, while the U.S. boasts a higher card payment adoption rate, it has fallen far behind Kenya and China in mobile payment adoption.

These observations raise concerns about the efficiency and innovativeness of the U.S. payments system and lead to important research and policy questions. Why did devel-

¹Source: *Statistical Yearbook of Payment and Settlement of China*.

²Sources: Global Financial Inclusion (Global Findex) Database of the World Bank, and eMarketer. See Appendix I for the data details.

oping countries lag in adopting card payments but some of them leapfrog in adopting mobile payments? Have advanced economies lost their leadership in the payment area? What government policies, if any, should be considered to facilitate mobile payment development?

This paper addresses these questions. We first compile a novel dataset to examine the general adoption patterns of card and mobile payments across countries beyond the idiosyncratic cases of Kenya, China, and the U.S. We find that card payment adoption increases monotonically with per capita income. In contrast, the adoption of mobile payment shows a non-monotonic relationship with per capita income: increasing among low-income countries, decreasing among middle-income countries, and increasing again among high-income countries. Moreover, advanced economies and developing countries tend to adopt different mobile payment solutions. The former favor systems that are complementary to card usage, while the latter choose those that substitute for cards.

We then construct a theory to explain the early success of advanced economies in adopting card payment, and how their advantage in card payment later hinders the adoption of mobile payment. In our model, three payment technologies—cash, card, and mobile—arrive sequentially. Newer technologies lower the variable costs of making payments, but they require a fixed cost to adopt. When card arrives after cash, high-income consumers adopt earlier because they spend more on purchases, and thus can save more on the variable costs of payments.³ This explains the high adoption rate of card payments in rich countries. However, when mobile arrives after card, adoption incentives are different between existing card users and cash users. Because the incremental reduction in variable costs brought by mobile is smaller for card users than for cash users, the former face a higher income threshold to switch to mobile than the latter. As a result, the pre-mobile composition of cash and card users in each country leads to a non-monotonic relationship between mobile payment adoption and per capita income across countries. Moreover, to save on adoption costs, cash users favor mobile solutions that bypass card while card users prefer capitalizing on card. This explains why most developing countries choose Mobile Money (a card-substituting technology), whereas most advanced economies choose card-complementing mobile solutions such as Apple Pay.

³In our analysis, adopting card payment includes agents' decision to join the formal banking system combined with choosing card as the cost-effective payment solution.

Our model calibration matches cross-country adoption patterns of both card and mobile payments well. Based on the calibrated model, we conduct counterfactual and welfare analysis. We find that lagging behind in mobile payment adoption does not necessarily imply that advanced economies fall behind in overall payment efficiency, even though they may benefit less from the mobile payment innovation than some developing countries. Down the road, greater technological advances in mobile payments are needed for advanced economies to catch up in the payment race, and policy interventions require prudent social cost-benefit analyses.

By focusing on the role of income heterogeneity, our model largely abstracts from network externality considerations. This is an intentional modeling choice for the following reasons. First, while network effects may play a role in payment adoption, they are unlikely the driving forces for the non-monotonic relationship between mobile adoption and per capita income across countries. In fact, even if network effects could help explain developing countries' slow adoption of card, they would not simultaneously explain those countries' leapfrogging in adopting mobile. Second, given that our model aims to explain the steady-state payment adoption patterns rather than characterizing transitional paths, there is less need to elaborate on the feedback loops among agents. Also, our simplified approach, in the spirit of Occam's razor, allows us to fit the data well with a parsimonious model, which also facilitates the subsequent counterfactual and welfare analysis. Finally, we extend the model to a two-sided market setting in Section 6.3 and specify conditions under which the model indeed incorporates network effects between consumers and merchants regarding their payment choices. This interpretation allows us to discuss issues otherwise veiled in a one-sided market setting, such as multiple equilibria and the social versus private costs of adopting payment innovations. We point out that our model may serve as a first-order approximation if some of the conditions in Section 6.3 do not strictly hold and we leave a full-blown two-sided market model for future research.

Our paper contributes to several strands of literature. The first is the theories of payments. In recent years, a fast growing literature has been developed for studying market structure and pricing of retail payments, especially card payments (e.g., Rochet and Tirole, 2002, 2003, Wright, 2003, 2012, and Shy and Wang, 2011). However, most of those studies assume a static environment. Among very few exceptions, Alvarez and Lippi (2009, 2017) study consumer payment choices in dynamic settings, but they do not

consider sequential innovations and leapfrogging, which is the focus of this paper.

The second is comparative studies on adoption and usage of payment innovations. While there is an abundance of literature studying domestic payment patterns (e.g., Rysman, 2007, Klee, 2008, Wang and Wolman, 2016 for the U.S.), cross-country studies are rather scarce. We fill this gap by compiling a novel dataset to study cross-country adoption patterns of mobile versus card payments. Our dataset includes both developed and developing economies, which allows us to uncover and address the leapfrogging puzzle.

Our paper is also related to the literature on the rise of digital payments and FinTech payment firms. According to Berg et al. (2022), the rise of FinTech payment firms is one of the most significant changes to the financial industry over the last decade. This has had positive impacts on financial inclusion and welfare (e.g., Jack and Suri (2014) on mobile payments in Kenya and Muralidharan et al. (2016) on smartcard payments in India). Digital payment services provided by FinTech firms also transform the lending business (e.g., Parlour et al., 2021, Ghosh et al., 2021, Ouyang, 2021). Our paper complements those works in the sense that we take a structural approach to study how cost savings of different electronic payments affect payment efficiency and drive different adoption patterns across countries.

Our analysis also contributes to the literature of financial development. The tradeoff between fixed and variable costs in our model is consistent with the mechanism of financial development studied in Greenwood and Jovanovic (1990). In their framework, agents need to pay a fixed adoption cost for accessing financial markets in order to gain a higher return. High-income agents are willing to pay for the access earlier and low-income agents wait until their incomes reach a threshold level. More recently, Philippon (2019) shows that the nature of fixed versus variable costs in robo-advising is likely to democratize access to financial services. Our model shares a similar insight but we extend it to sequential payment innovations to explain a novel cross-country leapfrogging pattern.

Finally, our paper contributes to the literature of technology diffusion. For a long time, researchers have been interested in the relationship between technology adoption and the heterogeneity of potential adopters (e.g., Griliches, 1957). While some argue that the observed adoption lags are evidence of information or coordination frictions, Manuelli and Seshadri (2014) among others have shown that the speed of adoption can be well explained by the moving equilibrium of frictionless models. Moreover, in the presence of

sequential innovations, some firms could get stuck with old technologies due to their past investments in technology-specific learning (e.g., Parente, 1994, Jovanovic and Nyarko, 1996, and Klenow, 1998). Our paper extends this line of research to a new context where consumers make frictionless adoption decisions on sequential payment innovations. We show high-income consumers and countries could be overtaken by their low-income counterparts in adopting mobile payments due to their sunk investments in precedent card payment technologies. Taking the theory to data, our model matches the non-monotonic relationship between mobile payment adoption and per capita income across countries, which is a novel empirical finding to the existing literature (e.g., Comin and Hobijn, 2004).

The remainder of this paper is structured as follows. Section 2 provides the background of mobile payments and summarizes stylized facts from a novel dataset regarding cross-country adoption patterns. Section 3 introduces the model and solves for the equilibrium. Section 4 calibrates the model and explores model implications via counterfactual exercises. Section 5 conducts welfare and policy analyses. Section 6 extends the model for robustness checks and provides further discussions. Finally, Section 7 concludes.

2 Background and stylized facts

Following Crowe et al. (2010), we define a mobile payment to be a money payment made for a product or service through a mobile phone, regardless of whether the phone actually accesses the mobile network to make the payment. Mobile payment technology can also be used to send money from person to person.

The very first mobile payment transaction in the world can be traced back to 1997, when Coca-Cola in Helsinki came out with a beverage vending machine where users could pay for the beverage with just an SMS message. Around the same time, the oil company Mobil also introduced a Radio Frequency Identification (RFID) device called Speedpass that allowed its users to pay for fuel at gas stations. These two earliest examples of mobile payment services were both based on SMS, and the payments were made through a mobile account that was linked to the user’s device.

The mobile payment systems based on SMS soon evolved into the world’s first phone-based banking service, launched by the Merita Bank of Finland in 1997. Later, this mobile payment technology progressed with more user applications, such as buying movie

tickets, ordering pizza, and arranging travels. In 2007, Vodafone launched one of the largest mobile payment systems in the world. It was based on SMS/USSD text messaging technology and offered various kinds of macro and micro payments.⁴ Vodafone launched this service in Kenya and Tanzania with the cooperation of local telecom operators.

The year 2011 witnessed major technology firms like Google and Apple entering the field of mobile payment. Google became the first major company to come up with a digital mobile wallet solution, Google Wallet. The wallet used the Near Field Communication (NFC) technology and allowed the customers to make payments, redeem coupons, and earn loyalty points. In 2014, Apple launched its own mobile payment service in the U.S. called Apple Pay (compatible with iPhone 6), which allowed the users to simply tap their phone against a contactless payment card terminal at the point of sale, paying instantly. Before long, competitors to Apple, such as Google and Samsung, released their respective apps, Android Pay (later merged with Google Wallet and became Google Pay) and Samsung Pay, in the wake of Apple Pay's success.

As a cutting-edge payment innovation, mobile brings many additional benefits compared with preceding card technologies, lowering both the adoption costs and variable costs of making payments. First, given that mobile phones had been widely adopted in most countries before the arrival of mobile payment, the fixed cost for adopting mobile payment is small for consumers and merchants. Second, mobile payment is fast, convenient, and more secure. Apple Pay, for example, enables the users to pay without unlocking their phones, and the Touch/Face ID of an iPhone adds extra security to authenticate a purchase. Apple Pay also encrypts payment information by a tokenization technology, thus enhancing privacy and reducing the risk of fraud (Gupta et al., 2015). Third, as mobile payment technology becomes more widespread, markets develop a system of complementary goods and services that further enhance users' benefits, such as financial planning, rewards programs, and price competition (Crowe et al. 2010).⁵

⁴Short Message Service (SMS) and Unstructured Supplementary Service Data (USSD) are two methods used by telecom companies to allow users to send and receive text messages. With SMS, messages are sent to SMS centers, which store the message and then transmit the message to the recipient. In contrast, USSD makes a direct connection between text message senders and recipients, making it more responsive.

⁵Crowe et al. (2010) provides detailed discussions on the long-run benefits of mobile payments. For example, consumers could have their payments automatically logged in their financial planning software. Also, they could upload warranties and instructional videos at the time of purchase. Merchants could engage in sophisticated rewards programs, where consumers could access their status from their mobile device and receive alerts when they are close to rewards thresholds. Also, consumers could compare

2.1 Alternative mobile payment technologies

While there are many mobile payment solutions, they fall into two basic categories: either bypassing or complementing the existing bank-based payment card systems. Therefore, we name them card-substituting and card-complementing mobile payments, respectively.⁶ The former is mainly used in developing countries like Kenya, and the latter is popular in advanced economies like the U.S.

2.1.1 Card-substituting mobile payment

Card-substituting mobile payment is epitomized by Kenya’s M-PESA model. M-PESA is a mobile payment service launched by Safaricom and Vodafone in Kenya in 2007. M-PESA users can deposit money into an account in their phones and send balances to other users by SMS text messages. Hence, they can use a mobile phone to deposit and withdraw money, pay for goods and services, and transfer money to other users. To deposit and withdraw money, M-PESA users rely on M-PESA agents (e.g., shops, gas stations, or post offices). These agents are the analogs of the ATMs and bank branches in the banking system, allowing the M-PESA operation to bypass the banking system.

Following the success in Kenya, M-PESA was emulated in many other developing countries. This category of mobile payment methods is defined as “Mobile Money” by the Global System for Mobile Communications Association (GSMA) that must meet the following four conditions. First, the payment method must include transferring money as well as making and receiving payments using a mobile phone. Second, the payment method must be available to the unbanked (i.e., people who do not have access to a formal account at a financial institution). Third, the payment method must offer a network of physical transactional points (that can include agents) widely accessible to users. Fourth, mobile-banking-related payment services (such as Apple Pay and Google Wallet) that offer the mobile phone as just another channel to access a traditional banking product do not satisfy this definition of Mobile Money.

The global adoption of mobile money in 2018 is illustrated in Figure 2. The percentage

prices at nearby stores. If it is relatively easy to add new payment mechanisms to a mobile device and to switch among options, one should see new entry and innovation in this arena.

⁶In our analysis, the adoption of card payment includes agents’ decision to join the formal banking system combined with using card as the payment solution. In that sense, we could also name the two mobile payment categories bank-substituting and bank-complementing mobile payments, respectively.

number for each region refers to the share of mobile money users located in that region. The gray areas represent regions where mobile money is unavailable. Most users of mobile money are concentrating in developing countries, particularly sub-Saharan Africa (45.6%) and South Asia (33.2%). In contrast, mobile money is barely relevant for developed countries.⁷

Figure 2. GLOBAL ADOPTION OF MOBILE MONEY

2.1.2 Card-complementing mobile payment

By contrast, card-complementing mobile payment systems are typically deployed in developed countries. The popular types, including those created by technology firms (e.g., Apple, Google, Samsung), rely heavily on banking and payment card networks. Because of their use of a proximity communication technology (e.g., NFC or QR codes), these payment types are often referred to as mobile proximity payment services.

Apple Pay is a leading example. Apple Pay was launched in 2014 as one of the first mobile wallets – apps that enable people to connect credit cards, debit cards, and bank accounts to mobile devices to send and receive money. Of the major mobile wallet services – Google Pay (formerly Android Pay), Samsung Pay and Apple Pay — the Apple service is the largest in terms of user adoption and market coverage.

Apple Pay represents a secure and sanitary payment option, since the app uses the NFC technology to transmit an encrypted virtual account number to the point-of-sale payment terminal. Originally launched in the U.S., Apple Pay debuted in the U.K., Australia, and Canada in 2015, and expanded to China, Switzerland, France, Singapore,

⁷Source: GSMA (2018), “State of the Industry Report on Mobile Money.”

and Japan in 2016. By 2020, Apple Pay has become available in dozens of countries (marked dark blue in Figure 3), most of which are developed countries.⁸

Figure 3. GLOBAL AVAILABILITY OF APPLE PAY

2.2 Data and stylized facts

To study the global adoption pattern of mobile payments, we assembled a novel dataset on debit card and mobile payment adoption in 94 countries.⁹ The countries in our sample accounted for 91.4% of world GDP in 2017.

Our data are drawn from the following sources (See Appendix I for more details). First, the data on the adoption rates of card-substituting mobile payment services in 2017 are based on the Global Financial Inclusion (Global Findex) Database of the World Bank, which surveyed 76 countries with a visible presence of Mobile Money payment services. Second, the data on the adoption rates of card-complementing mobile payments around 2017, gathered from eMarketer, cover 23 countries with a visible presence of mobile proximity payment services. Merging the two mobile payment data sources yields a sample of 94 countries, including five countries covered in both data sources. We also collect the adoption rate of debit cards for the 94 countries in 2017 from the Global Findex Database of the World Bank. Finally, we obtain the data on per capita GDP and other variables for each country in our sample from the World Bank.

⁸Source: https://en.wikipedia.org/wiki/Apple_Pay#Supported_countries.

⁹Debit card ownership is a good measure of consumers who have become banked and have access to either debit or credit card technology because credit card users almost surely own debit cards. For robustness checks, we redid the empirical analysis using an alternative measure from the World Bank dataset on the percentage of the adult population (age 15 and above) using a debit or credit card to make a purchase in the past year and the results are very similar.

Figure 4. CARD AND MOBILE PAYMENT ADOPTION ACROSS COUNTRIES

Figure 4 plots the adoption rates of debit card and mobile payments against log per capita GDP in 2017. Fitting a simple linear regression line to the data shows that debit card adoption increases with per capita GDP across countries, while there appears no clear relationship between mobile payment adoption and per capita GDP.¹⁰

However, some subtle patterns of mobile payment adoption emerge as we delve further into the data. First, we distinguish mobile payment technologies used in each country in our sample. As shown in Figure 5A, most countries in the highest income group adopt card-complementing mobile payments, while most other countries choose card-substituting ones. Also, considering that mobile payment is a fairly recent technological innovation, it is possible that some countries may not have fully introduced it due to information or coordination frictions. We then leave out the observations that have a very low adoption rate (i.e., $<10\%$)¹¹ and fit the remaining data with a smooth nonparametric curve.¹² It becomes evident that mobile payment adoption displays a non-monotonic relationship with per capita GDP: increasing among countries with per capita GDP less than \$2,500, decreasing among countries with per capita GDP between \$2,500 and \$20,000, and increasing again among countries with per capita GDP greater than \$20,000.

¹⁰The regression results are reported in Appendix Table A1, where per capita GDP is found statistically significant in the card adoption regression but not in the mobile adoption regression. The card adoption model also shows a good fit for the data (adjusted $R^2 = 0.81$) while the mobile adoption model shows a poor fit (adjusted $R^2 = -0.01$).

¹¹Removing observations with mobile payment adoption rates below 10% only affects countries from the Global Findex Database that use Mobile Money payment services. Presumably, the eMarketer dataset on mobile proximity payment adoption has already applied a similar rule.

¹²The nonparametric fitting curve is based on a local-linear and local-constant kernel regression using the Epanechnikov kernel function.

Figure 5. CROSS-COUNTRY MOBILE PAYMENT ADOPTION

Informed by this nonparametric fitting, we divide the sample into three income groups: low-income countries (per capita GDP $< \$2,500$), middle-income countries ($\$2,500 \leq$ per capita GDP $\leq \$20,000$), and high-income countries (per capita GDP $> \$20,000$). We then add back a linear regression line to each income-country group. The results are shown in Figure 5B and corroborate the non-monotonic pattern. The regression results are reported in the Appendix and are robust for using a nonlinear regression model or excluding two outlier countries (Kenya and China) with exceptionally high mobile payment adoption rates (see Tables A1-A2 in Appendix II). The results also show that the non-monotonic mobile payment adoption pattern continues to hold even after controlling for a variety of additional factors that might be relevant for the adoption decisions (see Tables A3-A4 in Appendix II).

To sum up, the data suggests the following stylized facts on cross-country adoption patterns of card and mobile payments:

1. *Positive relationship between per capita income and card adoption.* – The adoption of card payments increases with per capita income across countries.
2. *Non-monotonic relationship between per capita income and mobile payment adoption.* – The adoption of mobile payments increases with per capita income among low- and high-income countries, but decreases with per capita income among middle-income countries.

3. *Overtaking in mobile payment adoption.* – Some low-income countries overtake high-income countries in adopting mobile payments.
4. *Different mobile payment technology choices across countries.* – Low- and middle-income countries primarily adopt card-substituting mobile payment technologies, while high-income countries adopt card-complementing ones.

In the rest of the paper, we construct a theory to explain these stylized facts and conduct counterfactual and welfare analyses. We also provide discussions on the outlier countries with exceptionally low or high mobile payment adoption rates in Section 6.

3 Model

In this section, we provide a model with sequential payment innovations to explain the stylized facts. We outline the model setup in Section 3.1 and characterize the equilibrium in Section 3.2.

3.1 Setup

Our model studies the adoption of payment technologies across countries. In each country, three payment technologies arrive sequentially, in the order of cash, card, and mobile.

Cash is a traditional paper payment technology, accessible to everyone in an economy.¹³ Using cash incurs a cost τ_h per dollar of transaction, which includes handling, safekeeping, and fraud expenses. In contrast, card and mobile are electronic payment technologies, each of which requires a fixed cost of adoption but lowers variable costs of doing transactions comparing with cash. We denote k_d and k_m as the one-time fixed adoption costs associated with card and mobile, respectively. Those include the resources spent on joining banking or mobile payment networks plus the costs of acquiring the hardware and software for making electronic payments. It is natural to assume $k_d > k_m$.¹⁴ The variable costs

¹³One could also assume a fixed adoption cost for cash. But given cash is the only payment option before electronic ones, its adoption is guaranteed, with the adoption cost paid by adopters or subsidized by the government.

¹⁴In our model context, k_d includes the costs of being banked plus choosing card as the payment instrument, which can be much higher than k_m , the cost of joining a mobile payment network (e.g., Mobile Money). The costs of adopting mobile payment do not have to include the costs of adopting a mobile phone given most consumers have already adopted a mobile phone for communication needs.

associated with using card and mobile are denoted as τ_d and τ_m per dollar of transaction, respectively. To capture the technology progress between cash, card, and mobile, we assume $\tau_h > \tau_d > \tau_m$.¹⁵

Figure 6. EXPONENTIAL INCOME DISTRIBUTION

Time is discrete with an infinite horizon. We consider an economy where agents' incomes are exogenous and heterogeneous (e.g., due to differences in productivity). Without loss of generality, we assume that income I_t at date t follows an exponential distribution across the population in the economy, with the cumulative distribution function (cdf) $G_t(I_t) = 1 - \exp(-I_t/\lambda_t)$. The exponential distribution has been shown fitting income distributions well (e.g., see Dragulescu and Yakovenko, 2001). Figure 6 presents an example of fitting the U.S. household income distribution in 2017 with an exponential distribution.¹⁶ Note that the exponential distribution has the mean λ_t and a fixed Gini coefficient at 0.5. Assuming an exponential income distribution allows our benchmark model to focus on the effect of per capita income while keeping the income inequality fixed, and we show later that our findings are robust under alternative distributional assumptions with flexible Gini coefficients.¹⁷ Over time, each agent's income grows at a constant rate

¹⁵The assumption $\tau_d > \tau_m$ captures the technology progress between card and mobile. Violating this assumption would yield a mobile payment adoption pattern different from the data. Note that if $\tau_d \leq \tau_m$, card users would never have incentives to adopt mobile payment. Still, some cash users may adopt mobile payment if k_m is sufficiently smaller than k_d , but they will later switch from mobile to card when their incomes grow sufficiently high.

¹⁶Source: The U.S. Census Bureau's 2017 Current Population Survey (CPS) Annual Social and Economic Supplement (ASEC) Research File, which reports household income in \$2,500 increments up to \$100,000.

¹⁷In Section 6.2, we extend the analysis by assuming a log-logistic income distribution and allow for

g , i.e., $I_{t+1} = I_t(1 + g)$, as does the mean income of the economy, i.e., $\lambda_{t+1} = \lambda_t(1 + g)$. We normalize the population size to unity.

An agent has a linear utility $u = c$, where c is her consumption. We assume there is no storage technology, so each agent consumes all her income net of payment costs each period. We also assume payment services and merchant services are provided via competitive markets.¹⁸ As a result, a consumer always uses her favorite payment technology and the private cost to the consumer equals the social cost.¹⁹

3.2 Equilibrium

Based on the model setup, we derive the equilibrium adoption patterns of cash, card, and mobile payment technologies as they arrive sequentially in an economy.

3.2.1 Cash payment

Cash is the only payment technology available in the economy before electronic payments are introduced. Cash is accessible to everyone, so the adoption rate is 100%. In such a cash economy, the value function V_h of an agent depends on her income I_t , and can be written as

$$V_h(I_t) = (1 - \tau_h)I_t + \beta V_h(I_{t+1}),$$

$$\text{where } I_{t+1} = I_t(1 + g),$$

and β is the discount rate. Accordingly, $V_h(I_{t+1}) = (1 + g)V_h(I_t)$, and we derive

$$V_h(I_t) = \frac{(1 - \tau_h)I_t}{1 - \beta(1 + g)}. \quad (1)$$

country-specific Gini coefficients, and our findings are essentially unchanged. This is consistent with the regression results in Appendix Tables A3 and A4 which show that the cross-country mobile payment adoption pattern is significantly correlated with per capita income but not the Gini index.

¹⁸In the presence of market power, the adoption cost paid by the users could be higher than the social costs of offering payment services. As a result, payment innovations would be underadopted, which may justify government interventions. However, assuming a competitive payment market is a useful benchmark which allows us to focus on the effect of income heterogeneity on payment adoption and provides a consistent cross-country comparison without being bogged down by payment market structure details across countries.

¹⁹Note that one could also assume that a fraction ψ of each consumer's spending has to be paid with cash even after the consumer has adopted electronic payments. In that case, some consumers would use multiple payment means, and we can rescale all variable costs of payment by a factor of $(1 - \psi)$ and the analysis is intact.

3.2.2 Card payment

At date T_d , the payment card technology arrives.²⁰ Each agent then compares card and cash technologies and decides whether to adopt card.

At any date $t \geq T_d$, the value function V_d of an agent who has income I_t and has adopted card can be written as

$$V_d(I_t) = (1 - \tau_d)I_t + \beta V_d(I_{t+1}),$$

which yields

$$V_d(I_t) = \frac{(1 - \tau_d) I_t}{1 - \beta(1 + g)}. \quad (2)$$

The availability of the card technology also changes the value function of cash users because it adds an option of adopting card in the future. Therefore, the value function of an agent who has income I_t and decides to continue using cash at date t would be

$$V_h(I_t) = (1 - \tau_h)I_t + \beta \max\{V_h(I_{t+1}), V_d(I_{t+1}) - k_d\}. \quad (3)$$

At any date $t \geq T_d$, an agent would adopt card if and only if

$$V_d(I_t) - k_d \geq V_h(I_t). \quad (4)$$

Therefore, Eqs. (2), (3), and (4) pin down the minimum income level I_d for card adoption, which requires

$$\frac{(1 - \tau_d) I_d}{1 - \beta(1 + g)} - k_d = (1 - \tau_h)I_d + \beta \left[\frac{(1 - \tau_d)(1 + g)I_d}{1 - \beta(1 + g)} - k_d \right].$$

Accordingly, an agent would have adopted card by date $t \geq T_d$ if and only if her income satisfies that

$$I_t \geq I_d = \frac{(1 - \beta)k_d}{(\tau_h - \tau_d)}. \quad (5)$$

The intuition of condition (5) is straightforward: An agent would adopt card if the flow benefit of adoption $(\tau_h - \tau_d)I_t$ can cover the flow cost $(1 - \beta)k_d$.

²⁰Given that everyone has access to cash, whether the arrival of card technology is anticipated or not does not affect the analysis.

The card payment adoption rate, $F_{d,t}$, is determined as

$$F_{d,t} = 1 - G_t(I_d) = \exp\left(-\frac{(1-\beta)k_d}{(\tau_h - \tau_d)\lambda_t}\right). \quad (6)$$

It follows immediately from Eq. (6) that the payment card adoption rate increases in per capita income (i.e., $\partial F_{d,t}/\partial \lambda_t > 0$).

3.2.3 Mobile payment

Mobile payment arrives after card as an unanticipated shock.²¹ In the following, we start with a scenario where only a card-substituting mobile payment technology (e.g., Mobile Money) is introduced, and we then proceed to another scenario where a card-complementing mobile payment technology (e.g., Apple Pay) also becomes available.

A card-substituting mobile payment technology. At date $T_m > T_d$, a card-substituting mobile payment technology arrives. This mobile payment technology allows users to replace or bypass the card technology, with a lower marginal cost $\tau_m < \tau_d < \tau_h$ and a lower fixed cost $k_m < k_d$. Each agent then compares three payment technologies (i.e., cash, card, and mobile) to make the payment adoption decision.

At any date $t \geq T_m$, the value function V_m of an agent who has income I_t and has adopted mobile can be written as

$$V_m(I_t) = (1 - \tau_m)I_t + \beta V_m(I_{t+1}),$$

which yields

$$V_m(I_t) = \frac{(1 - \tau_m)I_t}{1 - \beta(1 + g)}. \quad (7)$$

Because mobile is a superior payment technology than card, (i.e., $\tau_m < \tau_d$ and $k_m < k_d$), an agent who has not adopted card by date $T_m - 1$ (i.e., $I_{T_m-1} < I_d$) would no longer consider adopting card at date T_m and afterwards. Instead, they would adopt mobile payment at a date $t \geq T_m$ whenever

$$V_m(I_t) - k_m \geq V_h(I_t), \quad (8)$$

²¹This is a simplifying assumption, and we will relax it in Section 6.1 to consider anticipated arrival of mobile payments.

where the value function of a cash user $V_h(I_t)$ now becomes

$$V_h(I_t) = (1 - \tau_h)I_t + \beta \max\{V_h(I_{t+1}), V_m(I_{t+1}) - k_m\}. \quad (9)$$

Equations (7), (8), and (9) then pin down the minimum income level I_m for mobile payment adoption:

$$I_t \geq I_m = \frac{(1 - \beta)k_m}{(\tau_h - \tau_m)}. \quad (10)$$

Given $\tau_m < \tau_d < \tau_h$ and $k_m < k_d$, Eqs. (5) and (10) show $I_m < I_d$, so the fraction of agents who have switched from cash to mobile by date $t \geq T_m$ is

$$\begin{aligned} F_{h \rightarrow m, t} &= G_{T_m-1}(I_d) - G_t(I_m) = \exp(-I_m/\lambda_t) - \exp(-I_d/\lambda_{T_m-1}) \\ &= \exp\left(-\frac{(1 - \beta)k_m}{(\tau_h - \tau_m)\lambda_t}\right) - \exp\left(-\frac{(1 - \beta)k_d}{(\tau_h - \tau_d)\lambda_{T_m-1}}\right). \end{aligned} \quad (11)$$

An agent who has adopted card by date $T_m - 1$ (i.e., $I_{T_m-1} \geq I_d$) would adopt mobile payment at a date $t \geq T_m$ whenever

$$V_m(I_t) - k_m \geq V_d(I_t), \quad (12)$$

where the value function of a card user now becomes

$$V_d(I_t) = (1 - \tau_d)I_t + \beta \max\{V_d(I_{t+1}), V_m(I_{t+1}) - k_m\}. \quad (13)$$

Equations (7), (12), and (13) pin down the income level $I_{m'}$ above which agents would switch from card to mobile payment to be

$$I_t \geq I_{m'} = \frac{(1 - \beta)k_m}{(\tau_d - \tau_m)}. \quad (14)$$

So the fraction of agents who have switched from card to mobile by date $t \geq T_m$ is

$$\begin{aligned} F_{d \rightarrow m, t} &= 1 - G_t(I_{m'}) = \exp(-I_{m'}/\lambda_t) \\ &= \exp\left(-\frac{(1 - \beta)k_m}{(\tau_d - \tau_m)\lambda_t}\right) \end{aligned} \quad (15)$$

as long as some card adopters have not adopted mobile (i.e., $F_{d \rightarrow m, t} < F_{d, T_m-1}$). Other-

wise, $F_{d \rightarrow m, t} = F_{d, T_m - 1}$.

Combining Eqs. (11) and (15), the total fraction of agents who have adopted mobile payments by date $t \geq T_m$ is

$$\begin{aligned} F_{m, t} &= F_{h \rightarrow m, t} + F_{d \rightarrow m, t} = \exp(-I_m/\lambda_t) - \exp(-I_d/\lambda_{T_m-1}) + \exp(-I_{m'}/\lambda_t) \quad (16) \\ &= \exp\left(-\frac{(1-\beta)k_m}{(\tau_h - \tau_m)\lambda_t}\right) - \exp\left(-\frac{(1-\beta)k_d}{(\tau_h - \tau_d)\lambda_{T_m-1}}\right) + \exp\left(-\frac{(1-\beta)k_m}{(\tau_d - \tau_m)\lambda_t}\right) \end{aligned}$$

as long as $F_{d \rightarrow m, t} < F_{d, T_m - 1}$. Otherwise, $F_{m, t} = \exp(-\frac{I_m}{\lambda_t}) = \exp(-\frac{(1-\beta)k_m}{(\tau_h - \tau_m)\lambda_t})$. This result unveils the following subtle relationship between the mobile payment adoption rate and per capita income:

1. To trace how the mobile payment adoption evolves in a country over time, one can take the value of λ_{T_m-1} as given, so Eq. (16) yields $\partial F_{m, t} / \lambda_t > 0$. This suggests that a country's mobile payment adoption rate increases over time as more agents switch from cash or card to mobile due to their income growth.
2. To make a cross-country comparison at a point in time, however, one needs to take into account $\lambda_{T_m-1} = \lambda_t / (1 + g)^{t - T_m + 1}$. Accordingly, Eq. (16) can be written as

$$F_{m, t} = \exp\left(-\frac{(1-\beta)k_m}{(\tau_h - \tau_m)\lambda_t}\right) - \exp\left(-\frac{(1-\beta)k_d(1+g)^{t-T_m+1}}{(\tau_h - \tau_d)\lambda_t}\right) + \exp\left(-\frac{(1-\beta)k_m}{(\tau_d - \tau_m)\lambda_t}\right).$$

In light of this expression, the sign of $\partial F_{m, t} / \lambda_t$ depends on the level of λ_t . The fraction of cash-mobile switchers (as captured by the first two terms) could decrease in λ_t if λ_t is sufficiently large. That is because in a country with a larger λ_t , more agents would have been locked in by card when the mobile arrives. In contrast, the fraction of card-mobile switchers (as captured by the third term) always increases in λ_t . Therefore, the mobile payment adoption rate may display a non-monotonic relationship with per capita income across countries.

3. In the long run, due to income growth, all the card adopters would eventually switch to mobile (i.e., $F_{d \rightarrow m, t} = F_{d, T_m - 1}$). We then have $F_{m, t} = \exp(-\frac{I_m}{\lambda_t}) = \exp(-\frac{(1-\beta)k_m}{(\tau_h - \tau_m)\lambda_t})$, in which case the mobile payment adoption rate becomes strictly increasing in per capita income across countries (i.e., $\partial F_{m, t} / \partial \lambda_t > 0$).

The discussion makes it clear that $F_{d \rightarrow m, T_m} < F_{d, T_m-1}$ is a necessary condition for the leapfrogging of mobile payment adoption to occur at T_m . According to Eqs. (6) and (15), this requires $\frac{k_m}{\tau_d - \tau_m} > \frac{k_d(1+g)}{\tau_h - \tau_d}$, which ensures $I_{m'} > I_d(1+g)$. Therefore, given $\lambda_{T_m} = (1+g) \lambda_{T_m-1}$, only a fraction of the consumers who have adopted card by $T_m - 1$ would cross the income threshold for adopting mobile at T_m . If this condition is violated, the cost savings of mobile payment relative to card would be so large that all card users switch to mobile at T_m . As a result, the cross-country mobile adoption would display a rank-preserving pattern, that is, a country with a higher per capita income (and thus a higher card adoption rate) would always have a higher mobile adoption rate.

A card-complementing mobile payment technology. We now extend the model to consider another scenario where at the same time T_m , a card-complementing mobile payment solution (in addition to the card-substituting one) also becomes available. As an add-on upgrade to the existing card technology, this card-complementing mobile payment technology allows a card adopter to pay an upgrading cost k_m^a to get the mobile payment feature that lowers the variable cost of payments (i.e., $\tau_h > \tau_d > \tau_m$). This add-on technology requires a lower fixed cost than adopting the card-substituting mobile payment method (i.e., $k_m^a < k_m$).

In this scenario, agents who have adopted card before T_m would prefer adopting the card-complementing mobile payment technology because $k_m^a < k_m$, while agents who have not adopted card would bypass card and adopt the card-substituting mobile payment technology because $k_m < k_d + k_m^a$.

Therefore, agents who have switched from cash to mobile by date $t \geq T_m$ must have chosen the card-substituting mobile payment technology. Their fraction is given by Eq. (11) above. On the other hand, agents who have adopted card solve the value function:

$$V_d(I_t) = (1 - \tau_d)I_t + \beta \max[V_d(I_{t+1}), V_m(I_{t+1}) - k_m^a]. \quad (17)$$

As a result, those who have chosen the card-complementing mobile payment by date $t \geq T_m$ are those whose income have crossed the threshold

$$I_t \geq I_{m'}^a = \frac{(1 - \beta)k_m^a}{(\tau_d - \tau_m)}. \quad (18)$$

The fraction of these card-mobile switchers is

$$F_{d \rightarrow m, t} = 1 - G_t(I_{m'}^a) = \exp(-I_{m'}^a / \lambda_t) = \exp(-\frac{(1 - \beta)k_m^a}{(\tau_d - \tau_m)\lambda_t}), \quad (19)$$

as long as $F_{d \rightarrow m, t} \leq F_{d, T_m - 1}$, a result similar to what is derived in Eq. (15) except that k_m^a replaces k_m . Otherwise, $F_{d \rightarrow m, t} = F_{d, T_m - 1}$.

All together, the total fraction of mobile payment adopters by date $t \geq T_m$ is

$$\begin{aligned} F_{m, t} &= F_{h \rightarrow m, t} + F_{d \rightarrow m, t} = \exp(-I_m / \lambda_t) - \exp(-I_d / \lambda_{T_m - 1}) + \exp(-I_{m'}^a / \lambda_t) \quad (20) \\ &= \exp(-\frac{(1 - \beta)k_m}{(\tau_h - \tau_m)\lambda_t}) - \exp(-\frac{(1 - \beta)k_d}{(\tau_h - \tau_d)\lambda_{T_m - 1}}) + \exp(-\frac{(1 - \beta)k_m^a}{(\tau_d - \tau_m)\lambda_t}) \end{aligned}$$

as long as $F_{d \rightarrow m, t} < F_{d, T_m - 1}$. Otherwise, $F_{m, t} = \exp(-\frac{I_m}{\lambda_t}) = \exp(-\frac{(1 - \beta)k_m}{(\tau_h - \tau_m)\lambda_t})$.

Again, the mobile payment adoption rate $F_{m, t}$ may display a non-monotonic relationship with per capita income λ_t across countries. Once all the card adopters have adopted mobile, mobile payment adoption becomes strictly increasing with per capita income across countries.

4 Model calibration and implications

In this section, we calibrate the model to fit the cross-country card and mobile payment adoption patterns. We then conduct counterfactual analyses to explore the model implications regarding mobile payment options, income growth, and technological progress.

4.1 Model calibration

We first calibrate the model with two mobile payment options (i.e., the card-substituting and card-complementing ones) using the parameter values as shown in Table 1.²²

The unit of time is year, and we set 2017 as the year T_m when mobile payment became widely available. Following convention, we set the discount factor $\beta = 0.95$ and the annual income growth rate $g = 2\%$. According to an ECB study (Schmiedel et al., 2012) on retail payment costs in 13 participating countries, the average social cost of using cash is 2.3%

²²We show later (see Sections 4.2.1 and 5.2) that the parameter values in Table 1 also allow our calibrated model to fit data well under the alternative assumption that only one mobile payment option is offered in each country.

of the transaction value, while that of using debit cards is 1.4%, so we set the values of τ_h and τ_d accordingly. We then calibrate $k_d = 500$ to fit the cross-country card adoption pattern in 2017. Finally, we calibrate the mobile payment variable cost $\tau_m = 1.395\%$ ($< \tau_d$) and the fixed costs $k_m = 150$ ($< k_d$) and $k_m^a = 100$ ($< k_m$) to fit the cross-country mobile payment adoption pattern in 2017.²³

Table 1. Parameter Values for Model Calibration

Parameter	Value	Description	Source of Identification
β	0.95	Discount factor	Standard
g	2%	Income growth rate	Standard
τ_h	2.3%	Cash variable cost	Schmiedel et al. (2012)
τ_d	1.4%	Card variable cost	Schmiedel et al. (2012)
k_d	500	Card adoption cost	Cross-country card payment adoption pattern, Figure 7A
τ_m	1.395%	Mobile variable cost	Cross-country mobile payment adoption pattern, Figure 7B
k_m	150	Mobile adoption cost	Cross-country mobile payment adoption pattern, Figure 7B
k_m^a	100	Mobile add-on cost	Cross-country mobile payment adoption pattern, Figure 7B

Note that the parameter values we use to calibrate the model are for illustration purpose and can be adjusted. For example, adopting card requires an agent to join the banking system, which could bring additional costs or benefits other than making payments. However, as long as those costs or benefits are proportional to income (e.g., Greenwood and Jovanovic, 1990), the model could be readily re-calibrated. In fact, the equilibrium adoption rates in our model depend on the ratios of fixed adoption costs to the savings in the variable costs (e.g., $\frac{k_d}{\tau_h - \tau_d}$, $\frac{k_m}{\tau_h - \tau_m}$, $\frac{k_m}{\tau_d - \tau_m}$, and $\frac{k_m^a}{\tau_d - \tau_m}$). If we use alternative calibration values for the variable costs, we can rescale the fixed adoption costs accordingly to fit the data and the analysis is intact.²⁴

²³ Adopting card requires joining the formal banking sector, which is often deemed too expensive in many developing countries where a large fraction of population works in informal sectors. To discipline the calibration, we assume that all countries share the same model parameter values and the card-substituting and card-complementing mobile payment technologies share the same value of τ_m . Relaxing such assumptions would provide additional degrees of freedom and may allow the model to fit the data targets even better.

²⁴ Also note that in the model calibration, we treat per capita income/spending and per capita GDP interchangeable. In reality, per capita income/spending could be proportional to per capita GDP. To account for that, one can rescale the payment adoption costs (i.e., k_d , k_m , and k_m^a) by the same proportion without affecting the analysis and findings. Also, one may assume a fraction of each consumer's spending has to be paid with cash even after the consumer has adopted electronic payments, in which case one can do a similar rescaling to the fixed adoption costs to recalibrate the model and the analysis is intact.

As shown in Figure 7, our calibration results fit the data well and match the first three stylized facts identified above: (1) *Positive relationship between per capita income and card adoption*; (2) *Non-monotonic relationship between per capita income and mobile payment adoption*; (3) *Overtaking in mobile payment adoption*.

Figure 7. MODEL FIT WITH DATA

Figure 8 shows that our calibration also matches the fourth stylized fact: (4) *Different technology choice across countries*. In Figure 8, we decompose the fraction of total mobile payment adopters at $T_m = 2017$ (red dashed line) into cash-mobile switchers (green solid line) and card-mobile switchers (blue solid line) by per capita income. In addition, we compare the cash-mobile and card-mobile switchers with the fractions of previous cash users (green dotted line) and card users (blue dotted line) at $T_m - 1$. In low-income countries (i.e., $\lambda_{T_m} < \$2,500$) and middle-income countries (i.e., $\$2,500 \leq \lambda_{T_m} \leq \$20,000$), mobile payment adoption almost entirely relies on cash-mobile switchers who choose the card-substituting technology. In contrast, mobile payment adoption in most high-income countries (i.e., $\lambda_{T_m} > \$20,000$) relies on card-mobile switchers who pick the card-complementing technology. In fact, the upper envelope of the green solid line and blue solid line follows the red dashed line closely, which implies that the model-calibrated cash-mobile switchers in low- and middle-income countries and card-mobile switchers in high-income countries can directly match the cross-country mobile adoption data.

Figure 8. COMPOSITION OF MOBILE PAYMENT ADOPTERS

Moreover, Figure 8 is instrumental to explain the non-monotonic relationship between mobile payment adoption and per capita income. Since most agents in low-income countries are cash users, mobile payment adoption is primarily driven by adopting the card-substituting technology and the adoption rate increases in per capita income. By contrast, a higher fraction of agents in middle-income countries are middle-income card users who are locked in by the card technology (i.e., their incomes are not sufficiently high to justify switching to the card-complementing mobile payment technology). As the fraction of such locked-in card users increases in per capita income, mobile payment adoption decreases in per capita income among such middle-income countries. Finally, most agents in high-income countries are rich card users and their incomes are high enough to justify switching to the card-complementing mobile payment technology, so the adoption of mobile payment again increases in per capita income.

4.2 Model implications

After calibrating our model to match the cross-country mobile payment adoption pattern, we conduct several counterfactual exercises to illustrate the implications of the model.

4.2.1 Mobile payment options

Based on our calibrated model, we first investigate how the availability of different mobile payment options affect the cross-country adoption pattern. In Figure 9, the green dashed line depicts the mobile payment adoption pattern if only the card-substituting option is

available in each country. The blue dotted line depicts the adoption pattern if only the card-complementing option is available in each country. The red solid line shows the adoption pattern if both mobile payment options are available in each country.

Figure 9. MOBILE PAYMENT OPTIONS AND ADOPTION

This exercise quantifies the role played by fixed adoption costs associated with different mobile payment technologies, and the results in Figure 9 provide the following insights.

First, the availability of both card-substituting and card-complementing mobile payment options in each country raises the adoption rate, especially for high-income countries. As the figure shows, the red line is above the green dashed line and the blue dotted line.

Second, excluding the card-complementing mobile payment option in each country would not lead to drastic changes in the mobile payment adoption pattern across countries. It would push down adoption in high-income countries to some degree, but its effects on low- and middle-income countries would be almost entirely negligible.

Third, shutting down the card-substituting mobile payment option in each country, however, would overturn the cross-country adoption pattern. Mobile payment adoption would be increasing in per capita income. Essentially, this would kill mobile payment adoption in most low- and middle-income countries, and it slightly pushes down mobile payment adoption in high-income countries.

Finally, one could assume that a country may choose to provide only one mobile option out of the two, whichever would yield the higher adoption rate. In that case, the adoption pattern would be given by the upper envelope of the green dashed line and the blue dotted line, which is not very different from the red line. This suggests that

the parameter values we used in the calibration would allow the model to fit the cross-country mobile payment adoption pattern well even under the alternative assumption, and the results of the counterfactual analyses would be very similar.

4.2.2 Income growth

We now consider the effect of income growth. According to our theory, long-run income growth would eventually lift all the card adopters (who exist before date T_m) to cross the mobile payment adoption threshold. Once that happens, mobile payment adoption would be solely driven by cash-mobile switchers and the adoption rate would become monotonically increasing in per capita income. However, our quantitative exercise suggests that it would just take too long for income growth to overturn the non-monotonic mobile payment adoption pattern.

Figure 10. INCOME GROWTH AND MOBILE PAYMENT ADOPTION

Recall that we assume per capita income grows at 2% annually in each country. Figure 10 tracks each country by per capita income at T_m and plots mobile payment adoption rates at year T_m (red solid line), $T_m + 50$ (pink dashed line), $T_m + 100$ (green dotted line), and $T_m + 180$ (blue dash-dotted line). The figure shows that mobile payment adoption increases in every country as per capita income grows. Nevertheless, the adoption rate continues to be non-monotonic in per capita income. Ultimately, it takes about 180 years to converge to an adoption curve that strictly increases in per capita income.²⁵

²⁵In our model simulation, with the 2% annual income growth rate, all the agents who have adopted card by $T_m - 1$ would have crossed the mobile payment adoption threshold in 180 years. Note that this

Figure 11 decomposes mobile payment adopters into cash-mobile switchers (the left panel) and card-mobile switchers (the right panel). The figure shows that both cash-mobile switchers and card-mobile switchers increase in every country as per capita income grows over time. Eventually, once all the card users have adopted mobile payment at year $T_m + 180$ in every country, the remaining adoption is determined solely by cash-mobile switchers and the mobile payment adoption rate strictly increases in per capita income.

Figure 11. INCOME GROWTH AND MOBILE PAYMENT ADOPTERS

4.2.3 Technological progress

Compared with income growth, the effect of technological progress on mobile payment adoption is more striking. According to our theory, advanced economies are stuck with card payment primarily because the value added of mobile payment is not substantial enough to induce some middle-income card adopters to switch. Therefore, greater technological progress of mobile payment would not only increase the adoption in every country, but could also restore advanced economies to the leading positions in the mobile payment race if the technological progression becomes sufficiently large.

To evaluate the effect of technological progress on mobile payment, we conduct a counterfactual exercise by reducing the variable mobile payment cost τ_m . As shown in Figure 12, greater technological progress (i.e., smaller values of τ_m) promotes the mobile payment adoption rate in every country and advanced economies are especially benefitted.

calculation is only for illustrating the working of the model, and the process would speed up if our model introduces birth and death of agents.

If the technological progress is sufficiently large, mobile payment adoption becomes strictly increasing in per capita income across countries.

Figure 12. TECH PROGRESS AND MOBILE PAYMENT ADOPTION

Taking a step further, Figure 13 decomposes mobile payment adopters into cash-mobile switchers (the left panel) and card-mobile switchers (the right panel).

Figure 13. TECH PROGRESS AND MOBILE PAYMENT ADOPTERS

As shown by this figure, technological progress mainly boosts mobile payment adoption among previous card users because they enjoy more cost savings than cash users through a lower τ_m due to their higher income and spending. This explains why high-income countries benefit more. Therefore, should some major technological progress occur down

the road, advanced economies might see their mobile payment adoption jump up and they may even regain leading positions in the mobile payment race.

5 Welfare and policy analyses

In this section, we use our calibrated model to gauge payment efficiency and explore welfare and policy implications.

5.1 Payment efficiency

To identify winners and losers in the adoption of payment innovations, we conduct a welfare analysis based on the calibrated model. We first evaluate payment efficiency for individual agents and then for aggregate economies. For ease of notation, we denote each agent by her income level I (without the time subscript) in the analysis.

5.1.1 Individual agents

We first consider individual agents in a cash economy. Denote $\bar{V}_h(I)$ as the value function of an agent I who would permanently use cash payment. By Eq. (1), we know

$$\bar{V}_h(I) = \frac{(1 - \tau_h) I}{1 - \beta(1 + g)}, \quad (21)$$

so the present-value welfare of agent I , denoted by $\omega_t(I)$, equals $\bar{V}_h(I)$ for any $t < T_d$.

At date T_d , the card technology arrives. Denote $\bar{V}_d(I)$ as the value function of an agent I who would permanently use card payment. By Eq. (2), we know

$$\bar{V}_d(I) = \frac{(1 - \tau_d) I}{1 - \beta(1 + g)}. \quad (22)$$

The present-value welfare of agent I at date T_d , denoted by $\omega_{T_d}(I)$, depends on the agent's income and the corresponding card adoption decisions:

$$\omega_{T_d}(I) = \begin{cases} \bar{V}_d(I) - k_d & \text{if } I \geq I_d; \\ \bar{V}_h(I) + \beta^s \begin{pmatrix} \bar{V}_d(I(1 + g)^s) \\ -k_d - \bar{V}_h(I(1 + g)^s) \end{pmatrix} & \begin{array}{l} \text{if } \frac{I_d}{(1+g)^s} \leq I < \frac{I_d}{(1+g)^{s-1}}, \\ \text{for } s \in \{1, 2, 3, \dots\}. \end{array} \end{cases} \quad (23)$$

Note that $I_d = \frac{(1-\beta)k_d}{(\tau_h - \tau_d)}$ is given by Eq. (5). The top equation of (23) calculates the welfare of an agent whose income crosses the card adoption threshold at date T_d , and the bottom equation calculates the welfare of an agent who would adopt card at a future date.

At date T_m , the two mobile payment technologies arrive. Denote $\bar{V}_m(I)$ as the value function of an agent I who would permanently use mobile payment. By Eq. (7), we know

$$\bar{V}_m(I) = \frac{(1 - \tau_m) I}{1 - \beta(1 + g)}. \quad (24)$$

The present-value welfare of agent I at date T_m , denoted by $\omega_{T_m}(I)$, depends on the agent's income and the corresponding mobile payment adoption decisions:

$$\omega_{T_m}(I) = \begin{cases} \bar{V}_m(I) - k_m^a & \text{if } I \geq I_{m'}^a; \\ \bar{V}_d(I) + \beta^s \begin{pmatrix} \bar{V}_m(I(1+g)^s) \\ -k_m^a - \bar{V}_d(I(1+g)^s) \end{pmatrix} & \text{if } \max(\frac{I_{m'}^a}{(1+g)^s}, I_d(1+g)) \leq I < \frac{I_{m'}^a}{(1+g)^{s-1}}, \\ & \text{for } s \in \{1, 2, 3, \dots\}; \\ \bar{V}_m(I) - k_m & \text{if } I_m \leq I < I_d(1+g); \\ \bar{V}_h(I) + \beta^s \begin{pmatrix} \bar{V}_m(I(1+g)^s) \\ -k_m - \bar{V}_h(I(1+g)^s) \end{pmatrix} & \text{if } \frac{I_m}{(1+g)^s} \leq I < \frac{I_m}{(1+g)^{s-1}}, \\ & \text{for } s \in \{1, 2, 3, \dots\}. \end{cases} \quad (25)$$

Note that $I_m = \frac{(1-\beta)k_m}{(\tau_h - \tau_m)}$ is given by Eq. (10), and $I_{m'}^a = \frac{(1-\beta)k_m^a}{(\tau_d - \tau_m)}$ is given by Eq. (18). The top equation of (25) calculates the welfare of a card-mobile switcher whose income crosses the mobile adoption threshold at date T_m , and the second equation is the welfare of a card user who would adopt mobile at a future date. The third equation is the welfare of a cash-mobile switcher at date T_m , and the bottom equation is the welfare of a cash user who would adopt mobile at a future date.

Define the payment efficiency of an agent I , $x_t(I)$, as the ratio between the present value of welfare at date t with and without incurring the payment costs:

$$x_t(I) = \frac{\omega_t(I)}{\frac{I}{1-\beta(1+g)}}. \quad (26)$$

Note that the denominator, $\frac{I}{1-\beta(1+g)}$, is the first-best welfare in a frictionless economy without any payment costs, so $x_t(I)$ gauges the fraction of the first-best welfare level that

can be achieved by agent I under available payment technologies at date t .

Figure 14. PAYMENT EFFICIENCY BY INDIVIDUAL INCOME

Using the parameter values in Table 1, we can compare payment efficiency for individual agents at different income levels under each payment innovation. As in the previous section, we assume that mobile payment technologies arrive at $T_m = 2017$. We then assume that the card payment arrives at $T_d = T_m - 30$.²⁶ Figure 14 plots the payment efficiency of each agent for $t < T_d$ (i.e., cash only), $t = T_d$ (i.e., card becomes available), $t = T_m$ (i.e., mobile becomes available), according to their individual income level at T_m . For a comparison, we also plot a counterfactual case for $t = T_m$ assuming mobile does not become available then, which we denoted as $\tilde{x}_{T_m}$.

The blue dotted line in Figure 14 shows that every agent has the same payment efficiency when cash is the only payment means (i.e., $x_{t < T_d} = 1 - \tau_h$). Once the card technology arrives at T_d , the payment efficiency improves for every agent, and it increases in agents' income (as shown by the blue dashed curve). A similar pattern holds when the mobile payment arrives at T_m (as shown by the red solid curve). The intuition for why payment efficiency measures (i.e., x_{T_d} and x_{T_m}) increase in agents' income is as follows: It is always feasible for a higher-income agent to mimic a lower-income agent's adoption behavior. If that turns out to be the optimal decision, the higher-income agent enjoys

²⁶The large-scale introduction of debit cards in the U.S. started in the mid-1980s (see Hayashi, Li, and Wang, 2017), so we set $T_d = T_m - 30$. Note that the simulation results are robust if we use an alternative year for T_d because choosing an earlier (or later) T_d would not change anything except adjusting down (or up) the level of the payment efficiency x_{T_d} given that the card adoption cost k_d counts for a larger (or smaller) share of agents' income in an earlier (or later) year.

higher payment efficiency than her lower-income counterpart because the adoption cost (i.e., k_d , k_m , or k_m^a) counts for a smaller share of her income. But if mimicking is not the optimal decision, the higher-income agent must be able to achieve even higher payment efficiency by choosing a payment method different from her lower-income counterpart.

Figure 14 also illustrates how payment efficiency evolves across income levels over time. At date T_d , agents either pay or expect to pay in the future the fixed cost k_d to adopt card, and the agents whose incomes are at the card adoption threshold are indifferent between adopting card or not. Accordingly, the payment efficiency measure x_{T_d} is a continuous and increasing function of income. For any date $t \in (T_d, T_m)$, card users who have paid off k_d in the past no longer count the fixed cost in their payment efficiency measure, so $x_t = 1 - \tau_d$ for them. Meanwhile, cash users who just meet or have not met the card adoption threshold need to pay the fixed cost, so their payment efficiency x_t displays a jump at the card adoption threshold, as illustrated by the green dash-dotted curve $\tilde{x}_{T_m}$. For those cash users, their payment efficiency improves over time because the card adoption cost k_d accounts for a smaller share of their income levels as their incomes grow. Comparing the two curves x_{T_m} (the red solid one) and $\tilde{x}_{T_m}$ (the green dash-dotted one) shows that introducing mobile improves payment efficiency for every agent (though much more for cash users than for card users) and shrinks the jump at the card adoption threshold.²⁷

5.1.2 Aggregate economies

We now evaluate overall payment efficiency across countries by solving the present-value welfare of the aggregate economies, denoted by $W_t(\lambda_t)$, for three periods: $t < T_d$ (i.e., cash only), $t = T_d$ (i.e., card becomes available), and $t = T_m$ (i.e., mobile becomes available).

Recall that $\bar{V}_h(I)$ is the value function of an agent I who would permanently use the cash technology, given by Eq. (21). Accordingly, the present-value welfare of a pure cash economy $W_{t < T_d}$ equals

$$W_{h,t} = \int_0^\infty \bar{V}_h(I) dG_t(I). \quad (27)$$

²⁷For cash users, introducing mobile improves their payment efficiency substantially because of the much reduced adoption cost comparing with card (recall that $k_d = 500$ vs. $k_m = 150$). For card users, the red solid curve is indeed continuous and increasing in income, but the magnitude of increase is small so it is not easily seen in the figure (recall that card users' payment efficiency only improves slightly somewhere between $1 - \tau_d$ and $1 - \tau_m$, where $\tau_d = 1.4\%$ and $\tau_m = 1.395\%$).

Recall that $\bar{V}_d(I)$ is the value function of an agent I who would permanently use the card technology, given by Eq. (22). The present-value welfare of the economy at T_d is

$$\begin{aligned} W_{T_d} = & W_{h,T_d} + \int_{I_d}^{\infty} (\bar{V}_d(I) - k_d - \bar{V}_h(I)) dG_{T_d}(I) \\ & + \sum_{s=1}^{\infty} \int_{\frac{I_d}{(1+g)^s}}^{\frac{I_d}{(1+g)^{s-1}}} \beta^s (\bar{V}_d(I(1+g)^s) - k_d - \bar{V}_h(I(1+g)^s)) dG_{T_d}(I), \end{aligned} \quad (28)$$

where $I_d = \frac{(1-\beta)k_d}{(\tau_h - \tau_d)}$ is from Eq. (5). Note that the first term of the right-hand side of Eq. (28) is the present value of welfare for all the agents if they continue using cash forever. The second term is the additional welfare gains for card adopters at date T_d , and the last term is the additional welfare gains for future card adopters.

Recall that $\bar{V}_m(I)$ is the value function of an agent I who would permanently use the mobile payment technology, given by Eq. (24). We can then derive the present value of welfare for the economy at T_m to be

$$\begin{aligned} W_{T_m} = & \int_0^{I_d(1+g)} \bar{V}_h(I) dG_{T_m}(I) + \int_{I_m}^{I_d(1+g)} (\bar{V}_m(I) - k_m - \bar{V}_h(I)) dG_{T_m}(I) \\ & + \sum_{s=1}^{\infty} \int_{\frac{I_m}{(1+g)^s}}^{\frac{I_m}{(1+g)^{s-1}}} \beta^s (\bar{V}_m(I(1+g)^s) - k_m - \bar{V}_h(I(1+g)^s)) dG_{T_m}(I) \\ & + \int_{I_d(1+g)}^{\infty} \bar{V}_d(I) dG_{T_m}(I) + \int_{\max(I_{m'}^a, I_d(1+g))}^{\infty} (\bar{V}_m(I) - k_m^a - \bar{V}_d(I)) dG_{T_m}(I) \\ & + \sum_{s=1}^{\infty} \int_{\max(\frac{I_{m'}^a}{(1+g)^s}, I_d(1+g))}^{\max(\frac{I_{m'}^a}{(1+g)^{s-1}}, I_d(1+g))} \beta^s (\bar{V}_m(I(1+g)^s) - k_m^a - \bar{V}_d(I(1+g)^s)) dG_{T_m}(I), \end{aligned} \quad (29)$$

where $I_m = \frac{(1-\beta)k_m}{(\tau_h - \tau_m)}$ is given by Eq. (10), and $I_{m'}^a = \frac{(1-\beta)k_m^a}{(\tau_d - \tau_m)}$ is given by Eq. (18). Note that the first term of the right-hand side of Eq. (29) is the present-value welfare for all the cash users at $T_m - 1$ if they continue using cash at date T_m and forever. The second term is the additional welfare gains of cash-mobile switchers at date T_m , and the third term is the additional welfare gains for future cash-mobile switchers. The fourth term is the present-value welfare for all the card adopters at $T_m - 1$ if they continue using card at date T_m and forever. The fifth term is the additional welfare gains of card-mobile switchers at date T_m , and the last term is the additional welfare gains for future card-mobile switchers.

With the exponential income distribution, one can solve Eqs. (27), (28), and (29)

explicitly (see Appendix III for the solution details). Analogous to the discussions above, we define the payment efficiency of an economy, $X_t(\lambda_t)$, as the ratio between the present value of aggregate welfare with and without incurring payment costs at date t :

$$X_t(\lambda_t) = \frac{W_t(\lambda_t)}{\frac{\lambda_t}{1-\beta(1+g)}}. \quad (30)$$

Using the parameter values in Table 1, we can now compare payment efficiency across countries under each payment technology. As in the previous section, we assume that mobile payment technologies arrive at $T_m = 2017$, and the card payment technology arrives at $T_d = T_m - 30$. Figure 15 plots the payment efficiency of each economy for $t < T_d$, $t = T_d$, and $t = T_m$, according to their per capita income level at T_m .

Figure 15. PAYMENT EFFICIENCY BY PER CAPITA INCOME

As depicted in Figure 15, every country has the same payment efficiency when cash is the only payment means (i.e., $X_{t < T_d} = 1 - \tau_h$). Once the card technology arrives, the payment efficiency improves in every country, and the welfare improvement increases in per capita income across countries. Hence, high-income countries gain the most from the card payment adoption. The arrival of mobile payments also benefits every country, though disproportionately. The relative welfare gain $(X_{T_m} - X_{T_d}) / X_{T_d}$ peaks at a per capita income around \$1,600, but richest countries remain leaders in terms of overall payment efficiency. In contrast, the poorest countries do not gain much from either card or mobile payment innovations because most of their consumers are stuck with cash.

5.2 Policy implications

Our model sheds light on the welfare implications of public policies aimed at promoting mobile payment development. On the supply side, introducing mobile payments requires investment in the related infrastructures. Hence, it is important to evaluate the social return of such investment to guide policy decisions. Our model informs such decisions by quantifying the social benefit of introducing mobile payments given a country's per capita income level.²⁸

In doing so, one could use the model to compare per capita welfare gain from introducing mobile payments and its counterfactual counterpart. In the counterfactual scenario, no mobile payment is introduced, so card and cash continue to be the only payment options at date T_m . Per capita welfare of the counterfactual economy, denoted as $\tilde{W}_{T_m}$, is given by

$$\begin{aligned} \tilde{W}_{T_m} = & W_{h,T_m} + \int_{I_d(1+g)}^{\infty} (\bar{V}_d(I) - \bar{V}_h(I)) dG_{T_m}(I) + \int_{I_d}^{I_d(1+g)} (\bar{V}_d(I) - k_d - \bar{V}_h(I)) dG_{T_m}(I) \\ & + \sum_{s=1}^{\infty} \int_{\frac{I_d}{(1+g)^s}}^{\frac{I_d}{(1+g)^{s-1}}} \beta^s (\bar{V}_d(I(1+g)^s) - k_d - \bar{V}_h(I(1+g)^s)) dG_{T_m}(I), \end{aligned} \quad (31)$$

where $I_d = \frac{(1-\beta)k_d}{(\tau_h - \tau_d)}$ is from Eq. (5). Note that Eq. (31) is similar to Eq. (28) except that the income distribution is measured at date T_m (instead of T_d) and agents who have already adopted card before T_m no longer need to pay the card adoption cost k_d . Given the exponential income distribution, one can solve $\tilde{W}_{T_m}$ explicitly.

With the parameter values in Table 1, we calculate $W_{T_m} - \tilde{W}_{T_m}$ using Eqs. (29) and (31) to quantify per capita welfare gain from introducing two mobile payment options (i.e., card-substituting and card-complementing ones) in each country at $T_m = 2017$ in dollar value. We also calculate per capita welfare gain from introducing just one mobile payment option, denoted as $W_{T_m}^{Subs} - \tilde{W}_{T_m}$ for the card-substituting one and $W_{T_m}^{Comp} - \tilde{W}_{T_m}$ for the card-complementing one.²⁹ The results, plotted in the left panel of Figure 16, are

²⁸Our analysis focuses on the direct social benefit from improving payment efficiency. To the extent that payment innovations may have indirect social benefits (e.g., financial inclusion) or feedback impact on the broad economy, our calculation can be viewed as a lower bound.

²⁹Note that the calculation of $W_{T_m}^{Subs}$ is similar to W_{T_m} except that card-mobile switchers now need to pay a higher adoption cost k_m instead of k_m^a . The calculation of $W_{T_m}^{Comp}$, however, is quite different from W_{T_m} because any cash users now have to adopt card first before adopting mobile.

similar to the mobile payment options and adoption patterns shown in Figure 9.

Figure 16. PER CAPITA WELFARE GAIN (\$) FROM MOBILE PAYMENTS

According to the left panel of Figure 16, the per capita welfare gain from introducing both mobile payment options is low for the poorest countries (e.g., welfare gain is \$17 per capita for countries with per capita income at \$300) as well as for some relatively high-income countries (e.g., welfare gain is \$26 per capita for countries with per capita income at \$27,000). In contrast, the welfare gain peaks at \$106 per capita for countries with per capita income at \$1,800. For a rich country at the U.S.-level of per capita income (\$53,356 in 2017), the welfare gain is \$40 per capita.

The left panel of Figure 16 also suggests that the incremental welfare gain from introducing the second mobile payment option is relatively small and varies by per capita income. To make it clearer, in the right panel of Figure 16, we plot each country's per capita welfare gain from its more desirable choice of the two mobile payment options (i.e., $\max(W_{T_m}^{Subs} - \tilde{W}_{T_m}, W_{T_m}^{Comp} - \tilde{W}_{T_m})$), and we then plot the per capita welfare gain from adding the second option (i.e., $W_{T_m} - \max(W_{T_m}^{Subs}, W_{T_m}^{Comp})$). The result shows that countries with per capita income below \$16,000 would derive little per capita welfare gain ($< \$0.9$) from introducing card-complementing mobile payment as the second option. In contrast, a country with per capita income at \$50,000 could gain \$10 per capita by introducing card-substituting mobile payment as the second option, but that benefit would decline for countries with higher per capita income.

Based on the per capita welfare gain quantified in this exercise, one could calculate the total welfare gains for a country (i.e., per capita welfare gain $\times$ population) for introducing either one or two mobile payment options. Evaluating the welfare gains against the investment costs would then help determine the social return and the priority of making such investments.

On the demand side, our model’s policy implications are straightforward. Given that consumers make payment adoption and usage decisions that are socially efficient, providing a subsidy to elevate the equilibrium mobile payment adoption rate would cause welfare losses. In a similar vein, restricting cash use in favor of card or mobile payments would also lead to negative welfare consequences. These implications are consistent with the finding of Alvarez et al. (2022), who show that restricting cash usage distorts individual payment choices and causes substantial welfare losses in Mexico.

Of course, there are other policies to boost mobile payment adoption, e.g., policymakers may help reduce mobile payment fixed and/or variable costs by encouraging R&D or with improved regulations.³⁰ Our model can be readily applied to those scenarios to quantify the implied social benefits. Also, providing incentives to early adopters may align the expectations or create externalities to later adopters. We discuss one such consideration in the next section where the model is extended to a two-sided market setting.

6 Model extensions

Our model fits the average cross-country pattern of mobile payment adoption well. In this section, we extend the model for robustness checks and provide some further discussions.

6.1 Anticipation for mobile payments

We first relax the assumption that mobile payments arrive as an unanticipated shock. It is possible that at some stage in the 2000s or 2010s mobile payments were anticipated in some countries, which may have affected the incentives to adopt card. Extending our

³⁰To the extent that private firms may not internalize all the social welfare gains in their R&D decisions, government could enhance welfare by providing additional R&D incentives. Improving regulation can also help. For example, the Check 21 Legislation appears to have been instrumental in reducing the costs of checks via digitalization in the United States (see Humphrey and Hunt, 2013).

model to consider this scenario, we find that agents may hold off their card adoption in the hope of adopting mobile payments later, but that effect is quantitatively small.

Formally, we assume that at date $T_m - n$, agents receive news that mobile payments (i.e., card-substituting and card-complementing ones) may arrive at a Poisson rate ρ starting from next period.³¹ We then characterize how this information affects the income threshold of card adoption and the card and mobile adoption rates as follows.

Recall that the value of being a mobile payment user, $V_m(I_t)$, is given by Eq. (7). Once mobile payments have arrived, the value of being a cash user $V_h(I_t)$ is given by Eq. (9) and the value of being a card user $V_d(I_t)$ is given by Eq. (17). Starting from date $T_m - n$ until date $T_m - 1$ before mobile payments actually arrive, the value of being a cash user, denoted by $\tilde{V}_h(I_t)$, is given by

$$\tilde{V}_h(I_t) = (1 - \tau_h)I_t + \beta \left\{ \begin{array}{l} \rho \max [V_h(I_{t+1}), V_m(I_{t+1}) - k_m] \\ + (1 - \rho) \max [\tilde{V}_h(I_{t+1}), \tilde{V}_d(I_{t+1}) - k_d] \end{array} \right\}, \quad (32)$$

where $\tilde{V}_d(I_t)$ is the value of being a card user during this period, which is given by

$$\tilde{V}_d(I_t) = (1 - \tau_d)I_t + \beta \left\{ \begin{array}{l} \rho \max [V_d(I_{t+1}), V_m(I_{t+1}) - k_m^a] \\ + (1 - \rho) \tilde{V}_d(I_{t+1}) \end{array} \right\}. \quad (33)$$

Denote I_d^ρ the income threshold at which an agent is indifferent between adopting card and continuing with cash at any date between $T_m - n$ and $T_m - 1$. Given the mobile payment news, the income threshold of adopting card becomes higher, which means $I_d^\rho > I_d > I_m$. At income I_d^ρ , Eqs. (32) and (33) imply that

$$\tilde{V}_h(I_d^\rho) = (1 - \tau_h)I_d^\rho + \beta \left\{ \begin{array}{l} \rho [V_m(I_d^\rho(1 + g)) - k_m] \\ + (1 - \rho) [\tilde{V}_d(I_d^\rho(1 + g)) - k_d] \end{array} \right\}, \quad (34)$$

and

$$\tilde{V}_d(I_d^\rho) = (1 - \tau_d)I_d^\rho + \beta \left\{ \begin{array}{l} \rho \max [V_d(I_d^\rho(1 + g)), V_m(I_d^\rho(1 + g)) - k_m^a] \\ + (1 - \rho) \tilde{V}_d(I_d^\rho(1 + g)) \end{array} \right\}. \quad (35)$$

³¹Alternatively, if only card-substituting mobile payment is anticipated to come, one simply needs to replace k_m^a with k_m for card-mobile switchers in the analysis. The results are very similar.

Since agents at income I_d^ρ are indifferent between adopting card and continuing with cash, we have

$$\tilde{V}_d(I_d^\rho) - \tilde{V}_h(I_d^\rho) = k_d. \quad (36)$$

Inserting Eqs. (34) and (35) into Eq. (36) then yields

$$\begin{aligned} k_d - \beta(1 - \rho)k_d &= (\tau_h - \tau_d)I_d^\rho + \beta\rho \max[V_d(I_d^\rho(1 + g)), V_m((I_d^\rho(1 + g)) - k_m^a)] \\ &\quad - \beta\rho[V_m(I_d^\rho(1 + g)) - k_m], \end{aligned} \quad (37)$$

which pins down I_d^ρ , the new income threshold for adopting card (see Appendix IV for the solution details).

Given the solution of I_d^ρ , the model yields the adoption of card and mobile payment for any $t \geq T_m$ as follows. Note that agents would stop adopting card once mobile payments arrive at date T_m , so the card adoption rate is fixed at F_{d,T_m-1} for any date $t \geq T_m - 1$. To be specific, F_{d,T_m-1} is the fraction of agents whose incomes have crossed the card adoption threshold I_d at date $T_m - n - 1$ or the fraction of agents whose incomes have crossed the new card adoption threshold I_d^ρ at date $T_m - 1$, whichever is larger:

$$\begin{aligned} F_{d,t} &= F_{d,T_m-1} = \max(1 - G_{T_m-n-1}(I_d), 1 - G_{T_m-1}(I_d^\rho)) \\ &= \max\left(\exp\left(-\frac{I_d}{\lambda_{T_m-n-1}}\right), \exp\left(-\frac{I_d^\rho}{\lambda_{T_m-1}}\right)\right), \end{aligned}$$

where $\lambda_{T_m-1} = \lambda_{T_m-n-1}(1 + g)^n$.

The mobile payment adoption rate at any date $t \geq T_m$ comes from two sources. The first one is the cash-mobile switchers (i.e., the fraction of agents who have switched from cash to mobile by date t):

$$\begin{aligned} F_{h \rightarrow m,t} &= 1 - F_{d,T_m-1} - G_t(I_m) \\ &= \exp\left(-\frac{I_m}{\lambda_t}\right) - \max\left(\exp\left(-\frac{I_d}{\lambda_{T_m-n-1}}\right), \exp\left(-\frac{I_d^\rho}{\lambda_{T_m-1}}\right)\right). \end{aligned} \quad (38)$$

The second one comprises of the card-mobile switchers (i.e., the fraction of agents who have switched from card to mobile by date t):

$$F_{d \rightarrow m,t} = 1 - G_t(I_{m'}) = \exp\left(-\frac{I_{m'}}{\lambda_t}\right) \quad (39)$$

as long as some card adopters have not adopted mobile (i.e., $F_{d \rightarrow m, t} < F_{d, T_m - 1}$). Otherwise, $F_{d \rightarrow m, t} = F_{d, T_m - 1}$. Combining cash-mobile switchers (i.e., $F_{h \rightarrow m, t}$) and card-mobile switchers ($F_{d \rightarrow m, t}$) together then yields the overall mobile payment adopters.

Figure 17. ANTICIPATED MOBILE PAYMENTS AND ADOPTION PATTERNS

Using the set of parameter values in Table 1, we compute I_d^ρ and re-simulate the model. We consider two scenarios: Agents anticipate mobile payments 5 years (i.e., $n = 5$ and $\rho = 1/n = 0.2$) or 10 years (i.e., $n = 10$ and $\rho = 1/n = 0.1$) ahead of the actual arrival at $T_m = 2017$. The resulting card and mobile adoption patterns are plotted in Figure 17. In either scenario, an increased card adoption threshold satisfies $I_d^\rho > I_d(1 + g)^n$, which induces agents who otherwise would have adopted card between $T_m - n$ and $T_m - 1$ to postpone their card adoption. Once mobile payments arrive at date T_m , those delayed card adopters become additional mobile payment adopters. The earlier the anticipation, the more the delayed card adopters, and the higher the mobile payment adoption. However, as Figure 17 shows, comparing with our original model where mobile payments arrive unexpectedly, the effect of anticipation is quantitatively small.³²

³²Note that our findings are robust for any alternative values of $\rho \in [0, 1]$. When $\rho = 0$, we are back to the original model where mobile payments are not anticipated. When ρ is sufficiently large, as in the two examples we considered (i.e., $\rho = 0.1$, $n = 10$ or $\rho = 0.2$, $n = 5$), the new income threshold for card adoption satisfies $I_d^\rho > I_d(1 + g)^n$. As a result, all the agents who otherwise would have adopted card between $T_m - n$ and $T_m - 1$ hold off their card adoption until mobile payments actually arrive at date T_m . The same result would hold for any larger value of ρ . Therefore, for $n = 10$ or $n = 5$, the original model and the extended model provide the upper and lower bounds for card and mobile adoption rates for any $\rho \in [0, 1]$.

6.2 Income distribution and Gini coefficient

We have assumed an exponential income distribution in the model. While this assumption simplifies the analysis and fits the data well, one may wonder if our findings are robust under alternative distributional assumptions. Moreover, the property that the exponential distribution has a fixed Gini coefficient at 0.5 raises the question of how much of the cross-country payment adoption pattern is affected by country-specific Gini coefficients outside the model.

Figure 18. LOG-LOGISTIC INCOME DISTRIBUTION

To address these questions, we extend the model by assuming a log-logistic income distribution, which has also been popularly used in empirical studies (e.g., see Fisk, 1961). Figure 18 presents an example of fitting the U.S. household income distribution in 2017 with a log-logistic distribution. The log-logistic distribution has two parameters that capture the mean and the Gini coefficient separately.

Keeping everything else the same as our previous analysis, we now assume income I_t follows a log-logistic distribution in each economy, with the cumulative distribution function (cdf)

$$G_t(I_t) = 1 - \frac{1}{1 + \left(\frac{I_t}{\lambda_t}\right)^{\frac{1}{\eta}} \Phi},$$

where λ_t is the mean, η is the Gini coefficient, and $\Phi = \left(\frac{\pi\eta}{\sin(\pi\eta)}\right)^{\frac{1}{\eta}}$. Over time, each agent's income grows at a constant rate g , i.e., $I_{t+1} = I_t(1 + g)$, as does the mean income of the economy (i.e., $\lambda_{t+1} = \lambda_t(1 + g)$).

The model analysis is the same as before except that we now replace the card adoption equation (6) with Eq. (40) and replace the mobile adoption equation (20) with Eq. (41)

as follows:

$$F_{d,t} = \frac{1}{1 + \left(\frac{(1-\beta)k_d}{(\tau_h - \tau_d)\lambda_t} \right)^{\frac{1}{\eta}} \Phi}, \quad (40)$$

$$F_{m,t} = \frac{1}{1 + \left(\frac{(1-\beta)k_m}{(\tau_h - \tau_m)\lambda_t} \right)^{\frac{1}{\eta}} \Phi} - \frac{1}{1 + \left(\frac{(1-\beta)k_d}{(\tau_h - \tau_d)\lambda_{T_m-1}} \right)^{\frac{1}{\eta}} \Phi} + \frac{1}{1 + \left(\frac{(1-\beta)k_m^a}{(\tau_d - \tau_m)\lambda_t} \right)^{\frac{1}{\eta}} \Phi}. \quad (41)$$

Under the log-logistic distribution assumption, all the theoretical results we had before continue to hold. The remaining questions are how much the log-logistic distribution would affect the quantitative findings, and what role the country-specific Gini coefficients could play in explaining cross-country payment adoption patterns.

Figure 19. LOG-LOGISTIC MODEL FIT (GINI = 0.5).

To compare directly with our previous findings, we first assume a log-logistic income distribution with $\eta = 0.5$, the same value of the Gini coefficient under the exponential distribution, for every country in the sample. Using the same set of parameter values in Table 1, we re-simulate the model. The results, plotted in Figure 19, are similar to the model fitting pattern in Figure 7.

We then simulate the model again by allowing each country to have the actual country-specific Gini coefficient as observed in the data.³³ In Figure 20, we plot the results and

³³In the simulation, we use the average value of the Gini coefficient during the one decade before the sample period (i.e., 2007-2016) for each country. The resulting Gini coefficients range from 0.27 to 0.61 among our sample countries.

contrast them with those assuming a uniform $\text{Gini} = 0.5$. The comparison suggests that the cross-country card and mobile adoption patterns are mainly driven by the level of per capita income, and the difference in Gini coefficient only plays a small role quantitatively. This finding is consistent with the regression results reported in Appendix Tables A3 and A4, which show that the cross-country mobile payment adoption pattern is significantly correlated with per capita income but not the Gini index.

Figure 20. LOG-LOGISTIC MODELS (COUNTRY-SPECIFIC GINI VS. GINI = 0.5).

Furthermore, we redo the welfare and policy analysis with the log-logistic income distribution. The results are quantitatively very similar to what we obtained in the previous analysis.

6.3 Two-sided market considerations

It is well known in the literature that the payment market is two-sided. A payment technology needs to be adopted by both buyers and sellers for being widely used in the economy. Our model so far has been explicit about consumers' (buyers') side of the market but not much about the merchants' (sellers') side. We now extend the model to a two-sided market setting and explore the policy implications.

As before, consider that each consumer receives an income I_t at date t , and I_t follows an exponential distribution across the population of consumers. The income is used to purchase a numeraire good for consumption each period. The numeraire good is produced at a unit cost and distributed through competitive merchants. Conducting a transaction

between a merchant and a consumer requires using a payment technology $i \in \{h \text{ (cash)}, d \text{ (card)}, m \text{ (mobile)}\}$, for which the merchant (seller) and the consumer (buyer) each incurs a variable cost $\tau_{s,i}$ and $\tau_{b,i}$ per dollar of transactions, respectively. Merchants are each at a sufficiently large size, so the fixed cost for a merchant to adopt card or mobile payment technology is negligible on a per customer or per transaction basis. Assume merchants can price discriminate based on payment method, for example, by specializing in accepting a particular payment form or charging customers different prices based on payment instruments. Therefore, a competitive merchant accepting payment technology i would set price p_i for selling the numeraire good to break even:

$$p_i = \frac{1}{1 - \tau_{s,i}},$$

and a consumer using payment technology i at date t would purchase and consume the quantity $q_{i,t}$ of the good:

$$q_{i,t} = \frac{I_t(1 - \tau_{b,i})}{p_i} = I_t(1 - \tau_{b,i})(1 - \tau_{s,i}).$$

Assume that consumers need to pay k_d and k_m as the one-time fixed costs associated with adopting card and mobile payment technology, respectively. It is straightforward to see the new model setting is equivalent to our original model by modifying notations: For each payment technology $i \in \{h, d, m\}$, we simply need to redefine the variable cost τ_i such that

$$(1 - \tau_i) = (1 - \tau_{b,i})(1 - \tau_{s,i}) \implies \tau_i = \tau_{b,i} + \tau_{s,i} - \tau_{b,i}\tau_{s,i}.$$

Essentially, merchants pass on their payment costs to consumers through retail prices. As before, to capture the technology progress between cash, card, and mobile, we assume $\tau_h > \tau_d > \tau_m$ and $k_d > k_m$.

Extending our model interpretation to the two-sided market setting brings additional insights. First, the discussion makes it clear that one should take into account payment costs of both merchants and consumers in the analysis. In our competitive market setting, consumers ultimately bear the social costs of the payment instruments they use, and that is why we choose to calibrate the model using measures of the social costs of payment means.

Moreover, given that the payment market outcome depends on two sides’ decisions, multiple equilibria can arise. The market outcome we analyzed previously remains a valid equilibrium, but it is no longer the unique one. For example, there could exist another equilibrium where no merchants or consumers adopt a new payment technology because they each expect no adoption from the other side. This so-called “chicken-and-egg” dynamic often arises in network industries or for technologies featuring strong adoption complementarity, and coordination becomes an important issue (see e.g., Buera et al., 2021). In terms of mobile payments, we observe in the data that some countries have an adoption rate far below their peers with similar per capita income levels, which might signal certain coordination failures among relevant parties.³⁴ In those cases, appropriate policy interventions, such as coordinating standard setting or providing incentives to early adopters, may help align market expectations and enhance welfare.

This model extension suggests that our model can apply to a two-sided market setting where competitive players (e.g., payment service providers and merchants) pass on payment costs to consumers based on the payment methods they use. In the cases where market players have market power and price strategically across payment methods, our model may serve as a simplified first-order approximation (see Li et al., 2020 for a related analysis). We leave a full-blown two-sided market analysis for future research.

6.4 Additional factors

By focusing on the role played by income, our analysis has abstracted from some other factors that could also affect payment technology adoption. For example, the model calibration suggests that the extraordinary mobile payment adoption in Kenya and China (cf. Figure 4B) cannot be fully explained by our theory, so additional factors may also be at play. In fact, Jack and Suri (2014) highlight the role of M-PESA in urban-rural remittances in Kenya, which provides an important risk-sharing function.³⁵ In China, the two

³⁴For example, Aker, Prina and Welch (2020) show that mobile money has failed to take off in Niger because of a chicken-and-egg problem: Agents need to be widespread for the service to be useful, but putting agents everywhere is not viable until the service is widespread.

³⁵Recent studies find that the unique urban-rural remittance pattern in Kenya is an important factor to explain its exceptionally wide adoption of M-PESA. Accordingly, Kenya’s success in adopting mobile payment should be regarded as an outlier rather than normative (see Piper, Kelsey (September 11, 2020). *What Kenya can teach its neighbors — and the US — about improving the lives of the “unbanked.”* Vox). This is consistent with our model’s prediction, which underestimates the mobile payment adoption rate of Kenya but fits well the adoption rates of Kenya’s neighboring countries.

tech giants, Alibaba and Tencent, have developed their mobile payment services, Alipay and WeChat Pay, to strategically extend their business models to cross-sell consumer and business loan services based on payments data (Hau et al., 2019).

More broadly, income is unlikely the only factor that matters. Holding income fixed, exogenous variations in other factors may also affect a country’s payment technology adoption, such as the footprint of its banking system, the payments market structure and pricing, country-specific payment adoption and usage costs, as well as government policies and regulations. Also, while income is assumed to grow exogenously in our model, adopting better payment technologies may have feedback effect on income and generate second-order effects to some extent.

Nevertheless, focusing on income heterogeneity allows us to provide a parsimonious model to explain cross-country payment adoption patterns without being bogged down by idiosyncratic details. It is known from consumer surveys that income is a key factor explaining card adoption at household level (e.g., see Greene and Shy, 2022), which is consistent with our finding that per capita GDP positively correlates with card adoption across countries. It is striking to see such a positive relation absent for mobile payments, and our model offers a coherent explanation in the context of sequential payment innovations. Our calibrated model fits the average cross-country mobile payment adoption pattern well and provides a convenient framework for counterfactual and welfare analysis. Incorporating additional factors in the analysis may further enhance our understanding of payment adoption across countries, especially the variations in the data that are not accounted for by our theory, and those will be valuable venues for future research.

7 Conclusion

This paper provides a framework to explain the adoption of card and mobile payments within and across countries. With a novel dataset, we find that the adoption of mobile payment exhibits a non-monotonic relationship with per capita income. This is in contrast with card payment, for which the adoption increases monotonically in per capita income across countries. Also, countries choose different mobile payment solutions: Advanced economies favor those complementary to the existing card payments, while developing countries prefer those substituting cards.

Our theory provides a consistent explanation for these patterns. In our model, three payment technologies, cash, card, and mobile, arrive sequentially. Newer payment technologies lower the variable costs of conducting payments, but they require a fixed cost to adopt. Rich countries enjoy advantages in adopting card payments for replacing cash early on, but this success later hinders their adoption of the mobile payment innovation. Also, the fixed-cost considerations provide strong incentives for card-intensive countries to adopt mobile payment methods complementary to cards, while cash-intensive countries favor card-substituting mobile solutions.

Our model calibration matches cross-country data well. We find that lagging behind in mobile payment adoption does not necessarily mean that advanced economies fall behind in overall payment efficiency. Rather, slower adoption can be an optimal choice given that the incremental benefit of switching from card to the current mobile payment technology is not large enough. Down the road, greater technological advances are needed for advanced economies to catch up in the mobile payment race, and policy interventions require prudent social cost-benefit analyses.

While our paper focuses on consumers' choices of making payments, mobile payments may have broader impact. For example, it may affect financial inclusion and credit markets. Moreover, the rise of nonbank payment service providers, particularly FinTech firms, may pose new challenges to financial stability and regulations. Those would be interesting topics for continuing research (see Goldstein et al. (2019) for a general discussion). Last but not least, our analysis is related to other financial or non-financial innovations. By deriving conditions for leapfrogging in the payment context, our findings shed light on the broad issue on rank-preserving versus leapfrogging in the adoption of new technologies.

References

- [1] Aker, Jenny C., Silvia Prina, and C. Jamilah Welch. (2020). “Migration, Money Transfers, and Mobile Money: Evidence from Niger.” *AEA Papers and Proceedings*, 110, 589-93.
- [2] Alvarez, Fernando, and Francesco Lippi. (2009). “Financial Innovation and the Transactions Demand for Cash.” *Econometrica*, 77(2), 363-402.
- [3] Alvarez, Fernando, and Francesco Lippi. (2017). “Cash Burns: An Inventory Model with a Cash-credit Choice.” *Journal of Monetary Economics*, 90, 99-112.
- [4] Alvarez, Fernando, David Argente, Rafael Jimenezc, and Francesco Lippi. (2022). “Cash: A Blessing or a Curse?” *Journal of Monetary Economics*, 125, 85-128.
- [5] Berg, Tobias, Valentin Burg, Jan Keil, and Manju Puri. (2022). “On the Rise of Payment Firms.” Working Paper.
- [6] Buera, Francisco, Hugo Hopenhayn, Yongseok Shin, and Nicholas Trachter. (2021). “Big Push in Distorted Economies.” NBER Working Paper No. 28561.
- [7] Comin, Diego, and Bart Hobijn. (2004). “Cross-country Technology Adoption: Making the Theories Face the Facts.” *Journal of Monetary Economics*, 51(1), 39-83.
- [8] Crowe, Marianne, Marc Rysman, and Joanna Stavins. (2010). “Mobile Payments in the United States at Retail Point of Sale: Current Market and Future Prospects.” *Review of Network Economics*, 9(4), Article 2.
- [9] Dragulescu, Adrian, and Victor M. Yakovenko. (2001). “Evidence for the Exponential Distribution of Income in the USA.” *European Physical Journal B*, 20(4): 585-589.
- [10] Fisk, Peter R. (1961). “The Graduation of Income Distributions.” *Econometrica*, 29 (2): 171-185.
- [11] Ghosh, Pulak, Boris Vallee, and Yao Zeng. (2021). “FinTech Lending and Cashless Payments.” Working Paper.
- [12] Goldstein, Itay, Wei Jiang, and G. Andrew Karolyi. (2019). “To FinTech and Beyond.” *Review of Financial Studies*, 32(5), 1647-1661.

- [13] Greene, Claire, and Oz Shy. (2022). “Payment Card Adoption and Payment Choice.” Federal Reserve Bank of Atlanta Working Paper.
- [14] Greenwood, Jeremy, and Boyan Jovanovic. (1990). “Financial Development, Growth, and the Distribution of Income.” *Journal of Political Economy*, 98(5), 1076-1107.
- [15] Griliches, Zvi. (1957). “Hybrid Corn: An Exploration in the Economics of Technical Change.” *Econometrica*, 25(4), 501-522.
- [16] Gupta, Sunil, Shelle Santana, and Margaret L. Rodriguez. (2016). “Apple Pay.” Harvard Business School Case 516-027.
- [17] Hau, Harald, Yi Huang, Hongzhe Shan, and Zixia Sheng. (2019). “How FinTech Enters China’s Credit Market.” *AEA Papers and Proceedings*, 109, 60-64.
- [18] Hayashi, Fumiko, Bin Grace Li, and Zhu Wang. (2017). “Innovation, Deregulation, and the Life Cycle of a Financial Service Industry.” *Review of Economic Dynamics*, 26, 180-203.
- [19] Humphrey, David B., and Robert Hunt. (2013). “Cost Savings from Check 21 Electronic Payment Legislation.” *Journal of Money, Credit and Banking*, 45(7), 1415-1429.
- [20] Jack, William, and Tavneet Suri. (2014). “Risk Sharing and Transactions Costs: Evidence from Kenya’s Mobile Money Revolution.” *American Economic Review*, 104(1), 183-223.
- [21] Jovanovic, Boyan, and Yaw Nyarko. (1996). “Learning by Doing and the Choice of Technology.” *Econometrica*, 64(6), 1299-1310.
- [22] Klee, Elizabeth. (2008). “How People Pay: Evidence from Grocery Store Data.” *Journal of Monetary Economics*, 55(3), 526-541.
- [23] Klenow, Peter. (1998). “Learning Curves and the Cyclical Behavior of Manufacturing Industries.” *Review of Economic Dynamics*, 1, 531-550.
- [24] Li, Bin Grace, James McAndrews, and Zhu Wang. (2020). “Two-sided Market, R&D, and Payments System Evolution.” *Journal of Monetary Economics*, 115, 180-199.
- [25] Manuelli, Rodolfo, and Ananth Seshadri. (2014). “Frictionless Technology Diffusion: The Case of Tractors.” *American Economic Review*, 104(4), 1368-91.

- [26] Muralidharan, Karthik, Paul Niehaus, and Sandip Sukhtankar. (2016). “Building State Capacity: Evidence from Biometric Smartcards in India.” *American Economic Review*, 106(10), 2895-2929.
- [27] Ouyang, Shumiao. (2021). “Cashless Payment and Financial Inclusion.” Working Paper.
- [28] Parente, Stephen. (1994). “Technology Adoption, Learning by Doing and Economic Growth.” *Journal of Economic Theory*, 63, 346-369.
- [29] Parlour, Christine A., Uday Rajan, and Haoxiang Zhu. (2021). “When FinTech Competes for Payment Flows.” *Review of Financial Studies*, forthcoming.
- [30] Philippon, Thomas. (2019). “On Fintech and Financial Inclusion.” NBER Working Paper No. 26330.
- [31] Rochet, Jean-Charles, and Jean Tirole. (2002). “Cooperation among Competitors: Some Economics of Payment Card Associations.” *RAND Journal of Economics*, 33(4), 549-570.
- [32] Rochet, Jean-Charles, and Jean Tirole. (2003). “Platform Competition in Two-Sided Markets.” *Journal of the European Economic Association*, 1(4), 990-1029.
- [33] Rysman, Marc. (2007). “An Empirical Analysis of Payment Card Usage.” *Journal of Industrial Economics*, 55(1), 1-36.
- [34] Schmiedel, Heiko, Gergana Kostova, and Wiebe Ruttenberg. (2012). “The Social and Private Costs of Retail Payment Instruments: A European Perspective.” Occasional Paper Series 137, European Central Bank.
- [35] Shy, Oz , and Zhu Wang. (2011). “Why Do Payment Card Networks Charge Proportional Fees?” *American Economic Review*, 101(4), 1575-1590.
- [36] Wang, Zhu, and Alexander Wolman. (2016). “Payment Choice and Currency Use: Insights from Two Billion Retail Transactions.” *Journal of Monetary Economics*, 84, 94-115.
- [37] Wright, Julian. (2003). “Optimal Card Payment Systems.” *European Economic Review*, 47, 587-612.
- [38] Wright, Julian. (2012). “Why Payment Card Fees Are Biased Against Retailers.” *RAND Journal of Economics*, 43(4), 761-780.

Appendix

I. Data sources.

The mobile payment data introduced in Section 2.2 are drawn from two sources. First, the data on the adoption rate for card-substituting mobile payment services in 2017 are based on the Global Financial Inclusion (Global Findex) Database of the World Bank, which surveyed 76 countries with a visible presence of Mobile Money payment services. The Global Findex database was launched in 2011 and has been published every three years since then. The 2017 version of the database is based on nationally representative surveys of more than 150,000 adults (age 15 and above) in 144 economies. Among the 144 economies, 76 economies (where the GSMA MMU database indicates that mobile money accounts were available at the time the survey was carried out) were surveyed for mobile money adoption: “To identify people with a mobile money account, the 2017 Global Findex survey asked respondents about their use of specific services available in their economy — such as M-PESA, MTN Mobile Money, Airtel Money, or Orange Money — and included in the GSM Association’s Mobile Money for the Unbanked (GSMA MMU) database. The definition of a mobile money account is limited to services that can be used without an account at a financial institution.”

Second, the data on the adoption rate for card-complementing mobile payments around 2017 were gathered from eMarketer’s public website. eMarketer is a market research company headquartered in New York City. Its report on “Proximity Mobile Payment Users Worldwide, 2019” estimates adult mobile proximity payment users (age 14+) in 23 countries where mobile proximity payments had a visible presence. According to the European Payments Council, “mobile proximity payments are mobile payments in which the payer and the payee are in the same location and where the communication between their devices takes place through a proximity technology (such as Near Field Communication (NFC), Quick Response (QR) codes, Bluetooth technology, etc.).” To be more specific, the adoption rate of mobile proximity payments in the eMarketer data is the adoption rate among mobile phone users, so we multiply that by the mobile phone ownership rate of each country (obtained from GSMA) to obtain the mobile proximity payment adoption rate in the population. As a sanity check, our estimate of the mobile payment adoption rate in the eMarketer data is 24.6% for the U.S., comparable to the

mobile payment adoption rate of 28.7% estimated from the U.S. Survey of Consumer Payment Choice conducted by the Federal Reserve in 2017.

II. Regression results.

Figure 5 in Section 2.2 shows that mobile payment adoption displays a non-monotonic relationship with per capita GDP: increasing among countries with per capita GDP less than \$2,500, decreasing among countries with per capita GDP between \$2,500 and \$20,000, and increasing again among countries with per capita GDP greater than \$20,000. Figure 20 redoes the exercise and shows that this pattern continues to hold if we exclude two outlier countries (Kenya and China) that have exceptionally high mobile payment adoption rates ($>60\%$).

Figure 20. Mobile Payment Adoption Pattern (excluding Kenya and China)

Table A1 reports the OLS regression results for card and mobile payment adoption related to Figures 4, 5B and 20B. Across the 94 countries in the sample, regression (1) indicates that the card adoption rate in 2017 is significantly and positively related to per capita GDP in 2017. In contrast, regression (2) suggests that the mobile payment adoption bears no significant relationship with per capita GDP for the same sample. In fact, the adjusted R^2 shows a negative value, which implies that the fit would be better if we simply run the regression with only a constant.

Table A1. Cross-Country Payment Adoption: OLS Regressions

	Card		Mobile	
	(1)	(2)	(3)	(4)
ln(GDP per capita)	0.186***	0.001	0.113**	0.091**
	(0.009)	(0.010)	(0.053)	(0.040)
ln(GDP per capita) $\times$ 1{Middle Income}			-0.220**	-0.175**
			(0.096)	(0.072)
ln(GDP per capita) $\times$ 1{High Income}			-0.087	-0.065
			(0.116)	(0.088)
1{Middle Income}			1.708**	1.344**
			(0.804)	(0.609)
1{High Income}			0.425	0.295
			(1.169)	(0.882)
Constant	-1.179***	0.163*	-0.497	-0.366
	(0.079)	(0.083)	(0.362)	(0.274)
Observations	94	94	59	57
Adjusted R^2	0.81	-0.01	0.07	0.08

Regressions in this table are based on OLS models. The dependent variable in regression (1) is debit card adoption rate in 2017. The dependent variables in regressions (2), (3) and (4) are mobile payment adoption rate in 2017. The independent variables include the GDP per capita of 2017 and a constant in regressions (1) and (2), plus two dummy variables (i.e., Middle Income and High Income) and their interaction terms with the GDP per capita in regressions (3) and (4). In regression (4), we exclude two outliers that have mobile payment adoption rates greater than 60% (i.e., Kenya and China). Standard errors are reported in the parentheses. *** denotes statistical significance at 1% level, ** at 5% level, and * at 10% level.

However, a subtle pattern of mobile payment adoption emerges once we remove countries with exceptionally low adoption rates of mobile payments (adoption rate $< 10\%$) and group the remaining ones by income. Regression (3) shows that mobile payment adoption increases in per capita GDP for low-income countries (per capita GDP $< \$2,500$) and high-income countries (per capita GDP $> \$20,000$), but decreases in per capita GDP for middle-income countries ($\$2,500 \leq \text{per capita GDP} \leq \$20,000$). Specifically, the coefficient estimate of ln(GDP per capita) for the low-income countries is 0.113 and statistically significant. This suggests that doubling per capita GDP would increase mobile payment adoption by 11.3% for the low-income countries. The coefficient estimate of ln(GDP per capita) $\times$ 1{High Income} is small and not statistically significant, suggesting that the marginal effect of per capita GDP on mobile payment adoption in high-income countries is not significantly different from that in low-income countries. On the other hand, we estimate the coefficient of ln(GDP per capita) $\times$ 1{Middle Income} to be -0.220 and sta-

tistically significant. This implies that the marginal effect of per capita GDP on mobile payment adoption in middle-income countries is significantly lower than that in low-income (and high-income) countries. The coefficient difference, (0.113-0.220), suggests that doubling per capita GDP is associated with a 10.7% reduction in mobile payment adoption rate among middle-income countries.

As a robustness check, we exclude two outlier countries (Kenya and China) with exceptionally high mobile payment adoption rates ($> 60\%$) in regression (4). The results are similar to regression (3) though the estimates are smaller in absolute values.

For additional robustness checks, we re-run the regressions using the fractional logit (FL) model to address the fractional nature of the dependent variables (i.e., card/mobile adoption rates), which are bounded by 0 and 1. The estimated marginal effects, shown in Table A2, are very similar to the OLS results in Table A1.

Table A2. Cross-Country Payment Adoption: FL Regressions

	Card		Mobile	
	(1)	(2)	(3)	(4)
$\ln(\text{GDP per capita})$	0.229***	0.001	0.106***	0.085***
	(0.012)	(0.008)	(0.039)	(0.032)
$\ln(\text{GDP per capita}) \times 1\{\text{Middle Income}\}$			-0.211***	-0.171***
			(0.076)	(0.059)
$\ln(\text{GDP per capita}) \times 1\{\text{High Income}\}$			-0.077	-0.058
			(0.083)	(0.077)
$1\{\text{Middle Income}\}$			1.644**	1.317***
			(0.651)	(0.505)
$1\{\text{High Income}\}$			0.347	0.237
			(0.831)	(0.787)
Observations	94	94	59	57

Regressions in Table A2 are based on the fractional logit (FL) models. The dependent and independent variables in the regressions are the same as those in Table A1. The coefficient estimates are expressed in terms of marginal effects evaluated at the means of the independent variables. Standard errors are reported in the parentheses. *** denotes statistical significance at 1% level, ** at 5% level, and * at 10% level.

Finally, we include additional control variables to the mobile payment adoption regression. The regressions in Table A3 exclude countries with adoption rates less than 10% and the regressions in Table A4 also exclude Kenya and China. The results in both tables support the findings in Tables A1 and A2 that mobile payment adoption has a non-monotonic relationship with per capita income.

Table A3. Mobile Payment Adoption: OLS Regressions with More Control Variables

	<i>Mobile</i>							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ln(GDP per capita)	0.113** (0.0532)	0.116** (0.0563)	0.132** (0.0648)	0.146* (0.0736)	0.164** (0.0716)	0.164** (0.0723)	0.160** (0.0733)	0.153** (0.0749)
ln(GDP per capita) $\times$ 1{Middle income}	-0.220** (0.0956)	-0.220** (0.0965)	-0.238** (0.103)	-0.250** (0.111)	-0.246** (0.107)	-0.243** (0.109)	-0.263** (0.117)	-0.278** (0.118)
ln(GDP per capita) $\times$ 1{High income}	-0.0872 (0.116)	-0.0862 (0.117)	-0.107 (0.125)	-0.108 (0.139)	-0.126 (0.134)	-0.125 (0.136)	-0.133 (0.138)	-0.134 (0.145)
1{Middle income}	1.708** (0.804)	1.710** (0.812)	1.863** (0.869)	1.976** (0.924)	1.924** (0.892)	1.902** (0.910)	2.048** (0.961)	2.161** (0.971)
1{High income}	0.425 (1.169)	0.420 (1.181)	0.613 (1.246)	0.621 (1.359)	0.841 (1.315)	0.836 (1.330)	0.872 (1.343)	0.861 (1.393)
Education		-0.0368 (0.242)	-0.0630 (0.249)	-0.190 (0.281)	-0.245 (0.272)	-0.234 (0.281)	-0.278 (0.296)	-0.239 (0.298)
Mobile phones			-0.0406 (0.0781)	-0.0764 (0.0860)	-0.0812 (0.0830)	-0.0855 (0.0871)	-0.0980 (0.0912)	-0.108 (0.0963)
Banking concentration				-0.0438 (0.127)	-0.0457 (0.123)	-0.0527 (0.130)	-0.0483 (0.131)	-0.0745 (0.135)
Bank ROA					4.491** (2.144)	4.357* (2.286)	3.862 (2.500)	3.679 (2.486)
Share of population above 65						-0.116 (0.629)	-0.110 (0.634)	-0.0922 (0.642)
Share of self-employed							-0.130 (0.253)	-0.166 (0.267)
Gini index								-0.0844 (0.347)
Constant	-0.497 (0.362)	-0.498 (0.365)	-0.565 (0.390)	-0.530 (0.500)	-0.705 (0.490)	-0.693 (0.499)	-0.533 (0.593)	-0.405 (0.605)
Observations	59	59	59	55	55	55	55	54
R^2	0.146	0.146	0.151	0.170	0.243	0.244	0.249	0.265
Adjusted R^2	0.065	0.048	0.034	0.025	0.092	0.072	0.056	0.050

The dependent variable in each regression is mobile payment adoption rate in 2017. The independent variables include those in Table A1 plus the additional ones: Education (World Bank education index), Mobile phones (number of mobile phones per capita), Banking concentration (assets of five largest banks as a share of total commercial banking assets), Bank ROA (bank return on assets), Share of population above 65 (share of population with age 65 and above), Share of self-employed (share of total employment that is self-employed), and Gini index. Most independent variables are dated by year 2017 except that Banking concentration, Bank ROA and Gini index are the averages between 2007-2016. *** denotes statistical significance at 1% level, ** at 5% level, and * at 10% level.

Table A4. Mobile Payment Adoption: OLS Regressions with More Control Variables
(Excluding Kenya and China)

	<i>Mobile</i>							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ln(GDP per capita)	0.0912** (0.0403)	0.0975** (0.0424)	0.100** (0.0492)	0.120** (0.0557)	0.134** (0.0540)	0.133** (0.0543)	0.128** (0.0548)	0.109* (0.0540)
ln(GDP per capita) $\times$ 1{Middle income}	-0.175** (0.0724)	-0.175** (0.0729)	-0.178** (0.0786)	-0.188** (0.0841)	-0.184** (0.0810)	-0.175** (0.0825)	-0.199** (0.0877)	-0.199** (0.0856)
ln(GDP per capita) $\times$ 1{High income}	-0.0654 (0.0875)	-0.0625 (0.0883)	-0.0662 (0.0943)	-0.0883 (0.105)	-0.103 (0.101)	-0.0982 (0.102)	-0.109 (0.103)	-0.0789 (0.104)
1{Middle income}	1.344** (0.609)	1.347** (0.613)	1.376** (0.662)	1.443** (0.704)	1.390** (0.678)	1.317* (0.690)	1.484** (0.721)	1.477** (0.703)
1{High income}	0.295 (0.882)	0.278 (0.889)	0.313 (0.941)	0.510 (1.023)	0.676 (0.988)	0.660 (0.994)	0.704 (1.000)	0.433 (0.998)
Education		-0.0941 (0.185)	-0.0992 (0.191)	-0.184 (0.216)	-0.205 (0.208)	-0.171 (0.215)	-0.218 (0.223)	-0.149 (0.215)
Mobile phones			-0.00723 (0.0597)	-0.0264 (0.0660)	-0.0269 (0.0636)	-0.0380 (0.0659)	-0.0521 (0.0683)	-0.0401 (0.0697)
Banking concentration				0.0379 (0.0966)	0.0324 (0.0930)	0.0129 (0.0976)	0.0175 (0.0982)	-0.0307 (0.0973)
Bank ROA					3.557** (1.680)	3.199* (1.766)	2.655 (1.891)	2.492 (1.807)
Share of population above 65						-0.332 (0.474)	-0.332 (0.476)	-0.248 (0.462)
Share of self-employed							-0.158 (0.192)	-0.146 (0.194)
Gini index								0.135 (0.250)
Constant	-0.366 (0.274)	-0.366 (0.276)	-0.378 (0.296)	-0.485 (0.377)	-0.627* (0.369)	-0.593 (0.374)	-0.399 (0.442)	-0.333 (0.433)
Observations	57	57	57	53	53	53	53	52
R^2	0.162	0.167	0.167	0.186	0.263	0.272	0.284	0.323
Adjusted R^2	0.080	0.066	0.048	0.039	0.109	0.098	0.092	0.115

The dependent variable in each regression is mobile payment adoption rate in 2017. The independent variables include those in Table A1 plus the additional ones: Education (World Bank education index), Mobile phones (number of mobile phones per capita), Banking concentration (assets of five largest banks as a share of total commercial banking assets), Bank ROA (bank return on assets), Share of population above 65 (share of population with age 65 and above), Share of self-employed (share of total employment that is self-employed), and Gini index. Most independent variables are dated by year 2017 except that Banking concentration, Bank ROA and Gini index are the averages between 2007-2016. *** denotes statistical significance at 1% level, ** at 5% level, and * at 10% level.

III. Present-value welfare of aggregate economies.

Given the exponential distribution $G_t(I)$, Eq. (27) yields that the present-value welfare of a cash economy at date t ($< T_d$) is

$$W_{t < T_d} = \int_0^\infty \bar{V}_h(I) dG_t(I) = \frac{(1 - \tau_h) \lambda_t}{1 - \beta(1 + g)}. \quad (42)$$

Given the exponential distribution $G_{T_d}(I)$, Eq. (28) yields that

$$\begin{aligned} W_{T_d} &= \frac{(1 - \tau_h) \lambda_{T_d}}{1 - \beta(1 + g)} + \left(\frac{\tau_h - \tau_d}{1 - \beta(1 + g)} \right) \int_{I_d}^\infty I dG_{T_d}(I) - k_d \int_{I_d}^\infty dG_{T_d}(I) \\ &+ \sum_{s=1}^\infty \beta^s \left(\frac{(\tau_h - \tau_d)(1 + g)^s}{1 - \beta(1 + g)} \right) \int_{\frac{I_d}{(1+g)^s}}^{\frac{I_d}{(1+g)^{s-1}}} I dG_{T_d}(I) - k_d \sum_{s=1}^\infty \beta^s \int_{\frac{I_d}{(1+g)^s}}^{\frac{I_d}{(1+g)^{s-1}}} dG_{T_d}(I) \\ &= \frac{(1 - \tau_h) \lambda_{T_d}}{1 - \beta(1 + g)} + \left(\frac{\tau_h - \tau_d}{1 - \beta(1 + g)} \right) \exp\left(-\frac{I_d}{\lambda_{T_d}}\right) (\lambda_{T_d} + I_d) - k_d \exp\left(-\frac{I_d}{\lambda_{T_d}}\right) \\ &+ \sum_{s=1}^\infty \beta^s \left(\frac{(\tau_h - \tau_d)(1 + g)^s}{1 - \beta(1 + g)} \right) \left(\begin{aligned} &\exp\left(-\frac{I_d}{(1+g)^s \lambda_{T_d}}\right) (\lambda_{T_d} + \frac{I_d}{(1+g)^s}) \\ &- \exp\left(-\frac{I_d}{(1+g)^{s-1} \lambda_{T_d}}\right) (\lambda_{T_d} + \frac{I_d}{(1+g)^{s-1}}) \end{aligned} \right) \\ &- \sum_{s=1}^\infty \beta^s \left(\exp\left(-\frac{I_d}{(1+g)^s \lambda_{T_d}}\right) - \exp\left(-\frac{I_d}{(1+g)^{s-1} \lambda_{T_d}}\right) \right) k_d. \end{aligned}$$

Denote that ϕ satisfies $\frac{I_{m'}^a}{(1+g)^\phi} > I_d(1+g)$ and $\frac{I_{m'}^a}{(1+g)^{\phi+1}} \leq I_d(1+g)$. Eq. (29) implies

$$\begin{aligned} W_{T_m} &= \frac{(1 - \tau_h)}{1 - \beta(1 + g)} \int_0^{I_d(1+g)} I dG_{T_m}(I) + \frac{(\tau_h - \tau_m)}{1 - \beta(1 + g)} \int_{I_m}^{I_d(1+g)} I dG_{T_m}(I) - k_m \int_{I_m}^{I_d(1+g)} dG_{T_m}(I) \\ &+ \sum_{s=1}^\infty \beta^s \left(\frac{(\tau_h - \tau_m)(1 + g)^s}{1 - \beta(1 + g)} \right) \int_{\frac{I_m}{(1+g)^s}}^{\frac{I_m}{(1+g)^{s-1}}} I dG_{T_m}(I) - k_m \sum_{s=1}^\infty \beta^s \int_{\frac{I_m}{(1+g)^s}}^{\frac{I_m}{(1+g)^{s-1}}} dG_{T_m}(I) \\ &+ \frac{(1 - \tau_d)}{1 - \beta(1 + g)} \int_{I_d(1+g)}^\infty I dG_{T_m}(I) + \left(\frac{(\tau_d - \tau_m)}{1 - \beta(1 + g)} \right) \int_{I_{m'}^a}^\infty I dG_{T_m}(I) - k_m^a \int_{I_{m'}^a}^\infty dG_{T_m}(I) \\ &+ \sum_{s=1}^\phi \beta^s \frac{(\tau_d - \tau_m)(1 + g)^s}{1 - \beta(1 + g)} \int_{\frac{I_{m'}^a}{(1+g)^s}}^{\frac{I_{m'}^a}{(1+g)^{s-1}}} I dG_{T_m}(I) - k_m^a \sum_{s=1}^\phi \beta^s \int_{\frac{I_{m'}^a}{(1+g)^s}}^{\frac{I_{m'}^a}{(1+g)^{s-1}}} dG_{T_m}(I) \\ &+ \beta^{\phi+1} \frac{(\tau_d - \tau_m)(1 + g)^{\phi+1}}{1 - \beta(1 + g)} \int_{I_d(1+g)}^{\frac{I_{m'}^a}{(1+g)^\phi}} I dG_{T_m}(I) - k_m^a \beta^{\phi+1} \int_{I_d(1+g)}^{\frac{I_{m'}^a}{(1+g)^\phi}} dG_{T_m}(I). \end{aligned}$$

Given the exponential distribution $G_{T_m}(I)$, this yields

$$\begin{aligned}
W_{T_m} = & \frac{(1 - \tau_h)}{1 - \beta(1 + g)} \left(\lambda_{T_m} - \exp\left(-\frac{I_d(1 + g)}{\lambda_{T_m}}\right)(\lambda_{T_m} + I_d(1 + g)) \right) \\
& + \frac{(\tau_h - \tau_m)}{1 - \beta(1 + g)} \left(\exp\left(-\frac{I_m}{\lambda_{T_m}}\right)(\lambda_{T_m} + I_m) - \exp\left(-\frac{I_d(1 + g)}{\lambda_{T_m}}\right)(\lambda_{T_m} + I_d(1 + g)) \right) \\
& - k_m \left(\exp\left(-\frac{I_m}{\lambda_{T_m}}\right) - \exp\left(-\frac{I_d(1 + g)}{\lambda_{T_m}}\right) \right) \\
& + \sum_{s=1}^{\infty} \beta^s \left(\frac{(\tau_h - \tau_m)(1 + g)^s}{1 - \beta(1 + g)} \right) \left(\begin{aligned} & \exp\left(-\frac{I_m}{(1+g)^s \lambda_{T_m}}\right)(\lambda_{T_m} + \frac{I_m}{(1+g)^s}) \\ & - \exp\left(-\frac{I_m}{(1+g)^{s-1} \lambda_{T_m}}\right)(\lambda_{T_m} + \frac{I_m}{(1+g)^{s-1}}) \end{aligned} \right) \\
& - k_m \sum_{s=1}^{\infty} \beta^s \left(\exp\left(-\frac{I_m}{(1 + g)^s \lambda_{T_m}}\right) - \exp\left(-\frac{I_m}{(1 + g)^{s-1} \lambda_{T_m}}\right) \right) \\
& + \frac{(1 - \tau_d)}{1 - \beta(1 + g)} \exp\left(-\frac{I_d(1 + g)}{\lambda_{T_m}}\right)(\lambda_{T_m} + I_d(1 + g)) \\
& + \left(\frac{(\tau_d - \tau_m)}{1 - \beta(1 + g)} \right) \exp\left(-\frac{I_{m'}^a}{\lambda_{T_m}}\right)(\lambda_{T_m} + I_{m'}^a) - k_m^a \exp\left(-\frac{I_{m'}^a}{\lambda_{T_m}}\right) \\
& + \sum_{s=1}^{\phi} \beta^s \frac{(\tau_d - \tau_m)(1 + g)^s}{1 - \beta(1 + g)} \left(\begin{aligned} & \exp\left(-\frac{I_{m'}^a}{(1+g)^s \lambda_{T_m}}\right)(\lambda_{T_m} + \frac{I_{m'}^a}{(1+g)^s}) \\ & - \exp\left(-\frac{I_{m'}^a}{(1+g)^{s-1} \lambda_{T_m}}\right)(\lambda_{T_m} + \frac{I_{m'}^a}{(1+g)^{s-1}}) \end{aligned} \right) \\
& - k_m^a \sum_{s=1}^{\phi} \beta^s \left(\exp\left(-\frac{I_{m'}^a}{(1 + g)^s \lambda_{T_m}}\right) - \exp\left(-\frac{I_{m'}^a}{(1 + g)^{s-1} \lambda_{T_m}}\right) \right) \\
& + \beta^{\phi+1} \frac{(\tau_d - \tau_m)(1 + g)^{\phi+1}}{1 - \beta(1 + g)} \left(\begin{aligned} & \exp\left(-\frac{I_d(1+g)}{\lambda_{T_m}}\right)(\lambda_{T_m} + I_d(1 + g)) \\ & - \exp\left(-\frac{I_{m'}^a}{(1+g)^{\phi} \lambda_{T_m}}\right)(\lambda_{T_m} + \frac{I_{m'}^a}{(1+g)^{\phi}}) \end{aligned} \right) \\
& - k_m^a \beta^{\phi+1} \left(\exp\left(-\frac{I_d(1 + g)}{\lambda_{T_m}}\right) - \exp\left(-\frac{I_{m'}^a}{(1 + g)^{\phi} \lambda_{T_m}}\right) \right).
\end{aligned}$$

IV. Solving the new card adoption threshold I_d^ρ .

With mobile payments being anticipated, I_d^ρ is the new income threshold at which an agent is indifferent between adopting card and continuing with cash at any date between $T_m - n$ and $T_m - 1$. To solve I_d^ρ , we can use Eq. (37):

$$\begin{aligned}
k_d - \beta(1 - \rho)k_d = & (\tau_h - \tau_d)I_d^\rho + \beta\rho \max[V_d(I_d^\rho(1 + g)), V_m((I_d^\rho(1 + g)) - k_m^a)] \\
& - \beta\rho[V_m(I_d^\rho(1 + g)) - k_m].
\end{aligned}$$

We use a guess-and-verify method. We first guess that $I_d^\rho(1+g) \geq I_{m'}^a = \frac{(1-\beta)k_m^a}{(\tau_d-\tau_m)}$. Equation (37) then yields

$$k_d - \beta(1-\rho)k_d = (\tau_h - \tau_d)I_d^\rho + \beta\rho(k_m - k_m^a),$$

which pins down

$$I_d^\rho = \frac{k_d - \beta(1-\rho)k_d - \beta\rho(k_m - k_m^a)}{(\tau_h - \tau_d)}.$$

We then check if the guess $I_d^\rho(1+g) \geq I_{m'}^a$ indeed holds. With the model parameter values given in Table 1 and for any $\rho \in [0, 1]$, we find that the guess does not hold.

We then guess $I_d^\rho(1+g) < I_{m'}^a = \frac{(1-\beta)k_m^a}{(\tau_d-\tau_m)}$. Equation (37) now yields

$$\begin{aligned} k_d - \beta(1-\rho)k_d &= (\tau_h - \tau_d)I_d^\rho + \beta\rho V_d(I_d^\rho(1+g)) - \beta\rho[V_m(I_d^\rho(1+g)) - k_m] \\ \implies (1-\beta)k_d + \beta\rho(k_d - k_m) &= (\tau_h - \tau_d)I_d^\rho + \beta\rho[V_d(I_d^\rho(1+g)) - V_m(I_d^\rho(1+g))]. \end{aligned}$$

From Eq. (7), we know

$$V_m(I_d^\rho(1+g)) = \frac{(1-\tau_m)(1+g)I_d^\rho}{1-\beta(1+g)}.$$

From Eq. (17), we have

$$\begin{aligned} V_d(I_t) &= (1-\tau_d)I_t + \beta \max[V_d(I_{t+1}), V_m(I_{t+1}) - k_m^a] \\ \implies V_d(I_t) &= \sum_{x=0}^J \beta^x (1-\tau_d)I_t(1+g)^x + \beta^{J+1} \frac{(1-\tau_m)I_t(1+g)^{J+1}}{1-\beta(1+g)} \\ \implies V_d(I_d^\rho(1+g)) &= \sum_{x=0}^J [\beta^x (1-\tau_d)I_d^\rho(1+g)^{x+1}] + \beta^{J+1} \frac{(1-\tau_m)I_d^\rho(1+g)^{J+2}}{1-\beta(1+g)}, \end{aligned}$$

where J satisfies $I_d^\rho(1+g)^{J+1} < I_{m'}^a = \frac{(1-\beta)k_m^a}{(\tau_d-\tau_m)}$ and $I_d^\rho(1+g)^{J+2} \geq I_{m'}^a = \frac{(1-\beta)k_m^a}{(\tau_d-\tau_m)}$. Therefore, I_d^ρ can be solved by

$$(1-\beta)k_d + \beta\rho(k_d - k_m) = (\tau_h - \tau_d)I_d^\rho + \beta\rho \left[\sum_{x=0}^J [\beta^x (1-\tau_d)I_d^\rho(1+g)^{x+1}] + \beta^{J+1} \frac{(1-\tau_m)I_d^\rho(1+g)^{J+2}}{1-\beta(1+g)} - \frac{(1-\tau_m)(1+g)I_d^\rho}{1-\beta(1+g)} \right],$$

and we verify that $I_d^\rho(1+g) < I_{m'}^a = \frac{(1-\beta)k_m^a}{(\tau_d-\tau_m)}$ indeed holds.