Working Paper Series

Dealer costs and customer choice

WP 23-13

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November 2023

Abstract

We introduce a model to explain how an increase in intermediation costs leads to structural changes in the corporate bond market. We state three facts on corporate bond markets after the Dodd-Frank act: (1) an increase in customer liquidity provision through prearranged matches, (2) a paradoxical decrease in measured illiquidity, and (3) an increase in the illiquidity component on the yield spread. Investors take longer to finish a trade and require higher illiquidity premium even though measured illiquidity decreased. We introduce a search and matching model which explains these facts. It also suggests the possibility of multiple equilibria and financial instability when dealers face high costs to intermediate transactions.

JEL classification: D53, G12, G18.

Keywords: over-the-counter markets, intermediation costs, liquidity, corporate bonds, Volcker rule, post-2008 regulation.

December 6, 2023. Updates in this link.

*The views in this paper are those of the authors and not necessarily the views of the Federal Reserve Bank of Richmond, the Federal Reserve System, or Barclays Corporate & Investment Bank. Dyskant participated in this work in his personal capacity and not as part of his employment with Barclays. We benefited from discussions with Briana Chang, Veronica Guerrieri, Zhiguo He, Yesol Huh, Ben Lester, Dmitry Orlov, Sebastien Plante, Erwan Quintin, Andreas Rapp, Philip Reny, Roberto Robatto, Sang Seo, Cathy Zhang and participants at several seminars. Silva thanks the hospitality of the Kenneth C. Griffin Department of Economics at the University of Chicago, where part of this article was written. This work was partially funded by Fundação para a Ciência e a Tecnologia (UID/ECO/00124/2019, UIDB/00124/2020 and Social Sciences DataLab, PINFRA/22209/2016), POR Lisboa and POR Norte (Social Sciences DataLab, PINFRA/22209/2016).

1 Introduction

We introduce a model to explain three observations on the corporate bond market: (1) the increase in customer-customer prearranged matches intermediated by dealers, for which customers provide liquidity to other customers; (2) the decrease in measured illiquidity, even though market participants indicate more difficulty in trading; and (3) the increase in the importance of illiquidity for the yield spread. To explain these observations, we introduce a search and matching models with elements of Lagos and Rocheteau (2009) and Hugonnier, Lester, and Weill (2022). The model generates the three stated observations. Moreover, the model implies the possibility of multiple equilibria and financial crises.

In the model, dealers face intermediation costs to facilitate trading. When the intermediation cost increases, customers accept to wait until they are matched with another customer. The intermediation cost creates a customer-dealer and a customer-customer market. The customer-dealer market represents inventory trade and the customer-customer market represents customer liquidity provision as described by Choi et al. (2023). In the customer-dealer market, customers do not incur higher search costs. They are willing to pay a higher price for immediacy. In the customer-customer market, customers might pay a smaller price for the asset (or sell for a higher price), but they take longer to trade.

An important ingredient of the model is the ability of investors to choose to participate in customer-dealer or customer-customer markets—in the spirit of Guerrieri et al. (2010), agents direct their search in financial markets. An increase in dealer costs makes investors look for other investors to trade, which represents customer liquidity provision intermediated by dealers. For a large enough increase in dealer costs, this change in composition decreases the aggregate bid-ask spread in equilibrium. Standard indicators of illiquidity rely on observed transactions. Therefore, a decrease in the aggregate bid-ask spread implies an improvement in standard indicators of liquidity. The model then generates a change in the structure of the corporate bond market together with improvement of indicators of market illiquidity.

As the arrangement of customer-customer matches takes longer to be executed, the actual frictions in the model increase. We use the model to propose a new measure of illiquidity. The measure obtained from the model takes into account equilibrium prices, search frictions, and the fraction of the market that engages in inventory trade or customer liquidity provision.

After the 2007-2009 financial crisis, several regulations were enacted with the objective of avoiding future financial crises. The US enacted the Dodd-Frank Wall Street Reform and Consumer Protection Act in 2010. To some extent, the Dodd-Frank Act and similar regulations in other countries accomplished their goal. This was highlighted by the ability of the financial sector to resist the fluctuations caused by the recent pandemic. However, the focus of academics, practitioners and government officials turned to how such regulations affect the financial sector in normal times, when the economy is not under distress. There are indications that the form in which trades take place in over-the-counter markets has changed after the regulations were put in place. We propose amodel of trading in over-the-counter markets to analyze these changes.[1]

The improvement in the traditional measures of market liquidity after the 2008 financial crisis has been documented by Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018). We confirm this finding with recent data for the BPW and Amihud liquidity measures (respectively after Bao, Pan, and Wang 2011 and Amihud 2002). We focus on the corporate bond market, a market for which over-the-counter trading is the usual method of trade. The improvement in measured market liquidity could suggest that regulations had a minor impact on financial market liquidity. However, we show that the impact of illiquidity on the yield spread of corporate bonds increased. While markets seem to be more liquid, the cost of illiquidity increased.[2]

In addition to the changes in the illiquidity measures, Bessembinder et al. (2018) and Choi et al. (2023) report a decrease in dealer trade frequency. Especially, Choi et al. (2023) indicate a change in the composition of the provision of liquidity. The provision of liquidity has increasingly been made by customers rather than dealers. In practice, there is a perceived movement of customers from dealers as final trade counterparties to other customers.

The model builds on Duffie, Gârleanu, and Pedersen (2005), Lagos and Rocheteau (2009)

[1]During a 2015 congressional hearing, for example, Rep. French Hill questioned the Federal Reserve Chair at the time, Janet Yellen, on whether regulations were to blame for the deterioration of liquidity on different bond markets. Yellen replied: "I am not ruling out the possibility that regulations could play a role here, it is simply we have not been able to understand through a lot of different factors and we need to look at it more to sort out just what is going on and what the different influences are, but I am not ruling that out."

[2]We obtain similar findings on the importance of illiquidity for the yield spreads as Li and Yu (2023) and Wu (2023), which worked in coincident and independent papers. Our empirical results are different in some aspects (such as our focus on the BPW and Amihud measures) and they complement their findings. However, our focus is on the model to explain the movements in the corporate bond market.

(LR) and Hugonnier, Lester, and Weill (2022) (HLW). Depending on the parameters, it implies LR or HLW as particular cases. Time is continuous. There is an asset that pays dividends over time. There are two types of traders: customers and dealers. Dealers have access to a competitive inter-dealer market where the asset is traded at an equilibrium price. Customers trade in a decentralized way with dealers or with other customers. There are search frictions when customers search for a trade counterparty. These search frictions are different for finding another dealer or customer. In particular, the search friction to find a dealer is smaller than the search friction to find another customer. Customers are heterogeneous in the valuation of the asset.

Each asset has a stochastic maturity date and a stochastic issue opportunity. The heterogeneity in asset valuation implies gains to trade. Assets trade hands over time. We characterize the stationary distribution of asset holdings as well as the equilibrium bid-ask spreads. We then show that the empirical behavior of market illiquidity and its correlation with yield spreads can be rationalized by the model when we interpret the Dodd-Frank act as causing an increase in intermediation costs.

The increase in costs faced by dealers increases the equilibrium bid-ask spread of dealers. Customers that do not have the asset but have a high valuation of it still trade with dealers. They pay a high ask price because they want to find a trade counterparty fast. Similarly, customers that have the asset but have a low valuation of the asset also look for dealers to trade. They accept a lower bid price as they want to sell the asset fast. On the other hand, customers that have intermediary valuations of the asset avoid trading with dealers. They wait to be matched with other customers to avoid the surcharge in the form of large bid-ask spreads. Empirically, Choi et al. (2023) find that matched customers in fact pay lower spreads. We show that an increase in trade costs increases the number of customer-customer trades. As a result, the measured bid-ask spread decreases as well as measured illiquidity measures.

The paper is organized as follows. Section 2 states the observations on the corporate bond market after 2008. Section 3 describes the model. Section 4 characterizes the equilibrium and defines a measure of illiquidity obtained from the model. Section 5 discusses the implications of the model for the illiquidity measures and for the liquidity premium. Section 6 concludes.

2 Changes in the corporate bond market

We call attention to three observations about the corporate bond market: (1) the increase in the importance of trades customer-customer relative to trades customer-dealer (what we call customer-customer trades are risk-free principal trades, as explained below); (2) the decrease in the values of illiquidity measures; and (3) the increase in the liquidity premium. We describe in detail these three observations below. Our goal with the model in section 3 is to explain these observations.

2.1 Trade composition and perception of illiquidity

Our first observation is the change in the composition of trades and increased perception of illiquidity in the corporate bond market. As documented by Choi et al. (2023), after the regulations that followed the 2008 financial crisis, it is more common to find trades for which customers are matched with other customers instead of trades for which dealers use their inventory to provide liquidity. Dealers facilitate both forms of trade. However, when customers are matched with other customers, customers provide liquidity. In this case, the dealer does not use its inventory of bonds. It is a customer that provides liquidity to other customers either as a seller or a buyer.

The changes in the composition of trades have been connected with the enactment of regulations that affect depository institutions, such as banks, with access to the Federal Reserve as a lender of last resort or to FDIC insurance (Adrian et al. 2017, Bao et al. 2018, Choi et al. 2023; Duffie 2012 pointed out some risks of the new regulations). Especially, the Volcker rule prohibits banks from engaging in proprietary trading, that is, trading that uses the inventory of assets purchased earlier with the intention of profiting from a higher sale price. The Volcker rule is part of the Dodd-Frank Wall Street Reform and Consumer Protection Act. The Dodd-Frank act was enacted in July 2012. The Volcker rule was put into effect in July 2015 after a period of transition. The objective of the Volcker rule is to limit risk taking of protected institutions. However, as the rule prohibits proprietary trade, it decreases incentives of maintaining an inventory of assets and increases incentives of finding matches between customers for direct trades without changes in inventory.

Choi et al. (2023) classify trades as being the result of a match customer-customer (DC-DC), a match customer-dealer intermediated by an interdealer (DC-ID) and inventory trades. They focus on over-the-counter trades in the corporate bond market. The classification is made with TRACE data, using dealer identifiers, counterparty pair types, and the time record of the trades. DC-DC and DC-ID trades have matches identified within a period of 15 minutes. Inventory trades are not matched with the opposing side, which implies that the asset is held after the trade as inventory. Since 2011, they find an increase in the fraction of customer-customer trades.

Customer-customer trades require a longer search and matching process for the execution of the trade. Suppose that a customer contacts a dealer to sell a certain asset. This customer demands liquidity. The conventional assumption would be that the dealer would buy the asset and thus provide liquidity. Instead, especially for trades equal to or larger than 1 million dollars, it is more frequent now that the dealer uses its relationships with other clients to find a customer willing to purchase the asset. The second customer provides liquidity. This process can take time. The whole search process initiated when the dealer was contacted about the intention to sell from the first customer.

An evidence that customers indeed provide liquidity is that they are compensated for it (Choi et al. 2023). Customers that buy the asset when the first contact was a demand for sale pay smaller or even negative spreads. The same happens for customers that sell the asset when the trade was initiated by a demand to buy. In the same direction, Giannetti et al. (2023) find that bond mutual funds engage in more liquidity provision since 2015, and that the performance of funds with strategies of liquidity provision has improved. Rapp and Waibel (2023) show that regulatory costs are associated with the use of the client network for the provision of liquidity. As we discuss below, the decrease in spreads charged to customers implies a decrease in bid-ask spreads and an improvement of liquidity measures, even though these DC-DC trades could take longer to be executed.

There is evidence that the search process can be costly. Transactions datasets such as TRACE contain only the final outcome of successful transactions. It is then not possible to measure the duration of the whole search process. Using data from electronic platforms, Kargar et al. (2023) find that a substantial number of requests for quote are not promptly fulfilled. If the quote is not fulfilled initially, it takes on average from 2 to 3 days for a trade to be finalized. Another indicator of the need match customers is the advent of electronic platforms to facilitate matching between trade counterparties (Hendershott et al. 2021).

The Volcker rule does not allow proprietary trading, but allows trading to facilitate transactions that were driven by customers. The law recognizes the role of dealers in the functioning of markets. Dealers cannot transact in a way intended to make profits based on the increase in the price of the asset, but they can profit from bid-ask spreads. As a result, a change in the market structure, with customer liquidity provision more frequent, would imply higher transaction costs for those trades that are executed with the inventory of dealers.

In fact, Choi et al. (2023) find that inventory trades have a transaction cost 60% higher than before the financial crisis. According to the classification above, inventory trades do not require a match of another dealer or customer to be executed. These trades are faster to be finalized. Therefore, the higher transaction cost reflects a higher premium on immediacy after the change in the market structure.

Early evidence that the new regulations affected markets was shown by Adrian et al. (2017) and Bao et al. (2018). Adrian et al. found that the ability to intermediate customer trades of affected institutions decreased. Bao et al. note that dealers affected by the Volcker rule have been the main liquidity providers. They found that the illiquidity of bonds in time of stressed bonds has increased after the Volcker rule. As stated above, there was an increase in the fraction of liquidity provided by customers, but only with a more costly matching procedure. Therefore, the increase in illiquidity during stress events can be explained by a change in the structure of markets toward costly matching.

There is therefore evidence that the structure of the corporate bond market has changed toward the prearrangement of trades between customers. This prearrangement is made to save on inventory of securities. As a result, the complete trade from the first contact until transaction becomes costly and protracted.

The changes in market structure, however, are not fully captured by standard measures of illiquidity. These measures do not take into account the time for the arrangement of matches. They use recorded prices at the final moment of the trade. We next discuss the behavior of the illiquidity measures over time.

2.2 Illiquidity measures

Our second observation is the improvement of the illiquidity measures since 2008. This improvement is surprising given the changes in the market structure, as discussed above. Trade in over-the-counter markets has moved toward prearranged matching of customers instead of a faster trade using existing dealer inventory. It is more frequent to observe the provision of liquidity made by customers. Given that these trades take longer to be executed, it is surprising to observe an improvement of measured illiquidity. As we argue later, the source of the difference is the fact that these measures use observed trading records. We later offer an alternative measure of illiquidity implied by the model in section 3.

We discuss the behavior of two measures of illiquidity: the γ measure, proposed by BPW, and the Amihud measure, proposed by (Amihud, 2002). Figure 1 shows the evolution of the measures over time.

The γ measure (BPW) is given by the covariance of subsequent price changes. The γ_i measure for bond i is defined as

$$\gamma_i = -\mathrm{Cov}(\Delta p_{it}, \Delta p_{it+1}), \tag{1}$$

where $\Delta p_{it} = p_{it} - p_{it-1}$ and p_{it} is the logarithm of the clean price P_{it} of bond i on trade t. The clean price is the bond price minus accrued interest since the last coupon payment. We require a bond to have at least ten pairs of consecutive annualized-returns to estimate γ_i.

The objective of the measure is to extract a transitory component from observed prices. This transitory component is interpreted as the impact of illiquidity, as efficient markets with no trading frictions imply uncorrelated returns. Let $p_t = f_t + u_t$, where f_t corresponds to a fundamental component, equal to the value of the asset with no market frictions, and u_t corresponds to the transitory component, uncorrelated with the fundamental value. If the fundamental value f_t follows a random-walk process, then (1) implies that γ depends only on the transitory component u_t.

In addition to γ, we estimate the Amihud measure (Amihud 2002). The Amihud measure for each bond is given by the average of absolute returns divided by the volume of trades,

$$\mathrm{AMD}_{id} = \frac{1}{N_{id}} \sum_{j=1}^{N_{id}} \frac{|r_{ij}|}{V_{id}}, \tag{2}$$

where N_{id} is the number of available returns r_{ij} of bond i on day d, and V_{id} is the volume of trade of bond i on day d in millions of dollars. We require at least two trades on each day to estimate AMD_{id}.

High Amihud measure means high price change per unit of volume, that is, high impact or order flow. Liquid markets should not show large changes in price relative to volume. Therefore, a high Amihud measure is interpreted as lack of market liquidity. Table 1 reports the correlations between γ, Amihud and other variables.[3]

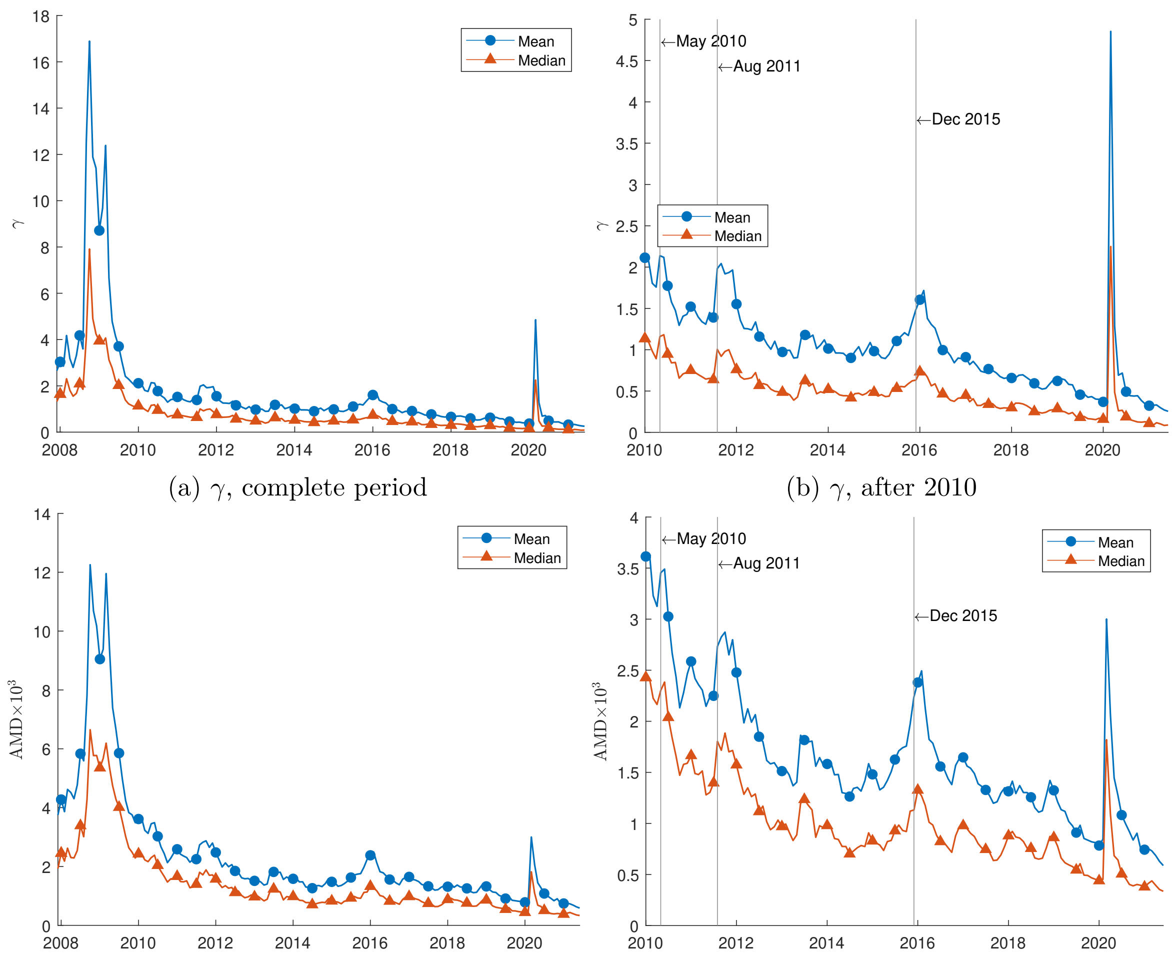
Table 1: Correlations between illiquidity measures and other variables

	γ	AMD	Spread	CDS	Volume	Frequency	Maturity	Age	Turnover	ZTD
γ	1.00									
AMD	.466	1.00								
Spread	.385	.444	1.00							
CDS	.290	.347	.816	1.00						
Volume	−.002	−.055	.040	.056	1.00					
Frequency	.047	.196	.146	.140	.420	1.00				
Maturity	.163	.149	.092	−.026	.097	−.052	1.00			
Age	.017	.109	.079	.056	−.202	−.001	−.075	1.00		
Turnover	.013	−.008	.125	.127	.588	.303	.110	−.209	1.00	
ZTD	−.051	−.198	−.080	−.088	−.199	−.356	.084	.016	−.034	1.00

Correlations between our main illiquidity measures, γ and AMD, and other commonly-used liquidity metrics, the spread, and the CDS. Data description in appendix B. Spread is the corporate bond yield spread with respect to the US Treasury with the same maturity (appendix B). Maturity is the issue's time to maturity. Maturity and age are calculated in years at the last business day of each month. Turnover is the traded volume divided by the amount outstanding. ZTD is the percentage of zero-trading days.

We define a measure of aggregate market level illiquidity over time by taking the median, mean or volume-weighted average of bond level measures in each cross-section. γ and AMD increase when liquidity worsens. Both illiquidity measures strongly increased during the financial crisis up until the first half of 2009. After the crisis, liquidity in the corporate bond market gradually improved. The covid shock was large but brief and did not affect the trend. Figure 1 show the aggregate measures for the corporate bond market γ and AMD over time.

[3] Additional measures of illiquidity are given, among others, by Mahanti et al. (2008) and Dick-Nielsen et al. (2012). Mahanti et al. present a liquidity measure based on the accessibility of the issues. Dick-Nielsen et al. introduce a measure computed by an average of different illiquidity measures.



(a) γ, complete period

(b) γ, after 2010

(c) AMD, complete period

(d) AMD, after 2010

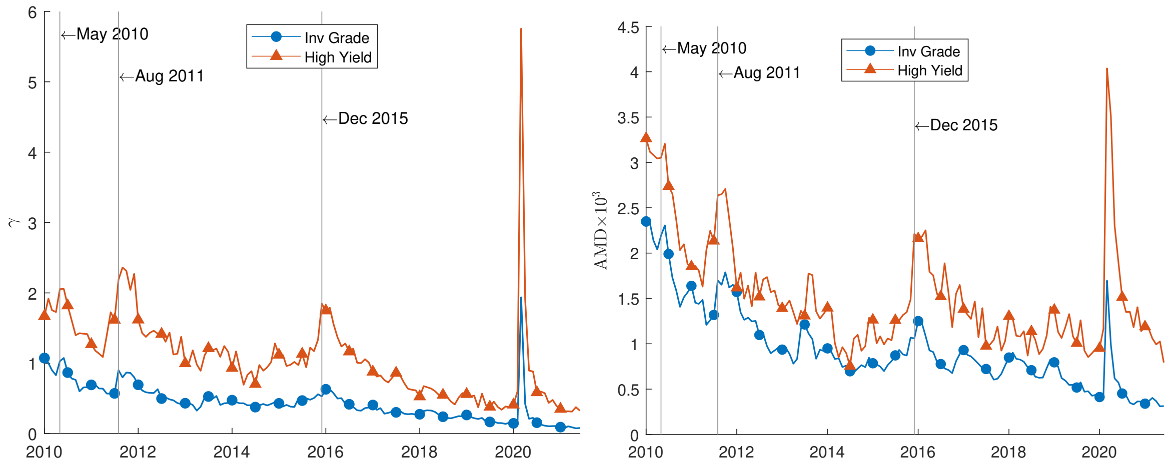
Figure 1: Time series of γ and AMD illiquidity measures. Periods: (1) May 2010, when the Dodd-Frank bill passed the U.S. Senate, and the European debt crisis deepened with the ECB announcement of the Securities Market Programme and Greece's bailout; (2) August 2011, when the U.S. had its credit rating downgraded; and (3) December 2015, when the Fed raised interest rates for the first time since the financial crisis.

It is possible to identify significant events in the corporate debt market that create local peaks in the time-series of the illiquidity measures. Two of these local peaks can be associated with events related to credit and regulatory changes reminiscent of the financial crisis. The first local peak can be associated with the passing of the Dodd-Frank bill by the US Senate and the deepening of the European debt crisis around May 2010, when Greece agreed to a bailout and the European Central Bank announced the Securities Market Programme. The second local peak occurs around the downgrade of the US credit rating in August 2011. The third peak in illiquidity follows the decision of the Federal Reserve in December 2015 to raise interest rates. This increase in interest rates was the first increase after a period of 7 years of low interest rates close to zero.

Different from the previous events, the decision of the FOMC to raise rates is not a credit event. Its effects on corporate bond liquidity can be understood by the microstructure of the corporate bond market. The predominance of over-the-counter trading in the corporate bond market provides dealers with an important role in supplying liquidity. To pursue their activity, dealers must hold inventory, thereby incurring costs and risk. The ability with which liquidity suppliers can manage their inventories affects market liquidity (Comerton-Forde et al. 2010). The increase in illiquidity in this period is also consistent with the changes in liquidity provision discussed above and the funding liquidity channel discussed in Brunnermeier and Pedersen (2009) and in Boudt et al. (2017).

As interest rates rise, it becomes more costly and riskier for dealers to maintain corporate bonds in their inventories. Fleming and Remolona (1999) state that the release of major macroeconomic changes by the FOMC worsens liquidity as it affects inventory controls. Chordia et al. (2001) show that an increase in short term interest rate negatively impacts liquidity because of the increase in inventory costs. Anderson and Stulz (2017) show that bond illiquidity is higher around extreme VIX changes more recently than it was before the crisis. These results indicate that an increase in interest rates should have a higher impact on illiquidity for riskier bonds. In accordance with this view, figure 2 shows that the Fed tightening had a higher impact on illiquidity for high yield bonds.

According to our first point, liquidity provision by dealers has been replaced by customer liquidity provision. On the other hand, rather than a measured decrease in liquidity, the



(a) γ (b) AMD

Figure 2: γ and AMD illiquidity measures after the 2008 financial crisis according to the corporate bond rating. Description of the selected periods in Fig 1.

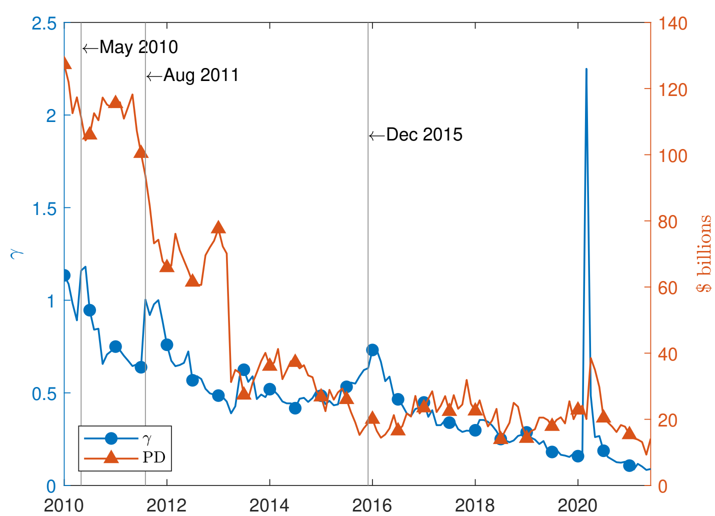


Figure 3: Median γ (left) and median Primary Dealers monthly net positions in corporate debt instruments (right). Primary Dealers net positions in billions of U.S. Dollars; data from the New York Fed. Description of the selected periods in Fig 1.

indicators or illiquidity show a declining trend. Figure 3 shows the decrease in inventories together with a decrease in the liquidity measures. The figure shows the net position of Primary Dealers in corporate debt instruments (commercial papers, bonds, notes and debentures) and illiquidity over time. We explain this apparent paradox with the model of section 3.

Also used as a measure of liquidity, turnover has declined after the 2008 financial crisis. We calculate daily turnover for an individual bond by dividing the amount traded in each day by the amount outstanding at the end of the corresponding month. We then define the monthly turnover measure for an individual bond by the median of its daily turnover. Figure 4 shows the median turnover of all bonds as an aggregate measure (the behavior looks similar for the mean turnover). The figure also shows the moving average of 12 months.

The turnover rate decreased consistently after August 2009. The turnover 12-month average decreased from 13.8% in August 2009 to 7.8% in August 2017. This decline is consistent with the decrease in inventories as discussed above. As we show later, the decline in turnover is consistent with our results in Proposition 3, which states that an increase in intermediation costs, such as the one induced by the Dodd-Frank regulations, leads to a decrease in turnover.

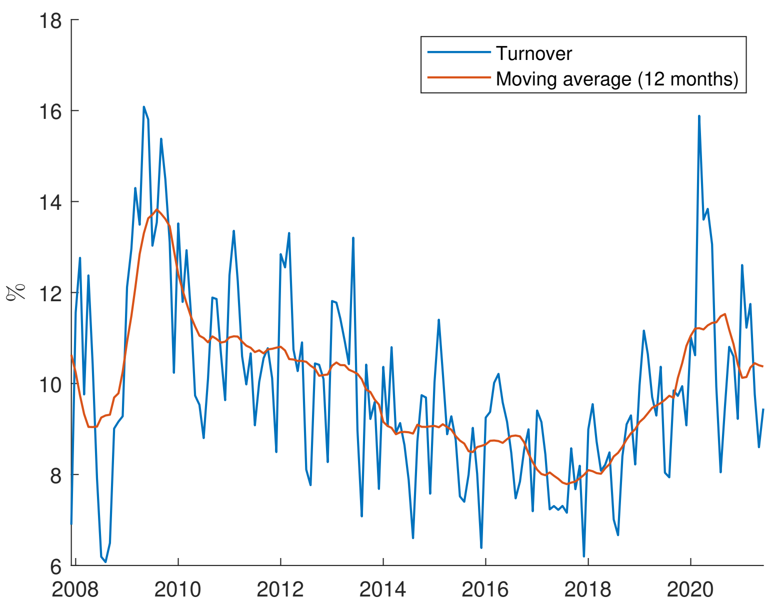


Figure 4: Monthly turnover (mean from daily values). Turnover has decreased since 2009.

2.3 Liquidity premium

Our third observation is the increase in the premium on illiquidity. We measure illiquidity with monthly data on γ and AMD. We regress yield spreads on the illiquidity measures and bond characteristics as controls such as risk (CDS), volume, and maturity. We find that the coefficient on γ increased 5.7 times from 2007 to 2021 and that the coefficient on AMD increased 4.6 times in the same period (table 2).[4]

We run a Fama-MacBeth regression (Fama and MacBeth 1973) of corporate yield spreads on γ and AMD. In the first step, we estimate N cross-sectional regressions for each month, where N is the number of issues. In the second step, we average the coefficients over the T periods in the sample. The t-statistics are calculated with standard errors corrected for serial correlation by Newey-West (Newey and West 1987). Our sample implies 115 monthly cross-sections from a sample of 3,073 bonds and 139,168 bond-month observations, as stated in appendix B. We estimate the coefficients for the complete period December 2007–June 2021 and the following four sub-periods: (1) the financial crisis period from December 2007 to December 2009; (2) the post-crisis period from January 2010 to November 2015; (3) the rate normalization period from December 2015 to February 2020; and (4) the COVID-19 pandemic period from March 2020 to June 2021. Table 2 summarizes our results. As the results with γ and AMD are consistent with each other, we focus our discussion on the illiquidity measure γ.

We start our analysis by looking at the full sample period. Taken individually, the coefficients on the illiquidity measures are smaller only to the variables reflecting the issuer's coupon and credit quality (CDS and credit rating). The first three lines show the results with the illiquidity measures and CDS taken individually. Illiquidity is an important element for customers to consider in the corporate bond market.

The inclusion of controls maintains the economic and statistical significance of illiquidity. The coefficient on AMD for the regression with all controls (line 5 of table 2) implies that an increase of one standard deviation of the AMD of an issue increases corporate yield spreads

[4] Li and Yu (2023) and Wu (2023), in independent work, also find an increase in the coefficients related to illiquidity. They use the bid-ask spread as a measure of illiquidity. Wu (2023) use monthly data, as we do. Li and Yu (2023) uses quarterly data; they find an increase of four times of the coefficient on the bid-ask spread. We find a larger increase in the coefficient on illiquidity using γ and AMD and monthly data.

Table 2: Corporate Yield Spreads on γ, AMD and controls

γ	AMD	CDS	EqVol.	Cpn	IG	Call	Volume	Freq.	Maturity	Age	Turnover	ZTD	Constant	Adj.R^2	Obs.
Panel A: Complete Period, December 2007–June 2021															
.429													1.41	.181	196,345
[11.01]													[7.86]		
	.278												1.37	.149	196,345
	[9.58]												[8.09]		
		.787											.664	.646	196,345
		[32.11]											[8.32]		
.143		.535	.548	.175	−.851	−.042	−.467	.423	.010	.004	.010	.370	.297	.791	196,345
[9.61]		[20.38]	[9.12]	[11.93]	[−9.20]	[−1.82]	[−4.39]	[8.68]	[2.01]	[.70]	[5.73]	[6.40]	[3.04]		
	.098	.537	.579	.179	−.862	−.058	−.410	.281	.015	−.005	.012	.603	.237	.792	196,345
	[8.70]	[19.59]	[9.71]	[11.33]	[−9.26]	[−2.11]	[−3.96]	[6.12]	[2.39]	[−1.05]	[6.79]	[6.66]	[2.49]		
Panel B: Crisis, December 2007–December 2009															
.151													3.33	.085	16,687
[4.56]													[6.19]		
	.102												3.28	.052	16,687
	[3.96]												[7.58]		
		.935											1.48	.744	16,687
		[20.33]											[5.75]		
.042		.758	.877	.290	−1.32	−.188	−.295	.009	−.046	.058	.009	.720	.638	.791	16,687
[4.37]		[16.19]	[5.86]	[4.97]	[−3.07]	[−2.16]	[−1.02]	[.18]	[−3.13]	[3.09]	[1.84]	[3.74]	[1.28]		
	.036	.764	.876	.300	−1.32	−.233	−.196	−.062	−.048	.045	.010	1.28	.544	.792	16,687
	[3.62]	[10.42]	[6.26]	[5.77]	[−3.89]	[−2.42]	[−.56]	[−1.60]	[−2.88]	[4.27]	[3.06]	[3.56]	[1.40]		
Panel C: Post-Crisis, January 2010–November 2015															
.327													1.20	.169	90,727
[10.37]													[18.13]		
	.211												1.21	.120	90,727
	[9.85]												[17.22]		
		.827											.446	.668	90,727
		[67.56]											[18.52]		
.098		.541	.392	.146	−.692	−.049	−.424	.460	.018	−.003	.008	.157	.228	.801	90,727
[9.50]		[23.55]	[5.97]	[16.05]	[−14.86]	[−2.01]	[−4.51]	[13.67]	[7.43]	[−1.81]	[6.26]	[3.00]	[3.85]		
	.069	.547	.419	.148	−.709	−.065	−.411	.377	.023	−.010	.009	.317	.187	.800	90,727
	[8.64]	[22.93]	[6.19]	[16.83]	[−15.27]	[−2.86]	[−4.66]	[14.99]	[10.38]	[−4.22]	[8.30]	[6.78]	[3.12]		
Panel D: Rate Normalization, December 2015–February 2020															
.606													.858	.225	68,997
[16.40]													[23.32]		
	.368												.818	.192	68,997
	[10.20]												[22.42]		
		.644											.592	.584	68,997
		[19.13]											[25.85]		
.225		.440	.569	.150	−.738	.010	−.393	.535	.026	−.011	.011	.390	.199	.776	68,997
[13.06]		[16.52]	[5.63]	[19.55]	[−16.23]	[0.97]	[−1.75]	[6.15]	[23.51]	[−8.08]	[2.58]	[8.58]	[2.57]		
	.150	.432	.622	.154	−.754	.008	−.328	.326	.033	−.021	.013	.585	.132	.778	68,997
	[9.06]	[16.38]	[6.22]	[18.94]	[−16.70]	[0.76]	[−1.45]	[3.43]	[36.09]	[−7.77]	[3.19]	[10.27]	[1.74]		

Individual bond yield spreads regressed on the illiquidity measures γ and AMD and other variables for different periods. Coefficients estimated using Fama-MacBeth and standard errors corrected by Newey-West. T-statistics reported in square brackets. γ and AMD are the illiquidity measures detailed in section B. AMD multiplied by 10^3. EqVol. is the annualized volatility of the issuer's equity returns and Cpn is the issue's coupon. IG is 1 if the bond is Investment Grade and 0 otherwise. Call is 1 if the bond is callable and 0 otherwise. Volume is calculated as the total \$ amount traded $\times 10^{-11}$. Frequency in thousands of trades. Maturity and Age calculated in years at the last business day of the month. Turnover is the monthly median of daily volume divided by amount outstanding and ZTD is the percentage of zero-trading days. Adj. R^2 is the time series average of cross-sectional adjusted-R^2's.

Table 2: Corporate Yield Spreads on γ, AMD and controls (continued)

γ	AMD	CDS	EqVol.	Cpn	IG	Call	Volume	Freq.	Maturity	Age	Turnover	ZTD	Constant	Adj.R^2	Obs.
Panel E: COVID-19 Pandemic, March 2020–June 2021															
.748													.1.11	.242	19,934
[11.38]													[5.18]		
	.566												.878	.293	19,934
	[13.53]												[5.49]		
		.832											.592	.591	19,934
		[14.88]											[4.22]		
.239		.463	.658	.208	−1.19	.049	−1.17	.544	.015	−.003	.021	.708	.383	.799	19,934
[8.62]		[6.10]	[5.49]	[8.47]	[−10.97]	[1.30]	[−5.50]	[4.53]	[2.65]	[−.45]	[6.75]	[4.03]	[5.50]		
	.165	.474	.686	.203	−1.16	.038	−.999	.246	.020	−.010	.022	.870	.315	.801	19,934
	[4.54]	[4.99]	[5.32]	[7.82]	[−9.49]	[.89]	[−4.58]	[2.29]	[4.99]	[−1.82]	[8.00]	[5.22]	[3.66]		

Individual bond yield spreads regressed on the illiquidity measures γ and AMD and other variables for different periods. Coefficients estimated using Fama-MacBeth and standard errors corrected by Newey-West. T-statistics reported in square brackets. γ and AMD are the illiquidity measures detailed in section B. AMD multiplied by 10^3. EqVol. is the annualized volatility of the issuer's equity returns and Cpn is the issue's coupon. IG is 1 if the bond is Investment Grade and 0 otherwise. Call is 1 if the bond is callable and 0 otherwise. Volume is calculated as the total \$ amount traded $\times 10^{-11}$. Frequency in thousands of trades. Maturity and Age calculated in years at the last business day of the month. Turnover is the monthly median of daily volume divided by amount outstanding and ZTD is the percentage of zero-trading days. Adj. R^2 is the time series average of cross-sectional adjusted-R^2's.

28 basis points. This increase is equivalent to 13% of the average yield spread. The results are even more substantial for γ. An increase of one standard deviation in the γ of an issue is associated with an increase in the yield spread of 40 basis points. This increase corresponds to 18% of the average yield spread.[5]

Turnover also shows a robust, positive coefficient, contrasting with that of volume. In some sense this result is intriguing given that volume is an element of the turnover and that the two measures, as a result, have a strong correlation. As shown in table 1, volume and turnover have a correlation of 0.727. One explanation, which is consistent with our model, is that turnover better captures search frictions since trade volume can increase artificially with the amount of bonds outstanding while turnover corrects for such changes.

Credit risk, captured by the CDS spread, is the most relevant pricing factor of corporate spreads. An increase of 100 basis points to an issuer's CDS is associated with an increase in yield spreads of 59 basis points. The credit quality of an issuer is not exclusively priced by its CDS. The credit rating also plays an important role in the pricing of yield spreads. Investment grade bonds have spreads substantially lower, by 85 basis points, than their lower

[5] We consider the one standard deviation change to be the time series average of cross-sectional standard deviations within each interval. According to table 3, for the complete period, one standard deviation of γ corresponds to 3.931. Similarly, we consider the average yield spread to be the time series mean of cross-sectional averages within that period. These and other metrics are detailed in table 3.

grade peers.[6]

Among the additional controls, trade frequency, in particular, yields a positive coefficient. That is, bonds that trade more frequently have on average higher spreads. In contrast, volume traded has a negative slope. These results combined indicate that bonds with larger average size per trade have on average tighter spreads.[7]

We now analyze yield spreads and illiquidity over time. As shown in table 2, the coefficient on illiquidity has been increasing over time. So, although the illiquidity measures have been decreasing over time, their importance on the credit spread has been increasing. We relate this observation to the changes in regulations with the Dodd-Frank act. Illiquidity is most relevant both in its ability to explain the variation of yield spreads and in its contribution to basis points in relative terms.

The explanatory power of illiquidity is larger for later periods. In terms of absolute basis points, a one standard deviation increase to an issue's illiquidity is associated with an increase of about 30 basis points to yield spreads during the monetary tightening period. This value compares to 50 basis-points during the financial crisis and 18 basis-points during the post-crisis period.

Although the impact of a one standard deviation increase in illiquidity is smaller in absolute terms after the rate normalization (30 versus 50 basis points during the financial crisis), the impact is higher in relative terms. The higher impact in relative terms occurs because the corporate yield spreads decreased during the period. A one standard deviation increase in bond illiquidity is associated with an increase in the average yield spread of 12% during the financial crisis and 11% in the post-crisis periods. After the normalization, an increase of one standard deviation in bond illiquidity is associated with an increase in the average yield spread of 18%.

[6]See, for example, Forte and Pena (2009) for the role of CDS for corporate bonds.

[7]In the same direction, Chordia et al. (2000) find a positive correlation between spreads and the number of individual transactions, and a negative correlation between spreads and individual volume.

3 Model

3.1 Environment

Our model describes over-the-counter markets as an economy in which agents take decisions under search frictions. It builds on the literature initiated by Duffie et al. (2005).

Agents, time, goods and assets There are two types of agents in the economy: a measure one of infinitely-lived customers and a measure one of infinitely-lived dealers. Time is continuous and infinite. All agents discount the future at rate $r > 0$, and have access to a transferable utility technology. There is an endogenous supply $s \geq 0$ of assets. A unit of the asset pays a unit flow of dividend goods, which cannot be traded—that is, the agent holding an asset consumes its dividend good. Customers can hold either zero or one unit of the asset, and dealers do not hold assets. We refer to customers holding an asset as owners, and to those not holding an asset as non-owners.

Preferences Customers are heterogeneous in the utility ν that they derive from consuming the dividend flow of the asset. We refer to ν as the customer utility type. Types are fixed over time, common knowledge, independent across customers, and initially drawn from the cumulative distribution F. The distribution F has support $\mathbb{R}$, and a continuous density $f(\cdot) > 0$. Moreover, we assume that $\int \nu^2 f(\nu) d\nu < \infty$ and that there is no free disposal of assets. The assumption that the distribution of types has unbounded support is convenient because we do not have to consider corner solutions. However, we can obtain our main results also with a bounded support $[\underline{\nu}, \bar{\nu}]$ if we assume that the density is sufficient low at the extremes. Similarly, the assumption of no free disposal can be replace with the assumption that the measure of agents with $\nu < 0$ is sufficiently low.

Decentralized market There is a decentralized asset market in the style of Duffie et al. (2005), where trade occurs in one of two ways: customers can chose to search for a dealer, as in LR, or to search for another customer, as in HLW. Customers cannot search for both simultaneously. The contact rate is λ_D when searching for a dealer, and $\lambda_C/2 > 0$ when searching for another customer. Hence, customers searching for a dealer will meet one at rate λ_D, whereas those searching for another customer will meet one at rate λ_C (with arrival rate $\lambda_C/2 > 0$, the customer finds another customer, and with arrival rate $\lambda_C/2 > 0$ another customer finds him). Customers searching for other customers do not meet a customer searching for dealers. We refer to trades between customers without dealer intermediation as CC trades, and trades intermediated by dealers as DC trades.

The inter-dealer market Dealers have access to a competitive inter-dealer market where assets are traded at an endogenous price p. When a dealer meets a customer, the dealer bargains over the possible gains from trade coming from either buying an asset from the customer, and reselling it at price p, or buying it at the price p, and selling it to the customer. However, there is an intermediation cost $\tau \geq 0$ that has to be paid by the dealer if the dealer buys or sells that asset, even if the asset only stays in the balance sheet for an infinitesimal amount of time. As a result, the net price by which the dealer sells an asset is $p - \tau$, and the net price by which the dealer buys an asset is $p + \tau$.

Bargaining Customers trade with dealers using a Nash bargaining protocol where the bargaining power of the customer is $\theta_D \in [0, 1]$, and customers trade with other customers using a Nash bargaining protocol where the bargaining power of the non-owner is $\theta_C^n \in [0, 1]$, and the bargaining power of the asset owner is $\theta_C^o = 1 - \theta_C^n$.

We assume the following relation between search and bargaining parameters:

Assumption 1. $\lambda_D \theta_D > \max\{\lambda_C \theta_C^o, \lambda_C \theta_C^n\}$.

Assumption 1 guarantees that customers are better off searching for dealers than searching for other customers when the intermediation cost τ is equal to zero. That is, in the absence of intermediation costs, the dealers have a superior technology for trade.

Asset supply Assets mature and customers produce new assets following two Poisson distributions. With Poisson arrival rate $\mu > 0$, the asset matures. Which means that the asset disappears from the economy. With Poisson arrival rate $\eta > 0$, a customer can issue a new asset at no cost. Similar to Bethune et al. (2022), the existence of asset maturity and issuance implies that a steady state with positive trade emerges even without time-varying types. Adding time-varying types in our model is not as straightforward as it would be in their model, however, and we discuss this later in the paper.

3.2 Value functions and reservation value

We now describe how agents evaluate future payoffs and assets. We focus on steady-state equilibria and omit time subscripts. Let $\Phi^o(\nu)$ and $\Phi^n(\nu)$ denote the cumulative distribution of owners and non-owners. Since each owner holds exactly one unit of the asset, the measure of assets is $s = \int d\Phi^o$. Let $\{\Omega_D^o, \Omega_C^o\}$ and $\{\Omega_D^n, \Omega_C^n\}$ be two partitions of $\mathbb{R}$. The set Ω_D^o represents the owners that search for dealers and Ω_C^o represents the set of owners that search for non-owners. Analogously, Ω_D^n represents the set of non-owners that search for dealers and Ω_C^n represents the set of non-owners that search for owners. We denote the search partitions of customers by $\mathcal{P} = \{\Omega_D^o, \Omega_C^o, \Omega_D^n, \Omega_C^n\}$, and assume that customers of the same type make the same search decisions.

Denote the value functions of owners and non-owners searching for a dealer by $V_D^o(\nu)$ and $V_D^n(\nu)$ and the value functions of owners and non-owners searching for other customers by $V_C^o(\nu)$ and $V_C^n(\nu)$. The value function of customers is the maximum between the searching choices. That is, $V^o(\nu) = \max\{V_D^o(\nu), V_C^o(\nu)\}$ and $V^n(\nu) = \max\{V_D^n(\nu), V_C^n(\nu)\}$. The reservation value of a customer, $\Delta(\nu)$, is the compensation that makes a customer indifferent between holding and not holding an asset. That is, $\Delta(\nu) = V^o(\nu) - V^n(\nu)$.

Searching for dealers The value function of an owner of type ν searching for dealers is

$$rV_D^o(\nu) = \nu - \mu\Delta(\nu) + \lambda_D\theta_D \max\{(p-\tau) - \Delta(\nu), 0\}. \tag{3}$$

The first term of the value function, ν, is the utility flow of holding the asset. The second term, $-\mu\Delta(\nu)$, is the loss of the reservation value due to asset maturity. The third term, $\lambda_D\theta_D \max\{(p-\tau) - \Delta(\nu), 0\}$, is the profit of an owner when meeting a dealer. When trading with a dealer, an owner sells the asset if the inter-dealer price for the asset minus the intermediation cost cost τ is larger than the reservation value of the owner. If the owner sells the asset, the gains from trade are $(p - \tau) - \Delta(\nu)$ and the owner keeps a share θ_D of it. If the owner does not sell the asset, the gains from trade are zero.

The value function of a non-owner of type ν searching for dealers is

$$rV_D^n(\nu) = \eta\Delta(\nu) + \lambda_D\theta_D \max\{\Delta(\nu) - (p+\tau), 0\}. \tag{4}$$

The first term of the value function, $\eta\Delta(\nu)$, is the gain of the reservation value obtained from an issuance opportunity. The second term, $\lambda_D\theta_D \max\{\Delta(\nu) - (p+\tau), 0\}$, is the profit of an non-owner when meeting a dealer. An non-owner buys an asset from a dealer if the inter-dealer asset price plus the intermediation cost is smaller than the reservation value of the non-owner. If the non-owner buys the asset, the gains from trade are $\Delta(\nu) - (p+\tau)$ and the non-owner keeps a share θ_D of it. If the non-owner does not buy the asset, the gains from trade are zero.

Searching for customers The value function of an owner of type ν searching for a non-owner is

$$rV_C^o(\nu) = \nu - \mu\Delta(\nu) + \lambda_C\theta_C^o \int_{\Omega_C^n} [\Delta(\tilde{\nu}) - \Delta(\nu)]\mathbb{1}_{\{\Delta(\tilde{\nu})>\Delta(\nu)\}} d\Phi^n(\tilde{\nu}). \tag{5}$$

The first term of the value function, ν, is the utility flow of holding the asset. The second term, $-\mu\Delta(\nu)$, is the loss of the reservation value because of asset maturity. The third term is the expected profits of an owner when meeting a non-owner. When trading with a non-owner of type $\tilde{\nu}$, an owner of type ν sells the asset if the reservation value of his counterparty, $\Delta(\tilde{\nu})$, is higher than the reservation value of the owner. That is, if $\Delta(\tilde{\nu}) > \Delta(\nu)$. The gains from trade are $\Delta(\tilde{\nu}) - \Delta(\nu)$ and the owner keeps a share θ_C^o of it. We obtain the expected value of the gains from trade by integrating it in $\tilde{\nu}$ over Ω_C^n using the distribution of owners $\Phi^n(\tilde{\nu})$.

The value function of a non-owner of type ν searching for an owner is

$$rV_C^n(\nu) = \eta\Delta(\nu) + \lambda_C\theta_C^n \int_{\Omega_C^o} [\Delta(\nu) - \Delta(\tilde{\nu})]\mathbb{1}_{\{\Delta(\nu)>\Delta(\tilde{\nu})\}} d\Phi^o(\tilde{\nu}). \tag{6}$$

The first term of the value function, $\eta\Delta(\nu)$, is the expected gain of reservation value because of an asset issuance. The second term is the expected profit of a non-owner when searching for an owner. When trading with an owner of type $\tilde{\nu}$, a non-owner of type ν buys the asset if his reservation value, $\Delta(\nu)$, is higher than the reservation value of the owner. That is, if

$\Delta(\nu) > \Delta(\tilde{\nu})$. The gains from trade are $\Delta(\nu) - \Delta(\tilde{\nu})$ and the non-owner keeps a share θ_C^n of it. We obtain the expected value of the gains from trade by integrating it in $\tilde{\nu}$ over Ω_C^o using the distribution of owners $\Phi^o(\tilde{\nu})$.

Value functions, reservation value and the optimal searching choice The value functions V^o and V^n of a customer type ν satisfy

$$V^o(\nu) = \max\{V_D^o(\nu), V_C^o(\nu)\} \quad \text{and} \quad V^n(\nu) = \max\{V_D^n(\nu), V_C^n(\nu)\}, \tag{7}$$

and the reservation value function satisfies

$$\Delta(\nu) = V^o(\nu) - V^n(\nu). \tag{8}$$

We characterize the search partition $\mathcal{P} = \{\Omega_D^o, \Omega_C^o, \Omega_D^n, \Omega_C^n\}$ in the following way. For Ω_D^o, an owner searches for a dealer if it yields higher value than searching for a non-owner. In the same way, for Ω_D^n, a non-owner searches for a dealer if it yields higher value then searching for an owner. We have

$$\Omega_D^o = \{\nu \in \mathbb{R};\ V_D^o(\nu) \geq V_C^o(\nu)\} \quad \text{and} \quad \Omega_D^n = \{\nu \in \mathbb{R};\ V_D^n(\nu) \geq V_C^n(\nu)\}. \tag{9}$$

Analogously,

$$\Omega_C^o = \{\nu \in \mathbb{R};\ V_C^o(\nu) > V_D^o(\nu)\} \quad \text{and} \quad \Omega_C^n = \{\nu \in \mathbb{R};\ V_C^n(\nu) > V_D^n(\nu)\}, \tag{10}$$

where we assume that customers search for dealers when indifferent between searching for a dealer or other customers, and search for other customers only when they are strictly better off doing so than searching for dealers. For the equilibrium class that we consider, this assumption is without loss of generality because there is a measure zero of customers that are indifferent between the two and trade in equilibrium.

3.3 Inter-dealer market clearing

The interdealer market clears when the measure of owners finding dealers to sell an asset is equal to the measure of non-owners finding dealers to buy an asset. That is,

$$\lambda_D \int_{\Omega_D^o} \mathbb{1}_{\{\Delta(\nu)<p-\tau\}} d\Phi^o(\nu) = \lambda_D \int_{\Omega_D^n} \mathbb{1}_{\{\Delta(\nu)>p+\tau\}} d\Phi^n(\nu). \tag{11}$$

The left-hand side describes the measure of owners that want to sell the asset at the inter-dealer market price net of the intermediation cost, $p-\tau$, and find a dealer. The right-hand side have an analogous description for the sellers that want to buy an asset at the inter-dealer market price added the intermediation cost, $p+\tau$. As the search intensity parameter is the same for potential sellers and buyers, this parameter cancels out from the formula. We will use this equation to find the equilibrium price p^* in this market.

3.4 The distribution of assets

The cumulative distribution of owners is given by Φ^o and the cumulative distribution of non-ownersis given by Φ^n. The change over time of the distribution of owners Φ^o satisfies

$$\begin{aligned}
\dot{\Phi}^o(\nu) &= \eta\Phi^n(\nu) - \mu\Phi^o(\nu) \\
&- \lambda_D \int_{-\infty}^{\nu} \mathbb{1}_{\{\tilde{\nu}\in\Omega_D^o, \Delta(\tilde{\nu})<p-\tau\}} d\Phi^o(\tilde{\nu}) + \lambda_D \int_{-\infty}^{\nu} \mathbb{1}_{\{\tilde{\nu}\in\Omega_D^n, \Delta(\tilde{\nu})>p+\tau\}} d\Phi^n(\tilde{\nu}) \\
&- \lambda_C \int_{-\infty}^{\nu} \int_{\nu}^{\infty} \mathbb{1}_{\{\tilde{\nu}\in\Omega_C^o, \hat{\nu}\in\Omega_C^n, \Delta(\hat{\nu})>\Delta(\tilde{\nu})\}} d\Phi^n(\hat{\nu}) d\Phi^o(\tilde{\nu}) \\
&+ \lambda_C \int_{-\infty}^{\nu} \int_{\nu}^{\infty} \mathbb{1}_{\{\tilde{\nu}\in\Omega_C^o, \hat{\nu}\in\Omega_C^n, \Delta(\tilde{\nu})>\Delta(\hat{\nu})\}} d\Phi^o(\hat{\nu}) d\Phi^n(\tilde{\nu}),
\end{aligned} \tag{12}$$

where $\dot{\Phi}^o(\nu) = 0$ for all ν in an steady-state equilibrium. On the right-hand side of (12), the first term accounts for the inflow of owners that issue an asset. The second term accounts for the outflow of owners because of asset maturity. The third and fourth terms account for owners searching for dealers. The third term for the outflow of owners with type below ν searching for dealers and that sell their asset and the fourth for the inflow of non-owners with type below ν searching for dealers and that buy an asset. The fifth and sixth terms account for customers searching for other customers. The fifth term for the outflow of owners with type below ν searching for other costumers, and that sell their asset to non-owners of type above ν. The sixth term for the inflow of non-owners with type below ν searching for other costumers, and that buy an asset from owners of type above ν.

We will see that the last term in equation (12) will be equal to zero in equilibrium. The reason is that, in equilibrium, the reservation value function Δ is monotonically increasing in the utility type ν. As a result, the measure of non-owners with type $\tilde{\nu}$ and owners with type $\hat{\nu}$ such that $\Delta(\tilde{\nu}) > \Delta(\hat{\nu})$ and $\tilde{\nu} < \hat{\nu}$ is equal to zero.

As the measure of customers, F, is exogenous, the measures of owners and non-owners satisfy the equilibrium condition

$$\Phi^o(\nu) + \Phi^n(\nu) = F(\nu). \tag{13}$$

All assets in the economy are held by owners. So, the stock of assets is $s = \Phi^o(\infty)$.

3.5 Equilibrium

We define a symmetric stationary equilibrium in the following way.

Definition 1. *An equilibrium is a family of value functions, reservations value, price, distributions and partitions,* $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \mathcal{P}\}$ *satisfying equations* (3)–(13).

An equilibrium can be a complicated object. To simplify it further, let $\Omega_C = \Omega_C^o = \Omega_C^n = (\nu_l, \nu_h)$ and $\Omega_D = \Omega_D^o = \Omega_D^n = (-\infty, \nu_l] \cup [\nu_h, \infty)$. Notice that we can have $\Omega_C^o = \Omega_C^n$ as we have some customers of type ν holding the asset and other customers of the same type that do not hold the asset. Define the following class of equilibrium.

Definition 2. *A symmetric stationary equilibrium* $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \mathcal{P}\}$ *is regular if* $\Omega_C = \Omega_C^o = \Omega_C^n = (\nu_l, \nu_h)$ *and* $\Omega_D = \Omega_D^o = \Omega_D^n = (-\infty, \nu_l] \cup [\nu_h, \infty)$ *for some* $\nu_l, \nu_h \in \mathbb{R}$ *satisfying* $\nu_l \leq \nu_h$ *with strict inequality if* $\tau > 0$*, and the reservation value* Δ *is continuous and strictly increasing.*

We use the notation $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ instead of $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \mathcal{P}\}$ when referring to a regular equilibrium since ν_l and ν_h characterize $\mathcal{P}$. Figure 5 illustrates the partition of a regular equilibrium.

What motivates looking for an equilibrium with the characteristics of a regular equilibrium is the following. Customers with type close to each other, inside $\Omega_C = (\nu_l, \nu_h)$, choose to

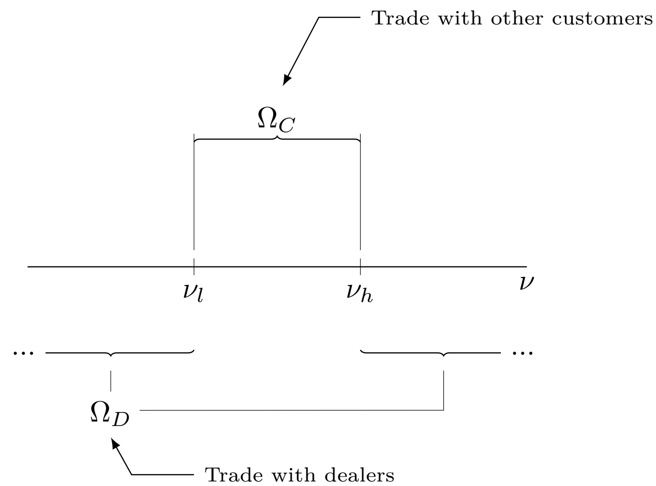


Figure 5: Partition $\{\Omega_D, \Omega_C\}$ in a regular equilibrium.

trade among themselves to avoid the cost τ that has to be paid when trading with dealers because they do not gain much from trading to justify the cost. Customers with extreme types, that is, outside $\Omega_C = (\nu_l, \nu_h)$, are in a hurry to trade and they are willing to cover higher dealer cost. These customers have very low or very high values for ν. Customers of type $\nu \leq \nu_l$ that hold the asset search for dealers to sell their asset. Customers of type $\nu \geq \nu_h$ that do not have the asset search for dealers to buy an asset.

We also impose that $\nu_l < \nu_h$ when the intermediation cost is strictly positive, $\tau > 0$. The reason is that we can always build an equilibrium where customers do not search for costumers because they expect other customers to do the same. In this case, the probability of finding a customer is zero so customers may as well search for dealers. The assumption that $\nu_l < \nu_h$ if $\tau > 0$ rules out equilibria built on this sort of weak inequality. In the next section, we characterize a regular equilibrium and provide conditions that it exists.

4 Equilibrium

A regular equilibrium has two blocks. Given ν_l and ν_h, customers type $\nu \in \Omega_D = (-\infty, \nu_l] \cup [\nu_h, \infty)$ act as in LR. Customers type $\nu \in \Omega_C = (\nu_l, \nu_h)$ act as in HLW. We can solve these two blocks separately using the tools developed in these papers.

The challenge is to characterize ν_l and ν_h that are consistent with the equilibrium equations (9) and (10). That is, to find ν_l and ν_h such that customers searching for dealers, with low types $\nu \leq \nu_l$ or high types $\nu \geq \nu_h$, are not better off searching for other customers. Analogously, that customers searching for customers, with intermediary types $\nu_l < \nu < \nu_h$, are not better off searching for dealers.

4.1 Solving the LR block

The reservation value of a type-ν customer searching for a dealer is $\Delta(\nu) = V_D^o(\nu) - V_D^n(\nu)$. The value functions, $V_D^o(\nu)$ and $V_D^n(\nu)$, of a type-ν customer searching for a dealer when holding and not holding an asset are stated in equations (3) and (4). Taking the difference between the two equations to isolate $\Delta(\nu)$ yields the following lemma.

Lemma 1 (Reservation value, dealer market)**.** *Consider a regular equilibrium* $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ *and the set of utility types* $\Omega_D = (-\infty, \nu_l] \cup [\nu_h, \infty)$*. Then, the reservation value* $\Delta(\nu)$ *satisfies*

$$\Delta(\nu) = \begin{cases} \sigma_D[\nu + \lambda_D \theta_D (p - \tau)], & \nu \leq \nu_l \\ \sigma_D[\nu - \lambda_D \theta_D (p + \tau)], & \nu \geq \nu_h \end{cases} \tag{14}$$

where

$$\sigma_D = \frac{1}{r + \eta + \mu + \lambda_D \theta_D}. \tag{15}$$

Moreover, $\Delta(\nu_l) \leq p - \tau$ *and* $\Delta(\nu_h) \geq p + \tau$*.*

The derivative of the reservation value with respect to ν is given by σ_D. As HLW, we can interpret σ_D as the local surplus at ν. It captures the trade surplus generated if the asset of an agent type ν is transferred to an agent type $\nu + d\nu$. The local surplus is independent of ν for all $\nu \notin (\nu_l, \nu_h)$. That is, because all agents that search for a dealer face the same price after bargaining and intermediation costs, the trade surplus in this case is constant.

In addition to the interpretation above, we interpret σ_D as an indicator of market friction. To see this, suppose that there are no search frictions when the agent searches for a dealer. In the model, when $\lambda_D \to \infty$. In this case, $\Delta(\nu)$ is constant in ν at $p - \tau$ or $p + \tau$ and so $\sigma_D = 0$. Higher values of σ_D are associated with higher search frictions. We will later find an analogous value of search frictions for the customer-customer market.

We now turn to the distributions of owners and non-owners Φ^o and Φ^n. From Lemma 1, owners of type $\nu \leq \nu_l$ always sell to dealers whereas non-owners of type $\nu \geq \nu_h$ always buy from dealers. Moreover, non-owners of type $\nu \leq \nu_l$ are inactive. They have a small reservation value $\Delta(\nu)$. It neither compensates for them paying the price asked by dealers nor searching for other customers. It does not compensate searching for other customers because they would have to find a customer with the asset and with an even smaller reservation value so that this other customer would like to sell the asset. In the same way, owners of type $\nu \geq \nu_h$ are inactive. Their high reservation value does not compensate the bid price made by dealers. It also unlikely that this owner could find another customer with even higher reservation value that would like to purchase the asset. So, they do not search for other customers. Therefore, non-owners of type $\nu \leq \nu_l$ and owners of type $\nu \geq \nu_h$ are inactive.

Lemma 2 (Distributions, dealer market)**.** *A regular equilibrium* $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ *is such that the cumulative distribution of owners satisfies*

$$\Phi^o(\nu) = F(\nu) - \Phi^n(\nu) = \frac{\eta F(\nu)}{\eta + \mu + \lambda_D}, \tag{16}$$

for all $\nu \leq \nu_l$*, and*

$$\Phi^o(\nu) = F(\nu) - \Phi^n(\nu) = \frac{\eta}{\eta + \mu} - \frac{(\eta + \lambda_D)[1 - F(\nu)]}{\eta + \mu + \lambda_D} \tag{17}$$

for all $\nu \geq \nu_h$*. Moreover,* ν_l *and* ν_h *satisfy*

$$\eta F(\nu_l) = \mu[1 - F(\nu_h)] \quad \textit{and} \quad \Phi^o(\nu_l) + \Phi^n(\nu_h) = \frac{\mu}{\eta + \mu}. \tag{18}$$

It is useful to define $\nu^* = F^{-1}\big[\mu/(\mu + \eta)\big]$. When $\nu_l = \nu_h = \nu^*$, all customers trade with dealers and ν^* is the marginal customer—customers type $\nu > \nu^*$ buy assets and customers type $\nu < \nu^*$ sell assets. When we decrease ν_l below ν^*, reducing the measure of customers that sell assets, we have also to increase ν_h in order to keep the market clearing in the inter-dealer market. That is, the asset inflow into the market,

$$\lambda_D \Phi^o(\nu_l) = \lambda_D \frac{\eta F(\nu_l)}{\eta + \mu + \lambda_D},$$

equals the asset outflow from the market,

$$\lambda_D[\Phi^n(\infty) - \Phi^n(\nu_h)] = \lambda_D \left[\frac{\mu}{\eta + \mu} - \Phi^n(\nu_h)\right] = \lambda_D \frac{\mu[F(\infty) - F(\nu_h)]}{\eta + \mu + \lambda_D} = \lambda_D \frac{\mu[1 - F(\nu_h)]}{\eta + \mu + \lambda_D}.$$

4.2 Solving the HLW block

For customers searching for other customers, the value functions and reservation values are obtained in the following way. For the value functions, from equations (5) and (6), the value function of a type $\nu \in \Omega_C$ customer holding an asset satisfies

$$rV_C^o(\nu) = \nu - \mu\Delta(\nu) + \lambda_C \int_{\nu}^{\nu_h} \theta_C^o[\Delta(\tilde{\nu}) - \Delta(\nu)]d\Phi^n(\tilde{\nu}), \text{ and} \tag{19}$$

$$rV_C^n(\nu) = \eta\Delta(\nu) + \lambda_C \int_{\nu_l}^{\nu} \theta_C^n[\Delta(\nu) - \Delta(\tilde{\nu})]d\Phi^o(\tilde{\nu}). \tag{20}$$

Combined with the definition of reservation value, $\Delta(\nu) = V_C^o(\nu) - V_C^n(\nu)$, given in equation (8), the equations above imply the following lemma.

Lemma 3 (Reservation value, customer-customer market)**.** *A regular equilibrium* $\{V^o,\ V^n,\ \Delta,\ p,\ \Phi^o,\ \Phi^n,\ \nu_l,\ \nu_h\}$ *satisfies*

$$\Delta(\nu) = \Delta(\nu_l) + \int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu})d\tilde{\nu} \tag{21}$$

for all $\nu \in (\nu_l, \nu_h)$*, and*

$$\sigma_C(\nu) = \frac{1}{r + \mu + \eta + \lambda_C\Big\{\theta_C^o\big[\Phi^n(\nu_h) - \Phi^n(\nu)\big] + \theta_C^n\big[\Phi^o(\nu) - \Phi^o(\nu_l)\big]\Big\}}. \tag{22}$$

for almost all $\nu \in (\nu_l, \nu_h)$*.*

The trade surplus between a seller of type ν and buyer of type $\nu + d\nu$ is approximately equal to $\sigma_C(\nu)d\nu$, thus providing us with the interpretation discussed in HLW of $\sigma_C(\nu)d\nu$ as the local surplus. The function $\sigma_C(\nu)$ discounts the additional utility $d\nu$ by the discount rate r, the likelihood that the asset will mature μ, the loss in the likelihood of issuing an asset η, and the loss in option value from either meeting another buyer with higher valuation $\lambda_C\theta_C^o\big[\Phi^n(\nu_h) - \Phi^n(\nu)\big]$, or finding another seller with lower valuation, $\lambda\theta_C^n\big[\Phi^o(\nu) - \Phi^o(\nu_l)\big]$.

We now turn to the distributions Φ^o, Φ^n among customers searching other customers.

Lemma 4 (Distributions, customer-customer market)**.** *A regular equilibrium* $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ *is such that the cumulative distribution of owners satisfies*

$$\tilde{\Phi}^o(\nu) = F(\nu) - F(\nu_l) - \tilde{\Phi}^n(\nu) \\ - \tfrac{\mu+\eta+\lambda_C[F(\nu_h)-F(\nu)-s_C]}{2\lambda_C} + \tfrac{\sqrt{\{\mu+\eta+\lambda_C[F(\nu_h)-F(\nu)-s_C]\}^2+4\lambda_C\eta\big[F(\nu)-F(\nu_l)\big]}}{2\lambda_C}, \tag{23}$$

$\nu \in (\nu_l, \nu_h)$*, where* $\tilde{\Phi}^o(\nu) \equiv \Phi^o(\nu) - \Phi^o(\nu_l)$ *and* $\tilde{\Phi}^n(\nu) \equiv \Phi^n(\nu) - \Phi^n(\nu_l)$*, and*

$$s_C \equiv \Phi^o(\nu_h) - \Phi^o(\nu_l) = \frac{\eta}{\mu+\eta}\big[F(\nu_h) - F(\nu_l)\big]. \tag{24}$$

Figure 6 shows a representation of the reservation value as function of the utility type ν. Lemma 1 implies that $\Delta(\nu)$ is linear for $\nu \leq \nu_l$ and $\nu \geq \nu_h$. Moreover, $\Delta(\nu_l) \leq p - \tau$ and $\Delta(\nu_h) \geq p + \tau$. That is, owners with $\nu \leq \nu_l$ choose sell to dealers and non-owners with $\nu \geq \nu_h$ choose to buy from dealers. Lemmas 3 and 4 imply the nonlinear shape of Δ in (ν_l, ν_h). Customers that trade with dealers have $\nu \leq \nu_l$. Customers that trade with other customers have $\nu \in (\nu_l, \nu_h)$.

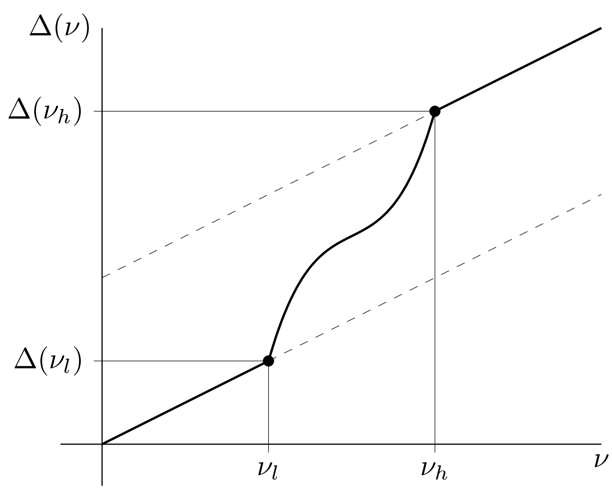


Figure 6: Reservation value as function of the customer type, $\Delta(\nu)$. Customers that trade with dealers have $\nu \leq \nu_l$. Customers that trade with other customers have $\nu \in (\nu_l, \nu_h)$.

4.3 Solving for ν_l and ν_h

The results in sections 4.1 and 4.2 establish necessary conditions for the equilibrium objects $V^o, V^n, \Delta, p, \Phi^o, \Phi^n, s, \nu_l$ and ν_h—that is, equations (4)–(24). In those equations, the equilibrium objects can all be written as functions of ν_l and ν_h. In this section, we provide necessary conditions on ν_l and ν_h and also show that these conditions, combined with equations (4)–(24), are not only necessary but also sufficient for a regular equilibria, and therefore it provides a full characterization.

Lemma 5. *A regular equilibrium* $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ *satisfies*

$$2\tau\lambda_D\theta_D = \int_{\nu_l}^{\nu_h} \frac{\sigma_C(\nu) - \sigma_D}{\sigma_D} d\nu. \tag{25}$$

Moreover,

$$\begin{aligned} p &= \Delta(\nu_l) + \tau + \frac{\lambda_C\theta_C^o}{\lambda_D\theta_D} \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu}) d\tilde{\nu} d\Phi^n(\nu) \\ &= \Delta(\nu_h) - \tau - \frac{\lambda_C\theta_C^n}{\lambda_D\theta_D} \int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} \sigma_C(\tilde{\nu}) d\tilde{\nu} d\Phi^o(\nu). \end{aligned} \tag{26}$$

Lemmas 2 to 5 establish necessary conditions that are satisfied in all regular equilibria. In the proposition below, we show that these conditions are not only necessary but sufficient. Therefore they fully characterize a regular equilibrium.

Proposition 1. *If a family* $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ *is a regular equilibrium, the family* $\{\Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ *satisfies equations* (14)–(26)*. Reversely, if* $\{\Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ *satisfies equations* (14)–(26)*, then* $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ *is a regular equilibrium where the value functions* V^o *and* V^n *are constructed using equations* (3)–(7)*.*

Proposition 1 gives us an easy way to solve for an equilibrium. Given ν_l and ν_h, we can substitute our solutions for Φ^o, Φ^n and σ_C into equation (25) to write then as functions of ν_l and ν_h. We then define the function $g : (-\infty, \nu^*] \to [\nu^*, \infty)$ as $g(\nu_l) = F^{-1}\left[(\mu - \eta F(\nu_l))/\mu\right]$, which uses the results in Lemma 2, to define implicitly ν_h as a function of ν_l. That is, $\nu_h = g(\nu_l)$. An equilibrium ν_l then solves

$$G(\nu_l) = \tau, \tag{27}$$

where $G : (-\infty, \nu^*] \to \mathbb{R}$ is given by

$$G(\nu_l) \equiv \frac{1}{2\lambda_D \theta_D} \int_{\nu_l}^{g(\nu_l)} \frac{\sigma_C(\nu; \nu_l, \nu_h) - \sigma_D}{\sigma_D} d\nu, \tag{28}$$

Note that the function G satisfies $G(\nu^*) = 0$ and $G(\nu_l) > 0$ for ν_l sufficiently small.

After obtaining a ν_l that satisfies $G(\nu_l) = \tau$, we can then obtain all the other equilibrium objects using equations (3)–(7) and (14)–(26). Moreover, as the first part of proposition 1 establishes sufficiency, all regular equilibrium can be obtained in this way.

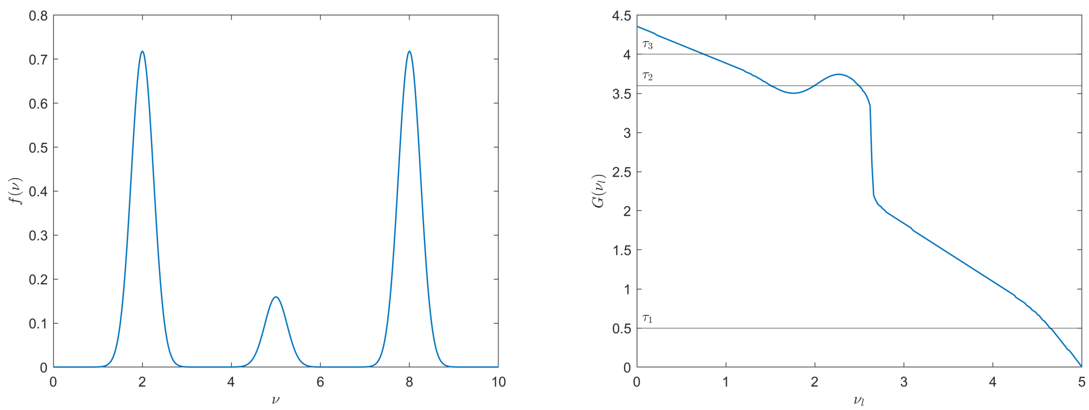
An important result is that proposition 1 does not imply uniqueness. We can have multiple equilibria because G may not be monotone. The intuition is the following. When many customers decide to search for customers instead of for dealers, then the probability of matching is higher and the gain of searching for customers increases. More customers search for customers if they are convinced that others will follow this strategy. This behavior can lead to multiple equilibria. But we can show that it only happens when intermediation costs are sufficiently high.

Proposition 2. *There exists $\bar{\tau} > 0$ such that a regular equilibrium is unique for $\tau \in [0, \bar{\tau})$.*

Proposition 2 establishes that strategic complementarity is not strong enough to generate multiplicity when the intermediation cost τ is close to zero. Assumption 1 implies that searching for dealers is in general preferable than searching for customers. If τ is small enough, no matter how many customers search for customers, it is still preferable to search for dealers. Multiplicity only happens if τ is large enough so that the measure of customers searching for customers affects the decision to search for dealers or for customers.

Figure 7 shows the different types of equilibria. The figure shows the results from simulations with $r = 0.05$, $\mu = \eta = 0.3$, $\theta_D = \theta_C^n = \theta_C^o = 0.5$, $\lambda_D = 3$ and $\lambda_C = 1$. The distribution of utility types, in panel 7a, is a combination of three normal distributions.[8] Panel 7b shows the function G and characterizes the equilibrium ν_l for given values of τ. The equilibrium is unique around τ_1 and τ_3. For $\tau = \tau_1$, the equilibrium ν_l is unique and approximately equal to 4.6. For $\tau = \tau_3$, it is for ν_l around 0.7. For $\tau = \tau_2$, we have three equilibria associated with ν_l approximately equal to 1.5, 2 and 2.5.

[8] $f(\nu) = 0.45N(2, 0.25) + 0.1N(5, 0.25) + 0.45N(8, 0.25)$.



(a) Density of types f (b) Function G

Figure 7: Economy with different equilibrium patterns according to the intermediation cost τ. Small customer-customer market for τ_1. Large customer-customer market for τ_3. Multiplicity of equilibria for τ_2.

An interpretation of G is that it is a proxy for the expected difference in valuation of an owner with ν_l that wants to sell the asset and a non-owner with ν_h that wants buy the asset, both using CC trades. A large difference in valuation, high G, implies that a non-owner might need to pay a substantial amount to the owner to acquire the asset. If G is high relative to τ, it is better to switch from CC to DC trade. A buyer might pay $p + \tau$ net of bargaining in a DC trade, but it would still be smaller than the expected price to pay in a CC trade. This case is such that $G(\nu_l) > \tau$. A switch from CC to DC implies an increase in ν_l and a smaller interval (ν_l, ν_h).

$G(\nu_l)$ decreases with ν_l if the valuation of agents that engage in CC trades gets closer to each other as ν_l increases. This is the case in figure 7 around τ_3. Consider an increase in τ. It would decrease the gain of DC trades. The equilibrium ν_l would decrease and the set of CC trades would increase. Similarly, an increase in λ_C would make CC trades more effective. It would imply a downward shift in G. For $\tau = \tau_3$, it would decrease ν_l and increase the set of CC trades. We can apply the same reasoning to changes around τ_1.

The G function in this example, however, increases for certain values of ν_l as (ν_l, ν_h) includes the higher density of utility types around 2 and 8 in the limits of the interval. Surprisingly, an increase in τ_2 would increase the equilibrium around $\nu_l^* = 2$. The measure of customers trading with dealers would increase with the intermediation cost τ. Similarly, an increase in λ_C, which makes CC trades more effective, would shift G downward and decrease the set of CC trades.

These effects are related with the instability of equilibrium for $\nu_l = 2$. For this equilibrium, suppose that a small set of agents to the left of $\nu_l = 2$ switch their decisions from DC trades to CC trades. The set of agents in CC trades would increase to $(\nu_l - \epsilon, \nu_h')$, where $\nu_h' = g(\nu_l - \epsilon)$. We would then have $G(\nu_l - \epsilon) < G(\nu_l) < \tau_2$, which implies that it is beneficial for an agent to the left of $\nu_l - \epsilon$ also to switch from DC to CC trades. All agents to the left would behave in the same way, which would increase further the set of agents in CC trades, until the equilibrium with ν_l around 0.7 is reached.

At $\tau = \tau_2$ and ν_l around 1.5, the equilibrium is stable. A switch from DC to CC trades of a small set of agents to the left of $\nu_l = 1.5$ would increase G. It would be better to return to DC trades. The same reasoning can be applied to a switch from CC to DC trades to the right of $\nu_l = 1.5$ and also to the other stable equilibrium for $\tau = \tau_2$ around $\nu_l = 2.5$.

A stable equilibrium is therefore associated with a region in which G is decreasing and an unstable equilibrium with a region in which G is increasing. A small perturbation in the set of agents in CC or DC trades in regions where G is decreasing would make agents return to the previous decision of the counterparty. However, in regions where G is increasing, such perturbation would make agents switch the trading counterparty permanently toward a new equilibrium. For these reasons, when necessary to derive results, we focus on the region where G is decreasing and so of a stable equilibrium.

4.4 A measure of illiquidity

The interpretation of σ_D and σ_C as measures of trade distortions leads to a natural definition of a measure of illiquidity. σ_D and σ_C are the derivatives of the reservation value Δ with respect to ν. As discussed above, σ_D and σ_C given in (15) and (22) increase if the trade distortions increase. If λ_D or λ_C go to infinity, which means that there are no search frictions, then σ_D or σ_C go to zero.

A natural measure of illiquidity obtained from the model is therefore given by a weighted average of the frictions faced by all investors in the market. That is, define a measure of illiquidity by

$$\sigma = \int_{-\infty}^{\infty} \sigma_i(\nu)\, dF(\nu), \tag{29}$$

where $\sigma_i(\nu) = \sigma_D(\nu)$ when $\nu \in (-\infty, \nu_l] \cap [\nu_h, +\infty)$ and $\sigma_i(\nu) = \sigma_C(\nu)$ when $\nu \in (\nu_l, \nu_h)$. Given the expressions of σ_D and σ_C in (15) and (22), we see that the measure of illiquidity depends on the interest rate and characteristics of the asset (μ and η) as well as on the measure of investors that engage in different forms of trades.

5 Turnover, illiquidity, and liquidity premium

Several indicators are used to measure illiquidity in financial markets. Here we focus on turnover and the bid-ask spread. We also discuss the liquidity premium—that is, the compensation required to induce investors to purchase an asset that is less liquid.

Turnover is defined by the ratio of the volume of assets traded over a certain period to the amount outstanding of the asset. High turnover ratio indicates high trading activity and is associated with liquidity. Denote turnover by T. The value of turnover implied by the model is given by

$$\begin{aligned} T \equiv & \frac{\lambda_D \left\{\Phi^o(\nu_l) + [\Phi^n(\infty) - \Phi^n(\nu_h)]\right\} + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} d\Phi^n(\tilde{\nu})\, d\Phi^o(\nu)}{\int \Phi^o(\nu)\, d\nu} \\ = & \frac{2\lambda_D \Phi^o(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} \, d\Phi^n(\tilde{\nu})\, d\Phi^o(\nu)}{\eta/(\eta+\mu)}. \end{aligned} \tag{30}$$

The volume of assets sold by customers to dealers is $\lambda_D \Phi^o(\nu_l)$ and the volume of assets bought by customers from dealers is $\lambda_D[\Phi^n(\infty) - \Phi^n(\nu_h)]$. As market clearing implies that $\lambda_D \Phi^o(\nu_l) = \lambda_D[\Phi^n(\infty) - \Phi^n(\nu_h)]$, we have that the total volume of bonds traded between customers and dealers is $2\lambda_D \Phi^o(\nu_l)$. The total volume of bonds traded between customers is $\lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)$. Finally, the amount of bonds outstanding is $s = \int d\Phi^o(\nu) = \frac{\eta}{\eta+\mu}$.

Another measure of liquidity is the difference between the bid and ask prices. That is, the difference between the bid on the asset made by the dealer when the customer wants to sell an asset and the price asked by the dealer when the customer wants to buy an asset.

In our model, we define the bid-ask price as $|p^{buy} - p^{sell}|$, where p^{sell} is the value received by the customer when the customer sells the asset, and p^{buy} is the value paid by the customer when the customer purchases the asset. Usually, $p^{sell} < p^{buy}$.[9] A common interpretation is that the market is less liquid when $|p^{buy} - p^{sell}|$ increases. We now calculate the value of $|p^{buy} - p^{sell}|$ as implied by the model, and show that this is not always the case.

In a regular equilibrium, the average price paid by customers buying from dealers is

$$\begin{aligned} p_D^{buy} &= \frac{\int_{\nu_h}^{\infty} [\theta_D(p+\tau) + (1-\theta_D)\Delta(\nu)] d\Phi^n(\nu)}{\Phi^n(\infty) - \Phi^n(\nu_h)} \\ &= \frac{\int_{\nu_h}^{\infty} \frac{(1-\theta_D)\nu + (r+\eta+\mu+\lambda_D\theta_D)(p+\tau)}{r+\eta+\mu+\lambda_D\theta_D} d\Phi^n(\nu)}{\Phi^n(\infty) - \Phi^n(\nu_h)} = p + \tau + \frac{\int_{\nu_h}^{\infty} \frac{(1-\theta_D)\nu}{r+\eta+\mu+\lambda_D\theta_D} d\Phi^n(\nu)}{\Phi^n(\infty) - \Phi^n(\nu_h)}, \end{aligned} \tag{31}$$

and the average price paid by customers buying from customers is

$$p_C^{buy} = \frac{\int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} [\theta_C^n \Delta(\nu) + \theta_C^o \Delta(\tilde{\nu})] d\Phi^o(\tilde{\nu}) d\Phi^n(\nu)}{\int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} d\Phi^o(\tilde{\nu}) d\Phi^n(\nu)}. \tag{32}$$

So the average price paid by customers is

$$\begin{aligned} p^{buy} &= \mathbb{P}[\text{buy from dealer}] p_D^{buy} + \mathbb{P}[\text{buy from customer}] p_C^{buy} \\ &= \frac{\lambda_D[\Phi^n(\infty) - \Phi^n(\nu_h)] p_D^{buy} + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} d\Phi^o(\tilde{\nu}) d\Phi^n(\nu) p_C^{buy}}{\lambda_D[\Phi^n(\infty) - \Phi^n(\nu_h)] + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} d\Phi^o(\tilde{\nu}) d\Phi^n(\nu)} \end{aligned} \tag{33}$$

Similarly, the average price received by customers selling to dealers is

$$\begin{aligned} p_D^{sell} &= \frac{\int_{-\infty}^{\nu_l} [\theta_D(p-\tau) + (1-\theta_D)\Delta(\nu)] d\Phi^o(\nu)}{\Phi^o(\nu_l)} \\ &= \frac{\int_{-\infty}^{\nu_l} \frac{(1-\theta_D)\nu + (r+\eta+\mu+\lambda_D\theta_D)(p-\tau)}{r+\eta+\mu+\lambda_D\theta_D} d\Phi^o(\nu)}{\Phi^o(\nu_l)} = p - \tau + \frac{\int_{-\infty}^{\nu_l} \frac{(1-\theta_D)\nu}{r+\eta+\mu+\lambda_D\theta_D} d\Phi^o(\nu)}{\Phi^o(\nu_l)}, \end{aligned} \tag{34}$$

and the average price received by customers selling to customers is

$$p_C^{sell} = \frac{\int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} [\theta_C^n \Delta(\tilde{\nu}) + \theta_C^o \Delta(\nu)] d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)}{\int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)}. \tag{35}$$

[9] For example, if the customer sells the asset to a dealer, p^{sell} is the bid made by the dealer to buy the asset. If the customer purchases the asset from a dealer, p^{buy} is the price asked by the dealer for the asset.

So the average price received by customers selling assets is

$$p^{sell} = \mathbb{P}[\text{sell to dealer}]p_D^{sell} + \mathbb{P}[\text{sell to customer}]p_C^{sell}$$
$$= \frac{\lambda_D \Phi^o(\nu_l) p_D^{sell} + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu) p_C^{sell}}{\lambda_D \Phi^o(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)}. \tag{36}$$

The bid-ask spread is then given by

$$\text{BA} \equiv p^{buy} - p^{sell} = \frac{\lambda_D \Phi^o(\nu_l)(p_D^{buy} - p_D^{sell})}{\lambda_D \Phi^o(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)}$$
$$= \frac{2\lambda_D \Phi^o(\nu_l)\tau + \lambda_D(1-\theta_D)\frac{\int_{\nu_h}^{\infty} \nu d\Phi^n(\nu) - \int_{-\infty}^{\nu_l} \nu d\Phi^o(\nu)}{r+\eta+\mu+\lambda_D\theta_D}}{\lambda_D \Phi^o(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)}. \tag{37}$$

The second and third equalities above are obtained using the inter-dealer market clearing condition and the structure of the customer-customer market. The inter-dealer market clearing condition implies that $\Phi^o(\nu_l) = \Phi^n(\infty) - \Phi^n(\nu_h)$. As every asset sold by a customer is bought by another customer in the customer-customer market, we have that $p_C^{sell} = p_C^{buy}$.

5.1 The impact of τ on turnover

In regions with a unique equilibrium, an increase in intermediation cost decreases the measure of customers trading with dealers. As dealers under our assumptions have a more efficient matching technology, this force tends to decrease turnover when intermediation costs increase. We call this the *dealer-efficiency* impact on turnover. However, there is another force. When an asset is intermediated by dealers, there is not a long chain of trades to allocate the asset. The asset goes from an investor with $\nu \leq \nu_l$ to a dealer, then another dealer, and then to an an investor with $\nu \geq \nu_h$. In customer-to-customer trades, the asset can potentially be allocated by a long chain of customers until it gets to an investor with very high ν_h and ceases to be traded. The length of this chain tends to increase turnover when intermediation costs increase and more investors chose customer-customer trade. We call this the *customer-chain* impact on turnover.

Proposition 3. *The turnover is always decreasing in τ in a neighborhood of $\tau = 0$. Moreover, if the search technologies satisfy $\lambda_C \leq \lambda_D \leq \frac{\sqrt{2}-1}{\sqrt{2}}(\eta + \mu)$, then the turnover is decreasing in*

τ in all regions with a unique equilibrium.

For low intermediation costs (τ close to zero), the *dealer-efficiency* impact on turnover dominates the *customer-chain* impact. Turnover decreases when the intermediation cost increases. In this case, the measure of investors choosing customer-customer trade is small. It takes too long to form a chain of trades and the asset is likely to mature before it is formed. For τ not close to zero, it is possible that the *customer-chain* impact on turnover dominates the *dealer-efficiency* impact. For this to happen, the search technology has to be sufficiently efficient so that assets do not mature, or possible buyers get an issuance opportunity, before the asset is traded and the chain is formed. Formally, we show that whenever $\lambda_C \leq \lambda_D \leq \frac{\sqrt{2}-1}{\sqrt{2}}(\eta+\mu)$ the *dealer-efficiency* impact dominates the *customer-chain* impact on turnover.

5.2 The impact of τ on bid-ask spreads

There are two ways in which an increase in the intermediation cost of dealers, τ, impacts the bid-ask spread: the *intensive* margin and the *extensive* margin.

On the intensive margin, customers have to compensate dealers for their costs to intermediate the trade. As a result, an increase in τ implies an increase in the bid-ask spread charged by dealers. On the extensive margin, the higher bid-ask spread charged by dealers drive customers to trade with other customers instead of trading with dealers (customer matching). Since we assume that trading with customers is slow but has zero intermediation cost for customers, the extensive margin reduces the average bid-ask spread.

These two forces move the bid-ask spread in opposite directions and therefore the impact of τ on the average bid-ask spread is ambiguous. However, we find that an increase in τ from $\tau = 0$ implies an initial increase in the bid-ask spread, which could be interpreted as a decrease in liquidity. The intensive margin dominates in this case. On the other hand, for a high enough value of τ, the bid-ask spread decreases. In this case, the extensive margin dominates. The bid-ask spread would decrease, which could be interpreted as an improvement in liquidity.

Proposition 4. *The following holds regarding the behavior of bid-ask spreads.*

i. There exists $\tilde{\tau} > 0$ such that the bid-ask spread is increasing in τ for all $\tau \in [0, \tilde{\tau})$.

ii. There exist $\tau_0, \tau_1 > 0$, with $\tau_0 < \tau_1$, and associated regular equilibria $\mathcal{E}_0$ and $\mathcal{E}_1$, such that the bid-ask spread in $\mathcal{E}_0$ is strictly bigger than the bid-ask spread in $\mathcal{E}_1$.

Proposition 4 has two parts. Part 1 asserts that, for small intermediation costs, the bid-ask spread has the expected behavior—a higher costs for dealers to intermediate transactions increases the bid-ask spread in the market. Part 2 shows that, on the other hand, the bid-ask spread is not monotone in τ. This result is illustrated in Figure 8.

The intuition for Proposition 4 is the following. When τ is not very high, most trades are DC trades. In this case, an increase in τ increases the trading cost for most customers. This force increases the average bid-ask spread, which is the first part of the proposition. However, as stated in the second part of the proposition, if τ increases enough, many customers stop trading with dealers and the bid-ask spread actually decreases. In fact, the bid-ask spread converges to zero as τ goes to infinity.

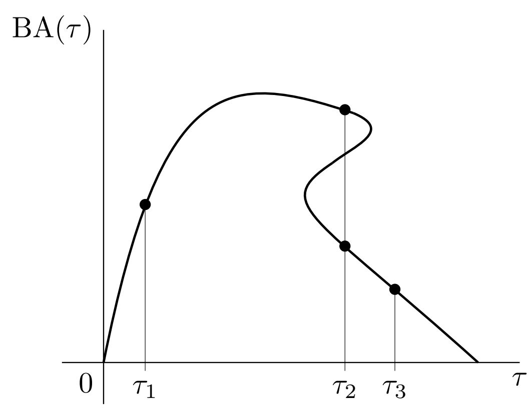


Figure 8: An increase in the intermediation cost parameter τ_1 increases the bid-ask spread. τ_2 implies multiplicity of equilibrium. An increase in τ_3 implies a decrease in the bid-ask.

Figure 8 shows a possible relation between the bid-ask spread and τ. It is compatible with the function G in figure 7b. For small values of τ, the bid-ask spread is increasing in τ. For intermediary values of τ, there can be multiplicity of equilibrium and corresponding multiplicity of bid-ask spreads. For large values of τ, the bid-ask spread is decreasing τ. A decreasing bid-ask spread as the intermediation cost τ increases is a result of the higher number of customer-customer transactions.

5.3 The impact of τ on liquidity premium

The presence of bid-ask spreads in financial markets raises concerns about the potential for increased customer trading costs, which could lead to lower prices or wider spreads. We refer to the increase in prices implied by higher illiquidity as the liquidity premium.

In our model, the liquidity premium is best measured as a function of σ_D and σ_C of customers, as discussed in section 4.4. However, these parameters are not directly observable in practice. Instead, we employ empirical measures of illiquidity, such as those proposed by Bao et al. (2011).

One challenge with these empirical measures is that they capture the overall bid-ask spread in the market. As discussed in the previous section, this spread can either narrow or widen due to a composition effect when intermediation costs increase. We show here that increases in dealer intermediation costs can mechanically inflate or deflate the liquidity premium due to their reliance on the bid-ask spread.

To understand this non-monotonic relationship, consider two economies, A and B, characterized by identical parameters θ_D, λ_C, λ_D, μ, η, and r. Assume that $\theta_C^o = \theta_C^n = \theta_C = \frac{1}{2}$, $\mu = \eta$, $\tau^A = \tau$, and $\tau^B = \tau + d\tau$, where $d\tau \geq 0$ is a small perturbation and $\bar{\nu}^A > \bar{\nu}^B$. Furthermore, suppose that both economies possess a unique equilibrium, implying the existence of strictly decreasing functions $G_A(\nu_l)$ and $G_B(\nu_l)$ associated with economies A and B, respectively. This uniqueness of equilibrium allows us to define the bid-ask spread as a function of the intermediation cost, resulting in $\mathrm{BA}^A = \mathrm{BA}(\tau)$ and $\mathrm{BA}^B = \mathrm{BA}(\tau + d\tau)$.

As discussed previously, the empirical measures of illiquidity employed in practice partially capture the bid-ask spread. Let's assume that the illiquidity measure is given by the following equation:

$$L(\tau) = \bar{L}(\tau) + \alpha BA(\tau)$$

where $\bar{L}(\tau)$ represents other aspects of illiquidity that increase with intermediation costs, implying $\bar{L}'(\tau) > 0$, and $\alpha > 0$ determines the proportion of the bid-ask spread captured by the illiquidity measure. Consequently, the liquidity premium can be expressed as

$$LP(\tau) = \frac{p^A(\tau) - p^B(\tau + d\tau)}{\bar{L}(\tau + d\tau) + \mathrm{BA}(\tau + d\tau) - \mathrm{BA}(\tau) - \bar{L}(\tau)}, \tag{38}$$

where $p^A(\tau)$ and $p^B(\tau + d\tau)$ denote equilibrium prices in economies A and B.

Proposition 5. *Assume that $\bar{L}'(\tau) = d\bar{L} > 0$ and $d\tau > 0$ is sufficiently small. Then, the liquidity premium is non-monotone in τ. Moreover, the liquidity premium for low τ is lower than the liquidity premium for high τ. That is, $LP(0) < \lim_{\tau \to \infty} LP(\tau) = \frac{\bar{\nu}^A - \bar{\nu}^A}{d\bar{L}\, d\tau}$.*

Proposition 5 underscores the inherent non-monotonic relationship between liquidity premium and intermediation costs, which extends to any measure that captures bid-ask spreads. This proposition also yields a key prediction of the model: as an economy transitions from low to high intermediation costs, the liquidity premium tends to increase. This non-monotonic behavior in the liquidity premium stems from its dependence on measures that capture the bid-ask spread.

6 Conclusions

We propose a model to explain structural changes observed in the corporate bond market since the 2008 financial crisis. It has been identified that a larger fraction of trades executed in a way that dealers do not need to maintain asset inventory. In these trades, customers provide liquidity to other customers. These trades happen more frequently for larger amounts. They take longer to be executed. At the same time, standard measures of illiquidity show a decrease in illiquidity since 2008 whereas investors require a higher illiquidity premium. The model explains these at first sight conflicting observations.

The model combines Lagos and Rocheteau (2009) and Hugonnier, Lester, and Weill (2022). Lagos and Rocheteau study trades between customers and dealers. Hugonnier, Lester, and Weill study trades between customers and customers. We combine the two models two include the decision of a customer to trade with a dealer with another costumers. Both models study decisions on OTC markets with search frictions, as in Duffie et al. (2005).

In the model, τ represents the intermediation cost paid by dealers. We interpret the regulations in Dodd-Frank, which include increased capital requirements, increased reporting requirements, and increased restrictions on trading activities, as an increase in τ. The regulations made it more expensive for dealers to provide liquidity. We then examine the equilibrium outcomes from the model.

When the intermediation cost of dealers increases, customers seek liquidity from other customers, increasing the number of customer-customer trades and decreasing the number of customer-dealer trades. In the context of the model, the measure of customers in the customer-customer market increases. The average bid-ask spread, which considers the final transaction prices, decreases. However, the average trade becomes more costly. A measure of illiquidity based on final prices would imply a decrease in illiquidity, as we find empirically.

The model allows us to propose a new measure of illiquidity. This measure takes into account the distortions caused by the search frictions, as well as the bargaining power, number of customers engaged in customer-customer or customer-dealer trades and other variables. An increase in the measure of agents that engage in customer-customer trades increases the value of this comprehensive measure of illiquidity.

The model implies the possibility of multiple equilibria and financial crises. Depending on τ, there can be an equilibrium with a small number of customer-customer trades and an another one with a large number of customer-customer trades. This is so because the decision to direct search on one market or the other depends on the expected number of agents that engage in the same activity. Both equilibria are stable. A movement from one equilibrium to another would cause abrupt changes in the market that can be perceived as financial crises.

The 2008 financial crisis generated a strong response in financial regulations. Our results indicate a way to connect the changes in regulations with changes in the structure of financial markets. Especially, in the structure of a market heavily based on over-the-counter trades such as the corporate bond market.

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A Proofs

Proof of Lemma 1

Proof. Take the difference between equations (3) and (4) to obtain

$$\begin{aligned} r\Delta(\nu) &= \nu - \mu\Delta(\nu) - \eta\Delta(\nu) \\ &\quad + \lambda_D\theta_D \max\{p - \tau - \Delta(\nu), 0\} - \lambda_D\theta_D \max\{\Delta(\nu) - p - \tau, 0\}, \end{aligned} \tag{39}$$

which implies that

$$\implies \Delta(\nu) = \frac{\nu + \lambda_D\theta_D\left[p + \tau + \max\{\Delta(\nu), p - \tau\} - \max\{\Delta(\nu), p + \tau\}\right]}{r + \nu + \mu + \lambda_D\theta_D} \tag{40}$$

for all types $\nu \in (-\infty, \nu_l] \cup [\nu_h, \infty)$. Equation (40) is associated with a functional operator satisfying all Blackwell's conditions for a contraction. Then, by the contraction mapping theorem, there is a unique function Δ satisfying the equation (40). Also note that if $\tau = 0$, then the results follow directly from equation (40). So we focus on the case with $\tau > 0$.

Since Δ is strictly increasing and continuous, we must have that $\Delta(\nu_l) \leq p - \tau$. To see this, notice first that, if $p - \tau < \Delta(\nu_l) < p + \tau$, then the customer would not trade with a dealer because of transaction costs, as the reservation value of a potential seller is higher than the highest bid price of a dealer, $p - \tau$, and the reservation value of a potential buyer is smaller than the lowest ask price of a dealer, $p + \tau$. The last terms in equations (3) and (4) would be zero. Therefore, searching for a dealer is equivalent to be inactive. In this case, the customer would be better off searching for customers type $\nu \in (\nu_l, \nu_h)$ to obtain a share of the gains from trade. This implies that $\nu_l \notin \Omega_D$, which is a contradiction. Implicit in this argument is the fact that the densities of Φ^o and Φ^n are bounded away from zero in the set (ν_l, ν_h) because of issuance and maturity (see proof of Lemma 2), and $\nu_l \neq \nu_h$ (which holds by assumption on a regular equilibrium with $\tau > 0$).

Moreover, if $\Delta(\nu_l) \geq p + \tau$, then either $p - \tau < \Delta(\nu) < p + \tau$ for some customer type $\nu \in \Omega_D$ or $\Delta(\nu) \geq p + \tau$ for all customer type $\nu \in \Omega_D$. The first cannot hold because again it would imply $\nu \notin \Omega_D$. The second would be inconsistent with inter-dealer market clearing because all customers searching for a dealer would want to buy assets as their reservation value would be greater than or equal to the highest ask price.

Therefore, we must have $\Delta(\nu_l) \leq p - \tau$. An analogous argument applies for ν_h in the opposite direction. That is, $\Delta(\nu_h) \geq p + \tau$. With $\Delta(\nu_l) \leq p - \tau$ and $\Delta(\nu_h) \geq p + \tau$, we can solve for the max relations in equation (40), which then implies equation (14). ■

Proof of Lemma 2

Proof. First note that equation (12) implies

$$\dot{\Phi}^o(\infty) = \eta\Phi^n(\infty) - \mu\Phi^o(\infty) \tag{41}$$

as, when ν goes to infinity, both the inflow and outflow from trading goes to zero. Then, from $\dot{\Phi}^o(\infty) = 0$ and equation (13), we have that

$$\eta[F(\infty) - \Phi^o(\infty)] - \mu\Phi^o(\infty) = 0 \iff \Phi^o(\infty) = \frac{\eta}{\eta + \mu}, \tag{42}$$

which characterizes the total supply of assets $s = \Phi^o(\infty)$, equal, by definition, to the measure of owners. This also establishes that the measure of non-owners is given by

$$\Phi^n(\infty) = F(\infty) - \Phi^o(\infty) = 1 - \Phi^o(\infty) \implies \Phi^n(\infty) = \frac{\mu}{\eta + \mu}. \tag{43}$$

Consider now the case $\nu \leq \nu_l$. According the law of motion for Φ^o, given by equation (12), we have

$$\dot{\Phi}^o(\nu) = \eta\Phi^n(\nu) - \mu\Phi^o(\nu) - \lambda_D\Phi^o(\nu), \tag{44}$$

as no customer with $\tilde{\nu} \leq \nu \leq \nu_l$ will either search for other customers or purchase the asset from a dealer. Substituting $\Phi^n(\nu) = F(\nu) - \Phi^o(\nu)$ and setting $\dot{\Phi}^o(\nu) = 0$ implies

$$\Phi^o(\nu) = \frac{\eta F(\nu)}{\eta + \mu + \lambda_D}, \quad \nu \leq \nu_l. \tag{45}$$

Consider now the case $\nu \geq \nu_h$. In this case, it is useful to work with the measure of non-owners of type above ν, $\Phi^n(\infty) - \Phi^n(\nu)$. Using equations (13) and (12), we have

$$\begin{aligned}
0 &= \dot{\Phi}^n(\infty) - \dot{\Phi}^n(\nu) && (46)\\
&= -\eta[\Phi^n(\infty) - \Phi^n(\nu)] - \lambda_D[\Phi^n(\infty) - \Phi^n(\nu)] + \mu[\Phi^o(\infty) - \Phi^o(\nu)] && (47)\\
&= -\eta[1 - s - F(\nu) + \Phi^o(\nu)] - \lambda_D[1 - s - F(\nu) + \Phi^o(\nu)] + \mu[s - \Phi^o(\nu)] && (48)\\
&= -(\eta + \lambda_D)[1 - F(\nu)] + (\eta + \mu + \lambda_D)[s - \Phi^o(\nu)] && (49)
\end{aligned}$$

$$\implies s - \Phi^o(\nu) = \frac{(\eta + \lambda_D)[1 - F(\nu)]}{\eta + \mu + \lambda_D}. \tag{50}$$

As $s = \frac{\eta}{\eta+\mu}$, we have

$$\Phi^o(\nu) = \frac{\eta}{\eta+\mu} - \frac{(\eta+\lambda_D)[1-F(\nu)]}{\eta+\mu+\lambda_D}, \quad \nu \geq \nu_h. \tag{51}$$

Now let us show that $\eta F(\nu_l) = \mu[1-F(\nu_h)]$. According to the market clearing condition (11) and Lemma 1,

$$\Phi^o(\nu_l) = \int_{-\infty}^{\infty} \mathbb{1}_{\{\nu\in\Omega_D^n, \Delta(\nu)>p+\tau\}} d\Phi^n(\nu) \implies \Phi^o(\nu_l) = \Phi^n(\infty) - \Phi^n(\nu_h). \tag{52}$$

We know that $\Phi^o(\nu_l) = \frac{\eta F(\nu_l)}{\eta+\mu+\lambda_D}$. Moreover,

$$\begin{aligned} \Phi^n(\infty) - \Phi^n(\nu_h) &= F(\infty) - \Phi^o(\infty) - [F(\nu_h) - \Phi^o(\nu_h)] \\ &= 1 - s - \left[F(\nu_h) - \frac{\eta}{\eta+\mu} + \frac{(\eta+\lambda_D)[1-F(\nu_h)]}{\eta+\mu+\lambda_D}\right] = \frac{\mu[1-F(\nu_h)]}{\eta+\mu+\lambda_D}. \end{aligned} \tag{53}$$

Thus, $\eta F(\nu_l) = \mu[1-F(\nu_h)]$. Finally, the result that $\Phi^o(\nu_l) = \frac{\mu}{\eta+\mu} - \Phi^n(\nu_h)$ comes from equation (52) and the fact that $\Phi^n(\infty) = F(\infty) - \Phi^o(\infty) = 1 - \frac{\eta}{\eta+\mu} = \frac{\mu}{\eta+\mu}$. ■

Proof of Lemma 3

Proof. By taking the difference between equations (19) and (20), we know that the reservation value satisfies

$$\Delta(\nu) = \frac{\nu + \lambda_C \int_{\nu}^{\nu_h} \theta_C^o[\Delta(\tilde{\nu}) - \Delta(\nu)] d\Phi^n(\tilde{\nu}) - \lambda_C \int_{\nu_l}^{\nu} \theta_C^n[\Delta(\nu) - \Delta(\tilde{\nu})] d\Phi^o(\tilde{\nu})}{r+\mu+\eta}. \tag{54}$$

Moreover, because Δ is continuous and monotone, equation (54) implies that Δ is Lipschitz continuous in the interval (ν_l, ν_h). To see this, note that we can rearrange equation (54) to show that

$$\left|\frac{\Delta(\nu+t) - \Delta(\nu)}{t}\right| \leq \left|\frac{1 + 2\lambda_C \sup_x f(x)[\Delta(\nu_h) - \Delta(\nu_l)]}{r+\mu+\eta}\right|$$

for all ν and $\nu + t$ in the interval (ν_l, ν_h), where f is the density of the distribution F. Given that Δ is Lipschitz continuous in the interval (ν_l, ν_h), Δ is differentiable almost everywhere in the interval (ν_l, ν_h) and satisfies $\Delta(\nu) = \Delta(\nu_l) + \int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu}) d\tilde{\nu}$, where $\sigma_C(\nu)$ denote the derivative of Δ. Using this result, take the derivative on both sides of equation (54) to obtain

$$\sigma_C(\nu) = \frac{1 - \lambda_C \theta_C^o [\Phi^n(\nu_h) - \Phi^n(\nu)]\sigma_C(\nu) - \lambda_C \theta_C^n [\Phi^o(\nu) - \Phi^o(\nu_l)]\sigma_C(\nu)}{r + \mu + \eta}. \tag{55}$$

We then obtain $\sigma_C(\nu)$ by rearranging the equation above. ■

Proof of Lemma 4

Proof. We have $\dot{\tilde{\Phi}}^o(\nu) = \dot{\Phi}^o(\nu) - \dot{\Phi}^o(\nu_l)$. From equation (12), we have

$$\begin{aligned}
\dot{\tilde{\Phi}}^o(\nu) &= \eta\tilde{\Phi}^n(\nu) - \mu\tilde{\Phi}^o(\nu) - \lambda_C \int_{\nu_l}^{\nu} \int_{\nu}^{\nu_h} d\Phi^n(\hat{\nu}) d\Phi^o(\tilde{\nu}) \\
&= \eta\tilde{\Phi}^n(\nu) - \mu\tilde{\Phi}^o(\nu) - \lambda_C \tilde{\Phi}^o(\nu) \big[\Phi^n(\nu_h) - \Phi^n(\nu)\big] \\
&= \eta\tilde{\Phi}^n(\nu) - \mu\tilde{\Phi}^o(\nu) - \lambda_C \tilde{\Phi}^o(\nu) \big[F(\nu_h) - F(\nu)\big] + \lambda_C \tilde{\Phi}^o(\nu) \big[\tilde{\Phi}^o(\nu_h) - \tilde{\Phi}^o(\nu)\big] \\
&= \eta\big[F(\nu) - F(\nu_l)\big] - \tilde{\Phi}^o(\nu) \Big\{\mu + \eta + \lambda_C \big[F(\nu_h) - F(\nu) - \tilde{\Phi}^o(\nu_h)\big]\Big\} - \lambda_C \tilde{\Phi}^o(\nu)^2. \quad (56)
\end{aligned}$$

We can then solve the quadratic equation above with $\dot{\tilde{\Phi}}^o(\nu) = 0$ to obtain equation (23). Equation (24) is obtained by the solution of the quadratic equation for $\nu = \nu_h$. ■

Proof of Lemma 5

Proof. In a regular equilibrium, customers of type ν_l are indifferent between searching for dealers or customers. The reason is that equations (3)–(6) and the continuity of Δ imply the continuity of V_C^o, V_C^n, V_D^o and V_D^n. As a result,

$$\begin{aligned}
rV_C^o(\nu_l) &= \nu_l - \mu\Delta(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \theta_C^o [\Delta(\nu) - \Delta(\nu_l)] d\Phi^n(\nu) \\
&= \nu_l - \mu\Delta(\nu_l) + \lambda_C \theta_C^o \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu}) d\tilde{\nu} d\Phi^n(\nu) \\
&= \nu_l - \mu\Delta(\nu_l) + \lambda_D \theta_D [p - \tau - \Delta(\nu_l)] = rV_D^o(\nu_l). \quad (57)
\end{aligned}$$

Which implies that

$$p = \Delta(\nu_l) + \tau + \frac{\lambda_C \theta_C^o}{\lambda_D \theta_D} \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu}) d\tilde{\nu} d\Phi^n(\nu). \tag{58}$$

And similarly,

$$
\begin{aligned}
rV_C^n(\nu_h) &= \eta\Delta(\nu_h) + \lambda_C \int_{\nu_l}^{\nu_h} \theta_C^n[\Delta(\nu_h) - \Delta(\nu)]d\Phi^o(\nu) \\
&= \eta\Delta(\nu_h) + \lambda_C\theta_C^n \int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} \sigma_C(\tilde{\nu})d\tilde{\nu}d\Phi^o(\nu) \\
&= \eta\Delta(\nu_h) + \lambda_D\theta_D[\Delta(\nu_h) - p - \tau] = rV_D^n(\nu_h). \qquad (59)
\end{aligned}
$$

Which implies that

$$
p = \Delta(\nu_h) - \tau - \frac{\lambda_C\theta_C^n}{\lambda_D\theta_D} \int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} \sigma_C(\tilde{\nu})d\tilde{\nu}d\Phi^o(\nu). \qquad (60)
$$

Equalizing the above two price equations and using lemma 3 we obtain

$$
\begin{aligned}
2\tau\lambda_D\theta_D = {} & \lambda_D\theta_D \int_{\nu_l}^{\nu_h} \sigma_C(\nu)d\nu \\
& - \lambda_C\theta_C^n \int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} \sigma_C(\tilde{\nu})d\tilde{\nu}d\Phi^o(\nu) - \lambda_C\theta_C^o \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu})d\tilde{\nu}d\Phi^n(\nu). \qquad (61)
\end{aligned}
$$

Applying integration by parts in the last two terms we obtain

$$
\begin{aligned}
2\tau\lambda_D\theta_D = {} & \lambda_D\theta_D \int_{\nu_l}^{\nu_h} \sigma_C(\nu)d\nu \\
& - \lambda_C \int_{\nu_l}^{\nu_h} \Big\{\theta_C^n\big[\Phi^o(\nu) - \Phi^o(\nu_l)\big] + \theta_C^o\big[\Phi^n(\nu_h) - \Phi^n(\nu)\big]\Big\}\sigma_C(\nu)d\nu. \qquad (62)
\end{aligned}
$$

From the definition of $\sigma_C(\nu)$, we have $\lambda_C\{\theta_C^o\big[\Phi^n(\nu_h) - \Phi^n(\nu)\big] + \theta_C^n\big[\Phi^o(\nu) - \Phi^o(\nu_l)\big]\} = \frac{1}{\sigma_C(\nu)} - (r + \mu + \eta)$. Substituting above and rearranging implies

$$
2\tau\lambda_D\theta_D = \int_{\nu_l}^{\nu_h} [(r + \mu + \eta + \lambda_D\theta_D)\sigma_C(\nu) - 1]d\nu. \qquad (63)
$$

As $r + \mu + \eta + \lambda_D\theta_D = 1/\sigma_D$, we obtain

$$
2\tau\lambda_D\theta_D = \int_{\nu_l}^{\nu_h} \frac{\sigma_C(\nu) - \sigma_D}{\sigma_D} d\nu. \qquad (64)
$$

This concludes the proof. ■

Proof of Proposition 1

Proof. The necessity of equations (14)–(26) are established in Lemmas 2–5. So let us focus on the sufficiency. Consider a family $\{\Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ satisfying equations (14)–(26) and value functions V^o and V^n constructed using equations (3)–(7) given the family $\{\Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$. Let us show that the family $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ is a regular equilibrium—that is, it satisfies equations (3)–(13) and definition 2.

Equations (3)–(7)**:** These equations are satisfied by the construction of V^o and V^n.

Equations (9)–(10)**:** First let us show that $V^o_D(\nu) \geq V^o_C(\nu)$ for all $\nu \leq \nu_l$.

$$V^o_D(\nu) \geq V^o_C(\nu) \Longleftrightarrow \lambda_D\theta_D[(p-\tau) - \Delta(\nu)] \geq \lambda_C\theta^o_C \int_{\nu_l}^{\nu^h} [\Delta(\tilde{\nu}) - \Delta(\nu)] d\Phi^n(\tilde{\nu})$$
$$\Longleftrightarrow (p-\tau) - \Delta(\nu) \geq \tfrac{\lambda_C\theta^o_C}{\lambda_D\theta_D} \int_{\nu_l}^{\nu^h} [\Delta(\tilde{\nu}) - \Delta(\nu_l)] d\Phi^n(\tilde{\nu})$$
$$+\tfrac{\lambda_C\theta^o_C}{\lambda_D\theta_D}[\Phi^n(\nu_h) - \Phi^n(\nu_l)][\Delta(\nu_l) - \Delta(\nu)].$$

From equation (26) we know that $\frac{\lambda_C\theta^o_C}{\lambda_D\theta_D}\int_{\nu_l}^{\nu^h}[\Delta(\tilde{\nu}) - \Delta(\nu_l)]d\Phi^n(\tilde{\nu}) = (p-\tau) - \Delta(\nu_l)$, therefore

$$V^o_D(\nu) \geq V^o_C(\nu) \iff \Delta(\nu_l) - \Delta(\nu) \geq \frac{\lambda_C\theta^o_C}{\lambda_D\theta_D}[\Phi^n(\nu_h) - \Phi^n(\nu_l)][\Delta(\nu_l) - \Delta(\nu)].$$

Assumption 1 implies that $\frac{\lambda_C\theta^o_C}{\lambda_D\theta_D} \in (0,1)$. From the equations (16) and (17) we have that $\Phi^n(\nu_h) - \Phi^n(\nu_l) = \frac{\mu[F(\nu_h) - F(\nu_l)]}{\eta+\mu} \in [0,1)$. Using equation (14) in Lemma 1, we have that

$$V^o_D(\nu) \geq V^o_C(\nu) \Longleftrightarrow \nu_l - \nu \geq \frac{\lambda_C\theta^o_C}{\lambda_D\theta_D}[\Phi^n(\nu_h) - \Phi^n(\nu_l)][\nu_l - \nu].$$

We can then see that $V^o_D(\nu) \geq V^o_C(\nu)$ holds. Moreover, it holds with strictly inequality for all $\nu < \nu_l$. The proofs that $V^n_D(\nu) \geq V^n_C(\nu)$ for all $\nu \leq \nu_l$; $V^o_D(\nu) \leq V^o_C(\nu)$ for all $\nu \in (\nu_l, \nu_h)$; and $V^n_D(\nu) \leq V^n_C(\nu)$ for all $\nu \in (\nu_l, \nu_h)$ are analogous.

Equation (8)**:** Let us start with $\nu \leq \nu_l$. In this case we have that $V^o(\nu) = V_D^o(\nu)$ and $V^n(\nu) = V_D^n(\nu)$ based on equation (9). Then, from equations (3) and (4) we have that

$$\begin{aligned}
V^o(\nu) - V^n(\nu) &= \frac{\nu + \lambda_D\theta_D(p-\tau) - (\eta + \mu + \lambda_D\theta_D)\Delta(\nu)}{r} \\
&= \frac{(r + \eta + \mu + \lambda_D\theta_D)\Delta(\nu) - (\eta + \mu + \lambda_D\theta_D)\Delta(\nu)}{r} = \Delta(\nu).
\end{aligned}$$

The result for $\nu \geq \nu_h$ is analogous. For $\nu \in (\nu_l, \nu_h)$ we have that

$$\begin{aligned}
r[V^o(\nu) - V^n(\nu)] = \nu - (\eta + \mu)\Delta(\nu) + \lambda_C \int_{\nu}^{\nu_h} \theta_C^o[\Delta(\tilde{\nu}) - \Delta(\nu)]d\Phi^n(\tilde{\nu}) \\
- \lambda_C \int_{\nu_l}^{\nu} \theta_C^n[\Delta(\nu) - \Delta(\tilde{\nu})]d\Phi^o(\tilde{\nu})
\end{aligned}$$

Replacing equation (21) and applying integration by parts we get

$$\begin{aligned}
r[V^o(\nu) - V^n(\nu)] &= \nu - (\eta + \mu)\Delta(\nu_l) - (\eta + \mu)\int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu})d\tilde{\nu} \\
&+ \lambda_C \int_{\nu}^{\nu_h} \theta_C^o[\Phi^n(\nu_h) - \Phi^n(\tilde{\nu})]\sigma_C(\tilde{\nu})d\tilde{\nu} - \lambda_C \int_{\nu_l}^{\nu} \theta_C^n[\Phi^o(\tilde{\nu}) - \Phi^o(\nu_l)]\sigma_C(\tilde{\nu})d\tilde{\nu} \\
&= \nu - (\eta + \mu)\Delta(\nu_l) + \lambda_C \int_{\nu}^{\nu_h} \theta_C^o[\Phi^n(\nu_h) - \Phi^n(\tilde{\nu})]\sigma_C(\tilde{\nu})d\tilde{\nu} \\
&- \int_{\nu_l}^{\nu} \left\{\eta + \mu + \lambda_C\theta_C^n[\Phi^o(\tilde{\nu}) - \Phi^o(\nu_l)]\right\} \sigma_C(\tilde{\nu})d\tilde{\nu} \\
&= \nu - (\eta + \mu)\Delta(\nu_l) + \lambda_C \int_{\nu}^{\nu_h} \theta_C^o[\Phi^n(\nu_h) - \Phi^n(\tilde{\nu})]\sigma_C(\tilde{\nu})d\tilde{\nu} \\
&- \nu + \nu_l - \int_{\nu_l}^{\nu} \left\{r + \lambda_C\theta_C^o[\Phi^n(\nu_h) - \Phi^n(\tilde{\nu})]\right\} \sigma_C(\tilde{\nu})d\tilde{\nu} \\
&= \nu_l - (r + \eta + \mu)\Delta(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \theta_C^o[\Phi^n(\nu_h) - \Phi^n(\tilde{\nu})]\sigma_C(\tilde{\nu})d\tilde{\nu} + r\Delta(\nu).
\end{aligned}$$

Now we can replace equation (26) to obtain

$$\begin{aligned}
r[V^o(\nu) - V^n(\nu)] &= \nu_l - (r + \eta + \mu)\Delta(\nu_l) + \lambda_C\theta_C^o[p - \tau - \Delta(\nu_l)] + r\Delta(\nu) \\
&= \nu_l + \lambda_C\theta_C^o(p - \tau) - (r + \eta + \mu + \lambda_C\theta_C^o)\Delta(\nu_l) + r\Delta(\nu) = r\Delta(\nu),
\end{aligned}$$

where the last equality we obtained using equation (14) applied to $\Delta(\nu_l)$.

Equation (11): The left-hand side of Equation (11) is given by

$$\lambda_D \int_{\Omega_D^o} \mathbb{1}_{\{\Delta(\nu)<p-\tau\}} d\Phi^o(\nu) = \lambda_D \int_{-\infty}^{\nu_l} d\Phi^o(\nu) = \lambda_D \Phi^o(\nu_l).$$

The right-hand side is

$$\lambda_D \int_{\Omega_D^n} \mathbb{1}_{\{\Delta(\nu)>p+\tau\}} d\Phi^n(\nu) = \lambda_D \int_{\nu_h}^{\infty} d\Phi^n(\nu) = \lambda_D \left[\Phi^n(\infty) - \Phi^n(\nu_h)\right].$$

Therefore, we have market clearing if, and only if, $\Phi^o(\nu_l) = \Phi^n(\infty) - \Phi^n(\nu_h)$. This equation holds because, from equation (17), $\Phi^o(\infty) = \frac{\eta}{\eta+\mu} \Longrightarrow \Phi^n(\infty) = 1 - \Phi^o(\infty) = \frac{\mu}{\eta+\mu}$, and, from equation (18), $\frac{\mu}{\eta+\mu} - \Phi^n(\nu_h) = \Phi^o(\nu_l)$.

Equation (12): First, consider $\nu \leq \nu_l$. Then, equation (12) is given by

$$\dot{\Phi}^o(\nu) = \eta\Phi^n(\nu) - \mu\Phi^o(\nu) - \lambda_D\Phi^o(\nu) = \eta F(\nu) - (\eta - \mu - \lambda_D)\Phi^o(\nu).$$

Equation (16) states that $\Phi^o(\nu) = \frac{\eta F(\nu)}{\eta-\mu-\lambda_D}$. Thus, $\dot{\Phi}^o(\nu) = \eta F(\nu) - \eta F(\nu) = 0$. Consider now $\nu \geq \nu_h$. Then, equation (12) is given by

$$\begin{aligned}\dot{\Phi}^o(\nu) &= \eta\Phi^n(\nu) - \mu\Phi^o(\nu) - \lambda_D\Phi^o(\nu_l) + \lambda_D[\Phi^n(\nu) - \Phi^n(\nu_h)] \\ &= (\eta + \lambda_D)F(\nu) - (\eta + \mu + \lambda_D)\Phi^o(\nu) - \lambda_D[\Phi^o(\nu_l) + \Phi^n(\nu_h)].\end{aligned}$$

Using equations (17) and (18) we then have

$$\begin{aligned}\dot{\Phi}^o(\nu) &= (\eta + \lambda_D)F(\nu) + (\eta + \lambda_D)[1 - F(\nu)] - \frac{\eta(\eta + \mu + \lambda_D)}{\eta + \mu} - \frac{\lambda_D\mu}{\eta + \mu} \\ &= \eta + \lambda_D - \frac{\eta(\eta + \mu) + \lambda_D(\eta + \mu)}{\eta + \mu} = \eta + \lambda_D - (\eta + \lambda_D) = 0.\end{aligned}$$

Finally, let us consider $\nu \in (\nu_l, \nu_h)$. In this case we have

$$\begin{aligned}\dot{\Phi}^o(\nu) &= \eta\Phi^n(\nu) - \mu\Phi^o(\nu) - \lambda_D\Phi^o(\nu_l) - \lambda_C[\Phi^o(\nu) - \Phi^o(\nu_l)][\Phi^n(\nu_h) - \Phi^n(\nu)] \\ &= \eta[\Phi^n(\nu) - \Phi^n(\nu_l)] - \mu[\Phi^o(\nu) - \Phi^0(\nu_l)] + \eta\Phi^n(\nu_l) - \mu\Phi^o(\nu_l) - \lambda_D\Phi^o(\nu_l) \\ &\quad - \lambda_C[\Phi^o(\nu) - \Phi^o(\nu_l)][F(\nu_h) - F(\nu)] + \lambda_C[\Phi^o(\nu) - \Phi^o(\nu_l)][\Phi^o(\nu_h) - \Phi^o(\nu)].\end{aligned}$$

We have shown that $\eta\Phi^n(\nu_l) - \mu\Phi^o(\nu_l) - \lambda_D\Phi^o(\nu_l) = 0$ when considering the case $\nu \leq \nu_l$. By using this result and the notation $\tilde{\Phi}^o(\nu) = \Phi^o(\nu) - \Phi^0(\nu_l)$ and $s_C = \tilde{\Phi}^o(\nu_h)$, we obtain

$$\begin{aligned}
\dot{\tilde{\Phi}}^o(\nu) &= \eta[F(\nu) - F(\nu_l)] - (\eta + \mu)\tilde{\Phi}^o(\nu) \\
&\quad - \lambda_C\tilde{\Phi}^o(\nu)[F(\nu_h) - F(\nu)] + \lambda_C\tilde{\Phi}^o(\nu)\tilde{\Phi}^o(\nu_h) - \lambda_C\tilde{\Phi}^o(\nu)^2 \\
&= \eta[F(\nu) - F(\nu_l)] - (\eta + \mu)\tilde{\Phi}^o(\nu) \\
&\quad - \tilde{\Phi}^o(\nu)\left\{\eta + \mu + \lambda_C[F(\nu_h) - F(\nu) - s_C]\right\} - \lambda_C\tilde{\Phi}^o(\nu)^2.
\end{aligned}$$

The distribution $\tilde{\Phi}^o(\nu)$, as defined in equation (23), is the positive root of the equation above. Therefore, $\dot{\tilde{\Phi}}^o(\nu) = 0$.

Equation (13)**:** This is directly stated in equations (16), (17) and (23).

We showed that the family $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ is an equilibrium. That is, that it satisfies equations (3)–(13). It is easy to see that it must also be a regular equilibrium because equation (26) implies that $\nu_l \leq \nu_h$ with strict inequality if $\tau > 0$, and equations (14) and (21) imply that Δ is continuous and strictly increasing. ■

Proof of Proposition 2

Proof. First note that equation (26) is necessarily satisfied by all regular equilibrium. Therefore, it suffices to show that in a neighborhood of $\tau = 0$, there is a unique pair (ν_l, ν_h) satisfying equation (26) for all τ in this neighborhood. Equation (26) can be rewritten as

$$G(\nu_l) = \frac{1}{2\lambda_D\theta_D}\int_{\nu_l}^{g(\nu_l)}\left[\frac{\sigma_C(\nu;\nu_l,g(\nu_l))}{\sigma_D} - 1\right]d\nu = \tau.$$

When $\tau = 0$, then $G(\nu_l) = \tau$ implies that $\nu_l = g(\nu_l) = \nu_h = \nu^*$. That is because $\frac{\sigma_C(\nu;\nu_l,g(\nu_l))}{\sigma_D} - 1$ is bounded away from zero. To see this notice that

$$\begin{aligned}
&\frac{\sigma_C(\nu;\nu_l,g(\nu_l))}{\sigma_D} - 1 > 0 \\
\Leftrightarrow\ &\frac{r + \mu + \eta + \lambda_D\theta_D}{r + \mu + \eta + \lambda_C\Big\{\theta_C^o\left[\Phi^n(\nu_h) - \Phi^n(\nu)\right] + \theta_C^n\left[\Phi^o(\nu) - \Phi^o(\nu_l)\right]\Big\}} > 1 \\
\Leftrightarrow\ &\lambda_D\theta_D > \lambda_C\Big\{\theta_C^o\left[\Phi^n(\nu_h) - \Phi^n(\nu)\right] + \theta_C^n\left[\Phi^o(\nu) - \Phi^o(\nu_l)\right]\Big\}.
\end{aligned}$$

But note that $\lambda_C\Big\{\theta_C^o[\Phi^n(\nu_h)-\Phi^n(\nu)]+\theta_C^n[\Phi^o(\nu)-\Phi^o(\nu_l)]\Big\} < \lambda_C \max\{\theta_C^o,\theta_C^n\} < \lambda_D\theta_D$, which implies that $\frac{\sigma_C(\nu;\nu_l,g(\nu_l))}{\sigma_D}-1$ is bounded away from zero. As a result, we can only have $G(\nu_l)=0$ if the limits in the integral are the same. Then, since $G(\cdot)$ is continuous, it suffices to show that it is strictly monotone in a neighborhood $(\bar{\nu}_l,\nu^*]$. Note also that $G(\cdot)$ is differentiable and that

$$\begin{aligned}G'(\nu_l) &= \frac{1}{2\lambda_D\theta_D}\left\{g'(\nu_l)\left[\frac{\sigma_C(g(\nu_l);\nu_l,g(\nu_l))}{\sigma_D}-1\right]-\left[\frac{\sigma_C(\nu_l;\nu_l,g(\nu_l))}{\sigma_D}-1\right]\right\}\\ &+\frac{1}{2\lambda_D\theta_D}\int_{\nu_l}^{g(\nu_l)}\frac{1}{\sigma_D}\left[\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_l}+g'(\nu_l)\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_h}\right]d\nu.\end{aligned}$$

The first term on the right-hand is negative since $g'(\nu_l)=-\frac{\eta f(\nu_l)}{\mu f(g(\nu_l))}$, and $\frac{\sigma_C(\nu;\nu_l,g(\nu_l))}{\sigma_D}-1$ is bounded away from zero. Moreover, using the definition of σ_C, we can bound it above by

$$-\frac{1}{2\lambda_D\theta_D}\left[\frac{r+\eta+\mu+\lambda_D\theta_D}{r+\eta+\mu+\lambda_C\max\{\theta_D^o,\theta_D^n\}}-1\right].$$

Therefore, to establish that $G'(\nu_l)<0$ in a neighborhood of $(\bar{\nu}_l,\nu^*]$ we just have to show that the second term converges to zero when $\nu_l\nearrow\nu^*$. Since $g(\nu_l)\to\nu^*$ when $\nu_l\nearrow\nu^*$, it suffices to show that the terms inside the integral, $\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_l}$ and $\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_h}$, are bounded. We can write the first term as below.

$$\begin{aligned}\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_l} &= -\sigma_C(\nu;\nu_l,g(\nu_l))^2\lambda_C\frac{\partial\Big\{\theta_C^o[\Phi^n(\nu_h)-\Phi^n(\nu)]+\theta_C^n[\Phi^o(\nu)-\Phi^o(\nu_l)]\Big\}}{\partial\nu_l}\\ &= -\sigma_C(\nu;\nu_l,g(\nu_l))^2\lambda_C\frac{\partial\Big\{\theta_C^o[\Phi^n(\nu_h)-\Phi^n(\nu_l)+\Phi^n(\nu_l)-\Phi^n(\nu)]+\theta_C^n\tilde{\Phi}^o(\nu)\Big\}}{\partial\nu_l}\\ &= -\sigma_C(\nu;\nu_l,g(\nu_l))^2\lambda_C\frac{\partial\Big\{\theta_C^o\frac{\mu[F(\nu_h)-F(\nu_l)]}{\eta+\mu}-\theta_C^o[F(\nu)-F(\nu_l)]+\tilde{\Phi}^o(\nu)\Big\}}{\partial\nu_l}\\ &= -\sigma_C(\nu;\nu_l,g(\nu_l))^2\lambda_C\left\{\theta_C^o\frac{\eta f(\nu_l)}{\eta+\mu}+\frac{\partial\tilde{\Phi}^o(\nu)}{\partial\nu_l}\right\}.\end{aligned}$$

The first term in parenthesis is bounded. The second term is obtained by applying the implicit function theorem to equation (56) and it yields

$$\frac{\partial\tilde{\Phi}^o(\nu)}{\partial\nu_l}=-\frac{\eta f(\nu_l)\left[1+\frac{\lambda_C\tilde{\Phi}^o(\nu)}{\eta+\mu}\right]}{\Big\{\mu+\eta+\lambda_C[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)]\Big\}+2\lambda_C\tilde{\Phi}^o(\nu)},$$

which is also bounded. Similarly for $\frac{\partial \sigma_C(\nu;\nu_l,g(\nu_l))}{\partial \nu_h}$,

$$\frac{\partial \sigma_C(\nu;\nu_l,g(\nu_l))}{\partial \nu_h} = -\sigma_C(\nu;\nu_l,g(\nu_l))^2\lambda_C \frac{\partial\left\{\theta_C^o\frac{\mu[F(\nu_h)-F(\nu_l)]}{\eta+\mu} - \theta_C^o[F(\nu)-F(\nu_l)] + \tilde{\Phi}^o(\nu)\right\}}{\partial \nu_h}$$
$$= -\sigma_C(\nu;\nu_l,g(\nu_l))^2\lambda_C \left\{\theta_C^o\frac{\mu f(\nu_h)}{\eta+\mu} + \frac{\partial\tilde{\Phi}^o(\nu)}{\partial \nu_h}\right\}.$$

Again, the first term in parenthesis is bounded. The second term is

$$\frac{\partial\tilde{\Phi}^o(\nu)}{\partial \nu_h} = -\frac{\mu f(\nu_h)\frac{\lambda_C\tilde{\Phi}^o(\nu)}{\eta+\mu}}{\left\{\mu+\eta+\lambda_C\big[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\big]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)},$$

which is also bounded. Therefore, $G(\cdot)$ is strictly monotone in a neighborhood $(\bar{\nu}_l,\nu^*]$ with $G'(\nu)<0$ in this neighborhood. If we then define the neighborhood $[0,\bar{\tau})$, where $\bar{\tau}=G(\bar{\nu}_l)$, we can conclude that there is a unique regular equilibrium for any $\tau\in[0,\bar{\tau})$. ■

Proof of Proposition 3

Proof. We know that there exists neighborhood $[0,\bar{\tau})$ of $\tau=0$ that regular equilibrium is unique. Moreover, because $G(\nu^*)=0$, we must have $G'(\nu_l)\leq 0$ for any $\nu_l<\nu^*$ with an unique equilibrium in a neighborhood around it. To see this note that if $G'(\nu_l)>0$ for an ν_l with $G(\nu_l)=\tau$, then there exists $\nu_l'>\nu_l$ such that $G(\nu_l')>G(\nu_l)$. But then $G(\nu_l')>G(\nu_l)\geq G(0)$ and we can conclude by continuity that there must be another ν_l'' with $G(\nu_l'')=\tau$. This is a contradiction since we started assuming that there is an unique regular equilibrium at ν_l.

Consider then any (τ^0,τ^1) and (ν_l^0,ν_l^1) such that all $\tau\in(\tau^0,\tau^1)$ is associated with a unique regular equilibrium at some $\nu_l(\tau)\in(\nu_l^0,\nu_l^1)$. Since these regular equilibrium are characterized by $G(\nu_l)=\tau$ and $G'(\nu_l)\leq 0$, we must then have that $\nu_l(\tau)$ is decreasing in τ for all $\tau\in(\tau^0,\tau^1)$. Therefore, to obtain that the turnover is decreasing in τ in the neighborhood (τ^0,τ^1), it suffices to show that it is increasing in ν_l.

From equation (30) we have that

$$T = \frac{2\lambda_D\Phi^o(\nu_l)+\lambda_C\int_{\nu_l}^{\nu_h}\int_{\nu}^{\nu_h}d\Phi^n(\tilde{\nu})d\Phi^o(\nu)}{\frac{\eta}{\eta+\mu}} = \frac{2\lambda_D\Phi^o(\nu_l)+\lambda_C\int_{\nu_l}^{\nu_h}\tilde{\Phi}^o(\nu)d\tilde{\Phi}^n(\nu)}{\frac{\eta}{\eta+\mu}}.$$

From equation (16) we have $\Phi^o(\nu_l)=\frac{\eta F(\nu_l)}{\eta+\mu+\lambda_D}$, which is increasing in ν_l with its derivative given by $\frac{\partial\Phi^o(\nu_l)}{\partial\nu_l} = \frac{\eta f(\nu_l)}{\eta+\mu+\lambda_D}$. For the second term we have that

$$\begin{aligned}\frac{\partial\int_{\nu_l}^{\nu_h}\tilde{\Phi}^o(\nu)d\tilde{\Phi}^n(\nu)}{\partial\nu_l} &= -\cancel{\tilde{\Phi}^o(\nu_l)\phi^n(\nu_l)} + g'(\nu_l)\tilde{\Phi}^o(\nu_h)\phi^n(\nu_h)\\ &+\int_{\nu_l}^{\nu_h}\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}\phi^n(\nu) + \frac{d\phi^n(\nu)}{d\nu_l}\tilde{\Phi}^o(\nu)d\nu.\end{aligned}$$

Again for the first term inside the integral

$$\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l} = \frac{\partial\tilde{\Phi}^o(\nu)}{\partial\nu_l} + g'(\nu_l)\frac{\partial\tilde{\Phi}^o(\nu)}{\partial\nu_h} = \frac{-\eta f(\nu_l)}{\Big\{\mu+\eta+\lambda_C\big[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\big]\Big\}+2\lambda_C\tilde{\Phi}^o(\nu)}.$$

Now we can apply the implicit function theorem to equation (56) and obtain that

$$\phi^o(\nu) = \frac{[\eta+\lambda_C\tilde{\Phi}^o(\nu)]f(\nu)}{\Big\{\mu+\eta+\lambda_C\big[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\big]\Big\}+2\lambda_C\tilde{\Phi}^o(\nu)}.$$

Thus,

$$\begin{aligned}\frac{d\phi^o(\nu)}{d\nu_l} &= -\frac{2\lambda_C\eta f(\nu)\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}}{\Big[\Big\{\mu+\eta+\lambda_C\big[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\big]\Big\}+2\lambda_C\tilde{\Phi}^o(\nu)\Big]^2}\\ &+\lambda_C f(\nu)\frac{\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}\Big[\Big\{\mu+\eta+\lambda_C\big[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\big]\Big\}+2\lambda_C\tilde{\Phi}^o(\nu)\Big]}{\Big[\Big\{\mu+\eta+\lambda_C\big[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\big]\Big\}+2\lambda_C\tilde{\Phi}^o(\nu)\Big]^2}\\ &-\lambda_C f(\nu)\frac{2\lambda_C\tilde{\Phi}^o(\nu)\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}}{\Big[\Big\{\mu+\eta+\lambda_C\big[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\big]\Big\}+2\lambda_C\tilde{\Phi}^o(\nu)\Big]^2}\\ &= \frac{\lambda_C f(\nu)\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}-2\lambda_C\phi^o(\nu)\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}}{\Big\{\mu+\eta+\lambda_C\big[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\big]\Big\}+2\lambda_C\tilde{\Phi}^o(\nu)} = \frac{\lambda_C[\phi^n(\nu)-\phi^o(\nu)]\frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}}{\Big\{\mu+\eta+\lambda_C\big[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\big]\Big\}+2\lambda_C\tilde{\Phi}^o(\nu)}.\end{aligned}$$

Also note that from the above we obtain $\phi^n(\nu)$ and $\frac{d\phi^n(\nu)}{d\nu_l}$ from the identity $f(\nu) = \phi^o(\nu) + \phi^n(\nu)$. Then we obtain that

$$\begin{aligned}\phi^n(\nu) &= f(\nu) - \frac{[\eta+\lambda_C\tilde{\Phi}^o(\nu)]f(\nu)}{\Big\{\mu+\eta+\lambda_C\big[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\big]\Big\}+2\lambda_C\tilde{\Phi}^o(\nu)}\\ &= \frac{\Big[\{\mu+\lambda_C\big[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\big]\}+\lambda_C\tilde{\Phi}^o(\nu)\Big]f(\nu)}{\Big\{\mu+\eta+\lambda_C\big[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\big]\Big\}+2\lambda_C\tilde{\Phi}^o(\nu)}.\end{aligned}$$

Now we can write that

$$
\begin{aligned}
\frac{\partial \int_{\nu_l}^{\nu_h} \tilde{\Phi}^o(\nu) d\tilde{\Phi}^n(\nu)}{\partial \nu_l} &= g'(\nu_l)\tilde{\Phi}^o(\nu_h)\phi^n(\nu_h) + \int_{\nu_l}^{\nu_h} \frac{d\tilde{\Phi}^o(\nu)}{d\nu_l}\phi^n(\nu) + \frac{d\phi^n(\nu)}{d\nu_l}\tilde{\Phi}^o(\nu) d\nu \\
&= -\tilde{\Phi}^o(\nu_h)\frac{\eta f(\nu_l)}{\mu+\eta+\lambda_C\tilde{\Phi}^o(\nu_h)} \\
&\quad - \eta f(\nu_l)\int_{\nu_l}^{\nu_h} \frac{\phi^n(\nu) + \frac{\lambda_C[\phi^o(\nu)-\phi^n(\nu)]\tilde{\Phi}^o(\nu)}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)} d\nu.
\end{aligned}
$$

With this equation in hand we now can show that $\frac{dT}{d\nu_l} \geq 0$, which is equivalent to show that

$$
\frac{2\lambda_D \eta f(\nu_l)}{\eta+\mu+\lambda_D} \geq
$$

$$
\frac{\lambda_C\tilde{\Phi}^o(\nu_h)\eta f(\nu_l)}{\mu+\eta+\lambda_C\tilde{\Phi}^o(\nu_h)} + \lambda_C\eta f(\nu_l)\int_{\nu_l}^{\nu_h} \frac{\phi^n(\nu) + \frac{\lambda_C[\phi^o(\nu)-\phi^n(\nu)]\tilde{\Phi}^o(\nu)}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)} d\nu,
$$

which happens if, and only if,

$$
\frac{2\lambda_D}{\eta+\mu+\lambda_D} \geq
$$

$$
\frac{\lambda_C\tilde{\Phi}^o(\nu_h)}{\mu+\eta+\lambda_C\tilde{\Phi}^o(\nu_h)} + \lambda_C\int_{\nu_l}^{\nu_h} \frac{\phi^n(\nu) + \frac{\lambda_C[\phi^o(\nu)-\phi^n(\nu)]\tilde{\Phi}^o(\nu)}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)} d\nu.
$$

Note that

$$
\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu) = \mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^o(\nu)+\tilde{\Phi}^o(\nu)\right].
$$

Then we have that

$$\begin{aligned}
&\int_{\nu_l}^{\nu_h} \frac{\phi^n(\nu)+\frac{\lambda_C[\phi^o(\nu)-\phi^n(\nu)]\tilde{\Phi}^o(\nu)}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]}}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^o(\nu)+\tilde{\Phi}^o(\nu)\right]} d\nu \\
&= \int_{\nu_l}^{\nu_h} \frac{f(\nu)}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]} d\nu \\
&- \int_{\nu_l}^{\nu_h} \frac{\phi^o(\nu)\left\{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]\right\}-\lambda_C[\phi^o(\nu)-\phi^n(\nu)]\tilde{\Phi}^o(\nu)}{\left\{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]\right\}^2} d\nu \\
&= \int_{\nu_l}^{\nu_h} \frac{f(\nu)}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]} d\nu - \int_{\nu_l}^{\nu_h} \frac{d}{d\nu}\left[\frac{\tilde{\Phi}^o(\nu)}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]}\right] d\nu \\
&= \int_{\nu_l}^{\nu_h} \frac{f(\nu)}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]} d\nu - \frac{\tilde{\Phi}^o(\nu_h)}{\mu+\eta+\lambda_C\tilde{\Phi}^o(\nu_h)}.
\end{aligned}$$

So again it suffices to show that

$$\begin{gathered}
\frac{2\lambda_D}{\eta+\mu+\lambda_D} \geq \\
\frac{\lambda_C\tilde{\Phi}^o(\nu_h)}{\mu+\eta+\lambda_C\tilde{\Phi}^o(\nu_h)} + \lambda_C \int_{\nu_l}^{\nu_h} \frac{\phi^n(\nu)+\frac{\lambda_C[\phi^o(\nu)-\phi^n(\nu)]\tilde{\Phi}^o(\nu)}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)} d\nu = \\
\frac{\lambda_C\tilde{\Phi}^o(\nu_h)}{\mu+\eta+\lambda_C\tilde{\Phi}^o(\nu_h)} + \lambda_C \int_{\nu_l}^{\nu_h} \frac{f(\nu)}{\mu+\eta+\lambda_C[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)]} d\nu - \frac{\lambda_C\tilde{\Phi}^o(\nu_h)}{\mu+\eta+\lambda_C\tilde{\Phi}^o(\nu_h)} \\
\iff \frac{2\lambda_D}{\eta+\mu+\lambda_D} \geq \int_{\nu_l}^{\nu_h} \frac{\lambda_C f(\nu)}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]} d\nu \\
\iff \int_{\nu_l}^{\nu_h} \left[\frac{2\lambda_D}{\eta+\mu+\lambda_D} - \frac{\lambda_C[F(\nu_h)-F(\nu_l)]}{\mu+\eta+\lambda_C[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)]}\right] dF(\nu) \geq 0.
\end{gathered}$$

First note that the above inequality holds in a neighborhood of $\tau = 0$ because it implies that $\nu_l \approx \nu_h$. Moreover, if $\lambda_C \leq \lambda_D \leq \frac{\sqrt{2}-1}{\sqrt{2}}(\eta+\mu)$, we must have that

$$\frac{2\lambda_D}{\eta+\mu+\lambda_D} \geq \frac{\lambda_C[F(\nu_h)-F(\nu_l)]}{\mu+\eta+\lambda_C[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)]}$$

holds. We can see this by comparing the two functions $\frac{2x}{\eta+\mu+x}$ and $\frac{ax}{\eta+\mu+bx}$, where $a = F(\nu_h) - F(\nu_l)$ and $b = \tilde{\Phi}^n(\nu_h) - \tilde{\Phi}^n(\nu) + \tilde{\Phi}^o(\nu)$. Both functions are strictly increasing in x, and equal zero at $x = 0$. Moreover, the derivative of the first function is strictly greater than the derivative of the second one for $x \leq \frac{\sqrt{2}-1}{\sqrt{2}}(\eta+\mu)$, which implies that

$$\frac{2\lambda_D}{\eta+\mu+\lambda_D} > \frac{\lambda_D[F(\nu_h)-F(\nu_l)]}{\mu+\eta+\lambda_D\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]} \geq \frac{\lambda_C[F(\nu_h)-F(\nu_l)]}{\mu+\eta+\lambda_C\left[\tilde{\Phi}^n(\nu_h)-\tilde{\Phi}^n(\nu)+\tilde{\Phi}^o(\nu)\right]}.$$

This concludes the proof. ■

Proof of Proposition 4

Proof. The bid-ask spread, defined in equation (37), is

$$\text{BA} = \frac{2\lambda_D\Phi^o(\nu_l)\tau + \lambda_D(1-\theta_D)\frac{\int_{\nu_h}^{\infty}\nu d\Phi^n(\nu)-\int_{-\infty}^{\nu_l}\nu d\Phi^o(\nu)}{r+\eta+\mu+\lambda_D\theta_D}}{\lambda_D\Phi^o(\nu_l)+\lambda_C\int_{\nu_l}^{\nu_h}\int_{\nu_l}^{\nu}d\Phi^n(\tilde{\nu})d\Phi^o(\nu)}.$$

To see the first part note that for $\tau = 0$ we know that $\nu_l = \nu_h = \nu^*$ and the equilibrium is unique. So we can take the derivative of BA in equation (37) with respect to τ evaluated at $\tau = 0$ and show that it has to be strictly positive. Given that all the functions are differentiable, we must have this derivative strictly positive in a neighborhood of $\tau = 0$.

We have that

$$\frac{dBA}{d\tau} = \frac{\partial BA}{\partial\tau} + \frac{\partial BA}{\partial\nu_l} \times \frac{\partial\nu_l}{\partial\tau}.$$

For the first term in the right-hand side we have

$$\frac{\partial BA}{\partial\tau} = \frac{2\lambda_D\Phi^o(\nu_l)}{\lambda_D\Phi^o(\nu_l)+\lambda_C\int_{\nu_l}^{\nu_h}\int_{\nu_l}^{\nu}d\Phi^n(\tilde{\nu})d\Phi^o(\nu)},$$

which implies that $\frac{\partial BA}{\partial\tau} = 2$ when evaluated at $\tau = 0$ since in this case $\nu_l = \nu_h = \nu^*$. For the second term we have that

$$\begin{aligned}\left.\frac{\partial BA}{\partial\nu_l}\right|_{\substack{\tau=0,\\ \nu_l=\nu^*}} &= \lambda_D(1-\theta_D)\frac{\frac{d}{d\nu_l}\left[\frac{\int_{\nu_h}^{\infty}\nu d\Phi^n(\nu)-\int_{-\infty}^{\nu_l}\nu d\Phi^o(\nu)}{r+\eta+\mu+\lambda_D\theta_D}\right]\lambda_D\Phi^o(\nu_l)}{\left[\lambda_D\Phi^o(\nu_l)+\lambda_C\int_{\nu_l}^{\nu_h}\int_{\nu_l}^{\nu}d\Phi^n(\tilde{\nu})d\Phi^o(\nu)\right]^2} \\ &\quad - \lambda_D(1-\theta_D)\frac{\frac{\int_{\nu_h}^{\infty}\nu d\Phi^n(\nu)-\int_{-\infty}^{\nu_l}\nu d\Phi^o(\nu)}{r+\eta+\mu+\lambda_D\theta_D}\frac{d}{d\nu_l}\left[\lambda_D\Phi^o(\nu_l)+\lambda_C\int_{\nu_l}^{\nu_h}\int_{\nu_l}^{\nu}d\Phi^n(\tilde{\nu})d\Phi^o(\nu)\right]}{\left[\lambda_D\Phi^o(\nu_l)+\lambda_C\int_{\nu_l}^{\nu_h}\int_{\nu_l}^{\nu}d\Phi^n(\tilde{\nu})d\Phi^o(\nu)\right]^2}.\end{aligned}$$

Note that

$$\frac{d}{d\nu_l}\left[\int_{\nu_h}^{\infty}\nu d\Phi^n(\nu)-\int_{-\infty}^{\nu_l}\nu d\Phi^o(\nu)\right]=\frac{d}{d\nu_l}\left[\int_{\nu_h}^{\infty}\frac{\nu\mu f(\nu)}{\eta+\mu+\lambda_D}d\nu-\int_{-\infty}^{\nu_l}\frac{\nu\eta f(\nu)}{\eta+\mu+\lambda_D}d\nu\right]$$
$$=\frac{-\nu_h\mu f(\nu_h)g'(\nu_l)-\nu_l\eta f(\nu_l)}{\eta+\mu+\lambda_D}=\frac{[\nu_h-\nu_l]\eta f(\nu_l)}{\eta+\mu+\lambda_D},$$

which is zero when evaluated at $\nu_l=\nu_h=\nu^*$. Moreover,

$$\frac{d}{d\nu_l}\left[\lambda_D\Phi^o(\nu_l)+\lambda_C\int_{\nu_l}^{\nu_h}\int_{\nu_l}^{\nu}d\Phi^n(\tilde{\nu})d\Phi^o(\nu)\right]$$

is positive in a neighborhood of $\tau=0$ since this is basically turnover, which we showed in the previous proof that is increasing in ν_l in a neighborhood of $\tau=0$. Thus, $\left.\frac{\partial BA}{\partial\nu_l}\right|_{\substack{\tau=0,\\ \nu_l=\nu^*}}\leq 0$. We also have shown that $\frac{\partial\nu_l}{\partial\tau}<0$ in a neighborhood of $\tau=0$. Therefore, we have that

$$\left.\frac{dBA}{d\tau}\right|_{\substack{\tau=0,\\ \nu_l=\nu^*}}=\underbrace{\left.\frac{\partial BA}{\partial\tau}\right|_{\substack{\tau=0,\\ \nu_l=\nu^*}}}=2+\underbrace{\left.\frac{\partial BA}{\partial\nu_l}\right|_{\substack{\tau=0,\\ \nu_l=\nu^*}}}_{\leq 0}\times\underbrace{\left.\frac{\partial\nu_l}{\partial\tau}\right|_{\substack{\tau=0,\\ \nu_l=\nu^*}}}_{\leq 0}>0.$$

This proves the first part of the proposition. Namely, that the bid-ask spread is increasing in τ in a neighborhood of $\tau=0$.

In order to show the second part of the proposition, it suffices to show that BA converges to zero as τ converges to infinity. First lets us show that ν_l converges to $-\infty$ when τ converges to infinity.

In equilibrium we must have that

$$G(\nu_l)=\frac{1}{2\lambda_D\theta_D}\int_{\nu_l}^{g(\nu_l)}\frac{\sigma_C(\nu;\nu_l,\nu_h)-\sigma_D}{\sigma_D}d\nu=\tau.$$

The term $\frac{\sigma_C(\nu;\nu_l,\nu_h)-\sigma_D}{\sigma_D}$ is bounded below by $\frac{\lambda_D\theta_D-\lambda_C\max\{\theta_C^n,\theta_C^o\}}{\eta+\mu+\lambda_D\theta_D}$, and above by $\frac{\lambda_D\theta_D}{\eta+\mu+\lambda_D\theta_D}$. Therefore, as τ converges to infinity, in order to obtain an equilibrium we must have ν_l converging to $-\infty$, and $\nu_h=g(\nu_l)$ converging to ∞.

Consider now the formula for the bid-ask spread,

$$\text{BA} = \frac{2\lambda_D \Phi^o(\nu_l)\tau + \lambda_D(1-\theta_D)\frac{\int_{\nu_h}^{\infty} \nu d\Phi^n(\nu) - \int_{-\infty}^{\nu_l} \nu d\Phi^o(\nu)}{r+\eta+\mu+\lambda_D\theta_D}}{\lambda_D \Phi^o(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu_l}^{\nu} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)}.$$

We can see that $\int_{\nu_h}^{\infty} \nu d\Phi^n(\nu) - \int_{-\infty}^{\nu_l} \nu d\Phi^o(\nu)$ converge to zero since $\int_{-\infty}^{\infty} \nu^2 f(\nu) d\nu$ is bounded. To show that $\Phi^o(\nu_l)\tau = \frac{\eta F(\nu_l)\tau}{\eta+\mu+\lambda_D}$ converges to zero is not as simple because $F(\nu_l)$ converges to zero and τ converges to infinity. But note that

$$\lim_{\tau \nearrow \infty} F(\nu_l)\tau = \lim_{\tau \nearrow \infty} \frac{\tau}{1/F(\nu_l)} = \lim_{\tau \nearrow \infty} \frac{1}{f(\nu_l)\frac{d\nu_l}{d\tau}/F(\nu_l)^2} = \lim_{\nu_l \searrow -\infty} \frac{G'(\nu_l)F(\nu_l)^2}{f(\nu_l)}.$$

And, as we have established in the of Proposition 2,

$$\begin{aligned} G'(\nu_l) = & \frac{1}{2\lambda_D\theta_D}\left\{ g'(\nu_l)\left[\frac{\sigma_C(g(\nu_l);\nu_l,g(\nu_l))}{\sigma_D} - 1\right] - \left[\frac{\sigma_C(\nu_l;\nu_l,g(\nu_l))}{\sigma_D} - 1\right]\right\} \\ & + \frac{1}{2\lambda_D\theta_D}\int_{\nu_l}^{g(\nu_l)} \frac{1}{\sigma_D}\left[\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_l} + g'(\nu_l)\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_h}\right] d\nu. \end{aligned}$$

So we only need to show that each of the terms above, when multiplied by $\frac{F(\nu_l)^2}{f(\nu_l)}$, converges to zero as ν_l converges to $-\infty$. Let us start showing that $\frac{F(\nu_l)^2}{f(\nu_l)}$ converges to zero. Note that

$$0 \le \lim_{\nu_l \searrow -\infty} -\nu_l F(\nu_l) = \lim_{\nu_l \searrow -\infty} \int_{-\infty}^{\nu_l} |\nu_l| f(\nu) d\nu \le \lim_{\nu_l \searrow -\infty} \int_{-\infty}^{\nu_l} |\nu| f(\nu) d\nu = 0,$$

where the equality in the end comes from the fact that $\int \nu^2 f(\nu) d\nu$ is finite. Therefore we can conclude that $\lim_{\nu_l \searrow -\infty} \nu_l F(\nu_l) = 0$. But then

$$0 = \lim_{\nu_l \searrow -\infty} -\nu_l F(\nu_l) = \lim_{\nu_l \searrow -\infty} \frac{-\nu_l}{1/F(\nu_l)} \overset{\text{L'Hôpital}}{=} \lim_{\nu_l \searrow -\infty} \frac{-1}{-f(\nu_l)/F(\nu_l)^2} = \lim_{\nu_l \searrow -\infty} \frac{F(\nu_l)^2}{f(\nu_l)}.$$

Now let us look the individual terms of $G'(\nu_l)$. We have that

$$\lim_{\nu_l \searrow -\infty} \frac{F(\nu_l)^2}{f(\nu_l)}\left[\frac{\sigma_C(\nu_l;\nu_l,g(\nu_l))}{\sigma_D} - 1\right] = 0$$

because $\left[\frac{\sigma_C(\nu_l;\nu_l,g(\nu_l))}{\sigma_D}-1\right]$ is bounded. We have that

$$\begin{aligned}&\lim_{\nu_l\searrow-\infty}\frac{F(\nu_l)^2}{f(\nu_l)}g'(\nu_l)\left[\frac{\sigma_C(g(\nu_l);\nu_l,g(\nu_l))}{\sigma_D}-1\right]\\&=\lim_{\nu_h\nearrow-\infty}\frac{[1-F(\nu_h)]^2}{f(\nu_h)}\left[\frac{\sigma_C(\nu_h;g^{-1}(\nu_h),\nu_h)}{\sigma_D}-1\right]=0\end{aligned}$$

because again $\left[\frac{\sigma_C(\nu_h;g^{-1}(\nu_h),\nu_h)}{\sigma_D}-1\right]$ is bounded and we can show that $\lim_{\nu_h\nearrow-\infty}\frac{[1-F(\nu_h)]^2}{f(\nu_h)}=0$ in the same fashion that we showed that $\lim_{\nu_l\searrow-\infty}\frac{F(\nu_l)^2}{f(\nu_l)}=0$. Moreover,

$$\begin{aligned}0&\le\int_{\nu_l}^{g(\nu_l)}\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_l}d\nu=\int_{\nu_l}^{g(\nu_l)}\sigma_C(\nu;\nu_l,g(\nu_l))^2\lambda_C\left\{\theta_C^o\frac{\eta f(\nu_l)}{\eta+\mu}-\frac{\partial\tilde{\Phi}^o(\nu)}{\partial\nu_l}\right\}d\nu\\&=\int_{\nu_l}^{g(\nu_l)}\left\{\theta_C^o\frac{\lambda_C\eta\sigma_C(\nu;\nu_l,g(\nu_l))^2}{\eta+\mu}+\frac{\lambda_C\eta\sigma_C(\nu;\nu_l,g(\nu_l))^2\left[1+\frac{\lambda_C\tilde{\Phi}^o(\nu)}{\eta+\mu}\right]}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}\right\}f(\nu_l)d\nu\\&\le\int_{\nu_l}^{g(\nu_l)}\left\{\theta_C^o\frac{\lambda_C\eta\sigma_C(\nu;\nu_l,g(\nu_l))^2}{\eta+\mu}+\frac{\lambda_C\eta\sigma_C(\nu;\nu_l,g(\nu_l))^2\left[1+\frac{\lambda_C\tilde{\Phi}^o(\nu)}{\eta+\mu}\right]}{\left\{\mu+\eta+\lambda_C\left[F(\nu_h)-F(\nu)-\tilde{\Phi}^o(\nu_h)\right]\right\}+2\lambda_C\tilde{\Phi}^o(\nu)}\right\}f(\nu)d\nu\end{aligned}$$

which is bounded because the term in brakets is bounded and $\int_{\nu_l}^{g(\nu_l)}f(\nu)d\nu\le 1$. Therefore, we have that

$$\lim_{\nu_l\searrow-\infty}\frac{F(\nu_l)^2}{f(\nu_l)}\int_{\nu_l}^{g(\nu_l)}\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_l}d\nu=0.$$

The proof that

$$\lim_{\nu_l\searrow-\infty}\frac{F(\nu_l)^2}{f(\nu_l)}\int_{\nu_l}^{g(\nu_l)}g'(\nu_l)\frac{\partial\sigma_C(\nu;\nu_l,g(\nu_l))}{\partial\nu_h}d\nu=0.$$

is analogous. With that, we can conclude that $\lim_{\tau\nearrow\infty}BA=0$, which implies the second part of Proposition 4, and concludes the proof. ■

Proof of Proposition 5

Proof. First not that, due to the assumed symmetry in the parameters, we have that $p^A(\tau)=$

$\sigma_D \bar{\nu}^A$ and $p^B(\tau + d\tau) = \sigma_D \bar{\nu}^A$. Therefore, the liquidity premium can be written as

$$LP(\tau) = \frac{\bar{\nu}^A - \bar{\nu}^B}{d\bar{L} + \mathrm{BA}(\tau + d\tau) - \mathrm{BA}(\tau)}.$$

We have shown in the proof of Proposition 4 that $\frac{dBA}{d\tau} > 0$ in a neighborhood of $\tau = 0$, which implies that $\mathrm{BA}(\tau + d\tau) - \mathrm{BA}(\tau) > 0$ and $LP(\tau) < \frac{\bar{\nu}^A - \bar{\nu}^B}{d\bar{L}}$.

We have also show that $\lim_{\tau \nearrow \infty} BA = 0$. Since we know that $BA(\tau) > 0$ in a neighborhood of $\tau = 0$, this implies that $\frac{dBA}{d\tau} < 0$ in for some τ, which implies that $\mathrm{BA}(\tau + d\tau) - \mathrm{BA}(\tau) < 0$ and $LP(\tau) > \frac{\bar{\nu}^A - \bar{\nu}^B}{d\bar{L}}$. Moreover, because

$$\lim_{\tau \nearrow \infty} BA(\tau + d\tau) - BA(\tau) = \lim_{\tau \nearrow \infty} BA(\tau + d\tau) - \lim_{\tau \nearrow \infty} BA(\tau) = 0,$$

we have that $\lim_{\tau \nearrow \infty} LP(\tau) = \frac{\bar{\nu}^A - \bar{\nu}^B}{d\bar{L}}$. This concludes te proof. ■

B Data

We use corporate bonds transactions data from the TRACE Enhanced (ETRACE) database from January 2005 to June 2021. This initial data set provides us with a total of 171,140,493 trades as well as with 283,250 uniquely-identifiable bonds.[10]

We use a procedure based in Dick-Nielsen (2009) and Dick-Nielsen (2014) to filter out errors, cancellations, reversals and double counting as well as transactions missing individual CUSIP identification. We subsequently drop trades missing yield information and trades that are either on a when-issued basis, in a non-secondary Market, with a special condition, automatic give-ups, or in equity-linked notes.[11]

[10] The Trade Reporting and Compliance Engine (TRACE) is the "FINRA-developed vehicle that facilitates the mandatory reporting of over-the-counter secondary market transactions in eligible fixed income securities." The bond transactions report was implemented in different phases. It started with Phase I, on July 2002, for investment grade bonds and with issue size greater than or equal to $1 bi, and it continued later with the requirements expanded in Phase II in 2003. The complete implementation occurred in 2005, with Phase III. The report of corporate bond transactions is mandatory for all broker-dealers FINRA members. Therefore, Phase III virtually contains complete coverage of all public transactions. For consistency of the selection into the dataset, our dataset focus on Phase III. The Enhanced TRACE differs from the Standard TRACE in that it discloses more detailed information in individual transactions, e.g., actual trade size.

[11] To remove any potentially erroneous trades still remaining in the database, we also add a price filter for trades with prices deviating more than 25% from the daily average. This procedure cleans only about 0.1% of the trades.

To avoid having many bonds in our sample that trade only momentarily, we add the following two conditions: (1) the bond must have existed in ETRACE for at least one complete year; and (2) the bond must have traded at least 75% of its relevant trading days (BPW and Anderson and Stulz 2017). Bonds must also have sufficient trades to satisfy the conditions, as defined in the following section, necessary to calculate their individual illiquidity measures. Having applied all these trade-based criteria, we are left with 55,753,160 transactions in 5,410 unique issues.

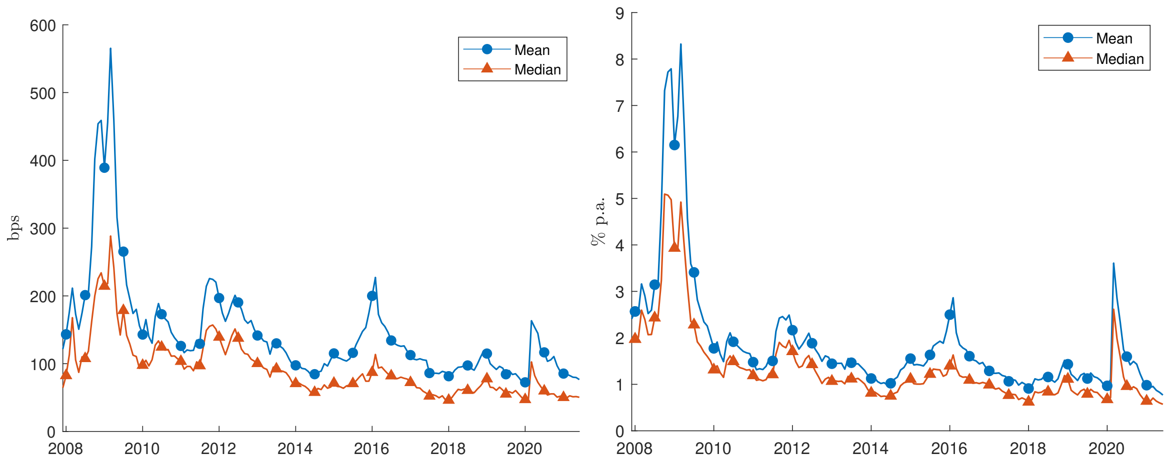
We use Bloomberg to collect bond information on issuance and maturity dates, provisions, coupons, currency denomination, amount outstanding, and ratings. We use the amount outstanding of each issue at the last business day of each month. A bond is defined as investment grade if its rating is greater than or equal to BBB– from S&P and Fitch or Baa3 from Moody's. We first use the rating from Standard & Poor's; if this rating is unavailable, we use the rating from Fitch; and if this rating is unavailable, we use the rating from Moody's.[12]

We exclude trades that took place outside the range of issuance and maturity dates of an issue, and bonds for which the outstanding amount at the last business day of that specific month was zero. Defaulting bonds are eliminated from the sample for as long as they are considered in default, and so are bonds with missing information. We only keep in our sample callable or non-provisional, fixed-rate bonds issued in the US. Callable bonds comprise a significant portion of our sample. Removing these bonds would negatively impact the quality of our results. Instead, we control our results for callability by introducing a dummy to our model. At this stage, our sample consists of 45,026,565 trades in 4,255 individual bonds.

We calculate the individual yield spread as the difference between the yield of the corporate bond and the yield of the government bond with the same maturity, as in BPW. The constant maturity yield curve is obtained from the Federal Reserve Bank of St. Louis FRED dataset. We use linear interpolation to calculate the yield of the government bond matching the exact maturity of the corporate bond. The monthly cross-sectional yield spread of a corporate bond is then calculated as the average daily spread in the month.

We use the Eikon dataset to collect each issuer's daily 5-year Credit Default Swap (CDS) quotes, which we use to proxy for the issuer's credit risk. Our measure of credit risk for each

[12]Although we use a different order based on data availability, this process is similar to Dick-Nielsen et al. (2012).



(a) CDS (b) Yield spreads

Figure 9: 5-Year Credit Default Swaps and yield spreads for individual corporate bond issues in the sample.

monthly cross-section is the average of the issuer's end-of-day CDS spreads. As this data is sufficiently large for the bonds in our database from December 2007, we redefine our sample period to begin in December 2007. We use stock prices to calculate the annualized equity return volatility of each issuer. Bonds missing CDS and equity volatility data are excluded from our dataset. We collect the daily stock prices of the issuers from CRSP.

Our final bond sample consists of 32,435,392 trades in 3,073 unique issues, which are distributed over a period of 115 months starting from December 2007. In total, we have 139,168 combinations of bond-month observations. The number of observations varies between monthly cross-sections depending on, among other things, newly-issued and matured bonds, trade frequency, and issues satisfying our selection criteria in the observed cross-section. Our final sample is predominantly composed by investment grade bonds.

We separate our sample period in three time intervals characterized by different macroeconomic conditions: (1) the financial crisis period from December 2007 to December 2009; (2) the post-crisis period with historically low interest rates from January 2010 to November 2015; and (3) the monetary tightening period from December 2015 to June 2017. Figure 9 shows the CDS and the yield spreads over time of the bonds in the sample. Table 3 presents a summary of our data together with the illiquidity measures described in the next section.

Table 3: Summary statistics

	Complete Period Dec '07–June '17			Crisis Dec '07–Dec '09			Post-Crisis Jan '10–Nov '15			Monetary Tightening Dec '15–June '17		
Observations	139,168			17,165			91,424			30,579		
Investment Grade	87%			86%			87%			87%		
N. of Bonds	3,073			1,134			2,688			1,905		
Callable	33%			33%			33%			31%		
N. of Firms	416			227			388			310		
N. of Trades	32,435,392			6,078,875			20,192,250			6,164,267		
	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD
γ	2.225	1.044	3.931	5.776	2.623	11.648	1.284	.639	1.842	1.067	.480	1.586
AMD ($\times 10^3$)	2.933	1.755	3.945	6.497	3.789	9.603	2.003	1.255	2.386	1.717	.947	2.325
Spread	2.163	1.543	2.179	4.160	2.856	4.634	1.598	1.202	1.387	1.646	1.090	1.909
CDS ($\times 10^{-2}$)	1.674	1.063	2.008	2.712	1.529	3.837	1.385	.972	1.375	1.390	.789	1.969

This table reports a summary of our sample variables together with a summary of the main variables calculated. The observations are the bond-month combinations. The mean, median and standard deviation are the time-series averages of the respective cross-sectional measures within each sub-period. Spread is the corporate bond yield spread detailed in section B. γ and AMD are the illiquidity measures detailed in section B.

C Additional empirical results

C.1 The determinants of bond illiquidity

Given the importance of bond-level illiquidity for the yield spread, we now study the determinants of illiquidity for a particular bond. We regress our illiquidity measures on a list of varying characteristics such as CDS, time to maturity, age, volume, frequency and issuer's equity return volatility and credit rating. We also make use the intrinsic characteristics of a bond such as coupon paid, issuance size and whether the bond is callable or not. Our model consists in regressing pooled OLS estimations with two-dimensional clustered standard errors. Our results have strong statistical and economical significance as we discuss next. Results are reported in table 4.

We find that the credit risk of the issuer and the time to maturity are the most important characteristics of a particular bond to explain variations in illiquidity of an issue. Illiquidity is positively correlated to both credit risk and time to maturity.

When the CDS of an issuer widens by 100 basis points, we find that the γ's of its bonds increases by .901. This increase corresponds to about 40% of the average γ reported in table 3. Similarly, we find that one additional year in time to maturity increases γ by .165, which corresponds to 7% of the average γ.

Table 4: γ and AMD on bond characteristics

	γ			AMD		
CDS	.817		.901	.697		.719
	[6.75]		[6.50]	[7.89]		[6.93]
Maturity		.161	.165		.128	.139
		[11.02]	[11.84]		[7.87]	[9.29]
Age			$-.002$			.117
			[$-.12$]			[5.42]
Coupon			.078			.079
			[2.14]			[2.17]
Volume			$-.750$			-2.01
			[-3.78]			[-7.32]
Frequency			.510			3.24
			[2.15]			[12.82]
ln(Issuance Size)			$-.787$			-1.17
			[-8.31]			[-8.59]
EqVol.			.481			.308
			[1.38]			[1.70]
IG			2.18			2.19
			[5.05]			[6.78]
Call			$-.318$			$-.053$
			[-3.08]			[$-.50$]
Constant	.526	.666	2.27	1.39	1.57	5.01
	[4.26]	[6.15]	[3.28]	[10.17]	[13.06]	[5.94]
Adj.R^2	.089	.032	.142	.111	.035	.226
Obs.	139,168	139,168	139,168	139,168	139,168	139,168

Bond-level illiquidity measures regressed on bond characteristics. We run a pooled OLS regression with standard-errors clustered by bond and month. T-statistics in square brackets. γ and AMD are the illiquidity measures detailed in section B. AMD is multiplied by 10^3. Maturity is the issue's time to maturity. Maturity and age are calculated in years at the last business-day of each month. Volume is calculated as the total \$ amount traded $\times 10^{-11}$ and frequency is in thousands of trades. Issuance size is in \$ millions. EqVol. is the issuer's annualized equity return volatility. IG is 1 if the bond is Investment Grade and 0 if otherwise. Call is 1 if the bond is callable and 0 if otherwise.

The variables representing trading activity, that is, volume and frequency, show contrasting results. Bonds with greater volume are more liquid, but so are bonds that trade less frequently. These results support the view that bonds with larger average size per trade are potentially more liquid. Callable bonds show as more liquid for both γ and AMD than issues with no embedded options.

The results for the credit rating and coupon are surprising. We find that investment-grade bonds have substantially higher γ's. This contrasts with the time-series measures observed in figure 2, which shows, instead, that investment-grade bonds have smaller γ's. Once CDS and credit quality are considered simultaneously, as in table 4, we obtain that the effect of credit quality on γ captured more strongly by the CDS than by the credit rating. This effect occurs because high-yield bonds have, on average, significantly higher CDS spreads than investment-grade issues. The coupon paid also presents a non-intuitive positive slope, which implies that bonds that pay higher coupons are less liquid.

The results for γ and AMD point in general to the same direction. An exception is the coefficient for age, negative, not statistically significant, for γ, but positive and strongly significant for AMD. A positive coefficient for age indicates a market preference for on-the-run securities as oppose to older, off-the-run issues from the same firm. This finding is consistent with the on-the-run and off-the-run spread (see, for example, Krishnamurthy 2002).

In summary, our results indicate that the main determinants of illiquidity of a particular bond are credit risk and time to maturity. We address in the next section whether corporate bond liquidity can be affected by aggregate factors, common to all bonds.

C.2 Commonality in liquidity

We now investigate which factors affect aggregate liquidity of corporate bonds. We analyze factors such as financial stability, economic conditions, the term structure, and market volatility. To avoid having our results driven by few issues with exceptionally high illiquidity, we focus on market illiquidity as measured by the median γ and AMD of individual bonds.[13]

We proxy financial conditions by the NFCI Index, for which historical data is retrieved from the Chicago Fed database.[14] The volatility measures for the equities market and the

[13]BPW also consider the median as aggregate market liquidity for similar reasons.

[14]The National Financial Conditions Index (NFCI) captures "financial conditions in money markets, debt

benchmark 10-year Treasury interest rate are respectively given by the Cboe's VIX and TYVIX indices. We retrieve historical data for both indices from FRED.[15] Additionally, we retrieve from FRED historical the difference between the 10-year and 2-year constant maturity Treasury rates as a measure of the slope of the yield curve. The corporate debt net position data of Primary Dealers is retrieved from the NY Fed and monthly medians are used as observations. We use the sum of individual bonds total volume traded and number of transactions as an indication of market volume and frequency. We regress monthly changes in aggregate market illiquidity on monthly changes in financial, economic and market measures.

When financial conditions tighten (NFCI increases), we find that corporate bond market liquidity decreases. Aggregate financial conditions are closely related with corporate bond market liquidity. Table 5 reports the results across different market illiquidity measures.

About changes in the term structure of interest rates. We find that a higher slope of the yield curve is associated with an increase in corporate bond market illiquidity. When the yield curve steepens, corporate bond liquidity lowers.

Another important element contributing to corporate bond market illiquidity is the balance sheet of Primary Dealers. The net position of Primary Dealers in corporate debt instruments is positively correlated to market illiquidity. The results are similar for γ and AMD. This result supports the evidence that the post-crisis liquidity provision in corporate debt instruments has been shifting from traditional Primary Dealers to other market players.

Volatility in equity markets yields a positive slope, as expected. However, its statistical significance weakens in the presence of control variables. We find that illiquidity increases with the VIX. Alternatively, the volatility of interest rates, proxied by the benchmark 10-year Treasury note, has a negative slope. When TYVIX is regressed alone, the coefficient is positive but with a small t-statistics of .13 and a nearly null adjusted-R^2.

We observe a relation between trading activity measures and illiquidity at the aggregate level similar to the observed at the individual bond level. Illiquidity decreases with volume and increases with frequency. Although presenting a negative sign, volume, however, is not statistically significant for γ as it is for AMD.

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and equity markets and the traditional and 'shadow' banking systems."

[15]In FRED, the data for the Cboe's 10-Year US Treasury Note Volatility Index, TYVIX, is referred to by its legacy ticker, VXTYN.

Table 5: Monthly changes in aggregate market illiquidity on monthly changes in macro variables

	$\Delta\gamma$									Δ AMD								
Aggr. by	Median			Mean			Vol. Weight			Median			Mean			Vol. Weight		
ΔNFCI	2.14		1.73	5.48		4.82	3.30		2.57	1.49		1.73	2.99		2.69	2.07		1.69
	[4.32]		[8.10]	[6.11]		[6.97]	[1.92]		[1.59]	[7.98]		[8.62]	[7.20]		[6.78]	[8.56]		[5.38]
ΔVIX		.051	.012		.113	.008		.062	.004		.031	.012		.062	.006		.045	.009
		[2.51]	[1.97]		[2.16]	[.62]		[1.44]	[.15]		[2.45]	[1.91]		[2.32]	[1.01]		[2.15]	[.93]
ΔYCurve		1.00	.622		1.00	.468		.451	.578		.420	.622		.463	.206		.576	.463
		[1.95]	[3.74]		[1.08]	[.85]		[.63]	[.77]		[2.24]	[4.72]		[.94]	[.66]		[2.92]	[2.58]
ΔPrimaryDealers			.015			.012			−.022			.015			.006			0
			[2.49]			[1.57]			[−1.08]			[2.61]			[1.18]			[.00]
ΔTYVIX			−.091			−.043			.420			−.091			−.154			−.025
			[−4.83]			[−.28]			[1.16]			[−4.82]			[−3.81]			[−.45]
ΔVolume			−.190			−.626			.374			−.190			−.838			−.480
			[−1.71]			[−1.19]			[.44]			[−1.64]			[−2.37]			[−2.84]
ΔFrequency			.434			1.04			−.143			.434			.970			.517
			[2.41]			[1.65]			[−.21]			[2.28]			[2.56]			[2.24]
Constant	.012	−.003	−.031	.036	−.005	.037	.019	−.006	−.019	.004	−.007	.031	.008	−.015	−.002	.002	−.013	−.009
	[.66]	[−.09]	[1.98]	[.91]	[−.07]	[.98]	[.25]	[−.08]	[−.22]	[.24]	[−.22]	[2.17]	[.33]	[−.25]	[−.08]	[.18]	[−.42]	[−.47]
Adj.R^2	.401	.394	.734	.471	.236	.540	.111	.034	.182	.471	.283	.678	.478	.234	.732	.422	.265	.534

Results of changes in aggregate market illiquidity on changes in macro variables. The market illiquidity can be aggregated by the median, mean or volume-weighted average of individual illiquidity measures within each of the 115 monthly cross-sections covered in our sample. Therefore, we have 114 observations of monthly changes in aggregate market liquidity. Coefficients are estimated using OLS and standard errors are corrected by Newey-West. T-statistics are reported in square brackets. γ and AMD are the illiquidity measures detailed in section B. AMD is multiplied by 10^3. NFCI is the Chicago Fed's National Financial Conditions Index. VIX and TYVIX are respectively the Cboe's volatility indices for the equities market and the benchmark 10-year Treasury Note. YCurve is the yield curve slope measured as the spread between the 10-year and the 2-year constant maturity yields. PrimaryDealers is the monthly median of corporate debt net positions in billions of U.S. dollars of Primary Dealers reported by the NY Fed. Volume is the total \$ amount traded in a month $\times 10^{-13}$ and Frequency the total number of trades in a month $\times 10^{-5}$.