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Minority Inflation, Unemployment, and Monetary Policy

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Abstract

Our paper addresses the heterogeneous effects of monetary policy on households of different races. The cyclical volatility of real income differs significantly for households of different races and income levels, reflecting differential exposure to fluctuations in employment and consumer prices. All Black households are disproportionately affected by employment fluctuations, whereas price volatility is only particularly pronounced for Black households with income above the national median. The latter face 40 percent higher price volatility than both poorer households of the same race and white households of similar income. To evaluate the effects of policy, we propose a New Keynesian framework with heterogeneous exposure to employment and price volatility. We find that an accommodative monetary stance generates asymmetric outcomes within race groups. Low-income households experience unemployment stabilization benefits, while high-income ones incur real income volatility costs. Differences are especially large among Black households. Reducing the volatility of unemployment by 1 percentage point engenders a 1.17 percentage point reduction in overall income volatility for poorer Black households, but an increase of 0.6 percentage points in income volatility for richer Black households.

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1 Introduction

Disparities in economic well-being between Black and white Americans have proven exceptionally persistent.[1] Several decades after the passing of the Civil Rights legislation, Black Americans remain more likely to be unemployed than white Americans, are less likely to access unemployment insurance, their earnings are on average lower, and so are their wealth accumulation rates, home-ownership rates, and access to credit.[2] A significant portion of these gaps remains unexplained after accounting for observable factors such as age, education, or income.[3] In this paper, we further document racialized disparities in households' exposure to business cycles and inflationary shocks and how such differences translate into heterogeneous effects of monetary policy.

A robust policy debate on how to best address economic disparities between Black and white Americans is ongoing. To the extent that these gaps extend to a variety of labor market variables, the discussion has given rise to calls for monetary policy to take racialized differences into account.[4] Our paper contributes directly to this debate by measuring the differential impacts of alternative monetary policy strategies on different racial groups.

We start with a conceptual insight. While the potential for monetary policy to affect long-run unemployment rates is open for debate, workhorse models imply that it can play a significant role in stabilizing unemployment over the business cycle. Similarly, while real wages may not depend on monetary policy over the long run, they might be sensitive to fluctuations in inflation at higher frequencies. Accordingly, we investigate whether monetary policy is an effective means of reducing racial disparities in real income *volatility* and show how volatility relates to households' well-being.

[1]Here and throughout the text, we follow the New York Times editorial board standard in capitalizing "Black" and not "white". As they explain, "[w]hite does not represent a shared culture and history in the way Black does, and also has long been capitalized by hate groups." We also follow sociological and medical literature in distinguishing between racial and racialized differences. To quote from the American Heart Association (Taylor, 2015), "racial differences are understood to be clinical, biological, genetic, or epigenetic factors associated with disease risk, outcome, or treatments. Racialized disparities are caused by social factors that vary in prevalence in population groups."

[2]See Daly, Hobijn and Pedtke (2020); Skandalis, Marinescu and Massenkoff (2022); Hurst, Rubinstein and Shimizu (2024); Derenoncourt, Kim, Kuhn and Schularick (2024).

[3]As noted in Cajner, Radler, Ratner and Vidangos (2017), and further documented in Section 3.

[4]In particular, these calls include explicit legislation to this effect proposed in Congress, the Federal Reserve Racial and Economic Equity Act (2022). This reads, "The Board of Governors and the FOMC shall exercise all duties and functions in a manner that fosters the elimination of disparities across racial and ethnic groups with respect to employment, income, wealth, and access to affordable credit." Furthermore, the Statement on Longer-Run Goals and Monetary Policy Strategy adopted by the Federal Reserve in 2024, states that it pursues "maximum employment (as) a *broad-based and inclusive* goal" (Statement on Longer-Run Goals and Monetary Policy Strategy, 2024, italics added)

Next, we provide empirical context. We do this by investigating the components of real income volatility plausibly affected by monetary policy. First, we offer a comprehensive review of racialized differences in unemployment rates. While the level differences are well-known, one less-appreciated fact is that (conditional on the levels) the cyclical properties of Black and white unemployment are similar. Similar unemployment cyclicality, together with a higher unemployment level, implies that Black families are overall more exposed to fluctuations in aggregate employment.

Then, we document that the average price volatility facing Black families is higher than for white families. This difference is entirely accounted for by above-median-income Black families, who face 40 percent higher price volatility than both poorer households of the same race and white households of similar income levels. Higher price volatility for richer Black households' consumption bundle implies that these families are more exposed to fluctuations in aggregate prices than any other group.

When we investigate why high-income Black families face disproportionately more volatile prices than their white counterparts, we find that transportation and visible goods play an important role. Such consumption patterns are consistent with the findings of a well-developed literature on conspicuous consumption and an acceleration in Black suburbanization (Charles, Hurst and Roussanov, 2009; Bartik and Mast, 2021). The signaling value of visible consumption and longer commutes plausibly account for richer Black families' specific consumption patterns, which feature disproportionate expenditures on vehicles, vehicle-related goods, and gasoline. This unique pattern of consumption exposes them to higher price volatility than both poorer Black families and white families of similar income. We see this evidence as a primary reason in favor of explicitly distinguishing between income and race when evaluating the heterogeneous effects of monetary policy.

After documenting empirical differences in real income volatility (either because of fluctuations in employment or prices), we quantify the idea that monetary policy has a role in addressing them. In particular, we ask whether a more accommodative monetary policy — one that tolerates larger volatility in inflation in exchange for greater stability in unemployment — can disproportionately benefit Black Americans.

We propose a model in which Black and white households have different incomes and consume different consumption bundles accordingly, with richer Black households' consumption more concentrated in goods with more flexible prices (such as fuel and vehicles). At the same time, the wages of all households are nominally sticky, but, as in the data, aggregate demand for labor of Black households fluctuates more strongly than for white ones. The model thus captures the greater exposure of Black households to *both* inflationary shocks and aggregate unemployment fluctuations.

A trade-off arises for the monetary authority, which can be expressed as a negative relationship between changes in unemployment and changes in prices (i.e., as Phillips curves). As a result, any policy stabilizing unemployment would benefit households with high unemployment rates, but potentially harm those with consumption baskets concentrated in flexibly priced goods. Given similar degrees of wage flexibility, and the fact that Black households are, on average, more exposed to *both* unemployment and price volatility, the relative gains of more accommodative monetary policy for Black households are qualitatively ambiguous.

Within the context of our model, we derive a set of sufficient statistics to assess the trade-offs in real income volatility. We show that, for any given household, the relative gains from accommodative monetary policy, in terms of stability of income, are (i) positively related to the household's long-run unemployment rate; and, (ii) inversely related to the difference between the slope of price and wage Phillips Curves faced by that specific household.

The sufficient statistics we propose allow us to adjudicate the net gains from accommodative policy. Quantifying the relevant trade-offs for different racial and income groups requires measures of each group's long-run unemployment rate and group-specific wage and price Phillips Curves. We provide, therefore, race- and income-specific estimates of those Phillips Curve coefficients which are, to the best of our knowledge, new to the literature. We obtain these estimates through a novel and intuitive empirical strategy, which exploits the persistent aggregate demand shortfall in the aftermath of the Great Recession as an instrumental variable.

With the requisite statistics at hand, we examine the trade-off between unemployment and price stabilization through a racialized lens. We find that accommodative monetary policy is particularly beneficial for low-income Black households, since those have very high unemployment rates. It is also beneficial for low-income white households, though to a lesser extent. The policy is, instead, detrimental to high-income households of any race. This is true even though high-income Black households have almost twice the unemployment rates as white households of similar income, because they also face significantly higher price volatility than these households. Indeed, the relative benefits they receive from unemployment stabilization is more than offset by their higher sensitivity to price fluctuations. Differences between poorer and richer households are especially pronounced among Black households. Reducing the volatility of unemployment by 1 percentage point engenders a 1.17 percentage point *reduction* in overall real income volatility for poorer Black households, but an *increase* of 0.6 percentage points in real income volatility for richer Black households.

The break-down by income reveals what is hidden by a purely race-based lens. Pooling households by race implies virtually no benefit (nor cost) from stabilizing unemployment for white households — but a clear relative gain to Black ones. A pure racial lens, therefore, under-emphasizes both the benefits of greater unemployment stabilization to low-income white households, and the costs to high-income Black households.

Contribution to the literature Our paper primarily contributes to an emerging literature assessing how minority or historically marginalized groups are affected by a variety of monetary policy decisions. This literature includes insightful recent work by Bartscher, Schularick, Kuhn and Wachtel (2022) and Nakajima (2023), who study racial inequality in income and wealth empirically and quantitatively, respectively.[5] Other recent papers, such as Cairó and Lipton (2023) and Lahcen, Baughman and van Buggenum (2023), study the racial unemployment gap, while Kuhn, Schularick and Steins (2020), Derenoncourt, Kim, Kuhn and Schularick (2023) and Derenoncourt et al. (2024) highlight the role of the wealth gap. Our work emphasizes heterogeneity in real income volatility, instead, and investigates the contribution of both unemployment and price volatility differentials to fluctuations in real income.[6]

We also contribute to a growing literature on the measurement of inflation inequality. Several studies estimate inflation inequality by calculating group-specific consumption expenditures from the Consumer Expenditure Survey (Hobijn and Lagakos, 2005; Jaravel, 2019; Cravino, Lan and Levchenko, 2020; Del Canto, Grigsby, Qian and Walsh, 2023). Recent studies use the Nielsen data, that covers consumer packaged goods, and exploit group-specific consumption expenditures and prices paid (Kaplan and Schulhofer-Wohl, 2017; Argente and Lee, 2021). All these papers study inflation inequality across *income* groups. Our paper is one of the first papers focusing on inflation inequality across *race* groups. To this point, our paper emphasizes the need to study racial inequality in inflation *separately* from income inequality. Indeed, we find that racialized differences in the volatility of prices are, in fact, significantly more pronounced among high-income households of different races than across race groups

[5] A parallel strand of the literature focuses more generally on how concerns for inequality may inform monetary policy decisions, including Acharya, Challe and Dogra (2023), Bilbiie, Monacelli and Perotti (2024), and McKay and Wolf (2023) among others. Bergman, Born, Matsa and Weber (2024) emphasize heterogeneous effects of monetary policy on workers with different levels of labor force attachment, but abstract from race *per se*.

[6] Ganong, Jones, Noel, Greig, Farrell and Wheat (2020) show that "income volatility has a substantial welfare cost for all groups, and further that the cost is substantially larger for Black and Hispanic households than it is for White households." Our work complements their findings as we offer direct evidence of racialized differences in the consumption basket.

overall.

Our work also relates to a vast theoretical and quantitative literature on monetary policy: Christiano, Eichenbaum and Trabandt (2016), Galí, Smets and Wouters (2012), Gertler, Sala and Trigari (2008) and Krause and Lubik (2007) also formally model unemployment within New Keynesian setups. Our model is itself a two-agent new Keynesian model, with heterogeneity in exposure to labor market fluctuations.

Structure of the paper Section 2 introduces our conceptual framework to anchor the empirical analysis. This follows in section 3, where we investigate in detail differences in unemployment (section 3.3) and prices (section 3.4). Section 4 fully develops our model into a quantitative framework, and proposes a set of sufficient statistics to assess the net gains from different policy stances (section 4.5). We calibrate and estimate all the relevant parameters of the model in section 5, and illustrate our policy analysis in section 5.2. Section 6 concludes.

2 Real income

We start by describing how changes in real economic activity and inflation affect the degree to which the income of different groups fluctuates. First, we lay out our definitions for real income and specify the appropriate price indices. Then, we propose an approximation for the volatility of real income, which has intuitive appeal and a clean welfare interpretation under conditions discussed in more detail in section 4.5. The definitions laid out in this section provide the conceptual basis to empirically account for heterogeneity in real income volatility across people. A key source of heterogeneity is that different groups may consume different consumption baskets, leading to differing exposure to inflationary shocks. In our application, those groups will be either race or income (or both), but the framework can be readily extended to other cuts of the data.

2.1 Definitions

We follow the convention adopted by most statistical agencies of constructing price indices by taking observed expenditure shares as weights. We also focus on labor income, since it is the most relevant component of income for a large fraction of the population, including many who have illiquid assets that cannot be readily used to smooth the effects of labor income shocks.[7] The key contribution that we detail here is the construction of price indices that are group-specific.

At a given time t, we define the **real labor income** of a household i in group k to be

$$Y_t^k = \frac{\mathcal{W}_t^k}{P_t^k}(1 - u_t^k), \tag{1}$$

where Y_t^k is *real* income, $\mathcal{W}_t^k$ is *nominal* wage, P_t^k is a price index and u_t^k is the unemployment rate.

Price indices P_t^k reflect prices of the same underlying set of goods but differ across groups because of differing weights. In particular,

$$\frac{P_t^k}{P_{t^*}^k} = \sum_j \alpha_j^k \frac{P_t^j}{P_{t^*}^j} \tag{2}$$

where P_t^j is the price of good j in year t, and $\alpha_j^k = \frac{P_{t^*}^j C_{t^*}^{kj}}{\sum_j P_{t^*}^j C_{t^*}^{kj}}$ is the share of expenditures in a reference year t^*. Weights are group-specific, reflecting different consumption baskets. Note that while the price index is group-specific, prices of individual goods j are not.[8] We also allow for wages and unemployment rates to be group-specific, which may stem from differences in human capital and geography, or be understood as a function of labor market discrimination.

For our analysis, it is worth focusing on the following log-linear approximation to real income:

$$\widehat{Y}_t^k \simeq \widehat{\mathcal{W}}_t^k - \widehat{P}_t^k - \frac{1}{1 - u^k}(u_t^k - u^k), \tag{3}$$

with

$$\widehat{P}_t^k = \sum_j \alpha_j^k \widehat{P}_t^j,$$

where the hat denotes log deviations from average. The approximation becomes better as those deviations approach zero.

[7] This lens complements the one by Bartscher et al. (2022), for instance, who focus on wealth and asset accumulation effects.

[8] Empirically, we verify that this is not a bad approximation after all when applied to price indices by racial groups. See table 2 and related discussion.

2.2 Volatility

Households care about income volatility, as in most circumstances, it will influence the volatility of their consumption. We can approximate the **volatility of real income** of group k by[9]

$$var\left(\frac{Y_1^k}{Y^k}\right) \simeq var(\widehat{Y}_1^k) = var\left(\widehat{\mathcal{W}}_1^k - \widehat{P}_1^k - \frac{1}{1-u^k}(u_1^k - u^k)\right)$$

where var is the variance. It follows immediately from the expression above that the variance of real income depends not only on the variances of unemployment rate, nominal wages, and prices, but also on their covariances. Therefore, measuring group-specific income volatility requires measuring *group-specific* nominal wages, prices, and unemployment rates for each group, a task we undertake in Section 3 below. Monetary policy has an impact on both variances and covariances, so our analysis of its effects on income volatility will take both into consideration (section 4).

3 Racialized volatility gaps

3.1 Data

We construct real labor income measures using a variety of microdata sources. In particular, we use data on wages, unemployment rates, and prices for the years 1998-2019, using the Current Population Survey (CPS), the BLS-CPI, and the Consumer Expenditure Survey (CES). We complement our analysis of unemployment and wages with data from the American Community Survey (ACS), the 2000 Census, and the Occupational Employment and Wages Statistics (OEWS). Throughout, we define "Black" as all individuals in the data who identify as "African-American or Black, alone" and "white" as all CPS respondents who identify as "white alone."[10]

To help isolate the direct role of race from the effects of other socioeconomic characteristics that correlate with (and may be caused by) race, we create what we refer to as a "counterfactual white" population or a "synthetic control." We reweigh white households in our various samples to reproduce basic demographic characteristics of Black households along gender, age, marital status, educational attainment, and occupation.[11] In other words, we build

[9] See proposition 1 in Section 4.5.

[10] Inclusion of respondents who identify as multi-racial does not change our results, but we restrict our definition to single-race individuals, not Hispanic/Latinos, to be consistent with the literature (see, for example, Cajner et al. (2017)).

[11] We use 4 educational categories: less than high school, high school diploma or equivalent, some college

counterfactual average measures of white households' inflation, wages, unemployment, and real income, that would hold if white households had the same educational, age, and gender composition as Black households. The re-weighted white households can be interpreted as a "synthetic control" that matches various characteristics of Black households — but not their race.

3.2 Real income

We document sizeable racialized disparities in the cyclical sensitivity of real labor income, explore its component parts, and show that those disparities cannot be easily traced to differences in observable socio-economic characteristics between Black and white households. Furthermore, absent perfect insurance, those disparities lead to larger fluctuations in disposable resources for Black households, and plausibly in consumption and well-being.[12]

Our measure of real income follows the discussion in section 2.1 and is equal to $Y_t^k = \mathcal{W}_t^k/P_t^k \times (1 - u_t^k)$, where k takes values B or w for Black or white, Y_t^k is real labor income, $\mathcal{W}_t^k/P_t^k$ is the average real wage for individuals in group k, and u_t^k is the unemployment rate for group k. Table 1 shows the standard deviation of real income, as well as of income's components.[13]

On average, we find that real income is more volatile for Black households than for white ones in the period 1998-2019. The first column of table 1 shows that the standard deviation of real income is 35% higher for Black households below the median income than for white households in the same income bracket. The difference is even larger for households whose income is above the national median, with richer Black households facing 96% more volatile real income than white ones of similar income.

For households below median income, both the volatility of the unemployment rate and that of real wages are about 30-35% higher for Black than for white households. For households above the median income, the volatility of the unemployment rate and that of real wages

but no degree, 4-year college and above. We use 5 occupational categories corresponding to 1-digit SOC codes.

[12]In terms of the decomposition of welfare in Dávila and Schaab (2022), the disparities in the cyclicality of welfare imply gains from stabilization policy that go beyond macroeconomic stability to include increased insurance.

[13]We compute hourly wages using information on weekly earnings, weeks worked, and hours worked from the Current Population Survey Annual Social and Economic Supplement (CPS-ASEC) for the years 1998-2019. We exclude part-time workers, so to divide earnings by 40, and winsorize wage values at the top and bottom 0.5%, to avoid influence from outliers. In all calculations, we use log wages residualized on gender, age and age square, education, state of residence, class of worker, and occupation at the SOC 1-digit level (5 categories). We then divide by CPI and obtain real wages.

Table 1: Standard deviations of the components of real labor income

	real income	unemployment	real wage	CPI
(a) Black, low Y	2.24%	1.55%	2.00%	1.41%
(b) White, low Y	1.66%	1.15%	1.53%	1.43%
(c) Black, high Y	2.58%	0.80%	2.64%	2.03%
(d) White, high Y	1.31%	0.48%	1.47%	1.41%
ratio (a/b)	1.35	1.35	1.30	0.98
ratio (c/d)	1.96	1.68	1.81	1.44

Note: all monthly data series are detrended according to Hodrick and Prescott (1997), with the recommended rescaling factor in Ravn and Uhlig (2002). Real income is constructed by authors according to equation 1. Wages are computed for full-time workers, expressed in logs, and residualized on gender, education, age, state, class of worker, and 1-digit SOC occupations. Source: CPS-ASEC and CPI microdata 1998-2019.

are 68% and 81% higher, respectively, for Black than for white households. Prices are about equally as volatile for poorer households of different races. For richer households, instead, there is a significant divide: prices are 44% more volatile for Black households with income above the median, vis-a-vis white ones with similar income.

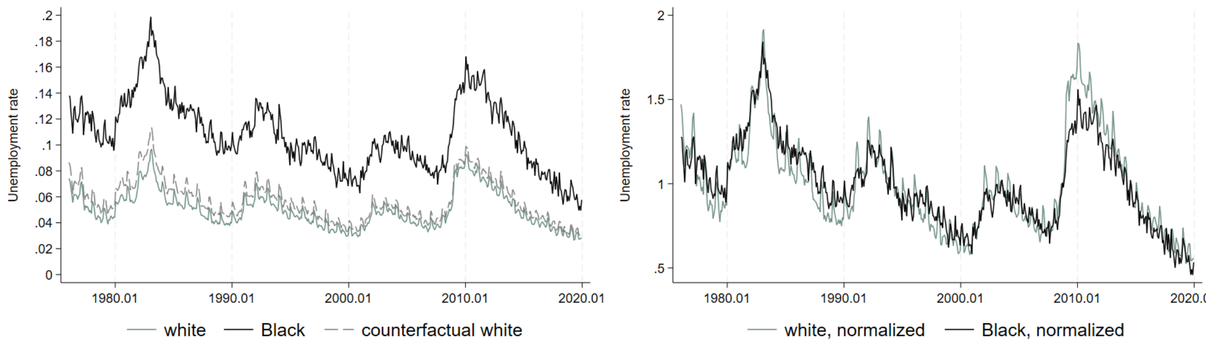
We now discuss unemployment and prices in more detail, providing context for the disparities we found in the data.

3.3 Unemployment

Overall, the unemployment rate for Black prime-age workers is more volatile than for corresponding white workers — this is true for both workers in low-income households and richer ones. The higher volatility of Black unemployment can be traced to its overall higher level. Figure 1 depicts the unemployment rate for Black and white workers for the period 1978-2019 from the Current Population Survey (CPS). From the figure, it is apparent that Black unemployment is close to twice as large as white unemployment throughout the period (as documented, among many others, by Bayer and Charles (2018), Chetty et al. (2020), Derenoncourt and Montialoux (2021)). Furthermore, Black and white unemployment exhibit remarkably similar movements when normalized by their own averages $(u_{k,t}/\bar{u}_k)$, where $\bar{u}_k$ is average unemployment for group k. Since $std(1-u_{k,t}) = std(u_{k,t}) = \bar{u}_k std\left(u_{k,t}/\bar{u}_k\right)$, those facts together imply that the high average unemployment rate for Black households implies higher volatility in their *employment* rate.

The ratio of unemployment rates shown in Figure 1 is remarkably stable over time, and approximately equal to 2, thus suggesting that monetary policy and other cyclical stabilization tools are unlikely to affect it. In our analysis, we therefore take the gap between

Figure 1: The level of unemployment is always higher for Black than white people, but fluctuations over the cycle are similar.



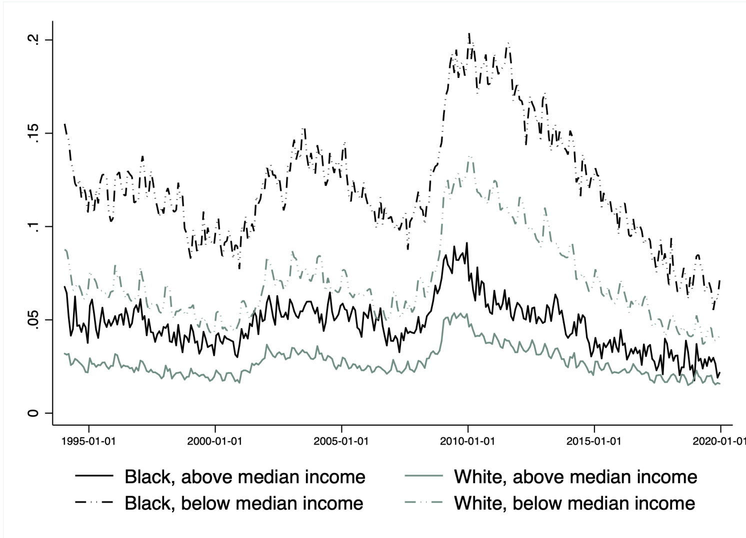
Note: left panel in percentage points, right panel as a ratio the series' own mean. Source: CPS microdata 1976-2019.

unemployment rates as *structural* to monetary policy.

Little of the racialized disparity in unemployment rates is accounted for by observables. The gap in the data is over and above what would be implied by differences in socio-economic variables such as age, gender, marital status, educational attainment, and occupation of employment (Cajner et al., 2017). Figure 1 illustrates this point by depicting three time-series. Black unemployment (solid black line), white unemployment (solid teal line), and the unemployment rate that white workers would have if they had (i) the demographic and occupational composition of *Black* workers, but (ii) the unemployment rate of *white* workers in each specific demographic-occupational group ("counterfactual white", dashed grey line). The counterfactual white unemployment rate is barely distinguishable from the actual unemployment rate of white workers in the data. We take this as further evidence that differences in observable characteristics do not explain the unemployment gap between Black and white workers.

Remarkably, unemployment for Black workers remains about twice as large as that for white workers even when we focus only on households who rarely experience unemployment spells to begin with, that is, those with income above the national median. Figure 2 depicts unemployment rates by race and household income. The unemployment rate for Black workers whose household income is below the national median ("poor") is about twice as large as for workers who are poor but white. Similarly, the unemployment rate for Black workers whose household income is *above* the national median ("rich") is also about twice as large as rich white workers.

Figure 2: Unemployment rate for by race and household income.



Note: the median household income threshold is computed nationwide, year-by-year. Source: CPS-ASEC microdata 1976-2019.

3.4 Inflation

We now show how the incidence of inflation differs across households. We document that the prices of goods and services consumed by Black households are more volatile than those consumed by white households. Furthermore, practically all of the difference is accounted for by disparities between households of different races with income above the national median.

Table 2 presents the duration of prices, the frequency of price changes, and the standard deviation of inflation for Black and white households of any income bracket. To construct the table, we combine three sources of data. The first is the US Consumer Expenditure Survey (CES), from which we obtain expenditure shares across detailed product categories for Black and white households. We use the consumption basket of Black and white households in 2015, which is neither in recession nor boom. The second data source is the measures of price stickiness constructed by Nakamura and Steinsson (2008), who report the frequency of price adjustment for detailed product categories in the US Consumer Price Index (CPI) for 1998-2005. The third source is the item-level consumer price data from the US Bureau of Labor Statistics (BLS) from January 1998 to December 2020. Those are the most finely disaggregated consumer prices publicly available. Combining the first and the second data sources, 472 Universal Classification Code (UCC) categories in the CES are matched to 254 Entry Level Items (ELIs) in Nakamura and Steinsson (2008). Combining the first and the third data sources, 352 UCC categories in the CES are matched to 138 Item Strata in the

Table 2: Black households are more exposed to price volatility, mostly on account of different consumption baskets.

	Black	white	counterfactual white
Duration of all prices (in months)	8.07	8.51	8.45
Duration of regular prices (in months)	12.12	12.59	12.63
Frequency of all price changes (in %)	30.16	28.67	29.13
Frequency of regular price changes (in %)	23.94	22.87	23.37

	Black	white	Δ_{B-w}
Standard deviation of CPI (CES 1998-2020)	2.48%	2.30%	7.8
Standard deviation of CPI (Nielsen 2004-2020)	0.84%	0.74%	13.5
— *different consumption basket*			*9.3*
— *different price changes*			*4.2*

Note: this table reports the weighted mean duration of prices, the weighted mean frequency of price changes, and the standard deviation of the 12-month log change in CPI for Black and white households. Source: CPI and ACS microdata. Nielsen data on a quarter-to-quarter basis from Lee (2022).

BLS data.[14]

The top panel of table 2 reports the mean duration of prices weighted by consumption shares in each category. Rows labeled "all prices" include sales, while those labeled "regular prices changes" exclude sales. The mean duration is 8.07 months for Black households and 8.51 months for white households, which means that the prices of the goods consumed by Black households are less sticky. Duration of prices in a product category can be converted to frequency of price changes.[15] The mean frequency of all price changes is such that 30.16% of prices change in a month for Black households and 28.67% for white households (third row of table 2). Excluding sales, the results are qualitatively similar. The bottom panel of table 2 shows the standard deviation of CPI for Black and white households calculated from January 1998 to December 2019. The standard deviation is 2.48 percent for Black households and 2.30 percent for white households. We conclude that Black households' consumption bundles are tilted towards goods that have more frequent price adjustments and, accordingly, that they face price volatility that is 8 percent higher than the one faced by white households.

[14] We use the most recent concordance: `www.bls.gov/cpi/additional-resources/ce-cpi-concordance.htm`

[15] A constant hazard of λ of price change implies a monthly probability of a price change equal to $f = 1 - e^{-\lambda}$. This implies $\lambda = -\ln(1-f)$ and $d = 1/\lambda = -1/\ln(1-f)$.

How much of the racialized difference in price volatility is accounted for by different goods and services, and how much by different frequencies of price changes within group categories? To answer this question, Lee (2022) uses "a longitudinal panel of about 60,000 U.S. households that continually provide information about what products they buy and when and where they make purchases".[16] The data spans the period 2004-2020. During these years, the standard deviation of CPI was 0.84 for Black households and 0.74 for White households. In other words, Black households faced 13.5 percent higher inflation volatility, a value comparable to the 8 percent difference this paper documents from CES. The Nielsen panel, however, also allows researchers to decompose differences in total volatility across races into two components: one that originates in race-specific consumption baskets and one that instead takes into account race-specific price changes. Most of the different inflation volatility between Black and white households is accounted for by different consumption baskets (69 percent). Different price changes also contribute to the racialized gap in price volatility, but their contribution is quantitatively smaller (31 percent).

3.4.1 Income differences v. racialized differences

The findings described in the previous section are consistent with Cravino et al. (2020). They find that the prices of the goods and services consumed by high-income households are stickier and less volatile than those consumed by middle-income households. Part of our finding, therefore, reflects overall income differences between Black and white households.[17] We next show more explicitly that, in addition, there is a distinct racialized component in consumption patterns, thus in the volatility of consumption prices.

We compare households of different races in the same income group; below or above median income ($62.6k). We find that racialized differences in the volatility of prices are entirely accounted for by differences between Black and white households who have income above the median (table 3). In particular, while the volatility of prices is very similar for poor families, regardless of their race, for higher-income families, Black ones face 3.45 percent volatility v. 2.43 for white families (a two-fifths increase).

We then inspect consumption patterns more closely (table 4) between high-income Black and white households. Among households whose income is above the national median, a large part of the differences in price volatility is accounted for by the fact that Black families

[16] In this data, race is self-reported and about 10 percent of households identify as Black, with over 80 percent reporting "white" instead.

[17] Indeed, when reweighing consumption baskets of white by Black's socio-economic characteristics such as gender, age, marital status, and education, we find that the gap between Black and white CPI volatility is somewhat smaller, as shown in the last column of table 2.

Table 3: Volatility of prices for lower-income households is similar across race groups. On the other hand, richer Black households experience higher volatility than poorer Black households and white households of similar income.

	Below median income		Above median income	
	Black	white	Black	white
Standard deviation of CPI	2.44	2.48	3.45	2.43
% HHs in income bracket	*65*	*45*	*35*	*55*

Note: The table reports the mean duration of prices, weighted by expenditure shares, and the standard deviation of the 12-month log change in CPI for Black and white households in the same income group. Median income is 62.6k per household. Source: CPI-CES microdata 1998-2019.

spend disproportionately more than white families on vehicle-related goods and services. These include gasoline, motor vehicle maintenance and servicing, vehicle leasing, and motor vehicle insurance. What are the reasons behind these disparities? Prior literature suggests that two race-specific factors are at play.

Visible consumption is one. Charles et al. (2009) document that Black and Hispanic families allocate a disproportionate share of their income to goods signaling their socio-economic status, such as clothing, jewelry, and cars. While race-specific preferences may play a role, Charles et al. (2009) emphasize visible consumption as a way to overcome statistical discrimination and stereotypes that conflate being Black with being poor. Specific Black consumers may want to signal they don't fit in this statistical simplification, perhaps to receive better service while shopping or on the road, for instance. These consumers do so via the consumption of goods typically associated with high income — one of them is larger, more luxurious cars (which consume larger amounts of gasoline and whose parts and repairs are more costly than smaller cars).

The legacy of a long history of housing discrimination also plausibly contributes to expenditure differences. Fu, Rolheiser and Severen (2024) document, for instance, how Black workers commute longer times to and from work, partially as a result of geographical segregation (nowadays *de facto*, but *de jure* in the past). Racialized differences in commuting persist when zooming in on higher-income workers who commute primarily by car (as opposed to public transport). Bartik and Mast (2021) also document the accelerating suburbanization of Black households, especially those with higher incomes. As richer Black households move out from city centers, their commute lengthens disproportionately since they tend to remain in larger cities. Therefore, their consumption of vehicle-related goods also increases disproportionately. The recent and accelerating suburbanization of (richer) Black households may, therefore, contribute to these households' disproportionate expenditure on gasoline and related goods.

Table 4: Prices change more frequently for goods on which high-income Black households spend more relative to high-income white households.

Category	share Black	share white	share difference (Black-white)	duration all prices	duration regular prices
Top 10, larger expenditure shares by high-income Black households					
Gasoline	12.89%	8.55%	4.34%	0.5	0.5
Limited Service Meals and Snacks	5.53%	4.19%	1.34%	13.8	15.8
Motor Vehicle Maintenance and Servicing	1.28%	0.79%	0.48%	8.4	8.8
Vehicle Leasing	0.99%	0.51%	0.48%	1.8	1.8
Motor Vehicle Insurance	3.19%	2.71%	0.48%	11.8	11.8
Wireless Phone Service	1.79%	1.32%	0.47%	7.2	7.2
Electricity	2.02%	1.58%	0.44%	2.1	2.1
Laundry and Dry Cleaning Services	0.65%	0.26%	0.39%	29.7	33.0
Nonfrozen Noncarbonated Juices and Drinks	1.02%	0.64%	0.38%	2.9	8.1
Cable and Satellite Television Service	2.53%	2.15%	0.37%	7.3	7.6
Mean				**8.5**	**9.7**
Median				**7.2**	**7.8**
Top 10, larger expenditure shares by high-income white households					
Club Dues and Fees For Participant Sports/Group Exercises	0.68%	0.98%	-0.31%	7.4	11.2
Veterinarian Services	0.14%	0.50%	-0.36%	11.0	11.0
Purchase Of Pets, Pet Supplies, Accessories	0.07%	0.44%	-0.36%	13.6	23.6
Household Decorative Items and Clocks	0.07%	0.44%	-0.37%	6.0	51.9
Day Care and Preschool	0.62%	1.05%	-0.43%	14.0	14.0
New Car and Truck Purchase	0.43%	0.87%	-0.43%	2.7	2.7
Alcoholic Beverages Away From Home	0.13%	0.59%	-0.46%	18.6	19.5
Pet Food	0.25%	0.77%	-0.52%	4.1	14.2
Full Service Meals and Snacks	3.43%	4.61%	-1.18%	19.2	19.5
Outboard Motors and Powered Sports Vehicles	0.00%	1.60%	-1.60%	8.7	11.7
Mean				**10.5**	**17.9**
Median				**9.8**	**14.1**

Note: This table reports the categories with the largest differences in expenditure shares between Black and white households and the frequency of price changes. Averages and medians are equal weighted across the ten top categories for each group. Nakamura and Steinsson (2008) provides duration for three different gasoline (regular, mid-grade, and premium) but the CES only provides the total consumption of gasoline. We take the average duration across three categories.

4 Monetary policy framework

We now set up a model with sticky prices and wages, and workers with different exposure to unemployment and inflation fluctuations.[18] The structural model provides the basis for the Phillips curve estimates and quantification of policy trade-offs in Section 5.

The framework takes the vantage point of a monetary authority that does not have instruments to affect racial differences in preferences and labor market institutions and can only influence aggregate outcomes.

Relative to a standard new Keynesian framework, the model features:

1. Multiple sectors with different degrees of price stickiness.

2. Heterogeneous groups of workers with different consumption baskets and exposure to unemployment fluctuations (Black and white workers, with different levels of income).

3. A structural gap in cyclical unemployment fluctuations modeled after the empirically observed gap in cyclical unemployment risk between Black and white workers.

The model has two periods, indexed $\{0, 1\}$, with output and consumption happening entirely in the second period. In the first period (period 0), some households and firms choose wages and prices that they will charge in the second period (period 1). In period 1, there is then a cost-push shock to which the monetary authority reacts. The remaining firms and households choose wages and prices, determining the equilibrium level of inflation, unemployment, and real income for different groups.

4.1 Households

There are four large households, indexed by their race and income level, $k \in \{B, W\} \times \{H, L\}$. To allow for wage stickiness and unemployment, we follow the basic structure in Galí et al. (2012). Each of these households consist of a double-continuum of workers indexed by $i \in [0, 1]$, representing the type of labor service provided by the worker, and $s \in [0, 1]$ determining their disutility from work. In particular, workers of type s have utility from leisure proportional to g(s), with g increasing in s.

[18] We present the main model equations in the text, but refer the reader to appendix A for details and derivations.

4.1.1 Consumption

In period $t = 1$, household members consume varieties (indexed v) produced in J different sectors $j = \{1, ..., J\}$. Those are aggregated into a consumption composite through a CES over varieties, nested into a Cobb-Douglas aggregator over sectors:

$$C_1^k = \prod_{j=1}^{J} \left(C_1^{kj}/\alpha^{kj}\right)^{\alpha^{kj}}, \quad \text{with} \quad C_1^{kj} = \left[\int \left[C_1^{kj}(v)\right]^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, j \in \{1, ..., J\},$$

where C_1^k is the *household-specific* consumption composite, and C_1^{kj} are sector-specific composites of varieties. ν is the elasticity of substitution between varieties.

The share of each sector's goods consumed by household k is denoted by α^{kj} and is a household-specific parameter capturing the observation that consumption composites are different for different types of households. Because of this, households experience different inflation rates.

Households assign the same consumption to all members. Furthermore, for any given variety of labor i, the household's head always selects members with the lowest utility of leisure. Given these assignment rules, households choose wages and consumption of different goods to maximize the aggregated utility function:

$$\frac{1}{1-\rho} \left(C_1^k\right)^{1-\rho} \int_i H(N_1^k(i))^{1-\rho} di$$

where $N_1^k(i)$ is the total amount demanded of labor of type i supplied by household of type k and $\rho > 0$ is the coefficient of relative risk aversion.[19] Furthermore, $H(N_1^k(i)) = \int g(s)(1 - N_1^k)ds = \int_{N_1^k(i)}^1 g(s)ds$ captures the utility of leisure given employment of each type i and optimal assignment of types s to work or not. Households maximize aggregated utility, subject to the aggregated budget constraint

$$\sum_j P_1^{kj} C_1^{kj} = \int N_1^k(i) \mathcal{W}_1^k(i) di,$$

where $P_1^j = \left[\int p_1^j(v)^{1-\nu} dv\right]^{\frac{1}{1-\nu}}$ is the price of the cost-minimizing bundle of varieties within each sector with $p_1^j(v)$ is the price of variety v in sector j.

Utility maximization also implies the *household-specific* price index:

[19] See the appendix A.1 for details of derivation.

$$P_1^k = \prod_{j=1}^{J} \left(P_1^j\right)^{\alpha^{kj}}$$

The index is household-specific because their consumption baskets differ. To the extent that prices of different goods react differently to inflation, this leads to different exposures to inflationary shocks.

4.1.2 Wage setting

The wage setting follows Galí et al. (2012). For tractability, the setup includes an additional layer of employment agencies that hire differentiated worker services and transform them into a homogeneous input used by final goods producers. Workers have market power vis-a-vis those employment agencies and set wages optimally.[20]

Employment agency firms hire workers of different types i and households k, and aggregate their services into one homogeneous input used by the final goods producers according to a CES production function with elasticity of substitution ε. Demand for variety of labor i supplied by household k is $N_1^k(i) = \left(\mathcal{W}^k(i)/\mathcal{W}^k\right)^{-\varepsilon} N_1^k$, where $\mathcal{W}_1^k(i)$ is the nominal wage of type i workers and $\mathcal{W}_1 = [\int \left(\mathcal{W}_1^k(i)\right)^{1-\varepsilon} di]^{\frac{1}{1-\varepsilon}}$ is the wage index for household k implied by cost-minimization by the employment agency, with N_1^k is the overall demand for labor of type k by the employment agency.

The model captures the differential behavior of unemployment rates through the hiring behavior of employment agencies. Namely, employment agencies are constrained to hiring workers of different groups in an exogenously fixed proportion. In particular, for any group k,

$$\frac{u_1^k}{u^k} = \frac{u_1}{u}$$

This constraint ensures that the ratio unemployment gaps between groups remain constant over the business cycle, which they mostly do in the data (figure 2). It is a stark but straightforward, reduced-form way to encode the various sources of racial and income disparities, including possible discrimination, that lead to persistent gaps in employment rates in the data — which are taken to be invariant to monetary policy.

[20] As discussed by Galí et al. (2012), those assumptions are to be interpreted not as realistic features of the economy but as modeling devices to tractably embed nominal wage rigidities (and resulting equilibrium unemployment) in the model. Similar devices are standard in New Keynesian models with wage stickiness.

Labor types differ in the timing of wage-setting. A fraction θ^k of worker types (say, indexed $i \leq \theta^k$ or below) chooses $t = 1$ wages in $t = 0$, whereas the remaining types choose wages in $t = 1$.

Using hats to denote log deviations from perfect foresight values, household choices imply the following wage Phillips Curve, where φ is the elasticity of the marginal rate of substitution between consumption and leisure, and N is the perfect foresight employment level of the household.[21]

$$\pi_1^{\mathcal{W},k} - \mathbb{E}_0 \pi_1^{\mathcal{W},k} = -\frac{(1-\theta^k)(1+\varphi)}{\theta^k(1+\epsilon\varphi)}(u_1^k - u^k). \tag{4}$$

where $\pi_1^{\mathcal{W},k}$ is nominal wage inflation for household of type k, between periods 0 and 1.

If there are no shocks in $t = 1$, wages set in $t = 0$ remain optimal, and workers work the amount they prefer at those wages. Then both unemployment and its natural rate are equal to the perfect foresight value u. If, however, labor demand is lower than expected, workers setting wages in $t = 1$ then choose lower wages, but those who chose their wages in $t = 0$ cannot. This leads to unemployment rising above, and wage inflation falling below, what was previously expected.

In equilibrium, the unemployment gaps for different groups are proportional to the aggregate unemployment gap, and this is proportional to the deviation between the unemployment rate and its perfect forecast value, u. It follows that the wage Phillips Curve can be written as a function of the aggregate unemployment rate:

$$\pi_1^{\mathcal{W},k} = \mathbb{E}_0 \pi_1^{\mathcal{W},k} - \psi^{\mathcal{W},k}(u_1 - u). \tag{5}$$

where u_1 is the aggregate unemployment rate and $\psi^{\mathcal{W},k} = \frac{(1-\theta^k)(1+\varphi)}{\theta^k(1+\epsilon\varphi)}\frac{u^k}{u}$ is a function of parameters and the perfect foresight value of the unemployment rates.

4.2 Final Goods Firms

Within each sector $j \in \{1, ..., J\}$, monopolistic firms produce final varieties for households and distribute profits to a separate class of firm owners who consume all the profits. They also incur labor hiring costs, which are higher if the unemployment rate is low.[22] Because of this, marginal costs may depend on the unemployment rate over and above its impact on

[21] We provide a derivation in the appendix A

[22] See Ravenna and Walsh (2008) for a model with this characteristic.

wages, potentially leading to real wages that decline as unemployment gaps tighten. The (nominal) marginal cost of production is

$$MC_1 = e^{\tilde{\eta}_1}\mathcal{M}(u_1 - u)\bar{\mathcal{W}}_1,$$

for all $j \in \{1, ..., J\}$, where $\bar{\mathcal{W}}_1$ is the price of the homogeneous input sold by the employment agencies. Furthermore, $\mathcal{M}' < 0$, implying that higher unemployment rates reduce hiring costs, leading to lower marginal costs. Finally, $\tilde{\eta}_1$ is an exogenous cost-push shock, capturing other sources of marginal cost fluctuations, such as international commodity prices.

Firms choose (nominal) prices and satisfy the demand for their products at the chosen price. All proceeds are rebated to and consumed by a separate class of firm owners. Within each sector j, a fraction ϑ^j of producers sets their price in $t = 0$ and the remaining fraction $1-\vartheta^j$, sets their price in $t = 1$.

Optimal price setting, together with optimal wage setting, and proportional unemployment gaps, imply the system of sectoral Phillips Curves.[23]

$$\pi_1^j = \mathbb{E}_0\pi_1^{\bar{\mathcal{W}}} - \psi^j\left(u_1 - u - \eta_1\right)$$

where $\pi_1^{\bar{\mathcal{W}}}$ is the increase in the homogeneous labor input, itself a weighted average of wage changes, η_t is the cost-push shock $\tilde{\eta}_t$, normalized to simplify notation, and $\psi^j \equiv (1-\vartheta^j)\left(\zeta + \sum_k n^k\psi^{\mathcal{W},k}\right)$ with n^k the expected proportion of workers of type k. Sectoral inflation responds more to unemployment to the extent that the fraction of firms setting prices late $1-\vartheta^j$ is larger. For all sectors, sectoral inflation rises relative to expected wage inflation when either the unemployment rate is low, or when there is a cost-push shock.

Recall that the cost of living index for household k is given by a weighted average of the sectoral prices, with sector-specific weights given by the group-specific consumption shares α^{kj}. The sectoral Phillips curves can then be aggregated into a group-specific price-Phillips Curve as

$$\pi_1^{p,k} \equiv \sum_j \alpha^{kj}\pi_1^j = \mathbb{E}_0\pi_1^{\bar{\mathcal{W}}} - \psi^k(u_1 - u - \eta_1), \tag{6}$$

where $\psi^k = \sum_j \alpha^{kj}\psi^{kj}$.

[23] Again, see the appendix A for details of the derivation.

4.3 Monetary Policy

The Central Bank chooses monetary policy to influence the various inflation rates — through the effect of policy on unemployment via the Phillips Curves. It trades off inflation and unemployment by letting unemployment rise above its natural rate if inflation is high. In particular, the Central Bank adopts a policy rule tying a weighted average of group-specific inflation rates to a weighted average of group-specific unemployment rates, as follows:

$$\sum_k \phi^k \pi_1^{p,k} = \sum_k \omega^k (u_1^k - u^k) - \tilde{\varepsilon}_1, \tag{7}$$

where $\tilde{\varepsilon}_1$ is a contractionary monetary policy shock and we normalize $\sum_k \omega^k = 1$.

In equilibrium, unemployment rates are proportional to one another $(u_1^k/u^k = u_1/u)$, so that the rule simplifies to

$$\sum_k \phi^k \pi_1^{p,k} = \Omega(u_1 - u) - \tilde{\varepsilon}_1,$$

with $\Omega \equiv \sum_k \frac{u^k}{u} \omega^k$.

The generic rule in (7) can accommodate different views and proposals. Monetary policy is less accommodative if coefficients on $\pi_1^{p,k}$ become larger on average; vice versa, it can reflect more accommodative policy if the opposite is true.

As far as the Central Bank taking demographic characteristics into account when setting policy, we note that a monetary policy rule that is neutral with respect to race and income sets the weights ϕ^k and ω^k in proportion to the population of different groups. Since, by definition, u_1 is the aggregate unemployment, such a neutral policy would imply $\Omega = 1$. Deviations from this benchmark can be interpreted as deviations from a policy that is neutral with respect to race and income.

For instance, policy targeting the unemployment rate for Black workers, in place of average unemployment, would imply setting $\omega^{w,H} = \omega^{w,L} = 0$ and $\omega^{B,H} + \omega^{w,L} = 1$. Because unemployment among Black workers is twice as large as among white workers, this would imply Ω that is twice as large. Under such a rule, the monetary authority would be willing to tolerate twice as large fluctuations in inflation in exchange for the same volatility in aggregate unemployment. It follows that a mandate to stabilize unemployment rates for racial minorities is equivalent to a mandate to more strongly stabilize *aggregate* unemployment.[24]

[24] This point was also made in the context of a different model by Nakajima (2023).

4.4 Stabilization trade-offs

We now consider the impact of a cost-push shock η_1 on unemployment and the various inflation rates. Combining the policy rule (7) with the Phillips Curves (4) and (5), and the determinant of price index anchoring (2), yields

$$\begin{aligned}
&\pi_1^{p,k} = \psi^{p,k}(1-\Phi)\eta_1 - \psi^{p,k}\varepsilon_1, \\
&\pi_1^{\mathcal{W},k} = -\psi^{\mathcal{W},k}\Phi\eta_1 - \psi^{\mathcal{W},k}\varepsilon_1, \text{ and} \\
&u_1 - u = \Phi\eta_1 + \varepsilon_1.
\end{aligned} \tag{8}$$

where ε_1 is the monetary policy shock $\tilde{\varepsilon}_1$, normalized for notational simplicity, and $\Phi \in [0,1]$ is the sensitivity of unemployment to the cost-push shock under the chosen monetary policy rule, with $\partial\Phi/\partial\Omega < 0$.[25] It follows that, as the monetary authority puts more weight on unemployment rather than inflation, unemployment fluctuates less with the cost-push shock than it would otherwise. To gain intuition, consider two polar cases. In one case, monetary policy aims to perfectly stabilize unemployment and $\Phi \to 0$. Then, the cost-push shock is allowed to translate entirely into inflation. In the other polar case, $\Phi \to 1$, and inflation is completely stabilized. Instead, it is unemployment volatility that absorbs the effect of the cost shock in its entirety. The trade-off between unemployment and inflation volatility depends on the Phillips Curve parameter $\psi^{p,k}$.

Abstracting from monetary policy shocks, real wage changes are given by

$$\pi_1^{\mathcal{W},k} - \pi_1^{p,k} = -\left[(1-\Phi)\psi^{p,k} + \Phi\psi^{\mathcal{W},k}\right]\eta_1,$$

so that, in all scenarios, the real wage declines in response to a cost-push shock. This happens because increases in costs lead to higher prices and, through a contractionary response of monetary policy (for $\Phi > 0$), also lower nominal wages.

Putting it all together, we can now consider the impact of a cost-push shock on real income. In the model, real wages vary with the difference between relative wage and price inflation, so that $\widehat{\mathcal{W}}_1^k - \widehat{P}_1^k = \pi_1^{\mathcal{W},k} - \pi_1^{p,k}$. Plugging in the expressions in (8) into (1) then yields real income in $t = 1$ as a function of the change in the expected path of the cost-push shocks

$$\widehat{Y}_1^k \simeq -\left((1-\Phi)\psi^{p,k} + \Phi\psi^{\mathcal{W},k} + \frac{1}{1-u^k}\frac{u^k}{u}\Phi\right)\eta_1 \tag{9}$$

[25] In particular, $\Phi = \frac{\sum_{kj}\phi^k\alpha^{kj}(1-\vartheta^j)(\zeta+\sum_k n^k\psi^{\mathcal{W},k})}{\Omega+\sum_{kj}\phi^k\alpha^{kj}(1-\vartheta^j)\left(\zeta+\sum_{k'} n^{k'}\psi^{k',\mathcal{W}}\right)}$

The cost-push shock affects real income, but the choice of the policy rule — reflected in Φ — can buffer or amplify that impact, making income more or less volatile for any household of type k.

4.5 Volatility and Welfare

We can use equation (9) to calculate a second-order approximation for the variance of income — a measure of its volatility. Within our model, this approximation also characterizes utility losses when households are strongly risk-averse. The result is summarized in the following proposition, and we offer a proof in the appendix:

Proposition 1. *Let $\mathbb{E}\eta_1 = 0$ and $var(\eta_1) = \sigma^2$. Also , suppose $var(\varepsilon_1) = 0$, so all $t = 1$ variables are only functions of the realization of η_1. Then, for small σ^2,*

$$var\left(\frac{Y_1^k}{Y^k}\right) = var\left(\widehat{Y}_1^k\right) + \mathcal{O}(\sigma^3),$$

where $\widehat{Y}_1^k$ is defined in equation (9), and $\mathcal{O}(\sigma^3)$ denotes terms that decline at rate σ^3 or more as σ approaches zero.

Moreover, denote by U^k the utility function for members of household k. Then,

$$\mathbb{E}_0 \int U^k\left(C_1^k, N_1^k(i,s)\right) dsdi \propto -\rho \times var\left(\widehat{Y}_1^k - \frac{b^k}{w^k}\widehat{N}_1^k\right) + \left(\mathbb{E}\left[U\left(C_1^k, \int N_1^k(i)di\right)\right]^{\frac{1}{1-\rho}}\right)^{1-\rho} + \mathcal{O}(\sigma^3) \quad (10)$$

where $b^k = |U_N^k(C_1^k, N^k)|/U_C^k(C_1^k, N^k)$ is the perfect foresight ratio of the marginal utility of leisure to the marginal utility of consumption.[26]

The proposition establishes two main points. The first is that the variance of income is proportional to the variance of the log-linear approximation of income around $\eta_1 = 0$. The second is that if the household does not value leisure at the margin (so that $b^k = 0$), there is a component of utility that declines with the variance of real income, the more so the larger is their risk aversion.[27] More generally, uncertainty will also distort prices and wages, leading

[26] Proof of the proposition is given in appendix B.

[27] Ganong, Jones, Noel, Greig, Farrell and Wheat (2020) also find that "the[ir] model implies that income volatility has a substantial welfare cost for all groups. [...] Three sufficient statistics are required to calculate the welfare cost [of income volatility]: the elasticity of consumption with respect to temporary income shocks,

to welfare losses on average. Average welfare losses depend on the dispersion of prices and wages induced by nominal stickiness, which induces misallocation and is captured in the second term in equation (10).[28]

We calculate the standard deviation of $\widehat{Y}_1$, following the first part of Proposition 1, using (9). Then we can evaluate different policy rules in terms of how they affect real income volatility — which, following the second part of 1, has welfare implications for each household k.

One way to express the relevant policy trade-offs is through the relationship between changes in real income volatility triggered by η_1 and variation in the monetary authority's stance. When the Central Bank, for instance, allows the volatility of the unemployment rate to increase following a cost-push shock (a less accommodative policy stance), how much of the increased unemployment volatility passes through to real income volatility? These changes can be expressed as follows:

$$\frac{d\sigma(\widehat{Y}_1^k)}{d\sigma(u_1)} = \underbrace{\frac{1}{1-u^k}\frac{u^k}{u}}_{\substack{\text{unemployment}\\\text{channel}}} - \underbrace{\left(\psi^{p,k} - \psi^{\mathcal{W},k}\right)}_{\text{price channel}} \tag{11}$$

where $\sigma(\widehat{Y}_1^k)$ is the standard deviation of real income and $\sigma(u_1)$ the standard deviation of unemployment induced by η_t.

As policy allows for more unemployment volatility, the volatility of real income changes in response to two channels. The first one is the *unemployment channel*. It captures the direct effect of fluctuations in the overall unemployment rate and increases in proportion to the average rate of unemployment for the group under consideration. White households face an unemployment rate similar to the national unemployment rate on average ($u^w/u \simeq 1$). Because of that, increases in the volatility of the unemployment rate translate roughly one-to-one to increases in the volatility of real income through the unemployment channel.[29] In contrast, Black households face twice the national rate of unemployment on average ($u^B/u \simeq 2$). This implies that the same increase in the volatility of the national unemployment rate translates into close to twice the increase in the volatility of their real income through the unemployment channel.

A second channel is the *price channel*. It captures how stabilizing unemployment can destabilize real income through its effects on inflation and real wages. If $\psi^{p,k} < \psi^{\mathcal{W},k}$, prices are

the coefficient of relative risk aversion, and the variance of temporary income shocks."

[28]Higher variance in log income around a constant average log income level may also lead to welfare gains because of Jensen's inequality. This effect is smaller than the one emphasized here so long as $\rho > 1$.

[29]We abstract from income heterogeneity in this discussion for ease of exposition.

less sensitive to unemployment fluctuation than wages. In that case, low unemployment leads to real wage gains, so households gain from lower unemployment through both channels. Minimizing fluctuations in unemployment will, in that case, lead unambiguously to minimizing fluctuations in real income. If, however, $\psi^{p,k} > \psi^{\mathcal{W},k}$, prices are more sensitive to unemployment than wages, and lower unemployment leads to real wage losses. In that case, the price channel enters with the opposite sign as the unemployment channel. By stabilizing unemployment, then, the monetary authority accepts more volatile inflation. With $\psi^{p,k} > \psi^{\mathcal{W},p}$, the empirically relevant case, the monetary authority then faces the following trade-off: by dampening the effect of cost-push shocks on unemployment, it can protect households from income fluctuations tied to unemployment, but in exchange for unstable real wages when they are employed. The net effect on real income volatility is ambiguous and has to be assessed quantitatively.

As we will see, the ambiguity is especially problematic for proposals to introduce racial considerations into monetary policy. It is true that all Black households indeed face higher unemployment rates than white ones regardless of income. However, we also found that richer Black Americans face substantially higher volatility of prices for their consumption basket than white Americans of similar income and also than poorer Black Americans. This multilayered heterogeneity implies that the racialized effects of monetary policy are intertwined with disparities in income, complicating the effort of focusing on trade-offs with a broad brush.

5 Quantitative Analysis

Both group-specific unemployment rates and price and wage Phillips Curve coefficients affect the Central Bank's policy trade-offs. In this section, we assess all of these factors.

Group-specific average unemployment rates can be estimated in a straightforward way: by taking long-run averages of the unemployment series by group based on microdata from household surveys.

The estimation of Phillips Curve coefficients is more and more challenging. We leverage information from the Great Recession and its aftermath to circumvent some of the identification problems and obtain reliable estimates.

5.1 Estimation of Phillips curves

5.1.1 Empirical strategy

The model in Section 4 implies a system of group-specific Phillips Curves, trading off aggregate unemployment with group-specific price and wage inflation.[30] We estimate their empirical analogues:

$$\pi_t^{c,k} = \alpha + \psi^{c,k}(u_{t-12} - u_{t-12}^*) + \beta^k \mathbf{X}_t + \xi_t^k \tag{12}$$

where $c \in \{p, \mathcal{W}\}$ denotes whether those are price or wage Phillips Curves, k denotes the race-by-income group, $\pi_t^{c,k}$ is 12-month inflation, u_{t-12}^* is the long-run structural unemployment trend as measured by the Congressional Budget Office, and $\mathbf{X}_t$ is a vector of controls. These, together with the error term ξ_t^k, incorporate the direct effect of the cost-push shock η_t on price inflation, labor or goods market frictions that affect price or wage inflation for any given level of the unemployment rates, as well as any measurement error in any of the variables.[31]

The main challenge for estimating Phillips Curves, in general, is that a monetary authority that stabilizes inflation induces a correlation between the unemployment rate and the cost-push shocks in the residuals, biasing the OLS estimates of Phillips Curve coefficients towards zero (McLeay and Tenreyro, 2020; Fitzgerald et al., 2024). Within the model laid out in Section 4, the shock η affects price inflation through its direct effect on marginal costs, but the monetary authority reacts to stabilize inflation so that the unemployment rate also reacts. The net result is that, as one can see from equations (8), cost-push shocks generate a positive co-movement between unemployment and inflation that is mediated by the monetary policy choice summarized in Φ. Therefore, in order to estimate the Phillips Curves, one needs access to a monetary shock that affects the unemployment rate directly and only affects inflation indirectly through its effect on unemployment. Within the context of the model, this role is played by the monetary policy shock ε_t.[32]

To obtain an unbiased estimate of the Phillips Curve coefficients, we leverage the unemployment spike following the collapse of Lehman Brothers in 2008 and its long aftermath.

[30] The Phillips Curves derived from the two-period model do not include a forward expectational term that emerges in multi-period Calvo models. For these frameworks, one can write Phillips Curves as a function of current unemployment analogous to the ones we have here following the steps Hazell et al. (2022). In such settings, the slope of the Phillips Curve depends on the persistence of underlying shocks.

[31] Frictions are modeled, as it is typical, as shocks to elasticity parameters.

[32] In particular, as one can see from equation (8), ε affects inflation and unemployment in proportion to the Phillips Curve coefficients. Equation (8) also implies that OLS would give an *unbiased* estimate of $\psi^{\mathcal{W},k}$, since wages are not directly affected by the cost-push shock η_t.

Specifically, we follow Mian and Sufi (2014), Kehoe et al. (2019) and Davis and Haltiwanger (2019) in interpreting the initial unemployment spike in late 2008 as caused by the impact of housing wealth losses on household demand for goods and services. That initial spike reverted slowly over the course of nine years because aggregate demand remained depressed as households rebuilt their wealth, and the monetary authority was constrained by the Zero Lower Bound. Altogether, we interpret the persistent unemployment rate post-2008 crisis as largely explained by a monetary shock ε_t rather than an endogenous reaction of monetary policy to a cost-push shock η_t.[33]

Operationally, the assumptions above motivate an instrumental variable approach. Specifically, we regress each of the group-specific 12-month price inflation rates on the unemployment gap, lagged 12 months, and controls, using a dummy for the period of unemployment above the natural rate post-Lehman (September 2008-January 2017) as an instrument for the unemployment gap. The estimator takes the following intuitive form:

$$\widehat{\psi}^{p,k} = \frac{\mathbb{E}\left[\pi_t^{p,k} | t \in \text{Sep2008-Jan2017}\right] - \mathbb{E}\left[\pi_t^{p,k} | t \notin \text{Sep2008-Jan2017}\right]}{\mathbb{E}\left[u_{t-12} - u^*_{t-12} | t \in \text{Sep2008-Jan2017}\right] - \mathbb{E}\left[u_{t-12} - u^*_{t-12} | t \notin \text{Sep2008-Jan2017}\right]}.$$

Abstracting from controls, the estimated Phillips curve coefficient is the difference between the average inflation rates in the post-Lehman period relative to inflation in other parts of the sample, divided by the difference between average unemployment gaps in those two periods. Formally, the proposed instrument is *valid* so long as the error terms are expected to average to zero over the 101 months within the post-Lehman period, and the 163 months in its complement (including η_t, net of oil shocks added as controls). In other words, the proposed instrument is *valid* so long as one interprets the extended period of high unemployment after the Great Recession *without* relying on long-lasting labor or goods market distortions, or prolonged cost-push shocks. To more comprehensively control for cost-push shocks, we further include in the specification two years worth of oil-supply shocks identified by Känzig (2021) and 12 seasonal dummies. More generally, we assume that cost shocks to various inputs and/or changes in market frictions — both captured by the residuals — were short-lived enough. This implies that they can be safely assumed not to have a long-lasting

[33] In particular, distortions to corporate and financial markets caused by the crisis were quickly resolved given large scale government interventions (e.g., bailouts and stress tests), and the evidence does not point to distortions in labor markets playing an oversized role in determining unemployment fluctuations over that period (Şahin et al., 2014).

explanatory power over the persistent unemployment gap in the years following the financial crisis.

We estimate Phillips Curve coefficients for prices and wages relevant for different groups. We also separately estimate the difference in coefficients, allowing us to obtain comparisons between the sensitivity to unemployment of inflation rates relevant for Black and white households, as well as for the difference between wage and price inflation.[34] These are depicted in table 5.

Table 5: The slope of the price Phillips curves by race and income.

	$\pi^{p,k}$		$\pi^{p,k} - \pi^{p,\text{Black}}$	
	OLS	IV	OLS	IV
white, low income	-0.27 (0.097)	-1.16 (0.37)	-0.0005 (0.0046)	-0.015 (0.016)
Black, low income	-0.27 (0.097)	-1.15 (0.37)	–	–
white, high income	-0.21 (0.096)	-1.06 (0.37)	0.049 (0.04)	0.43 (0.13)
Black, high income	-0.26 (0.13)	-1.50 (0.50)	–	–

Note: Estimated coefficients for the Phillips curves in equation (12) by race and income, for absolute and relative price inflation (columns 2 and 3, or 4 and 5, respectively). In columns 3 and 5, the unemployment gap is instrumented using a dummy variable for whether monetary policy was at the zero lower bound at the date of interest. Seasonal dummies and oil shocks controls à la Känzig (2021) used throughout. Newey-West robust standard errors with 48 lags. See text for details. Source: CPS and CPI data 1998-2019.

5.1.2 Estimates

Table 5 shows the estimated coefficients for the slope of price Phillips Curves using price indices based on the consumption bundles of different types of households. In our estimates, we find that, in absolute value, those are steepest for high-income Black households and least steep for high-income white households, with low-income households in between. The two last columns show the difference between coefficients for white and Black households within the same income category. Under the IV specification, the estimates are statistically

[34] For those, we take the relative inflation as the dependent variable, so as to difference out the part of the residual which is common to the indices being compared.

significant from each other. Furthermore, the difference is economically sizable among high-income households but not among low-income ones (in line with the volatility measures summarized in Table 1).

The magnitudes of our estimated coefficients are comparable to coefficients for Phillips Curves previously estimated using the overall price index. In line with McLeay and Tenreyro (2020), OLS estimates are biased downward due to monetary policy seeking higher inflation when output is below potential. The IV estimates are at a similar order of magnitude, but higher overall, than the ones obtained by Hazell, Herreño, Nakamura and Steinsson (2022), when using a full set of controls and instrumenting for local relative prices of tradeables. Including shelter puts the estimates in these two papers closer together.[35]

Table 6 presents estimates of the slope of wage Phillips Curves estimated in the same way the price Phillips Curves. The first two columns show OLS and IV estimates. Both OLS and IV estimates are similar, in line with equation 8, which indicates that the cost-push shock η affects wage inflation proportionally to the Phillips Curve coefficient.

While the wages of low-income workers appear to be more sensitive to unemployment, there is not a meaningful racial difference for the same level of income. Furthermore, the estimated coefficients are smaller than the respective price coefficients. The net effect (shown in columns 3 and 4) is that increased unemployment has a positive (and statistically significant) effect on *real* wages. That net effect of unemployment on real wages is roughly similar for all groups except for high-income Black households. For those, the net effect is about 50% as large.[36]

[35] In particular, Hazell et al. (2022) find $\psi = 0.339$. However, as they recognize, this value does not include shelter. Using coarser data, Hazell et al. (2022) also calculate a coefficient that takes shelter inflation into account and find shelter inflation to be about 2.5 times higher than their previously reported estimates, bringing their estimates closer to ours. Furthermore, as Hazell et al. (2022) also recognize, their estimates fall within the lower end of the range found by the literature.

[36] The low sensitivity of wages to unemployment is in line with findings by Grigsby, Hurst and Yildirmaz (2021). The estimates they provide use ADP administrative payroll data and are, as noted by the authors, superior both to estimates from survey data sources and to those from official administrative sources. Grigsby et al. (2021) also provide evidence that wages of new hires are no less rigid than wages of incumbent workers through a careful matching estimation exercise.

Table 6: The slope of wage Phillips Curves by race and income.

	$\pi^{\mathcal{W},k}$		$\pi^{p,k} - \pi^{\mathcal{W},k}$	
	OLS	IV	OLS	IV
white, low income	-0.19 (0.05)	-0.30 (0.10)	-0.09 (0.09)	0.86 (0.31)
Black, low income	-0.31 (0.09)	-0.28 (0.07)	-0.04 (0.13)	0.87 (0.39)
white, high income	-0.13 (0.03)	-0.15 (0.30)	0.08 (0.09)	0.92 (0.37)
Black, high income	-0.24 (0.10)	-0.15 (0.09)	0.18 (0.17)	1.34 (0.56)

Note: Estimated coefficients for the Phillips curves in equation (12) by race and income, for nominal and real wages (columns 2 and 3, or 4 and 5, respectively). In columns 3 and 5, the unemployment gap is instrumented using a dummy variable for whether monetary policy was at the zero lower bound at the date of interest. Seasonal dummies and oil shocks controls à la Känzig (2021) used throughout. Newey-West robust standard errors with 48 lags. See text for details. Source: CPS-ASEC and CPI data 1998-2019.

5.2 Heterogeneous effects of accommodating monetary policy

As discussed in section 4.5, equation (11) provides a criterion to evaluate whether a monetary authority can reduce real income volatility for different groups, by allowing unemployment to react more to supply shocks. In particular, recall

$$\frac{d\sigma(\widehat{Y}_1^k)}{d\sigma(u_1)} = \underbrace{\frac{1}{1-u^k}\frac{u^k}{u}}_{\substack{\text{unemployment}\\ \text{channel}}} - \underbrace{\left(\psi^{p,k} - \psi^{\mathcal{W},k}\right)}_{\text{price channel}}$$

and consider that our estimation finds that $\psi^{p,k} > \psi^{\mathcal{W},k}$ — so there is a non-trivial trade-off between the two channels. By stabilizing unemployment in the face of supply shocks, the monetary authority reduces unemployment volatility through the unemployment channel, but increases it through the price channel.

Table 7 shows the effects of accommodating monetary policy on real income volatility for

	Unemployment channel	Price channel	Total effect
White, low income	1.14	-0.86	0.28
Black, low income	2.04	-0.87	1.17
White, high income	0.41	-0.91	-0.50
Black, high income	0.75	-1.35	-0.60
All white	0.73	-0.89	-0.16
All Black	1.53	-1.06	0.48

Table 7: Effects of reduction in unemployment volatility on components of income volatility, $(\sigma(\widehat{Y}_1^k)/\sigma(u_1))$ as defined in equation (11), for different groups.

different groups by race and income.[37] The first column shows the *unemployment channel*. This measures the extent to which more volatile unemployment implies more volatile real income through each household's exposure to unemployment fluctuations. For an average household, the unemployment channel is close to 1 by construction. Since the unemployment rates for different household types are different in levels but move in lock-step with the cycle, the unemployment channel on income is amplified for households that have higher average unemployment rates (such as Black and low-income households). It follows that, for instance, for Black low-income households, the contribution of the unemployment channel to income volatility is more than five times larger than for white high-income households.

The second column shows the *price channel*. This is given by the difference between the slope of the wage Phillips Curve and the slope of the price Phillips Curve faced by each household. It is generally negative, so that it countervails the unemployment channel: a *less* accommodative monetary policy can bring *less* real income volatility, through greater price stability. The price channel has a similar magnitude for all but one group: Black, high-income households. These are very strongly exposed to inflation fluctuations, due to the concentration of their consumption baskets on flexible price goods (notably fuel).

The third column shows the Total effect, obtained from adding the second and third columns. The sign essentially follows income. Low-income individuals, whether Black or white, have more to gain from a more accommodative monetary policy; on the other hand, high-income individuals have more to gain from a policy that favors inflation stability.

Recent literature suggests that, as inflation rises, the slope of the Phillips curve may also

[37] To compute these, we use the population shares of each subgroup in the population of working age, and excluding households who are not Black nor white, who are multiracial, and those of any race living in group quarters. Median income thresholds are computed nation-wide, year by year, and used to classify respondent's household income accordingly (where "low" and "high income" denote below or above the median).

	Baseline	Slope by 2	Slope by 3
White, low Income	0.28	-0.58	-1.44
Black, low Income	1.17	0.30	-0.57
White, high Income	-0.50	-1.41	-2.32
Black, high Income	-0.60	-1.95	-3.30
All white	-0.16	-1.05	-1.94
All Black	0.48	-0.58	-1.64

Table 8: Effect of reduction in unemployment volatility on income volatility. The last two columns consider how the total effect changes as the slope of the Phillips Curves are multiplied by 2 and 3, respectively.

increase.[38] Table 8 compares our baseline results to scenarios in which the slope of the Phillips curve is steeper, by a factor 2 or 3. Such scenarios reflect the possibility of Phillips Curves changing their slopes as inflation increases, output gaps tighten, or expectations become unanchored (Benigno and Eggertsson, 2023). When the Phillips Curve coefficients are twice as large, this is enough to make poor white households prefer a policy of stabilizing inflation over unemployment. Poor Black households remain the only group that prefers unemployment stabilization in this case, given how large their exposure to unemployment stabilization is. Finally, when the slope of Phillips Curves is multiplied by three, all groups prefer inflation stabilization over unemployment stabilization.

5.2.1 Bias assessment from understating heterogeneity

Our analysis offers several insights for decision-makers evaluating whether monetary policy can, or even *should*, be used with its distributional effects in mind.

We find that monetary policy decisions have potentially heterogeneous effects on the real income volatility of different households, but these effects do not fall neatly along racial categories. More accommodative policy, tolerating larger fluctuations in inflation vis-a-vis more stable unemployment, disproportionately reduces real income volatility for Black households whose income is below the national median (as well as low-income white households, but to a smaller extent). The very same accommodative policy, however, *increases* the volatility of real income for Black households whose income is at or above the national median (and white households of similar income levels, too). Richer Black households are more exposed to both unemployment and inflation volatility, and the net effect of more accommodative policies is an increase in income instability for these households. Such heterogeneous effects

[38] See Forbes et al. (2022) for recent worldwide evidence. Changes in slope are consistent with menu costs (Alvarez et al., 2019), unanchoring of expectations (Carvalho et al., 2022) or non-linearities in labor markets (Benigno and Eggertsson, 2024).

are masked if we consider averages only — when we find barely a negative effect for white households and a relatively sizable positive one for Black ones. The latter hides the fact that stabilization of unemployment benefits poorer Black households to such an extent that it masks the overall negative income volatility effect that the same policies has for richer Black households.

6 Conclusions

Our paper addresses a pressing policy question: are there heterogeneous effects of monetary policy actions on the well-being of households of different races? If so, how and why?

We offer three insights in this respect. First, we note that income volatility is both a well-accepted target of monetary policy and plausibly an important part of households' well-being.

Second, we note that real income volatility is affected both by employment volatility and by fluctuations in the prices of a household's consumption basket. We show that Black families, on average, are more exposed to both types of volatility than white families are. However, it is *poorer* Black families who have the largest differential on the employment margin; on the other hand, it is *richer* Black families who experience the most significant divergence in the volatility of prices. In other words, there is important heterogeneity in the sources of real income volatility across levels of income, even within households of the same race.

Third, we evaluate whether such heterogeneity is quantitatively important. It is. We estimate a significant and sizable differential between the trade-off faced by richer Black families v. richer white families, as estimated by the slopes of race- and income-specific Phillips curves. On the other hand, no such difference arises for poorer families. Accordingly, accommodative monetary policy — tolerating higher volatility in prices in exchange for more stable unemployment — has heterogeneous effects for families of the same race but different income levels. Black families with higher-than-median income bear the brunt of increased price volatility (poorer Black families, instead, benefit the most from stable unemployment). All in all, a policy that might have been advocated as disproportionately benefiting Black households ends up advantaging poor households regardless of their race (while disproportionately disadvantaging Black households with above-median income).

Our paper points out that there are significant racialized aspects to American households' experiences of the labor market and of the goods market. Unemployment rates for Black families are higher and more volatile than for white families, even conditional on income, education, and several other observable characteristics. This gap is persistent and large.

Higher volatility of prices, as faced by higher-income Black households, also has its roots in racialized differences in housing and consumption. All in all, we conclude that race is an important consideration in the background of monetary policy actions, distinct from income, education, occupation, and other factors. Yet, we do not find that a solution is tasking the monetary authority to pursue unemployment stabilization more aggressively than what targeting *average* unemployment would do. Indeed, "targeting the Black unemployment rate" would not necessarily result in muting racialized differences in economic outcomes. Instead, it would exacerbate divergence by income levels. We see this result as emphasizing that, while race is a fundamental aspect of American households' economic experiences, there is significant diversity among Black families, that also shapes their employment and consumption outcomes. Because of this, our results encourage policymakers and thought leaders interested in alleviating inequality to carefully consider the richness of socio-economic circumstances underlying race.

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ONLINE APPENDIX

A Model

There are two periods indexed $t \in \{0, 1\}$. A cost-push shock $\tilde{\eta}$ and a monetary policy shock $\tilde{\varepsilon}$ are realized in period 1. Wages are set by households and prices by firms. There are nominal frictions because a fraction of the nominal prices and wages are set in period $t = 0$, before the value of the shocks are known. We focus on production, employment, and consumption outcomes that occur in period $t = 1$.

In order to describe price and wage inflation between periods $t = 0$ and $t = 1$ firms and households start out with *nominal* prices and wages preset at initial values W_0 and P_0. Those can be understood as prices and wages relevant for allocations in $t = 0$ but that are otherwise irrelevant for $t = 1$ allocations.

We describe in detail the price and wage-setting decisions of households and firms.

A.1 Households

There are four large households, indexed by $k \in \{B, w\} \times \{H, L\}$. Each of these households consists of a double-continuum of workers indexed by $i \in [0, 1]$, representing the type of labor service provided by the worker, and $s \in [0, 1]$ determining their disutlity from work. Within a given household k, all workers supplying a type of labor i change their wages simultaneously and select a common wage. Their nominal wage is given by $\mathcal{W}_1^k(i)$. Workers of type s have a utility from leisure that increases with s.

Each worker in household k consumes the same quantity C_1^k of a family-specific consumption composite. This is a composite of varieties (indexed v) produced in J different sectors $j = \{1, ..., J\}$ given by:

$$C_1^k = \prod_{j=1}^{J} \left(C_1^{kj}\right)^{\alpha^{kj}}, \quad \text{with} \quad C_1^{kj} = \left[\int \left[C_1^{kj}(v)\right]^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, j \in \{1, ..., J\}.$$

where C_1^{kj} are the sector-specific composites of varieties $C_1^{kj}(v)$ consumed by household k, α^{kj} is the household-specific share parameter governing demand for composite good k, and ν is the elasticity of substitution between varieties.

When specifying preferences, we allow α^{kj}, the share of goods consumed from each sector, to be household-specific. This captures the observation that consumption bundles are different for different types of households, so that they are exposed to different degrees of price flexibility and inflationary shocks.

Households take the price of individual varieties $p_1^j(v)$ within each sector j as given.

Cost minimization implies that household k consumption of sectoral composite C_1^{kj} satisfies $C_1^{kj}(v) = \left(p_1^j(v)/P_1^j\right)^{-\nu} C_1^{kj}$ where $P_1^j = \left[\int p_1^j(v)^{1-\nu} dv\right]^{\frac{1}{1-\nu}}$ is the price index for goods produced in sector j. Furthermore, cost minimization also pins down the consumption aggregate purchased from each sector j for a given overall consumption composite C_1^k. This satisfies $C_1^{kj} = \alpha^{kj} \frac{P_1^k}{P_1^j} C_1^k$ where now

$$P_1^k = \prod_{j=1}^{J} \left(\frac{P_1^j}{\alpha^{kj}} \right)^{\alpha^{kj}} \tag{13}$$

is a household-specific price index.

Utility for each household member is a function of consumption (which is equal for all members) and their leisure. We assume that the marginal utility of leisure for each household member increases in consumption. This implies that preferences satisfy the King-Plosser-Rebelo conditions, being consistent with a balanced growth path.

$$\frac{1}{1-\rho} \left(C_1^k\right)^{1-\rho} g(s)(1 - N_1^k(i,s)) ds di,$$

where $N_1^k(i,s) \in \{0,1\}$ is labor supplied by worker of type (i,s), $g(s)$ increases with s and $\rho > 0$ determines the risk aversion of households.

The household maximizes average utility for its members. Its period utility is therefore

$$\frac{1}{1-\rho} \left(C_1^k\right)^{1-\rho} \int g(s)(1 - N_1^k(i,s)) ds di,$$

Its period budget constraint is

$$P_1^k C_1^k = \int N_1^k(i,s) \mathcal{W}_1^k(i) ds di + \Pi_1^k.$$

where Π_1^k are profits distributed to households of type k. .

While $\mathcal{W}_1^k(i)$ is a choice variable for the household, how many of its members work given $\mathcal{W}_1^k(i)$ is not, that is, it is constrained by the labor demand function for type of labor service i given by $N_1^k(i)$:

$$\int N_1^k(i,s)ds = N_1^k(i)$$

Substituting equalized consumption, the constraint on hours worked and using the fact that households will optimally choose to have workers of each type with lower s (and lower utility of leisure) work, we have that the period utility reduces to

$$\int U(C_1^k, N_1^k(i))di = \frac{1}{1-\rho}\left(C_1^k\right)^{1-\rho}\int_i\int_{N_1(i)}^1 g(s)dsdi = \frac{1}{1-\rho}\left(C_1^k\right)^{1-\rho}\int_i H^{1-\rho}(N_1^k(i))di,$$

where $H(N) \equiv \left[\int_N^1 g(s)ds\right]^{\frac{1}{1-\rho}} = [G(1)-G(N)]^{\frac{1}{1-\rho}}$, where G is the integral of g. Note that $H(N)$ declines with N.

When choosing wage for type of worker i, they face iso-elastic residual demand $N_1^k(i) = \left(\mathcal{W}_1^k(i)/\mathcal{W}_1^k\right)^{-\epsilon} N_1^k$ where N_1^k is the total number of workers of all types in household k in demand, $\mathcal{W}_1^k = \left[\int \mathcal{W}_1^k(i)^{\frac{\epsilon-1}{\epsilon}}di\right]^{\frac{\epsilon}{\epsilon-1}}$ is the aggregate wage index for labor offered by household k, and ϵ is the elasticity of substitution between workers providing different services i. This residual demand for labor is analogous to Galí et al. (2012) and others and can be derived from the demand of a layer of employment agency firms.

We assume that while workers of type i can coordinate on their wage setting, there is no coordination across types, so that the household takes $\mathcal{W}_1^k$ as given.

Worker types $i < \theta^k$ chooses $t = 1$ wages in $t = 0$, whereas the remaining types choose wages in $t = 1$. Because types are uniformly distributed, there is, for each family k, a θ^k fraction of early wage setters and a $1-\theta^k$ fraction of late wage setters. In particular, for all $i < \theta^k$ household k chooses $\mathcal{W}_1^k(i)$ to maximize

$$\mathbb{E}_0\left[\int U(C_1^k, N_1^k(i))di\right]$$

subject to the aggregated budget constraint

$$P_1^k C_1^k = \int N_1^k(i)\mathcal{W}_1^k(i)di + \Pi_1^k$$

and the residual demand function $N_1^k(i) = \left(\mathcal{W}_1^k(i)/\mathcal{W}_1^k\right)^{-\epsilon} N_1^k$, while taking the wage index $\mathcal{W}_1^k$ as given. For $i \geq \theta^k$ the problem is analogous but without the expectation operator in the objective function, since they choose wages with full information. The consolidated household also chooses aggregate consumption in $t = 1$.

We can rewrite the budget constraint as a function of employment and aggregate wages only:

$$P_1^k C_1^k = \int N_1^k(i)^{1-\frac{1}{\epsilon}} \left(N_1^k\right)^{\frac{1}{\epsilon}} \mathcal{W}_1^k$$

For the fraction θ^k choosing wages ex-ante, the optimal choice of $\mathcal{W}_1^k(i)$ is the same, as is also the case for the fraction $1 - \theta^k$ choosing in $t = 1$. Denote variables specific to early price-setters by $i =$ early and late price-setters by $i =$ late. Then, the first-order conditions with respect to $N_1^k(\text{early})$ and C_1^k can be combined to get

$$\mathcal{W}_1^k(\text{early}) = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_0 \left[\left(C_1^k\right)^{1-\rho} \left(H(N_1^k(\text{early}))\right)^{-\rho} H'(N_1^k(\text{early})) \right]}{\mathbb{E}_0 \left[\frac{H(N_1^k(\text{early}))^{1-\rho} \left(C_1^k\right)^{-\rho}}{P_1^k} \right]},$$

for the θ^k fraction of wages selected early, in $t = 0$ and

$$\mathcal{W}_1^k(\text{late}) = \frac{\epsilon}{\epsilon - 1} \frac{\left(C_1^k\right)^{1-\rho} H'(N_1^k(\text{late}))}{\frac{H(N_1^k(\text{late}))\left(C_1^k\right)^{-\rho}}{P_1^k}},$$

for the $1 - \theta^k$ fraction that set wages in t=1, which can be simplified to

$$\frac{\mathcal{W}_1^k(\text{late})}{P_1^k} = \frac{\epsilon}{\epsilon - 1} \frac{C_1^k H'(N_1^k(\text{late}))}{H(N_1^k(\text{late}))}.$$

Define

$$mrs_1^k(i) = \frac{C_1^k H'(N_1^k(i))}{H(N_1^k(i))}.$$

Let $w_1^k(i) \equiv \frac{\mathcal{W}_1^k(\text{late})}{P_1^k}$ denote real wages of type i. Lastly, let hatted variables denote deviations from $t = 0$ values for nominal prices and wages, and perfect foresight $t = 1$ values for quantities.[39] Then, log-linearizing the expressions above yields

[39] More formally, we linearized around the value quantities obtained from the model if shocks have zero variance.

$$\widehat{w}_1^k(\text{early}) + \pi_1^{p,k} = \mathbb{E}_0(\widehat{mrs}_1(\text{early}) + \pi_1^{p,k}),$$

and

$$\widehat{w}_1^k(\text{late}) = \widehat{mrs}_1(\text{late})$$

where $1+\pi_1^{p,k} = P_1^k/P_0^k$ (so that $\pi_1^{p,k} \simeq \widehat{P_{1k}}$ since we are log-linearizing around $t=0$ values). We add inflation on both sides to account for households choosing nominal wages, taking nominal MRS into account.

Also, note that, for both $i \in \{\text{early}, \text{late}\}$,

$$\begin{aligned}
\widehat{mrs}_1^k(i) &= \varphi \widehat{N}_1^k(i) + \widehat{C}_1^k \\
&= -\epsilon\varphi\left(\widehat{w}_1^k(i) - \widehat{w}_1^k\right) + \varphi \widehat{N}_1^k + \widehat{C}_1^k \\
&\equiv -\epsilon\varphi(\widehat{w}_1^k(i) - \widehat{w}_1^k) + \widehat{mrs}_1^k
\end{aligned}$$

where $\varphi \equiv H''(N)N/H'(N) - H'(N)N/H(N)$ is the elasticity of the mrs's with respect to N close to its perfect foresight value, and the second equation follows from the labor demand equation, and the last equation defines $\widehat{mrs}_1^k$, an aggregate index for the marginal rate of substitution of household of type k.

Combining expressions, we obtain

$$\widehat{w}_1^k(\text{early}) + \pi_1^{p,k} = \mathbb{E}_0\left[\epsilon\varphi\widehat{w}_1^k + \widehat{mrs}_1 - \epsilon\varphi\widehat{w}_1^k(\text{early}) + \pi_1^{p,k}\right]$$

for early wage setters,

$$\widehat{w}_1^k(\text{late}) = \epsilon\varphi\widehat{w}_1^k + \widehat{mrs}_1 - \epsilon\varphi\widehat{w}_1^k(\text{late})$$

for late ones.

Solving for $\widehat{w}_1^k(i)$, this can be written compactly as

$$\widehat{w}_1^k(i) + \pi_1^{p,k} = \mathbb{E}_i \left[\frac{\epsilon\varphi}{1+\epsilon\varphi} \widehat{w}_1^k + \frac{1}{1+\epsilon\varphi} \widehat{mrs}_1^k + \pi_1^{p,k} \right], \ i \in \{0,1\}$$

for $i \in \{\text{early}, \text{late}\}$, with $\mathbb{E}_{\text{early}} = \mathbb{E}_0$ and $\mathbb{E}_{\text{late}} = \mathbb{E}_1$.

The aggregate wage index satisfies (after log-linearizing)

$$\widehat{w}_1^k = \theta^k \widehat{w}_1^k(\text{early}) + (1-\theta^k)\widehat{w}_1^k(\text{late}).$$

Substituting out $\widehat{w}_1^k(i)$,

$$\widehat{w}_1^k = \theta^k \left(\mathbb{E}_0 \left[\frac{\epsilon\varphi}{1+\epsilon\varphi} \widehat{w}_1^k + \frac{1}{1+\epsilon\varphi} \widehat{mrs}_1^k + \pi_1^{p,k} \right] - \pi_1^{p,k} \right) + (1-\theta^k) \left[\frac{\epsilon\varphi}{1+\epsilon\varphi} \widehat{w}_1^k + \frac{1}{1+\epsilon\varphi} \widehat{mrs}_1^k \right]$$

which we can rewrite as

$$\widehat{w}_1^k - \mathbb{E}_0 \widehat{w}_1^k = \mathbb{E}_0 \left[\frac{1}{1+\epsilon\varphi} \left(\widehat{mrs}_1^k - \widehat{w}_1^k \right) + \pi_1^{p,k} \right] - \pi_1^{p,k} + \frac{1-\theta^k}{\theta^k} \frac{1}{1+\epsilon\varphi} \left(\widehat{mrs}_1^k - \widehat{w}_1^k \right). \quad (14)$$

Note that, taking expectations on both sides we can verify that

$$\mathbb{E}_0 w_1^k = \mathbb{E}_0 \widehat{mrs}_1^k$$

so the expression simplifies to

$$\widehat{w}_1^k - \mathbb{E}_0 \widehat{w}_1^k = \mathbb{E}_0 \left[\pi_1^{p,k} \right] - \pi_1^{p,k} + \frac{1-\theta^k}{\theta^k} \frac{1}{1+\epsilon\varphi} \left(\widehat{mrs}_1^k - \widehat{w}_1^k \right).$$

Note further that we can write $\widehat{w}_1^k = \pi_1^{\mathcal{W},k} - \pi_1^{p,k}$, where $1 + \pi_1^{\mathcal{W},k} = \frac{\mathcal{W}_1^k}{\mathcal{W}_0^k}$ is the nominal wage inflation between periods 0 and 1. It follows that wage inflation satisfies

$$\widehat{\pi}_1^{\mathcal{W},k} - \mathbb{E}_0 \widehat{\pi}_1^{\mathcal{W},k} = \frac{1-\theta^k}{\theta^k} \frac{1}{1+\epsilon\varphi} \left(\widehat{mrs}_1^k - \widehat{w}_1^k \right).$$

Define $\widehat{N}_1^{n,k} \equiv \frac{1}{\varphi}\left(\widehat{w}_1^k - \widehat{C}_1^k\right)$ as the "natural" rate of employment, or the employment that would be obtained if households could freely choose how much to work at the prevailing wage index. If $\widehat{N}_1^k = \widehat{N}_1^{n,k}$, then $\widehat{mrs}_1 = \widehat{w}_1$ and wage inflation equals its expected value. More generally, we can verify that $\widehat{N}_1^{n,k} - \widehat{N}_1^k = \frac{1}{\varphi}\left(\widehat{w}_1^k - \widehat{C}_1^k\right) - \widehat{N}_1^k = \frac{1}{\varphi}\left(\widehat{w}_1^k - \widehat{C}_1^k - \varphi\widehat{N}_1^k\right) = \frac{1}{\varphi}\left(\widehat{w}_1^k - \widehat{mrs}_1^k\right)$ It follows that $\widehat{mrs}_1^k - \widehat{w}_1^k = \varphi(\widehat{N}_1^k - \widehat{N}_1^{n,k})$, so that

$$\pi_1^{\mathcal{W},k} - \mathbb{E}_0\pi_1^{\mathcal{W},k} = \frac{1-\theta^k}{\theta^k(1+\epsilon\varphi)}\varphi(\widehat{N}_1^k - \widehat{N}_1^{n,k}). \tag{15}$$

Taking expectations from both sides, we can verify from above that early households expect the unemployment rate to equal its natural level.

$$\mathbb{E}_0 N_1^k = \mathbb{E}_0 N_1^{n,k}.$$

Lastly, from the budget constraint

$$\widehat{C}_1^k = \widehat{w}_1^k + \widehat{N}_1^k$$

Recall that

$$\widehat{N}_1^{n,k} - \widehat{N}_1^k = \frac{1}{\varphi}\left(\widehat{w}_1^k - \widehat{C}_1^k\right) - \widehat{N}_t^k$$

so that

$$\widehat{N}_1^{n,k} - \widehat{N}_1^k = \frac{1+\varphi}{\varphi}\widehat{N}_1^k$$

Now, given $N_1^k = \bar{N}_1^k(1-u_1^k)$, we have that $\widehat{N}_1^k = -\frac{1}{1-u^k}(u_1^k - u^k)$

so that

$$\widehat{N}_1^k - \widehat{N}_1^{n,k} = -\frac{1+\varphi}{\varphi}(u_1^k - u^k)$$

and we can re-express the wage Phillips Curve as a function of the unemployment rate,

$$\pi_1^{\mathcal{W},k} - \mathbb{E}_0 \pi_1^{\mathcal{W},k} = -\frac{(1-\theta^k)(1+\varphi)}{\theta^k(1+\epsilon\varphi)}(u_1^k - u^k). \tag{16}$$

A.2 Employment Agencies

Workers sell their labor to a layer of employment agencies. They purchase the varieties of labor and aggregate them into a single homogeneous good that is then sold to firms at a price $\bar{\mathcal{W}}$.

The production function is

$$\bar{N}_1 = \sum_k \left[\int \left(N_1^k(i) \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}},$$

where $N_1^k(i)$ is the quantity provided of variety i of labor by workers of type k.

Profit maximization implies the iso-elastic residual demand curves

$$N_1^k(i) = \left(\mathcal{W}_1^k(i) / \mathcal{W}_1^k \right)^{-\epsilon} N_1^k,$$

where N_1^k is the total number of workers of type k in demand and $\mathcal{W}_1^k = \left[\int \mathcal{W}_1^k(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$.

Employment agencies cannot choose which group to hire from. Rather, they hire among the different types in proportion to their equilibrium employment. That is, firms take the ratios $\frac{N_1^k}{\sum_k N_1^k}$ as equal to their availability in equilibrium, which can be written in terms of the unemployment rates u_1^k:

$$N_1^k = (1 - u_1^k)\bar{N}^k.$$

For the unemployment rates u_1^k, we impose the stable ratios in the data, which we take as reflecting deep structural features affecting unemployment across races:

$$\frac{u_1^k}{u^k} = \frac{u_1}{u} \tag{17}$$

where u_1 is the aggregate unemployment rate, and variables without subscript denote their long-term averages.

Zero profits imply the price of the employment agency service

$$\bar{\mathcal{W}}_1 = \sum_k \frac{N_1^k}{\sum_{k'} N_1^{k'}} \mathcal{W}_1^k.$$

A.3 Final goods firms

Firms produce final varieties for households and distribute profits to a separate class of firm owners who consume all the profits. They are monopolists who choose their (nominal) prices and satisfy the demand for their products at the chosen price. Within each sector $j \in \{1, ..., J\}$, a fraction ϑ^j of producers sets their price in $t = 0$ and the remaining fraction $1 - \vartheta^j$, sets their price in $t = 1$.

Firms use labor in production purchased from employment agency firms. They also incur labor hiring costs, which are higher if the unemployment rate is low.[40] This means that marginal costs are sensitive to the unemployment rate over and above its impact on wages, potentially leading to prices that react more to the business cycle than wages. The (nominal) marginal cost of production is

$$MC_1 = e^{\tilde{\eta}_1} \mathcal{M}(u_1 - u) \bar{\mathcal{W}}_1,$$

where $\bar{\mathcal{W}}_1 = \sum_k \frac{N_1^k}{\sum_{k'} N_1^{k'}} \mathcal{W}_1^k$, for all $j \in \{1, ..., J\}$, $\mathcal{M}' < 0$. Finally, $\tilde{\eta}_1$ is an exogenous cost-push shock, capturing other sources of marginal cost fluctuations such as international commodity prices.

Producers selling a variety v face residual demand for their goods given by $\left(P_1^j(v)/P_1^j\right)^{-\nu} Y_1^j$, where Y_1^j is total output of good j in the economy.

When setting prices at time t, their problem can be stated as

$$\max_{P_1^j(v)} \mathbb{E}_v \left[\left(P_1^j(v)/P_1^j\right)^{-\nu} Y_1^j \left(P_1^j(v) - MC_1\right) \right],$$

where $P_1^j(i)$ is the price that all firms changing prices in sector j choose whether they are changing early ($i =$ early) or late ($i =$ late), and MC_1 is their marginal cost of production (in nominal terms). The first order condition for $P_1^j(v)$ (after some rearranging) is

[40] See Ravenna and Walsh (2008) for a model with this characteristic

$$0 = \mathbb{E}_v \left[\left(P_1^j(v)/P_1^j \right)^{-\nu} Y_1^j \left[-(\nu - 1) + \nu MC_1 \frac{1}{P_1^j(v)} \right] \right] \Rightarrow$$
$$P_1^j(v) = \frac{\nu}{\nu - 1} \mathbb{E}_v \left[\frac{\left(P_1^j\right)^\nu Y_1^j}{\mathbb{E}_v \left[\left(P_1^j\right)^\nu Y_1^j \right]} MC_1 \right].$$

The sectoral price level is given by

$$\left(P_1^j\right)^{1-\nu} = \vartheta^j \left(P_1^j(\text{early})\right)^{1-\nu} + (1 - \vartheta^j) \left(P_1^j(\text{late})\right)^{1-\nu}.$$

Using lower case to write the expression above in real terms (deflating by sector-specific price index P_1^j and using $1 + \pi_1^j = P_1^j / P_0^j$,

$$P_1(v) = \frac{\nu}{\nu - 1} \mathbb{E}_v \left[\frac{\left(p_1^j\right)^\nu Y_1^j}{\mathbb{E}_v \left[\left(p_1^j\right)^\nu Y_1^j \right]} (1 + \pi_1^j) mc_1^j \right].$$

Using hats to denote log-deviations from $t = 0$ and $\pi_1^j = \frac{P_1^j}{P_0^j}$ for inflation in sector j, we have

$$\pi_1^j = \vartheta^j \mathbb{E}_0 \left[\widehat{mc}_1^j + \pi_1^j \right] + (1 - \vartheta^j) \left(\widehat{mc}_1^j + \pi_1^j \right)$$

From the expression for the nominal marginal cost (A.3), we have that

$$\widehat{mc}_1^j = \tilde{\eta}_1 - \zeta(u_1 - u) + \sum_k n^k \pi_1^{\mathcal{W},k} - \pi_1^j,$$

where $\zeta = \mathcal{M}'/\mathcal{M}$ is the semi-elasticity of hiring costs per unit produced with respect to the unemployment rate and $n^k \equiv N^k/N$ is the steady-state ratio of workers of type k.

Recall that $\mathbb{E}_0(u_1 - u_1^n) = 0$. Also, assuming $\mathbb{E}_0 \tilde{\eta} = 0$, it follows that

$$\mathbb{E}_0 \widehat{mc}_1^j = \sum_k n^k \mathbb{E}_0 \pi_1^{\mathcal{W},k} - \mathbb{E}_0 \pi_1^j.$$

It follows that

$$\pi_1^j = \vartheta^j \sum_k n^k \mathbb{E}_0 \pi_1^{\mathcal{W},k} + (1-\vartheta^j)\left(\tilde{\eta}_1 - \zeta(u_1 - u) + \sum_k n^k \pi_1^{\mathcal{W},k}\right). \tag{18}$$

Using the expression for the household-specific price index (13), we can obtain the household-specific inflation rate

$$\pi_1^{p,k} = \sum_j \alpha^{kj}\pi_1^j = \sum_j \alpha^{kj}\left(\vartheta^j \sum_k n^k \mathbb{E}_0 \pi_1^{\mathcal{W},k} + (1-\vartheta^j)\left(\tilde{\eta}_1 - \zeta(u_1 - u) + \sum_k n^k \pi_1^{\mathcal{W},k}\right)\right). \tag{19}$$

Firms generate profits, which are distributed to households proportionately so that households of type k receive a fraction f^k of profits:

$$\bar{N}^k \Pi_1^k = f^k \Pi_1^j(v) \ \forall \{k, j, v\}$$

A.4 Monetary Policy

The Central Bank trades off inflation and unemployment. In particular, the Central Bank follows a general monetary policy rule

$$\sum_k \phi^k \pi_1^{p,k} = \sum_k \omega^k (u_1^k - u^k) - \tilde{\varepsilon}_1,$$

where $\tilde{\varepsilon}_1$ is a contractionary monetary policy shock. The Central Bank trades off a weighted average of inflation rates experienced by different types of households against a weighted average of their unemployment rates.

Given

In equilibrium, unemployment rates are proportional to one another $\left(\frac{u_1^k}{u^k} = \frac{u_1}{u}\right)$, so that the rule simplifies to

$$\sum_k \phi^k \pi_1^{p,k} = \Omega(u_1 - u) - \tilde{\varepsilon}_1,$$

with $\Omega \equiv \sum \frac{u^k}{u}\omega^k$.

Using the household-specific price Phillips Curves (19) derived above, the monetary policy rule can be expressed as

$$\sum_{kj} \phi^k \alpha^{kj} \left(\vartheta^j \mathbb{E}_0 \pi_1^{\mathcal{W}} + (1-\vartheta^j)\pi_1^{\mathcal{W}}\right) = \left[\Omega + \zeta \sum_{kj} \phi^k \alpha^{kj}(1-\vartheta^j)\right](u_1 - u) - \sum_{kj} \phi^k \alpha^{kj}(1-\vartheta^j)\tilde{\eta}_1 - \tilde{\varepsilon}_1,$$

where $\pi_1^{\mathcal{W}} \equiv \sum n^k \pi_1^{\mathcal{W},k}$. Applying the wage Phillips Curves and rearranging, this simplifies further to (using the fact that $\sum_j \alpha^{kj} = 1$ and normalizing $\sum_k \phi^k = 1$)

$$\left[\Omega + \left(\zeta + \sum_{k'} n^{k'} \psi^{\mathcal{W},k'}\right) \times \sum_{kj} \phi^k \alpha^{kj}(1-\vartheta^j)\right](u_1 - u) = \mathbb{E}_0 \pi_1^{\mathcal{W}} + \tilde{\eta}_1 \sum_{kj} \phi^k \alpha^{kj}(1-\vartheta^j) + \varepsilon_1,$$

where $\psi^{\mathcal{W},k} \equiv \frac{1-\theta^k}{\theta^k}\frac{\varphi}{1+\epsilon\varphi}\frac{1}{u}$ is the coefficient for the wage Phillips Curve (16) with aggregate unemployment substituted for group unemployment.

The expression above can be written more compactly as

$$u_1 - u = \Psi \mathbb{E}_0 \left(\pi_1^{\mathcal{W}}\right) + \tilde{\Phi}\tilde{\eta}_1 + \Psi \tilde{\varepsilon}_1,$$

with

$$\Psi = \left[\Omega + \left(\zeta + \sum_{k'} n^{k'} \psi^{\mathcal{W},k'}\right) \times \sum_{kj} \phi^k \alpha^{kj}(1-\vartheta^j)\right]^{-1}$$

and

$$\tilde{\Phi} = \Psi \sum_{kj} \phi^k \alpha^{kj}(1-\vartheta^j).$$

Taking expectations on both sides and recalling that $\mathbb{E}_0(u_1 - u) = 0$, we have that $\mathbb{E}_0 \pi_1^{\mathcal{W}} = 0$, allowing the expression to simplify further to

$$u_1 - u = \Phi \eta_1 + \varepsilon_1, \tag{20}$$

where, for notational simplicity, we renormalize $\varepsilon = \Psi\tilde{\varepsilon}_1$ and $\eta = \frac{\tilde{\eta_1}}{\zeta+\sum_k n^k \psi^{\mathcal{W},k}}$ and $\Phi = \tilde{\Phi}(\zeta + \sum_k n^k \psi^{\mathcal{W}}) = \frac{(\zeta+\sum_k n^k\psi^{\mathcal{W},k})\times\sum_{kj}\phi^k\alpha^{kj}(1-\vartheta^j)}{\Omega+\left(\zeta+\sum_{k'} n^{k'}\psi^{\mathcal{W},k'}\right)\times\sum_{kj}\phi^k\alpha^{kj}(1-\vartheta^j)}$

Note that $\Phi \in \{0,1\}$, and declines with Ω, the weight given to unemployment fluctuations in the policy rule.

A.5 Solving for Equilibrium Inflation and Unemployment

A.5.1 Solving for expected sectoral and wage inflation

From above we have

$$\mathbb{E}_0 \sum_k n^k \pi_1^{\mathcal{W}^k} = 0.$$

So that, in symmetric equilibrium with $\mathbb{E}_0 \widehat{w}_1^k$ identical for all k, we have that $\widehat{w}_1^k = 0$. Furthermore, taking expectations on both sides of the price Phillips Curve (18), we have that, sector j,

$$\mathbb{E}_0 \pi_1^j = \sum_k n^k \mathbb{E}_0 \pi_1^{\mathcal{W},k} = 0.$$

We also have that $\pi^{p,k} = \sum_j \alpha^{kj}\pi^j$, so that

$$\mathbb{E}_0 \pi^{p,k} = 0.$$

A.5.2 System Reduction and Aggregation

The system has a recursive structure, with $\pi_1^{\mathcal{W},k} - \mathbb{E}_0\pi_1^{\mathcal{W},k}$ depending on deviations between unemployment and the natural rate for each group and $\pi_1^{kj} - \mathbb{E}_0\pi_1^{kj}$ depending on both that and $\pi_1^{\mathcal{W},k} - \mathbb{E}_0\pi_1^{\mathcal{W},k}$. We can use that structure to write inflation for each group $\pi^{p,k}$ as a function of the aggregate unemployment rate.

In particular, employment agencies hire workers proportionately to their groups, so that $u_1^k/u^k = u_1/u$, where we drop the k superscripts to denote aggregate unemployment rates. It follows that the wage Phillips Curve can be written compactly as

$$\pi_1^{\mathcal{W},k} - \mathbb{E}_0\pi_1^{\mathcal{W},k} = -\psi^{\mathcal{W},k}(u_1 - u). \tag{21}$$

where

$$\psi^{\mathcal{W},k} \equiv \frac{(1-\theta^k)(1+\varphi)}{\theta^k(1+\epsilon\varphi)} \frac{u^k}{u}$$

Furthermore, from equation (16) and $u_1^k - u^k = \frac{u^k}{u}(u_1 - u)$, we have that

$$\pi_1^j = -(1-\vartheta^j)\left(\tilde{\eta}_1 + (\zeta + \sum_k n^k \psi^{\mathcal{W},k})(u_1 - u)\right),$$

We can also rewrite the sectoral Phillips Curves more compactly as

$$\pi_1^j = -\psi^{p,j}\left(u_1 - u - \eta_1\right),$$

where

$$\psi^{p,j} = (1-\vartheta^j)\left(\zeta + \sum_k n^k \psi^{\mathcal{W},k}\right),$$

and

$$\eta_1 = \frac{\tilde{\eta}_1}{\zeta + \sum_k n^k \psi^{\mathcal{W},k}}.$$

Lastly, inflation for group k is given by a weighted average of inflation in each of the sectors:

$$\pi_1^{p,k} = \sum_j \alpha^{kj} \pi_1^j$$

so that

$$\pi_1^{p,k} = -\psi^{p,k}\left(u_1 - u - \eta_1\right), \tag{22}$$

where $\psi^{p,k} \equiv \sum_j \alpha^{kj}\psi^j = \left(\xi + \sum_k n^k \psi^{\mathcal{W},k}\right) \sum_j \alpha^j (1-\vartheta^j)$.

A.5.3 Equilibrium inflation and unemployment

We can now write equilibrium price and wage inflation as a function of shocks. In particular, given the monetary policy rules pinning down $u_1 - u$ as a function of shocks through equation (20), and the wage and price Phillips Curves (equations (21) and (22), respectively) we have that

$$\pi_1^{p,k} = \psi^{p,k}\left[(1-\Phi)\eta_1 - \varepsilon_1\right],$$

and

$$\pi_1^{\mathcal{W},k} = -\psi^{\mathcal{W},k}(\Phi\eta_1 + \varepsilon_1).$$

Now we can define real labor income for household in group k as

$$Y_1^k = \frac{\mathcal{W}_1^k}{P_1^k} N_1^k = \frac{\mathcal{W}_1^k}{P_1^k}(1-u_1^k)\bar{N}^k$$

so that we can approximate

$$\begin{aligned}
\widehat{Y}_1 &= \pi_1^{\mathcal{W},k} - \pi_1^{p,k} - \frac{1}{1-u^k}(u_1^k - u^k) \\
&= \pi_1^{\mathcal{W},k} - \pi_1^{p,k} - \frac{1}{1-u^k}\frac{u^k}{u}(u_1 - u) \\
&= -\left[\psi^{\mathcal{W},k}\Phi + \psi^{p,k}(1-\Phi) - \frac{u^k}{u}\frac{1}{1-u^k}\right]\eta_1 - \left(\psi^{\mathcal{W},k} - \psi^{p,k} - \frac{u^k}{u}\frac{1}{1-u^k}\right)\varepsilon_1
\end{aligned}$$

B Second Order Approximations (proof of Proposition 1)

Proposition 1 describes properties for a second-order approximation for the income variance and welfare approximation.

As a preliminary, consider $V:\mathbb{R}\rightarrow\mathbb{R}$ and $f:\mathbb{R}\rightarrow\mathbb{R}$ to be twice differentiable. Let $\hat{\eta}$ be a random variable with $\mathbb{E}\hat{\eta}=0$ and $\mathbb{E}\hat{\eta}^2=1$, where $\mathbb{E}$ is the expectations operator. Let $\sigma>0$ be a scalar. Let $dV\left(e^{f(\sigma\hat{\eta})}\right)=V\left(e^{f(\sigma\hat{\eta},\sigma)}\right)-V\left(e^{f(0,0)}\right)$ be the change in V associated with a change in σ away from zero for any given $\hat{\eta}$. Then, using a second-order Taylor expansion around $\sigma=0$,

$$\begin{aligned}
d\mathbb{E}V\left(e^{f(\sigma\hat{\eta},\sigma)}\right)=\mathbb{E}dV\left(e^{f(\sigma\eta,\sigma)}\right)&=\mathbb{E}\left[V'e^f f_{\sigma\eta}\eta\sigma+\frac{1}{2}\left[V''\left(e^f f_{\sigma\eta}\right)^2\hat{\eta}^2\right]\sigma^2\right]\\
&+\frac{1}{2}\mathbb{E}V'e^f f_{\sigma\eta}^2\hat{\eta}^2\sigma^2\\
&+\frac{1}{2}\mathbb{E}\left[V'e^f\left(f_{\sigma\eta,\sigma\eta}\hat{\eta}^2+f_{\sigma,\sigma}\right)\sigma^2\right]+\mathcal{O}(\sigma^3)\\
&=\frac{1}{2}\left[\left(V''e^f+V'\right)\left(f_{\sigma\eta}\right)^2+V'\left(f_{\sigma\eta,\sigma\eta}+f_{\sigma,\sigma}\right)\right]e^f\sigma^2+\mathcal{O}(\sigma^3)
\end{aligned}$$

where for notational simplicity, we omit the arguments of f, V and their derivatives when evaluated at $\sigma=0$, and $\mathcal{O}(\sigma^3)$ denotes all terms that decline with σ at a faster rate than σ^3. Further, the first equality incorporates the fact shown by Schmitt-Grohé and Uribe (2004) show that $f_\sigma=f_{\sigma\eta,\sigma}=0$.

Furthermore,

$$\begin{aligned}
dV\left(\mathbb{E}e^{f(\sigma\hat{\eta},\sigma)}\right)&=V'\mathbb{E}e^f f_{\sigma\eta}\eta\sigma+\frac{1}{2}V''\left(\mathbb{E}e^f f_{\sigma\eta}\hat{\eta}\right)^2\sigma^2\\
&+\frac{1}{2}V'\mathbb{E}e^f\left(f_{\sigma\eta}\right)^2\eta^2\sigma^2+\frac{1}{2}\mathbb{E}V'e^f(f_{\sigma\eta,\sigma\eta}+f_{\sigma,\sigma})\hat{\eta}^2\sigma^2+\mathcal{O}(\sigma^3)\\
&=\frac{1}{2}V'\left[\left(f_{\sigma\eta}\right)^2+f_{\sigma\eta,\sigma\eta}+f_{\sigma,\sigma}\right]e^f\sigma^2+\mathcal{O}(\sigma^3)
\end{aligned}$$

So that, combining the expressions and using $\mathbb{E}\hat{\eta}=0$ and $\mathbb{E}\hat{\eta}^2=1$,

$$d\mathbb{E}V\left(e^{f(\sigma\hat{\eta},\sigma)}\right)=dV\left(\mathbb{E}e^{f(\sigma\hat{\eta},\sigma)}\right)+\frac{1}{2}V''\left(e^f f_{\sigma\eta}\right)^2\sigma^2+\mathcal{O}(\sigma^3)$$

Since $\mathbb{E}V\left(e^{f(0,0)}\right) = V\left(\mathbb{E}e^{f(0,0)}\right) = V\left(e^{f(0,0)}\right)$, we can write this more simply as

$$\mathbb{E}V\left(e^{f(\sigma\hat{\eta},\sigma)}\right) = V\left(\mathbb{E}e^{f(\sigma\hat{\eta},\sigma)}\right) + \frac{1}{2}V''\left(e^f f_{\sigma\eta}\right)^2\sigma^2 + \mathcal{O}(\sigma^3)$$

Variance For any $f(\sigma\hat{\eta},\sigma)$ the variance of $e^{f(\sigma\hat{\eta},\sigma)}$ is given by

$$var\left(e^{f(\sigma\hat{\eta},\sigma)}\right) = \mathbb{E}\left(e^{f(\sigma\hat{\eta},\sigma)}\right)^2 - \mathbb{E}^2\left(e^{f(\sigma\hat{\eta},\sigma)}\right)$$

so that, taking a second order approximation around $\sigma = 0$ (in which case $var\left(e^{f(\sigma\hat{\eta},\sigma)}\right) = 0$),

$$var\left(e^{f(\sigma\hat{\eta}),\sigma}\right) = d\mathbb{E}\left(e^{f(\sigma\hat{\eta},\sigma)}\right)^2 - d\left(\mathbb{E}\left(e^{f(\sigma\hat{\eta})},\sigma\right)\right)^2 + \mathcal{O}(\sigma^3)$$

Applying the formula above with $V(x) = x^2$,

$$var\left(e^{f(\sigma\hat{\eta},\sigma)}\right) = \left(e^f f_{\sigma\eta}\right)^2\sigma^2 + \mathcal{O}(\sigma^3)$$

Note that since, in the absence of monetary shocks, all variables in $t = 1$ are functions only of the realization of η_1, we can write $Y_1^k = e^{f^Y(\eta_1,\sigma)}$ with $\eta_1 = \sigma^2\hat{\eta}_1$. It follows that $\widehat{Y}_1^k = f_\eta^Y(0,0)\eta^1$, so that $\mathbb{E}\widehat{Y}_1^k = 0$ and $var(\widehat{Y}_1^k) = \left(f_\eta^Y(0,0)\right)^2$. It follows that

$$var(Y_1^k(\eta_1,\sigma)) = \left(Y^k\right)^2 var(\widehat{Y}_1^k) + \mathcal{O}(\sigma^3),$$

or

$$var\left(\frac{Y_1^k(\eta_1,\sigma)}{Y^k}\right) = var(\widehat{Y}_1^k) + \mathcal{O}(\sigma^3),$$

where $Y^k = Y_1^k(0,0)$ proving the first part of the proposition

Utility Noting that $N_1^k(i)$ may differ whether the corresponding wages are set early or late, we can write the household utility as a function of η, as

$$V(U) = \frac{U^{1-\rho}}{1-\rho}$$

where

$$U = u\left(C_1^k(\eta), \int N_1^k(i,s)(\eta) di ds\right) = u(C_1^k(\eta), \Delta_1^k N_1^k(\eta)) = C_1^k(\eta) H\left(\Delta_1^k(\eta) N_1^k(\eta)\right)$$

and

$$\Delta_1^k(\eta) = \left[\theta^k \left(\frac{\mathcal{W}_1^k(\text{early})(\eta)}{\mathcal{W}_1^k(\eta)}\right)^{-\epsilon} + (1-\theta^k)\left(\frac{\mathcal{W}_1^k(\text{late})(\eta)}{\mathcal{W}_1^k(\eta)}\right)^{-\epsilon}\right]$$

Let $f^u(\eta,\sigma) = \log u\left(C_1^k(\eta,\sigma), H\left(\Delta_1^k(\eta,\sigma) N_1^k(\eta,\sigma)\right)\right)$. Then, from above we can write

$$\begin{aligned} d\mathbb{E}V\left(e^{f^u(\eta,\sigma)}\right) - dV\left(\mathbb{E}e^{f^u(\eta,\sigma)}\right) &= \frac{1}{2}V''\left(e^{f^u(0,0)} f_\eta^u\right)^2 \sigma^2 + \mathcal{O}(\sigma^3) \\ &= -\frac{1}{2}\rho V\left(f_\eta^u\right)^2 \sigma^2 + \mathcal{O}(\sigma^3) \end{aligned}$$

Also, we have that

$$f_{\sigma\eta}^u = \frac{u_c}{u}\frac{\partial C}{\partial \eta} + \frac{u_N}{u}\Delta\frac{\partial N}{\partial \eta} + \frac{u_N}{u}\frac{\partial \Delta}{\partial \eta},$$

where, again, to save on notation, we omit the arguments of the functions, all of which are evaluated at $(\eta,\sigma) = (0,0)$.

Further, we can verify that $\Delta_1^k = 1$ since in that case $\mathcal{W}_1^k(\text{early}) = \mathcal{W}_1^k(\text{late}) = \mathcal{W}_1^k$ and $\frac{\partial \Delta_1^k}{\partial \eta} = 0.$, so that the expression simplifies to

$$f_\eta^u(0) = \frac{u_c}{u}\frac{\partial C}{\partial \eta} + \frac{u_N}{u}\frac{\partial N}{\partial \eta},$$

From the household budget constraint, we have that

$$C_1^k(\eta) = w_1^k(\eta)\Delta_1^k(\eta) N_1^k(\eta)$$

so that, again noting the properties of Δ

$$\frac{\partial C_1^k}{\partial \eta} = \frac{\partial w_1^k}{\partial \eta} N_1^k + w_1^k \frac{\partial N_1^k}{\partial \eta}$$

Using this to substitute out $\frac{\partial C_1^k}{\partial \eta}$ in the expression for $f^{u\prime}(0)$ and noting that $u_N < 0$ we can write this as

$$f^{u\prime}(0) = \frac{u_c C^k}{u} \left(\frac{\partial \ln w_1^k}{\partial \eta} + \frac{w^k - b^k}{w^k} \frac{\partial \ln N_1^k}{\partial \eta} \right)$$

where $b_1^k \equiv \frac{|U_N^k|}{U_c^k}$ is the perfect foresight value of leisure in terms of consumption for household k. Thus, utility rises with wages and with employment to the extent that wages are higher than that value. Since $Y_1^k = C_1^k = w_1^k N_1^k$, We can also rewrite this as

$$f^{u\prime}(0) = \frac{u_c C^k}{u} \left(\frac{\partial \ln Y_1^k}{\partial \eta} - \frac{b^k}{w^k} \frac{\partial \ln N_1^k}{\partial \eta} \right)$$

Lastly, given that $U(C, N) = CH(N)$, we have that $u_c C/u = 1$ and it follows that

$$d\mathbb{E}V\left(e^{f^u(\sigma\widehat{\eta})}\right) - dV\left(\mathbb{E}e^{f^u(\sigma\widehat{\eta})}\right) = -\frac{\rho}{2} V \left(\frac{\partial \ln Y_1^k}{\partial \eta} - \frac{b^k}{w^k} \frac{\partial \ln N_1^k}{\partial \eta} \right)^2 \sigma^2 + \mathcal{O}(\sigma^3)$$

Adopt the notation

$$\widehat{Y}_t = \frac{\partial \ln Y_1^k}{\partial \eta} \eta = f_Y' \eta$$

etc to rewrite the expression as

$$\frac{d\mathbb{E}V\left(e^{f^u(\sigma\widehat{\eta})}\right) - dV\left(\mathbb{E}e^{f^u(\sigma\widehat{\eta})}\right)}{V} = -\frac{\rho}{2} var \left(\widehat{Y}_t - \frac{b^k}{w^k} \widehat{N}_t \right) + \mathcal{O}(\sigma^3)$$