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Taxation, Compliance, and Clandestine Activities

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Taxation, Compliance, and Clandestine Activities*

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Abstract

We investigate the trade-off policymakers face between raising tax revenues for public good provision and mitigating the distortionary effects of taxation when individuals can evade taxes and allocate work hours between legal and clandestine (illicit) activities. These distortions lower the constrained optimal tax rate and result in the under-provision of the public good. This under-provision problem is mitigated when surplus from the audit agency is seamlessly transferred to the taxing authorities. Extensions of the basic model incorporate agent heterogeneity and a more general specification of the concealment cost function for infringements.

Keywords: Taxation, evasion, compliance, legal and illicit activities, public goods, externalities
JEL Classification: H2, H4, K10

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1 Introduction

Income tax evasion is a pervasive problem. Developing countries with weak institutions and limited enforcement resources are particularly hampered by revenue losses due to such evasion. Existing theoretical and empirical models of tax evasion have typically examined evasion associated with legal economic activity, where the underlying activity itself is not socially harmful. Yet a substantial share of tax evasion is linked to clandestine or illicit activities that both erode tax revenues and generate additional social harm.[1] Almost by definition, income from illicit activities cannot be reported to tax authorities, as the underlying illegal activity might be revealed. Examples of illicit activities that remain hidden include the production of illegal drugs and the trade in illegal weapons. In other cases, income from illicit activities could, in principle, be reported to tax authorities, but hiding part of this income might be more profitable. Examples include evading regulations regarding construction (e.g., infrastructure that is not code-compliant and hence may lead to public harm in the event of a catastrophic event like an earthquake or a storm) or product safety. In addition, violations of environmental regulations belong to this group. Polluting the environment is illegal, but the profits from this production can be reported to tax authorities without revealing the illicit elements in the production process. Although the public economics literature, such as Slemrod and Yitzhaki (2002), has extensively analyzed the tax evasion problem, it has not yet fully explored the relationship of this issue with the incentives to engage in illegal activities and the provision of public goods.

Suppose that an income tax is used to finance a public good. Higher tax rates sub-

[1] In the paper, illegal activities refer to those forbidden by law or statute. We use the term "clandestine" or "illicit" activities to refer to practices that are socially harmful or institutionally disapproved, which are, in addition, conducted in violation of existing regulations.

stantially increase incentives for tax evasion and increase the relative payoffs of illicit activities. Hence, more time will be devoted to such activities, amplifying public harm. Consequently, an income tax needed to finance a public good must appropriately account for incentives for tax evasion and engagement in illegal activities.

The present paper has two primary purposes. The first one is to develop a micro-founded framework that analyzes the effects of an income tax used to finance the provision of a public good on tax evasion and the allocation of time across legal and illicit activities. The second primary purpose is to characterize constrained optimal taxes and auditing intensity in the presence of tax evasion and illicit opportunities. In both cases, we analyze how different institutional arrangements affect tax revenue and welfare.

This paper has two primary objectives. First, it develops a micro-founded framework to analyze how an income tax, used to finance a public good, affects tax evasion and the allocation of time between legal and illicit activities. Second, it characterizes constrained-optimal taxes and auditing intensity in the presence of tax evasion and illicit opportunities. In both cases, the analysis shows how alternative institutional arrangements influence tax revenue and welfare.

We start with a basic model that considers identical individuals devoting a given endowment of time to leisure, legal, and illegal activities. Income from legal and illicit sources, net of tax payments on declared income, fines, and expenses incurred to conceal income, is spent on a consumption good. The income tax creates an incentive to substitute away from legitimate income to covert illegal income-earning opportunities outside the ambit of income taxation. The agent incurs a concealment cost to evade taxes on legal income or hide illegal income. The concealment costs are affected differently by the two types of income. Furthermore, we allow for the possibility that the concealment cost function may be characterized by substitutability (i.e., concealing a given amount of legal income may become more costly due to a marginal increase in illegal income and vice versa) or complementarity. The introduction of the general concealment cost function enables us to explain various types of observed behavior, including why some individuals report income accurately, while others may, in contrast, overreport their legal earnings.

The government raises revenues from the income tax on reported income. An audit agency collects fines on tax evasion and illegal income. The analysis considers different possibilities about the transferability of the audit agency's surplus (of fines over audit costs) to the government for public good provision. The analysis first considers the case where the audit level is exogenously given. In this environment, the government chooses a constrained optimal income tax to balance the marginal social damage from the tax-induced expansion of the illegal good to the net marginal social benefit from the public good funded by the additional revenues (including taxes or any surpluses transferred by the audit agency) associated with the higher tax rate. The analysis is then extended to consider joint optimal choices of the tax rate and audit level.

Section 2 thoroughly reviews the related literature. While elements of tax evasion, illegality, endogenous labor supply, income concealment efforts, adverse externalities, and tax-financed public goods have been investigated in the literature, we are not aware of a unified treatment of these issues within the context of second-best choice of policies.[2] As we show in this paper, the interaction between different relevant distortions is

[2]Concealment actions are modeled in our paper through a concealment cost function. In general, concealment costs represent the resources individuals expend to hide their illegal activities. These costs can include time and effort spent on record-keeping or creating false documents to avoid detection, financial resources used to pay for offshore accounts, bribes, or other evasion methods, and psychological costs associated with the guilt of engaging in illegal activities or being caught. Concealment costs will likely vary with the income hidden from each source. We will explore this relationship in detail in the following

critically important in characterizing the constrained optimal taxes (and the departures from efficient public good provision), highlighting the importance of a unified approach.

The general model is presented in section 3, and a benchmark central planner allocation is derived in section 4. In sections 5 and 6, we analyze a stripped-down version of the model to develop some key insights. In the baseline scenario, the government chooses an inefficiently low tax rate (i.e., the public good is underprovided compared to a social planner optimum) when the audit agency surplus cannot be used for public good provision. The lack of transferability is an extreme case of deadweight losses associated with coordination failure between different government agencies. Turning to the other polar case of full transferability, we show that the optimal tax rate is higher and the under-provision of the public good less severe (compared to the non-transferability context). Indeed, we demonstrate that over-provision of the public remains a possibility in the full-transferability case.

The level of under-provision of the public good is exacerbated as the strength of the negative externality, measured by the per-unit incidence in the production of the illegal good, increases. The worsening under-provision stems from the government holding back its income tax rate in recognition of the high externality costs of a tax-induced increase in illegal goods production. With a lower tax rate, the government's net revenue falls, so it cuts back on public goods provision. In this constrained environment, where there is no feasible instrument to eliminate the public bad, the tax rate alone cannot achieve the multiple goals of providing efficient levels of the public good and controlling the externalities.

When we extend the analysis to allow for the audit levels to be flexible, an increase

analysis.

in the detection probability increases legal income, decreases illegal income, decreases total (legal plus illegal) income, and increases reported income. This means that if audit levels can be adjusted upwards, the government can choose a higher tax rate to finance a larger level of the public good, somewhat mitigating the under-provision problem. In other words, audit probability works as a complement to the income tax rate.

Extensions of the basic model are considered in sections 7 and 8. Section 7 considers the interdependence of concealment costs of the two types of income. Section 8.1 allows for heterogeneity of agents to consider possible non-participation by some agents in illicit activities. Section 8.2 allows for separate legal and illegal income audits. Section 8.3 considers alternate penalty function formulations. While some interesting departures occur, the core insights from the basic model are largely preserved in these extensions. Section 9 provides concluding remarks.

2 Related Literature

There is a substantial literature on tax evasion. Several papers, including Slemrod and Yitzhaki (2002), Sandmo (2005), and Slemrod (2007), provide comprehensive overviews of the economics of tax evasion.[3] In this section, we discuss some work that overlaps with different aspects of our analysis.

Tax evasion and the choice between legal and illegal activities. The relationship between tax evasion and the shadow economy has been extensively explored in the literature, with numerous studies highlighting the link between individuals' choices of participating in legal or illegal activities and their tax compliance behavior. In general,

[3] The paper by Allingham and Sandmo (1972) was the first to introduce a theoretical model of tax evasion, analyzing how individuals decide to evade taxes and the associated costs, including the probability of detection and penalties. Andreoni, Erard, and Feinstein (1998) discusses various aspects of tax compliance and tax evasion. Although most articles deal with individual tax evasion, some approaches address the specific challenges of tax evasion at the firm level (Chen and Chu, 2005).

the production of illicit goods often relies on informal markets with minimal regulation and oversight, which facilitates tax evasion strategies. Several papers provide in-depth overviews of how tax evasion is closely associated with underground economic activities, contributing to the shadow economy's size; see, for instance, Schneider and Enste (2000), Buehn and Schneider (2012) and Schneider and Enste (2013). These articles explore various reasons for tax evasion, including high tax burdens, complex tax systems, and weak enforcement, which collectively lead to reduced government revenue, distorted competition, and increased inequality. Pitt and Lee (1981) examines individuals' choices between legal and illegal work in the context of tax evasion and provides valuable insights into how local economic conditions and industry-specific factors impact the propensity for tax evasion. Friedman et al. (2000) models the determinants of individual participation in unofficial (illegal) activities and finds remarkable variations between countries in tax evasion underscored by informal activities. Marjit and Mishra (2021) explores how political factors can affect the design of policies addressing tax evasion in the informal sector. In Choi and Thum (2005), corruption of government officials drives firms into illegal, unregulated production. Bandyopadhyay and Pinto (2017) focuses on illegal practices by firms in hiring undocumented immigrants, and examines how that affects the constrained optimal mix of state and federal policies that combine border enforcement measures and firm audits.

Choice between legal and illegal activities with endogenous labor supply. Early research on tax evasion often assumed a fixed labor supply, restricting individuals' ability to adjust their labor market behavior in response to changes in the underlying model parameters. Subsequent studies have relaxed this assumption. When labor supply becomes endogenous, tax policies may have unintended consequences. Studies that focus on the relationship between tax evasion and labor supply include, among others, Pencavel (1979), Baldry (1979), Hillman and Katz (1984), Cowell (1985), and Tabbach (2005). The core argument in this line of research is that the structure of the tax system can influence individuals' decisions about how much to work, how to allocate working hours between legal and illegal activities, and whether to evade taxes. The main conclusion is that when labor supply decisions are endogenous, changes in the tax rate and other model parameters have ambiguous effects on labor supply due to the interplay of income and substitution effects.[4]

Tax evasion and the cost of concealing income. Concealing taxable income from tax authorities requires dedicating wasteful real resources, including time, effort, and monetary compensation to individuals or organizations facilitating this process.[5] Slemrod (1994) and Cremer and Gahvari (1994) analyze tax evasion in models that explicitly include the cost of concealing income. Both papers conclude that the concealment technology may affect the progressivity of the tax system. Canta, Cremer, and Gahvari (2024) show that when individuals differ in their ability to conceal income, tax evasion can interact with redistributive policies in ways that reduce distortions. Bayer (2006) develops a model in which the taxpayer and the tax authority engage in a concealment-detection game. Using experimental methods, Bayer and Sutter (2009) focus on the excess burden from resource waste in detection and concealment. Balafoutas et al. (2015) look at inefficiencies arising from collusive behavior in markets. Unlike our paper, existing models have typically abstracted from the costs of simultaneously concealing income from legal

[4]For instance, Tabbach (2005) considers the allocation of time between legal activities, criminal activities, and leisure. Individuals, as a result, may adjust along the three margins in response to changes in tax rates and criminal returns. The impact of taxation on crime is, therefore, not always straightforward.

[5]The costs referred to here do not include potential penalties or stigma costs.

and illegal activities.[6]

Illegal activities that produce negative externalities. It is crucial to differentiate between economic activities in informal markets to evade taxes and illegal activities that generate negative externalities and directly harm society. Participation in illegal activities may entail a private benefit for those directly involved but may also create broader costs to society. For instance, Flores (2016) focuses on the optimal choice of law enforcement policies when illegal markets create both consumption and violence-related externalities. The most common examples of illegal goods that generate negative externalities considered in the literature concern drug use and trade (Becker, Murphy, and Grossman (2006), Mejia and Restrepo (2016)), and organized crime (Garoupa (2000)). Other illegal activities that may generate negative externalities include cybercrime and illicit activities that degrade the environment, such as illegal deforestation, wildlife trade, and dumping (Chimeli and Soares (2017), Assunção, Gandour, and Rocha (2023)). Overall, the discussions around externalities emphasize the need for a comprehensive public policy approach considering the societal impacts of illegal goods production and tax evasion.

Tax evasion and the provision of public goods. The relationship between tax rates and tax evasion becomes more complicated when public goods are considered. Tax evasion limits the government's revenue-raising capacity and affects the optimal provision of public goods. The importance of including public goods in the analysis was earlier raised by Kolm (1973) and Sandmo (1981). Falkinger (1991), and later Balestrino and Galmarini (2003) derive conditions under which tax evasion influences the optimal level of public goods. Several authors have explored the inverse relationship, suggesting

[6]The paper by Marjit, Mishra, and Mitra (2021) is somewhat related to the issue of incurring concealment costs to evade taxes. In their paper, firms use false (Sham) litigation to appeal against their stipulated tax burden to defer tax payments. The higher return earned by investing in the informal economy compensates the firms for interest-inclusive penalties in the event of an adverse court judgment.

that taxpayer attitudes toward public goods may shape tax compliance behavior. For instance, Cowell and Gordon (1988) state that individuals underreport income not only to reduce tax liability but also to express dissatisfaction with public good provision. Finally, (Besley and Persson (2013)) note that the effectiveness of tax administration, rule of law, and political incentives, are critical determinants of a government's ability to affect tax compliance, and provide public goods, which ultimately, facilitate economic development.

3 The Model

This section presents the theoretical framework, which we use in the subsequent sections 4 through 6, respectively, to describe and analyze the social planner optimum (first-best outcome), an individual agent's choices, and the government's constrained optimal choices. As discussed earlier, section 7 and sections 8.1 through 8.3 build on this framework to examine various extensions.

Individuals. Consider a continuum of identical individuals with measure one. An individual has a unit time endowment of which ℓ units are devoted to leisure, n_1 units to the production of legal goods, and n_2 units to the production of an illicit good.[7] Assume linear production functions $y_i = w_i n_i$, $w_i > 0$, $i = 1, 2$, such that the respective wage rates in the legal and illegal sectors are set parametrically at w_1 and w_2, and y_i also represents an individual's income from good i.[8] The utility of leisure and work is represented by the function $h(\ell)$, $h' > 0, h'' < 0$, where $\ell = 1 - (n_1 + n_2)$.

The individual declares $x \geqslant 0$ as taxable income and pays an income tax tx, where

[7] In the model, the "illicit" good generates a private benefit but also a negative externality. A good is legally defined as "illicit" when the negative external harm is larger than the private benefit. This issue is further clarified in section 4 where we discuss the first-best outcome of the model.

[8] Qualitatively similar results are obtained with $y_i = y_i(n_i)$, $y_i' > 0$, and $y_i'' \leqslant 0$.

the tax rate is $0 \leqslant t \leqslant 1$. The total amount of unreported legal income is $z = y_1 - x$, while the total amount of unreported income is $\tilde{z} = y_1 + y_2 - x$.[9]

An individual has to incur some cost to conceal legal income (i.e., to engage in tax evasion) and to conceal any income earned from the illegal goods sector.[10] We represent overall concealment cost as $\gamma(z, y_2) = g_z z^2/2 + g_2 y_2^2/2 + g_{z2}|z|y_2,\ g_z, g_2 > 0$.[11] Furthermore, g_{z2} can be zero or of either sign because misreporting income from one source could make it easier or more difficult to misreport income from another source, or the two concealment costs may be independent of each other. The benchmark model assumes independence ($g_{z2} = 0$), while section 7 allows for complementarity ($g_{z2} < 0$) or substitutability ($g_{z2} > 0$).

Turning to the illegal good, recall that the illegality arises out of adverse social externalities associated with the aggregate production of the good $Y_2 = \int y_2(j)dj$. This externality is measured by the function $\Psi(Y_2)$ where $\Psi(Y_2) = \Psi_0 Y_2$, $\Psi_0 > 0$.[12]

An individual derives constant marginal utility of unity from consumption of either the legal or the illegal good in addition to the utility from leisure $h(\ell)$. The individual also derives utility from the public good $\Phi(G)$ – with $\Phi(G)$, $\Phi' > 0$, $\Phi'(0) \to +\infty$, and $\Phi'' < 0$

9While x is assumed non-negative, z is not subject to such constraint. In principle, the government may observe an individual's consumption but not how income is generated. In the presence of tax evasion and the possibility of receiving income from illicit activities, the income sources can only be verified after being audited. So, while we allow $z \lessgtr 0$, we still restrict $\tilde{z} \geqslant 0$. Otherwise, the government would immediately realize that reported income exceeds observed consumption.

10Several papers include the cost of concealing income, e.g., Slemrod (1994). It could be assumed that, without such concealment effort, infringements are immediately observed by the government even in the absence of an audit.

11The cost function assumes that the cost of hiding legal and illegal income is different. This function is flexible enough to include a wide range of relevant cases. It is assumed that the Hessian of $\gamma(z, y_2)$ is positive-definite: $g_z > 0$, $g_2 > 0$, and $g_z g_2 - g_{z2}^2 > 0$.

12The analytics are amenable to a more general formulation with $\Psi'(Y_2) > 0$ and $\Psi''(Y_2) \leqslant 0$. However, the assumed linearity allows us to parameterize the magnitude of the externality by the constant Ψ_0. This parametrization is particularly helpful for simulations (presented later) where we vary the magnitude of the harmful externality to see the corresponding effects on the endogenous variables.

– and disutility $\Psi(Y_2)$ from the negative externality related to the illegal good. Therefore, the individual's utility function is $u = c + h(\ell, n_1, n_2) + \Phi(G) - \Psi(Y_2)$, where c denotes the level of private consumption of the two goods. The individual's budget constraint requires that c equals the income of the individual from the production of the two goods net of tax payments, concealment (cost) related expenses, and expected fines F (defined below). Thus, we have $c = y_1 + y_2 - tx - \gamma - F$, where $y_i = w_i n_i$, and $\ell = 1 - (n_1 + n_2)$. Section 5 describes the individual's choices of time (income) allocated to the production of the two goods and income declared (i.e., x) or alternately income not reported ($z = y_1 - x$). The individual makes these choices knowing there is a probability of being audited in which event the true income levels will be discovered and the infringements penalized. We turn to the latter issue below which describes the different scenarios of fines and characterizes penalties for tax evasion and participation in illegal goods production.

Fines and penalties. The scheme of fines and penalties faced by individuals caught underreporting or earning income from illegal sources plays a critical role in our analysis. We will, therefore, consider several alternative possibilities. Expected fines depend on both the audit probability and the penalties. Let p_z and p_2 be the probabilities of detection of tax evasion and illegal income, respectively, while α and β are the respective penalties for the two types of infringements. Expected fines depend on the aforementioned audit probabilities and penalties, as well as the extent of evasion of taxable income $tz = t(y_1 - x)$ and illegal income y_2. Therefore, we can represent the expected fine by $F \equiv F(y_1, y_2, x, t, p_z, p_2, \alpha, \beta)$, where the function can take any arbitrary form and may not be continuous or differentiable. Furthermore, we assume that the function treats tax evasion of legal income and the earnings of illegal income separately.[13]

In principle, the penalty for misreporting legal income may differ depending on whether taxable income was under- or over-reported. We therefore allow for the possibility of a penalty $\alpha^N \geqslant 0$ when $z < 0$ and $\alpha^P \geqslant 0$ when $z > 0$.[14] Assuming that $t \geqslant 0$, the total expected fine is the sum of the expected fines on evaded taxes and illegal income earned. Accordingly, the total expected fine takes the form:

$$F = \begin{cases} -p_z \alpha^N tz + p_2 \beta y_2 & \text{if} \quad z < 0 \\ p_2 \beta y_2 & \text{if} \quad z = 0 \\ p_z \alpha^P tz + p_2 \beta y_2 & \text{if} \quad z > 0 \end{cases}, \tag{1}$$

The basic model uses a simplified version of F, where a tax audit also uncovers the extent of illegal income earned such that $p = p_z = p_2$. We extend the analysis later, allowing p_z and p_2 to differ.[15] We discuss next how the fine function can represent different institutionally determined scenarios.

We assume that $\beta \geqslant 1$ such that if audited an individual's illegal income y_2 is confiscated by the government, and the government also collects an additional penalty $(\beta - 1)y_2$. We describe below four likely scenarios for fines based on different values assumed by α^N and α^P:

1. If there is no penalty for overreporting legal income ($z < 0$), then $\alpha^N = 0$, and the expected fine is $F = p_2 \beta y_2$ when $z \leqslant 0$, and $F = p_z \alpha^P tz + p_2 \beta y_2$ when $z > 0$.[16]

2. If the underreporting and the overreporting of legal income are equally penalized,

[13]The government takes the functional form of F as given. It may choose detection probabilities p but cannot manipulate the punishment function. For a discussion of why fines and detection probabilities cannot be set independently, see Andreoni (1991).

[14]Yitzhaki (1974) extends the original Allingham-Sandmo model by assuming that the penalty is a function of the tax evaded by the individual.

[15]For instance, p_2 may supplement p_z so that $p_2 = p_z + p_2'$.

[16]Typically, $\alpha^P \geqslant 1$, so that if found underreporting, individuals pay the evaded tax tz plus a penalty $tz(\alpha^P - 1)$, proportional to the tax evaded (on the amount that exceeds legal income).

then the expected fine parameter for legal income (inclusive of complete payment of evaded taxes) is $\alpha^N = \alpha^P = \alpha > 1$, such that total fine paid is $F = p(\alpha t|z| + \beta y_2)$.

3. If overreported income is credited back to the individual and, in addition, the individual is rewarded, then $\alpha^N < -1$. Assuming that the reward for underreporting is the same as the punishment for overreporting, $-\alpha^N = \alpha^P = \alpha > 1$, and $F = p\alpha tz + p\beta y_2$.

4. If there is no reward for overreporting but a simple credit of the overpaid tax, then $\alpha^N = -1$, and $\alpha^P = \alpha > 1$.[17]

For ease of reference in the following analysis, it is useful to note the following expressions for the partials of the fine function

$$F_1 = -F_x = F_z = \begin{cases} -p\alpha^N t & \text{if} \quad z < 0 \\ 0 & \text{if} \quad z = 0 \\ p\alpha^P t & \text{if} \quad z > 0 \end{cases}, \quad \text{and} \quad F_2 = p\beta, \tag{2}$$

where the subindices denote partial derivatives. The following partials related to the effects of policy variables are useful as well:

$$F_t = \begin{cases} -p\alpha^N z & \text{if} \quad z < 0 \\ 0 & \text{if} \quad z = 0 \\ p\alpha^P z & \text{if} \quad z > 0 \end{cases}, \quad \text{and} \quad F_p = \begin{cases} -\alpha^N tz + \beta y_2 & \text{if} \quad z < 0 \\ \beta y_2 & \text{if} \quad z = 0 \\ \alpha^P tz + \beta y_2 & \text{if} \quad z > 0 \end{cases}, \tag{3}$$

Government budget constraint. The government collects income taxes from all individuals. Total taxes are $T = \int tx(j)dj$. The government also provides a public G and runs an audit agency.

In general, the government can target resources to detect tax evasion and engagement in illegal activities, which determine the respective audit probabilities p_z and p_2.

[17] In practice, however, it can be thought that $\alpha^N < 1$ because the amount is typically returned, if at all, with substantial delay.

Increasing these probabilities is costly, and this cost is denoted by a convex function $\theta = \theta(p_z, p_2)$, with $\theta_z > 0$ for $p_z > 0$, and $\theta_2 > 0$ for $p_2 > 0$. Also, we assume that $\theta_z = 0$ for $p_z = 0$, and $\theta_2 = 0$ for $p_2 = 0$. We allow for the two types of audits to be independent (i.e., $\theta_{z2} = 0$) or complementary (i.e., $\theta_{z2} < 0$). However, for clarity of exposition of our core analysis, we assume that the same audit detects both the evasion of legal income and the amount of illegal income, such that $p = p_z = p_2$. Under this assumption, the audit cost function assumes a simpler form: $\theta = \theta(p)$, $\theta' > 0$ when $p > 0$, and $\theta' = 0$ when $p = 0$. The more general form of the audit cost function is considered in section 8.2.

We assume that the audit agency's costs have to be funded by its fine collections, such that $F \geqslant \theta$. Suppose there is a surplus, $F > \theta$. In that case, there is a continuum of possibilities, ranging from the surplus being lost in bureaucratic red tape (a deadweight loss) to the entire surplus being transferred to the government for public good provision. Accordingly, the overall government budget constraint for public good provision is $G \leqslant T + \mu[F - \theta(p)]$, where $\mu = 0$ captures zero transfer of surplus,[18] $\mu \in (0, 1)$ partial transfer of surplus, and $\mu = 1$ full transfer of surplus from audit agency to public good provision.[19]

[18]This case can also be viewed as the audit agency functioning entirely independently of the rest of the government.

[19]Note that the analysis would not be fundamentally altered when we relax the self-financing constraint. One might imagine a more complex political-economy setting where the audit agency's budget increases when more cases of tax evasion and illicit production are uncovered. Even though much more complex in the modeling, this would technically lead to a similar budget constraint as in our self-financing setting. Some studies, such as Slemrod and Yitzhaki (1987) and Slemrod and Yitzhaki (2002), have explored various aspects of tax system administration, offering a rationale for considering a separate budget for the audit agency when determining the tax rate, but more importantly, the audit probability.

4 Social planner's problem

In this section, we consider the solution of the social planner's program as a benchmark. The population of identical individuals is assumed to have unit mass such that we can represent aggregate variables simply by their individual counterparts such that social welfare (following a utilitarian representation) is: $W = \int u(j)dj = c + h(\ell) + \Phi(G) - \Psi(y_2)$, with $c = y_1 + y_2 - G$, $y_i = w_i n_i$ for $i = 1, 2$, and $\ell = 1 - (n_1 + n_2)$. Maximization of social welfare given the preferences, technology, and time endowment yields the following first-order conditions (FOC) for the first-best allocation:

$$y_1 : 1 - h'/w_1 \leqslant 0, \quad y_2 : 1 - h'/w_2 - \Psi' \leqslant 0, \quad G : -1 + \Phi' = 0. \tag{4}$$

Consider the cases of y_1 and $G > 0$ at the first-best social optimum, such that from equation (4) we get $h' = w_1$ and $\Phi' = 1$. The former equality simply says that efficiency requires that the marginal productivity of labor in producing y_1 be equal to the marginal utility of leisure, while the latter equality requires marginal benefit from the public good to equal the marginal cost of its provision (in terms of the lost marginal utility of private goods' consumption). It is important to note here for later reference that if $\Phi'(G) > 1$ at any second-best equilibrium, then the social optimization tells us that the public good is under-provided at that equilibrium.[20] Similarly, $\Phi'(G) < 1$ suggests over-provision at a second-best equilibrium.

We next turn to a discussion of y_2, the "illegal good". Suppose that the relative returns to the production of goods y_1 and y_2 are such that $w_2 > w_1$.[21] The latter would justify the allocation of time to the production of y_2.[22] Suppose, however, that

[20] This is because the form of the social welfare function suggests that if we hold (y_1, y_2) constant at the second-best equilibrium, a small increase in public good provision will raise social welfare.

[21] We will maintain this assumption throughout the rest of the analysis.

[22] Notice that if we allow a corner solution for y_2, then using $h' = w_1$, the FOC for the choice of y_2 yields

the magnitude of the adverse externality Ψ_0 is sufficiently large such that $\Psi_0 > \frac{(w_2 - w_1)}{w_2}$. This inequality suggests that for social efficiency, a government will want the complete elimination of y_2. In other words, when the externality produced by the production of y_2 is sufficiently large (relative to its productive contribution), then $y_2 = 0$ is socially optimal, or y_2 is an "illegal good".

However, in a second-best world characterized by high enforcement costs and distortive taxation as discussed in the following sections, the government's capacity to restrict the production of illegal goods and simultaneously provide an efficient level of public goods is limited. The central purpose of this paper is precisely to characterize a social welfare-minded government's constrained optimal policies and associated equilibrium inefficiencies, keeping in mind the feasible policy instruments at hand and the government's budget constraint.

5 Individual Optimization: Baseline Case

For clarity of analysis and exposition, we consider the baseline case where $g_{z2} = 0$, $p = p_z = p_2$, $p\alpha^P < 1$ and $p\beta < 1$. Using the various relationships defined in section 3, the representative individual's utility function can be reduced to $u(y_1, y_2, z) = y_1 + y_2 - tx - F(y_1, y_2, z) - \gamma(z, y_2) + h(\ell) + \Phi(G) - \Psi(Y_2)$, with $x = y_1 - z$ and $\ell = 1 - \frac{y_1}{w_1} - \frac{y_2}{w_2}$.

An atomistic individual ignores the effects of her choice on public good provision (G) and the adverse externality from the aggregate production of the illegal good (Ψ), such that assuming interior solutions, the individual's FOC's for the choice of $y_i, i = 1, 2$, yield

$$y_1 : w_1(1 - t) = h', \quad y_2 : w_2(1 - p\beta - g_2 y_2) = h'. \tag{5}$$

$\Psi' \geqslant \frac{(w_2 - w_1)}{w_2}$. Using the linear form $\Psi(Y_2) = \Psi_0 Y_2$, we have a corner solution $y_2 = 0$ if $\Psi_0 \geqslant \frac{(w_2 - w_1)}{w_2}$. When $w_2 < w_1$, then given that $\Psi_0 > 0$, the corner solution always obtains. It is also evident from an individual's FOCs described in the next section that if $w_2 < w_1$, then in the absence of policy intervention, there is no reason for an individual to allocate time to the production of y_2.

The first expression in (5) represents the equalization of the marginal after-tax return from labor in the legal good to the marginal utility of leisure. The second expression equates the return of labor from illegal production net of expected penalties and concealment costs (related to illegal income) to the marginal utility of leisure.

The following FOCs characterize an optimum for z, for the cases of tax evasion ($z > 0$) and overreporting ($z < 0$), respectively,

$$z > 0 : \ t(1 - p\alpha^P) - g_z z = 0, \quad z < 0 : \ t(1 + p\alpha^N) - g_z z = 0. \tag{6}$$

Lemma 1 below proves that overreporting can be ruled out in the current modeling environment using the different fine function possibilities discussed in section 3.

Lemma 1. *Assuming that* $\alpha^P = \alpha \geqslant |\alpha^N| \geqslant 1$, $1 - p\alpha > 0$ *and* $g_{z2} = 0$, *there must be some tax evasion (i.e.,* $z > 0$*) at any positive tax rate* $t > 0$.

Proof. Let us first consider the possibility of over-reporting of income (i.e., $z < 0$). When $\alpha^N > 0$ and $z < 0$, the left-hand-side (LHS) of the last equality in (6) is strictly positive, so a solution $z < 0$ is not possible. When $\alpha^N \leqslant 0$, $|\alpha^N| \leqslant \alpha \Rightarrow \alpha^N \geqslant -\alpha \Rightarrow 1 + p\alpha^N \geqslant 1 - p\alpha > 0$. Therefore, even in this case, the LHS of the last equality of (6) is strictly positive when $z < 0$, ruling out this solution.

Turning to non-negative values of z and using $\alpha^P = \alpha$, notice that $u_z = t\left(1 - p\alpha^P\right) - g_z z \Rightarrow u_{z_{|z=0}} = t\left(1 - p\alpha\right) > 0$, such that there is always some net benefit from tax evasion (i.e., a corner solution at $z = 0$ can be ruled out). Thus, the only optimum that is possible in this environment is an interior optimum for $z > 0$, where the marginal benefit of tax evasion net of expected penalty, $t\left(1 - p\alpha^P\right)$, is equated to the marginal concealment cost, $g_z z$. □

Given lemma 1, which ensures that $z > 0$, we ignore α^N for the rest of the analysis (of the basic model). For simplicity of exposition, we use $\alpha^P = \alpha > 1$. Therefore, the fine function described in section 3 takes the form $F = p(\alpha t z + \beta y_2)$. Proposition 1 describes the solutions in his case.

Proposition 1. *If $g_{z2} = 0$, then an increase in the income tax rate t will reduce legal income y_1 and declared legal income x, and raise illegal income y_2. Declared legal income falls by more than legal income, so that unreported legal income, $z = y_1 - x$, rises.*

Proof. Using equations (5) and (6), we have $w_1(1-t) = w_2(1 - p\beta - g_2 y_2)$, and $g_z z = t(1 - p\alpha)$, which yields

$$z = \frac{t(1-p\alpha)}{g_z}, \quad y_2 = \frac{(1-p\beta)w_2 - (1-t)w_1}{g_2 w_2}. \tag{7}$$

From equation (5), we have $h'(l) = w_1(1-t)$. We can invert the h' function to get $\ell[w_1(1-t)]$, and then use $\ell[w_1(1-t)] = 1 - (y_1/w_1) - (y_2/w_2)$ and the expression for y_2 from equation (7) to solve for y_1:

$$y_1 = w_1\left(1 - \frac{y_2}{w_2} - \ell[w_1(1-t)]\right). \tag{8}$$

Using equations (7) and (8) it is easy to check that:

$$\frac{\partial y_1}{\partial t} < 0, \quad \frac{\partial y_2}{\partial t} > 0, \quad \frac{\partial z}{\partial t} > 0, \quad \frac{\partial x}{\partial t} < 0. \tag{9}$$

□

Note that in this case, the solution for z is determined independently of y_1 and y_2. Also, when $(1-p\beta)w_2 > w_1$, $y_2 > 0$ for every $0 \leqslant t \leqslant 1$. The marginal utility of income from y_2 evaluated at the FOC for $y_1 > 0$, $[w_1(1-t) = h']$, is $u_2 = (1-p\beta)w_2 - w_1(1-t) - g_2 y_2$. So whenever $(1-p\beta)w_2 > w_1$ and $t \geqslant 0$, u_2 is increasing in y_2 at $y_2 = 0$. This means that some amount of production of the illegal good is always privately profitable in this case. Finally, note that given w_1 and w_2, the time allocation between leisure and the production of legal and illegal activities is determined only by the tax rate t, since $n_1 + n_2 = 1 - \ell[w_1(1-t)]$. We denote the solutions in this case as $y_1 \equiv y_1(t,p)$, $y_2 \equiv y_2(t,p)$ and $z \equiv z(t,p)$. Substituting the solutions into the utility, we obtain the indirect utility function $v \equiv v(t,p,G,Y_2)$.

Going back to the comparative statics with respect to t, it follows that a small increase in the income tax rate dissuades effort allocation to the production of the legal good and simultaneously incentivizes a higher allocation of time to the production of the illegal good and leisure. An increase in t also increases ℓ and reduces $(n_1 + n_2)$ since $\frac{\partial n_1}{\partial t} + \frac{\partial n_2}{\partial t} = \frac{\partial y_1}{\partial t}\frac{1}{w_1} + \frac{\partial y_2}{\partial t}\frac{1}{w_2} < 0$. For subsequent reference, note that y_2 and z are linear in t, and under fairly weak assumptions about the functional form of $h'(l)$, y_1 and x are concave in t.

Consider next the financing of the public good based on the total revenue collected by the government. We can show that the total revenue function $T + \mu\left[F - \theta\left(p\right)\right]$ is a concave function of t.[23] Tax revenue $T = tx$, is a concave function of t. Since $\left(\frac{dT}{dt}\right)_{|t=0} = x > 0$, $\frac{dT}{dt}$ is positive for sufficiently low tax rates. The penalty function F is increasing and convex in t. However, it follows that for $G\left(t\right) \equiv T + \mu\left[F - \theta\left(p\right)\right]$, $G''\left(t\right) < 0$. Thus, $G\left(t\right)$ is an increasing function of t for sufficiently low tax rates.

6 Government Policy Choices

This section analyzes the government's policy choices. We begin by obtaining the income tax rate that maximizes welfare for given values of the audit probability. Next, we allow the government to choose the audit probability as well. We continue with section 5's assumptions $g_{z2} = 0$, $p_z = p_2 = p$, $p\alpha < 1$, and $p\beta < 1$.

[23] All relevant derivations are included in Appendix A.1

6.1 Choosing the tax rate

The focus of this section is on a constrained optimal tax that is necessary to finance the public good while keeping in mind the distortionary effect of this taxation on the production of the two goods and on tax evasion.

It is useful to highlight at this point the inability of the tax rate to achieve multiple objectives simultaneously. Consider the example $\Psi(y_2) = \Psi_0 y_2$ and $w_1 \geqslant w_2(1-\Psi_0)$, which implies that the socially optimal value of y_2 is equal to zero. Suppose, however, that $(1-p\beta)w_2 > w_1$. Then, some amount of production of the illegal good is always privately profitable as long as t is non-negative. The private incentive to produce y_2 could be eliminated in this case by subsidizing the production of the legal activity y_1 (i.e., a negative tax on y_1).[24] While this negative tax eliminates the production of the illegal good, it will likely fail to enable the government to provide a (socially desirable) positive amount of public goods when an income tax is the sole source of fiscal revenue.

As it is well known, a single policy instrument cannot simultaneously achieve both goals. The optimal choice of t, therefore, has to assess the trade-offs between a higher tax rate, which ultimately finances the public good, and the incentives to participate in the illegal market. In what follows, unless otherwise stated, we assume $(1-p\beta)w_2 > w_1$ and $0 \leqslant t \leqslant 1$.

6.1.1 Analysis

The government chooses t anticipating the effect of the tax rate on atomistic individuals' behavior to maximize the utilitarian social welfare function $W \equiv \int v di = C + H - \Gamma + \Phi(G) - \Psi(Y_2)$. As discussed earlier, the government's budget constraint implies that

[24] When $t \leqslant 0$, the solution for z is negative, specifically $z = \frac{t(1+p\alpha^N)}{g_z} < 0$, and y_2 is still the same as in equation (7). The government can therefore set the tax (subsidy) rate $t = 1 - \frac{(1-p\beta)w_2}{w_1} < 0$ so that $y_2 = 0$.

$G = T + \mu[F - \theta(p)]$, $T = tx$, $0 \leqslant \mu \leqslant 1$, and $F \geqslant \theta(p)$.

For the basic analysis, we assume that the audit agency's constraint is not binding [i.e., $F > \theta(p)$], such that the associated Lagrangian multiplier λ is zero.[25]

From the envelope theorem, $\partial v/\partial t = -(x + F_t)$, where $x = y_1 - z \geqslant 0$. Given our assumption here that $p_z = p_2 = p$, and given proposition 1, which states that $z > 0$, the fine function presented in equation (1) reduces to $F(t, z, y_2) \equiv p(\alpha t z + \beta y_2)$, such that $F_t = p\alpha z \geqslant 0$. Using these facts, the FOC for the government's optimization problem at an interior optimum ($t > 0$) reduces to:

$$\frac{\partial W}{\partial t} \equiv -(x + F_t) + \Phi' \frac{dG}{dt} - \Psi' \frac{\partial y_2}{\partial t} = 0, \quad \Rightarrow \quad \Phi' \frac{dG}{dt} = (x + F_t) + \Psi' \frac{\partial y_2}{\partial t}. \tag{10}$$

where the LHS of the last equation in (10) is the social benefit from the additional public good that can be provided from the extra revenues that are collected (from taxes and fines) from raising the tax rate. The first term on the RHS of the last equation in (10) measures the marginal utility loss from a higher tax burden. The second RHS term represents the utility loss from higher expected fines. The last RHS term measures the utility loss from the negative externality associated with a tax-induced expansion of the illegal good (from the comparative static results in (9) we have $\frac{\partial y_2}{\partial t} > 0$).

At a constrained optimum, the government balances the social marginal gain from greater public good provision with the aforementioned social marginal costs, where the latter arises from the dual distortions of tax evasion and illegal goods production. Furthermore, notice that equation (10) requires that $\frac{dG}{dt} > 0$, such that a strictly positive constrained optimal tax rate is also Laffer-efficient. Using $G(t) \equiv tx + \mu[F - \theta(p)]$, we

[25] Appendix B.1 considers a binding audit constraint (i.e., $\lambda > 0$).

get $\frac{dG}{dt} \equiv x + t\frac{dx}{dt} + \mu\frac{dF}{dt}$. Substituting the RHS of the latter equation in (10) we get

$$(\Phi' - 1)(x + F_t) = \Psi' \frac{\partial y_2}{\partial t} + \Phi' \left(F_t - t\frac{\partial x}{\partial t} - \mu \frac{dF}{dt} \right). \tag{11}$$

Case 1: $\mu = 0$**.** Recall from section 3 that $\mu = 0$ represents the case where the audit agency's surplus, $F - \theta(p)$, is completely lost in the bureaucratic process involving the transfer and is a deadweight loss. In this case, the public good is funded entirely through tax collections. When $\mu = 0$, $\frac{dG}{dt} \equiv x + t\frac{dx}{dt}$, such that equation (10) reduces to

$$\frac{\partial W}{\partial t} \equiv -(x + F_t) + \Phi' \underbrace{\left(x + t\frac{\partial x}{\partial t} \right)}_{\partial G/\partial t} - \Psi' \frac{\partial y_2}{\partial t} = 0. \tag{12}$$

Proposition 2. *If the audit agency's surplus cannot be transferred to the government for public good provision ($\mu = 0$), then the public good G is underprovided (i.e., $\Phi' > 1$).*

Proof. When $\mu = 0$,

$$(\Phi' - 1)(x + F_t) = \Psi' \frac{\partial y_2}{\partial t} + \Phi' \left(F_t - t\frac{\partial x}{\partial t} \right). \tag{13}$$

Since $x \geqslant 0$, and when $g_{z2} = 0$, $z > 0$, then $F_t = p\alpha z > 0$ [from equation (3)], $\frac{\partial y_2}{\partial t} > 0$, and $\frac{\partial x}{\partial t} < 0$ [from the comparative static results (9)]. Thus, equation (10) implies that at the constrained optimal tax equilibrium we have, which violates the efficiency condition for the public good outlined in equation (4). Given that the marginal benefit of the public good Φ' exceeds the unit marginal cost of provision (in terms of utility loss from private good consumption), we have underprovision of the public good at this constrained optimal tax equilibrium. □

Case 2: $\mu = 1$**.** When $\mu = 1$, the audit agency's surplus is fully transferred to the government for public good provision. Does this increased revenue source act as a substitute for taxation and reduce the tax rate, or does it have a complementary effect? In addition, what does the optimal tax rule in the full transferability situation have to say about the efficiency of public good provision? These questions are addressed in proposition 3 below.

Proposition 3. *Suppose the audit agency's surplus is fully transferable to the government, i.e.,* $\mu = 1$.

(i) The optimal income tax rate is higher than under $\mu = 0$.

(ii) If, in addition, (a) the illegal activity is "illicit" in the planner's sense, i.e. $\Psi_0 \geqslant \frac{w_2 - w_1}{w_2}$*, and (b) the illegal activity is privately attractive at the margin, i.e.* $(1 - p\beta)w_2 > w_1$*, then the public good is under-provided at the constrained optimum:* $\Phi'(G^*) > 1$.

Proof. (i) Recall that $F(t, z, y_2) \equiv p(\alpha t z + \beta y_2)$, such that $\frac{dF}{dt} = p\left[\alpha\left(z + t\frac{\partial z}{\partial t}\right) + \beta\frac{\partial y_2}{\partial t}\right] > 0$, because from equation (9) we have $\frac{\partial z}{\partial t} > 0$ and $\frac{\partial y_2}{\partial t} > 0$. Using equation (10) and the implicit function rule to obtain

$$\frac{\partial t}{\partial \mu} = -\frac{\frac{\partial^2 W}{\partial t \partial \mu}}{\frac{\partial^2 W}{\partial t^2}} = -\frac{\Phi' \frac{dF}{dt}}{\frac{\partial^2 W}{\partial t^2}} > 0, \tag{14}$$

because $\frac{d^2 W}{dt^2} < 0$ from the second-order condition of the government's optimization problem. Hence, the optimal tax rate must be higher under full transferability ($\mu = 1$) than under no transferability ($\mu = 0$).

(ii) When $\mu = 1$, $\frac{dG}{dt} \equiv x + t\frac{\partial x}{\partial} + \frac{dF}{dt}$, such that equation (10) reduces to

$$\left.\frac{\partial W}{\partial t}\right|_{\mu=1} \equiv -(x + F_t) + \Phi'\left(x + t\frac{\partial x}{\partial t} + \frac{dF}{dt}\right) - \Psi'\frac{\partial y_2}{\partial t} = 0. \tag{15}$$

Using $\frac{dF}{dt}$ in equation (15) and reorganizing terms we get

$$(\Phi' - 1)(x + F_t) = (\Psi' - \Phi' p\beta)\frac{\partial y_2}{\partial t} + \Phi' \underbrace{\left[(1 - p\alpha)t\frac{\partial z}{\partial t} - t\frac{\partial y_1}{\partial t}\right]}_{>0}. \tag{16}$$

The sign of the RHS of (16) depends on the relative magnitudes of the two terms. Note that $p\alpha < 1$ and from equation (9) we have $\frac{\partial z}{\partial t} > 0$ and $\frac{\partial y_1}{\partial t} < 0$ so the last term on the RHS of equation (16) is positive. This positive second term pushes towards underprovision of G.

Consider next the first term. Under (a) and (b), we have $w_1 \geqslant (1-\psi_0)w_2$ and $(1-p\beta)w_2 > w_1$. With linear externalities, this gives $\Psi_0 > p\beta$. Therefore, $\Psi_0 - \Phi' p\beta \geqslant p\beta(1-\Phi')$. Suppose, to the contrary, that $\Phi'(G^*) < 1$. Then $1-\Phi' > 0$, so the first RHS term in (16) is also positive. However, LHS is negative in this case, a contradiction. Thus $\Phi'(G^*) > 1$, i.e., the public good is underprovided.[26] □

The intuition behind part (i) of Proposition 3 is the following. Recall that $\frac{dG}{dt} \equiv x + t\frac{\partial x}{\partial t} + \mu\frac{dF}{dt}$, such that when $\mu > 0$, there is an additional marginal contribution ($\mu\frac{dF}{dt} > 0$) of the tax rate towards the funding of the public good that comes from additional fines collected. This contribution is scaled by μ and is therefore magnified at higher values of μ. Therefore, as would be expected, we find that a greater degree of transferability (μ) raises the optimal tax rate.

We turn next to the reason behind part (ii). Note that two conditions are critical: (a) the planner regards the activity as socially harmful (so it generates a negative externality), and (b) individuals still find it privately profitable to engage in it despite expected penalties. Together, these imply that the marginal social damage from illegal production is larger than the expected fine per unit of illegal activity. In other words, each additional unit of illegal output imposes more harm than the government recoups in revenues. As

[26]Note that the FOC (16) can be rewritten as

$$(\Phi'-1)\underbrace{\left(x + p\alpha z + p\beta\frac{\partial y_2}{\partial t}\right)}_{>0} = (\Psi' - p\beta)\frac{\partial y_2}{\partial t} + \Phi'\underbrace{\left[(1-p\alpha)t\frac{\partial z}{\partial t} - t\frac{\partial y_1}{\partial t}\right]}_{>0}. \tag{17}$$

This means that if the marginal damage from the externality exceeds the expected penalty ($\Psi' \geqslant p\beta$), the RHS is unambiguously positive, and the public good is underprovided ($\Phi' > 1$). For instance, when $\Psi(y_2)$ is linear in y_2, i.e., $\Psi(y_2) = \Psi_0\, y_2$, this happens when $\Psi_0 \geqslant p\beta$.

a result, when the government chooses taxes to balance public good provision against these external costs, the first-order condition forces the equilibrium to occur at a point where the marginal value of the public good still exceeds unity, $\Phi'(G^*) > 1$.

6.1.2 How does the optimal tax depend on the intensity of the externality?

We answer the question above by using the aforementioned linear form for the externality, $\Psi(y_2) = \Psi_0 y_2$, where the parameter Ψ_0 is a measure of the intensity with which the illegal good creates the negative externality.

Proposition 4. *Assuming a linear externality function and a given audit probability, the constrained optimal tax rate is monotonically declining in the intensity of the externality. As a result, the provision of the public good also decreases.*

Proof. The effect of a change in Ψ_0 on the optimal tax rate is given by

$$\frac{\partial t}{\partial \Psi_0} = -\frac{\frac{\partial^2 W}{\partial t \partial \Psi_0}}{\frac{\partial^2 W}{\partial t^2}} = -\frac{-\frac{\partial y_2}{\partial t}}{\frac{\partial^2 W}{\partial t^2}}. \tag{18}$$

Using the SOC corresponding to equation (10) and noting that equation (9) ensures that the illegal income rises with the income tax rate (i.e., $\frac{\partial y_2}{\partial t} > 0$), equation (18) indicates that the optimal tax decreases with the intensity of the externality, i.e., $\frac{\partial t}{\partial \Psi_0} < 0$. Note that $\frac{dG}{\Psi_0} = \frac{dG}{dt}\frac{dt}{d\Psi_0}$. Since at the optimum $\frac{dG}{dt} > 0$ and $\frac{\partial t}{\partial \Psi_0} < 0$, then $\frac{dG}{\Psi_0} < 0$. □

By lowering the tax rate, the government trades off some benefit of public good provision against the benefit derived from lower externalities associated with a lower production level of the illegal good y_2. Also, considering that $\frac{dG}{\Psi_0} < 0$, if the public good is underprovided for relatively low levels of Ψ_0, then the underprovision will become more severe when the intensity of the negative externality becomes higher.

We present a numerical example below that traces the effects of a greater intensity of the externality on the optimal tax rate and the other endogenous variables at the optimal tax equilibrium. The outcomes are summarized in Figure 1. The exercise uses $y_i = w_i n_i$, $w_2 > w_1$, $h(\ell) = h_0 \ell^{h_1}$, and $\Phi(G) = \phi_0 G^{\phi_1}$, with $\phi_0 > 0$, $0 < \phi_1 < 1$, and $\Psi(Y_2) = \Psi_0 Y_2$, with $\Psi_0 > 0$. In the exercise, the intensity of the negative externality becomes stronger as Ψ_0 increases.[27] Also, $g_{z2} = 0$, $\alpha^P = \alpha$, and $\mu = \{0, 1\}$.[28] Throughout the exercise, p is fixed at $p = 0.07$.

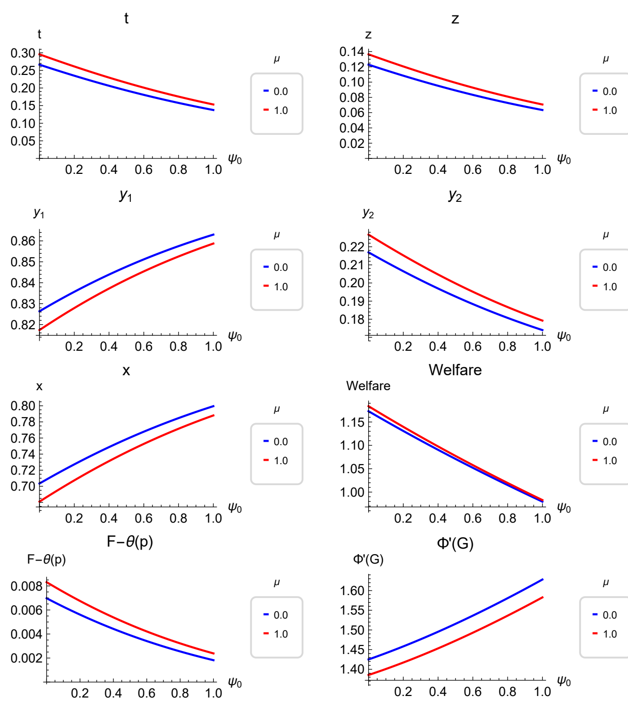

Figure 1: Solutions as a function of Ψ_0 for $\mu \in \{0, 1\}$.
Note: The first graph shows the optimal value of t for different values of Ψ_0 and $\mu \in \{0, 1\}$. The other graphs show the values of the corresponding variables evaluated at the optimal t.
The graphs use $p = 0.07$, $w_1 = 1$, and $w_2 = \frac{3}{2}$, $g_z = g_2 = 2$, $g_{z2} = 0$.

The numerical example serves to highlight the following key points. First, as the externality Ψ_0 increases, t smoothly declines, and therefore unreported legal income z smoothly declines. As a consequence, the level of underprovision increases as Ψ_0 increases, as stated in proposition 4. Second, a lower tax rate is associated with a higher

[27] All the numerical exercises considered in the paper use these functional forms.
[28] Since in the present case $z > 0$, only α^P is relevant.

production of the legal good y_1 and a lower production of the illegal good y_2. Third, welfare declines as Ψ_0 increases. Fourth, when the surplus of the audit agency is shared ($\mu = 1$), the optimal tax rate t is higher. Therefore, z, and y_2 are higher, and y_1 and x are lower. The level of underprovision of G is lower, and welfare is higher.[29]

6.2 How does the audit probability affect the optimal tax rate?

Relying only on a single policy variable, in this case, t, restricts the government's ability to achieve multiple goals. An additional tool for the government is allocating resources θ to increase the effectiveness of the audit technology. In our model, the latter translates into higher levels of p. In the subsequent analysis, we allow the government to choose different combinations $\{t, p\}$.

The following discussion focuses first on the effect of a change in p on the individuals' choices at a given tax rate. Next, we consider the impact of a change in p on the optimal tax rate. Finally, we consider jointly the optimal choice by the government of the tax rate and the audit probability and discuss the comparative static effect of a rise in the intensity of the externality from the illegal good.

Comparative statics with respect to p. We begin by examining how an individual's behavior changes when p changes.[30] We obtain:

$$\frac{\partial y_1}{\partial p} = -\frac{w_1}{w_2}\frac{\partial y_2}{\partial p} > 0, \quad \frac{\partial z}{\partial p} < 0, \quad \frac{\partial x}{\partial p} > 0. \tag{19}$$

[29]The exercise considers parameter values such that the audit agency's budget constraint $F \geqslant \theta(p)$ does not bind (i.e., $\lambda = 0$ at the optimal t). However, this budget constraint will hold with equality (and $\lambda > 0$) for sufficiently large values of Ψ_0. When this happens, the tax rate is implicitly determined by the constraint $F = \theta(p)$, and it no longer depends on Ψ_0. For the parameter values used in the numerical exercise, this tax rate is 0.08. Since the tax rate is always lower when $\mu = 0$, $F - \theta(p)$ will hold as an equality for a lower value of Ψ_0 than when $\mu = 1$.

[30]The formal derivations are included in Appendix A.2.

In other words, an increase in the audit probability shifts production from illegal to legal activities and induces individuals to report more income. Moreover,

$$\frac{\partial y_1}{\partial p}\frac{1}{w_1} + \frac{\partial y_2}{\partial p}\frac{1}{w_2} = 0, \quad \frac{\partial y_1}{\partial p} + \frac{\partial y_2}{\partial p} = -\frac{\beta(w_2 - w_1)}{g_2 w_2}, \tag{20}$$

which means that a higher p shifts labor from illegal to legal activities in the same amount, leading to a lower overall income $(y_1 + y_2)$, given $w_2 > w_1$. Additionally, note from equations (7) and (8) that all the variables are linear in p, so the second-order derivatives with respect to p are all zero, and the cross-partial derivatives are given by

$$\frac{\partial^2 y_1}{\partial p \partial t} = \frac{\partial^2 y_2}{\partial p \partial t} = 0, \quad \frac{\partial^2 z}{\partial p \partial t} < 0, \quad \frac{\partial^2 x}{\partial p \partial t} = -\frac{\partial^2 z}{\partial p \partial t} > 0. \tag{21}$$

The audit probability does not impact the effect of the tax rate on y_1 and y_2. However, it does reduce the positive impact that the tax rate has on z and the negative impact (in absolute value) that the tax rate has on x. The higher the audit probability, the smaller is the impact of a tax increase on the amount of misreported income. Finally, income taxes collected by the government, T, increase linearly in p, and the budget constraint of the audit agency, F, is concave in p.

Combinations of $\{t, p\}$ for given values of unreported legal income z and illicit income y_2. A given level of $\bar{z} > 0$ can be attained with different combinations of $\{t, p\}$. Consider the solution for z given by (7). Then the relationship between p and t for which $z(t, p) = \bar{z}$, $p = p(t)$, is given by

$$\left.\frac{\partial p}{\partial t}\right|_{z=\bar{z}} = \frac{(1 - \alpha p)}{\alpha t} > 0. \tag{22}$$

When t increases, unreported legal income z increases, so p needs to be raised in the amount stated by (22) to restore the original level of $\bar{z}$. In other words, the same unreported legal income z can be achieved with a higher tax rate as long as p is higher.

Hence, t and p are complements. Similarly, for $y_2(t, p) = \bar{y}_2$,

$$\left.\frac{\partial p}{\partial t}\right|_{y_2 = \bar{y}_2} = \frac{w_1}{\beta w_2} > 0. \tag{23}$$

An increase in t also increases y_2. To return y_2 to its original level, p must therefore increase.

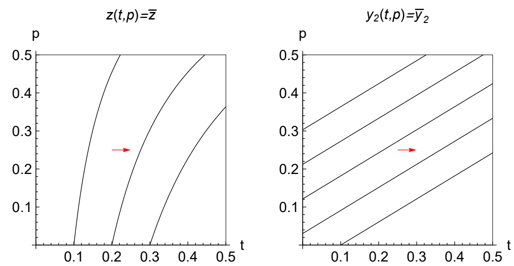

Figure 2: Isolevel curves $z(t, p) = \bar{z}$ and $y_2(t, p) = \bar{y}_2$.

When t is the only policy variable, the government chooses a relatively low tax rate to account for the negative externalities produced by good 2: a lower t implies a lower y_2. This, however, leads to underprovision of G. The ability to change p would allow the government to increase t and, therefore, G. As long as the higher t is coupled with a sufficiently high level of p, the government could keep y_2 at least at the same level as before. However, since providing p is costly, the government must balance the benefits and costs of raising p.

How does the optimal tax rate depend on p? In general,

$$\frac{\partial t}{\partial p} = -\frac{\frac{\partial^2 W}{\partial t \partial p}}{\frac{\partial^2 W}{\partial t^2}}, \tag{24}$$

where $\frac{\partial^2 W}{\partial t^2} < 0$. Under plausible conditions, discussed in Appendix B.2, $\frac{\partial^2 W}{\partial t \partial p} > 0$, which implies that $\frac{\partial t}{\partial p} > 0$. In other words, by raising p, the government can increase the optimal tax rate and reduce the severity of the underprovision of G.

The graphs in Figure 3 below show the solutions for different values of p and μ.

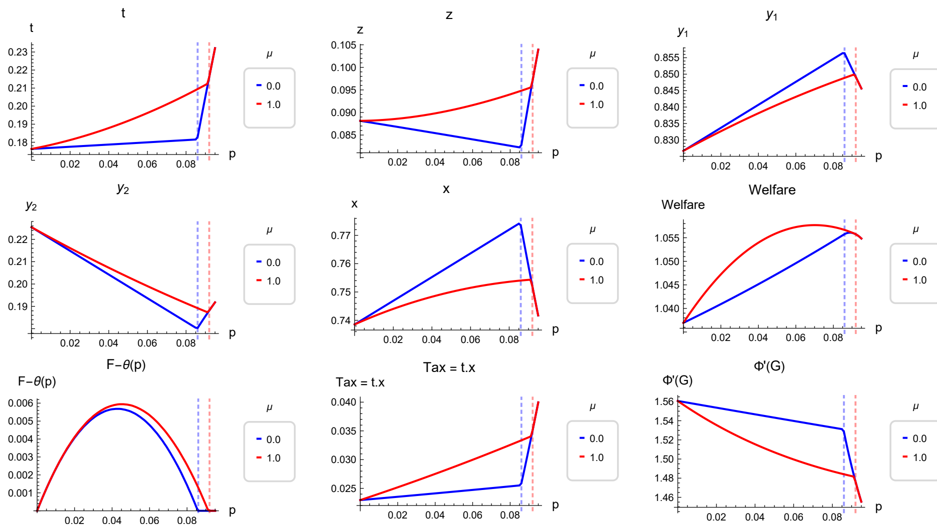

Figure 3: Solutions as a function of p and $\mu \in \{0, 1\}$.
The graphs consider the case $\Psi_0 > 1 - \frac{w_1}{w_2}$ ($\Psi_0 = 3/5$, $w_1 = 1$ $w_2 = 3/2$). The vertical red dashed line indicates the value of p at which $F - \theta(p) = 0$ when $\mu = 1$, and the vertical blue dashed line the value of p at which $F - \theta(p) = 0$ when $\mu = 0$.

The first graph shows the tax rate chosen by the government. The rest of the graphs show the values of the other endogenous variables evaluated at the solution tax rate. The numerical example highlights several observations. First, the tax rate increases as the audit probability p increases. Second, the tax rate is higher when $\mu = 1$ than when $\mu = 0$.

Third, the level of underprovision of G declines as p gets larger. To gain some intuition, consider the case $\mu = 0$ (the government relies only on income tax revenue to finance the public good), so that $G = T$. Evaluated at the optimal t, an increase in p affects T as follows:

$$\frac{dT}{dp} = t\frac{\partial x}{\partial p} + \left(x + t\frac{\partial x}{\partial t}\right)\frac{\partial t}{\partial p}.$$

A higher p directly increases reported legal income x and raises the tax collected by the government. This higher level of p also allows the government to choose a higher tax rate, and its impact on tax revenue is given by $\frac{\partial T}{\partial t} = x + t\frac{\partial x}{\partial t}$. Since in equilibrium, the government chooses a tax rate such that $\frac{\partial T}{\partial t} > 0$, tax revenue always increases as p rises. When $\mu = 1$ and $F - \theta(p) > 0$, the government can, in principle, achieve at least the same level of G as when $\mu = 0$. To the extent that the starting point is a level of G such that $\Phi' > 1$, the government will choose a higher G when $\mu = 1$, and the underprovision will be lower in this case.

Fourth, for $\mu = 1$, welfare is concave in p: as p increases, welfare rises, reaches a maximum, and then declines. The policymaker can, therefore, increase welfare by jointly increasing t and p (up to a certain point).

Fifth, raising p when it is relatively small allows the government to relax the agency's budget constraint $F - \theta(p)$ and increase welfare. However, increasing p is costly, so the agency's budget constraint eventually binds at a sufficiently large value of p, i.e., $F - \theta(p) = 0$, where F is evaluated at the optimal t when the audit probability is p. When this budget constraint binds, then the relationship between t and p becomes mechanical, determined by the equation $F - \theta(p) = 0$. The vertical dashed lines show the values of p where the agency's budget constraint is exactly met for each scenario, $\mu = \{0, 1\}$. The variables' behavior aligns in both scenarios once p reaches a value where $F - \theta(p) = 0$ for $\mu = 1$.

6.3 Choosing the tax rate and the audit probability

Suppose that the government simultaneously chooses the tax rate and the audit probability.[31] As in the previous case, we focus here on the case in which $\lambda = 0$.[32] We highlighted earlier the inability of the tax rate to achieve multiple objectives simultaneously.

[31] Slemrod and Yitzhaki (1987) consider a model where both t and p are choice variables in a standard tax evasion framework.

[32] Appendix B.3 describes the FOC when $\lambda > 0$.

For instance, when the government can choose both t and p, it can attain the combination $\{y_2, z\} = \{0, 0\}$ by choosing (i) $\{t, p\} = \{0, \frac{(w_2 - w_1)}{\beta w_2}\}$;[33] or (ii) $\{t, p\} = \{1 - \frac{(\alpha-\beta)w_2}{\alpha w_1}, \frac{1}{\alpha}\}$.[34] However, in choosing the optimal policy combination, the government also considers the provision of the public good G.

The FOC for the tax rate has already been outlined in equation (10). Turning to the choice of the audit probability, notice that using the envelope theorem, we have $\partial v/\partial p = -F_p = -(\alpha t z + \beta y_2) < 0$, such that the FOC for the audit probability reduces to

$$\frac{\partial W}{\partial p} = -F_p + \Phi' t \frac{\partial x}{\partial p} - \Psi' \frac{\partial y_2}{\partial p} + \Phi' \mu \left(\frac{dF}{dp} - \theta' \right). \tag{25}$$

The FOC for p depends on several different effects of the audit probability p on the relevant endogenous variables. These effects are:

(i) Reduced individual income: Individuals earn less because they are more likely to be penalized for tax evasion or the production of illegal goods (represented by the first term, $-F_p < 0$).

(ii) Increased public good provision: A higher p also discourages tax evasion, leading to more reported income and higher tax revenue for the government. This additional revenue can then be used to fund a greater provision of the public good (second term).

(iii) Reduced negative externality: As a higher p discourages illegal activities, the negative externality associated with those activities goes down (third term). This is a social benefit.

[33]Note that $p < 1$ requires $\beta > \frac{(w_2 - w_1)}{w_2} > 0$, but this is trivially satisfied since we already assumed that $\beta > 1$.

[34]Note that $t > 0$ requires $\frac{\beta}{\alpha} > \frac{(w_2 - w_1)}{w_2} > 0$, and $t < 1$ requires $\alpha > \beta$. Since $\alpha > 1$, $0 < p < 1$ in this case.

(iv) Impact on the audit agency budget: The higher p also affects the audit agency's budget. While it might lead to more penalty fees collected, it also comes with higher operational costs (fourth term). The impact on providing the public good ultimately depends on the extent of transferability of these resources, which is parameterized by $\mu \in [0, 1]$).[35]

Ultimately, the government will choose the policy combination $\{t, p\}$ that satisfies equations (10) and (25). Consider different cases depending on the value of μ.

Case 1: $\mu = 0$. It follows that,

$$\frac{\partial W}{\partial p} \equiv -F_p + \Phi' t \frac{\partial x}{\partial p} - \Psi' \frac{\partial y_2}{\partial p} = 0 \quad \Rightarrow \quad \Phi' t \frac{\partial x}{\partial p} - \Psi' \frac{\partial y_2}{\partial p} = F_p. \tag{26}$$

The LHS describes the positive impact of a higher p on utility. First, a higher p directly increases taxes collected by the government since individuals report a higher income. The latter tends to increase G . Second, it decreases y_2 and, as a result, the negative externalities. At the same time, a higher p negatively affects utility as shown on the RHS: disposable income declines since individuals pay higher fines for misreporting legal income and for producing the illegal good y_2. The fine, per se, is not a social cost but simply a transfer from the individual to the audit agency. However, due to $\mu = 0$, these transfers cannot be used for productive purposes and are therefore lost to society.

Case 2: $\mu = 1$. In this case,

$$\frac{\partial W}{\partial p} = 0 \quad \Rightarrow \quad \Phi' \left(t \frac{\partial x}{\partial p} + \frac{dF}{dp} - \theta' \right) - \Psi' \frac{\partial y_2}{\partial p} = F_p. \tag{27}$$

[35] As highlighted by Slemrod and Yitzhaki (1987) in a somewhat different framework, increasing the audit probability (or enforcement efforts) requires a larger amount of resources (such as hiring auditors). Instead of focusing on the revenue generated by the change in p, which is merely a transfer from individuals to the agency, they claim the optimal approach should balance the social benefit of reducing tax evasion with the actual enforcement costs.

Compared to the previous case with $\mu = 0$, now a higher p affects the government's revenue by affecting the collection of penalty fees, $\frac{dF}{dp}$, and by the cost of increasing p, θ'. The net impact of p on G is therefore $\frac{\partial G}{\partial p} = \left(t\frac{\partial x}{\partial p} + \frac{dF}{dp} - \theta'\right)$. We can rewrite (27) as

$$\frac{\partial W}{\partial p} \equiv (\Phi' - 1)F_p + \Phi'\left(t\frac{\partial x}{\partial p} - \theta'\right) - \Psi'\frac{\partial y_2}{\partial p}. \tag{28}$$

It follows, from (16), that $\Phi' > 1$ when $p = 0$, hence, the first term is positive. The second term is positive as $\frac{\partial x}{\partial p} > 0$ and $\theta'(0) = 0$. Also $-\Psi'\frac{\partial y_2}{\partial p} > 0$ for all p. As a result, at $p = 0$, $\frac{\partial W}{\partial p} > 0$.

When $\mu = 1$, the optimal policy combination $\{t, p\}$ satisfies

$$\frac{-(x + F_t) + \Phi'\left(x + t\frac{\partial x}{\partial t}\right) - \Psi'\frac{\partial y_2}{\partial t}}{-F_p + \Phi' t\frac{\partial x}{\partial p} - \Psi'\frac{\partial y_2}{\partial p}} = \frac{\frac{dF}{dt}}{\frac{dF}{dp} - \theta'}. \tag{29}$$

While $\frac{dF}{dt} > 0$, the function $[F - \theta(p)]$ is concave in p. Figure 3 shows that evaluated at the audit probability p that maximizes welfare, $\frac{d[F-\theta(p)]}{dp}$ is negative. This means the optimal p is larger than the one that maximizes $[F - \theta(p)]$.

Effect of Ψ_0 on $\{t, p\}$. How does the intensity of the negative externality affect the optimal combination $\{t, p\}$? We showed earlier that when the audit probability is fixed, an increase in the intensity of the negative externality generated by the production of y_2 decreases the optimal tax. We can similarly show that if t is fixed, an increase in the intensity of the negative externality entails an increase in p. As before, suppose the externality is $\Psi(y_2) = \Psi_0 y_2$, $\Psi_0 > 0$.

Lemma 2. *Assuming a linear externality function and a fixed tax rate, a rise in the intensity of the externality will increase the audit rate.*

Proof. If t is kept constant, then

$$\frac{\partial p}{\partial \Psi_0} = -\frac{\frac{\partial^2 W}{\partial p \partial \Psi_0}}{\frac{\partial^2 W}{\partial p^2}} = -\frac{-\frac{\partial y_2}{\partial p}}{\frac{\partial^2 W}{\partial p^2}} > 0. \tag{30}$$

since $\frac{\partial^2 W}{\partial p^2} < 0$, due to the SOCs, and $\frac{\partial y_2}{\partial p} < 0$ from equation (19). □

Recall, however, that t and p are complements. From equation (18), a higher Ψ_0 tends to reduce t, so the complementarity will tend to pull p down. If the direct effect of Ψ_0 shown in equation (30) dominates the cross effect of t on p, then p will rise. Similarly, from equation (30), a higher Ψ_0 tends to increase p. The complementarity will tend to push t up. Therefore, if the direct effect of Ψ_0 on t dominates the cross effect from a change in p, then the optimal tax rate will fall as Ψ_0 rises. The following proposition summarizes this result.

Proposition 5. *Assuming that direct effects dominate cross effects, a rise in the intensity of the externality will reduce the tax rate but raise the audit rate.*

Proof. See Appendix B.4. □

Figure 4 graphically shows how the solutions change as Ψ_0 increases. These solutions are compared to the scenario where the audit probability is held constant, in this case, at a relatively low level.

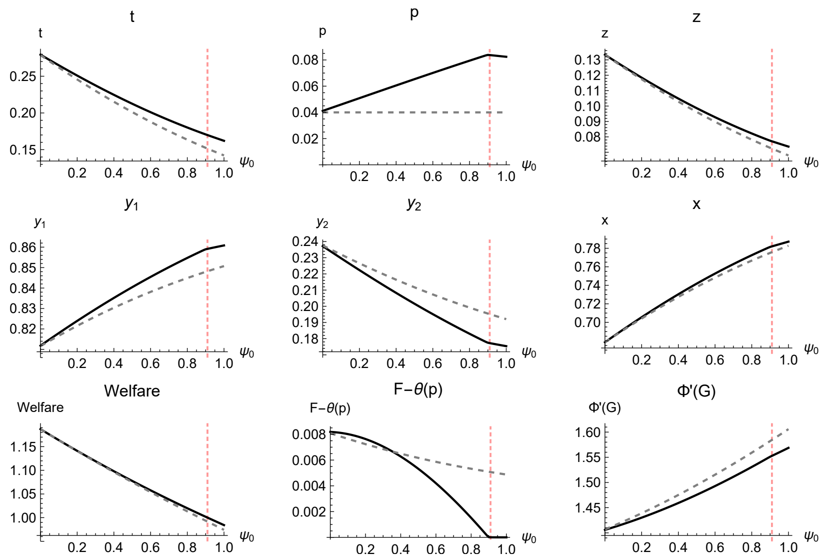

Figure 4: Solutions as a function of Ψ_0.
Note: The exercise assumes $g_{z2} = 0$ and $\mu = 1$. The gray dotted line shows the solutions when the audit probability is fixed at $p = 0.04$. The red vertical dotted line shows the value of Ψ_0 at which the audit agency's budget constraint binds, i.e., $F - \theta(p) = 0$.

Based on this exercise, a few remarks are worth emphasizing. First, as Ψ_0 increases, p increases, and t decreases. Second, as this happens, individuals switch from illegal to legal activities, and reported income x increases. Third, lower tax rates and higher audit probabilities increase the fiscal pressure on the budget constraint, and the provision of the public good shrinks. Fourth, for sufficiently large values of Ψ_0 (in the example, when $\Psi_0 \geqslant 0.91$), the budget constraint of the audit agency starts to bind at the optimal levels of t and p. Further increases in Ψ_0 that reduce the optimal tax rate would require a corresponding decrease in p to satisfy the audit agency's budget constraint $F - \theta(p) = 0$. Finally, suppose that the audit probability is kept constant at a relatively low level ($p = 0.04$ on the graph). As Ψ_0 increases, the difference in welfare between the optimal combination $\{t, p\}$ and the welfare when p is fixed also increases. This is driven by the fact that the optimal p increases as Ψ_0 increases. If the audit probability were fixed at a relatively large p (not shown in the graph), then the difference in welfare would decrease as Ψ_0 increases.

7 General Concealment Function

Consider the concealment function introduced earlier $\gamma(z, y_2) = g_z z^2/2 + g_2 y_2^2/2 + g_{z2}|z|y_2$, with $g_z, g_2 > 0$, and $g_{z2} \neq 0$.[36] While the analysis thus far focused on $g_{z2} = 0$, we now allow g_{z2} to be positive or negative. For now, we maintain the assumption $\alpha^N = \alpha^P = \alpha$, i.e., over and underreporting of legal income are equally penalized.[37] The concealment isocost curves of $\gamma(z, y_2)$ for $g_{z2} < 0, g_{z2} = 0$, and $g_{z2} > 0$ are depicted in Figure 5.

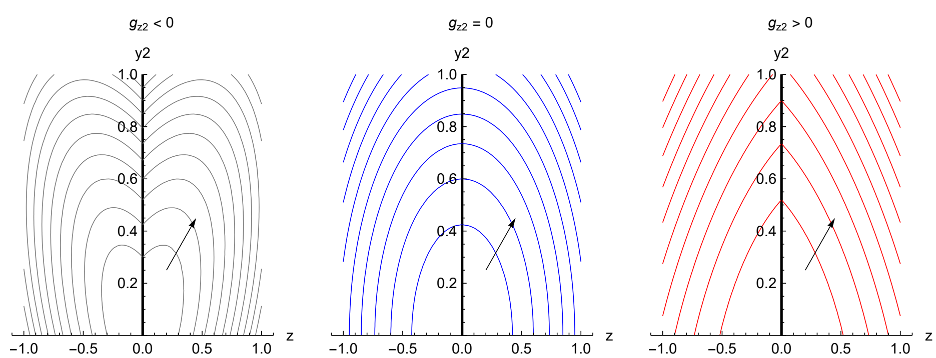

Figure 5: General concealment function. Different values of g_{z2}.

The figure shows undeclared legal income z on the horizontal axis and illicit income y_2 on the vertical axis. The concealment costs increase as we move outwards. When $g_{z2} = 0$ or $g_{z2} > 0$, a larger $|z|$, for a fixed y_2, always increases the concealment cost. However, when $g_{z2} < 0$, it is possible to observe a situation in which a slight increase in $|z|$ lowers the concealment cost so that the same cost can be attained with a higher y_2.

By considering a more general concealment function, we can rationalize a broader

[36] A full analysis of this case, along with all formal derivations, is provided in the Online Supporting Appendix C.

[37] We examine the implications of relaxing this assumption in section 8.3.

range of individual behaviors, including instances where individuals have no incentive to misreport or even overreport legal taxable income. Additionally, we can better understand the intricate interdependence between the ability to evade taxes and income generation from illegal sources.[38]

7.1 Individual Optimization

How does the individual optimization problem change in this context, considering that now z can be negative, positive, or zero? The following proposition characterizes the solutions depending on the sign of g_{z2}.

Proposition 6. *The solutions to the individual optimization problem, for given tax rates* t, *can be characterized as follows: (1) Suppose* $g_{z2} = 0$*: If* $t > 0$, *then* $z = z^P$ *and* $y_2 = y_2^P$*; (2) Suppose* $g_{z2} > 0$*: If* $0 < t \leqslant t^P$, *then* $z = 0$ *and* $y_2 = y_2^0$. *If* $t > t^P$, *then* $z = z^P$ *and* $y_2 = y_2^P$*; (3) Suppose* $g_{z2} < 0$*: If* $0 < t \leqslant t^N$, *then* $z = z^N$ *and* $y_2 = y_2^N$ *or* $z = z^P$ *and* $y_2 = y_2^P$. *If* $t > t^N$, *then* $z = z^P$ *and* $y_2 = y_2^P$. *The solutions* y_2^P, z^P, y_2^N, z^N, y_2^0, *and the threshold tax rates* t^N *and* t^P *are defined as follows:*

$$y_2^P = \frac{-g_{z2}A^P + g_z\Delta}{H} > 0, \quad y_2^N = \frac{g_{z2}A^N + g_z\Delta}{H} > 0, \quad y_2^0 = \frac{\Delta}{g_2 w_2} > 0, \tag{31}$$

$$z^P = \frac{g_2 A^P - g_{z2}\Delta}{H} > 0, \quad z^N = \frac{g_2 A^N + g_{z2}\Delta}{H} < 0, \tag{32}$$

where $A^P \equiv (1 - \alpha p)tw_2$, $A^N \equiv (1 + \alpha p)tw_2$, $\Delta \equiv [(1 - p\beta)w_2 - (1 - t)w_1]$, *and*

$$t^P = \frac{g_{z2}[(1 - p\beta)w_2 - w_1]}{(1 - \alpha p)g_2 w_2 - g_{z2} w_1} \geqslant 0, \quad t^N = \frac{-g_{z2}[(1 - p\beta)w_2 - w_1]}{(1 + \alpha p)g_2 w_2 + g_{z2} w_1} \geqslant 0. \tag{33}$$

Proof. See Online Supporting Appendix C. □

A few observations are worth highlighting. First, the tax rates t^N and t^P are the tax rates at which unreported legal income z changes from zero to positive (t^P when $g_{z2} > 0$)

[38] The technical details of this section can be found in the Online Supporting Appendix C.

or from negative to positive (t^N when $g_{z2} < 0$). These threshold tax rates and the sign and magnitude of g_{z2} are crucial for characterizing individuals' behavior and, therefore, the optimal level of t. Of course, when $g_{z2} = 0$, $t^N = t^P = 0$. Moreover, t^N and t^P are functions of p and g_{z2}, a factor that plays a crucial role in the subsequent analysis.

Second, when $g_{z2} > 0$, individuals will never choose a negative z. A negative z would imply over-reporting legal income, which increases both tax liability and the cost of hiding illegal income. It follows that in this case, individuals will report at most their entire legal income ($z \geqslant 0$). Specifically, when the tax rate is low enough ($t < t^P$), individuals report their entire legal income ($z = 0$). At low tax rates, the potential benefits from evading taxes on legal income are small relative to the direct cost of misreporting legal income and the additional cost it imposes on hiding illegal income. When the tax rate becomes sufficiently large ($t > t^P$), individuals will choose $z > 0$ (i.e., $z = z^P$), meaning they will underreport their legal income.

Third, when $g_{z2} < 0$, z and y_2 are complements in the sense that increasing one of them decreases the cost of concealing the other. In this case, $z^P > 0$ is a solution to the individual optimization problem for all $0 \leqslant t \leqslant 1$. However, when $0 < t \leqslant t^N$, z^N can also be a solution. When this is the case, the function z exhibits a discrete jump at $t = t^N$. That is, $z = z^N \leqslant 0$ for $0 \leqslant t \leqslant t^N$ and $z = z^P > 0$ for $t^N < t < 1$. The behavior of z affects other variables y_1, y_2, and x, impacting the welfare function, which will also show a jump at $t = t^N$. Thus, when determining the optimal tax rate, it's essential to consider that the highest welfare might be achieved at this non-differentiable point in the welfare function.

7.2 Optimal policy choices

In the case of the general concealment function, the analysis can get fairly complicated when $g_{z2} \neq 0$, because of the possibility of non-differentiability at t^N and t^P. Hence, we summarize the findings about optimal policy choices here while moving the details of the analysis to the Online Supporting Appendix C.

Optimal t for different values of p. The analysis in this case should also account for the fact that a change in p simultaneously shifts the curves and affects the threshold tax rates t^N and t^P, as explained earlier.[39] The outcomes, which depend on the sign of g_{z2}, can be summarized as follows.

When $g_{z2} > 0$, the conclusions are qualitatively similar to the baseline case of $g_{z2} = 0$. When $g_{z2} < 0$, the optimal tax rate may be achieved at a point in the welfare function that is non-differentiable. For instance, consider the outcomes when the solution is given by $\{y_1, y_2, z\} = \{y_1^N, y_2^N, z^N\}$.[40] In this case, the optimal tax rate declines as p increases for small values of p. The reason is that at these values of p, the optimal tax rate is $t = t^N$ and individuals choose $z = z^N = 0$. Since $\frac{\partial t^N}{\partial p} < 0$, there is a negative relationship between the audit probability and the optimal tax rate for small values of p. As p reaches a sufficiently large value, the audit agency's budget constraint becomes binding. At this value of p, the tax rate t is fully determined by the constraint $F - \theta(p) = 0$. An increase in p beyond this point would require the government to increase t to satisfy the constraint. Since this implies $t > t^N$, then individuals choose $\{y_1^P, y_2^P, z^P\}$. The optimal tax rate, therefore, discretely switches from $t = t^N$ to a value of t where the slope of the welfare function is zero. Further increases in p raise the optimal t.

[39] Figure C.7 in the Online Supporting Appendix C.4 shows the optimal tax rates (and associated values of the endogenous variables) for different values of p and g_{z2}.

[40] Figure C.9 in the Online Supporting Appendix C.4 compares the two solutions obtained when $g_{z2} < 0$.

Optimal t for different values of Ψ_0. We next consider how the strength of the externality, Ψ_0, influences the optimal tax rate and related variables. We analyze this relationship while keeping the audit probability p constant and considering different values of g_{z2}.

When $g_{z2} \geqslant 0$, the qualitative conclusions of the baseline model hold. However, if $g_{z2} < 0$, the optimal tax rate may exhibit a discontinuous jump at low levels of Ψ_0. For higher values of Ψ_0, the tax rate stabilizes at $t = t^N$. Specifically, when Ψ_0 is small, the optimal tax rate exceeds t^N, leading individuals to choose $\{y_1^P, y_2^P, z^P\}$. As Ψ_0 increases, the government reduces t, which in turn lowers y_2. Once Ψ_0 surpasses a critical value, welfare is maximized at $t = t^N$, where a discontinuity in the welfare function occurs. This pattern persists for larger Ψ_0, with the optimal tax rate remaining at t^N. Moreover, if p is set at a higher level, t still declines as Ψ_0 increases, but the optimal tax rate is higher, and the discontinuity occurs at a larger Ψ_0 due to the binding budget constraint of the audit agency.[41]

Optimal combination of t and p for different values of Ψ_0. When the government selects the optimal $\{t, p\}$ policy combination, the variables respond similarly across all values of g_{z2} to changes in Ψ_0: as Ψ_0 rises, t tends to decrease and p tends to increases (at least as long the audit agency's budget constraint does not bind.).[42] Figure C.14 in the Appendix illustrates how all variables react to changes in Ψ_0.

8 Extensions

This section explores extensions to the baseline model, incorporating new dimensions that affect the optimal tax rate and enforcement policies. We first introduce an extensive margin decision, allowing individuals to choose whether to engage in illegal activities.

[41] See Online Supporting Appendix C.4 for additional details.

[42] The details of the analysis are explained in the Online Supporting Appendix C.4.2.

Next, we examine how differing detection probabilities for evaded and illegal income impact optimal policy. We then analyze the role of alternative penalty structures, showing how they influence tax rates and enforcement effectiveness.

8.1 Extensive Margin

Suppose an individual decides whether to engage in illegal activities (extensive margin) and allocates time to each activity (intensive margin).[43] We model this decision using a random utility framework. An individual who does not engage in illegal activities has indirect utility $\tilde{v}^o = v^o(t, p, G, Y_2) + \varepsilon^o$, where

$$v^o(t, p, G, Y_2) = \max_{\{y_1, z\}}\{y_1 - tx + h(\ell) - (g_z/2)z^2 - p\alpha tz + \Phi(G) - \Psi(Y_2), \ell = 1 - n_1\}. \quad (34)$$

Here, $x = y_1 - z$ and $n_1 = y_1/w_1$. The solutions, denoted y_1^o and z^o, follow from Lemma 3 in the Online Supporting Appendix D.1.

If an individual engages in illegal activities, the corresponding solutions y_1, y_2, and z come from Proposition 1, with indirect utility $\tilde{v} = v(t, p, G, Y_2) + \varepsilon$. The random terms ε^o and ε follow independent Gumbel distributions, leading to the share of individuals engaging in illegal activities $\sigma = \frac{\exp(v)}{\exp(v)+\exp(v^o)}$. As a result, aggregate illegal activity is given by $Y_2 = \sigma y_2$.

Comparing the solutions in each case reveals that individuals engaged in both legal and illegal activities allocate less time to legal production than those solely in legal work. As a consequence, their reported legal income is also lower. In addition, individuals who participate in illegal activities experience a higher overall utility than those who engage solely in legal activities. The formal proof of these results is provided in the Online Supporting Appendix D.1.

[43] The complete analysis and formal derivations are included in the Online Supporting Appendix D.1.

8.2 Probability of Detection Differs by Type of Income

Previously, an increase in p allowed the government to detect both tax evasion and illegal income. Now, suppose detection differs: evaded income is detected with probability p_z and illegal income with probability p_2. Assuming $g_{z2} = 0$, individuals choose a positive level of z, and we set $\alpha^P = \alpha$. The solutions in this case are $y_2 = \frac{w_2(1-\beta p_2)-(1-t)w_1}{g_2 w_2}$, and $z = \frac{t(1-\alpha p_z)}{g_z}$. Since p_z affects only z and p_2 influences y_2, the changes in declared income x are $\frac{\partial x}{\partial p_z} = \frac{\alpha t}{g_z} > 0$, and $\frac{\partial x}{\partial p_2} = \frac{\beta w_1}{g_2 w_2} > 0$. The effectiveness of p_z in increasing reported income grows with t, while p_2's effect remains unchanged. When t is low, p_2 is more effective, but for sufficiently large t, p_z dominates.

We next examine how the externality strength Ψ_0 affects the optimal policy mix $\{t, p_z, p_2\}$ under different audit cost structures: (i) independent costs ($\theta_{z2} = 0$) and (ii) complementary costs ($\theta_{z2} < 0$). The optimal values are compared between an unconstrained scenario ($p_z \neq p_2$) and a constrained scenario ($p_z = p_2 = p$). We showed earlier that in the constrained case, the optimal audit probability increases with Ψ_0. In the unconstrained case, for scenario (i), an increase in Ψ_0 leads to a decrease in p_z and an increase in p_2, shifting the policy focus toward stricter monitoring of illegal income. In case (ii), where audit costs are complementary, both p_z and p_2 increase with Ψ_0. Notably, when Ψ_0 is low, the tax rate is lower in the unconstrained scenario than in the constrained one, indicating that greater flexibility in audit allocation allows for reduced reliance on taxation.[44]

8.3 Alternative Penalty Functions

We finally compare different penalty functions F that impose distinct penalties on misreported legal income depending on whether $z > 0$ or $z < 0$: (i) $\alpha^N = \alpha^P = \alpha$ (baseline

[44]Detailed derivations are provided in the Online Supporting Appendix D.2.

case); (ii) $\alpha^N = 0$, $\alpha^P = \alpha$ (no penalty for overreporting); (iii) $\alpha^N = -\alpha$, $\alpha^P = \alpha$ (reward for detected overreporting); and (iv) $\alpha^N = -1$, $\alpha^P = \alpha$ (reimbursement of overpayment).

We analyze how changes in the intensity of the negative externality Ψ_0 influence the optimal tax rate while keeping audit probability p fixed. In cases where $g_{z2} < 0$, the alternative penalty functions become relevant since overreporting ($z < 0$) may occur. These penalty structures affect both the threshold tax rate t^N and the responsiveness of z to changes in t when $z < 0$. Online Supporting Appendix D.3 provides detailed graphs illustrating these relationships.

In summary, when Ψ_0 is sufficiently large, penalty functions (ii), (iii), and (iv) enable the government to impose higher tax rates than in the benchmark case (i). The ranking of optimal tax rates follows: $t^N_{(i)} < t^N_{(ii)} < t^N_{(iv)} < t^N_{(iii)}$. These alternative penalty schemes also enhance audit agency revenue and overall welfare. The key insight is that allowing credits for overreported income can improve welfare in cases where $z < 0$ occurs. While higher tax rates lead to an increase in illegal good production (y_2), the additional tax revenue strengthens public good provision, resulting in net welfare gains.

9 Conclusions

A major contribution of this paper is to unify different related strands of the literature on optimal income taxes, tax evasion, provision of public goods, and containment of public "bads". While the framework of the paper applies to any nation, the substitution possibilities between legal income and effort expended on illicit production are especially pertinent for poorer nations with inadequate infrastructure to enforce the "rule of law". For example, we find that a lower audit probability encourages expansion of the illegal sector at the expense of the legal sector and raises tax evasion. Also, at a lower audit probability, the marginal evasion effect of a change in the income tax rate is larger, limiting the government's ability to raise the income tax rate to fund the public good adequately. Consequently, nations with poorer enforcement infrastructure are likely to suffer from inadequate public good provision and over-provision of the public bad.

A key factor of our analysis is the strength of the negative externality of the illegal good. The larger the strength of this negative externality, the greater the damage that the income tax does in terms of spurring the production of the illicit good. Accordingly, our baseline model finds a monotonic negative relationship between the constrained optimal tax rate and the strength of the externality.

Another insight gleaned from our baseline model is the importance of coordination between branches of the government, such that when the surplus of the audit agency can be seamlessly transferred to the government for public good provision, the optimal tax, the provision of the public good, and the welfare are all higher compared to a situation where such coordination is not possible. In addition, we show that when it is possible to optimally choose the audit probability, the inefficiencies arising out of the dual problems of tax evasion and the illegal market are somewhat mitigated.

The extensions considered in the paper throw further light on the main findings. Overall, the results in alternate settings align with the baseline model except for the case of complementarities in the concealment cost function, where multiple solutions may arise. Depending on the solution the economy is in, one obtains a negative or zero relationship between the optimal tax rate and the strength of the negative externality of the illegal good. Another extension considers heterogeneity among individuals who determine their participation (extensive margin) and effort (intensive margin) in the illicit market and finds that the income tax affects the two margins in the same direction – preserving the qualitative messages of our baseline model.

Finally, we note that we have had to make some limiting assumptions for focus and tractability. For example, we do not explicitly consider some possible general equilibrium interactions where policies can differentially distort relative prices (evasion may be less costly in some sectors than others) in different sectors. We have also abstracted from consideration of the impact on corporate behavior, where tax evasion can cause companies to alter their structure or operations by setting up financial subsidiaries or operating in tax havens. The latter consideration is critical when one considers the possibility of footloose organizations in an international setting, where enforcement or tax policy in one country can drive agents to another country, requiring international coordination to achieve global allocative efficiency. Such related issues remain on our agenda for future research.

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Technical Appendix

A Section 5 – Individual Optimization: Baseline Case

A.1 Comparative statics with respect to the tax rate

An increase in t decreases the return to the production of the legal good, incentivizing the production of the illegal good and the underreporting of legal income. Formally, the comparative statics for t are

$$\frac{\partial y_1}{\partial t} = \frac{w_1^2}{h''} - \frac{w_1}{w_2}\frac{\partial y_2}{\partial t} < 0, \quad \frac{\partial y_2}{\partial t} = \frac{w_1}{g_2 w_2} > 0, \quad \frac{\partial y_1}{\partial t}\frac{1}{w_1} + \frac{\partial y_2}{\partial t}\frac{1}{w_2} = \frac{w_1}{h''} < 0, \quad (35)$$

$$\frac{\partial z}{\partial t} = \frac{(1-\alpha p)}{g_z} > 0, \quad \frac{\partial x}{\partial t} = \frac{\partial y_1}{\partial t} - \frac{\partial z}{\partial t} = \frac{w_1^2}{h''} - \frac{w_1^2}{g_2 w_2^2} - \frac{(1-p\alpha)}{g_z} < 0. \quad (36)$$

Moreover, under the present assumptions,

$$\varepsilon_{z,t} \equiv \frac{\partial z}{\partial t}\frac{t}{z} = 1, \quad \varepsilon_{y_2,t} \equiv \frac{\partial y_2}{\partial t}\frac{t}{y_2} = \frac{t w_1}{w_2(1-p\beta)-(1-t)w_1} < 1. \quad (37)$$

While y_2 and z are linear in t (i.e., $\frac{\partial^2 y_2}{\partial t^2} = \frac{\partial^2 z}{\partial t^2} = 0$), for y_1 and x it follows that

$$\frac{\partial^2 y_1}{\partial t^2} = \frac{\partial^2 x}{\partial t^2} = \frac{w_1^3 h'''}{(h'')^3}. \quad (38)$$

Throughout the analysis, we assume $h(\ell)'' < 0$ and $h(\ell)''' > 0$.[45] When this is the case, both y_1 and x are concave in t, i.e., $\frac{\partial^2 y_1}{\partial t^2} = \frac{\partial^2 x}{\partial t^2} < 0$.

For future reference, taxes collected by the government $T = tx$ are a concave function of t:

$$\frac{d^2 T}{dt^2} = 2\frac{\partial x}{\partial t} + t\frac{\partial^2 x}{\partial t^2} < 0. \quad (39)$$

Therefore, for low values of t, $\frac{\partial T}{\partial t} > 0$, and for high values of t, $\frac{\partial T}{\partial t} < 0$. Moreover, the penalty function F is increasing and convex in t:

$$\frac{dF}{dt} = 2p\alpha z + p\beta\frac{w_1}{g_2 w_2} = \frac{2p\alpha t(1-p\alpha)}{g_z} + p\beta\frac{w_1}{g_2 w_2} > 0, \quad \frac{d^2 F}{dt^2} = \frac{2p\alpha(1-p\alpha)}{g_z} > 0. \quad (40)$$

This means that the government, by increasing the tax rate, can increase revenue since

[45] For the most frequently used functions $h(\ell)$, $h(\ell)''' > 0$. For example, when $h(\ell) = h_0 \ell^{h_1}$, $h_0 > 0$, $0 < h_1 < 1$, $h'(\ell) = h_0 h_1 \ell^{h_1 - 1} > 0$, $h''(\ell) = -h_0 h_1 (1-h_1)\ell^{h_1-2} < 0$, $h'''(\ell) = -(2-h_1)h''(\ell)/\ell > 0$. Similarly when $h(\ell) = h_0 \mathrm{Log}(\ell)$.

the collected penalties will be higher. Finally, note that

$$\frac{d^2T}{dt^2} + \frac{d^2F}{dt^2} = 2\frac{\partial x}{\partial t} + t\frac{\partial^2 x}{\partial t^2} + 2p\alpha\frac{\partial z}{\partial t} = 2\frac{\partial y_1}{\partial t} - 2(1-p\alpha)\frac{\partial z}{\partial t} + t\frac{\partial^2 x}{\partial t^2} < 0. \tag{41}$$

A.2 Comparative statics with respect to the audit probability

Formally, the comparative statics for p are given by:

$$\frac{\partial y_1}{\partial p} = \frac{\beta w_1}{g_2 w_2} = -\frac{w_1}{w_2}\frac{\partial y_2}{\partial p} > 0,\ \frac{\partial y_1}{\partial p}\frac{1}{w_1} + \frac{\partial y_2}{\partial p}\frac{1}{w_2} = 0,\ \frac{\partial y_1}{\partial p} + \frac{\partial y_2}{\partial p} = -\frac{\beta(w_2 - w_1)}{g_2 w_2} < 0 \tag{42}$$

$$\frac{\partial z}{\partial p} = -\frac{\alpha t}{g_z} < 0,\ \frac{\partial x}{\partial p} = \frac{\partial y_1}{\partial p} - \frac{\partial z}{\partial p} > 0. \tag{43}$$

In other words, an increase in the audit probability shifts production from illegal to legal activities and induces individuals to report more income. Note, however, that when $w_2 > w_1$, total production $(y_1 + y_2)$ declines. Note that

$$\varepsilon_{z,p} \equiv \frac{\partial z}{\partial p}\frac{p}{z} = \frac{-\alpha p}{1 - \alpha p} < 0.$$

Moreover,

$$\frac{\partial^2 y_1}{\partial p \partial t} = \frac{\partial^2 y_2}{\partial p \partial t} = 0, \quad \frac{\partial^2 z}{\partial p \partial t} = -\frac{\alpha}{g_z} < 0, \quad \frac{\partial^2 x}{\partial p \partial t} = -\frac{\partial^2 z}{\partial p \partial t} > 0. \tag{44}$$

The audit probability does not impact the effect of the tax rate on y_1 and y_2. However, it does reduce the positive impact that the tax rate has on z and the negative impact (in absolute value) that the tax rate has on x. This means that when the audit probability is higher, the same increase in t would have a smaller impact on the amount of misreported income.

Finally, the tax function is linear in p,

$$\frac{\partial T}{\partial p} = t\left(\frac{\beta g_z w_1 + \alpha g_2 t w_2}{g_2 g_z w_2}\right) > 0, \quad \frac{\partial^2 T}{\partial p^2} = 0,$$

and the function $F - \theta(p)$ is concave in p:

$$\begin{aligned} \frac{\partial[F - \theta(p)]}{\partial p} &= \frac{\beta[w_2(1 - 2p\beta) - w_1(1-t)]}{g_2 w_2} + \frac{(1 - 2p\alpha)\alpha t^2}{g_z} - \theta', \\ \frac{\partial^2[F - \theta(p)]}{\partial p^2} &= -\frac{2[\beta^2 g_z + (\alpha t)^2 g_2]}{g_2 g_z} - \theta'' < 0. \end{aligned}$$

Specifically, when $p = 0$, $[F - \theta(p)] = 0$, and $\partial[F - \theta(p)]/\partial p > 0$. Suppose $\theta = (\theta_0/2)p^2$, with $\theta_0 > 0$, and let

$$0 < p^* = \frac{\alpha g_2 t^2 w_2 + \beta g_z[w_2 - (1-t)w_1]}{2w_2(\alpha^2 t^2 g_2 + \beta^2 g_z + \theta_0 g_2 g_z/2)} < 1.$$

Then, when $p = 2p^*$, $F = 0$, and F reaches a maximum at $p = p^*$.

B Section 6 – Policy Choices

B.1 Choosing the tax rate: Analysis

Suppose the central government maximizes $W \equiv \int v di$ wrt t, or $W = C + H - \Gamma + \Phi(G) - \Psi(Y_2)$ with respect to t, where $C \equiv \int c di$, $H \equiv \int h(\ell) di$, $\Gamma \equiv \int \gamma(z) di$, $\Phi(G) \equiv \int \Phi(G) di$, $Y_2 \equiv \int y_2 di$, $T = \int tx di$, subject to

$$G = t(y_1 - z) + \mu[F - \theta(p)],\ 0 \leqslant \mu \leqslant 1, \text{ with } F \geqslant \theta(p) \quad [\lambda]. \tag{45}$$

Using the envelope theorem, $\partial v/\partial t = -(x + F_t)$, where $x = y_1 - z \geqslant 0$ and $F_t = p\alpha z \geqslant 0$. The FOC for t is

$$\begin{aligned} \frac{\partial W}{\partial t} &\equiv -(x + F_t) + \Phi' \frac{dG}{dt} - \Psi' \frac{\partial y_2}{\partial t} + \lambda \frac{dF}{dt} = 0, \\ &= -(x + F_t) + \Phi' \left(x + t\frac{\partial x}{\partial t} \right) - \Psi' \frac{\partial y_2}{\partial t} + (\Phi'\mu + \lambda)\frac{dF}{dt} = 0. \end{aligned} \tag{46}$$

The FOC can also be written as

$$\begin{aligned} &\Phi' \left(x + t\frac{\partial x}{\partial t} \right) + (\Phi'\mu + \lambda)\frac{dF}{dt} = (x + F_t) + \Psi' \frac{\partial y_2}{\partial t}, \\ &(\Phi' - 1)(x + F_t) = \Psi' \frac{\partial y_2}{\partial t} + \Phi' \left(F_t - t\frac{\partial x}{\partial t} - \mu \frac{dF}{dt} \right) - \lambda \frac{dF}{dt}. \end{aligned} \tag{47}$$

Suppose $\mu = 1$, and the constraint is binding at the optimal t ($\lambda > 0$). Since in this case $F = \theta(p)$, then the government does not receive any additional resources from the audit agency. Therefore, the binding budget constraint $F = \theta(p)$ plays a key role in determining t.

B.2 Discussion: How does the optimal tax rate depend on the audit probability?

In general,

$$\frac{\partial t}{\partial p} = -\frac{\partial^2 W/\partial t \partial p}{\partial^2 W/\partial t^2}, \tag{48}$$

where $\partial^2 W/\partial t^2 < 0$ and

$$\frac{\partial^2 W}{\partial t \partial p} = -\left(\frac{\partial x}{\partial p} + \frac{dF_t}{dp} \right) + \Phi'' \frac{dG}{dp}\frac{dG}{dt} + \Phi' \frac{d^2 G}{dt dp} - \Psi'' \frac{\partial y_2}{\partial p}\frac{\partial y_2}{\partial t} - \Psi' \frac{\partial^2 y_2}{\partial t \partial p} + \lambda \frac{d^2 F}{dt dp}. \tag{49}$$

When $g_{z2} = 0$, the following results hold:

$$\frac{\partial^2 y_2}{\partial t \partial p} = 0, \tag{50}$$

$$\frac{\partial y_2}{\partial t}\frac{\partial y_2}{\partial p} = -\frac{\beta w_1}{g_2^2 w_2} < 0, \tag{51}$$

$$\frac{\partial x}{\partial p} + \frac{dF_t}{dp} = \frac{\partial x}{\partial p} + p\alpha\frac{\partial z}{\partial p} + \alpha z = \frac{2t\alpha(1-\alpha p)}{g_z} + \frac{\beta w_1}{g_2 w_2} > 0, \tag{52}$$

$$\frac{dG}{dp} = \frac{\alpha t^2}{g_z} + \frac{t\beta w_1}{g_2 w_2} + \mu\left(\frac{dF}{dp} - \theta'\right), \tag{53}$$

$$\frac{d^2 G}{dt dp} = \frac{2\alpha t}{g_z} + \frac{\beta w_1}{g_2 w_2} + \mu\left[\frac{2t\alpha(1-2p\alpha)}{g_z} + \frac{\beta w_1}{g_2 w_2}\right]. \tag{54}$$

Additionally, if $\mu = 1$,

$$\frac{\partial^2 W}{\partial t \partial p} = \frac{d^2 G}{dt dp} - \left(\frac{\partial x}{\partial p} + \frac{dF_t}{dp}\right) + \Phi''\frac{dG}{dp}\frac{dG}{dt} + (\Phi' - 1)\frac{d^2 G}{dt dp} - \Psi''\frac{\partial y_2}{\partial p}\frac{\partial y_2}{\partial t} + \lambda\frac{d^2 F}{dt dp}. \tag{55}$$

Note that

$$\frac{d^2 G}{dt dp} = \frac{4t\alpha(1-p\alpha)}{g_z} + \frac{2\beta w_1}{g_2 w_2} > 0, \tag{56}$$

$$\frac{d^2 G}{dt dp} - \left(\frac{\partial x}{\partial p} + \frac{dF_t}{dp}\right) = \frac{2t\alpha(1-p\alpha)}{g_z} + \frac{\beta w_1}{g_2 w_2} > 0. \tag{57}$$

Suppose $\lambda = 0$ and $\Phi' > 1$. Then, all the terms in the previous expression are positive except for the third one. It follows, for example, that when Φ'' is relatively small, $\partial^2 W/\partial t \partial p > 0$, which implies that $\partial t/\partial p > 0$. In other words, by raising p, the government can increase the tax rate and reduce the severity of the underprovision of G, as shown in the graphs in the text.

The impact of p on T and $[F - \theta(p)]$ considering that the optimal t depends on p can be expressed as follows:

$$\frac{dT}{dp} = x\frac{\partial t}{\partial p} + t\left(\frac{\partial x}{\partial t}\frac{\partial t}{\partial p} + \frac{\partial x}{\partial p}\right) = t\frac{\partial x}{\partial p} + \left(x + t\frac{\partial x}{\partial t}\right)\frac{\partial t}{\partial p},$$

and

$$\frac{d[F - \theta(p)]}{dp} = \underbrace{\beta\left(y_2 + p\frac{\partial y_2}{\partial p}\right) + \alpha t\left(z + p\frac{\partial z}{\partial p}\right) - \theta'(p)}_{(i)} + \underbrace{\alpha p z\frac{\partial t}{\partial p}}_{(ii)}.$$

First, the function T is concave in t. For tax rates on the efficient side of the Laffer curve (i.e., tax rates such that $x + t\frac{\partial x}{\partial t}$), it follows that $\frac{\partial T}{\partial p} > 0$. Second, the function $F - \theta(p)$

is concave in p and reaches a maximum when expression (i) is equal to zero. When p positively affects t, however, the maximum is reached at a higher level of p, since (ii) is positive.

B.3 Choosing the tax rate and the audit probability

Suppose the government chooses both the tax rate and the audit probability. Consider the FOC for p. Using the envelope theorem, $\partial v/\partial p = -F_p$, we obtain

$$\frac{\partial W}{\partial p} \equiv -F_p + \Phi' \frac{\partial G}{\partial p} + \lambda \left(\frac{dF}{dp} - \theta' \right) - \Psi' \frac{\partial y_2}{\partial p} = 0, \tag{58}$$

$$= -F_p + \Phi' \left[t \frac{\partial x}{\partial p} + \mu \left(\frac{dF}{dp} - \theta' \right) \right] + \lambda \left(\frac{dF}{dp} - \theta' \right) - \Psi' \frac{\partial y_2}{\partial p}, \tag{59}$$

$$= -F_p + \Phi' t \frac{\partial x}{\partial p} - \Psi' \frac{\partial y_2}{\partial p} + (\Phi' \mu + \lambda) \left(\frac{dF}{dp} - \theta' \right). \tag{60}$$

Suppose that $\mu = \lambda = 0$. Then, the optimal policy combination $\{t, p\}$ satisfies

$$\frac{-(x + F_t) + \Phi' \left(x + t \frac{\partial x}{\partial t} \right)}{-F_p + \Phi' t \frac{\partial x}{\partial p}} = \frac{\frac{\partial y_2}{\partial t}}{\frac{\partial y_2}{\partial p}} = -\frac{w_1}{\beta w_2} < 0. \tag{61}$$

When $\mu = 1$, the optimal policy combination $\{t, p\}$ satisfies

$$\frac{-(x + F_t) + \Phi' \left(x + t \frac{\partial x}{\partial t} \right) - \Psi' \frac{\partial y_2}{\partial t}}{-F_p + \Phi' t \frac{\partial x}{\partial p} - \Psi' \frac{\partial y_2}{\partial p}} = \frac{\frac{dF}{dt}}{\frac{dF}{dp} - \theta'}. \tag{62}$$

While $\frac{dF}{dt} > 0$, the function $F - \theta(p)$ is concave in p.

B.4 Proof of Proposition 5

Proof. By differentiating the FOCs with respect to Ψ_0, we obtain

$$\frac{\partial t}{\partial \Psi_0} = -\frac{1}{D} \left(W_{t\Psi_0} W_{pp} - W_{p\Psi_0} W_{tp} \right) = -\frac{W_{pp}}{D} \left(W_{t\Psi_0} + \left. \frac{\partial t}{\partial p} \right|_{p \text{ exog.}} W_{p\Psi_0} \right), \tag{63}$$

$$\frac{\partial p}{\partial \Psi_0} = -\frac{1}{D} \left(W_{p\Psi_0} W_{tt} - W_{t\Psi_0} W_{tp} \right) = -\frac{W_{tt}}{D} \left(W_{p\Psi_0} + \left. \frac{\partial p}{\partial t} \right|_{t \text{ exog.}} W_{t\Psi_0} \right), \tag{64}$$

where $W_{tt} < 0$, $W_{pp} < 0$, $W_{tp} > 0$ (under the conditions discussed in Section B.2), $W_{t\Psi_0} < 0$, $W_{p\Psi_0} > 0$, and $D \equiv W_{tt} W_{pp} - (W_{tp})^2 > 0$. The signs of $\frac{\partial t}{\partial \Psi_0}$ and $\frac{\partial p}{\partial \Psi_0}$ depend on the signs of the expressions within the brackets in (63) and (64), respectively (since $-\frac{W_{tt}}{D} > 0$ and $-\frac{W_{pp}}{D} > 0$). The first term in (63), $W_{t\Psi_0}$, captures the direct impact of Ψ_0 on the marginal utility of t, which is negative. The second term captures the indirect effect on t through the impact of Ψ_0 on p. The expression $W_{p\Psi_0}$ describes the effect of a change in Ψ_0 on the marginal utility of p, which is positive, and $\left.\frac{\partial t}{\partial p}\right|_{p\text{ exog.}}$ the impact of a change in p on t when p is assumed exogenously determined, which is positive when $W_{tp} > 0$, as described in expression (22). When the first term, the direct effect, dominates the second term, the indirect effects, $\frac{\partial t}{\partial \Psi_0}$ is negative. A similar reasoning applies to $\frac{\partial p}{\partial \Psi_0}$. □

Taxation, Compliance, and Clandestine Activities

by

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March 2025

Supporting Information (Online Appendix)

C Section 7 – General Concealment Function

C.1 Proof of Proposition 6

Proof. In general, the FOCs at an interior solution for y_1, z and y_2 are given by:

$$y_1: \quad (1-t) - \tfrac{h'}{w_1} = 0 \quad \Rightarrow \quad w_1(1-t) = h', \tag{1}$$

$$\begin{array}{lccc} z<0: & (1+p\alpha)t - g_z z + g_{z2}y_2 = 0 & \Rightarrow & g_z z = (1+p\alpha)t + g_{z2}y_2, \\ z=0: & \begin{array}{c}(1+p\alpha)t + g_{z2}y_2 > 0 \\ (1-p\alpha)t - g_{z2}y_2 < 0\end{array} & \Rightarrow & \begin{array}{c}(1+p\alpha)t > -g_{z2}y_2 \\ (1-p\alpha)t < g_{z2}y_2\end{array}, \\ z>0: & (1-p\alpha)t - g_z z - g_{z2}y_2 = 0 & \Rightarrow & g_z z = (1-p\alpha)t - g_{z2}y_2, \end{array} \tag{2}$$

and

$$y_2: \begin{cases} z<0: & (1-p\beta-g_2y_2+g_{z2}z) - \frac{h'}{w_2} = 0 \quad \Rightarrow \quad w_2(1-p\beta-g_2y_2+g_{z2}z) = w_1(1-t), \\ z=0: & (1-p\beta-g_2y_2) - \frac{h'}{w_2} = 0 \quad \Rightarrow \quad w_2(1-p\beta-g_2y_2) = w_1(1-t), \\ z>0: & (1-p\beta-g_2y_2-g_{z2}z) - \frac{h'}{w_2} = 0 \quad \Rightarrow \quad w_2(1-p\beta-g_2y_2-g_{z2}z) = w_1(1-t). \end{cases} \tag{3}$$

As before, once we obtained y_2, the FOC $w_1(1-t) = h'(\ell)$ determines y_1, given that $\ell = 1-(n_1+n_2)$, and $n_1+n_2 = y_1/w_1 + y_2/w_2$. This means that $y_1 = 1 - h'^{-1}[w_1(1-t)] - \frac{w_1}{w_2}y_2$.

The solutions are obtained as follows. Suppose that $z<0$. Then, using the first equation of (2) and the first equation of (3) we solve for $\{y_2, z\} = \{y_2^N, z^N\}$. Suppose that an individual chooses $z>0$. Then, using the third equation of (2) and the third equation of (3), we can solve for $\{y_2, z\} = \{y_2^P, z^P\}$. Finally, suppose that $z=0$. Then, using the second equation of (2) and the second equation of (3), we obtain the solution $\{y_2, z\} = \{y_2^0, 0\}$.

The Hessian matrix (when z is positive or negative) is

$$H = \begin{bmatrix} h''/w_1^2 & h''/w_1 w_2 & 0 \\ h''/w_1 w_2 & h''/w_2^2 - g_2 & -\mathrm{sign}(z) g_{z2} \\ 0 & -\mathrm{sign}(z) g_{z2} & -g_z \end{bmatrix}, \tag{4}$$

and the SOCs for a maximum satisfy $|H_1| < 0$, $|H_2| > 0$, and $|H| \equiv h''(g_z g_2 - g_{z2}^2)/w_1^2 < 0$.

Throughout the analysis, we assume $(1 - p\beta)w_2 > w_1 > 0$, which implies that $y_2 > 0$ for all t.

The solutions depend on the threshold tax rates t^N and t^P. First, t^N is the tax rate at which $z^N = 0$, and t^P is the tax rate at which $z^P = 0$. Different cases arise depending on the g_{z2} value. If $g_{z2} = 0$, $t^N = t^P = 0$. When $g_{z2} > 0$, $t^P > 0$ (and $t^N < 0$), and when $g_{z2} < 0$, $t^N > 0$ (and $t^N < 0$).

Consider some preliminary analysis:

- Note that all variables are linear in t. This means that the behavior of z^N and z^P with respect to t can be characterized by their intercepts at $t = 0$ and their slopes, $\frac{\partial z^N}{\partial t}$ and $\frac{\partial z^N}{\partial t}$.[46]

- When $g_{z2} > 0$:

  - $z^N(0) > 0$ and $\frac{\partial z^N}{\partial t} > 0$. This means that $z^N < 0$ can never be a solution.
  - $z^P(0) < 0$. Therefore, a sufficient condition for $z^P > 0$ to be a solution is $\frac{\partial z^P}{\partial t} > 0$ and this happens when $(1 - \alpha p)g_2 w_2 - g_{z2} w_1 > 0$. In this case, $z^P > 0$ holds for $t > t^P$.
  - In sum, given that $\gamma(y_2, z)$ is strictly convex (i.e., $g_z g_2 > g_{z2}^2$), we need $(1 - \alpha p)g_2 w_2 - g_{z2} w_1 > 0$ when $g_{z2>0}$ so that $z^P > 0$ could be a solution. If $g_{z2} > 0$, z^P would be negative at low values of t (for instance, at $t = 0$), which means that z^P cannot be a solution. Hence, $\frac{\partial z^P}{\partial t} > 0$ is needed for z^P to be positive at a sufficiently high value of t (specifically $t > t^P$).

- When $g_{z2} < 0$

[46] Note that $z^N(0) = \frac{g_{z2}[(1-\beta p)w_2 - w_1]}{(g_z g_2 - g_{z2}^2)w_2}$, $\frac{\partial z^N}{\partial t} = \frac{g_2 w_2(1+\alpha p) + g_{z2} w_1}{w_2(g_z g_2 - g_{z2}^2)}$, and $z^P(0) = \frac{-g_{z2}[(1-\beta p)w_2 - w_1]}{(g_z g_2 - g_{z2}^2)w_2}$, $\frac{\partial z^P}{\partial t} = \frac{g_2 w_2(1-\alpha p) - g_{z2} w_1}{w_2(g_z g_2 - g_{z2}^2)}$. It follows that $z^N(0) + z^P(0) = 0$.

- $z^N(0) < 0$. Therefore, a sufficient condition for $z^N < 0$ to be a solution is $\frac{\partial z^N}{\partial t} > 0$. This happens when $(1+\alpha p)g_2w_2 + g_{z2}w_1 > 0$. In this case, $z^N < 0$ holds for $0 \leqslant t < t^P$.
- $z^P(0) > 0$ and $\frac{\partial z^P}{\partial t} > 0$. This means that $z^P > 0$ can be a solution for $t > 0$.
- $z^N < 0$ when $t = 0$, so z^N can be a solution. The slope $\frac{\partial z^N}{\partial t}$ determines how z^N changes when t changes, and the sign of this slope depends on the sign of $(1+\alpha p)g_2w_2 + g_{z2}w_1$. In other words, for $t^N > 0$ to exist when $g_{z2} < 0$, we need $(1+\alpha p)g_2w_2 + g_{z2}w_1 > 0$.
- The condition $(1+\alpha p)g_2w_2 + g_{z2}w_1 > 0$ holds when $g_2 > -g_{z2}$ since $(1+\alpha p)w_2 > w_1$ is always true. If the direct cost of increasing y_2 is greater than or equal to the direct cost of increasing z, i.e., $g_2 \geqslant g_z$, then $(1+\alpha p)g_2w_2 + g_{z2}w_1 > 0$, since $g_zg_2 > g_{z2}^2$ (by strict convexity of $\gamma(z, y_2)$).

Note that when $t = 0$, $y^N = y^P$. This is not true for z. Suppose $t = 0$ and $g_{z2} > 0$, then z^N cannot be a solution since $z^N < 0$. Suppose $g_{z2} < 0$, then $z^N < 0$ and z^P can be a solution at $t = 0$.

While the solutions depend on several parameters, the sign of g_{z2} is key.

Case (1): $g_{z2} = 0$. Shown before.

Case (2): $g_{z2} > 0$.

If $g_{z2} > 0$, then $z < 0$ cannot be a solution. From (2), note that if $z < 0$, then $g_zz = (1+p\alpha)t + g_{z2}y_2$. However, when $g_{z2} > 0$, it follows from this last equation that $z > 0$, which is a contradiction. This means that when $g_{z2} > 0$, either $z = 0$ or $z > 0$.

If $g_{z2} > 0$ and $t > t^P$, then $z > 0$. For $z > 0$ to be a solution, we need $g_zz = (1-p\alpha)t - g_{z2}y_2 > 0$, or $z = [(1-p\alpha)t - g_{z2}y_2]/g_z > 0$ (since $g_{z2} > 0$, $(1-p\alpha) > 0$). Moreover, for $z^P > 0$, we need $t > t^P$.

If $g_{z2} > 0$ and $0 < t \leqslant t^P$, then $z = 0$. When $t = 0$, $0 > -g_{z2}y_2$ and $0 < g_{z2}y_2$, so that $z = 0$. If $t > 0$, $(1+p\alpha)t > -g_{z2}y_2$ always holds, but $(1-p\alpha)t < g_{z2}y_2$ only holds for sufficiently low levels of t. Specifically, since when $z = 0$, $y_2 = y_2^0 = [w_2(1-p\beta) - w_1(1-t)]/(g_2w_2)$, it follows that $(1-p\alpha)t < g_{z2}y_2 \Leftrightarrow t \leqslant t^P$.[47]

Case (3): $g_{z2} < 0$.

[47] Note that if $g_{z2} > 0$, then $t^N < 0$. For $t^P > 0$, we need $(1-\alpha p)g_2w_2 > g_{z2}w_1$.

When $g_{z2} < 0$, $t^N > 0$ (and $t^P < 0$). In this case, z^N and z^P can be solutions for specific values of t. Specifically, if $t < t^N$, $z = z^N < 0$ or $z = z^P > 0$. At $t = 0$, $z^N = \frac{g_{z2}[(1-\beta p)w_2 - w_1]}{w_2(g_z g_2 - g_{z2}^2)} < 0$ and $z^P = \frac{-g_{z2}[(1-\beta p)w_2 - w_1]}{w_2(g_z g_2 - g_{z2}^2)} > 0$. For this to happen, $(1-\beta p)w_2 - w_1 > 0$.

Since $\frac{\partial z^N}{\partial t} > 0$, z^N increases with higher values of t and $z^N = 0$ at $t = t^N$. Also, $z^P > 0$ for $t > 0$. When $t \geqslant t^N$, $z^N \geqslant 0$, so z^N cannot be a solution.

When $t = t^N$, $z^N = 0$, and $z^P = \frac{-2g_2 g_{z2}[(1-\beta p)w_2 - w_1]}{(g_2 g_z - g_{z2}^2)(g_z w_1 + (1+\alpha p)g_{z2} w_2)} > 0$, so at t^N z jumps from 0 to z^P. □

Remarks

A few observations are worth highlighting. First, the tax rates t^N and t^P are the tax rates at which unreported legal income z changes from zero to positive (t^P when $g_{z2} > 0$) or from negative to positive (t^N when $g_{z2} < 0$). These threshold tax rates and the sign and magnitude of g_{z2} are crucial for characterizing individuals' behavior and, therefore, the optimal level of t. Of course, when $g_{z2} = 0$, $t^N = t^P = 0$.[48] Figure C.1 shows how t^N and t^P depend on g_{z2}: when $|g_{z2}|$ increases, both t^N and t^P increase.

[48]When $g_{z2} > 0$, for $z^P > 0$ to be a solution, we need $(1-\alpha p)g_2 w_2 - g_{z2} w_1 > 0$. If this condition is satisfied, t^P is positive and $\frac{\partial z^P}{\partial t} > 0$. Note that when $t = 0$, $z^P < 0$, which cannot happen. This means that for $z^P > 0$, the slope $\frac{\partial z^P}{\partial t}$ needs to be positive. If this is the case, the line z^P crosses the horizontal axis at $t = t^P < 1$, and $z^P > 0$ for $t > t^P$ (as long as $t^P < 1$). Intuitively, an increase in t directly increases the marginal utility of z by $(1 - p\alpha) > 0$. An increase in z raises the marginal cost of hiding illegal income. The increase in t also directly increases the marginal utility of y_2 by $\frac{w_1}{w_2} > 0$. A higher y_2 also entails a higher marginal cost of hiding y_2. Since $g_{z2} > 0$, then a higher level of z (respectively, y_2) increases further the marginal cost of hiding y_2 (respectively z). When $(1-\alpha p)g_2 w_2 - g_{z2} w_1 > 0$ the benefits of increasing z as a result of the increase in t dominate the direct and indirect costs of hiding z and y_2. When $g_{z2} < 0$, for $t^N > 0$, the condition $(1+\alpha p)g_2 w_2 + g_{z2} w_1 > 0$ needs to hold. Note that when $g_{z2} < 0$, if $t = 0$, then $z^N < 0$. When the condition $(1+\alpha p)g_2 w_2 + g_{z2} w_1 > 0$ is satisfied, $\frac{\partial z^N}{\partial t} > 0$, and when $t = t^N$ (considering that $0 \leqslant t^N < 1$), z^N would be zero. A similar intuition as in the case g_{z2} holds in this case as well.

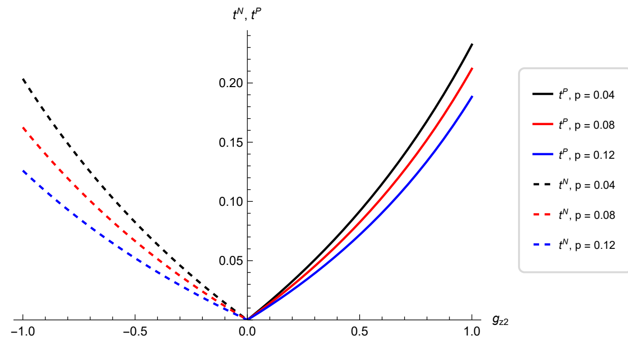


Figure C.1: t^N and t^P as a function of g_{z2}, for different values of p.

Second, the figure also shows that changes in p also affect t^N and t^P. In general, $\frac{\partial t^N}{\partial p}$ and $\frac{\partial t^P}{\partial p}$ cannot be unambiguously signed. However, note that $\frac{\partial t^N}{\partial g_{z2}} < 0$ and $\frac{\partial^2 t^N}{\partial g_{z2}\partial p} > 0$ evaluated at $g_{z2} = 0$, which means the function t^N rotates counterclockwise around $g_{z2} = 0$. It follows then that a higher p reduces t^N. When $g_{z2} > 0$, $\frac{\partial t^P}{\partial g_{z2}} > 0$ and the sign of $\frac{\partial^2 t^P}{\partial g_{z2}\partial p}$ evaluated at $g_{z2} = 0$ is the same as the sign of the expression $(\alpha - \beta)w_2 - \alpha w_1$. Therefore, when α and β do not differ too much from each other, $\left.\frac{\partial^2 t^P}{\partial g_{z2}\partial p}\right|_{g_{z2}=0}$ will tend to be negative, which means the function t^P rotates clockwise around $g_{z2} = 0$. As a result, an increase in p reduces t^P. Figure C.1 illustrates precisely this latter situation.

Third, when $g_{z2} > 0$, individuals will never choose a negative z. A negative z would imply over-reporting legal income, which increases both tax liability and the cost of hiding illegal income. It follows that in this case, individuals will report at most their entire legal income ($z \geqslant 0$). Specifically, when the tax rate is low enough ($t < t^P$), individuals report their entire legal income ($z = 0$). At low tax rates, the potential benefits from evading taxes on legal income are small relative to the direct cost of misreporting legal income and the additional cost it imposes on hiding illegal income. When the tax rate becomes sufficiently large ($t > t^P$), individuals will choose $z > 0$ (i.e., $z = z^P$), meaning they will underreport their legal income.

Fourth, when $g_{z2} < 0$, z and y_2 are complements in the sense that increasing one of them decreases the cost of concealing the other. In this case, $z^P > 0$ is a solution to the individual optimization problem for all $0 \leqslant t \leqslant 1$. However, when $0 < t \leqslant t^N$, z^N can also be a solution. Here, the function z exhibits a discrete jump at $t = t^N$. That is, $z = z^N \leqslant 0$ for $0 \leqslant t \leqslant t^N$ and $z = z^P > 0$ for $t^N < t < 1$. The behavior of z affects other variables y_1, y_2, and x, impacting the welfare function, which will also show a jump at $t = t^N$. Thus, when determining the optimal tax rate, it's essential to consider that the highest welfare might be achieved at this non-differentiable point in the welfare function.

C.2 Comparative static analysis

How are individuals' choices affected by changes in t and p? The impact of changes in t and p on $\{y_1, y_2, x, z\}$ and welfare is more complicated in the present case, particularly because the functions may become nondifferentiable at t^N and t^P, depending on the specific assumption on g_{z2}.

Comparative static analysis for t. The comparative static results in this case are given by

$$\frac{\partial y_2^N}{\partial t} = \frac{[g_z w_1 + (1+\alpha p) g_{z2} w_2]}{(g_2 g_z - g_{z2}^2) w_2}, \quad \frac{\partial z^N}{\partial t} = \frac{[g_{z2} w_1 + (1+\alpha p) g_2 w_2]}{(g_2 g_z - g_{z2}^2) w_2}, \tag{5}$$

$$\frac{\partial y_2^P}{\partial t} = \frac{g_z w_1 - (1-\alpha p) g_{z2} w_2}{(g_2 g_z - g_{z2}^2) w_2}, \quad \frac{\partial z^P}{\partial t} = \frac{(1-\alpha p) g_2 w_2 - g_{z2} w_1}{(g_2 g_z - g_{z2}^2) w_2}, \tag{6}$$

$$\frac{\partial y_2^0}{\partial t} = \frac{w_1}{g_2 w_2} > 0. \tag{7}$$

Moreover,

$$g_2 \frac{\partial y_2^P}{\partial t} + g_{z2} \frac{\partial z^P}{\partial t} = \frac{w_1}{w_2} > 0, \quad g_2 \frac{\partial y_2^N}{\partial t} - g_{z2} \frac{\partial z^N}{\partial t} = \frac{w_1}{w_2} > 0. \tag{8}$$

This means that, in each case, if one of the slopes is positive (negative), the other has to be negative (positive). In all cases,

$$\frac{\partial y_1}{\partial t} \frac{1}{w_1} + \frac{\partial y_2}{\partial t} \frac{1}{w_2} = \frac{w_1^2}{h''} < 0, \tag{9}$$

or equivalently $\frac{\partial(n_1+n_2)}{\partial t} < 0$, so the overall amount of work (in either legal or illegal activities) goes down as t increases.

Under the assumptions stated in Proposition 6, both z^N and z^P increase as t increases, i.e., $\frac{\partial z^N}{\partial t} > 0$ and $\frac{\partial z^P}{\partial t} > 0$ for $t > t^N$ and $t > t^P$, respectively. In general, however, the signs of $\frac{\partial y_2^N}{\partial t}$ and $\frac{\partial y_2^P}{\partial t}$ depend on g_{z2}.

If $g_{z2} > 0$, then $\frac{\partial y_2^N}{\partial t} > 0$ and the sign of $\frac{\partial y_2^P}{\partial t}$ is ambiguous.[49] However, it follows that when g_{z2} is sufficiently small, i.e., $0 < g_{z2} < \frac{g_z w_1}{(1-\alpha p) w_2}$, $\frac{\partial y_2^P}{\partial t} \geqslant 0$. Otherwise, $\frac{\partial y_2^P}{\partial t} \geqslant 0$. Note that when $g_{z2} > 0$, z is zero for $t \leqslant t^P$, which means that a higher level of t within that range does not affect z.

[49] If $g_{z2} > 0$, then $\frac{\partial y_2^N}{\partial t} > 0$, but y_2^N cannot is not a solution in this case.

If $g_{z2} < 0$, then $\frac{\partial y_2^P}{\partial t} > 0$, and the sign of $\frac{\partial y_2^N}{\partial t}$ is ambiguous. When $|g_{z2}|$ is sufficiently large, specifically, $-g_{z2} > \frac{g_z w_1}{(1+\alpha p) w_2}$, then $\frac{\partial y_2^N}{\partial t} < 0$. Otherwise, $\frac{\partial y_2^N}{\partial t} \geqslant 0$.

The following example illustrates the previous results. Figure C.2 shows the solution values of z, y_1, y_2, x and W for different tax rates t and values of g_{z2}. The figure considers the two solutions that may arise when $g_{z2} < 0$. The column on the left shows the results when $\{z, y_2\} = \{z^N, y_2^N\}$, and the column on the right when $\{z, y_2\} = \{z^P, y_2^P\}$. The threshold tax rates t^N and t^P affect how the variables respond to changes in t. In all cases, it is assumed that $|g_{z2}|$ is small enough so that $\frac{\partial y_2}{\partial t} > 0$.

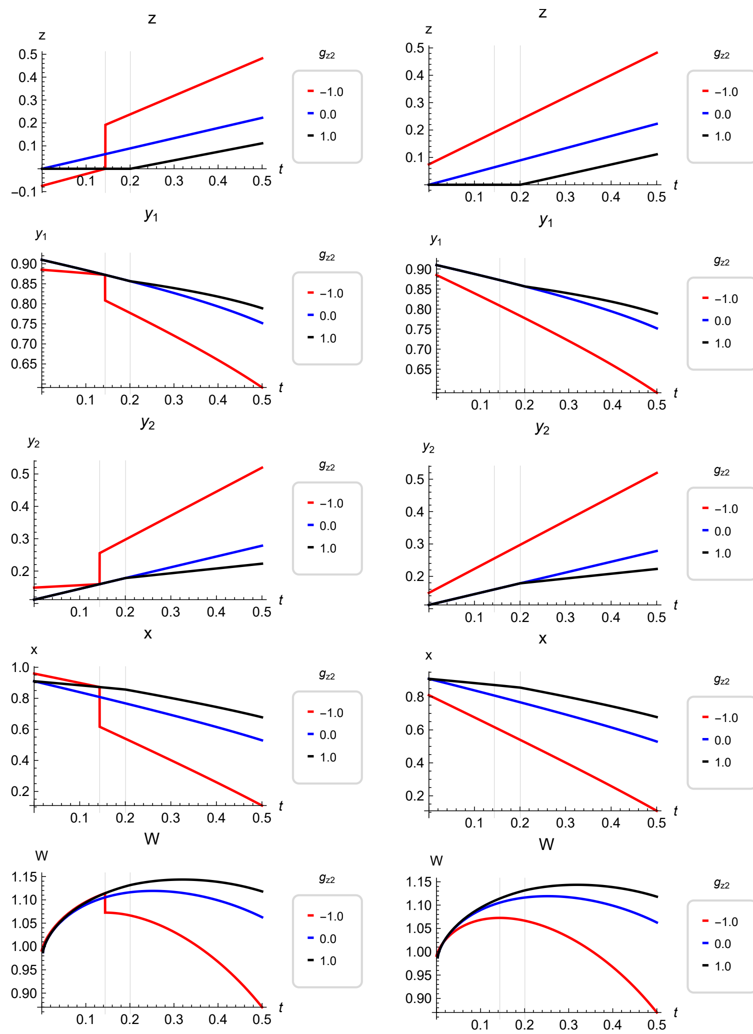


Figure C.2: Solutions $\{z, y_1, y_2, x, W\}$ for different tax rates t and several values of g_{z2}.
Note: The two columns show the two different solutions that arise when $g_{z2} < 0$. The column on the left shows the solutions when $z = z^N$ for $t < t^N$, and the right shows the solutions when $z = z^P$ for $t < t^N$. The vertical lines represent the values of t^N and t^P ($t^N = 0.14$ and $t^P = 0.20$ in this exercise).

A relevant consideration for determining the optimal value of t is that when $g_{z2} < 0$ and the solution is $\{y_1, y_2, z\} = \{y_1^N, y_2^N, z^N\}$, the maximum welfare could be reached at $t = t^N$, a non-differentiable point on the welfare function.

Comparative static analysis for p**.** The comparative static results for p are as follows:

$$\frac{\partial y_1}{\partial p} = -\frac{w_1}{w_2}\frac{\partial y_2}{\partial p},$$
$$\frac{\partial y_2}{\partial p} = \frac{(t\alpha g_{z2} - \beta g_z)}{(g_2 g_z - g_{z2}^2)}, \quad \frac{\partial y_2^0}{\partial p} = -\frac{\beta}{g_2} < 0,$$
$$\frac{\partial z^N}{\partial p} = -\frac{\partial z^P}{\partial p} = \frac{(t\alpha g_2 - \beta g_{z2})}{(g_2 g_z - g_{z2}^2)},$$
$$\frac{\partial x^N}{\partial p} = \frac{-\alpha t(g_{z2}w_1 + g_2 w_2) + \beta(g_z w_1 + g_{z2} w_2)}{w_2(g_2 g_z - g_{z2}^2)},$$
$$\frac{\partial x^P}{\partial p} = \frac{-\alpha t(g_{z2}w_1 - g_2 w_2) + \beta(g_z w_1 - g_{z2} w_2)}{w_2(g_2 g_z - g_{z2}^2)},$$
$$\frac{\partial x^0}{\partial p} = \frac{w_1\beta}{w_2 g_2} > 0.$$

In general, it follows that when $g_{z2} < 0$, $\frac{\partial y_1}{\partial p} > 0$ and $\frac{\partial y_2}{\partial p} < 0$, for both solutions y_2^N and y_2^P. Moreover, $\frac{\partial z^N}{\partial p} > 0$, $\frac{\partial z^P}{\partial p} < 0$. This means that while $\frac{\partial x^P}{\partial p} > 0$, the sign of $\frac{\partial x^N}{\partial p}$ is ambiguous. However, when $g_{z2} > 0$, the signs of all the derivatives are ambiguous. Specifically, the sign of $\frac{\partial z^P}{\partial p}$ is given by the sign of $(\beta g_{z2} - t\alpha g_2)$. When g_{z2} is sufficiently large, then $\frac{\partial z^P}{\partial p} > 0$ for all t. When g_{z2} is small, the sign also depends on t: if t is small, $\frac{\partial z^P}{\partial p} > 0$, and if t is large, $\frac{\partial z^P}{\partial p} < 0$.

Combining changes in t **and** p**.** It follows that

$$\frac{\partial^2 y_1}{\partial p \partial t} = -\frac{w_1}{w_2}\frac{\partial^2 y_2}{\partial p \partial t}, \tag{10}$$
$$\frac{\partial^2 y_2}{\partial p \partial t} = \frac{\alpha g_{z2}}{(g_2 g_z - g_{z2}^2)}, \quad \frac{\partial^2 y_2^0}{\partial p \partial t} = 0, \tag{11}$$
$$\frac{\partial^2 z^N}{\partial p \partial t} = -\frac{\partial^2 z^P}{\partial p \partial t} = \frac{\alpha g_2}{(g_2 g_z - g_{z2}^2)} > 0, \tag{12}$$
$$\frac{\partial^2 x^N}{\partial p \partial t} = \frac{-\alpha(g_{z2}w_1 + g_2 w_2)}{w_2(g_2 g_z - g_{z2}^2)}, \quad \frac{\partial^2 x^P}{\partial p \partial t} = \frac{-\alpha(g_{z2}w_1 - g_2 w_2)}{w_2(g_2 g_z - g_{z2}^2)}, \quad \frac{\partial^2 x^0}{\partial p \partial t} = 0. \tag{13}$$

In terms of cross-partial derivatives, it follows that $\frac{\partial^2 z^P}{\partial p \partial t} < 0$ for all g_{z2}. The other results depend on the sign of g_{z2}.

When $g_{z2} < 0$, $\frac{\partial^2 z^N}{\partial p \partial t} > 0$, $\frac{\partial^2 y_1}{\partial p \partial t} > 0$, and $\frac{\partial^2 y_2}{\partial p \partial t} < 0$. As a result, the sign of $\frac{\partial^2 x^N}{\partial p \partial t}$ is ambiguous. However, since $\frac{\partial^2 z^P}{\partial p \partial t} < 0$, $\frac{\partial^2 x^P}{\partial p \partial t} > 0$.

When $g_{z2} > 0$, $\frac{\partial^2 y_1}{\partial p \partial t} < 0$ and $\frac{\partial^2 y_2}{\partial p \partial t} > 0$. Since $\frac{\partial^2 z^N}{\partial p \partial t} > 0$, $\frac{\partial^2 x^N}{\partial p \partial t} < 0$. However, since

$\frac{\partial^2 z^P}{\partial p \partial t} < 0$, the sign of $\frac{\partial^2 x^P}{\partial p \partial t}$ is ambiguous.

Note that changes in p also affect the threshold values t^N and t^P. For instance, suppose that $g_{z2} > 0$. Then, given that $\frac{\partial^2 z^P}{\partial p \partial t} < 0$, the curve $z(t)$ rotates clockwise, and t^P declines.

Combinations of $\{t, p\}$ for given values of z and y_2. As we did previously, we examine how different combinations of (t, p) can be used to achieve fixed levels of z and y_2. Understanding this relationship is crucial when considering scenarios where the government can set both t and p. The relationship between t and p along $\bar{z}$ and $\bar{y}_2$ may be defined by correspondences rather than functions, depending on g_{z2}.

Suppose that $g_{z2} > 0$. In this case, individuals chose $z \geqslant 0$. A given level of $\bar{z} > 0$ can be attained with different combinations of (t, p). Consider the solution for z^P shown in (32). Then the slope of the relationship between p and t for which $z^P(t, p) = \bar{z} > 0$, i.e., $p = p(t)$, is given by

$$\left.\frac{\partial p}{\partial t}\right|_{z^P=\bar{z}} = \frac{-[(1-\alpha p)g_2 w_2 - g_{z2} w_1]}{(\beta g_{z2} - \alpha t g_2) w_2}. \tag{14}$$

Similarly, for $y_2(t, p) = \bar{y}_2$,

$$\left.\frac{\partial p}{\partial t}\right|_{y_2^P=\bar{y}_2} = \frac{g_z w_1 - (1-\alpha p) g_{z2} w_2}{(\beta g_z - \alpha t g_{z2}) w_2}. \tag{15}$$

In both cases, the slopes can be either negative or positive depending on the magnitudes of g_{z2} and t.

Since $(1-\alpha p)g_2 w_2 > g_{z2} w_1$ for all g_{z2} (which implies that $\frac{\partial z^P}{\partial t} > 0$), the sign of $\left.\frac{\partial p}{\partial t}\right|_{z^P=\bar{z}}$ depends on the sign of $(\beta g_{z2} - \alpha t g_2)$. When t is sufficiently low, specifically, $t < \frac{\beta g_z}{\alpha g_{z2}}$, the slope is negative; when $t > \frac{\beta g_z}{\alpha g_{z2}}$, the slope changes to positive.

Moreover, when $\frac{\partial y_2^P}{\partial t} > 0$, the numerator of $\left.\frac{\partial p}{\partial t}\right|_{y_2^P=\bar{y}_2}$ is positive. This means that when t is sufficiently low, specifically, $t < \frac{\beta g_z}{\alpha g_{z2}}$, the slope is positive; when $t > \frac{\beta g_z}{\alpha g_{z2}}$, the slope is negative. The opposite outcomes are obtained when $\frac{\partial y_2^P}{\partial t} < 0$, which tends to occur when g_{z2} is large.

Additionally, when $g_{z2} > 0$, z can also be zero. In particular, $z = 0$ for all combinations $\{t, p\}$ that satisfy $\{t > 0, p > 0, t \leqslant t^P(p)\}$. Moreover, in this case $\left.\frac{\partial p}{\partial t}\right|_{y_2^P=\bar{y}_2} = \frac{w_1}{\beta w_2}$, the same expression as in (23).

Figure C.3 shows two possible outcomes that may arise in this case.

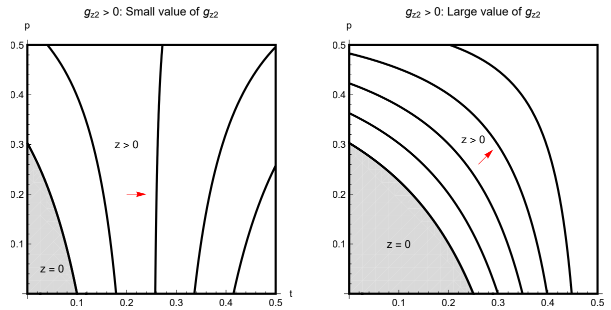


Figure C.3: $g_{z2} > 0$: $z(t,p) = \bar{z}$.

The graph on the left shows a case in which g_{z2} is relatively small. For low values of t, $\left.\frac{\partial p}{\partial t}\right|_{z^P=\bar{z}} < 0$, so t and p are "substitutes". When t is large enough, the slope becomes positive, so they are "complements".[50] The graph on the right shows a case in which g_{z2} is relatively large. For the relevant values of t, t and p are "substitutes", or $\left.\frac{\partial p}{\partial t}\right|_{z^P=\bar{z}} < 0$.[51] This means that an increase in t increases z, so p needs to go down to reach the original level of z. The figures also describe the set $\{t, p\}$ for which there is no tax evasion or $z = 0$ (the gray area).

Things are more complicated when $g_{z2} < 0$, since z can be either negative, zero, or positive, and multiple solutions may be observed. For $z > 0$, using $g_{z2} < 0$ in expressions (14) and (15), it follows that $\left.\frac{\partial p}{\partial t}\right|_{z^P=\bar{z}} > 0$ and $\left.\frac{\partial p}{\partial t}\right|_{y_2^P=\bar{y}_2} > 0$.

For $z < 0$:

$$\left.\frac{\partial p}{\partial t}\right|_{z^N=\bar{z}<0} = \frac{g_{z2}w_1 + (1+p\alpha)g_2 w_2}{(\beta g_{z2} - \alpha t g_2)w_2} < 0, \quad \left.\frac{\partial p}{\partial t}\right|_{y_2^N=\bar{y}_2} = \frac{g_z w_1 + (1+\alpha p)g_{z2} w_2}{(\beta g_{z2} - \alpha t g_2)w_2}. \tag{16}$$

Since $(1+\alpha p)g_2 w_2 + g_{z2}w_1 > 0$, then $\left.\frac{\partial p}{\partial t}\right|_{z^N=\bar{z}} < 0$. The sign of $\left.\frac{\partial p}{\partial t}\right|_{y_2^N=\bar{y}_2}$ depends on the sign of $\frac{\partial y_2^N}{\partial t}$: if $\frac{\partial y_2^N}{\partial t} > 0 (< 0)$, then $\left.\frac{\partial p}{\partial t}\right|_{y_2^N=\bar{y}_2} < 0 (> 0)$.

The graphs in Figure C.4 show the relationship $z(t,p) = \bar{z}$ for $g_{z2} < 0$. The graph on the left shows the case in which $z = z^N$ when $t \leqslant t^N$, and the graph on the right shows the case in which $z = z^P$ when $t \leqslant t^N$.

[50]Specifically, this happens when $t > \frac{\beta g_{z2}}{\alpha g_2}$, as shown in the Online Supporting Appendix C.2.
[51]For instance, if $\beta \geqslant \alpha$ and $g_2 \geqslant g_{z2}$, the t would need to be greater than one for the slope to change sign.

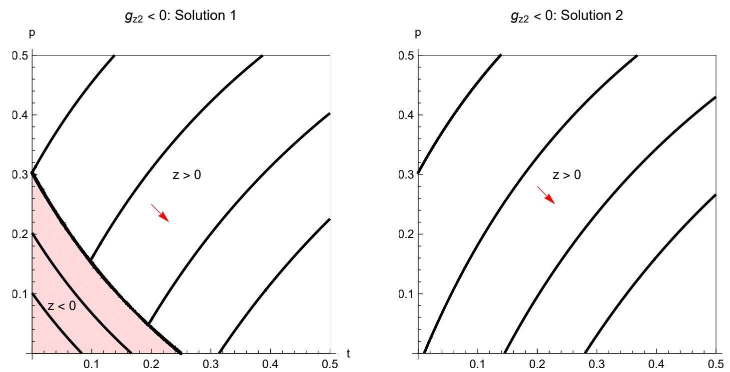


Figure C.4: $g_{z2} < 0$: $z(t, p) = \bar{z}$.
LEFT: $z = z^N$ when $t < t^N$. RIGHT: $z = z^P$ when $t < t^N$.

When $z > 0$, t and p are complements. This means that an increase in t requires an increase in p to maintain the same level of $\bar{z}$ (in other words, $\left.\frac{\partial p}{\partial t}\right|_{z^P=\bar{z}} > 0$). However, when $\bar{z} < 0$ (as depicted in the left-hand graph), t and p have an inverse relationship.

Numerical example. The following graphs illustrate some of the comparative static results discussed previously. Figure C.5 shows the solutions z, y_1, y_2, x and W for different tax rates t, audit probabilities p, and values of g_{z2}. The first column shows the solutions for $g_{z2} > 0$. The second column considers the two solutions that may arise when $g_{z2} < 0$. The solid lines describe the outcomes when $\{z, y_2\} = \{z^N, y_2^N\}$, and the dotted lines the outcomes when $\{z, y_2\} = \{z^P, y_2^P\}$. The threshold tax rates t^N and t^P, which depend on p, affect how the variables respond to changes in t. All cases assume that $|g_{z2}|$ is small enough so that $\frac{\partial y_2}{\partial t} > 0$.

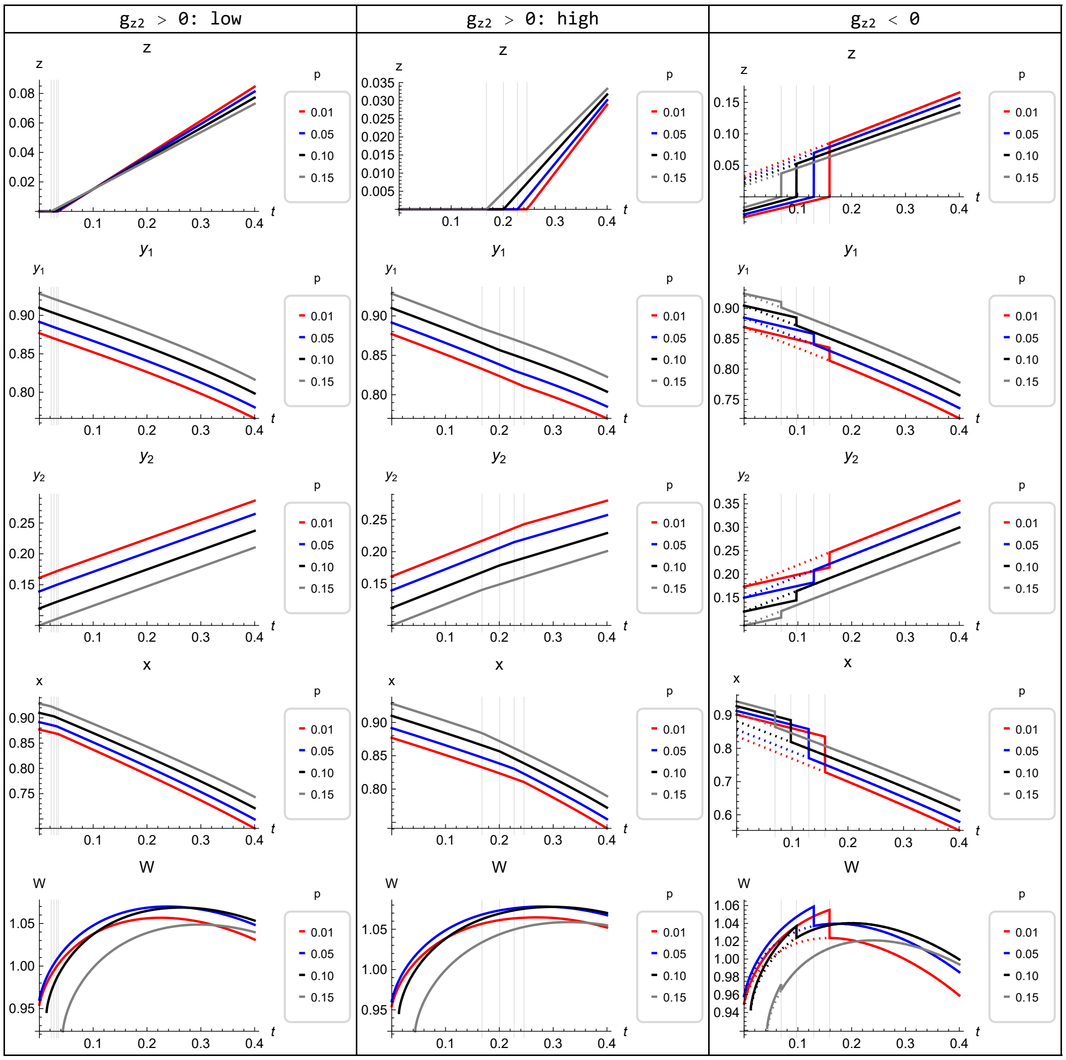


Figure C.5: Solutions $\{z, y_1, y_2, x, W\}$ for different tax rates t and different values of p.
Note: The first and second columns show the solutions for different values of $g_{z2} > 0$. The third column presents the two different solutions for $g_{z2} < 0$: the solid lines are the solutions when $\{y_1, y_2, z\} = \{y_1^N, y_2^N, z^N\}$ for $t < t^N$; the dotted lines are solutions when $\{y_1, y_2, z\} = \{y_1^P, y_2^P, z^P\}$ for $t < t^N$. The vertical lines represent the values of t^N and t^P in each case.

The following observations are worth noting. First, a change in p simultaneously affects the threshold values t^P and t^N and shifts the curves y_1, y_2, z, x and W. Suppose that $g_{z2} > 0$. Then, an increase in p decreases t^P and the slope $\frac{\partial z^P}{\partial t}$.[52] Consider the two cases for $g_{z2} > 0$ (first and second columns of Figure C.5). When $g_{z2} > 0$ is small (first

[52]Recall that from the comparative static analysis $\frac{\partial^2 z^P}{\partial t \partial p} < 0$.

column of Figure C.5), the curve z^P rotates at a low tax rate; when g_{z2} is sufficiently large second columns of Figure C.5), such rotation occurs at a high tax rate. Note that to the extent that $t < t^P$, $z = 0$, a change in p does not affect z. While, in general, the signs of $\frac{\partial y_1^P}{\partial p}$ and $\frac{\partial y_2^P}{\partial p}$ are ambiguous, when $\beta g_z > 0 \alpha g_{z2}$, $\frac{\partial y_2^P}{\partial p} < 0$ and $\frac{\partial y_1^P}{\partial p} > 0$.[53] As a result, a higher p shifts y_1 upwards and y_2 downwards (in both the first and second columns). The graph also shows that $\frac{\partial^2 W}{\partial t \partial p} > 0$, implying that the tax rate t that maximizes W is higher when p is larger.

Second, the behavior of the variables as t changes is particularly striking when $g_{z2} < 0$ and $\{y_1, y_2, z\} = \{y_1^N, y_2^N, z^N\}$ (solids lines in the third column of Figure C.5). All variables exhibit a discrete jump at $t = t^N$, including W. Such behavior is relevant when determining the optimal value of t since the maximum welfare may be reached at $t = t^N$, a non-differentiable point on the welfare function. Also, welfare is higher compared to the case in which $\{y_1, y_2, z\} = \{y_1^P, y_2^P, z^P\}$ (dotted lines) when $t < t^N$, and they coincide, of course, when $t \geqslant t^N$. The tax rate at which welfare is maximized may change abruptly for a sufficiently large p. When p is small, the optimal t equals t^N, a point where W is non-differentiable. An increase in p (in the graph, from $p = 0.01$ to $p = 0.05$) reduces t^N so the peak of W is reached at a lower tax rate. When p is increased further (in the graph, from $p = 0.05$ to $p = 0.10$), the optimal t is larger than t^N, and it is implicitly determined by the FOC $\frac{\partial W}{\partial t} = 0$. From here onward, a higher p leads to a higher optimal t.

C.3 Comment on multiple solutions/local optima

Suppose that $g_{z2} < 0$. For given values of G and Y_2, the individual's utility evaluated at $\{y_1, y_2, z\} = \{y_1^P, y_2^P, z^P\}$ is greater than or equal to the individual's utility evaluated at $\{y_1^N, y_2^N, z^N\}$ when $0 \leqslant t \leqslant t^N$, or $u^P \equiv u(y_1^P, y_2^P, z^P) \geqslant u(y_1^N, y_2^N, z^N) \equiv u^N$ for $0 \leqslant t \leqslant t^N$. Recall that when $t > t^N$, the solution is always $z = z^P$. First, note that $u^N = u^P$ when $t = 0$ or $t = t^N$. Second, $\left|\frac{\partial u^N}{\partial t}\right| > \left|\frac{\partial u^P}{\partial t}\right|$ at $t = 0$. This means that $u^P \geqslant u^N$ for $0 \leqslant t \leqslant t^N$.

However, when we include the effects of t on both G and $\Psi(Y_0)$, then welfare becomes larger when the solution is $\{y_1^N, y_2^N, z^N\}$, or $W^N \equiv W(G^N, Y_2^N) > W(G^P, Y_2^P) \equiv W^P$ for $0 \leqslant t \leqslant t^N$. These results are illustrated in Figure C.6.[54]

[53] This is the case shown in the figure.

[54] Consider as well the comparison of welfare levels between the two solutions in Figures C.9 or C.12.

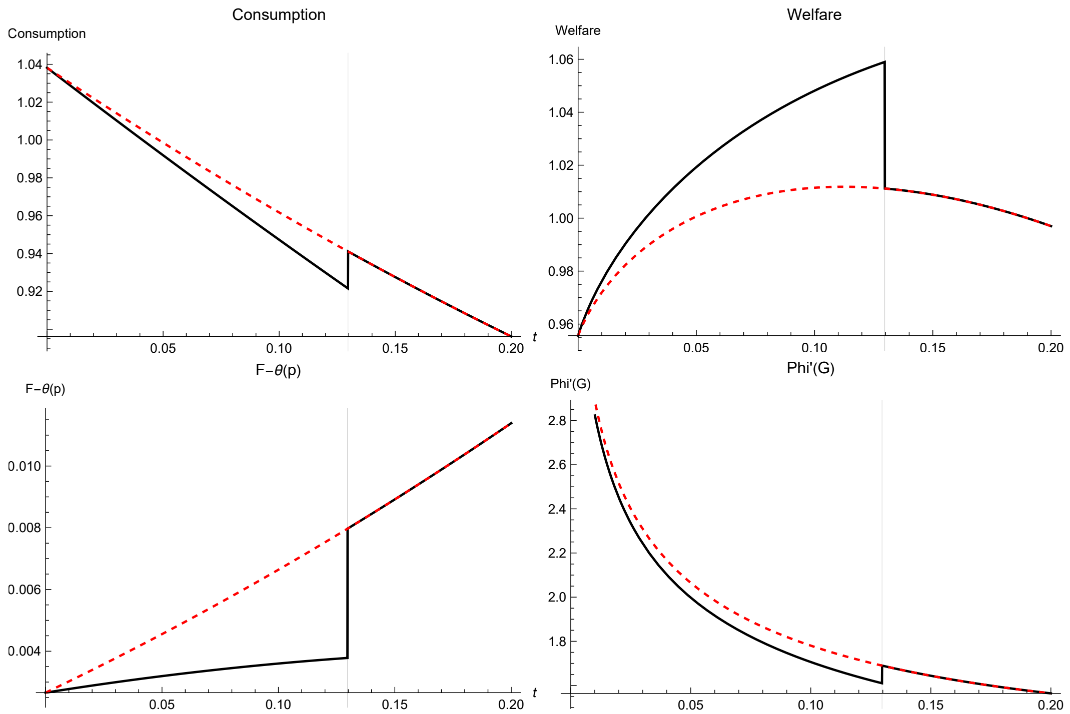


Figure C.6: Consumption (TOP LEFT), welfare (TOP RIGHT), audit agency's budget constraint (BOTTOM LEFT), marginal utility of the consumption of the public good (BOTTOM RIGHT) as a function of t when $g_{z2} = -0.75$.
The vertical line shows $t^N = 0.13$. The black solid lines show consumption and welfare as a function of t when $\{y_1, y_2, z\} = \{y_1^N, y_2^N, z^N\}$, and the dashed red line when $\{y_1, y_2, z\} = \{y_1^P, y_2^P, z^P\}$.
Note: The figures use $\Psi_0 = 0.40$, $p = 0.05$.

The graph on the top left compares consumption levels, excluding $\Phi(G)$ and $\Psi(Y_2)$. The graph on the top right compares overall welfare levels. The difference $W^N - W^P > 0$ when $0 < t < t^N$ is explained by the fact that $y_1^N > y_1^P$, $y_2^N < y_2^P$, $z^N < 0 < z^P$, so $x^N > x^P$. Overall, when $\{y_1^N, y_2^N, z^N\}$, the negative externality is smaller and taxes collected $(T = tx)$ are higher. The audit agency's budget constraint contributes a smaller amount of resources in this case, but to the extent that those contributions are not large (which is the case shown in the graph), the level of G will also be larger when $\{y_1^N, y_2^N, z^N\}$.

C.4 Optimal policy choices

In the case of the general concealment function, the analysis can get fairly complicated when $g_{z2} \neq 0$, because of the possibility of non-differentiability at t^N and t^P. We begin by examining how the optimal t depends on different variables, including the audit probability p, the magnitude of the negative externality Ψ_0 generated by the production of illegal goods, and the sign of g_{z2}. We next consider how the optimal policy combination $\{t, p\}$ is affected by Ψ_0 g_{z2}.

C.4.1 Choosing the tax rate

Solutions for different values of p. How does the optimal tax rate t depend on p? As previously discussed, the analysis is more complicated because p simultaneously shifts the curves and affects the threshold tax rates t^N and t^P. Figure C.7 below depicts the impact of changes in p on the relevant variables.

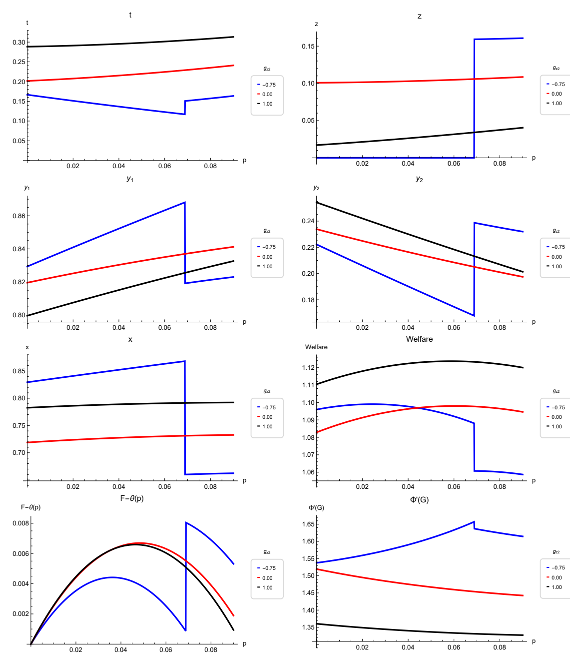


Figure C.7: Optimal values of t, y_1, y_2, z, and x, and the resulting levels of Welfare, $[F - \theta(p)]$ and $\Phi'(G)$, as a function p, for different values of g_{z2}.

Note: In all cases $\Psi_0 = 0.40$. When $g_{z2} < 0$, the figure only shows the solution for the case in which $z = z^N$ when $t < t^N$.

From the exercise, we can conclude the following. First, when g_{z2} is positive, the conclusions are similar to those previously discussed for the case where g_{z2} equals zero. Specifically, t increases gradually as p rises. Higher values of p allow the government to raise the provision of the public good G. Also, welfare increases with p up to a certain point and declines thereafter.

Second, when $g_{z2} < 0$, two key factors need to be taken into consideration: (i) we should account for the possibility that the optimal tax rate may be achieved at a point in the welfare function that is non-differentiable; (ii) two types of solutions may arise in this case when $t < t^N$: (i) $\{y_1^N, y_2^N, z^N\}$ or (ii) $\{y_1^P, y_2^P, z^P\}$. Figure C.7 focuses on case (i).[55] The graph shows that t declines as p increases when p is small. The reason is that at those values of p, the tax rate chosen by the government is $t = t^N$ and $z^N = 0$. Since $\frac{\partial t^N}{\partial p} < 0$, when p increase, t^N goes down. As p continues to rise, it reaches a value p^* such that $W^N[t^N(p^*), p^*] = \max_{t > t^N(p^*)} W^P[t, p^*]$, where W^N denotes welfare when individuals choose $\{y_1^N, y_2^N, z^N\}$ and W^P welfare when individuals choose $\{y_1^P, y_2^P, z^P\}$. A marginal increase in p at this point triggers a discontinuous jump to an optimal tax rate $t > t^N$. Increases in p beyond this point tend to increase the optimal tax rate.

Specifically, figure C.7 shows that when p is about 0.07, the maximum welfare value switches from $t = t^N$ to another in which t is the solution to $\frac{\partial W}{\partial t} = 0$. To the extent that $\frac{\partial t}{\partial p} > 0$, the optimal tax rate increases when p increases further (in the figure, this happens for values of p greater than 0.07). Figure C.8 highlights precisely this argument. When $p = 0.08, 0.09, 0.10, 0.11$, welfare is maximized at $t = t^N$; when $p = 0.12, 0.13, 0.14$, welfare is maximized at a tax rate at which $\frac{\partial WP}{\partial t} = 0$.

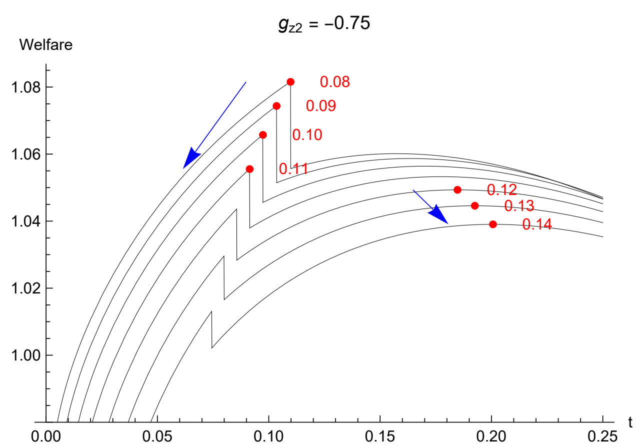


Figure C.8: Welfare as a function of t for different values of $p \in [0.08, 0.14]$ and $g_{z2} = -0.75$.
Note: The red dots indicate the maximum W for each value of p. The figures use $\Psi_0 = 0.33$.

Figure C.9 compares how the two solutions obtained when $g_{z2} < 0$, cases (i) and (ii), change as p increases.

[55] Figure C.9 compares cases (i) and (ii).

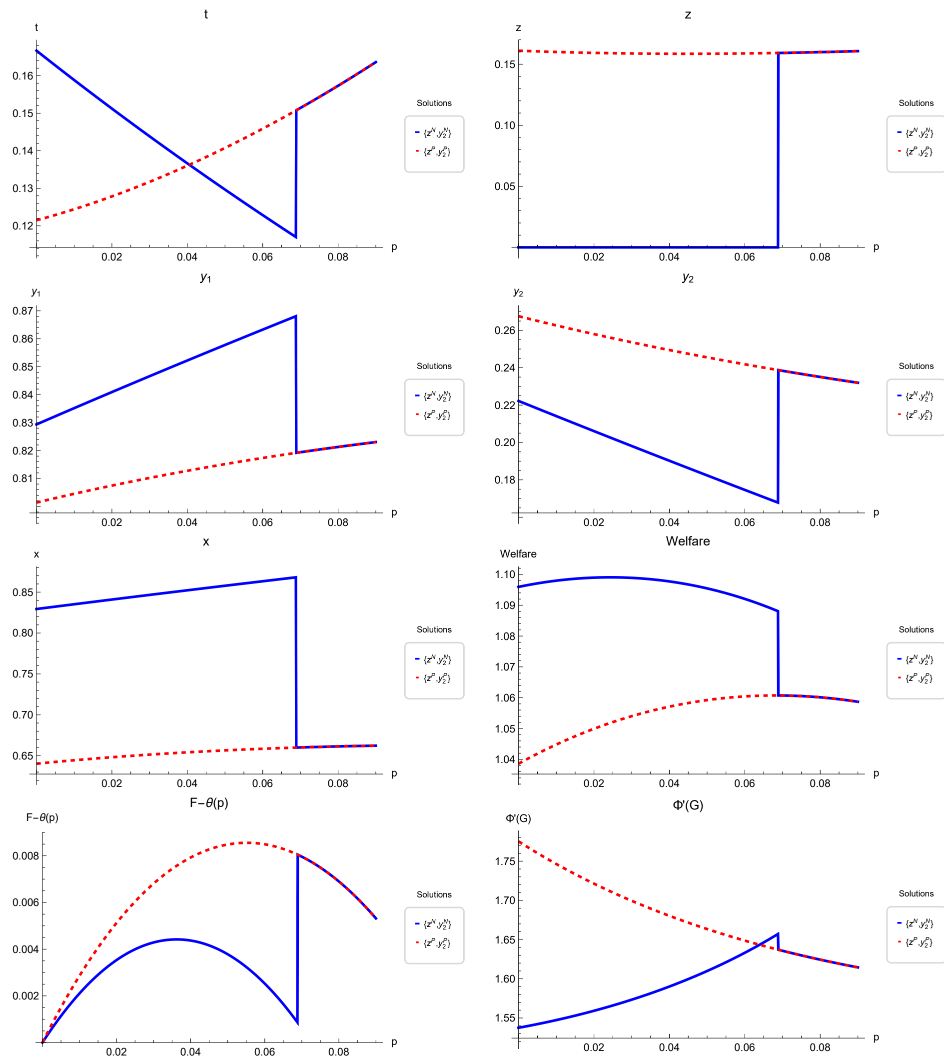


Figure C.9: Solutions for different values of p when $g_{z2} = -0.75$.
Note: The figures use $\Psi_0 = 0.40$.

The welfare in case (i), W^N (when the solution is $\{y_1^N, y_2^N, z^N\}$), is always greater than or equal to the welfare in case (ii), W^P (when the solution is $\{y_1^P, y_2^P, z^P\}$). The result is explained mostly by the fact that the tax rate is substantially lower, leading to a higher level of y_1 and a lower level of y_2^N. Note, however, that the level of underprovision of G in this case is much higher.

Solutions for different values of Ψ_0. We now examine the relationship between the strength of the externality Ψ_0, the optimal tax rate, and the variables of interest. Throughout the exercise, we keep the audit probability p constant while examining solutions for various values of g_{z2}. Figure C.10 summarizes the results.

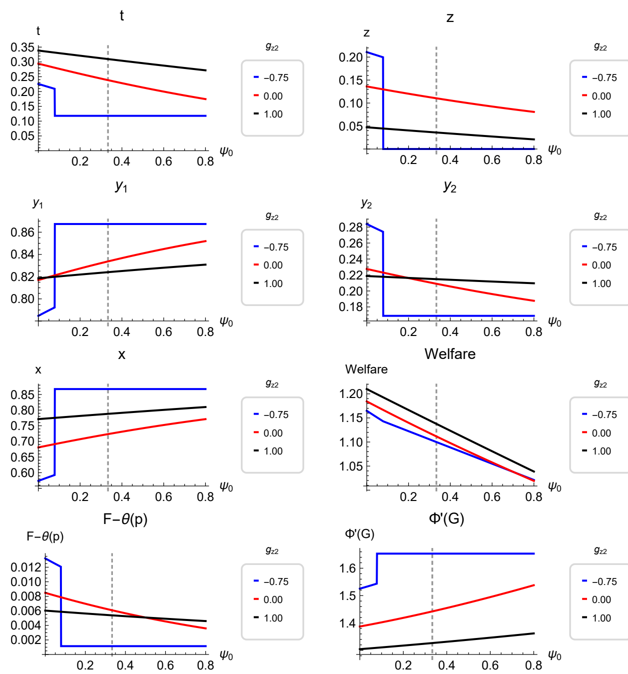
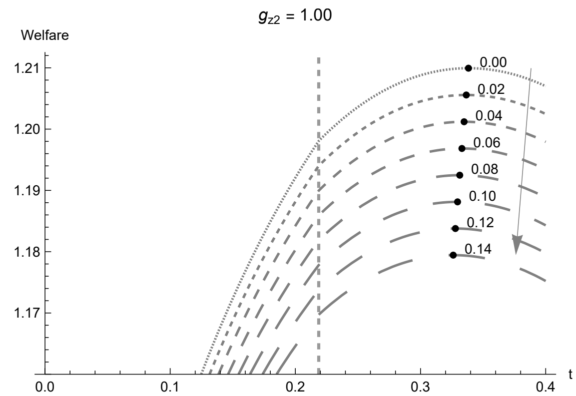
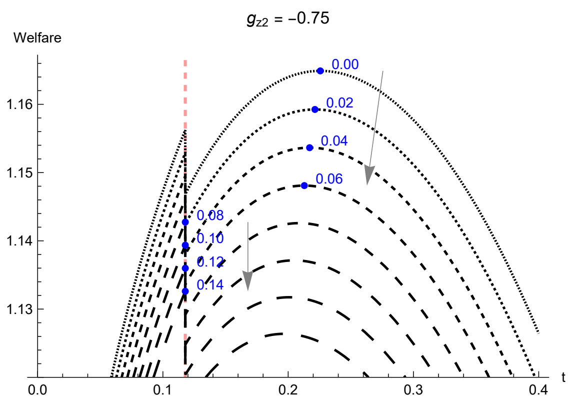


Figure C.10: Solutions for different values of Ψ_0, $g_{z2} = \{-0.75, 0.00, 1.00\}$, and $p = 0.0675$. The figures consider the $\{y_1^N, y_2^N, z^N\}$ for $t \leqslant t^N = 0.14$. The dotted vertical lines shows $\Psi_0 = 1 - \frac{w_1}{w_2} = \frac{1}{3}$.

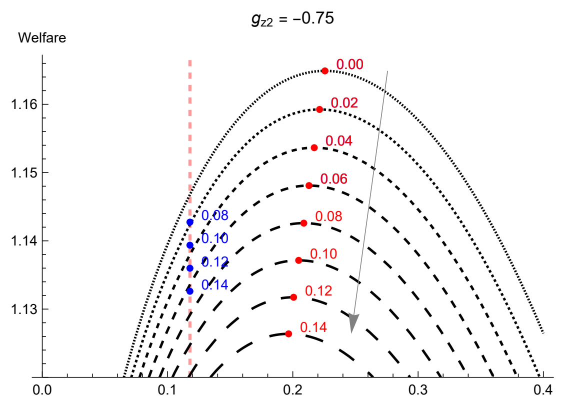
If g_{z2} is non-negative, the conclusions of the baseline model are qualitatively retained. As explained earlier, when $g_{z2} = 0$, $z > 0$, and the optimal value of t smoothly declines as Ψ_0 increases since $\frac{\partial t}{\partial \Psi_0} < 0$. This means that when the negative externality gets stronger, the government chooses a lower t, which reduces y_2. Figure C.11a illustrates how the maximum welfare changes when Ψ_0 increases from 0.00 to 0.14 and $g_{z2} = 1.00$. Note that in all cases, the highest welfare is reached at $t > t^P = 0.23$ (shown as the dashed vertical line in the graph).



(a) $g_{z2} = 1.0$. The dotted line shows the tax rate $t^P = 0.23$.



(b) $g_{z2} = -0.75$. The dotted line shows the tax rate $t^N = 0.12$. Solution with z^N for $t \leqslant t^N$.



(c) $g_{z2} = -0.75$. The dotted line shows the tax rate $t^N = 0.12$. Solution with z^P for $t \leqslant t^N$.

Figure C.11:
Welfare as a function of t for different values of Ψ_0 and $g_{z2} = -0.75, 1.00$.
Note: As Ψ_0 increases, the welfare function shifts downwards. The red dots indicate the maximum at each value of Ψ_0. The figures use $p = 0.0675$.

Next, suppose that $g_{z2} < 0$. Recall that in this case, multiple solutions may arise when $0 < t < t^N$: case (i) $\{y_1^N, y_2^N, z^N\}$, and case (ii) $\{y_1^P, y_2^P, z^P\}$. Figure C.10 focuses on case (i). In this scenario, the optimal t exhibits a jump at $\Psi_0 = 0.078$. When Ψ_0 is small (in figure, $\Psi_0 < 0.078$), the optimal tax rate is greater than t^N, so individuals choose $\{y_1^P, y_2^P, z^P\}$. Recall that this is a point where the optimal tax rate satisfies $\frac{\partial W^P}{\partial t} = 0$, and, as shown earlier, $\frac{\partial t}{\partial \Psi_0} < 0$. This means that when the negative externality gets stronger, the government chooses a lower t, which reduces y_2.

When Ψ_0 exceeds 0.078, the optimal tax rate jumps to $t^N = 0.12$ and is constant thereafter. Figure C.11b describes how this happens. As soon as Ψ_0 becomes greater than 0.078, welfare is now maximized at $t = t^N = 0.12$, a tax rate where the welfare function exhibits a discontinuity. This result continues to hold for larger values of Ψ_0: welfare declines as Ψ_0 increases but the maximum is still reached at $t = t^N$. Therefore, the optimal tax rate remains constant at $t = t^N$ for larger levels of Ψ_0.[56]

Figure C.12 replicates the graphs shown in Figure C.10 and compares the two solutions that may arise when $g_{z2} < 0$.

[56] If p is fixed at a higher level, t still declines as Ψ_0 increases, though the optimal t is higher. In this scenario, the discontinuity of the optimal t function (and other variables) occurs at a higher Ψ_0, likely due to a binding audit agency's budget constraint.

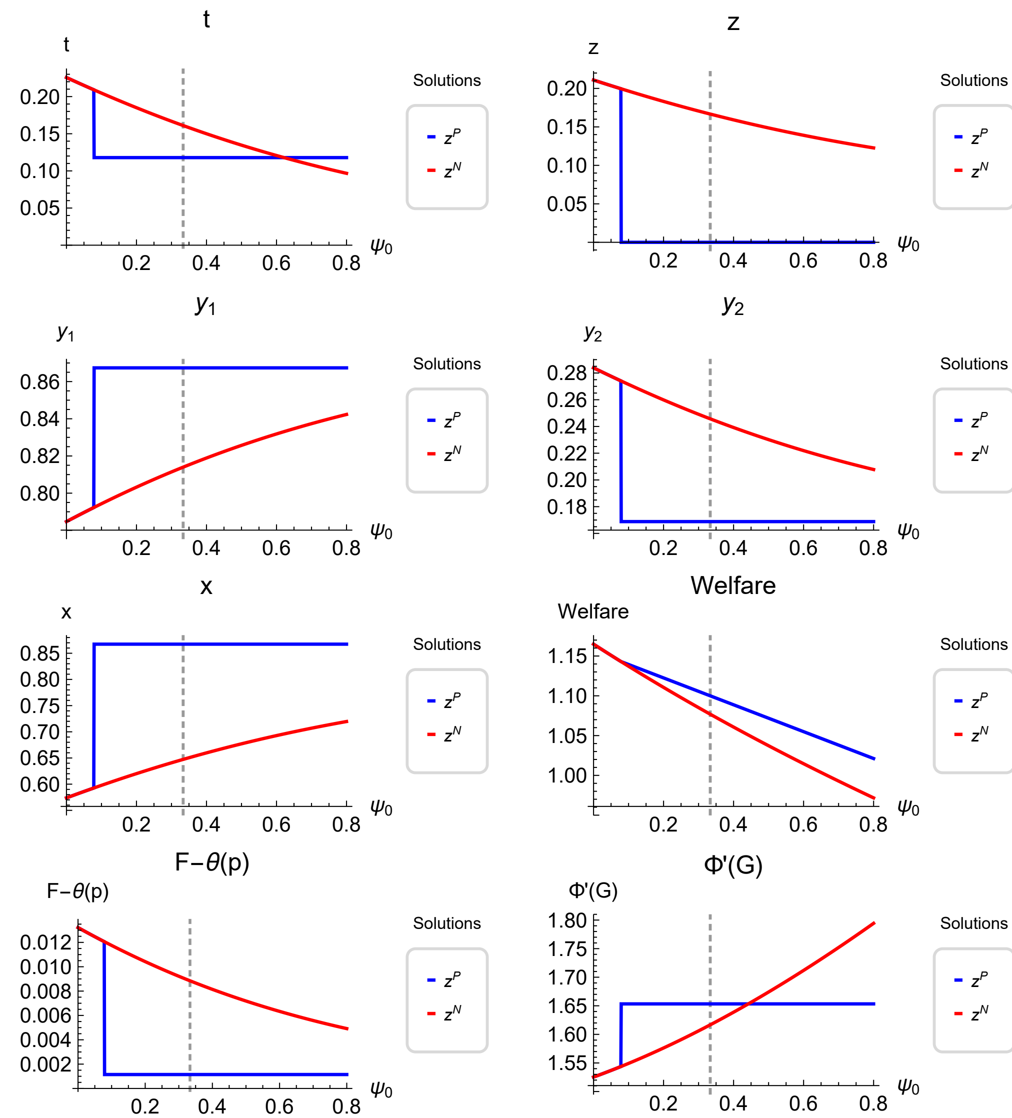


Figure C.12: Solutions for different Ψ_0, $g_{z2} = -0.75$, and $p = 0.0675$.
The graphs compare the outcomes at the two solutions that may arise when $g_{z2} < 0$ and $t \leqslant t^N$: $\{y_1^N, y_2^N, z^N\}$ (in blue) and $\{y_1^P, y_2^P, z^P\}$ (in red).

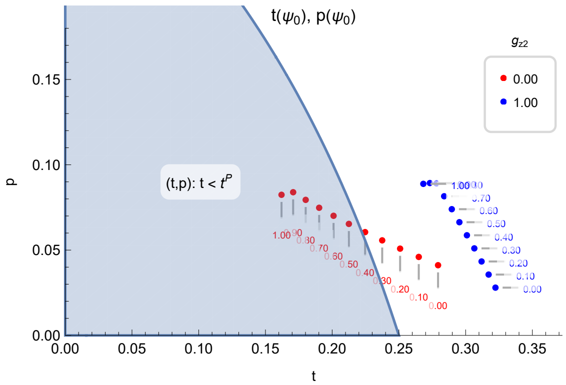
Consider case (ii). In this scenario, all the variables behave smoothly as Ψ_0 changes. The maximum welfare is reached at a tax rate that satisfies $\frac{\partial W^P}{\partial t} = 0$, where W^P is the welfare when the solution is $\{y_1^P, y_2^P, z^P\}$, and $\frac{\partial t}{\partial \Psi_0} < 0$. Note that when Ψ_0 is small, the optimal tax rate is greater than t^N, so the solutions in cases (i) and (ii) coincide. As Ψ_0 becomes sufficiently large, the optimal tax rate continues to decline smoothly when $\{y_1^P, y_2^P, z^P\}$, but it discretely declines to $t = t^N$ when $\{y_1^N, y_2^N, z^N\}$. In the latter case, the optimal tax rate remains at $t = t^N$ as Ψ_0 continues rising. Finally, note that the two solutions can be ranked in terms of their respective welfare levels: welfare when individuals choose $\{y_1^N, y_2^N, z^N\}$, W^N, is greater than welfare when individuals choose $\{y_1^N, y_2^N, z^N\}$, W^P, for optimal tax rates that satisfy $0 < t \leqslant t^N$.

C.4.2 Choosing both the tax rate and the audit probability

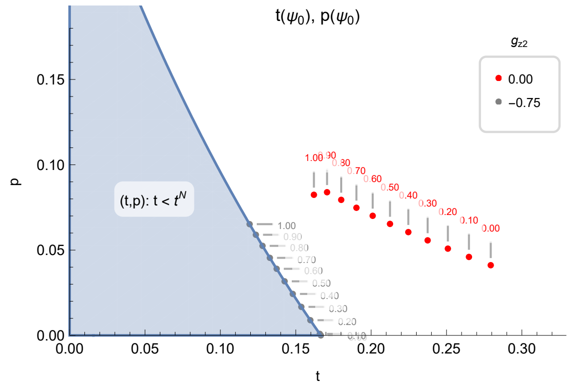
We now allow the government to choose the $\{t, p\}$ policy combination. Using a numerical example, we examine how this choice depends on Ψ_0 for the general concealment function considered in this section. Specifically, we obtain the solutions for different values of $g_{z2} < 0$ (when $\{y_1, y_2, z\} = \{y_1^N, y_2^N, z^N\}$) and compare them to the case $g_{z2} = 0$.

Using a numerical example, we examine how this choice depends on Ψ_0 for the general concealment function considered in this section. Specifically, we obtain the solutions for positive and negative values of g_{z2} and compare them to the case $g_{z2} = 0$. When $g_{z2} < 0$, we consider only one solution since the $\{t, p\}$ policy combinations in the different scenarios are such that $t > t^N$.

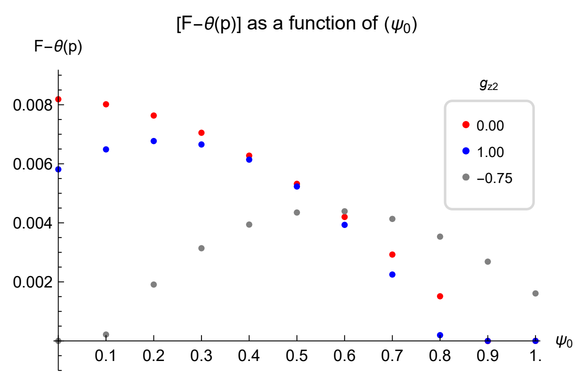
Figures C.13a and C.13b below show the combinations of $\{t, p\}$ that maximize welfare for values of Ψ_0 ranging from 0 to 1 and different values of g_{z2}. Figure C.13c describes the impact of those choices on the audit agency's budget constraint.



(a) $g_{z2} = \{0.00, 1.00\}$. The shaded area shows the combinations $\{t, p\} : t < t^P$. Red dots: solutions for different values of Ψ_0 and $g_{z2} = 0$. Blue dots: solutions for different values of Ψ_0 and $g_{z2} = 1.00$.



(b) $g_{z2} = \{0.00, -0.75\}$. The shaded area shows the combinations $\{t, p\} : t < t^N$. Red dots: solutions for different values of Ψ_0 and $g_{z2} = 0$. Grey dots: solutions for different values of Ψ_0 and $g_{z2} = -0.75$ and individuals choose $\{y_1^N, y_2^N, z^N\}$.



(c) $g_{z2} = \{0.00, 1.00, -0.75\}$.

Figure C.13: Policy combinations $\{t, p\}$ for different values of Ψ_0 and g_{z2}.

In all cases, p increases and t decreases as the intensity of the externality Ψ_0 rises, at least when the audit agency's budget constraint is not binding. If $g_{z2} > 0$, at the optimal combination of t and p, $z > 0$ since $t > t^P$. If $g_{z2} < 0$, $z = 0$ at the optimal policy combination since $t = t^N$. When Ψ_0 is sufficiently large, the audit agency's budget constraint $F - \theta(p)$ begins to bind.

Figure C.14 shows the outcomes for all relevant variables as a function of Ψ_0.

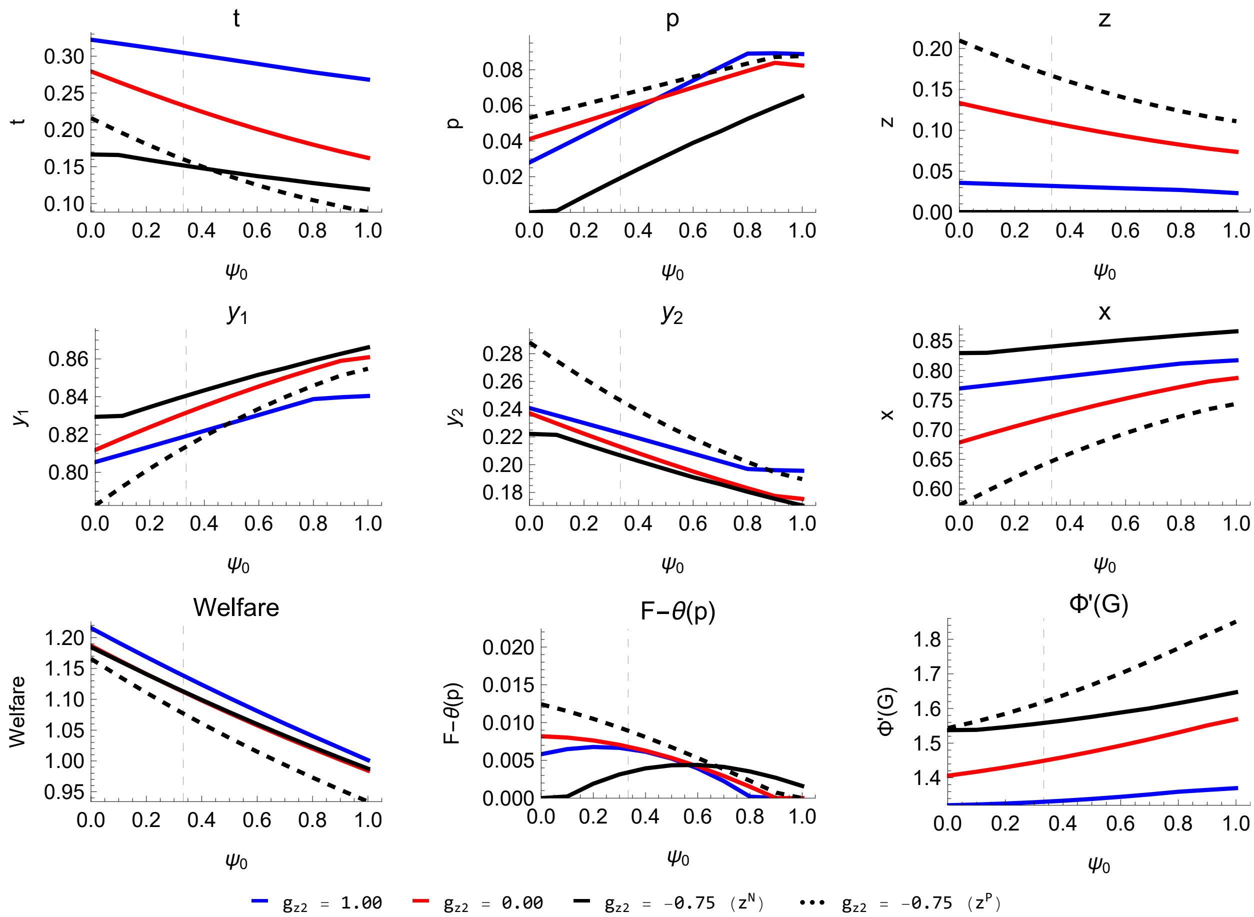


Figure C.14: Solutions as a function of Ψ_0.
The dashed vertical line is given by $\Psi_0 = \frac{(w_2 - w_1)}{w_2}$.

Qualitatively, the variables exhibit similar behavior for all values of g_{z2}: as Φ_0 increases, t declines, and p increases. As a result, z declines and y_2 declines, and y_1 and x increases. For a sufficiently large Ψ_0, given that t becomes small and p large, the audit agency's budget constraint, $[F - \theta(p)]$ starts to bind.

Comparing the two solutions when $g_{z2} < 0$, welfare is always higher if individuals choose $\{y_1^N, y_2^N, z^N\}$ as opposed to $\{y_1^P, y_2^P, z^P\}$. The optimal audit probability is lower in the former than in the latter case. The optimal tax rate when the solution is $\{y_1^N, y_2^N, z^N\}$ is always reached at $t = t^N$; it is lower in this case compared to the case $\{y_1^P, y_2^P, z^P\}$ when Ψ_0 is small and higher when Ψ_0 is large. Overall, $y_1^N > y_1^P$, $y_2^N < y_2^P$ and $z^N = 0 < z^P$. This translates into a larger amount of reported income ($x^N > x^P$). Even though the audit agency may contribute fewer resources to the provision of the public good (this happens when Ψ_0 is not too large), it follows that $G^N > G^P$.

D Extensions

D.1 Section 8.1 – Extensive Margin

Suppose an individual has to decide whether to engage in illegal activities (extensive margin) and choose the amount of time for each activity (intensive margin).[57] We formalize this decision through a random utility model.

D.1.1 Individuals who do not participate in illegal activities

Consider the decision of an individual who does not participate in illegal activities ($y_2 = 0$):

$$\max_{\{y_1, z\}} y_1 - t(y_1 - z) + h(\ell) - p\alpha t|z| - g_z z^2/2 + \Phi(G) - \Psi(Y_2), \tag{17}$$

where $x = y_1 - z$, $\ell = 1 - n_1$, and $n_1 = y_1/w_1$. In this case, unreported income $z = y_1 - x$. The solutions, denoted $y_1^o \equiv y_1^o(t,p)$ and, $z^o \equiv z^o(t,p) = \frac{t(1-p\alpha)}{g_z} > 0$, are derived in the Lemma 3 below.

Lemma 3. *The solutions of problem (17), denoted* y_1^o *and* z^o*, are (implicitly) given by* $(1-t)w_1 = h'(\ell^o)$, $\ell^o = 1 - n_1^o$ *and* $z = \frac{t(1-p\alpha)}{g_z} > 0$.

Proof. The FOCs for y_1 and z are

$$y_1: \quad (1-t) - h'/w_1 \leqslant 0, \tag{18}$$

$$z: \quad \begin{cases} t + p\alpha t - g_z z = 0, & \text{if } z < 0, \\ t - g_z z = 0, & \text{if } z = 0, \\ t - p\alpha t - g_z z = 0, & \text{if } z > 0. \end{cases} \tag{19}$$

At an interior solution for y_1, $(1-t)w_1 = h'$, so y_1 is independent of α and p. Moreover, $z = t(1-p\alpha)/g_z > 0$ since all the other cases cannot hold. □

The indirect utility is given by $\tilde{v}^o = v^o(t, p, G, Y_2) + \varepsilon^o$.

D.1.2 Individuals who participate in legal and illegal activities

Suppose that an individual also participates in illegal activities. In that case, the solutions are given by $y_1 \equiv y_1(t,p)$ $y_2 \equiv y_2(t,p)$ and $z \equiv z(t,p)$, derived in the proposition 1

[57]We still assume $g_{z2} = 0$, but the analysis can be extended for cases in which g_{z2} is different from zero.

Individuals who participate in both legal and illegal activities solve

$$\max_{\{y_1,y_2,z\}} y_1 + y_2 - t(y_1 - z) + h(\ell) - p(\alpha t|z| + \beta y_2) - (g_z z^2/2 + g_2 y_2^2/2) + \Phi(G) - \Psi(Y_2), \quad (20)$$

$$\ell = 1 - y_1/w_1 - y_2/w_2, \quad (21)$$

where we assumed $g_{z2} = 0$ and $\alpha^P = \alpha$. We have already shown that in this case, $z = y_1 - x > 0$. The FOCs for y_1, y_2 and z (at an interior solution) are

$$y_1: \quad (1-t) - h'/w_1 = 0, \quad (22)$$

$$y_2: \quad 1 - h'/w_2 - p\beta - g_2 y_2 = 0 \quad (23)$$

$$z: \quad t - p\alpha t - g_z z = 0. \quad (24)$$

This means that $z = \frac{t(1-p\alpha)}{g_z}$ and

$$(1-t)w_1 = (1 - p\beta - g_2 y_2)w_2 \quad \Rightarrow \quad y_2 = \frac{[(1-p\beta)w_2 - (1-t)w_1]}{g_2 w_2}. \quad (25)$$

For an interior solution, we need $(1-p\beta)w_2 \geqslant (1-t)w_1$ for all $0 \leqslant t \leqslant 1$. The solutions are given by $y_1 \equiv y_1(t,p)$ $y_2 \equiv y_2(t,p)$ and $z \equiv z(t,p)$ (similar to those derived from proposition 1). The indirect utility function is in this case $v(t,p,G,Y_2)$.

D.1.3 Comparing the solutions

The following proposition compares the solutions.

Proposition 7. *When* $g_{z2} = 0$, $n_1^o = n_1 + n_2$, $y_1^o = y_1 + \left(\frac{w_1}{w_2}\right) y_2$, $y_2 > 0$, $z^o = z$, $x^o = x + \frac{w_1}{w_2} y_2$, *and* $v - v^o = \frac{[(1-p\beta)w_2 - (1-t)w_1]^2}{2 g_2 w_2^2}$. *If* $(1-p\beta)w_2 > w_1$, *then* $y_1 < y_1^o < y_1 + y_2$, $x^o > x$, *and* $v > v^o$.

Proof. First, note that $w_1(1-t) = h'(\ell^o)$ and $w_1(1-t) = h'(\ell)$, so that $\ell^o = \ell$, or $n_1^o = n_1 + n_2$. Second, the latter implies that $(1 - y_1^o/w_1) = (1 - y_1/w_1 - y_2/w_2)$, then $y_1^o/w_1 = y_1/w_1 + y_2/w_2$. As a result, $y_1^o = y_1 + y_2 - [(w_2 - w_1)/w_2]y_2$, or $y_1^o = y_1 + (w_1/w_2)y_2$. Since $(1-p\beta)w_2 > w_1$ implies that $w_2 > w_1$, then $y_1 < y_1^o < y_1 + y_2$. Third, the solutions with and without illegal income are the same and equal $z^o = z = \frac{t(1-p\alpha)}{g_z} > 0$. Fourth, $y_1^o - z^o = y_1 + (w_1/w_2)y_2 - z$, or $x^o = x + (w_1/w_2)y_2$, which implies $x^o > x$. Substituting the solutions in the respective utility functions and calculating the difference give $v - v^o$. This difference only depends on y_2 since $z^o - z = 0$ and $y_1^o - y_1 = \frac{w_1}{w_2} y_2 > 0$. □

It follows from the proposition that if $(1-p\beta)w_2 > w_1$, then: (i) The production of legal activities by those who engage in both legal and illegal activities is less than the legal production of those who engage only in legal activities, which is still less than the total production from both legal and illegal activities ($y_1 < y_1^o < y_1 + y_2$); (ii) Individuals who engage only in legal activities report more legal income than those who participate in both legal and illegal activities ($x^o > x$); and (iii) Individuals who engage in both legal and illegal activities experience higher indirect utility than those who participate only in legal activities ($v > v^o$).

D.1.4 Extensive margin

An individual who only participates in legal activities is $\tilde{v}^o = v^o(t, p, G, Y_2) + \varepsilon^o$ and the utility of someone that participates in both types of activities is $\tilde{v} = v(t, p, G, Y_2) + \varepsilon$, where $v(t, p, G, Y_2)$ is the indirect utility function. The first term is a deterministic component given by the indirect utility functions defined earlier. The terms ε^o and ε are random components that vary across individuals (we suppress the subscripts to simplify notation). The variables capture the individual's idiosyncratic preferences for participating (or not) in illegal activities. We assume that ε and ε^o are independent Gumbel-distributed random variables.[58]

Given the assumptions on ε^o and ε, the share of individuals engaged in illicit activities becomes

$$\sigma = \frac{\exp(v)}{\exp(v) + \exp(v^o)},$$

so $(1 - \sigma)$ represents the share of those who decide not to participate in illegal activities. As a result $Y_2 = \sigma y_2$.

D.1.5 Comparative statics

How does the share of those participating in illegal activities σ change as t and p change? This is relevant because the aggregate production of illicit activities is given by $Y_2 = \sigma y_2$, so changes in t affect both the extensive margin (σ) and the intensive margin y_2. It follows, as stated in the following proposition, that both margins tend to move in the same direction when either t or p changes.

Proposition 8. *An increase in* t *raises both the time allocated to illegal activities and the number of individuals engaged in those activities. An increase in* p *has the opposite effect on both the intensive and extensive margins.*

[58] For convenience, we assume the Gumbel distributions have identical location and scale parameters. In particular, the location parameter equals 0, and the scale parameter equals 1. These assumptions do not affect our subsequent analysis in any substantial way.

Proof. Note that

$$\frac{\partial \sigma}{\partial t} = \sigma(1-\sigma)\left(\frac{\partial v}{\partial t} - \frac{\partial v^o}{\partial t}\right) \quad \Rightarrow \quad \frac{\partial \sigma}{\partial t} = \sigma(1-\sigma)y_2\frac{w_1}{w_2} > 0. \tag{26}$$

We showed earlier (equation (9)) that $\frac{\partial y_2}{\partial t} > 0$. This means that both the extensive (σ) and the intensive (y_2) margins move in the same direction when t increases so that $\frac{\partial Y_2}{\partial t} = \frac{\partial \sigma}{\partial t}y_2 + \frac{\partial y_2}{\partial t}\sigma > 0$. A similar argument can be used to show that $\frac{\partial \sigma}{\partial p} < 0$, $\frac{\partial y_2}{\partial p} < 0$, so $\frac{\partial Y_2}{\partial p} < 0$. □

D.1.6 Optimal tax rate

The government chooses the tax rate t that maximizes

$$v^E = \mathbb{E}\left[\max\{\tilde{v}, \tilde{v}^o\}\right] = \log\left[\exp(v) + \exp(v^o)\right] + \gamma, \quad \text{subject to} \quad T + F \geqslant G,$$

where[59]

$$\begin{aligned} T &= t[(1-\sigma)x^o + \sigma x] = tx + t\frac{w_1}{w_2}(1-\sigma)y_2, \\ F &= pt\alpha[(1-\sigma)z^o + \sigma z] + p\beta\sigma y_2 = pt\alpha z + p\beta Y_2. \end{aligned} \tag{27}$$

The FOC is

$$\frac{\partial v^E}{\partial t} \equiv \sigma\frac{\partial v^o}{\partial t} + (1-\sigma)\frac{\partial v}{\partial t} = 0 \quad \Rightarrow \quad \frac{\partial v}{\partial t} - \sigma\left(\frac{\partial v}{\partial t} - \frac{\partial v^o}{\partial t}\right) = 0. \tag{28}$$

Since $\left(\frac{\partial v}{\partial t} - \frac{\partial v^o}{\partial t}\right) = \frac{w_1}{w_2}y_2 > 0$, then the FOC satisfies

$$\frac{\partial v}{\partial t} = \frac{w_1}{w_2}\sigma y_2. \tag{29}$$

[59] γ is the Euler-Mascheroni constant.

D.2 Section 8.2 – Probability of Detection Differs by Type of income

So far, an increase in p allows the government to detect tax evasion and income generated from illegal activities. Suppose now that the detection technology is such that evaded income is detected with probability p_z and illegal income is detected with probability p_2. Moreover, to simplify the analysis, suppose that $g_{z2} = 0$, so that individuals choose a positive level of z (we also use $\alpha^P = \alpha$). In this case,

$$y_2 = \frac{w_2(1-\beta p_2) - (1-t)w_1}{g_2 w_2}, \quad z = \frac{t(1-\alpha p_z)}{g_z}, \tag{30}$$

and y_1 is implicitly defined by $(1-t)w_1 = h'(1 - w_1/y_1 - w_2/y_2)$. Note that since p_z does not affect y_2, it does not affect y_1 either, so $\partial x/\partial p_z = -\partial z/\partial p_z$. Additionally, p_2 does not have an impact on z, but it does affect y_2, and consequently, y_1, so $\partial x/\partial p_2 = \partial y_1/\partial p_2 = -w_1(\partial y_2/\partial p_2)/w_2$. This means that

$$\frac{\partial x}{\partial p_z} = \frac{\alpha t}{g_z} > 0, \quad \frac{\partial x}{\partial p_2} = \frac{\beta w_1}{g_2 w_2} > 0, \quad \frac{\partial^2 x}{\partial t \partial p_z} = \frac{\alpha}{g_z} > 0, \quad \frac{\partial^2 x}{\partial t \partial p_2} = 0. \tag{31}$$

The effectiveness of p_z in terms of its ability to incentivize higher levels of reported income x rises as the tax rate increases. However, the effectiveness of p_2 does not depend on t. When t is low (specifically, $t < \frac{\beta g_z w_1}{\alpha g_2 w_2}$), p_2 is more effective (i.e., $\partial x/\partial p_z < \partial x/\partial p_2$). But if t is sufficiently large, then p_z becomes more effective. The relative cost of each detection technology, captured by the function $\theta(p_z, p_2)$, will also matter when deciding the combination of p_z and p_2 that maximizes welfare.

D.2.1 Optimal policy combination of $\{t, p_z, p_2\}$ for different levels of Ψ_0

We showed earlier that if $p = p_z = p_2$ and t is fixed, the optimal audit probability p increases as the strength of the externality gets larger, i.e., $\frac{\partial p}{\partial \Psi_0} > 0$. This outcome is partly due to the pooling of audit resources. How do the results vary if the audit agency can separately oversee tax evasion and participation in illegal activities?

To examine how the strength of the externality Ψ_0 affects the optimal policy combination $\{t, p_z, p_2\}$, we consider two scenarios with different costs for providing audit probabilities p_z and p_2: (i) $\theta_{z2} = 0$ and (ii) $\theta_{z2} < 0$. In case (i), the cost of providing p_z is independent of the cost of providing p_2; in case (ii), p_z and p_2 are complements. The graphs below compare the optimal values of $\{t, p_z, p_2\}$ in the unconstrained case, i.e., when p_z and p_2 are not necessarily equal, to the corresponding solutions in the "constrained" case, i.e., when $p_z = p_2 = p$.

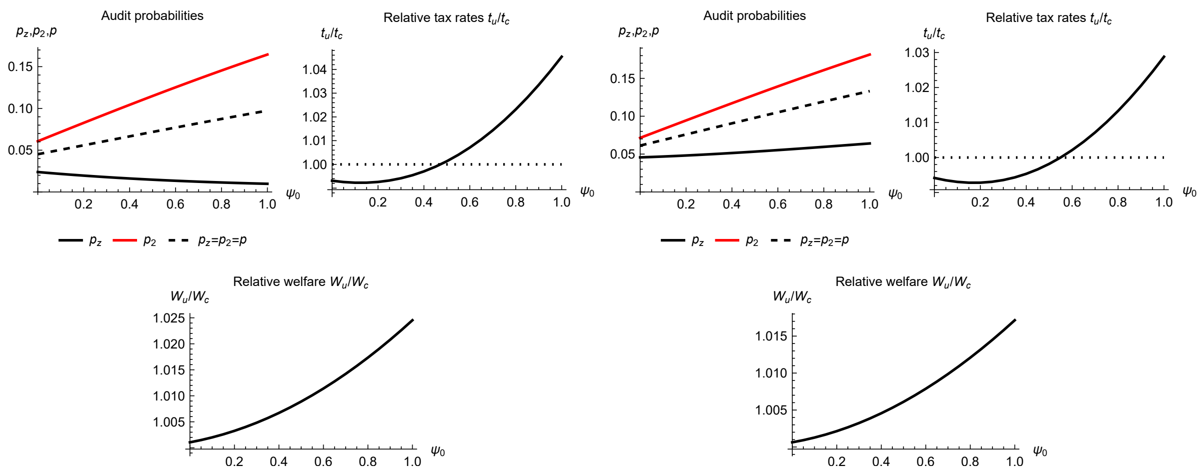


Figure D.15:

LEFT: Case (i): $\theta_{z2} = 0$. RIGHT: Case (ii): $\theta_{z2} = -2/3$.

Note: The subindex "u" denotes the "unconstrained" case in which p_z may differ from p_2; the subindex "c" denotes the "constrained" case in which $p_z = p_2 = p$.

The dashed black line in the graphs shows the optimal audit probability p. As explained earlier, p increases as Ψ_0 increases. In case (i), where the cost of providing audit probabilities p_z and p_2 is independent, an increase in Ψ_0 decreases p_z and increases p_2, and reduces t. This indicates that as the negative externality becomes stronger, the optimal policy shifts resources from the detection of tax evasion to the monitoring of illegal activities. In case (ii), when Ψ_0 increases, both p_z and p_2 increase. The increase in p_z as Ψ_0 gets larger is explained by the fact that higher levels of p_z reduce the cost of providing p_2. In both cases, the tax rates in the "unconstrained" scenario are lower than in the "constrained" scenario for low values of Ψ_0. It is important to note, however, that when t_u and t_c are graphed individually (not shown in the graphs above), both tax rates decline as Ψ_0 rises.

D.3 Section 8.3 – Alternative Penalty Functions

This section compares different functions F that penalize misreported legal income differently depending on whether $z > 0$ or $z < 0$:

(i) $\alpha^N = \alpha^P = \alpha$ (the same F that we have used in the analysis thus far);

(ii) $\alpha^N = 0$, $\alpha^P = \alpha$ (no punishment for overreporting);

(iii) $\alpha^N = -\alpha$, $\alpha^P = \alpha$ (reward for detected overreporting); and

(iv) $\alpha^N = -1$, $\alpha^P = \alpha$ (reimbursement of overpayment).

The subsequent analysis examines how changes in the intensity of the negative externality affect the optimal tax rate, keeping the audit probability p fixed. The analysis focuses on the case $g_{z2} < 0$ with $\{y_1, y_2, z\} = \{y_1^N, y_2^N, z^N\}$ when $t \leqslant t^N$. As shown earlier, $z \geqslant 0$ when $g_{z2} \geqslant 0$, which means that the alternative penalty functions are relevant only when $g_{z2} < 0$. Recall that in this case, z could be negative: there are two solutions, one of which is $z = z^N \leqslant 0$. Also, different values of α used in the penalty function F affect the threshold tax rate t^N, the value of t at which $z = 0$, and the slope $\frac{\partial z}{\partial t}$ when $z < 0$.

How does the optimal tax rate t depend on the specific penalty function? The following graphs summarize the results. Welfare levels and the audit agency's budget constraint $F - \theta(p)$ are expressed relative to the corresponding outcomes in the benchmark case (i) ($\alpha^N = \alpha^P = \alpha$).

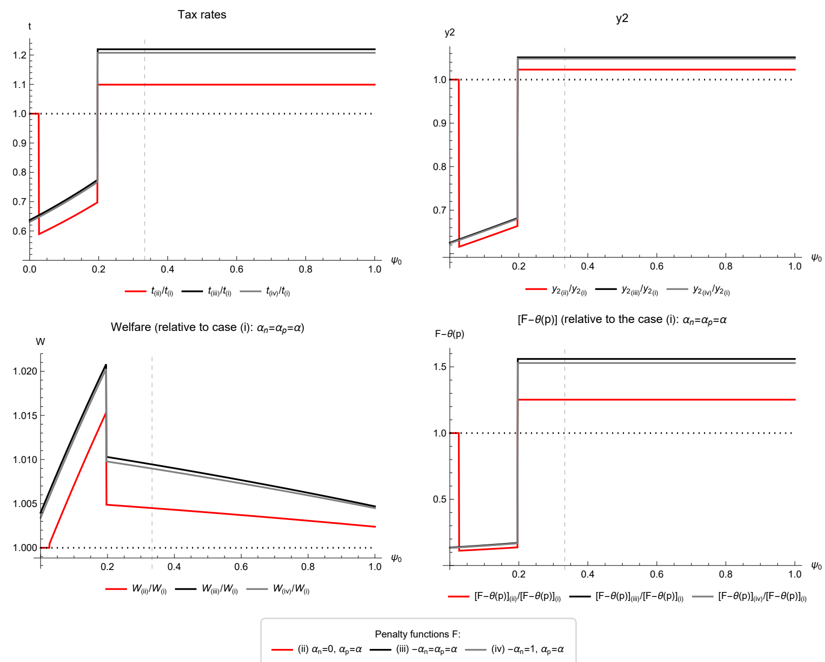


Figure D.16: Optimal tax rates, illegal activities, welfare, and audit agency's budget constraint for different penalty functions.
All the variables are expressed relative to the benchmark case (i) ($\alpha^N = \alpha^P = \alpha = 1.1$). In all cases, $p = 0.0675$, $g_{z2} = -0.75$, and $\{y_1^N, y_2^N, z^N\}$ for $t \leqslant t^N$. The dashed vertical lines is at $\Psi_0 = 1 - \frac{w_1}{w_2} = \frac{1}{3}$.

When Ψ_0 is sufficiently large, penalty functions that use (ii) $\alpha^N = 0$, $\alpha^P = \alpha$, (iii) $-\alpha^N = \alpha^P = \alpha$, (iii) $-\alpha^N = 1, \alpha^P = \alpha$, give the government the ability to impose higher tax rates than in the benchmark case (i). In this case, the optimal tax rates are reached at $t = t^N$:

$$t^N_{(i)} = \frac{-g_{z2}[(1-p\beta)w_2 - w_1]}{(1+\alpha p)g_2 w_2 + g_{z2} w_1}, \quad t^N_{(ii)} = \frac{-g_{z2}[(1-p\beta)w_2 - w_1]}{g_2 w_2 + g_{z2} w_1},$$
$$t^N_{(iii)} = \frac{-g_{z2}[(1-p\beta)w_2 - w_1]}{(1-\alpha p)g_2 w_2 + g_{z2} w_1}, \quad t^N_{(iv)} = \frac{-g_{z2}[(1-p\beta)w_2 - w_1]}{(1-p)g_2 w_2 + g_{z2} w_1}.$$

Moreover, these tax rates can be ranked as follows: $t^N_{(i)} < t^N_{(ii)} < t^N_{(iv)} < t^N_{(iii)}$, with the highest tax rate chosen in case (iii).

The alternative penalty functions also allow the audit agency to collect higher (net) revenue and attain higher levels of welfare. While the differences between cases (iii) and (iv) are minor, the exercise shows that it still pays off to return $\alpha^N > 1$ when the individual overpaid. Note, however, that the alternative penalty functions induce individuals to choose a higher level of the illegal good y_2.[60]

The main takeaway from this exercise is that in scenarios where individuals may overreport legal income (i.e., in situations where $z < 0$ is a possible outcome), the implementation of a penalty scheme that credits taxpayers when they overpay could improve welfare. While such a penalty function allows the government to impose higher tax rates, which eventually leads to higher levels of the illegal good y_2, the amount of resources devoted to finance the provision of the public good rises, and overall welfare increases.

[60]For large enough Ψ_0, the highest welfare is reached at $t = t^N$ in all cases (i) through (iv).