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Sovereign Default Intensity and Noise Bargaining

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Abstract

Hard sovereign defaults—defaults with large haircuts—are associated with deeper recessions, longer durations, and, as we show, larger devaluations than soft defaults. We rationalize these regularities by developing a single-proposer *noise bargaining* game and embedding it in a two-sector sovereign default model. Creditors weigh the sovereign's haircut offers against likely future offers and idiosyncratic valuation shocks. In short-lived recessions, creditors tend to reject large proposed haircuts, anticipating better terms as the economy recovers—endogenously correlating default intensity with adverse outcomes. Two years after default, our decomposition attributes nearly 80% of the observed output differentials to selection on different shock realizations.

JEL Codes: F34, C78, E32

Keywords: Default, Sovereign, Debt, Growth, Haircuts

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1 Introduction

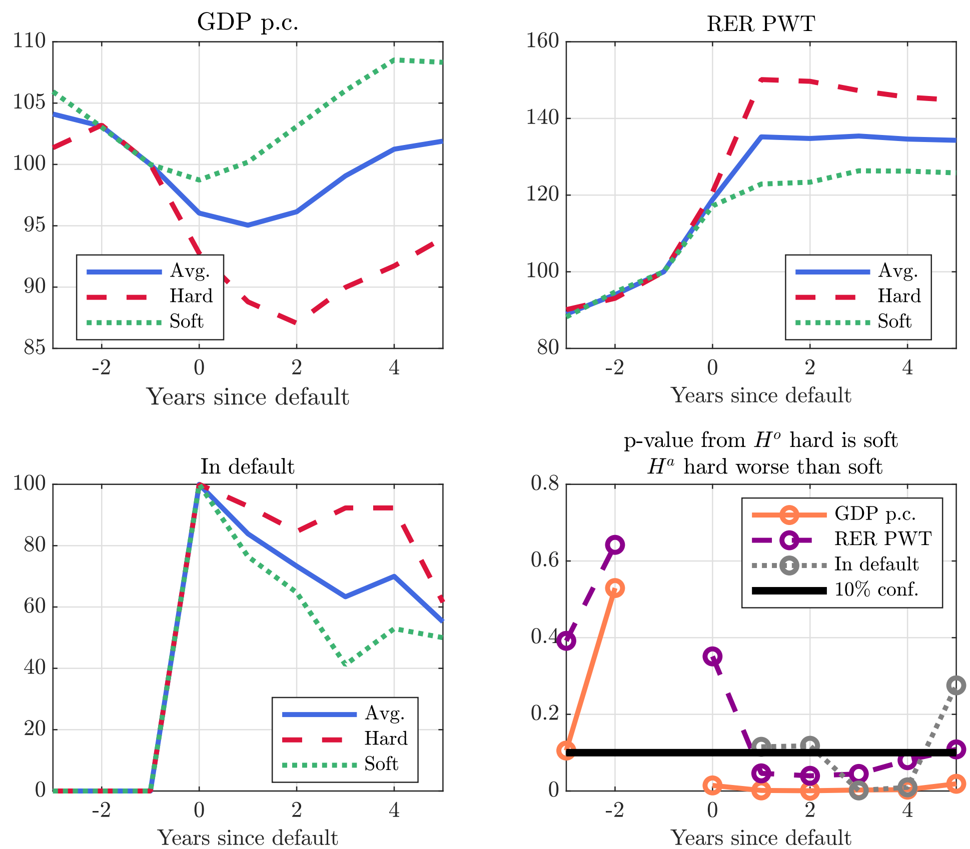
Research on sovereign default intensity has shown stark differences between hard defaults—those with large haircuts—and soft defaults. Hard defaults are associated with deeper and more persistent recessions (Trebesch and Zabel, 2017) and longer exclusion (Benjamin and Wright, 2009) from markets. We confirm these patterns and extend them, documenting hard defaults also feature larger real exchange rate (RER) depreciations (Figure 1).

Why do hard and soft defaults differ so dramatically? We answer this by developing a *noise bargaining* game and embedding it in a two-sector sovereign default model. The economy is subject to permanent and transitory shocks to its tradable goods sector, which change the output-costs of default and debt-GDP ratios. When attempting to restructure, the sovereign makes haircut offers, which creditors accept or reject based on idiosyncratic valuation shocks and expectations of future offers. This framework generates an endogenous mapping from the state of the economy to the type of restructuring that occurs.

Hard defaults require creditors to accept large haircut offers. In the model, this mostly happens when they expect large haircut offers will also occur in the future. Since the sovereign cannot commit to future offers, this will only be the case when the economy's trajectory is poor. Endogenously, this links hard defaults with adverse growth and RER trajectories, both of which increase debt to GDP making repayment more difficult. Default intensity and duration are linked due to arrears growth, which mechanically increases debt as duration increases, and valuation shocks, which generate delay. A decomposition exercise shows that most of the divergent GDP patterns are explained by different shocks rather than default costs prolonged by failed negotiations.

The rest of the paper proceeds as follows. We first discuss the relevant litera-

Figure 1: Hard and Soft Default Episodes



Note: GDP p.c. is real GDP per capita; RER PWT is the real exchange rate, which we obtain from the Penn World Tables (PWT); GDP and default status are from Trebesch and Zabel (2017); variables are defaulted by their values one year prior to default and multiplied by 100.

ture. Section 2 lays out the noise bargaining game and highlights the main mechanisms using a stylized model. Section 3 describes the quantitative model, and section 4 its parameterization. Section 5 establishes and interprets the model's replication of hard and soft default patterns. Section 6 concludes. The appendices provide model extensions, additional simulations, and technical details.

1.1 Related Literature

Our paper contributes to both the sovereign debt and bargaining literatures.

1.1.1 Sovereign debt restructurings

The empirical literature connecting default intensity with real outcomes motivates our work. Trebesch and Zabel (2017) document the divergent GDP patterns between hard and soft defaults.[1] The unpublished but influential paper Benjamin and Wright (2009) shows a positive correlation between haircuts and default duration. We additionally point out hard defaults are associated with greater RER depreciations. This complements the existing work on default intensity, while extending Hébert and Schreger (2017) and Augustin, Chernov, and Song (2020) that show default is associated with depreciations. Meyer, Reinhart, and Trebesch (2022) shows the total returns from two years before default to default are essentially identical across hard and soft defaults (Figure XII, p. 1666). This empirical result suggests a strong role for selection based on ex-post outcomes, which is what we find. Our model also reproduces this return behavior.

Our work is closely related to a literature on sovereign debt renegotiation. This began with Yue (2010), who introduced Nash bargaining over defaulted debt. This

[1] Relatedly, Asonuma and Trebesch (2016) highlight that real outcomes are much better in preemptive restructurings—restructurings that occur without missed payments, and these tend to be small haircut outcomes.

does not allow for endogenous delay in settlement. Benjamin and Wright (2009) introduced Rubinstein (1982)-style alternating offer bargaining with complete information. They say delay occurs in their model due to waiting for spreads on new issuances to fall (p. 2), which is an intuition that could also be applied to our model. But more concretely we show delay in our model arises from a contemporaneous tradeoff between smaller haircuts and higher acceptance rates and expectations of future haircut offers. An objective of their paper is to explain the connection between haircuts and delay, which we are able to explain largely through the simple mechanism of growth of debt in arrears.

Other work in this strand has emphasized the highly complicated restructuring process. Institutional details in the form of collective action clauses, judicial interpretation of pari passu or specific covenants (see Buchheit, Chabert, DeLong, and Zettelmeyer, 2019, for many of these details) means restructuring is a messy process. Some work has found these details don't matter (Bi, Chamon, and Zettelmeyer, 2016). But litigation shocks form the basis for the effective empirical strategy of Hébert and Schreger (2017). Quantitatively, Pitchford and Wright (2012) include multiple, bilateral negotiations between the sovereign and creditors. There, the sovereign has incentive to offer better terms to holdouts, which generates delays in restructuring. Our model captures the high degree of uncertainty associated with restructuring using valuation shocks. The idiosyncratic nature of these shocks preserves the fundamental incentives of creditors accept/reject decisions while simultaneously incorporating the uncertainty faced by all parties.

Recent work has also emphasized that default is partial in the sense that defaulted debt relative to payments due is always less than one. The Arellano, Mateos-Planas, and Ríos-Rull (2023) model explicitly keeps track of debt in arrears. Growth of debt in arrears plays a crucial role in our model, but we eliminate a continuous state variable (debt in arrears) by approximating how total debt evolves post default, including both interest on arrears and growth of missed coupons and principal payments.

1.1.2 Bargaining

A significant contribution of our paper is noise bargaining over haircuts, which generates delay and endogenous bargaining power with a single proposer in a tractable and intuitive way. Noncooperative bargaining has largely followed Rubinstein (1982) in using alternating offers. In this and many other bargaining models, immediate settlement occurs. As Merlo and Wilson (1995) analyze, the stochastic case is much more complicated and an agreement is not necessarily met right away. Benjamin and Wright (2009) has the proposer chosen stochastically with a time-varying pie, which generates delay. We generate delay through a single proposer subject to noise.

A large distinction in the bargaining literature is whether information is asymmetric. Grossman and Perry (1986) shows with incomplete information equilibrium delays can occur. Kennan and Wilson (1993) survey bargaining with private information. With asymmetric information, delay can occur as the proposer tries to learn about the underlying state or signal his type. In our approach, nature chooses shock realizations after the proposal, which could equivalently be asymmetric information if the shocks were known to creditors but private information. However, the mechanism for delay in our model is not learning or signaling but the proposer—knowing the distribution of idiosyncratic valuation shocks—trading off lower odds of acceptance with better terms conditional on acceptance.

One branch of the bargaining literature has focused on political applications has used single-proposer or persistent-proposer bargaining. Romer and Rosenthal (1978) is seminal in that strand, which had a single proposer making offers that if not accepted would result in a status quo. Primo (2002) endogenized the status quo and showed Romer and Rosenthal could be interpreted as bargaining where the single proposer makes a take it or leave it offer. In general, single proposer games can result in considerable bargaining power for the proposer.[2] In our setup, noise limits the sovereign's bargaining power.

2 Haircut determination through noise bargaining

Our model's haircut determination operates through a game of unilateral offers under imperfect information. To focus on this key contribution, we first describe this process and its implications in the context of a stylized model. We suppose the sovereign and creditor are risk neutral and that the defaulted debt stock b is short-term. The time discount factor for the sovereign and creditors is $\beta \in (0, 1)$. We look for a Markov perfect solution in pure strategies.

The timing is as follows. At the beginning of the period, the sovereign makes a haircut offer $\hat{h}$. (We use hats to distinguish off-equilibrium objects and non-hatted for on-equilibrium or expected values.) Shocks affecting creditor accept/reject decisions occur. Creditors accept or reject. If they accept, the sovereign pays the creditors $(1 - \hat{h})b$ and the game ends. If they reject, the sovereign suffers a default cost χy (the product of a proportional cost χ and output y) and the game proceeds to the next period.

To each possible haircut offer $\hat{h}$, the sovereign assigns a subjective expected acceptance rate $\hat{\alpha}$. After receiving the haircut offer $\hat{h}$ and after shocks are realized, creditors make a decision on whether to accept or reject the offer. The fundamental value of acceptance to creditors is $\hat{q}^A = 1 - \hat{h}$ (per unit of debt). The fundamental value of rejecting an offer is denoted q^D, which takes into account expected future haircut offers and acceptance rates. Absent any shocks, the creditors would choose

[2] This can be limited by changing the game structure like in Diermeier and Fong (2011) and Groseclose (2021) where rejected proposals cannot be reconsidered. This literature is well surveyed by the introduction to Groseclose.

to accept or reject based on the sign of $\hat{q}^A - q^D$. But we assume there are i.i.d. valuation shocks, ϵ^A and ϵ^D, that make the gain from accepting be $\hat{q}^A + \epsilon^A - q^D - \epsilon^D$.[3] This results in an ex-ante acceptance probability $\alpha(\hat{q}^A - q^D)$ for $\alpha : \mathbb{R} \to [0,1]$, which is necessarily increasing and right-continuous. This nests the perfect certainty case of $\alpha(\delta) = \mathbf{1}[\delta \geq 0]$, for instance. It also nests taste shocks with shape parameter σ, inducing $\alpha(\delta) = \exp(\sigma\delta)/(1 + \exp(\sigma\delta))$. Or a specification $\alpha(\delta) = \min\{\max\{0, \delta\}, 1\}^\sigma$ that has constant elasticity when interior. For Markov perfection, $\hat{\alpha}(\cdot) = \alpha(\cdot)$ in equilibrium.

The fundamental value to creditors of rejecting an offer is $q^D = \beta(q^D + \alpha(q^A - q^D))$, as rejection entails a one-period delay followed with a next-period restructuring probability of α. So, in equilibrium,

$$q^D = \frac{\beta\alpha}{1 - \beta(1 - \alpha)} q^A \leq q^A. \tag{1}$$

The fundamental value of accepting an offer per unit of debt is trivially $\hat{q}^A = 1 - \hat{h}$.

These basic creditor valuations already reveal an important insight. When is a large haircut offer likely to be accepted by creditors? It is when q^D is small, which occurs when $q^A = 1 - h$ is small, which means *expected future haircuts h* are large: Creditors accept large haircuts when the sovereign credibly offers large haircuts in the future. If this were not the case, creditors would reject the offer and wait for a future one that in expectation is better. Since we assume the sovereign cannot commit to future haircut offers, the only way to credibly offer large haircuts in the future is for the state of the economy to be such that large haircut offers will be optimal in the future. Thus, if the economy's expected outlook is poor for an extended duration, this is when large haircuts can be expected. If the economy is expected to recover soon, creditors will not accept large haircuts. Just from this,

[3] Formally, we assume creditors enjoy an additional payoff $\epsilon^A - \epsilon^D$ conditional on acceptance and zero otherwise.

hard defaults should be associated with persistently worse economic outcomes than soft defaults.

Now consider the sovereign's problem,

$$V = \max_{\hat{h}\in[0,1]} -\chi y + \beta[V + \hat{\alpha}(1-\hat{h}-q^D)(-(1-\hat{h})b - V)]. \tag{2}$$

The sovereign internalizes that smaller haircut offers $\hat{h}$ increase the cost $(1-\hat{h})b$ of restructuring but also increase acceptance rates $\hat{\alpha}$, reducing default costs χy in expectation. Letting ϵ_α denote the elasticity of α evaluated at $q^A - q^D$, the first order condition simplifies to $V = -b(q^A + \frac{1}{\epsilon_\alpha}(q^A - q^D))$ in equilibrium. Combining this, the Bellman equation $V = -\chi y + \beta(V + \alpha(-q^A b - V))$, and the creditor values, it is easy to characterize equilibrium haircuts as in proposition 1.

Proposition 1. *Any interior solution for haircuts is given by*

$$h = 1 - \frac{\epsilon_\alpha}{\epsilon_\alpha + 1} \cdot \frac{1}{1-\beta} \cdot \frac{\chi y}{b}. \tag{3}$$

with $\epsilon_\alpha > 0$ if $\alpha(\cdot)$ is strictly increasing and differentiable.

This characterization admits several important insights.

First, haircuts are increasing in debt to GDP ratios. This intuitive result arises from two facets of the model. One is that larger default costs χy increase the surplus from restructuring. The other is that the cost of restructuring is $(1-h)b$, so h only matters to the extent b is large.

This result underpins the main findings of the paper. It indicates that when GDP recovers faster, driving down debt to GDP, haircuts are smaller. Already this points to the importance of growth versus transitory shocks in generating hard and soft defaults: The sovereign will never recover from a negative growth shock, resulting in a permanently higher level of debt/GDP else equal and larger haircuts; in contrast, a transitory shock will fade, reducing debt/GDP and resulting in smaller haircuts. Thus, hard defaults should be associated with worse GDP trajectories, as in the data. This is the main mechanism underlying the data patterns for GDP in the model. Similar logical applies for depreciations. A real exchange rate depreciation causes debt to GDP to increase due to revaluation effects. If that is permanent, it will lead to larger haircuts. So hard defaults should be associated with greater devaluations, as in the data. It also means that interest on arrears, which grow debt to GDP over time, drives a positive correlation between haircuts and default duration, as observed in the data.

Second, the more noisy/less elastic α is, the larger equilibrium haircuts are. The marginal benefit from offering a smaller haircut is a greater acceptance rate. When the bargaining process is noisier, the marginal benefit is reduced, and the sovereign seeks a more aggressive haircut.

Third, a proportionate shift in α does not change haircuts, but it does extend the duration of bargaining and default. In the quantitative model, there will be a scale effect like this. This result shows that the scale effect will change how much debt can be sustained (by changing the expected net present value of default costs), but it will not directly affect haircut offers.

Fourth, increased patience reduces haircuts else equal. Larger patience increases the net present value of default costs, incentivizing escaping from default. This drives the sovereign to offer lower haircuts. A countervailing effect of creditor patience, evident in (1), makes q^D larger relative to q^A, which pushes down $q^A - q^D$ else equal. If α is constant elasticity, this effect does not matter; otherwise, it could drive haircuts in either direction.

A final key insight can be derived, which is that—under noise bargaining—the effective bargaining weight is controlled by how noisy the creditors' acceptance decision is. To see this, note that the surplus to the sovereign from reaching an agreement versus never reaching an agreement is $\frac{\chi y}{1-\beta} - (1-h)b$, while the creditors' surplus is $(1-h)b$. So the sovereign's share of the total surplus is

$$\frac{1}{\epsilon_\alpha + 1} = \frac{\text{Sovereign's surplus}}{\text{Total surplus}} = \frac{\frac{\chi y}{1-\beta} - (1-h)b}{\frac{\chi y}{1-\beta}}. \tag{4}$$

As bargaining becomes infinitely noisy, $\epsilon_\alpha \downarrow 0$, the sovereign's share of the surplus goes to one. He pushes for larger haircuts because the associated cost of a reduced acceptance rate goes to zero. As bargaining becomes noiseless, $\epsilon_\alpha \uparrow \infty$, the sovereign's share goes to zero.[4] In this case, smaller haircuts are rewarded by a larger acceptance rate. Equal bargaining weights correspond to $\epsilon_\alpha = 1$, such as when α is linear.

3 Quantitative model

The structure of the quantitative model can be summarized as follows. There are two types of goods, tradables and nontradables. The country is endowed with a stochastic amount of tradables and a deterministic amount of nontradables, both of which grow secularly over time.[5] The tradable endowment is subject to both growth shocks and transitory shocks. All debt is tradable-denominated. There are three types of agents:

- **Consumers**. These take all prices as given and choose the optimal level of tradable and nontradable consumption.
- **A sovereign**. This agent takes as given the behavior of consumers and credi-

[4]The limit case of totally noiseless bargaining results in the sovereign capturing all the surplus. If $\alpha(q^A - q^D) = \mathbf{1}[q^A - q^D \geq 0]$, then the sovereign offers the creditor reservation value $q^A = q^D$ and has his offer accepted. By (1), $q^D = \theta q^A$ for some endogenous $\theta \in [0,1)$. This can only happen if $q^A = q^D = 0$, implying $h = 1$: Haircuts are total. Adding noise to the creditor accept/reject decision weakly improves their outcomes.

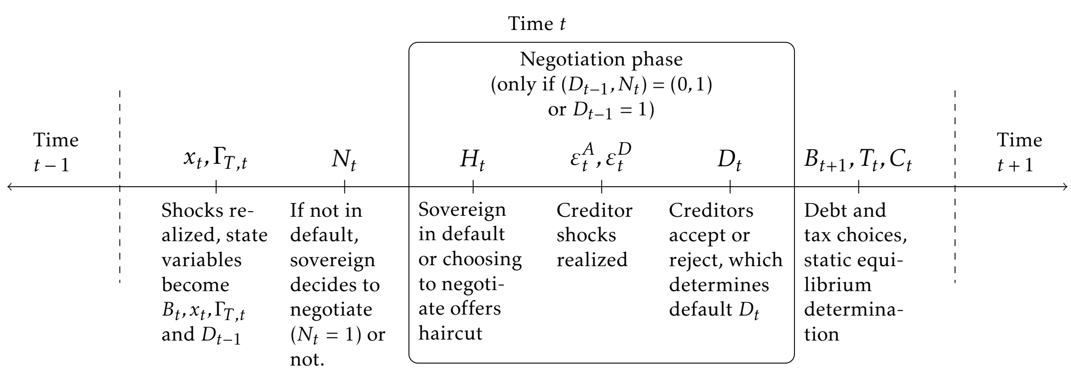
[5]The appendix provides an extension with production of nontradables that nests the benchmark.

tors and seeks to maximize consumer welfare using debt, lumpsum taxes of tradables, and sovereign debt negotation.[6]

- **Creditors**. These foreign agents own and competitively price all the sovereign debt, and they negotiate with the sovereign.

Restructurings are virtually identical to the stylized model. Specifically, during negotiations, the sovereign proposes a haircut amount that creditors decide to accept or reject. In their accept/reject decision, creditors take into account expected future haircut offerings, shocks, and sovereign choices. This is formally modeled as a dynamic game, and we will seek a Markov perfect equilibrium of that game. Figure 2 provides a timeline.

Figure 2: Timeline



3.1 Endowments

The nontradable endowment $Y_{N,t}$ grows deterministically at a rate μ with

$$Y_{N,t} = \Gamma_{N,t} \tag{5}$$

$$\Gamma_{N,t} = \mu \Gamma_{N,t-1}. \tag{6}$$

[6] The appendix provides an extension where the sovereign can also manipulate the real exchange rate through distortionary taxation, but it is never optimal to do so.

There is a "potential" tradable endowment $Y_{T,t}$ that has both transitory and permanent shocks.[7] It evolves according to

$$Y_{T,t} = z_t \Gamma_{T,t} \tag{7}$$

$$\Gamma_{T,t} = g_t \Gamma_{T,t-1} \tag{8}$$

$$\log z_t = \rho_z \log z_{t-1} + \varepsilon_{z,t}, \quad \varepsilon_{z,t} \overset{i.i.d.}{\sim} N(0, \sigma_z^2). \tag{9}$$

The transitory shock is z_t, while g_t is the growth shock. The growth shock evolves according to

$$\log g_t = (1-\rho_g)\log\mu + \rho_g \log g_{t-1} + (\rho_e - 1)\log e_{t-1} + \varepsilon_{g,t}, \quad \varepsilon_{g,t} \overset{i.i.d.}{\sim} N(0, \sigma_g^2) \tag{10}$$

for $\rho_e \in [0,1)$ where

$$e_t = \frac{\Gamma_{T,t}}{\Gamma_{N,t}}. \tag{11}$$

Incorporating a drift in g_t using $(\rho_e - 1)\log e_{t-1}$ ensures that the log difference between the trend components shrinks at rate ρ_e in expectation, thus keeping $\frac{\Gamma_{T,t}}{\Gamma_{N,t}}$ (which is e_t) stationary. Strictly speaking, this means the growth shocks are not permanent. However, we will set ρ_e close to one numerically, approximating permanence. Jointly, the vector $x_t := [z_t, g_t, e_t]'$ in logs follows a VAR(1) with correlated innovations.

When in default, a cost χ reduces the potential tradable endowment to the realized tradable endowment. Letting D_t denote default, the realized endowment is

$$(1 - D_t\chi)Y_{T,t}. \tag{12}$$

In our model, default is *not* a choice variable but rather the result of creditors reject-

[7] An important source of income in the countries studied in the related literature are commodity exports. The stochastic nature of the tradable endowment aims to capture fluctuations in commodity prices.

ing an offer tendered by the sovereign. Unlike in most of the literature, we will not need a flexibly parameterized default cost to reproduce the data's behavior. Second, because the default cost falls on tradables only, a default will itself generate a real exchange rate depreciation, consistent with the data.

3.2 The consumer problem

In describing the consumer problem, we focus on the no-default case where the tradable endowment is $Y_{T,t}$. The default case is the same but with $Y_{T,t}$ everywhere replaced by $(1-\chi_t)Y_{T,t}$. Preferences over consumption are given by a time separable utility $\mathbb{E}_t \sum_\tau \beta^{\tau-t} u(C_t)$ for C_t a CES aggregator of tradable and nontradable consumption:

$$C_t = \left(\alpha_T C_{T,t}^{\frac{\rho-1}{\rho}} + \alpha_N C_{N,t}^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}. \tag{13}$$

Here, $\rho > 0$ is the elasticity of substitution with the limit case of $\rho = 1$ given by Cobb-Douglas and smaller ρ making tradables and nontradables more complementary. In order to detrend the model, we need CRRA preferences over C_t with $u(C_t) = (1-\beta)\frac{C_t^{1-\sigma}}{1-\sigma}$.

We choose nontradables as the numeraire so $p_{N,t} \equiv 1$, and let $p_{T,t}$ denote the relative price of tradables at time t. Consumers take prices and lump-sum taxes T_t (paid in tradables) as given, and can neither borrow nor save, resulting in the budget constraint

$$\sum_{i\in\{T,N\}} p_{i,t} C_{i,t} = \sum_{i\in\{T,N\}} p_{i,t} Y_{i,t} - p_{T,t} T_t. \tag{14}$$

The consumer maximization problem (of choosing $\{C_{i,t}\}$ subject to $\{Y_{i,t}\}$) is static, and is characterized by the budget constraint and the first order condition (FOC)

$$p_{T,t} = \frac{\alpha_T}{\alpha_N}\left(\frac{C_{N,t}}{C_{T,t}}\right)^{1/\rho}. \tag{15}$$

The government will not consume any goods, only transferring resources to consumers. Therefore, market clearing for nontradables further requires that $C_{N,t} = Y_{N,t}$.

Define p_t as the price index

$$p_t = \left(\sum_{i \in \{T,N\}} \alpha_i^{\rho} p_{i,t}^{1-\rho} \right)^{\frac{1}{1-\rho}}. \tag{16}$$

Define Y_t so that $p_t Y_t = \sum_{i \in \{T,N\}} p_{i,t} Y_{i,t}$. One can verify the allocations

$$C_{i,t} = \alpha_i^{\rho} \left(\frac{p_{i,t}}{p_t} \right)^{-\rho} (Y_t - \frac{p_{T,t}}{p_t} T_t), \quad i \in \{T,N\} \tag{17}$$

satisfy the FOCs and the budget constraint. With these good-specific allocations, aggregate consumption is

$$C_t = \left(\sum_i \alpha_i C_{i,t}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} = Y_t - \frac{p_{T,t}}{p_t} T_t. \tag{18}$$

3.3 Real exchange rate determination

To determine the RER, consider a nominal exchange rate E giving the amount of foreign per unit of domestic. Assume the law of one price holds for tradables, $1/p^*_{T,t} = E/p_{T,t}$ or $E\frac{p^*_{T,t}}{p_{T,t}} = 1$ where the asterisk denotes foreign. The RER is the price of the domestic consumption good in terms of the foreign, $RER_t := \frac{E/p_t}{1/p^*_t}$. Following Uribe and Schmitt-Grohé (2017) with $p^*_{T,t}/p^*_t$ exogenous and normalized to 1,

$$RER_t := \frac{E/p_t}{1/p^*_t} \frac{p_{T,t}}{p_{T,t}} = \frac{E/p_t}{1/p^*_t} \frac{p_{T,t}}{E p^*_{T,t}} = \frac{p_{T,t}/p_t}{p^*_{T,t}/p^*_t} = \frac{p_{T,t}}{p_t}. \tag{19}$$

This says the RER is the price of tradables relative to the domestic basket. With these conventions, a depreciation is an RER increase.

3.4 Static equilibrium in the domestic model block

Since the consumer problem is static, we can characterize the equilibrium conditions for their optimality and market clearing conditional on a tax T_t and the shock $Y_{T,t}$. In repayment, the conditions are given by market clearing,

$$C_{N,t} = Y_{N,t},\ C_{T,t} = Y_{T,t} - T_t, \tag{20}$$

optimality,

$$RER_t = \frac{1}{p_t}\frac{\alpha_T}{\alpha_N}\left(\frac{Y_{N,t}}{Y_{T,t} - T_t}\right)^{1/\rho}, \tag{21}$$

and aggregation,

$$1 = \alpha_T^{\rho} RER_t^{1-\rho} + \alpha_N^{\rho} p_t^{\rho-1} \tag{22}$$

$$Y_t = RER_t Y_{T,t} + p_t^{-1} Y_{N,t} \tag{23}$$

$$C_t = \left(\alpha_T C_{T,t}^{\frac{\rho-1}{\rho}} + \alpha_N C_{N,t}^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}. \tag{24}$$

Some algebra gives two key expressions for use in the sovereign's problem:[8]

$$RER_t = \alpha_T\left(\alpha_T + \alpha_N\left(\frac{Y_{N,t}}{Y_{T,t} - T_t}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{1}{\rho-1}} \tag{25}$$

$$C_t = (Y_{T,t} - T_t)\alpha_T^{-\rho} RER_t^{\rho} \tag{26}$$

The conditions in default are the same but everywhere replacing $Y_{T,t}$ with $(1-\chi)Y_{T,t}$.

[8] We establish this in proposition 3 in the appendix.

3.5 Overview and timing of negotiation

We model the interaction between the sovereign and a large creditor (or multiple coordinating creditors) as an extensive form game with a Markov perfect equilibrium. The timing of the model, visualized in figure 2, is as follows:

1. The sovereign comes into the period with debt B_t.
2. Innovations are realized, resulting in the exogenous state variables z_t, g_t, e_t.
3. If in repayment, the sovereign decides whether to negotiate with creditors ($N_t = 1$) or not.
4. If the sovereign chooses to negotiate, a negotiation phase takes place:
   (a) The sovereign *proposes* a haircut $H_t \in [0,1]$.
   (b) Shocks affecting the accept/reject decision of creditors are realized.
   (c) Creditors decide to accept the offer or not.
   (d) If the offer is accepted, the sovereign is not in default; if the offer is rejected, the sovereign is in default and suffers an output cost.
5. The sovereign chooses debt issuance (if not in default), taxes or transfers, and agents consume. If in default, debt in arrears grows at rate $R^D \geq 1$.

In our model, a default occurs when a payment is missed, which occurs when the haircut offer H_t is not accepted.[9]

[9] The model allows a sovereign in good standing to offer a haircut $H_t > 0$ and have it be accepted, thereby restructuring the debt without a default. This is what Asonuma and Trebesch (2016) refer to as a *preemptive restructuring*. Consistent with their findings, preemptive restructurings in the model are associated with very mild effects on GDP. However, these restructurings are very rare in the model (creditor' acceptance decisions are noisy, so offering a haircut when not in default is risky).

3.6 The government problem

The sovereign levies taxes (or rebates if negative) T_t in tradable goods. The government internalizes the effect of its policies on prices and consumption allocations. Consequently, it knows that changing taxes T_t or a defaulting will change static equilibrium allocations as summarized in (25) and (26). To parsimoniously capture these equilibrium effects, we define

$$\psi(m) = \left(\alpha_T + \alpha_N \left(m^{-1}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{27}$$

so that $\psi((Y_{T,t} - T_t)/Y_{N,t})$ gives $(RER_t/\alpha_T)^\rho$. Note ψ is a monotone transformation of the RER. Since ψ is a decreasing function, a negative shock to tradables (else equal) makes them relatively more expensive ($p_{T,t}/p_t$ increases), driving up the real exchange rate and causing a depreciation. From (26),

$$C_t = (Y_{T,t} - T_t)\psi((Y_{T,t} - T_t)/Y_{N,t}). \tag{28}$$

At any point in time the sovereign has a stock of tradable-good-denominated debt B_t. The debt is long-term maturing at a geometric rate λ with a coupon κ on unmatured debt. We define $\tilde{\lambda} = \lambda + (1-\lambda)\kappa$ as the debt-service due per unit of debt. When not in default, the government can issue debt. Any net debt issuance $B_{t+1} - (1-\lambda)B_t$ is valued at $Q(B_{t+1}, x_t, \Gamma_{T,t})$ per unit. Consequently, the government budget constraint is

$$T_t + Q(B_{t+1}, x_t, \Gamma_{T,t})(B_{t+1} - (1-\lambda)B_t) = \tilde{\lambda}B_t. \tag{29}$$

When in default, debt grows at a rate R^D, capturing the accumulation of missed coupons and interest on the defaulted debt. Rather than separately model the stock of debt and debt in arrears, as done in Arellano et al. (2023), we combine these into a single state variable and approximate the cost using R^D, reducing the number of state variables.

In models of bargaining, restructuring, and long-term debt, a problem arises that was first pointed out by Hatchondo, Martinez, and Sosa-Padilla (2014). By issuing huge amounts of debt at time t just prior to default, a restructuring at $t+1$ will have the time t creditors own most of the debt. Thus, they will effectively be senior in negotiations and appropriate to themselves the vast majority of the restructured debt value. In fact, Hatchondo et al. show that this force can be so strong that the sovereign would issue an infinite amount of debt at an infinitesimal price, fully diluting existing debt holders. This behavior is pathological in that, in the data, there are many repeat buyers of debt, but in the model it as if each buyer only buys one time. Thus the model exaggerates the distinction between new and existing debt holders, making this strategy overly appealing. To obviate this, we impose a psychic cost

$$\Xi_t = \Gamma_{T,t}^{1-\sigma} \bar{\xi} \max\{0, \underline{q} - Q(B_{t+1}, x_t, \Gamma_{T,t})\}^2 \tag{30}$$

that penalizes for excessive dilution. Having the cost be continuous in Q aids convergence, and scaling by $\Gamma_{T,t}^{1-\sigma}$ allows detrending.

The recursive formulation of the sovereign's problem (keeping the time subscripts to aid in interpretation) conditional on repayment is

$$V^R(B_t, x_t, \Gamma_{T,t}) = \max_{B_{t+1}} u(C_t) - \Xi_t + \beta \mathbb{E}_{x_{t+1}|x_t} \left[\max_{N_{t+1} \in \{0,1\}} \left\{ \begin{array}{r} N_{t+1}\, V(B_{t+1}, x_{t+1}, g_{t+1}\Gamma_{T,t}) \\ +(1-N_{t+1})\, V^R(B_{t+1}, x_{t+1}, g_{t+1}\Gamma_{T,t}) \end{array} \right\} \right] \tag{31}$$

subject to (28), (29) and (30). Next period's negotiation decision N_{t+1} appears in the continuation utility and allows the sovereign to either continue in repayment next period or enter into negotiation. And V is the value associated with negotiation.

Hence, while in repayment, the sovereign can avoid the risk of failed negotiations and default by staying current on payments. Let the optimal savings policy and negotiation policies be denoted $\mathcal{B}(B,x,\Gamma)$ and $N(B,x,\Gamma_T)$, respectively.

The value conditional on default is

$$V^D(B_t,x_t,\Gamma_{T,t}) = u(C_t) + \beta \mathbb{E}_{x_{t+1}|x_t}\left[V(R^D B_t, x_{t+1}, g_{t+1}\Gamma_{T,t})\right] \tag{32}$$

$$\text{s.t. } C_t = (1-\chi)Y_{T,t}\psi(((1-\chi)Y_{T,t})/Y_{N,t}). \tag{33}$$

This embeds the endowment loss $\chi Y_{T,t}$ and the associated RER depreciation associated with default in $\psi(((1-\chi)Y_{T,t})/Y_{N,t})$. Note that in default, debt continues to grow at rate R^D, which reflects the growth of liabilities from continued missed principal and coupon payments as well as interest on debt in arrears. The continuation utility reflects that, when in default, negotiations will always take place next period.

The value of negotiation is

$$V(B_t,x_t,\Gamma_{T,t}) = \max_{\hat{H}_t\in[0,1]}\begin{bmatrix} A(\hat{H}_t;B_t,x_t,\Gamma_{T,t})V^R((1-\hat{H}_t)B_t,x_t,\Gamma_{T,t}) \\ +(1-A(\hat{H}_t;B_t,x_t,\Gamma_{T,t}))V^D(B_t,x_t,\Gamma_{T,t}) \end{bmatrix}, \tag{34}$$

where the sovereign internalizes the haircut offer's role in the probability of acceptance, A, an equilibrium object. Let the optimal policy be denoted $H(B,x,\Gamma_T)$.

Long-term debt models are notoriously difficult to solve due to convergence problems (Chatterjee and Eyigungor, 2012). To obtain convergence, we modify (31) and (34) to have an i.i.d. Type 1 Extreme Value shock assigned to each debt, haircut, and negotiation choice, and for parsimony use the same scale parameter for each shock.

3.7 Creditors' problem

Creditors haircut acceptance decision is influenced by two shocks. With a probability $1-\bar{\alpha}$, creditors reject any offer. This controls both the effective duration between offers and inefficiency in the bargaining process, but in the stylized model it does not change haircut offers. With a complementary probability, creditors accept or reject based on fundamental values of accepting, Q_t^A, or rejecting Q_t^D, an offer and idiosyncratic taste shocks $\epsilon_t^A, \epsilon_t^D$. The shocks ϵ_t^A and ϵ_t^D are i.i.d. Type 1 extreme value. These idiosyncratic valuation shocks control the elasticity of the probability of acceptance to the haircut, which plays a crucial role in bargaining, as shown in the stylized model. With these shocks, the ex-ante probability of accepting an offer $\hat{H}_t$ is

$$A(\hat{H}_t; B_t, x_t, \Gamma_{T,t}) = \bar{\alpha} \frac{1}{1 + e^{-(Q^A(\hat{H}_t, B_t, x_t, \Gamma_{T,t}) - Q^D(B_t, x_t, \Gamma_{T,t}))/\sigma_\alpha}}. \tag{35}$$

An accepted offer's fundamental value Q^A is given by

$$Q^A(\hat{H}_t, B_t, x_t, \Gamma_{T,t}) = (1-\hat{H}_t)\left(\tilde{\lambda} + (1-\lambda) Q(\underbrace{\mathcal{B}((1-\hat{H}_t)B_t, x_t, \Gamma_{T,t})}_{B_{t+1} \text{ if } \hat{H}_t \text{ accepted}}, x_t)\right), \tag{36}$$

which takes into account the haircut size and that the debt must be serviced at least once. An interesting feature of the model is that creditors take into account the effects of debt *concentration*: Q^A moves less than proportionately with $1-\hat{H}_t$ because larger haircuts reduce B_{t+1} (since $\mathcal{B}((1-\hat{H}_t)B_t, x_t)$ is typically monotone in its first argument) and thereby increase the market price $Q(B_{t+1}, x_t, \Gamma_{T,t})$. This debt concentration effect is the reverse of debt dilution.

The one-period-ahead debt pricing when in repayment is

$$Q(B_{t+1}, x_t, \Gamma_{T,t}) = \frac{1}{1+r^*}\mathbb{E}_{x_{t+1}|x_t}\begin{bmatrix} (1-N_{t+1})Q^A(0, B_{t+1}, x_{t+1}, \Gamma_{T,t+1}) \\ +N_{t+1}A_{t+1}Q^A(H_{t+1}, B_{t+1}, x_{t+1}, \Gamma_{T,t+1}) \\ +N_{t+1}(1-A_{t+1})Q^D(B_{t+1}, x_{t+1}, \Gamma_{T,t+1}) \end{bmatrix} \tag{37}$$

where

$$\begin{gathered} H_{t+1} = H(B_{t+1}, x_{t+1}, \Gamma_{T,t+1}),\ N_{t+1} = N(B_{t+1}, x_{t+1}, \Gamma_{T,t+1}), \\ \text{and } A_{t+1} = A(H_{t+1}; B_{t+1}, x_{t+1}, \Gamma_{T,t+1}). \end{gathered} \tag{38}$$

Creditors discount profits using the risk-free world interest rate r^*. Note that in pricing the rejected offer, the Markov policies H and A are used, consistent with the equilibrium concept.[10]

A rejected offer's value Q^D is given by

$$Q^D(B_t, x_t, \Gamma_{T,t}) = R^D\tilde{Q}(\underbrace{R^D B_t}_{B_{t+1}}, x_t, \Gamma_{T,t}). \tag{39}$$

where $\tilde{Q}$ is the continuation value of a unit of defaulted debt. The reason R^D multiplies Q^D is that Q^D is the price *per unit* of debt. $\tilde{Q}$ is analogous to Q but always features negotiation:

$$\tilde{Q}(B_{t+1}, x_t, \Gamma_{T,t}) = \frac{1}{1+r^*}\mathbb{E}_{x_{t+1}|x_t}\begin{bmatrix} A_{t+1}Q^A(H_{t+1}, B_{t+1}, x_{t+1}, \Gamma_{T,t+1}) \\ +(1-A_{t+1})Q^D(B_{t+1}, x_{t+1}, \Gamma_{T,t+1}) \end{bmatrix} \tag{40}$$

where A_{t+1} and H_{t+1} are as in (38).

One convenient feature of our setup is that the haircut H exactly corresponds to the Sturzenegger and Zettelmeyer (2008) (SZ) haircut measure.[11] The SZ haircut

[10] A subtle point is that the ϵ_t^A and ϵ_t^D shocks enter only via the acceptance probabilities. This can be achieved by assuming that creditors enjoy a valuation shock $\epsilon_t^A - \epsilon_t^D$ if they accept and nothing otherwise.

[11] There are two other primary measures of haircuts in the data. The first is a nominal haircut

H_{sz} is one minus the ratio of the net present value (NPV) of the new debt relative to the NPV of the old debt inclusive of interest on arrears, with both discounted using the IRR of the new debt. At the time of the debt exchange, the old debt with interest on arrears is summarized by B_t, while the new debt is $(1-\hat{H}_t)B_t$. Since these debt amounts prescribe exactly the same profile of payments ($\tilde{\lambda}$ next period, $(1-\lambda)\tilde{\lambda}$ in the second, and so on), the SZ haircut is

$$H_{sz} = 1 - \frac{\sum_{j=1}^{\infty}(1+r)^{-j}\tilde{\lambda}(1-\lambda)^{j-1}(1-\hat{H}_t)B_t}{\sum_{j=1}^{\infty}(1+r)^{-j}\tilde{\lambda}(1-\lambda)^{j-1}B_t} = \hat{H}_t \tag{41}$$

(where the r, though evidently irrelevant here, corresponds to the IRR from the new debt).

3.8 Equilibrium

An equilibrium is policies and values for the sovereign $\mathcal{B}, N, H, V, V^D, V^R$ and acceptance probabilities and values for creditors $A, Q^A, Q^D, Q, \tilde{Q}$ solving their respective problems that the other agents' policies and values as given. The detrended version of the model is shown in appendix C.

4 Estimation and calibration

This section describes how we determine the parameters of the model. We set some of the parameters apriori, others we estimate using a subset of the model's equations, and the rest we calibrate by matching moments. We focus on Argentina and use a quarterly time period.

measure which compares old and new debts at their face values. The second is a market haircut measure that compares the face value of the old debt with the market value of the new debt. Tomz and Wright (2013) say in their data all the haircuts deliver surprisingly similar results.

4.1 Exogenously determined parameters

We set the constant relative risk aversion (CRRA) parameter to 2 in line with most of the literature. We set the coupon payment $\kappa = .03$, the maturity rate $\lambda = .05$, and the real risk-free rate $r^* = .01$ following Chatterjee and Eyigungor (2012). The dilution penalty parameters from (30) have $\underline{q}$ corresponding to a 40% annual spread and $\bar{\xi} = 0.1$, which implies a substantial penalty.[12] We set the debt, haircut and negotiation taste shock scale parameter to the smallest value we found that still admitted convergence, 5×10^{-5}.[13] We fix the persistence of the transitory shock $\rho_z = .95$, which allows z to capture business cycle frequency movement. We set $\rho_e = .99$ so that the half-life of a $\epsilon_{g,t}$ shock is almost 20 years.

A crucial parameter is the rate on arrears. We set this to $R^D = 1.021$ to match the rate implied by κ, λ, and r^* when default lasts for 14 quarters. To establish this, let the debt in arrears be denoted A_t and good standing debt be G_t. Then debt in arrears evolves according to $A_t = A_{t-1}(1 + r^*) + (\lambda + \kappa(1 - \lambda))G_{t-1}$. The stock of debt in good standing dwindles at rate $1 - \lambda$: $G_t = (1 - \lambda)G_{t-1}$. So the total debt stock (arrears plus non-defaulted debt) grows according to

$$\underbrace{A_t + G_t}_{=B_t} = \underbrace{(A_{t-1} + G_{t-1})}_{=B_{t-1}}\left(1 + r^* \frac{A_{t-1}}{A_{t-1} + G_{t-1}} + \kappa(1 - \lambda)\frac{G_{t-1}}{A_{t-1} + G_{t-1}}\right). \tag{42}$$

The rate $R^D = 1.021$ matches the average interest rate after 14 quarters. Since growth rates begin at $\kappa(1 - \lambda) > r^*$, this approximation understates arrears growth early on and overstates it later.

[12] Our flow utility is $U(c) = (1-\beta)c^{1-\sigma}/(1-\sigma)$ with $\sigma = 2$, and average consumption is approximately one. So the value function is approximately one, $V \approx U(1)/(1 - \beta) = 1$. Decreasing the price from $\underline{q}$ to $\underline{q} - .01$ induces increases Ξ_t from 0 to 0.1, or about 10% of lifetime consumption.

[13] From the preceding footnote, each choice amounts to an influence of about 0.005% of lifetime consumption.

4.2 Estimation

We estimate the shock parameters $(\mu, \rho_g, \sigma_z, \sigma_g)$ and the CES parameters $(\theta_T, \alpha_N, \rho)$—where $\theta_T = \alpha_T/(\alpha_N + \alpha_T)$ is the share of tradables absent shocks and default—using time series for the real exchange rate, GDP in constant national prices, and default indicators.[14] These estimates are all conditional on a value of χ (and since we calibrate χ, we must reestimate the model many times). We incorporate results from the literature using priors to improve identification and efficiency. First, we center the estimate for the elasticity of substitution 0.5 but with substantial support in [.3,.7] in keeping with the survey in Akinci (2011). We center the share of tradables θ_T around the 40% used in Bianchi and Sosa-Padilla (2020). Last, we use a diffuse prior for σ_z that has a mode at 0.05, twice the size of in Arellano (2008) to account for the shock only hitting tradables.

The estimation equations are detrended and log-linearized versions of the endowment process (5-11), the static equilibrium equations (20-26) evaluated with $T_t = 0$, and a final equation that specifies a default policy, $\delta_t = .95\delta_{t-1} + .5\epsilon_{\delta,t}$ where $\delta_t = e^{D_t}$. We log-linearize with respect to RER_t, $y_{T,t} = Y_{T,t}/\Gamma_{T,t}$, $z_t, e_t, g_t, \Gamma_{T,t}$, and δ_t. So default has real effects, we log-linearize about $D_t = 1$ ($\delta_t = e^1$). We assume the measurement error of the log RER and GDP are 5% and 1%, respectively. We also allow for a small probability that default is mismeasured by using a measurement error of 25% for our default indicator (hence, for $D_t = 1$ to result in a no-default measurement requires a highly unlikely 4 standard deviation miss).[15]

Table 1 reports estimates of the posterior modes and the associated standard

[14] One of α_T or α_N is a normalization, which is identified from the average level of the RER in the data.

[15] We have annual default indicators throughout our quarterly sample and tried with some success to get the exact dates of default. However, the literature has measured default episodes in different ways, leading to very different results with respect to how long default episodes last and at what frequency it occurs. For example, for the 1980s in Mexico, criteria from Tomz and Wright (2013), Borenstein and Panizza (2009), Cruces and Trebesch (2013), and Arteta and Hale (2008) result in one, four, five and 23 measured defaults, respectively (Tomz and Wright, 2013, p. 256).

errors alongside the prior distributions with some of its summary statistics.

Table 1: Estimates from the linearized model

Parameter	Estimate	SE	Prior	Mode	Mean	Stdev.
ρ	0.844	0.032	Beta(20,20)	0.500	0.500	0.078
θ_T	0.211	0.004	Beta(30,44.5)	0.400	0.403	0.056
σ_z	0.023	0.019	Beta(2,20)	0.050	0.091	0.060
σ_g	0.048	0.011				
ρ_g	0.701	0.105				
μ	1.01	0.000				
α_T	0.151	0.013				

Note: this table reports posterior estimates of structural parameters from the linearized version of the model alongside their standard errors (SE) and prior distributions; estimated parameters include the elasticity of substitution between tradables and nontradables ρ, the tradable good share θ_T, the volatility of transitory shocks σ_z and growth shocks σ_g, growth's persistence ρ_g, the secular growth rate μ, and the CES weight on tradables α_T; when a prior is not listed for a given parameter, a flat prior is used.

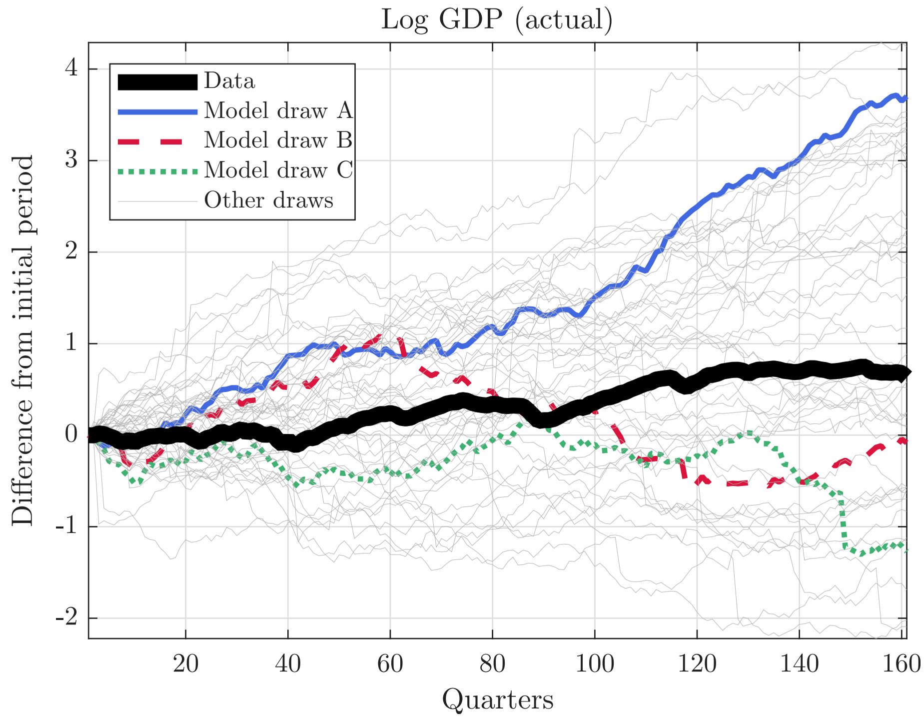
Figure 3 plots the data's path for log GDP alongside some simulated paths from the model. The data's path exhibits 10 year periods of stagnation punctuated by fast growth, along with periods of sharp declines. The estimated GDP process reproduces these features, having multi-year periods of sharp growth followed by decade long stagnations. The very precise estimate of the positive trend does mean these paths all grow secularly over time, but even after 160 quarters, cumulative GDP growth can be negative with a non-trivial probability.

The linearized model gives a few key elasticities. One is the elasticity of the RER to endowment shocks, -0.91. Another is the elasticity of the RER with respect to default, 0.66. The last is the elasticity of GDP to default, -0.17.

4.3 Calibration

The remaining four parameters are default costs χ, the discount factor β, the probability creditors reject any offer $\bar{\alpha}$, and the idiosyncratic valuation shocks σ_α. We

Figure 3: Simulated GDP paths



Note: this figure plots simulated GDP trajectories from the estimated model alongside actual Argentine GDP.

identify these by matching five moments: mean spreads and debt levels conditional on non-default periods, the mean and standard deviation of default duration, and time in default.[16] We do not target haircut levels directly (though they influence spreads) since we only have a handful of observations for Argentina.

The targeted and untargeted moments are displayed in Table 2. The model reproduces the targeted moments well and also reproduces a host of untargeted ones. Some of the key untargeted moments the model reproduces are the excess volatility of consumption, countercyclical net exports, countercyclical real exchange rates, and a large dispersion of haircuts.

In the appendix, we feed in the estimated path of shocks into the model and show the model tracks the data closely. Just prior to restructuring its 1982 default, the model predicts offered haircuts vary from an upper bound of just over 60% to approximately the data's haircut of 32.5%. The model's predicted haircut offers following the 2001 default vary from 20% to 50%, sharply below the data's haircut of 76.8%. However, we show the model can be brought even closer to the data by incorporating a real exchange rate peg, replicating the RER path in the 1990s when Argentina operated a fixed exchange rate. Being on the peg from 1991Q2 to 2001Q4 brings down the offered haircuts associated with the 1982 default (which was restructured in the early 1990s) to between 25% and 57%, comfortably encompassing the data's 32.5%, and increases the 2001-default haircut to 74%, just shy of the actual 76.8% haircut.

4.4 Shock responses

For the analysis of hard and soft defaults, it will be useful to understand the dynamics around transitory and permanent shocks. Figure 4 plots sample paths around

[16]The last moment aids in identification as the other four moments are conditional on either no default or default.

Table 2: Targeted and untargeted moments

Targeted moments	Model	Target	Parameter	Value
Debt to GDP | no default	0.97	0.96	χ	0.42
Spreads | no default	0.08	0.08	β	0.95
Log default duration	2.8	2.9	$\bar{\alpha}$	0.07
Log default duration s.d.	1.1	0.91		
Fraction of time in default	0.33	0.40	σ_α	0.31

Untargeted moments	Model	Target
Debt to GDP s.d. | no default	0.42	0.34
Debt to GDP	2.7	1.2
Debt to GDP s.d.	3.9	0.75
Debt service to GDP | no default	0.08	0.06
Debt service to GDP s.d. | no default	0.03	-
Spreads s.d. | no default	0.07	0.04
Haircut size	0.73	0.38
Haircut size s.d.	0.23	0.21
Corr. of haircut and duration	0.28	0.31
RER	1.7	1.8
RER s.d.	0.84	0.68
RER | no default	1.4	-
RER s.d. | no default	0.57	-
Log GDP s.d.*	0.24	0.09
S.d. log consumption / s.d. log GDP*	0.99	1.1
Corr. of spreads and log GDP*	-0.15	-0.34
Corr. of NX/GDP and log GDP*	-0.12	-0.46
Corr. of RER and log GDP*	-0.46	-0.36
Corr. of spreads and log GDP* | no default	-0.12	-0.47
Corr. of NX/GDP and log GDP* | no default	-0.20	0.10
Corr. of RER and log GDP* | no default	-0.35	0.15
Corr. of spreads and log GDP* | spreads<.2	-0.02	-0.51
Corr. of NX/GDP and log GDP* | spreads<.2	-0.18	-0.27
Corr. of RER and log GDP* | spreads<.2	-0.33	-0.25

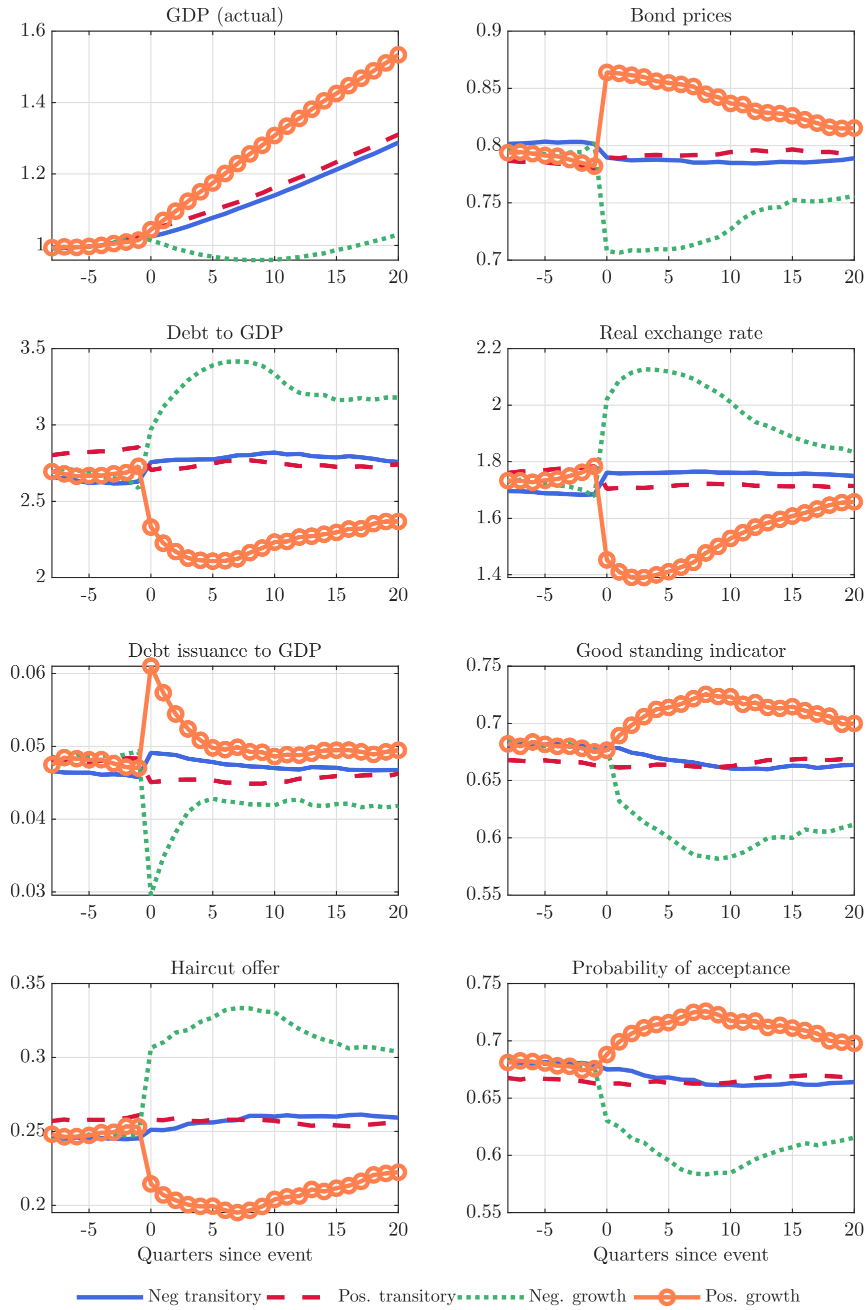
Note: this table reports targeted and untargeted moments for the model and data alongside the calibrated parameters; the targets for the haircut size and standard deviation are cross-country values computed from the Trebesch and Zabel (2017) dataset so naturally understate haircut mean and standard deviations for our economy calibrated to Argentina; * means the variable has been detrended using the Hamilton (2018) filter; debt to GDP is the debt stock divided by quarterly GDP; | means conditional.

growth and transitory shocks; specifically, it plots $\mathbb{E}[x_{t+h}|\delta_t]$ where x_{t+h} is a series of interest and δ_t denotes either a positive or negative shock for $\varepsilon_{g,t}$ or $\varepsilon_{z,t}$ that is between 1.5 to 2.5 standard deviations from zero. These averages include observations whether the sovereign is in default or not.

A positive growth shock (orange circles) leads to a pesistent expansion. The influx of tradable goods induces a large and prolonged RER appreciation, which reduces debt to GDP through revaluation. Bond prices improve as higher income makes debt less costly to service. Debt issuance increases substantially because the growth shock is persistent, resulting in expectations of higher future income the sovereign wishes to borrow against. If in default, the sovereign tends to restructure, which is why the good standing indicator increases. Haircut offers are smaller and the probability of acceptance is higher. A negative growth shock (green dots) is predictably like a positive shock but with the response inverted. It takes years for the economy to recover its pre-shock GDP level, partly because the shock increases default persistence and realized default costs.

Transitory shocks in comparison play a small role, acting mostly like small growth shocks. After a positive shock, debt to GDP and the RER both notably fall (the latter an appreciation) and bond prices improve. The default-oriented variables, like the good standing indicator, haircut offers, and the probability of acceptance are mostly unchanged by a positive shock, although worsen visibly after a negative shock. Unlike with growth, positive transitory shocks mean income is expected to be relatively small in the future, which induces less debt issuance.

Figure 4: Response to z and g innovations conditional on good standing



Note: this figure plots dynamic paths around shocks $\mathbb{E}[x_{t+h}|\delta_t]$ where x_{t+h} is the series indicated by the title and δ_t denotes either a positive or negative shock for $\varepsilon_{g,t}$ (the growth shock) or $\varepsilon_{z,t}$ (the transitory shock) that is between 1.5 to 2.5 standard deviations from zero; variation prior to the shock reflects imperfect shock discretization; averages include observations whether the sovereign is in default or not.

5 Equilibrium hard and soft defaults

5.1 Hard and soft defaults

Figure 5 plots the paths of several macroeconomic variables following hard (red dashed lines) and soft defaults (green dots), as distinguished by being above or below median haircut defaults. For completeness, we plot alongside the paths for average defaults (blue lines). Like in Trebesch and Zabel, GDP has not recovered in hard defaults even after 5 years. In contrast, soft defaults recover in around 2 years. Hard defaults are also associated with larger RER depreciations and larger times until restructuring. Like Meyer et al. (2022), there is almost no difference in spreads leading up to a default, with large return differential appearing ex post.

To understand the model's success in generating these patterns, it is useful to reconsider lessons from the stylized model. Recall that creditors only accept large haircut offers if they expect large haircut offers to be forthcoming in the future. Note that in the "haircut offer" panel of the figure where offers are highly persistent over time. Thus in hard defaults, the sovereign can credibly offer large haircut offers in the future.

But why does the sovereign persistently offer large haircuts (without being able to commit to them)? The reason is growth shocks. While both hard and soft defaults are triggered by negative growth shocks, in soft defaults those shocks go away and even turn positive. In hard defaults, the negative growth shock persists. This generates a large and persistent gap in the RER with hard defaults having much more depreciation. Although debt to GDP is similar when the default occurs, this depreciation also causes a revaluation that creates a gap. With larger debt to GDP, it is simply much more costly to offer a small haircut. This is why in the stylized model, haircuts were increasing in debt to GDP. As we will show later using IRFs, transitory shocks play a relatively minor roll.

In the stylized model, one way to write (3) is

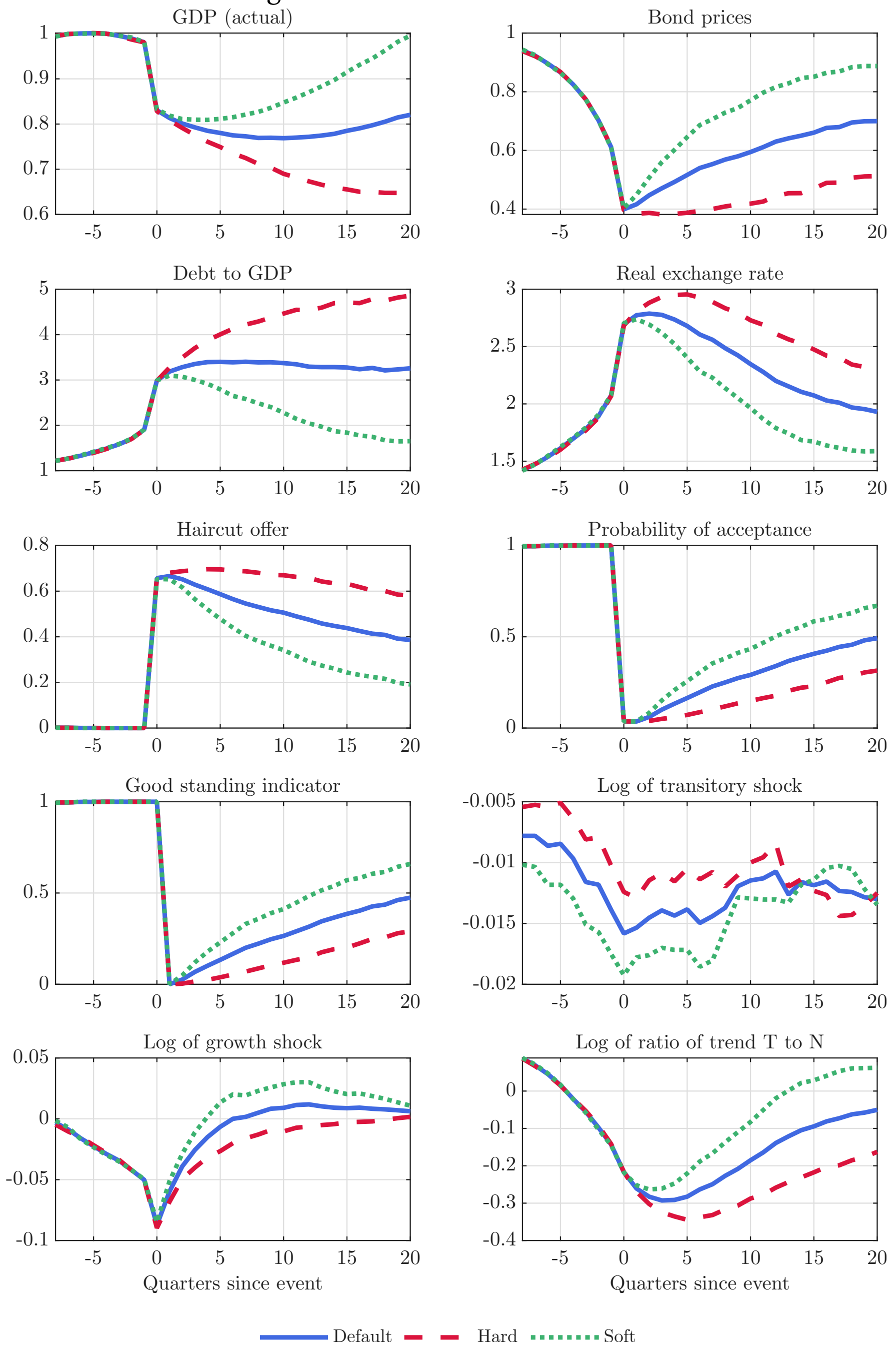
$$\text{Offered haircut} = 1 - \frac{\text{Acceptance rate elasticity}}{\text{Acceptance rate elasticity} + 1} \cdot \frac{\text{Default cost} \cdot \text{NPV(GDP)}}{\text{Debt}}. \quad (43)$$

This formula reflects (1) the total surplus bargained over is the net present value of default costs, (2) the creditor surplus is the value of restructured debt, and (3) the trade off the sovereign faces in offering haircuts. While not obvious, it also reflects that creditors are unwilling to accept high haircut offers if low ones are coming in the future. This formula helps interpret why hard defaults are associated with worse trajectories.

The distinction between hard and soft defaults is driven by different growth shocks. A bad growth shock, unlike an adverse transitory shock, lowers the net present value of GDP by a large amount, unlike a transitory shock. This is reflected in figure 4 where haircut offers were close to unchanged following transitory shocks but moved substantially after growth shocks. Because the NPV of GDP is lower, the total cost of default is lower. This shrinks the size of the total surplus in the bargaining problem, resulting in less surplus for both the sovereign and creditors. From the discussion around (4), recall the total surplus is the NPV of default costs ($\chi y/(1-\beta)$), the surplus to the creditors is the value of restructured debt ($(1-h)b$), and the surplus to the sovereign is the residual. Unless the acceptance rate elasticity changes significantly, a reduction in total surplus reduces the creditor surplus which occurs through larger haircuts.

Growth shocks also drive the distinction between hard and soft defaults because of the associated trajectory of future haircut offers. On average, the path for the NPV of GDP is expected to increase over time after a negative transitory shock hits. Following an i.i.d. growth shock, the expected path for the NPV of GDP is flat. And in the benchmark, which estimates the growth shock to be mildly persistent, the expected path is actually declining. According to (43), this generates an increas-

Figure 5: Hard and soft defaults



Note: this figure plots default events $\mathbb{E}[x_{t+h}|\delta_t]$ where x_{t+h} is the series indicated by the title and δ_t conditions on either a hard default, a soft default, or a default of any type (default intensity is determined by the recovery rate at the time of restructuring); averages include observations whether the sovereign is in default or not; defaults are predictable, leading to pre-event trends.

ing sequence of haircut offers, which can be seen in figure 5 for one year following a hard default. This makes creditors more willing to accept deep haircuts following severe negative growth shocks. This would generally create a counterfactual negative correlation between haircuts and default duration, but the benchmark model predicts a positive correlation. We will show this is accomplished through $R^D > 1$.

RER devaluations following adverse shocks amplify the dynamics. A negative shock increases debt when measured in consumption good, and growth shocks make this persistent. From (43), the larger debt stock results in larger haircuts. This arises in the stylized model because the total surplus is unaffected by more debt, which naturally leaves the creditor surplus—$(1-h)b$—unaffected. But with larger debt, haircuts must rise to hold this constant. A countervailing force is that RER devaluations effectively amplify default costs, which is a second-order effect.[17] This increases total surplus, acting to reduce haircuts. Numerically, the debt revaluation effect is larger.

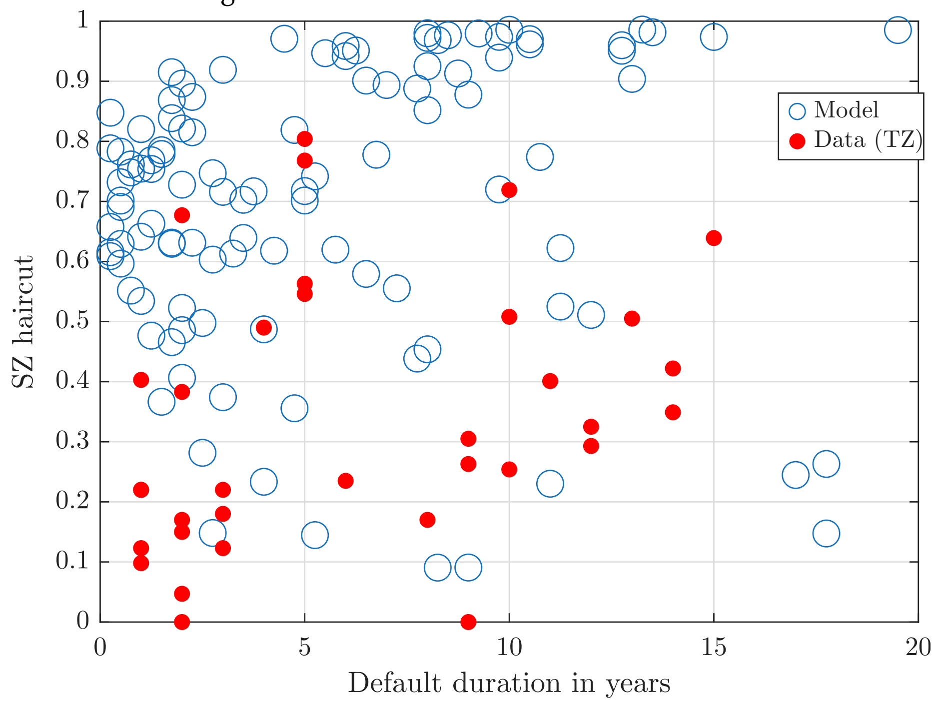
5.2 Duration and default intensity

One of the key contributions of the model is generating a strong link between haircut intensity and default duration. This was seen in the positive correlation between haircuts and duration in table 2. But it can be more clearly seen in figure 6, which plots simulation results against Trebesch and Zabel's data. Since Argentina is a serial defaulter that consistently borrows at high spreads, its haircuts are naturally larger than in the cross-section of countries from Trebesch and Zabel. However, the

[17] Devaluations indicate an increase in the relative scarcity of tradables, and default costs are denominated in tradables, so it is natural to expect this. Formally, it can be seen as follows. Consumption in default is GDP, so (33) can be written as $Y_t = (1-\chi)Y_{T,t}\psi((1-\chi)Y_{T,t}/Y_{N,t})$ where ψ is a monotone transformation of the RER. So the reduction in GDP associated with default as a share of no-default-cost-GDP is $1-(1-\chi)\psi((1-\chi)Y_{T,t}/Y_{N,t})/\psi(Y_{T,t}/Y_{N,t})$. A first order approximation of $\psi((1-\chi)Y_{T,t}/Y_{N,t})$ about $\chi = 0$ gives this effective default cost as $\chi - \psi'(Y_{T,t}/Y_{N,t})\chi(1-\chi)$ (which is greater than χ). For our estimated parameters, ψ is convex in the relevant region, so adverse shocks drive up default costs.

strong positive relationship is evident.

Figure 6: Haircut size and default duration



Note: this scatterplot compares the relationship between haircut size and default duration in the model versus the Trebesch and Zabel dataset; the data includes many countries but the model is calibrated to match Argentina, causing haircuts to be larger in the model than the data; however, in both the data and simulations, defaults with larger haircuts tend to last longer.

In the preceding section, we argued growth shocks could naturally result in the opposite pattern with faster settlement for high haircut defaults. In particular, hard defaults generate a sequence of haircut offers that first is increasing as the negative growth shock effects persist (figure 5). Thus, creditors else equal would be better accepting offers sooner rather than later. Conversely, in soft defaults, haircut offers are declining, and creditors could be better off accepting later rather than sooner.

The model overcomes this to generate a positive correlation between haircuts and duration through arrears growth $R^D > 1$. When table 3 shuts this down in the column labeled $R^D = 1$, the result goes away. Specifically, without debt in arrears growth, the model generates a negative correlation between haircuts and duration and much smaller haircuts. However, this barely matters to creditors, as spreads and debt when not in default are unchanged. Why? The total surplus in bargaining is the NPV of default costs. Growth of debt in arrears does not change this, and so the creditor surplus ($(1-h)b$ in the stylized model) is unchanged. But the increase in debt as default duration increases commensurately increases haircuts to hold the creditor surplus roughly constant. This generates a positive relationship between default duration and haircuts when $R^D > 1$. From the same logic, if instead $R^D = 1$, then debt won't be as large, but the amount going to creditors will be the same. Since creditors get the same amount ex post, they don't need to change terms ex ante. That's why spreads are mostly unaffected by varying R^D, and debt along with them.

Table 3: Summary statistics with and without growth of debt in arrears

Statistic	Bench.	$R^D = 1$
Debt to GDP | no default	0.973	1.00
Spreads | no default	0.080	0.082
Spreads s.d. | no default	0.073	0.074
Haircut size	0.731	0.547
Haircut size s.d.	0.225	0.263
RER	1.73	1.74
RER s.d.	0.838	0.838
Corr. of haircut and duration	0.280	-0.153
Log default duration	2.85	2.77
Fraction of time in default	0.327	0.329

Note: this table contrasts the benchmark calibration with a counterfactual scenario in which debt in arrears does not grow during default ($R_D = 1$).

5.3 Causation versus selection

Trebesch and Zabel (2017) view the gap in outcomes between hard and soft defaults as a causal effect of coercive negotiation tactics used in hard defaults. At the same time, they acknowledge the possibility of reverse causality. Using our model, we can quantify how much of the gap is causally driven by aggressive negotiation—reflected in larger haircut offers, lower probabilities of acceptance, prolonged default durations, and more realized default costs—and how much is attributable to selection based on different shocks hitting the economy.

We quantify the degree of causation versus selection in the gaps between hard and soft defaults by looking at the difference between realized GDP Y_t and potential GDP $\tilde{Y}_t$ (the GDP that would prevail absent default costs). The ratio of the hard-soft gap in potential GDP and actual GDP

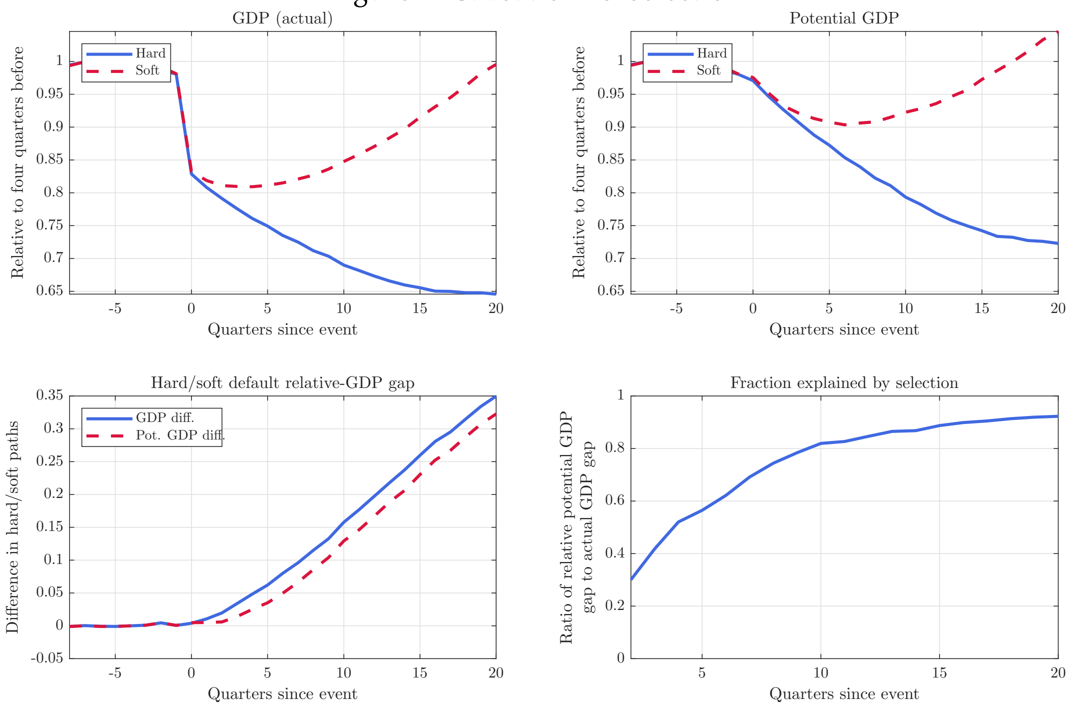
$$\theta_h = \frac{\text{Potential GDP Gap}}{\text{Actual GDP Gap}} = \frac{\mathbb{E}[\tilde{Y}^{soft}_{t+h}/\tilde{Y}^{soft}_{t-4} - \tilde{Y}^{hard}_{t+h}/\tilde{Y}^{hard}_{t-4}]}{\mathbb{E}[Y^{soft}_{t+h}/Y^{soft}_{t-4} - Y^{hard}_{t+h}/Y^{hard}_{t-4}]} \tag{44}$$

gives the fraction of the observed gap attributable to selection. For instance, if potential GDP always equaled GDP ($\{\tilde{Y}_t\} = \{Y_t\}$), then the observed difference would be entirely driven by selection based on shocks, not default costs; in that case, $\theta_h = 1$ for all h. Conversely, if potential GDP were the same in hard and soft defaults ($\{\tilde{Y}^{hard}_t\} = \{\tilde{Y}^{soft}_t\}$), then all the difference would be driven by default costs, resulting in no selection $\theta_h = 0$ for all h.[18] The top panels of figure 7 plot the paths for actual and potential GDP, the bottom left panel plots the numerator and denominator of (44), and the bottom right panel plots the degree of selection θ_h.

The estimates suggest that around 30% of the observed gap is driven by selection early on with selection increasingly responsible for the difference at longer horizons. Since shocks play such a large role in determining hard and soft default outcomes, it may be surprising that such a large share of the gap is causal. But the model captures the messiness of the data's restructuring process, which is filled

[18] At a deeper level, every model outcome is determined by the complete history of all shocks, including taste shocks that influence haircut offers and creditor accept/reject decisions. This metric captures selection based on fundamental endowment states.

Figure 7: Causation vs. selection



Note: this figure decomposes the observed gap in output between hard and soft defaults into causal effects of aggressive negotiation (causation) versus differences in underlying shocks (selection); the top panels compare the paths of actual GDP and potential GDP (a counterfactual output level that has no default costs) across hard and soft defaults; the bottom left panel shows the numerator and denominator of the selection metric θ_h in (44), while the bottom right panel plots the fraction of the gap explained by selection, θ_h; if potential GDP equals actual GDP, then all differences reflect selection ($\theta_h = 1$); if potential GDP is the same across hard and soft defaults, all differences reflect causation ($\theta_h = 0$); the estimates indicate that about 30% of the observed GDP gap is initially due to selection, but selection becomes increasingly dominant over longer horizons.

with noise that can unnecessarily prolong default and result in suboptimal outcomes for both the sovereign and creditors. As time goes on, this noise washes out, leaving selection as the primary driver. These results indicate a substantial role for both causation and selection.

6 Conclusion

This paper introduced noise bargaining, a tractable single-proposer framework that features endogenous delay and bargaining power. Acceptance rate elasticities—shaped by current offers, expectations of future offers, and idiosyncratic valuation shocks—determine how aggressively the proposer negotiates. Inelastic acceptance rates result in the proposer offering worse terms to the counterparty, which, all else equal, generates more delay.

Embedding this game in a quantitative sovereign default model accounts for the empirical regularities distinguishing hard and soft defaults. Creditors only accept large haircuts when they expect similarly poor offers in the future, a pattern that arises endogenously after negative growth shocks. Hard defaults therefore coincide with deeper and more protracted recessions and larger real depreciations. The positive correlation between delay and default intensity arises from accumulation of debt in arrears that renders debt unsustainably large in protracted defaults without a large haircut. Quantitatively, nearly four-fifths of post-default output differences reflect selection on underlying shocks rather than causal reductions in output, consistent with the absence of ex-ante yield differentials. Overall, sovereign default intensity emerges as the equilibrium outcome of a noisy bargaining game where income dynamics and arrears growth amplify small ex-ante differences into large ex-post heterogeneity.

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A Model extension: Labor supply and RER pegs

To incorporate production, suppose nontradable goods are produced with a linear technology $\Gamma_{N,t} L_t$ where L_t is labor. A competitive firm maximizes

$$\max_{L_t} p_{N,t} \Gamma_{N,t} L_t - w_t L_t. \tag{A.1}$$

So the nominal wage is $w_t = p_{N,t} \Gamma_{N,t}$. The real wage measured in nontradables is $w_t / p_{N,t} = \Gamma_{N,t}$.

We suppose flow utility is given by a CRRA function over CES of tradable and nontradable *net* of labor disutility

$$u\left(\mathcal{C}(C_{T,t}, C_{N,t} - G_t(L_t))\right), \tag{A.2}$$

where $\mathcal{C}$ is the CES aggregator from the benchmark and $G_t(L_t) = \Gamma_{N,t} g(L_t)$ is an increasing, strictly convex function. This is a variation of GHH preferences. To nest the benchmark model, we suppose that $G_t(L^*) = 0$ where $L^* = g'^{-1}(1)$ is the "natural rate" of labor supply. This means that at the undistorted labor choice, the labor disutility is zero. Without loss of generality, we can take $L^* = 1$ as a non-unitary L^* is isomorphic to $L^* = 1$ with $\Gamma_{N,t}$ scaled by a constant. Hence with the benchmark assumptions of undistorted labor, preferences reduce to $u(\mathcal{C}(C_{T,t}, C_{N,t}))$, like in the benchmark.

We introduce a possible distortion from a marginal tax on labor income $\tau_{L,t}$ that is rebated lump sum via $T_{N,t}$. With this assumption, L_t satisfies $G'(L_t) = (1 - \tau_{L,t}) \Gamma_{N_t}$ or $g'(L_t) = 1 - \tau_{L,t}$. To see this, consider the consumer budget constraint

$$p_{T,t} C_{T,t} + p_{N,t} C_{N,t} = p_{T,t} Y_{T,t} + (1 - \tau_{L,t}) w_t L_t - p_{T,t} T_t + T_{N,t}. \tag{A.3}$$

The FOCs with respect to C_t and L_t are

$$u' \cdot \mathcal{C}_2^{-\sigma} = \lambda_t p_{N,t} \tag{A.4}$$

$$G_t'(L_t) \cdot u' \cdot \mathcal{C}_2^{-\sigma} = \lambda_t (1 - \tau_{L,t}) w_t. \tag{A.5}$$

This gives $G_t'(L_t) = \frac{(1-\tau_{L,t})w_t}{p_{N,t}} = (1 - \tau_{L,t})\Gamma_{N,t}$. Additionally, market clearing of nontradables gives $C_{N,t} = Y_{N,t} = \Gamma_{N,t} L_t$.

For regularity, we assume $g'(0) = 0$. (A functional form that satisfies the assumptions is $g(L) = L^{1+\epsilon_L}/(1 + \epsilon_L) - 1$.) This implies a maximum feasible value of L, call it $\bar{L}$, with $L \in [0, \bar{L}]$ feasible and $L^* \in (0, \bar{L})$ maximizing $L - g(L)$. With these assumptions, $(C_{N,t} - G_t(L_t))/\Gamma_{N,t} = L_t - g(L_t)$ increases from $-g(0) > 0$ to L^* as $\tau_{L,t}$ varies from 1 to 0, and then falls from L^* to $\bar{L} - g(\bar{L}) = 0$ as $\tau_{L,t}$ increasingly subsidizes labor (pushing L_t to $\bar{L}$). The range of feasible values for $C_{N,t} - G_t(L_t)$ that the government can achieve varies from 0 to $\Gamma_{N,t} L^*$.

From the FOCs, the price tradables relative to nontradables (which are the numeraire) is

$$p_{T,t} = \frac{\alpha_T}{\alpha_N} \left(\frac{C_{N,t} - G_t(L_t)}{C_{T,t}} \right)^{1/\rho}. \tag{A.6}$$

The real exchange rate now depends on the ratio of $C_{T,t}$ to $C_{N,t} - G(L_t)$:

$$RER_t = \alpha_T \, \psi \left(\frac{Y_{T,t} - T_t}{Y_{N,t} - G_t(L_t)} \right)^{1/\rho} = \alpha_T \, \psi \left(\frac{(z_t - \tau_t)}{L_t - g(L_t)} \frac{\Gamma_{T,t}}{\Gamma_{N,t}} \right)^{1/\rho} = \alpha_T \, \psi \left(\frac{(z_t - \tau_t) e_t}{L_t - g(L_t)} \right)^{1/\rho}. \tag{A.7}$$

Since $L_t - g(L_t)$ has a feasible range of $[0, L^*]$, the real exchange rate range is limited. For $\rho \in (0, 1)$ like in the benchmark, $\psi : \mathbb{R}^{++} \to (0, \alpha_T^{\rho/(\rho-1)})$. For $\rho \in (1, \infty)$, $\psi : \mathbb{R}^{++} \to (\alpha_T^{\rho/(\rho-1)}, \infty)$. ψ is decreasing, and so when $L_t - g(L_t)$ approaches 0, RER_t approaches 0 if $\rho < 1$ and $\alpha_T^{\rho/(\rho-1)}$ if $\rho > 1$. Economically, this means that if $\rho < 1$, distortionary taxes applied to the nontradable sector reduce nontradable consumption and make tradables relatively more abundant, causing the real exchange rate to appreciate (fall toward zero) potentially infinitely. If $\rho > 1$, the intuition is similar, but the real exchange rate appreciation is bounded. Moreover, since ψ is decreasing, the real exchange rate is maximized for a given τ_t, z_t, e_t when $L_t - G(L_t)$ is maximized, which occurs at $L_t = L^*$. Hence, the real exchange rate for $\rho < 1$ must lie in $(0, RER_t^*]$ where RER_t^* is the "natural" real exchange rate, i.e., the distortionless RER, and RER_t^* is attained at zero taxes (resulting in $L_t = L^*$). For $\rho > 1$, the result is similar but the RER takes values in $(\alpha_T^{\rho/(\rho-1)}, RER_t^*]$. Let the lower bound be denoted $\underline{RER}$. For any ρ, the natural real exchange rate RER_t^* is

$$RER_t^* = \alpha_T\, \psi\left(\frac{Y_{T,t} - T_t}{\Gamma_{N,t} L^*}\right)^{1/\rho} = \alpha_T\, \psi\left(\frac{Y_{T,t} - T_t}{\Gamma_{N,t}}\right)^{1/\rho} = \alpha_T\, \psi\left((z_t - \tau_t)e_t\right)^{1/\rho}. \tag{A.8}$$

The value of $C_t = \mathcal{C}(C_{T,t}, C_{N,t} - G_t(L_t)) = (Y_{T,t} - T_t)\left(\frac{RER_t}{\alpha_T}\right)^{\rho}$ is increasing in RER_t. And since debt is denominated in tradables, the RER_t has no direct effect on debt issuance or net foreign transfers T_t. Hence, flow welfare is maximized, conditional on debt issuance, by setting the RER_t at its maximum of RER_t^*; welfare is highest in the undistorted economy.

However, we can impose suboptimal behavior of using a RER peg. Specifically, let $\widehat{RER}$ be a RER target, which is feasible whenever RER_t^* is larger than it. When the sovereign is heavily burdened by debt, the large transfers T_t make tradable consumption scarce, causing a RER appreciation. In this case, the sovereign can maintain the peg by increasing labor taxes, inefficiently decreasing nontradable output but making tradables relatively more abundant, depreciating the RER.

We suppose the sovereign in a peg sets $RER_t = \min\{\widehat{RER}, RER_t^*\}$. With $\rho < 1$, this policy is always feasible. When $RER_t = RER_t^*$, behavior is identical to the benchmark's. When $RER_t < RER_t^*$, the sovereign is using a distortionary tax to reduce nontradable output and cause the RER to appreciate (from RER_t^* to the smaller value $\widehat{RER}$).

The government problem in a peg is

$$V^R(B_t, x_t, \Gamma_{T,t}) = \max_{B_{t+1}} u(C_t) - \Xi_t + \beta \mathbb{E}_{x_{t+1}|x_t} \left[\max_{N_{t+1} \in \{0,1\}} \left\{ \begin{array}{c} N_{t+1} \, V(B_{t+1}, x_{t+1}, g_{t+1} \Gamma_{T,t}) \\ +(1 - N_{t+1}) \, V^R(B_{t+1}, x_{t+1}, g_{t+1} \Gamma_{T,t}) \end{array} \right\} \right]$$
$$\text{s.t. } (26), (29), (A.8), RER_t = \min\{\widehat{RER}, RER_t^*\}.$$

The value conditional on default is similar, noting default costs only reduce productivity in the tradable sector so the natural rate of labor is unchanged:

$$V^D(B_t, x_t, \Gamma_{T,t}) = u(C_t) + \beta \mathbb{E}_{x_{t+1}|x_t} \left[\bar{\alpha} V^D(B_{t+1}, x_{t+1}, g_{t+1} \Gamma_{T,t}) + (1 - \bar{\alpha}) V(B_{t+1}, x_{t+1}, g_{t+1} \Gamma_{T,t}) \right]$$
$$\text{s.t. } B_{t+1} = R^D B_t, RER_t = \min\{\widehat{RER}, RER_t^*\},$$

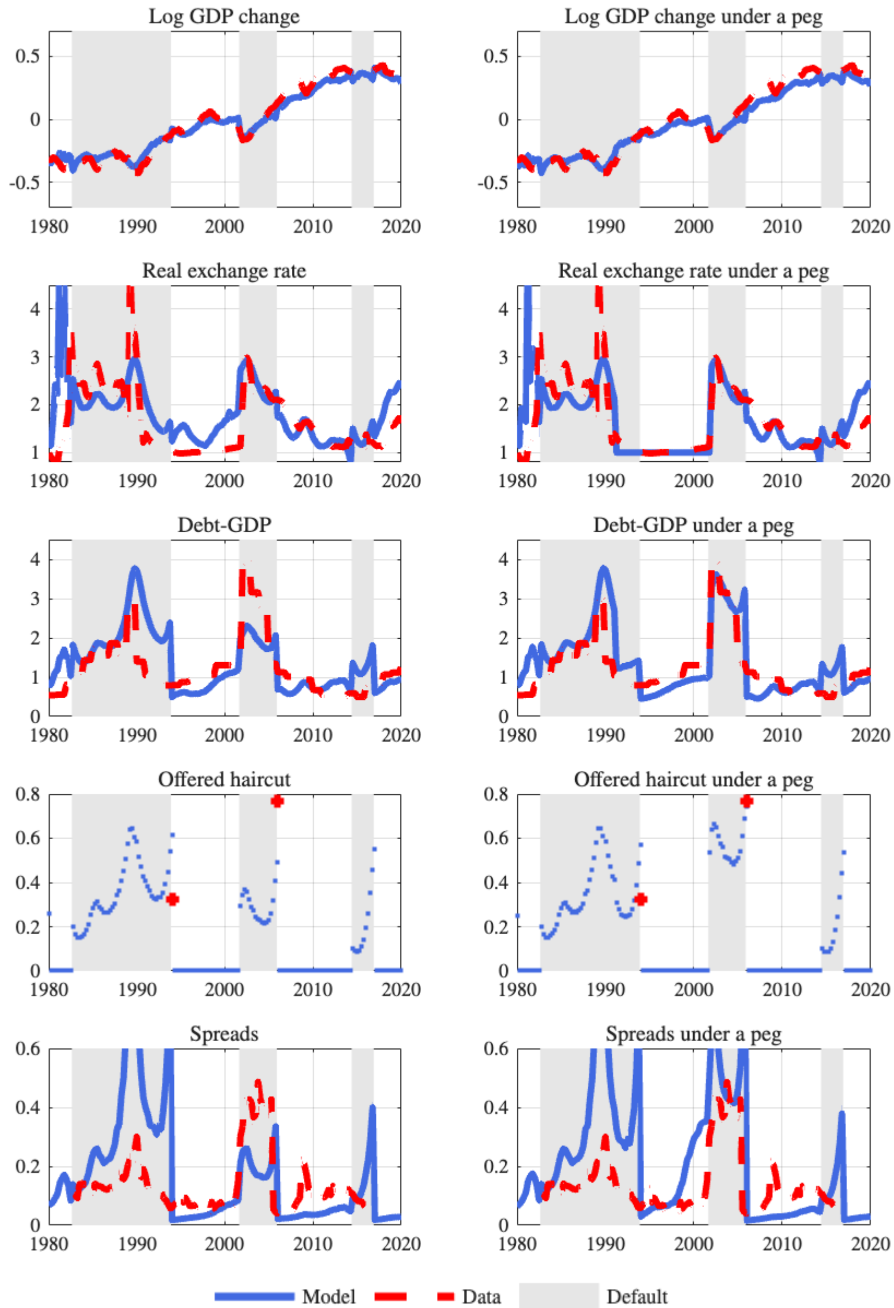
with C_t and RER_t^* analgous to (26) and (29), respectively, but for default.

B Argentina's hard and soft defaults

The left panels of figure 8 plot the simulated path of the economy given the estimated states from the Kalman filter while forcing default or repayment when the data had default or repayment, respectively. The model's behavior of log GDP and the RER (which are observables in the estimation) closely mirrors the data's counterparts. Debt/GDP tracks the data, capturing the run up in debt in the 1990s and jumps due to GDP and RER fluctuations. Spreads move with the data, but the model understates spreads in the late 2000s and early 2010s while overstating them in the 2014 default. However, the 2014 default was technical in nature, and the run up to it reflects uncertainty surrounding court decisions.

In the 2001Q4 default, the benchmark model underpredicts the 76.8% haircut in the data, while also understating the extent to which debt to GDP explodes in the default. However, this default was preceeded by an unusual period in Argentina's history. From 1991Q2 to 2001Q4, Argentina ran a currency board (Frank, 2004).

Figure 8: Path along estimated shocks



Note: this figure compares the data with model simulations; the simulations use estimated states from the Kalman filter force default or repayment when the data had default or repayment, respectively; the left panels are from the benchmark, while the right feature regime switches between pegged and floating exchange rates, mimicking Argentina's fixed exchange rate regime during the 1990s.

During this period, the currency board attempted to peg nominal exchange rate on a 1:1 basis with the USD, and in that period, the nominal exchange rate varied from 0.99 to 1. This also kept the RER between 1.31 and 0.98 and between 1.02 to .98 after 1993Q1 (as can be seen in the graph). However, eventually the currency board could not sustain the peg and abandoned it in 2002Q1, with the exchange rate exploding to 2.95 and a commensurate increase in inflation that caused debt to GDP to explode.

The model can capture this by adding a RER peg. As shown in Appendix A, when the nontradable good is produced using labor, the sovereign can artificially strengthen the RER by using a distortionary labor tax (which effectively makes the nontradable good scarce). We incorporate this feature by adding exogenous Markov regime switches between pegged and float states. When not pegged, the subjective probability of transitioning to a pegged regime is zero. But when in a pegged regime, there is a subjective 95% probability that the peg persists an additional quarter. We then simulate the model imposing the peg when one is present in the data.

The right panels of figure 8 plot the resulting series. Imposing these regime changes corrects the RER behavior and lets the model capture the magnitude of the debt-GDP increase from 2001 to 2002. The larger debt-GDP ratios result in larger haircut offers and spreads that are closer to the data's.

C The detrended model

We conjecture the form of the detrended problem and show its solution is a solution to the problem with trend. The conjectured solution deflates value functions by $\Gamma_{T,t}^{1-\sigma}$, consumption and debt choices by $\Gamma_{T,t}$, and leaves bond prices, haircuts, and accept/reject decisions unchanged.

We define detrended variables as

$$y_{T,t} \equiv Y_{T,t}/\Gamma_{T,t} \Rightarrow y_{T,t} = z_t, \tag{C.1}$$

$$y_{N,t} \equiv Y_{N,t}/\Gamma_{N,t} \Rightarrow y_{N,t} = 1, \tag{C.2}$$

$$c_t \equiv C_t/\Gamma_{T,t}, \tag{C.3}$$

$$\tau_t \equiv T_t/\Gamma_{T,t}, \tag{C.4}$$

$$\xi_t \equiv \Xi_t/\Gamma_{T,t}^{1-\sigma} = \bar{\xi}\max\{0, \underline{q} - q_t\}^2 \tag{C.5}$$

$$b_{t+1} \equiv B_{t+1}/\Gamma_{T,t} \Rightarrow b_t = g_t \frac{B_t}{\Gamma_{T,t}}. \tag{C.6}$$

Define the value conditional on repayment (recursively) as

$$\begin{aligned} &v^R(b_t, x_t, \hat{h}) = \\ &\max_{b_{t+1}} u(c_t) - \bar{\xi}\max\{0, \underline{q} - q_t\}^2 + \beta \mathbb{E}_{x_{t+1}|x_t} g_{t+1}^{1-\sigma} \max\{v(b_{t+1}, x_{t+1}), v^R(b_{t+1}, x_{t+1})\} \end{aligned} \tag{C.7}$$

$$\text{s.t. } c_t = (z_t - \tau_t)\psi((z_t - \tau_t)e_t) \tag{C.8}$$

$$\tau_t = -q_t(b_{t+1}, x_t)(b_{t+1} - (1-\lambda)\frac{b_t}{g_t}) + \tilde{\lambda}\frac{b_t}{g_t}. \tag{C.9}$$

Define the value conditional on default as

$$v^D(b_t, x_t) = u(c_t) + \beta \mathbb{E}_{x_{t+1}|x_t} g_{t+1}^{1-\sigma} v(b_{t+1}, x_{t+1}) \tag{C.10}$$

$$\text{s.t. } c_t = (1-\chi) z_t \psi((1-\chi) z_t e_t) \tag{C.11}$$

$$b_{t+1} = R^D \frac{b_t}{g_t}. \tag{C.12}$$

Define the value of negotiation as

$$v(b_t, x_t) = \max_{\hat{h}_t} \alpha(\hat{h}; b_t, x_t) v^R(b_t, x_t, \hat{h}) + (1 - \alpha(\hat{h}; b_t, x_t)) v^D(b_t, x_t). \tag{C.13}$$

The conjectured value function relationship is $\Gamma_{T,t}^{1-\sigma} v^R(b_t, x_t) = V^R(B_t, x_t, \Gamma_{T,t})$ for repayment and similarly for the other value functions (b_t and B_t are related as in (C.6)).

Define the detrended price schedule in repayment as

$$q(b_{t+1}, x_t) = \frac{1}{1+r^*}\mathbb{E}_{x_{t+1}|x_t}\begin{bmatrix} (1-n_{t+1})q^A(0, b_{t+1}, x_{t+1}) \\ +n_{t+1}(a_{t+1}q^A(h_{t+1}, x_{t+1}) + (1-a_{t+1})q^D(x_{t+1})) \end{bmatrix}, \tag{C.14}$$

where h, n, and a denote the corresponding policies from the detrended problems. Define the other detrended prices analogously. The conjectured price relationship is $q(b_{t+1}, x_t) = Q(B_{t+1}, x_t, \Gamma_{T,t+1})$ for repayment and similarly for the other price schedules (b_{t+1} and B_{t+1} are related as in (C.6)).[19]

Proposition 2. *A solution to the detrended problem is a solution to the problem with trend.*

Table 4 gives key measurements using variables with or without trend.

D Proofs

Proof of proposition 1. From (1), the creditors' fundamental value of rejecting an offer is $q^D = \beta\alpha(1-h)/(1-\beta(1-\alpha))$. So the sovereign's value function satisfies $V = \frac{-\chi y - \beta\alpha(1-h)b}{1-\beta(1-\alpha)} = \frac{-\chi y}{1-\beta(1-\alpha)} - q^D b$. Hence, $q^A - q^D = \frac{\epsilon_\alpha}{\epsilon_\alpha + 1} \cdot \frac{\chi y/b}{1-\beta(1-\alpha)}$. And also from (1), $q^A - q^D = \frac{1-\beta}{1-\beta(1-\alpha)}(1-h)$. Eliminating $q^A - q^D$ from these last two equations gives (3).

To establish $\epsilon_\alpha > 0$, we require α strictly increasing and differentiable. With this, $\epsilon_\alpha = 0$ gives $q^A - q^D = 0$. In that case, $q^D = \beta(q^D + \alpha(q^A - q^D))$ implies $q^D = \beta q^D$ so $q^D = q^A = 0$, implying $h = 1$, which is not interior. (1) shows $q^A - q^D \geq 0$ in any equilibrium, therefore an interior equilibrium must have $q^A - q^D > 0$. This gives $\epsilon_\alpha = \alpha'(q^A - q^D)/\alpha > 0$. □

[19] This is with $u(x) = x^{1-\sigma}/(1-\sigma)$. In the main text and code, we use $u(x) = (x^{1-\sigma} - 1)/(1-\sigma)$, which requires an additional adjustment.

Table 4: Measurement of key variables

Statistic	Not-detrended	Detrended
Real exchange rate	RER_t	$\alpha_T \psi((z_t - \tau_t)e_t)^{1/\rho}$
Price level	p_t	$\alpha_N^{\frac{\rho}{1-\rho}}(1 - \alpha_T^{\rho} RER_t^{1-\rho})^{\frac{1}{\rho-1}}$
Consumption	C_t	$c_t \Gamma_{T,t}$
GDP	Y_t	$(RER_t z_t + p_t^{-1}/e_t)\Gamma_{T,t}$
Debt level[a]	$RER_t B_t$	$RER_t(b_t/g_t)\Gamma_{T,t}$
Current account[b]	$RER_t T_t$	$RER_t \tau_t \Gamma_{T,t}$

Note: (a) Since this variable is denominated in tradables, to map to aggregate consumption units one multiplies by $p_{T,t}/p_t$, which is RER_t. (b) The current account is private savings less investment plus taxes less government expenditures, $(S-I)+(Taxes-G)$. Without private savings, capital or government expenditures, this is just taxes (measured in aggregate consumption unties), which is $RER_t T_t$.

Proof of proposition 2. We first show the budget constraints and value functions of the detrended problem can be mapped into a solution for the problem with trend. We will focus on repayment as the arguments for default are analogous. Consider the budget constraint in repayment. Beginning with (C.8), we have

$$c_t \Gamma_{T,t} = (z_t - \tau_t)\Gamma_{T,t} \psi((z_t - \tau_t)\frac{\Gamma_{T,t}}{\Gamma_{N,t}}) \Leftrightarrow C_t = (Y_{T,t} - T_t)\psi(\frac{Y_{T,t} - T_t}{Y_{N,t}}),$$

which is the same as (28) for the problem with trend. Likewise, beginning with (C.9) and multiplying by $\Gamma_{T,t}$,

$$T_t = -q_t(b_{t+1}, x_t)(B_{t+1} - (1-\lambda)\frac{b_t}{g_t}\Gamma_{T,t}) + \tilde{\lambda}\frac{b_t}{g_t}\Gamma_{T,t} \tag{D.1}$$

$$= -q_t(b_{t+1}, x_t)(B_{t+1} - (1-\lambda)\frac{B_t}{\Gamma_{T,t}}\Gamma_{T,t}) + \tilde{\lambda}\frac{B_t}{\Gamma_{T,t}}\Gamma_{T,t} \quad \because (C.6) \tag{D.2}$$

$$= -Q_t(B_{t+1}, x_t, \Gamma_{T,t})(B_{t+1} - (1-\lambda)\frac{B_t}{\Gamma_{T,t}}\Gamma_{T,t}) + \tilde{\lambda}\frac{B_t}{\Gamma_{T,t}}\Gamma_{T,t}, \tag{D.3}$$

which after simplification is (29). The last equality comes from the conjectured relationship between q and Q.

For the repayment value function, beginning with (C.7) and multiplying by $\Gamma_{T,t}^{1-\sigma}$ one has

$$\begin{aligned}
\Gamma_{T,t}^{1-\sigma} v^R(b_t, x_t) &= \max_{b_{t+1}} \Gamma_{T,t}^{1-\sigma} u(c_t) - \Gamma_{T,t}^{1-\sigma} \xi_t + \qquad \text{(D.4)} \\
&\quad \beta \mathbb{E}_{x_{t+1}|x_t} g_{t+1}^{1-\sigma} \Gamma_{T,t}^{1-\sigma} \max\{v(b_{t+1}, x_{t+1}), v^R(b_{t+1}, x_{t+1})\} \\
\Leftrightarrow V^R(B_t, x_t, \Gamma_{T,t}) &= \max_{b_{t+1}} u(C_t) - \Xi_t + \qquad \text{(D.5)} \\
&\quad \beta \mathbb{E}_{x_{t+1}|x_t} \Gamma_{T,t+1}^{1-\sigma} \max\{v(b_{t+1}, x_{t+1}), v^R(b_{t+1}, x_{t+1})\} \\
\Leftrightarrow V^R(B_t, x_t, \Gamma_{T,t}) &= \max_{B_{t+1}} u(C_t) - \Xi_t + \qquad \text{(D.6)} \\
&\quad \beta \mathbb{E}_{x_{t+1}|x_t} \max\{V(B_{t+1}, x_{t+1}, \Gamma_{T,t+1}), V^R(B_{t+1}, x_{t+1}, \Gamma_{T,t+1})\},
\end{aligned}$$

which is (31).

For the price schedule, subsituting the conjectured relationships into (C.14) gives (37) immediately. □

Proposition 3. (25) *and* (26) *hold.*

Proof. From (15),

$$p_{T,t} = \frac{\alpha_T}{\alpha_N} \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{1/\rho}. \qquad \text{(D.7)}$$

Dividing both sides by $p_{T,t}$, multiplying by p_t, and using $RER_t = p_{T,t}/p_t$,

$$p_t = RER_t^{-1} \frac{\alpha_T}{\alpha_N} \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{1/\rho}. \qquad \text{(D.8)}$$

Plugging that expression for p_t into (22), one arrives at

$$1 = \alpha_T^{\rho} RER_t^{1-\rho} + \alpha_N^{\rho} \left(RER_t^{-1} \frac{\alpha_T}{\alpha_N} \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{1/\rho} \right)^{\rho-1} \tag{D.9}$$

$$= RER_t^{1-\rho} \left(\alpha_T^{\rho} + \alpha_N^{\rho} \frac{\alpha_T^{\rho-1}}{\alpha_N^{\rho-1}} \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{\frac{\rho-1}{\rho}} \right) \tag{D.10}$$

$$\Rightarrow RER_t = \alpha_T \left(\alpha_T + \alpha_N \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{1}{\rho-1}}. \tag{D.11}$$

Then, beginning with the definition of the consumption aggregator,

$$C_t^{\frac{\rho-1}{\rho}} = \alpha_T (Y_{T,t} - T_t)^{\frac{\rho-1}{\rho}} + \alpha_N Y_{N,t}^{\frac{\rho-1}{\rho}} \tag{D.12}$$

$$= (Y_{T,t} - T_t)^{\frac{\rho-1}{\rho}} \left(\alpha_T + \alpha_N \left(\frac{Y_{N,t}}{Y_{T,t} - T_t} \right)^{\frac{\rho-1}{\rho}} \right) \tag{D.13}$$

$$= (Y_{T,t} - T_t)^{\frac{\rho-1}{\rho}} (RER_t/\alpha_T)^{\rho-1} \tag{D.14}$$

$$\Rightarrow C_t = (Y_{T,t} - T_t)(RER_t/\alpha_T)^{\rho}. \tag{D.15}$$

□