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Working Paper Series

Artificial Intelligence and Technological Unemployment

WP 26-01

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This Version: August 2025

How large are the effects of artificial intelligence (AI) on labor productivity and unemployment? We develop a labor-search model of technological unemployment where AI learns from workers, raises productivity, and displaces them if renegotiation fails. The model admits three steady states: no AI; some AI with limited capability, more job creation but higher unemployment; unbounded AI with endogenous growth and employment gains. Calibrated to U.S. data, the model implies a threefold productivity gain but a 23% employment loss, half within five years. Plausible parameters give rise to global and local indeterminacy with endogenous cycles in productivity and unemployment, underscoring the uncertainty of AI's impacts in line with a wide range of empirical findings. Equilibria are inefficient despite the Hosios condition; subsidizing jobs at risk of AI displacement is constrained optimal.

JEL Classification: E20, J20, J64, L20, O30, O40.
Keywords: generative artificial intelligence, technological unemployment, search and bargaining, endogenous growth, constrained efficiency, indeterminacy.
Acknowledgments: This paper is prepared for the Carnegie-Rochester-NYU Conference on Public Policy held on May 2-3, 2025. We are grateful for insightful comments by our discussant, Carter Braxton, as well as by George Alessandria, Mark Bils, Hugo Hopenhayn, Andreas Hornstein, Anton Korinek, Thomas Lubik, Zheng Liu, Pascual Restrepo, Guillaume Rocheteau, Ali Shourideh, Venky Venkateswaran, Liang Wang, Ariel Zetlin-Jones, and by participants of presentation at Richmond Fed, CEA, Academia Sinica, SAET, Wash U, National Chengchi U, UH Manoa, San Francisco Fed, UC Irvine and U of Macau. The views expressed in this article are those of the authors and do not necessarily reflect the views of the Federal Reserve System, the Board of Governors, or the regional Federal Reserve Banks. Correspondence: Ping Wang: pingwang@wustl.edu; Tsz-Nga Wong: 701 E. Byrd Street, Richmond, VA, 23219; (804) 697-8000; russell.wong@rich.frb.org.

1. Introduction

Recent advances in artificial intelligence (AI), particularly machine learning and generative AI (GenAI), represent a major technological shift with wide-ranging implications. The introduction of image-based AI in April 2022 and ChatGPT in November 2022 vividly demonstrated AI's transformative potential. McKinsey (2023) predicts that GenAI could potentially reduce human working hours by 60-70% in advanced economies. Similarly, Briggs and Kodnani (2023), in Goldman Sachs' report, estimate that widespread adoption could raise U.S. labor productivity growth by 1.5 percentage points annually over the next decade, adding 7% to global GDP. At the micro level, Brynjolfsson, Li, and Raymond (2025) show that access to a GenAI conversational assistant raised customer-support agents' productivity by roughly 14% on average. In contrast, Acemoglu (2024) presents a macroeconomic model suggesting a relatively modest aggregate productivity gain from automation (0.064% annual increase in TFP growth), despite substantial cost savings (14.4%). Moreover, recent work by Brynjolfsson, Chandar, and Chen (2025) shows the early-career workers in AI-exposed occupations have experienced a 13% relative decline in employment even after controlling for firm-level shocks.

These conflicting perspectives raise a question: what are the dynamic implications of GenAI within a unified framework of unemployment, labor productivity, and economic growth?

Our paper addresses this question by developing a labor-search model that explicitly incorporates GenAI as both a source of productivity growth and labor displacement. We model AI as a "learning-by-using" technology, consistent with the industry perspective that AI improves through usage and feedback. Specifically, AI capability improves through *reinforced learning from workers* when using AI for their jobs. This capability, in turn, raises workers' productivity but ultimately renders workers disaplaceble by AI. These dynamics make GenAI qualitatively distinct from earlier technologies—such as computers, software, or robotics—that lack such self-learning improvement.

The interdependence between AI and employment generates rich dynamic effects in the presence of labor-search frictions. In the Benchmark Model, where job finding rate is fixed, dynamics depend on two state variables—employment and AI capability—and yield three possible outcomes: (i) a some-AI steady state with limited AI capability, expanding job creation but declining employment; (ii) an unbounded-AI equilibrium with sustained productivity growth and higher employment, complementing the idea production framework by Aghion, Jones, and Jones (2019) (AJJ hereafter); and (iii) a no-AI equilibrium reverting to the standard model of Mortensen and Pissarides (1994) (MP hereafter) without AI taking off. Some-AI and unbounded-AI equilibria may co-exist (global indeterminacy), and convergence to the some-AI steady state may occur through an oscillatory path. And there could be uncountably many paths. Extending to the Full Model with endogenous job-finding rate driven by job creation, all three equilibria remain possible, coexisting with a continuum of oscillatory paths converging to the some-AI steady state (local indeterminacy).

The short-run cyclicality reflects the opposing forces of AI-induced job creation and destruction operating within a frictional labor market. The multiplicity in the short run and long run reflects a coordination failure between job-creating firms benefiting from AI-driven productivity gains and job-destroying firms adopting AI to displace workers. While short-run oscillations imply volatility in transition, a continuum of transition paths and multiple steady states indicate short- and long-run indeterminate outcomes. All suggest a highly unpredictable nature of AI development and its consequences.

Building on our theoretical results, we calibrate the model by targeting moments from recent empirical studies on AI adoption and productivity. The calibration yields a some-AI equilibrium with substantial productivity gains (366%) accompanied by a 23% employment loss—a phenomenon of *technological unemployment*. About half of the employment loss occurs within the first five years. Relative to the Benchmark Model, the Full Model generates a slower employment decline during transition, owing to higher job-finding rates, and faster growth in AI capability, driven by faster learning from employed workers. With higher replacement costs or stronger firm bargaining power, however, multiple equilibria may arise, including coexistence of the some-AI and unbounded-AI states. In the latter, employment rises modestly in the long run, resembling the labor-market effects of computer adoption: although AI may ultimately dominate human tasks, workers are not fully displaced, as rising AI-augmented productivity sustains employment, albeit at lower income shares.

We also analyze welfare. Even when the Hosios condition holds, equilibria are inefficient. The reason is that AI introduces new externalities beyond standard matching frictions: (i) job destruction through worker displacement; (ii) productivity enhancement for employed workers; (iii) feedback effects from AI learning that depend on the number of workers; and (iv) direct effects on matching surpluses and hence job-finding rates. These imply that Hosios' rule alone is insufficient. An optimal subsidy targeted to jobs exposed to AI adoption is welfare-improving: 26.6% in the short run and more than 50% in the long run.

Three main takeaways follow. First, AI induces technological unemployment: sizable employment losses occur despite productivity gains and expanded job creation. Second, unlike standard models, the interaction of AI and employment generates rich dynamics, including global and local indeterminacy and endogenous oscillations. Third, the Hosios condition is no longer sufficient; a targeted subsidy for AI-exposed jobs yields substantial welfare gains.

Literature Review Since GenAI's introduction in 2022, a wave of studies has examined its labor-market impacts: Korinek (2023) on LLMs in research, Trammell and Korinek (2023) on growth and labor share, Korinek and Stiglitz (2019) on inequality, Susskind (2024) on technological unemployment, and Nordhaus (2021) on broader growth issues.

Empirics Field experiments provide direct evidence: Brynjolfsson, Li, and Raymond (2025) find14% productivity gains (concentrated among low-skilled); Brynjolfsson, Chandar, and Chen (2025) find that AI-exposed occupations have experienced a 13% decline in employment even after controlling for firm-level shocks; Hui, Reshef, and Zhou (2024) document 2% job loess and 5.2% earnings decline; Liu, Xu, Nan, Li, and Tan (2023) show 24% fewer postings; Eloundou, Manning, Mishkin, and Rock (2023) estimate 23% of tasks exposed to AI (of which 23% can be profitable performed by AI); Noy and Zhang (2023) find 40% faster and 18% higher-quality writing with AI, and Peng, Kalliamvakou, Cihon, and Demirer (2023) show 55.8% faster coding.

More broadly, Brynjolfsson, Rock, and Syverson (2019) document slow diffusion, Agrawal, Gans, and Goldfarb (2019) survey impacts, Brynjolfsson, Mitchell, and Rock (2018) show machine learning suitability averages 3.47 (on a 1–5 scale) across 964 O*NET occupations, and Felten, Raj, and Seamans (2021) identify occupations (e.g., Genetic Counselors, Financial Examiners, Actuaries) and industries (Securities/Financial Investments, Accounting, Insurance), by linking AI-suitability to vacancy postings. Using exposure measures, Acemoglu, Autor, Hazell, and Restrepo (2022) find higher demand for skilled workers but lower overall hires. Survey evidence from Bick, Blandin, and Deming (2024) shows 24% weekly use, 11% daily, and 33.7% outside work.

Task Models Task models are often used to clarify that different technologies, like automation and AI, have distinct effects on labor demand, wages, and productivity (see a survey by Restrepo (2024)). Aghion et al. (2019) construct a task-based Roy model in which capital replaces workers once tasks are automated, while some tasks generate new ideas; they show that AI may enhance growth but may also lead to a technological singularity.

The most closely related work is Acemoglu (2024), who studies the macroeconomics of AI using the automation framework of Acemoglu and Restrepo (2018). In this setting, AI can deepen automation (reducing employment), generate new labor-intensive tasks (raising employment), and create complementarities that raise productivity. Acemoglu computes that AI raises TFP by at most 0.66%—equivalent to 0.064% annual growth—despite cost savings of 14.4%. In a related task-based model, Ales, Combemale, and Krishnan (2024) argue that GenAI is more general than prior automation technologies: its flexibility with complex and simple tasks enlarges the adoption zone relative to non-AI automation.

In frictionless task models, AI adoption reallocates tasks efficiently, generating little effect on aggregate employment. By contrast, in our search model where AI both creates and destroys jobs, labor-market reallocations—vacancy posting, job finding, renegotiation, and separations—are subject to frictions and delays. These frictions give rise to persistently high technological unemployment despite substantial productivity gains. Moreover, the high unemployment inefficiency persists in the long run.

Search Models Our paper also relates to a long literature on labor search and matching. Following Shimer (2005), it has been widely quantified to match aggregate labor-market data and extended to incorporate endogenous quits and layoffs. Methodologically, Lazaryan and Lubik (2019) show that discrete-time versions of DMP can feature global and local indeterminacy. In continuous time Mortensen (1999) obtain rich dynamics with increasing returns in productivity upon matching, including multiple equilibria with saddle paths, oscillations, and cycles. Along the lines of automation, Leduc and Liu (2024) study robots in a labor-search model, emphasizing short-run labor-share movements from replacement versus new task creation, identifying a countercyclical labor share consistent with data. Lu (2022) focus on the long-run effects of automation, finding that it reduces skilled but raises unskilled employment, potentially lowering aggregate unemployment. Similarly, Arnoud (2018) introduce wage renegotiation when automation arrives, leading to lower wages even when adoption is delayed—a feature our model shares.

There are a few more relevant search models if one may view AI evolution as a form of technological advancement. Mortensen and Pissarides (1998) show that innovation may generate creative destruction, destroying jobs and raising unemployment, while Postel-Vinay (2002) argue that such long-run destruction can coexist with short-run employment gains. Braxton and Taska (2023) calibrate a search model using Burning Glass and O*NET data, showing that match-embodied technological change explains over 45% of post-displacement earnings losses, especially in computer- and software-intensive occupations. Consistent with this, our counterfactual indicates that whereas exogenous productivity growth from computers reduced unemployment only modestly by 0.16 percentage points, AI's learning and displacement features induce a nearly 20% long-run employment loss, highlighting the profound labor-market consequences of AI relative to computers.

Unlike standard labor-search models, the presence of AI generates rich dynamics in which labor productivity, wages, and unemployment may evolve along a continuum of oscillatory paths in the short run and converge to multiple steady states in the long run. We can generate endogenous growth, but differently the growth is driven by the interaction between job creation and AI learning from workers.

2. Model

Consider a continuous-time labor search model in which an infinitely-lived worker seeks employment opportunities. At any instant, the worker is either employed or unemployed, with measures H_t and $1 - H_t$ respectively. When employed, the worker earns the wage rate w_t, determined through bargaining with his employer firm, in exchange for producing output y_t. Employment ends either randomly at an exogenous Poisson rate σ, or endogenously if the employer firm adopts AI to replace the worker. When unemployed, the worker enjoys a benefit rate $\ell_t = by_t$, $b \in (0, 1)$, reflecting leisure utility and unemployment benefits. Unemployed workers find a job and becomes employed at a Poisson rate α_t. Agents are risk neutral and discount future consumption at rate r.

Artificial Intelligence. In our model, artificial intelligence (AI) exhibits three essential characteristics. First, the capability of AI, denoted by A_t, enhances both the productivity and leisure of workers, implying $\partial y_t/\partial A_t \geq 0$. With $\partial y_t/\partial A_t > 0$, human capital per employed worker, herein measured by $y(A)$, rises with AI and hence AI is embodied in employed workers as long as the match continues. Yet, the AI stock remains even when a worker is fired, so AI is disembodied in this respect. Second, improved AI capability also increases the likelihood of job replacement through automation of tasks. Third, AI capability expands through a learning-by-using mechanism at the rate μ per employed workers. This captures the intelligence part of AI and without further remark we will use AI and GenAI interchangeably. Specifically, the knowledge acquired from each worker is shared within the AI system, the aggregate growth rate of AI capability is μH_t. However, AI's performance can deteriorate, and errors will eventually appear at an exogenous rate δ.[1] The law of motion for the capability of the AI is thus given by,

$$\frac{\dot{A}_t}{A_t} = \mu H_t - \delta, \text{ where } A_0 > 0. \tag{1}$$

Regarding the job-replacement feature of AI, there is an option to adopt AI for production without using workers, yielding $\pi^A y_t$ in *net* present value. When this replacement option arises, the firm renegotiates wages with the worker. If the renegotiation fails, AI replaces the worker, resulting in the worker's unemployment. If successful, the firm forgoes the replacement option, pays the renegotiated wages, and continues employment. The arrival rate of this replacement option for each matched firm-worker pair is $\rho\mu A_t$, proportional to AI's learning rate.[2] Thus, ρ captures the degree to which a job is exposed to AI-driven replacement risk. That is, the faster AI learns from employed workers, the greater the replacement probability is. Put differently, one may think of $\rho\mu$ as an effective exposure measure that is higher when exposing to a faster-learning AI. The standard labor-search model of Mortensen and Pissarides (1994) is the special case of $\rho = 0$ and $\partial y_t / \partial A_t = 0$.

Values. The value of an unemployed worker, U_t, is given by the following Hamilton-Jacobi-Bellman (HJB) formulation,

$$rU_t = \dot{U}_t + by_t + \alpha_t \left(W_t^0 - U_t \right). \tag{2}$$

The RHS consists of the flow of U_t, where the unemployed worker enjoys the growth of value while maintaining at the unemployed state, $\dot{U}_t$, the unemployment benefit by_t, and the change in value from unemployed (U_t) to employed (W_t^0) when he finds a job that happens at the rate α_t.

The dynamics for the employed worker's value is more involved since wages are renegotiated when firms threat to replace the worker with the AI. Denote w_t^n as the wage rate after the n-th renegotiation. The values of an employed worker, W_t^n, is given by,

$$rW_t^n = \dot{W}_t^n + w_t^n + \rho\mu A_t \left(1 - I_t^{n+1}\right) \left(W_t^{n+1} - W_t^n\right) + \left(\rho\mu A_t I_t^{n+1} + \sigma\right) \left(U_t - W_t^n\right). \tag{3}$$

The RHS consists of the flow of W_t^n, where the employed worker enjoys the growth of value,

[1] It includes various malfunctions like traditional security failings by malevolent attackers; safety malfunctions causing harm to a human being; and alignment failures. According Alex Stamos, "95% of the bugs in the AI system have not been invented yet."

[2] As it is typical in search and matching models, we assume that the firm cannot revisit the option once it has been passed, but the new options will be available to the firm with the same arrival rate.

$\dot{W}_t^n$, the wage rate w_t^n, the change in value to W_t^{n+1} from W_t^n after the $n+1$-th renegotiation, which happens at the rate ρA_t if the worker is not replaced (i.e., $I_t^{n+1}=0$). Otherwise, i.e., $I_t^{n+1}=1$, the worker becomes unemployed and his value changes to U_t from W_t^n. The worker can also become unemployed exogenously, happening at rate σ. Denote the worker's surplus as $S_t^{w,n}\equiv W_t^n-U_t$, which evolves according to:

$$rS_t^{w,n}=\dot{S}_t^{w,n}+w_t^n-by_t+\rho\mu A_t\left(1-I_t^{n+1}\right)S_t^{w,n+1}-\left(\rho A_t+\sigma\right)S_t^{w,n}-\alpha_t S_t^{w,0}. \tag{4}$$

Profit. The profit of firm after the n-th renegotiation, Π_t^n, is given by,

$$r\Pi_t^n=\dot{\Pi}_t^n+y_t-w_t^n+\rho\mu A_t\left[I_t^{n+1}\left(\pi^A y_t-\Pi_t^n\right)+\left(1-I_t^{n+1}\right)\left(\Pi_t^{n+1}-\Pi_t^n\right)\right]-\sigma\Pi_t^n. \tag{5}$$

The RHS consists of the flow of Π_t^n, where the firm enjoys the growth of profit, $\dot{\Pi}_t^n$, the output net of the wage rate, $y_t-w_t^n$, and the change in value when the AI presents an opportunity to replace the worker, which happens at the rate ρA_t. If the firm adopts the AI to replace the worker (i.e. $I_t^{n+1}=1$), its profit increases to $\pi^A y_t$ from Π_t^n. Otherwise, i.e., $I_t^{n+1}=0$, the firm renegotiates a lower wage sequence and its profit increases to Π_t^{n+1} from Π_t^n. Finally, the match can be destroyed exogenously at rate σ. In this case, the firm's value falls to zero from Π_t^n.

Replacement. The option of replacing worker with AI is taken if the wage renegotiation fails, i.e., the firm and worker cannot find a wage process such that both their continuation values within the match are weakly greater than their outside options. In other words, AI is adopt to replace the worker after the n-th renegotiation ($I_t^{n+1}=1$) if there does not exist any Π_t^n and W_t^{n+1} such that $\Pi_t^n\geq\pi^A y_t$ and $W_t^n\geq U_t$. Given the transferable utility in our model, it implies that

$$I_t^{n+1}=1 \text{ if and only if } \pi^A y_t>S_t^{n+1}\equiv\Pi_t^{n+1}+W_t^{n+1}-U_t, \tag{6}$$

where we refer S_t^n as to the joint surplus after the n-th renegotiation. Summing up the dynamics of the worker's surplus (4) and of the firm value (5) with the replacement decision given by (6). the law of motion for S_t^n is given by,

$$rS_t^n=\dot{S}_t^n+(1-b)\,y_t+\rho\mu A_t\max\left(\pi^A y_t,S_t^{n+1}\right)-\left(\rho\mu A_t+\sigma\right)S_t^n-\alpha_t S_t^{w,0}. \tag{7}$$

The following proposition shows S_t^n is renegotiation-proof.[3]

Proposition 1 ***(Renegotiation-Proof)*** *For any $n\geq 0$, we have $S_t^n=S_t$ and $I_t^n=I_t$.*

Proposition 1 shows that any further renegotiation will not change the joint surplus, i.e., S_t^n is independent of n for any $n>0$. It is because the option of replacing worker with AI

[3] Alll the proof can be found in the online Appendix.

only redistributes the value from the worker to firm. As a result, the decision of adopting AI to replace worker is also independent of n. Accordingly, we can suppress n and capture adoption decision by a simple indicator function I_t.

Unemployment. Proposition 1 simplifies the law of motion for the fraction of the employed workers, H_t, as:

$$\dot{H}_t = \alpha_t (1 - H_t) - \sigma H_t - \rho\mu A_t H_t I_t. \tag{8}$$

The change in employment, $\dot{H}_t$, consists of inflow of job-finding workers, $\alpha_t (1 - H_t)$, outflow of workers separated from jobs, σH_t, and replaced by AI, $\rho\mu A_t H_t I_t$. For each employed worker, he is more often threatened by AI replacement if AI learns faster (higher μ), AI is more capable (higher A_t), or his job has higher exposure to AI (higher ρ). Since the decision of adopting AI to replace worker is independent to n, the aggregate fraction of firms adopting AI if they have the option is also I_t. Thus, the total replacement rate of workers by AI is $\rho\mu A_t H_t I_t$, which is the source of *technological unemployment*.

Wage Bargaining. We assume the protocol of Nash bargaining such that the wage sequence is determined by,

$$\{w_t^n\} = \begin{cases} \arg\max \left(\Pi_t^0\right)^\theta \left(W_t^0 - U_t\right)^{1-\theta}, & \text{for } n = 0, \\ \arg\max \left(\Pi_t^n - \pi^A y_t\right)^\theta \left(W_t^n - U_t\right)^{1-\theta}, & \text{for } n > 0 \text{ and } I_t^n = 0, \end{cases} \tag{9}$$

where $\theta \in [0, 1]$ is the firm's bargaining power. When a firm and an unemployed worker match and negotiate for the new job (i.e., $n = 0$), the outside option to the worker is being unemployed with continuation value U_t, but the outside option to the firm is zero since no production takes place. Therefore the surpluses from the match are Π_t^0 to the firm and $W_t^0 - U_t$ to the worker. The protocol of Nash bargaining (9) splits their surpluses according to the firm's bargaining power θ. When a firm renegotiates with its worker after the n-th replacement option arrives, where $n > 0$, the firm's surplus from continuing the match (i.e., $I_t^n = 0$) becomes $\Pi_t^n - \pi^A y$ because now it can produce $\pi^A y$ without the worker as its outside option. Given the fact that w_t^n enters Π_t^n and W_t^n linearly, the bargaining solution of (9) is given by,

$$S_t^{w,n} = W_t^n - U_t = \begin{cases} (1-\theta) S_t, & \text{for } n = 0, \\ (1-\theta)\left(S_t - \pi^A y_t\right), & \text{for } n > 0 \text{ and } I_t^n = 0, \end{cases} \tag{10}$$

$$\Pi_t^n = \begin{cases} \theta S_t, & \text{for } n = 0, \\ \theta S_t + (1-\theta)\pi^A y_t, & \text{for } n > 0 \text{ and } I_t^n = 0, \end{cases} \tag{11}$$

where we have made use of the result $S_t = \Pi_t^n + W_t^n - U_t$ for all n from Proposition 1.

To solve the equilibrium wage, we focus on wage as the share of output, w_t^n / y_t (also known as the labor share) since output, and hence wage, can grow unboundedly in the presence of growing AI capability, The equilibrium wage is given by substituting the bargaining solution (10) in the dynamics of the worker's surplus (4):

$$\frac{w_t^n}{y_t} = \begin{cases} 1-\theta\left(1-b\right)+\left(1-\theta\right)\left(\theta\alpha_t s_t+\rho\mu A_t\pi^A\right), \text{ for } n=0, \\ 1-\theta\left(1-b\right)+\left(1-\theta\right)\left[\theta\alpha_t s_t-\left(r+\sigma-g_t\right)\pi^A\right], \text{ for } n>0 \text{ and } I_t^n=0, \end{cases} \tag{12}$$

where $s_t \equiv S_t/y_t$ is the joint surplus per output (referred as to surplus ratio hereafter) and $g_t \equiv \dot{y}_t/y_t$ is the growth rate of labor productivity. If the firm has all the bargaining power, i.e., $\theta = 1$, then the worker is always paid at his outside option such that $w_t^n = by_t$. If the firm has no bargaining power, i.e., $\theta = 0$, then the worker extracts all the joint surplus such that he is paid at $w_t^0 = y_t + \rho A_t \pi^A y_t$ for a new match and then renegotiated to $w_t^n = y_t - (r+\sigma-g_t)\,\pi^A y_t$ after the firm has threatened to replace him with the AI. To incentivize the firm not to replace the worker, the labor share has to been reduced by $(r+\sigma-g_t)\,\pi^A$, as shown in (12) as the bargaining outcome. The reduction in the labor share is increasing in the separation rate, σ, as the match is expected is last shorter under a higher σ. The reduction in the labor share is decreasing in the output growth rate, g_t, as the match will generate higher output growth so the firm is willing to pay higher wage after the renegotiation.

Roadmap. To summarize, the dynamics of this economy is completely characterized by $\dot{A}_t$ from (1), $\dot{H}_t$ from (8) and $\dot{s}_t$. By substituting $\dot{S}_t$ from (7) to the identity $\dot{s}_t = \dot{S}_t/y_t - g_t s_t$, the law of motion for the surplus ratio, $s_t \equiv S_t/y_t$, is given by,

$$\dot{s}_t = \left[r+\sigma+\alpha_t\left(1-\theta\right)-g_t\right]s_t-\left(1-b\right)-\rho\mu A_t\left(\pi^A-s_t\right)_+, \tag{13}$$

where $(X)_+ \equiv \max\{X, 0\}$. To structure our analysis clearly on this system, we examine three distinct specifications of the model.

First, we consider a *simple model* designed explicitly for intuitive understanding, where we assume labor productivity not influenced by the AI (i.e., $\partial y/\partial A = 0$) and the job finding rate exogenously fixed. This simplified framework admits closed-form equilibrium solutions, allowing us to explicitly characterize the constrained optimal allocations and transparently identify the sources of inefficiency in the labor market with AI. Additionally, we provide microfoundations for key parameters, specifically π^A and δ, useful to target the micro data we need to calibrate our model. We also conduct robustness checks against various alternative assumptions and extensions.

Second, in the *benchmark model* we consider more realistic process of labor productivity influenced by the AI while maintaining an exogenously fixed α. While also more intricate, we can still analytically characterize the local and global properties of the steady states. Most of key insights from the simple model persists, except that now the benchmark mode can generate a phenomenon of technological unemployment, and global indeterminacy can arise. It highlights

	Simple Model	Benchmark Model	Full Model
Labor productivity	$y_t = y$ Constant	$y_t = y(A_t)$ Increasing in AI	$y_t = y(A_t)$ Increasing in AI
Job-finding rate	$\alpha_t = \alpha$ Constant	$\alpha_t = \alpha$ Constant	$\alpha_t = \alpha(s_t)$ Free entry

Table 1: Summary of model specifications.

the uncertain long-run consequence due to the AI's positive impact to labor productivity.

Finally, the *full model* endogenizes job creation, where firms can create vacancies until the standard free-entry condition is met. Additionally, local indeterminacy can arise in the full model. It highlights that the mechanism of job creation and destruction by AI. As we will see, now local indeterminacy can arise, and the inefficiency loss cannot be fixed by the usual Hosios condition alone. It highlights the uncertain short-run consequence from the AI and the new policy implication. Table 1 summarizes the distinctive features of the simple model, the benchmark model and the full model.

3. Equilibrium Characterization

In this section, we characterize the equilibrium under the simple model, the benchmark model and the full model, respectively. By comparing the full model with the two simplified ones, we can gain insights toward understanding how adding AI's productivity effect and endogenous job finding depending on the joint surplus ratio alter the theoretical findings.

3.1. Simple Model

Consider the simple model with constant labor productivity, $y_t = y$ and constant job-finding rate, $\alpha_t = \alpha$. Given the result of Proposition 1, the dynamics of the simple model is completely characterized by the following system:

$$\dot{A}_t = (\mu H_t - \delta) A_t, \tag{14}$$

$$\dot{H}_t = \alpha (1 - H_t) - \sigma H_t - \rho\mu A_t H_t I_t, \text{ where } I_t = 1 \text{ iff } s_t < \pi^A, \tag{15}$$

$$\dot{s}_t = [r + \sigma + (1-\theta)\alpha] s_t - (1-b) - \rho\mu A_t \left(\pi^A - s_t\right)_+ . \tag{16}$$

At the steady state, provided it exists, the growth rates $\dot{A}_t/A_t$, $\dot{H}_t/H_t$, $\dot{s}_t/s_t$ and I_t converge to constant values, denoted as $A_\infty \equiv \lim_{t\to\infty} A_t$ and similarly for H_∞, s_∞ and I_∞. It follows from equations (15) and (16) the convergence of $\dot{H}_t/H_t$ and $\dot{s}_t/s_t$ implies H_∞ and s_∞ to finite values. However, equation (14) admits the possibility of unbounded A_∞ with an endogenous growth rate; for convenience we still refer such situation as to a steady state. The following proposition

	AI	Unemployment	Wage
Some-AI Steady State	dA_∞	$d(1-H_\infty)$	dw_∞^0
$d\mu$	+	+	+
$d\delta$	−	−	−
Unbounded-AI Steady State	dg_∞^A	$d(1-H_\infty)$	dw_∞^1
$d\alpha$	+	−	−
$d\sigma$	−	+	−

Table 2: Comparative Statics.

classifies steady states into the different types, which depend crucially on the long-run AI adoption decision and AI capability.

Proposition 2 ***(Steady State and Comparative Statics).*** *Suppose a steady state exists.*

a.(Some AI) If the steady state features finite AI capability, i.e., $g_\infty^A = 0$, then $I_\infty = 1$, $A_\infty = \frac{1}{\rho\mu}\left[\alpha\left(\frac{\mu}{\delta}-1\right)-\sigma\right]$ and $H_\infty = H^{AI} \equiv \frac{\delta}{\mu}$.

b.(Unbounded AI) If the steady state features infinite AI capability, i.e., $g_\infty^A > 0$ and $A_\infty = \infty$, then $I_\infty = 0$ and $H_\infty = H^{MP} \equiv \frac{\alpha}{\alpha+\sigma}$.

c.(No AI) If the steady state features zero AI capability, i.e., $g_\infty^A < 0$ and $A_\infty = 0$, then $H_\infty = H^{MP} \equiv \frac{\alpha}{\alpha+\sigma}$.

d. The comparative statics are given by Table 2.

If $I_\infty = 1$, i.e., AI is adopted to replace workers in the long run, then the long-run growth rate of the AI capability is zero, $\dot{A}_\infty/A_\infty = 0$, with a bounded-AI capability given by $A_\infty = A^{\text{AI}}$ and the steady-state level of employment is $H_\infty = H^{\text{AI}} \equiv \frac{\delta}{\mu}$, which will be referred to as the some-AI steady state. Notably, throughout all three models, the law of motion of A along pins down the some-AI steady-state H, driven solely by AI learning and error rates (μ, δ), independent of the exposure rate ρ; nonetheless, there will be a dynamic employment effect of ρ along the transition to be examined in the quantitative section.

If $I_\infty = 0$, i.e., AI is not adopted to replace workers in the long run, then there are three subcases. In the first subcase, the long-run growth rate of the AI capability is positive, $g_\infty^A > 0$, with $A_\infty = \infty$. The steady-state level of employment is $H_\infty = H^{\text{MP}}$, which is the same as the standard Mortensen-Pissarides model without AI, although AI capability grows unboundedly. In this case, the level of employment is high enough to for AI to learn to improve its capability, i.e., $g_\infty^A = \mu H^{\text{MP}} - \delta > 0$, resulting in the growth of AI at the endogenous rate g_∞^A. We refer this as to the unbounded-AI equilibrium.

In the second subcase of $I_\infty = 0$, the steady-state level of employment is still $H_\infty = H^{\text{MP}}$, but the long-run growth rate of the AI capability is negative, $g_\infty^A = \mu H^{\text{MP}} - \delta < 0$. In this case, the level of employment is not high enough to compensate for its deterioration in capability. We refer this as to the no-AI steady state. In the third subcase of $I_\infty = 0$, we have $g_\infty^A = \mu H^{\text{MP}} - \delta = 0$. In this degenerate situation, the steady-state level of AI capability is indeterminate.

The premise of Proposition 2 is that a steady state exists and is unique such that comparative statics are well-defined. The following proposition confirms that and provides the condition for each type of steady state to emerge. It also characterizes the path of I_t that is a key to determine which types of equilibrium emerges.

Proposition 3 ***(Global Determinacy).*** *The steady state exists and is unique.*

a.(Some AI) If $\frac{1-b}{r+\sigma+\alpha(1-\theta)} < \pi^A$*, then the equilibrium AI adoption is* $I_t = 1$ *for all* t*. The steady state is given by* $H_\infty = H^{AI}$ *with and* $A_\infty = A^{AI}$*.*

b. If $\frac{1-b}{r+\sigma+\alpha(1-\theta)} \geq \pi^A$*, then the equilibrium AI adoption is* $I_t = 0$ *for all* t*. The steady state is given by* $H_\infty = H^{MP}$ *and*

b-i.(Unbounded AI) $A_\infty = \infty$ *if* $(\mu - \delta)\,\alpha > \delta\sigma$*;*

b-ii.(No AI) $A_\infty = 0$ *if* $(\mu - \delta)\,\alpha < \delta\sigma$*;*

b-iii.(Degenerate) $A_\infty \in (0, \infty)$ *if* $(\mu - \delta)\,\alpha = \delta\sigma$*.*

Proposition 3 provides a complete characterization of the equilibrium in the simple model. First, the simple model is globally determinate, i.e., the equilibrium will eventually converge to a unique allocation, given by one of the four steady states. Second, Proposition 3 characterizes the conditions for each type of steady state to emerge, which depends on two sufficient statistics, s_∞ and π^A. Note that the condition does not depend on the worker's exposure to AI, ρ, although the equilibrium path of $\dot{s}_t$ does. It is because the equilibrium path of $\dot{A}_t$ and $\dot{H}_t$ only depend on I_t and I_t only depends on the ranking of s_t and π^A, rather than their levels. Proposition 3 establishes the result that the ranking of s_t and π^A in the simple model does not depend on ρ. In particular, we have either $s_t < \pi^A$ for all t or $s_t \geq \pi^A$ for all t. Thus, distinguishing the ranking of s_t and π^A in the steady state is sufficient to characterize the entire ranking of ranking of s_t and π^A along the equilibrium path. Finally, note that the steady-state ranking of s_t and π^A does not depend on ρ, because, from the equation of $\dot{s}_t$ (16), ρ does not affect the steady state s_∞ when $s_\infty \geq \pi^A$, and it does only when $s_\infty < \pi^A$, i.e., when the ranking of s_∞ and π^A is given.

Having characterizing the global dynamics leading to each type of the steady state, we now provide further characterization about the local dynamics around the steady state.

Proposition 4 ***(Local Determinacy).*** *In the neigborhood of the steady state* (A_∞, H_∞)*, there exists a unique saddle path of* $\{A_t, H_t\}$ *converging to* (A_∞, H_∞)*. Furthermore, the path converging to some-AI steady state is a spiral if*

$$\mu\alpha < 4\delta^2 \left(1 - \frac{\delta\,(\alpha + \sigma)}{\mu\alpha}\right). \tag{17}$$

The dynamical system of (14), (15) and (16) consists of one jump variable s_t and two predetermined variables A_t and H_t. Thus, such a system is locally determinate if there are two stable manifolds (negative real part of eigenvalues) for the predetermined variables and one unstable manifold (positive real part of eigenvalue) that determines the initial value of the jump variable. Proposition 4 shows it is always the case, no matter which type of the steady state arises. Spiral happens when the eigenvalues are complex, which occurs when the AI's learning rate is moderate such that condition (17) is satisfied. The dynamical system of (14), (15) resembles that of predator-prey model (Lotka 1925, Volterra 1926). As employment (prey) grows, AI's capability (predator) grows as well as it learns faster. However, when the AI's capability becomes sufficiently highly, it also replace employed workers at a faster rate so employment falls. But then AI's learning slows down so AI's capability falls due to its deterioration. This cycle repeats with diminishing magnitude until the steady state is reached. This mechanism is only plausible when the AI learning rate is not too high nor too low.

3.1.1. Constrained Efficiency

Define the (utilitarian) welfare of the economy at t as:

$$\mathcal{W}_t = \int_t^{\infty} e^{-r(\tau-t)} \left[\left\{yH_\tau + by\left(1-H_\tau\right) + \pi^A y\rho\mu A_\tau H_\tau I_\tau\right\}\right] d\tau, \tag{18}$$

which consists of the discount sum of the aggregate flow of the production yH_τ, of unemployment benefit $by\left(1-H_\tau\right)$, and of replacement gain from AI $\pi^A y\rho\mu A_\tau H_\tau I_\tau$. Denote the constrained optimal allocation $\{H_t^p, A_t^p, I_t^p\}_{t=0}^{\infty}$ as the allocation that maximizes the welfare subject to the search friction and AI evolution, i.e.,

$$\begin{aligned} \mathcal{W}_0^* \;&=\; \max_{\left\{H_t^p, A_t^p, I_t^p\right\}} \int_0^{\infty} e^{-rt}\left\{yH_t^p + by\left(1-H_t^p\right) + \pi^A y\rho\mu A_t^p H_t^p I_t^p\right\} dt, \\ &\quad\; \text{subject to (1) and (8) given } H_0 \text{ and } A_0. \end{aligned} \tag{19}$$

In the simple model, the constrained optimal welfare can be recursively expressed as $\mathcal{W}_t^* = \mathcal{W}\left(H_t, A_t\right)$, which is the solution to the following HJB equation:

$$r\mathcal{W}\left(H_t, A_t\right) = \max_{I_t^p \in [0,1]} \left\{ \begin{array}{l} yH_t + by\left(1-H_t\right) + \pi^A y\rho\mu A_t H_t I_t^p \\ +\mathcal{W}_H\left(H_t, A_t\right)\left[\alpha\left(1-H_t\right) - \sigma H_t - \rho\mu A_t H_t I_t^p\right] \\ +\mathcal{W}_A\left(H_t, A_t\right) A_t\left(\mu H_t - \delta\right) \end{array} \right\}, \tag{20}$$

where $\mathcal{W}_H\left(H_t, A_t\right) \equiv \partial\mathcal{W}\left(H_t, A_t\right)/\partial H_t$ and $\mathcal{W}_A\left(H_t, A_t\right) \equiv \partial\mathcal{W}\left(H_t, A_t\right)/\partial A_t$. Define the social values of employment and of AI as:

$$\omega_t^H \equiv \frac{1}{y}\mathcal{W}_H\left(H_t^p, A_t^p\right), \; \omega_t^A \equiv \frac{1}{y}\mathcal{W}_A\left(H_t^p, A_t^p\right).$$

Applying an envelop theorem on (20) with respect to H_t and A_t, the laws of motion of ω_t^H and ω_t^A are given by,

$$\dot{\omega}_t^H = (r+\sigma+\alpha)\,\omega_t^H - (1-b) - \rho\mu A_t^p\left(\pi^A - \omega_t^H\right)_+ - \mu A_t^p \omega_t^A, \tag{21}$$

$$\dot{\omega}_t^A = (r-\mu H_t^p + \delta)\,\omega_t^A - \rho\mu H_t^p\left(\pi^A - \omega_t^H\right)_+. \tag{22}$$

The following proposition characterizes the constrained optimal allocation of the simple model, which depends on the social values of employment.

Proposition 5 ***(Constrained Optimum).*** *The constrained optimal allocation features AI adoption, i.e., $I_t^p = 1$, if and only if $\omega_t^H < \pi^A$. In particular, if $\frac{1-b}{r+\sigma+\alpha} \geq \pi^A$, the constrained optimal allocation is given by $I_t^p = 0$ for all t. Otherwise, $I_t^p = 1$ for a sufficiently large t. The comparison between the equilibrium allocation and the constrained optimal allocation is as follows.*

a. If $\pi^A \leq \frac{1-b}{r+\sigma+\alpha}$, then AI adoption in the equilibrium is constrained optimal, i.e., $I_t = I_t^p = 0$.

b. If $\frac{1-b}{r+\sigma+\alpha} < \pi^A \leq \frac{1-b}{r+\sigma+\alpha(1-\theta)}$ then AI is under-adopted in the equilibrium, i.e., $0 = I_t \leq I_t^p$.

c. If $\frac{1-b}{r+\sigma+\alpha(1-\theta)} < \pi^A$, then AI is over-adopted in the equilibrium, i.e., $1 = I_t \geq I_t^p$.

Recall that AI is adopted ($I_t = 1$) in the equilibrium when $s_t < \pi^A$ but AI is adopted in constrained optimal allocation ($I_t^p = 1$) when $\omega_t^H < \pi^A$. So the equilibrium AI adoption is not constrained optimal when $s_t < \pi^A \leq \omega_t^H$ or $\omega_t^H < \pi^A \leq s_t$, over-adoption in the formal case but under-adoption in the latter. Proposition 5 characterizes the condition when over-adoption or under-adoption happens. In particular, when $\pi^A \leq \frac{1-b}{r+\sigma+\alpha}$, AI is not adopted in either equilibrium, and the equilibrium is constrained optimal (Proposition 5a). If $\pi^A > \frac{1-b}{r+\sigma+\alpha}$, then AI is eventually adopted to replace workers in the constrained optimal allocation. Recall in Proposition 3b that AI is never adopted if $\frac{1-b}{r+\sigma+\alpha(1-\theta)} \geq \pi^A$, so AI is under-adopted in Proposition 5b. It is because the firm has too much bargaining power such that the unemployed worker's value is suppressed, leading the joint surplus from the match is greater than the value of replacement by AI On the other hand, AI is always adopted to replace workers if $\frac{1-b}{r+\sigma+\alpha(1-\theta)} < \pi^A$ (Proposition 3a), so AI is over-adopted (Proposition 5c). In this case, the worker has too much bargaining power, leading the firm to over-adopt AI to replace workers.

The above logic suggests that the social planner might want to regulate bargaining power to implement the constrained optimal allocation – a similar idea proposed by Hosios (1990) for the standard labor search model. The following proposition establishes the Hosios condition for our simple model of AI

Proposition 6 ***(Hosios Condition under Exogenous Job Finding Rate).*** *Suppose $\frac{1-b}{r+\sigma+\alpha} < \pi^A$, i.e., the equilibrium in the simple model is not constrained optimal. If there exists firm's bargaining power $\theta_t^p \in [0,1]$ that implements the constrained optimal allocation with the equilibrium, then θ_t^p is given by,*

$$\theta_t^p = \frac{\mu A_t^p \omega_t^A}{\omega_t^H}. \tag{23}$$

Unlike the typical Hosios condition, in the presence of AI the optimal bargaining power is no longer constant. This is because the constrained optimal adoption I_t^p could be time-varying, but equilibrium adoption for any constant bargaining power must be time-invariant, according to Proposition 1. Thus, the social planner needs to change the firm's bargaining power over time. Recall employment generates a positive externality on AI capability by improving its learning. The level of externality is related to the ratio $\mu A_t^p \omega_t^A / \omega_t^H$, depending on the AI learning rate and the ratio between the social values of AI and of employment. Thus, employment should not be replaced if the externality is sufficiently high. To implement that in the equilibrium, the joint surplus must be high such that AI is not adopted to replace worker (Proposition 1). Recall that the joint surplus is increasing in the firm's bargaining power by suppressing the unemployed worker's value, thus, the optimal firm's bargaining power is regulated such that (23) is satisfied.

3.2. Benchmark Model

Now consider the benchmark model where AI capability improves labor productivity such that

$$y_t = y(A_t), \tag{24}$$

where we assume $y'(A_t) \geq 0$, $y''(A_t) \leq 0$ and $y(0) = y$. Denote the AI elasticity of productivity as $\varepsilon_y \equiv Ay'(A)/y(A)$, where we suppress its potential dependence on A_t. The output growth in the benchmark model is thus $g_t = \varepsilon_y g_t^A$. The dynamical system becomes:

$$\dot{A}_t = (\mu H_t - \delta) A_t, \tag{25}$$

$$\dot{H}_t = \alpha(1 - H_t) - \sigma H_t - \rho\mu A_t H_t I_t, \text{ where } I_t = 1 \text{ iff } s_t < \pi^A, \tag{26}$$

$$\dot{s}_t = [r + \sigma + (1-\theta)\alpha - \varepsilon_y(\mu H_t - \delta)] s_t - (1-b) - \rho\mu A_t \left(\pi^A - s_t\right)_+. \tag{27}$$

The benchmark model still features the same four types of steady state as in the simple model, summarized in Lemma 2. But the characterization is modified by the following proposition.

Proposition 7 *(Steady States).* *In the benchmark model, the steady state exists but may not be unique.*

a.(Some AI) If $\frac{1-b}{r+\sigma+\alpha(1-\theta)} < \pi^A$*, then there exists a steady state with* $H_\infty = H^{AI}$ *and* $A_\infty = A^{AI}$ *where AI is adopted to replace worker eventually, i.e.,* $I_\infty = 1$

b. Suppose $\frac{1-b}{r+\sigma+\alpha(1-\theta)-\varepsilon_y(\mu H^{MP}-\delta)} \geq \pi^A$*.*

b-i.(Unbounded AI) If $(\mu - \delta)\alpha > \delta\sigma$*, then there exists a steady state with* $H_\infty = H^{MP}$ *and* $A_\infty = \infty$*.*

b-ii.(No AI) If $(\mu - \delta)\alpha < \delta\sigma$*, then there exists a steady state with* $H_\infty = H^{MP}$ *and* $A_\infty = 0$*.*

b-iii.(Degenerate) If $(\mu - \delta)\alpha = \delta\sigma$*, then there exists a steady state with* $H_\infty = H^{MP}$ *and* $A_\infty \in (0, \infty)$*.*

b-iv. In all the subcases of b, the AI adoption is not adopted eventually, i.e., $I_\infty = 0$. *Furthermore, if* $H_0 > H^{MP}$, *then* $I_t = 0$ *for all* t.

c. Proposition 4 still holds in the benchmark model.

Global Indeterminacy, Local Determinacy. Multiple steady states are now plausible in the benchmark model, which happens when the conditions in Proposition 7a and 7b-i are both satisfied, i.e.,

$$0 < r + \sigma + \alpha(1-\theta) - \frac{1-b}{\pi^A} \leq \varepsilon_y \left(\frac{\mu\alpha}{\alpha+\sigma} - \delta \right). \tag{28}$$

When AI improves labor productivity, the benchmark model becomes globally indeterminate – given an initial state (H_0, A_0), the economy can converge to either the some-AI steady state or the unbounded-AI steady state. According to the above condition. It could occur when the AI elasticity of productivity ε_y is sufficiently high, or the firm's bargaining power θ is sufficiently high.

Intuitively, global indeterminacy occurs in the benchmark model because a path of growing or constant labor productivity can be self-fulfilling in the steady state. If firms expect high surplus ratio in the future such that $s_\infty > \pi^A$, firms will not adopt AI to replace workers, then the steady-state employment H_∞ will be high, then AI can learn faster. If the AI elasticity of productivity ε_y is sufficiently high, then the higher AI growth rate will yield sufficiently high growth in labor productivity that supports the expectation of a higher surplus ratio, $s_\infty \geq \pi^A$, in the first place. Thus, the non-adoption of AI can be self-fulfilling in the steady state. Similarly, low surplus ratio, $s_\infty < \pi^A$, can be self-fulfilling, leading to adoption of AI to replace workers in the steady state.

To see the role of the firm's bargaining power θ in global indeterminacy, note that θ enters the ODE of the surplus ratio, (27), because the flow of the unemployed workers' value involves their expected gain in value when they find a job, captured by $\alpha_t(1-\theta)s_t$, where they obtain a faction $1-\theta$ from the surplus of the next job which they find at the rate α_t. As a result, a higher θ leads a lower flow to the unemployed workers, similar to the effect of a lower unemployment benefit b, implying a lower outside option and hence a higher surplus for the current job. In this case, a higher θ will imply s_t more sensitive to the labor productivity of the job, making agents vulnerable to the self-fulfilling mechanism we described above.

Technological Unemployment. In some-AI steady state, comparative statics demonstrate a phenomenon of technological unemployment. Specifically, improvements in the AI learning rate (higher μ) or reductions in AI's error rate (lower δ) increases the labor productivity, $y\left(A^{\mathrm{AI}}\right)$ by raising the steady-state level of AI capability, A^{AI}. It also increases the long-run wage, w^0_∞. Nonetheless, the same parameter shifts also raise the unemployment rate, $1 - H^{\mathrm{AI}}$. Put differently, more capable AI curtails more employment—even as those who remain employed benefit from higher wages with a higher labor productivity. This outcome follows the intuition from the simple model. Once AI has displaced a substantial portion of the workforce, further decline in employment constrains AI's learning capabilities and thus prevents further expansion of AI until the some-AI steady state is reached. Thus, more capable AI displaces more employment at the some-AI steady state. By contrast, technological unemployment does not emerge in the unbounded-AI steady state. There, a higher AI growth rate is instead associated with lower unemployment rate, making the phenomenon of technological unemployment implausible in that regime.

Constrained Efficiency. In the benchmark model, the welfare can be recursively expressed as the solution to the following HJB equation:

$$r\mathcal{W}\left(H_t, A_t\right) = \max_{I_t^p \in [0,1]} \left\{ \begin{array}{l} y\left(A_t\right) H_t + by\left(A_t\right)\left(1 - H_t\right) + \pi^A y\left(A_t\right) \rho A_t H_t I_t^p \\ + \mathcal{W}_H\left(H_t, A_t\right)\left[\alpha\left(1 - H_t\right) - \sigma H_t - \rho A_t H_t I_t^p\right] \\ + \mathcal{W}_A\left(H_t, A_t\right) A_t\left(\mu H_t - \delta\right) \end{array} \right\}. \tag{29}$$

Since labor productivity is varying in the AI capability, we scale the welfare by output as $\omega_t \equiv \mathcal{W}\left(H_t^p, A_t^p\right)/y\left(A_t\right)$. The HJB equation for ω_t is given by,

$$\left(r - g_t\right)\omega_t = \dot{\omega}_t + H_t + b\left(1 - H_t\right) + \pi^A \rho A_t H_t I_t^p.$$

Since labor productivity is increasing in AI capability in the benchmark model, modify the social values of employment and of AI as:

$$\begin{aligned} \omega_t^H &\equiv \frac{\partial}{\partial H_t}\left[\frac{\mathcal{W}\left(H_t^p, A_t^p\right)}{y\left(A_t\right)}\right] = \frac{\mathcal{W}_H\left(H_t^p, A_t^p\right)}{y\left(A_t\right)}, \\ \omega_t^A &\equiv \frac{\partial}{\partial A_t}\left[\frac{\mathcal{W}\left(H_t^p, A_t^p\right)}{y\left(A_t\right)}\right] = \frac{\mathcal{W}_A\left(H_t, A_t\right)}{y\left(A_t\right)} - \frac{\varepsilon_y\left(A_t\right)}{A_t}\omega_t. \end{aligned}$$

The first order condition implies the constrained optimal adoption of AI given by,

$$I_t^p = 1 \text{ if and only if } \omega_t^H < \pi^A, \tag{30}$$

where the laws of motion of ω_t^H and ω_t^A are given by applying an envelop theorem with respect to H_t and A_t:

$$\dot{\omega}_t^H = \left(r + \sigma + \alpha - g_t\right)\omega_t^H - \left(1 - b\right) - \rho\mu A_t^p\left(\pi^A - \omega_t^H\right)_+ - \mu A_t^p \omega_t^A, \tag{31}$$

$$\dot{\omega}_t^A = \left(r + \delta - g_t - g_t^A\right)\omega_t^A - \rho\mu H_t^p\left(\pi^A - \omega_t^H\right)_+. \tag{32}$$

To implement the constrained optimal adoption of AI, i.e., $I_t = I_t^p$, the social planner wants to regulate θ_t^p such that the surplus ratio equal to the social value of employment, $s_t = \omega_t^H$. By comparing the ODE of $\dot{s}_t$ and $\dot{\omega}_t^H$, the social-optimal bargaining power θ_t^p is the same as (23) in the simple model. It is because the social planner and the firm internalize the growth prospect of AI on output in the same way, captured by the term $-\varepsilon_y(A_t)(\mu H_t - \delta)s_t$ in (27) for firms and $-\varepsilon_y(A_t)(\mu H_t - \delta)\omega_t^H$ in (31) for the social planner. Thus, no modification to the social-optimal bargaining power is needed when AI improves labor productivity and Proposition **6** continues to hold true.

3.3. Full Model

Now consider the full model where firms can create vacancies such that job finding rate become endogenous.

Job Creation. Firms can create a job with vacancy cost κy_t, which is filled at rate f_t. Like in the standard labor-search model, given the job-finding rate α_t, a constant-returns-to-scale matching function implies there exist a decreasing function Λ relating the job-finding rate and the vacancy-filling rate such that $\alpha_t = \Lambda(f_t)$, $\Lambda' < 0$.[4] In the equilibrium, firms will create new jobs until the free-entry condition, $\kappa y_t = f_t \Pi_t$. Using Proposition 1 and the bargaining solution (10), we can rewrite the free-entry condition as:

$$\alpha_t = \alpha(s_t) \equiv \Lambda\left(\frac{\kappa}{\theta s_t}\right). \tag{33}$$

The dynamics of the full model becomes:

$$\begin{aligned} \dot{A}_t &= (\mu H_t - \delta) A_t, && (34)\\ \dot{H}_t &= \alpha(s_t)(1 - H_t) - \sigma H_t - \rho\mu A_t H_t I_t, \text{ where } I_t = 1 \text{ iff } s_t < \pi^A, && (35)\\ \dot{s}_t &= [r + \sigma + (1-\theta)\alpha(s_t) - \varepsilon_y(\mu H_t - \delta)] s_t - (1-b) - \rho\mu A_t \left(\pi^A - s_t\right)_+. && (36) \end{aligned}$$

A steady-state surplus ratio solves $\dot{s}_\infty = 0$, which is given by the fixed point to

$$s_\infty = \frac{(1-b) + \rho\mu A_\infty \left(\pi^A - s_\infty\right)_+}{r + \sigma + (1-\theta)\alpha(s_\infty) - \varepsilon_y(\mu H_\infty - \delta)}. \tag{37}$$

The full model also features the same four types of steady state, but we no longer have the analytical characterization. Consider the some-AI steady state, i.e., $H_\infty = H^{\text{AI}} \equiv \delta/\mu$, where

[4]For an example of Cobb-Douglas matching function $m = v^\xi u^{1-\xi}$, where $u = 1 - H$ is the unemployment rate and v is the vacancy rate, the vacancy-filling rate is $f = m/v = (v/u)^{\xi-1}$ and thejob-finding rate is $\alpha = m/u = (v/u)^\xi = \Lambda(f) \equiv f^{1-1/\xi}$.

the surplus ratio $s_\infty = s^{\mathrm{AI}}$ is the fixed point to the following:

$$s^{\mathrm{AI}} = \frac{(1-b) + \left(\frac{\mu}{\delta} - 1\right)\alpha\left(s^{\mathrm{AI}}\right)\pi^A}{r + \left(\frac{\mu}{\delta} - \theta\right)\alpha\left(s^{\mathrm{AI}}\right)}.$$

For AI to be adopted at the steady state, it is necessary that $I_\infty = 1$, which requires $s^{\mathrm{AI}} < \pi^A$. On the other hand, consider the unbounded-AI steady state or the no-AI steady state,. i.e., $I_\infty = 0$ and the employment rate H_∞ and the surplus ratio s_∞ are the fixed point to the following:

$$H_\infty = \frac{\alpha(s_\infty)}{\alpha(s_\infty) + \sigma}, \ s_\infty = \frac{1-b}{r + \sigma + \alpha(s_\infty)(1-\theta) - \varepsilon_y(\mu H_\infty - \delta)}.$$

For AI not to be adopted at the steady state, it is necessary that $I_\infty = 0$, which requires $s_\infty \geq \pi^A$. The unbounded steady state features $\dot{A}_\infty > 0$, i.e., $\mu H_\infty > \delta$, vice versa for the no-AI steady state. Note that unlike the benchmark model, the employment levels of the unbounded steady state and of the no-AI steady state are now different because the job-finding rate is endogenously determined as $\alpha(s_\infty)$ in the full model, instead of being constant in the benchmark model. To differentiate them, we label the unbounded-AI stead by AJJ instead (e.g.., $H_\infty = H^{\mathrm{AJJ}}$, where $\mu H^{\mathrm{AJJ}} > \delta$), to be compared with the endogenous growth equilibrium in Aghion et al. (2019). Before shifting our attention to endogenous growth, we would like to compare briefly the some-AI and unbounded-AI equilibria.

Proposition 8 ***(Comparison Across Steady States).*** *Suppose the some-AI steady state and the unbounded-AI steady state coexists. Then the unemployment rate is lower in the unbounded-AI steady state, i.e.,*

$$\alpha\left(s^{AJJ}\right) > \alpha\left(s^{AI}\right),\ H^{AJJ} > H^{AI}.$$

However, the labor share is lower in the unbounded-AI steady state if the firm's bargaining power θ is sufficiently low.

The lower labor share is due to the threat of AI exposure via ρ and, as a result of renegotiation, the wage share is lower. Similar finding arises in the automation model of Arnoud (2018).

Unbounded AI and Endogenous Growth. In the unbounded-AI equilibrium, AI capability grows endogenously at the rate $g_t^A = \mu H_t - \delta$. The following proposition compares the asymptotic growth rate under different AI learning rate and AI failure rate.

Proposition 9 ***(Endogenous Growth).*** *The unbounded-AI features endogenous growth, with an asymptotic growth rate at $g_A = \mu H^{AJJ} - \delta$, which is increasing in the AI learning rate μ and factors raising the job finding rate, but decreasing in AI failure rate δ.*

In contrast to AJJ where endogenous growth is a result of new idea discovery, it is in our model a result of AI learning from employed workers. Thus, when AI can learn faster from workers (or fails less often), the AI capability grows faster as well, which leads to higher productivity growth and employment. A higher growth in AI is sustainable because AI has more employed workers to learn from.

Constrained Efficiency. In the full model, the welfare is measure by,

$$\mathcal{W}_t = \int_t^\infty e^{-r(\tau-t)} \left[\left\{y\left(A_\tau\right) H_\tau + \left[b - \kappa\Theta\left(\alpha_t\right)\right] y\left(A_\tau\right)\left(1-H_\tau\right) + \pi^A y\left(A_\tau\right) \rho\mu A_\tau H_\tau I_\tau\right\}\right] d\tau. \tag{38}$$

With the endogenous job-finding rate α_t, the total vacancy cost is $\kappa y\left(A_\tau\right)\Theta\left(\alpha_t\right)\left(1-H_t\right)$, where $\Theta\left(\alpha_t\right)$ is ratio between the measure of vacancies posted and the unemployment rate (also known as the market tightness), given by,

$$\Theta\left(\alpha_t\right) \equiv \frac{\alpha_t}{\Lambda^{-1}\left(\alpha_t\right)}, \text{where } \Theta'\left(\alpha_t\right) = \frac{1}{\xi_t f_t} \tag{39}$$

and ξ_t is the vacancy elasticity of the matching function. For example, the Cobb-Douglas matching function $m_t = v_t^\xi\left(1-H_t\right)^{1-\xi}$ implies $f_t = m_t/v_t = \left[v_t/\left(1-H_t\right)\right]^{\xi-1}$ and $\alpha_t = m_t/\left(1-H_t\right) = \Lambda\left(f_t\right)$, where $\Lambda\left(f\right) = f^{-\xi/(1-\xi)}$ and hence $\Theta\left(\alpha\right) = \alpha^{1/\xi}$.

Denote $\mathcal{W}_t^*$ as the maximal welfare attained by the constrained-optimal allocation $\{H_t^p, A_t^p\}$. By scaling the constrained-optimal welfare by output as $\omega_t \equiv \mathcal{W}_t^*/y\left(A_t\right)$, the HJB equation for ω_t is given by,

$$\left(r-g_t\right)\omega_t = \dot{\omega}_t + H_t + \left[b - \kappa\Theta\left(\alpha_t\right)\right]\left(1-H_t\right) + \pi^A\rho\mu A_t H_t I_t. \tag{40}$$

The first order condition implies the constrained optimal adoption of AI and of job-filling rate given by,

$$I_t^p = 1 \text{ if and only if } \omega_t^H < \pi^A, \tag{41}$$

$$\kappa\Theta'\left(\alpha_t^p\right) = \omega_t^H, \tag{42}$$

where the laws of motion of ω_t^H and ω_t^A are given by applying an envelop theorem with respect to H_t and A_t:

$$\dot{\omega}_t^H = \left[r + \sigma + \alpha_t - \varepsilon_y\left(\mu H_t^p - \delta\right)\right]\omega_t^H - \kappa\Theta\left(\alpha_t\right) - (1-b) - \rho\mu A_t^p\left(\pi^A - \omega_t^H\right)_+ - \mu A_t^p \omega_t^A, \tag{43}$$

$$\dot{\omega}_t^A = \left[r + \delta - \left(1+\varepsilon_y\right)\left(\mu H_t^p - \delta\right)\right]\omega_t^A - \rho\mu H_t^p\left(\pi^A - \omega_t^H\right)_+. \tag{44}$$

If it is constrained optimal not to adopt AI at the steady state, i.e., $I_\infty^p = 0$, then the social value of employment solves:

$$\omega_{\infty}^{H}=\frac{\kappa\Theta\left(\alpha_{\infty}^{p}\right)+(1-b)}{r+\sigma+\alpha_{\infty}^{p}-\varepsilon_{y}\left(\mu H_{\infty}^{p}-\delta\right)},$$

where the constrained-optimal job-finding rate and employment rate are given by,

$$\kappa\Theta'\left(\alpha_{\infty}^{p}\right)=\omega_{\infty}^{H},\ H_{\infty}^{p}=\frac{\alpha_{\infty}^{p}}{\alpha_{\infty}^{p}+\sigma}.$$

It is necessary that

$$\omega_{\infty}^{H}\geq\pi^{A}. \tag{45}$$

In the spirit of Hosios condition, the following proposition specifies the bargaining power and transfer policy that can implement the constrained optimal allocation with the equilibrium allocation.

Proposition 10 ***(Hosios Condition and Optimal Subsidy).*** *Suppose there exists firm's bargaining power $\theta_t^p\in[0,1]$ and (per-output) subsidy T_t to firm-worker matches that implements the constrained optimal allocation with the equilibrium, then θ_t^p and T_t are given by,*

$$\theta_t^p = \xi_t, \tag{46}$$

$$T_t = \mu A_t^p\omega_t^A\geq 0, \tag{47}$$

where the strict inequality holds if $I_\tau^p=1$ for some positive measure of $\tau\geq t$.

The (per-output) subsidy rate T_t can be financed by lump-sum tax on every agent. Whoever, the firm or the employed worker, receives the subsidy will not alter the allocation, because the level of wage will be accommodated in the outcome of Nash bargaining, like the standard irrelevance of tax incidence.

Unlike the simple model and the benchmark model, the Hosios condition in the full model echoes the standard Hosios condition that fixes the firm's bargaining power to the vacancy elasticity of matching, $\theta_t^p=\xi$. It is because in the full model, the social planner wants to induce firms to create vacancies by internalizing the positive externality every vacancy create to workers, like in the standard labor-search model without AI. However, setting $\theta_t^p=\xi$ is not enough because there is another source of inefficiency, coming from the suboptimal adoption of AI. Since constrained optimal adoption of AI is achieved by firms if the surplus ratio equal to the social value of employment, $s_t=\omega_t^H$, direct transfers to the firm-worker match by subsidizing output $(1+T_t)\,y_t$ is essential to target $s_t=\omega_t^H$. To implement the constrained optimal allocation, the level of subsidy equal to the flow of positive externality every firm-worker match engendered on AI, captured by the per-worker arrival rate, μA_t^p, times the social value of AI, ω_t^A.

<table>
<tr><th></th><th>Simple Model</th><th>Benchmark Model</th><th>Full Model</th></tr>
<tr><td>Uniqueness</td><td>Global and local determinacy (Prop. 3 and 4)</td><td>Local determinacy, global indeterminacy possible (Prop. 7)</td><td>Global and local indeterminacy possible (Figure 2 and 4)</td></tr>
<tr><td>Dynamics</td><td colspan="3">Spiral possible for the some-AI steady state (Prop. 4 and 7; figure 3)</td></tr>
<tr><td>Policy implications</td><td colspan="2">Regulate bargaining powers w.r.t. modified Hosios condition (Prop. 6)</td><td>Standard Hosios condition and subsidies to jobs exposed to AI risks (Prop. 10)</td></tr>
</table>

Table 3: Summary of theoretical findings.

3.4. Taking Stock

To conclude, other than the degenerate case, we obtain three types of the equilibrium: some-AI, unbounded-AI and no-AI. The simple model is global and local determinate. Global indeterminacy becomes possible in the benchmark model with AI influencing labor productivity, whereas in the most general full model when the job finding rate is endogenized both global and local indeterminacy are possible to arise in equilibrium. The intuition is rooted on self-fulfilling prophecies as a result of two dynamic interaction channels between firms and workers in the presence of AI: (i) learning-productivity channel: AI learns from workers and workers becomes more productive with AI capability expansion; (ii) matching externality: the "thickness" of workers induces firms' vacancy creations which justifies workers search and matching. When the learning-productivity channel is absent (simple model), indeterminacy cannot arise. In the benchmark model with the learning-productivity interactions, the second channel is not strong enough due to exogenous job finding rates, thus yielding global but not local indeterminacy. Both types of indeterminacy emerge in the full model with endogenous job finding rates. Around the some-AI steady state, spirals are possible. Due to various externalities through search, matching and AI adoption, the equilibrium cannot achieve constrained optimum. In both the simple and the benchmark model, to restore constrained optimum, it requires a modified Hosios condition to regulate bargaining powers. In the full model, to achieve constrained optimum requires the original form of Hosios' rule together with a subsidy policy to jobs exposed to AI. A summary of main theoretical findings over the three models is provided in Table 3.

4. Calibration

Overview. To quantify the impact of AI, we compute the equilibrium path starting from $t = 0$ to $t = \infty$ in the full model, which is completely characterized by the first order ODE system of $\left\{\dot{A}_t, \dot{H}_t, \dot{s}_t\right\}$ stated by equations (34), (35) and (36). The parameters used in the ODE system are calibrated directly or indirectly with the features of the U.S. labor market and empirical findings from the micro data of generative AI we detail below. To solve the ODE system of $\left\{\dot{A}_t, \dot{H}_t, \dot{s}_t\right\}$, we also need the boundary conditions, one for each of them (since the

ODE system is first order): we have the predetermined variable H_0 directly from the data of employment, the predetermined variable A_0 calibrated to match the estimated impact of AI adoption, and the jump variable s_0 calibrated to match the initial job-filling rate in the data.

Before we dive in the details, it is important to notice the challenge of calibrating a model with new technologies like AI. In the typical calibration exercises in the literature, it is innocuous and technically convenient to assume the economy in the neighborhood of the steady state, so the model's parameters are calibrated to match its steady state with the empirical moments, and its dynamics can be well approximated by linearization around the steady state. This assumption is no longer innocuous with AI, as its emergence only happened very recently. Since AI's long-run impacts remain to be seen in the data, we are reluctant to interpret the current economy as the steady state of our model. Instead, we choose to calibrate the initial state of our model, and let the model predicts the AI's long-run impact in its steady state. But it implies two challenges. First, we need to solve and calibrate the entire transitional dynamics of the model, because the initial state of the jump variable s_0 is endogenous. Second, we need check any possibility of multiple equilibrium paths since global and local indeterminacy can arise, as we have shown in the theory section. If it does happen, then we need to decide an equilibrium selection criterion. We will detail how we handle these two challenges in the next section, after we have explained our choice of empirical moments below.

Before AI. Under our continuous time setting, we normalize a unit interval of time as one year. We assume the economy was initially at the Mortensen-Pissarides steady state without AI ($A_{0_-} = 0$), where the unemployment rate has attained its long-run rate $1 - H_{0_-} = \frac{\sigma}{\alpha_{0_-} + \sigma}$. We follow the labor search literature to assume the long-run unemployment rate of 5%, i.e., $H_{0_-} = 0.95$. The pre-AI labor productivity is normalized at $y_{0_-} = y(0) = 1$. Consider that AI is invented at $t = 0$ such that the level of AI capability jumps from zero to A_0, inheriting the level of employment $H_0 = H_{0_-}$.

AI Elasticity of Productivity. To recognize this in our model, consider a simple constant-elasticity function:

$$y_t = y(A_t) = \begin{cases} A_t^{\varepsilon_y} & A_t \geq A_0, \\ 1 & A_t < A_0. \end{cases} \tag{48}$$

where each worker's productivity rises in AI with a constant elasticity ε_y when AI capability A_t exceeds the breakthrough level A_0, otherwise, output is constant and normalized to one. From micro evidence in the absence of any general equilibrium effect, Czarnitzki, Fernández, and Rammer (2023) find the AI semi-elasticity of sales (their proxy for productivity) as 8.285, with the average AI intensity of AI using firms as 0.129. Thus we set $\varepsilon_y = 0.129 \cdot 8.285 = 1.069$. Notably, our production function is by construction constant returns as it is linear in H_t, but labor productivity is rising more than proportionately over AI capability.

Initial AI Capability. Brynjolfsson, Li, and Raymond (2025) estimate that workers' productivity increases by 14% after their employer has adopted AI. To match that AI adoption effect, we set $y(A_0)/y(0) = 1.14$, which implies $A_0 = 1.130$ following $\varepsilon_y = 1.069$. The initial state of the economy is thus given by $(H_0, A_0) = (0.95, 1.130)$.

AI Exposure. The rate at which human can be replaced, ρ, can be measured by the rate of suitability for machine learning (SML) in Brynjolfsson et al. (2018): on 1-5 scale, SML averages 3.47, which is about $(3.47-1)/(5-1) = 61.8\%$. That is, $\rho = 0.618$.

Cost of AI. In our model, π^A captures the net present value of the firm's benefit from replacing workers with AI for a job. Denote ϕ as the per-output cost of adopting AI for replacing workers. Given the AI failure rate δ, we can express $\pi^A = (1-\phi)/(r+\delta)$, thus we can back out π^A from ϕ. In practice, ϕ may be measured by firms' cost share in AI inputs reported by the industry consulting profession. In some consulting reports, data on GenAI is not separated, we use the figures of all AI as a conservative measure. According to data from Menlo Ventures' report by Tully, Redfern, and Xiao (2024), U.S. business average annual spending on generative AI is \$8.025 billions between 2023 and 2024. Total U.S. business sales was about \$1851 billion in 2023; thus, the business spending on generative AI to sales ratios $\frac{8.025}{1851} = 0.43\%$ in 2023. That is, we set $\phi = 0.0043$, implying $\pi^A = 1.949$.

AI Learning Rate. Differently from software or equipment capital, AI improves its capability by learning-by-using. In our model, each capability unit of AI learns from the workers at the rate μH_t, where AI learning rate μ may be measured directly by micro evidence. Using machine learning rate, it is commonly found to be 0.001 (based on hourly iteration duration), ranging from 0.0001 to 0.01 (cf. Goodfellow, Bengio, and Courville (2016); Ning, Iradukunda, Zhang, and Zhu (2021)). The survey by Crane, Green, and Soto (2025) found that worker's AI adoption rates range from 20% to 40%, so we use 30%. Assuming that AI learns continually from workers for 40 hours per week and 50 weeks per year without counting vacation/sick time or overtime/work at home (the former two lower work hours but the latter two raise them), its learning rate is $\mu H_0 = -0.3 \cdot 50 \cdot 40 \cdot \ln(1-0.001)$, from which we obtain $\mu = 0.632$.

AI Error Rate. In the model, AI also fails at the rate δ. This failure rate can be backed out using the half life of an AI – under Moore's law of, the half life is 1.5 years. Thus, the decay rate is given by δ and by Moore's law, $\delta = -(\ln 0.5)/1.5 = 0.462$. Alternatively, the error rate reported in the literature of natural language processing ranged from 2% to 15%. If we assume that the AI error rate is 4% within a month, then we have $\delta = -12\ln(1-0.04) = 0.489$ which is also close to the number given by the Moore's law.

To calibrate remaining parameters of the labor market, we follow the standard practices in the labor search literature.

Discount Rate. We set $r = \ln(1.05) = 0.049$ to target annual discount rate of 5%.

Job-Finding rate and Separation Rate. Following Shimer (2005), we construct the job-finding rate based on the unemployment flow from the Current Population Survey (CPS). Before the pandemic period, the average monthly job-finding rate is 28% in 2019, which is also close to the average job-finding rate from 2016 to 2023. Thus, we take the steady-state monthly job-finding rate before AI as 28%. In continuous time, the corresponding Poisson rate is computed as $\alpha_{0_-} = -12\log(1-0.28) = 3.942$. We set $\sigma = 0.207$ to match the pre-AI long-run unemployment rate of 5%. From the CPS data, the average monthly job finding in 2023 is also 28%. So, we will calibrate the vacancy cost κ such that in the equilibrium of the

	Parameters	Values	Targets
ε_y	AI elasticity of productivity	1.069	Czarnitzki et al. (2023)
A_0	Initial AI capability	1.130	Brynjolfsson, Li, and Raymond (2025)
ρ	Worker exposure to AI	0.618	Brynjolfsson et al. (2018)
ϕ	Unit cost to replace workers by AI	0.0043	Tully et al. (2024)
μ	AI learning rate	0.632	Ning et al. (2021)
δ	AI error rate	0.462	Moore's law of 18 month half life, monthly error rate 4%
r	Discount rate	0.049	Annual discount rate 5%
κ	Cost of posting vacancies	0.077	2023 average monthly job-finding rate 28%
σ	Job separation rate	0.207	Long-run unemployment rate without AI 5%
ξ	Vacancy elasticity of matching	0.5	Petrongolo and Pissarides (2001)
θ	Firm's bargaining power	0.5	Hosios (1990)
b	Nonemployment income ratio	0.319	Replacement rate 40%, wage income ratio 0.6, leisure consumption share 32.8%

Table 4: Calibration parameters and targets.

full model, we have $\alpha_0 = \Lambda\left(\frac{\kappa}{\theta s_0}\right) = 3.942$.

Matching Elasticity and Bargaining Power. Assume the standard Cobb-Douglas matching function $m = v^{\xi}u^{1-\xi}$, where we can normalize the matching efficiency to one by accommodating the vacancy-posting cost κ. We set the vacancy elasticity of matching as $\xi = 0.5$ (see the survey by Petrongolo and Pissarides (2001)). In the sensitivity analysis, we will use the high value of $\xi = 0.765$ in Hall (2005) or the low value of $\xi = 0.28$ in Shimer (2005). Since Hosios rule applies in our full model, as well as in the standard labor search model, we follow the literature by assuming that bargaining in the labor market is efficient such that $\theta = \xi = 0.5$.

Replacement Rate. The unemployed workers' leisure is assumed to consist of components, the standard unemployment benefit without AI b^{MP} and the leisure value from AI ℓ^{AI} such that $b = b^{\text{MP}} + \ell^{\text{AI}}$. The labor search literature typically assume replacement rate of 40%, in line with interpreting that as the ratio of unemployment benefit to wage, as in Shimer (2005) and Hall (2005). Targeting the wage ratio at the ICT sector labor cost share at 60%, we have $b^{\text{MP}} = 0.4 \cdot 0.6$. For ℓ^{AI}, since we cannot directly measure the leisure value from AI, we assume that any increase in the value of leisure commensurates the increase in the technology-related leisure expenditures. According to the 2023 Connected Consumer Survey conducted by the Deloitte Center, the expenditure share for Technology, Media & Telecommunications is about $1.55\% = 805/51850$. The expenditure share in leisure consumption (e.g. entertainment and travel) is about 4.7% in 2023. Thus, we have $\ell^{\text{AI}} = \frac{1.55}{4.7} \cdot b_0 = 0.328 \cdot b_0$ and hence $b = (1 + 0.328) \cdot 0.4 \cdot 0.6 = 0.319$. Note that the (leisure-included) replacement rate may take higher values in some studies, for example, 82% in Bils, Chang, and Kim (2012) and 95.5% in Hagedorn and Manovskii (2008). We will thus perform robustness check with 90% (leisure-included) replacement rate, which gives $b = 0.9 \cdot 0.6 = 0.54$.

The model parametrization is summarized in Table 4.

Untargeted moments and sources	Model
AI elasticity of vacancy = 0.16, Acemoglu et al. (2022)	0.162
10-year average output growth rate = 7%, Briggs and Kodnani (2023)	7.08%
Excess daily return to AI-exposed firms: -25 to 40bp, Eisfeldt, Schubert, and Zhang (2023)	10bp

Table 5: Model predictions on the untargeted moments.

5. Quantitative Analysis

By now, we have completed the calibration. In this section, we will turn to examining equilibrium patterns and dynamic properties, which depends crucially on AI learning and failing rate and worker replacement costs. Recall the job finding rate function in the Full model, which under a Cobb-Douglas match technology, is given by,

$$\alpha_t = \Lambda\left(s_t\right) = \left(\frac{\theta}{\kappa} s_t\right)^{\frac{\xi}{1-\xi}},$$

where ξ is the vacancy elasticity of matching and κ the entry cost of workers.

Suppose there exists a some-AI steady state such that $\Lambda\left(s_0\right) = \alpha_0$. Using the numerical algorithm detailed in the Appendix, we can compute a some-AI steady state in the full model, where we found the following fixed point: $H^{\text{AI}} = 0.732$; $\alpha^{\text{AI}} = 5.643$; $s^{\text{AI}} = 0.8735$; $A^{\text{AI}} = 4.7661$, where $\pi^A = 1.949$. The above fixed point is unique under the parameters calibrated to the targets in Table 4, thus the calibrated full model is *global determinate*. The necessary and sufficient condition for the existence of a some-AI steady state is $1.949 = \pi^A > s^{\text{AI}} = 0.8735$. Thus, a some-AI steady state indeed exists such that its equilibrium path matches all the empirical moments in Table 4. Note that unlike H_0 or A_0, s_0 is a jump variable, which is endogenously determined in the equilibrium at $s_0 = 0.609$ such that the equilibrium path converges to the some-AI steady state. Calibrating κ such that the initial job-finding rate of a equilibrium path leading to the some-AI steady state is $\Lambda\left(s_0\right) = \alpha_0 = 3.942$, which gives $\kappa = \theta s_0 \alpha_0^{\xi/(\xi-1)} = 0.077$. In the Online Appendix, we show that the calibration parameters rule out the existence of the unbounded steady state.

Finally, the eigenvalues of the Jacobian matrix of equations (34), (35) and (36) are -6.452, -0.077 and 6.578, i.e., the system is *local determinate* as the only positive eigenvalue uniquely pins down the equilibrium path of the only jump variable s_t leading to the some-AI steady state.

Untargeted Moments. The calibrated model implies labor productivity, y_t, rising 7.08% over the first 10 years along the transition path, consistent with the projected AI output growth over a decade from 2023 by Briggs and Kodnani (2023), even this moment is not targeted in the calibration. The calibrated model also implies that the AI elasticity of vacancy is on average equal to 0.16 over the first 5 years, consistent with the estimation by Acemoglu et al. (2022) using the Burning Glass data. We will perform a robustness check in the appendix to various parameter values. As expected, the steady state values are more sensitive only to the AI learning rate, AI error rate and AI exposure rate. Finally, firms exposed to AI earn positive excess daily returns that are about 10 basis points (bp) higher than the initial state before the AI take-off, a magnitude that lies within the range estimated by Eisfeldt et al. (2023).

Transitional Dynamics of Technological Unemployment. In some-AI steady state, there is an employment loss of 23%, accompanied by an AI capability improvement of 321% and a productivity gain of 366%. The sizable employment loss is due to a large worker replacement effect under the calibrated AI learning and error rates that support AI adoption ($I_\infty = 1$), while the job creation effect is relatively modest. Over the first 5 years, this technological unemployment under the matching elasticity of 0.5 implies an AI elasticity of search worker of -0.16 and an AI elasticity of vacancy of 0.16, where the latter is consistent with the result in Acemoglu et al. (2022). In the online Appendix, we provide further discussion on the role of AI inducing employment loss.

From the transitional dynamics shown in Figure 1, it can be seen that the employment loss is significant at the early stage and, as AI capability improves, it enhances worker productivity remarkably, thereby replacing workers only at a moderate rate. Such changes are most prominent over the first 5 years along the transition, where the employment loss amounts to 11.5 percentage points, the AI capability improvement about 45.5% and the output gain about 49.3%.[5] The growth in AI continues into the medium run; over the first 10 years the AI capability improvement accumulates to 94.0% and the output gain about 103%.

Figure 1 also illustrates AI's impact on job creation. Compared to the case of exogenous job-finding rate (the benchmark model), we note that in the full model with endogenous job finding rates depending on matching surpluses, the fraction of employed workers falls slower because the job finding rate is higher as firms create more vacancies to take advantage of the labor productivity growth driven by AI. AI capability rises more rapidly in the shorter run along the transition, because AI learns faster with more employed workers.

Possibility of Global Indeterminacy. In the full model, global indeterminacy may arise with coexistence of some-AI and unbounded steady states. Figure 2 reports the combination of parameters (and the rest calibrated to empirical targets in Table 4) that will lead to global indeterminacy. Given the value of firm's bargaining power θ, some-AI and unbounded steady states coexist in any economy when AI cost of replacement (in log) is between the blue line and the red line, i.e., $\phi \in [\phi_{\min}, \phi_{\max}]$. For example, global indeterminacy is plausible when the firm's bargaining power raises to $\theta = 0.95$, given $\phi = 0.0043$ in Table 4. Otherwise, if $\phi < \phi_{\min}$, then only some-AI steady state exists; if $\phi > \phi_{\max}$, then only unbounded-AI equilibrium exists.

[5]The model's prediction of 11.5% short-run loss of employment due to AI is in line with the range of layoff reported (5%-15%). According to a report by Forbes on February 12, 2025, "Meta has officially begun layoffs, cutting approximately 3,600 employees, or 5% of its global workforce. Many affected workers have pushed back, arguing that the company prioritizes AI-driven efficiency over human labor. Workday laid off 1,750 employees (8.5% of its workforce) while increasing investments in AI. Salesforce cut 1,000 jobs to pivot toward AI-driven solutions. Dell slashed 2,500 jobs (10% of its workforce) as it shifted toward AI-powered infrastructure. Intel cut over 15,000 jobs (15% of its workforce) as it pivots toward AI-driven computing and semiconductor advances. Electronic Arts (EA) laid off 775 employees (6% of its workforce) to prioritize AI and machine learning in game development."

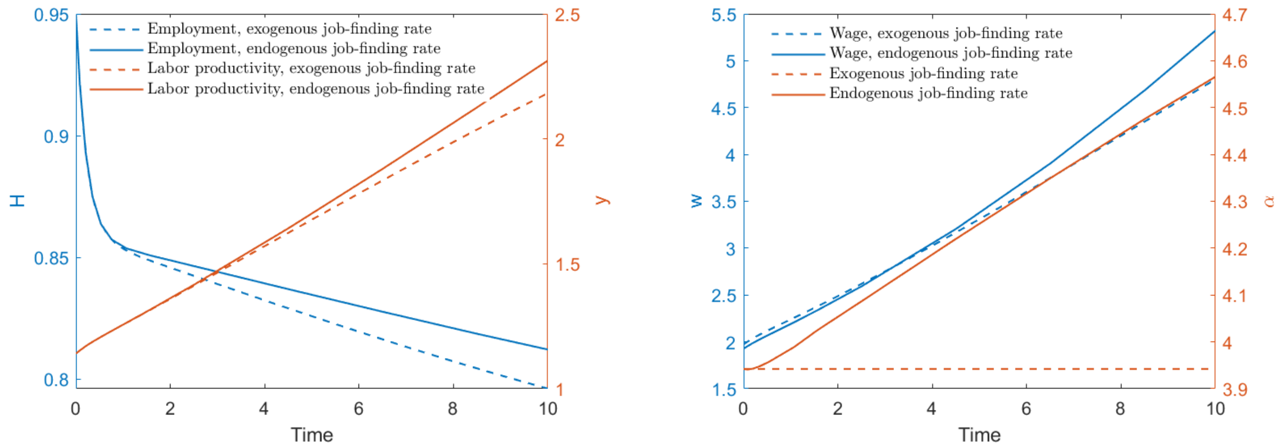

Figure 1: Simulated time series of employment and AI capability.

The region of global indeterminacy is larger when the firm's bargaining power is higher. For example, with $\theta = 0.9$, unbounded-AI equilibrium exists if ϕ exceeds 0.3. The higher values of firm's bargaining power are in line with Hagedorn and Manovskii (2008) (in which $\theta = 0.948$).

Intuitively, under plausibly higher values of firm's bargaining power and replacement rate, in addition to the some-AI scenario, firms can exercise adequately lower wages by renegotiations without laying off workers while continuously expanding AI with unbounded capability through AI learning. In other words, the current AI economy as we see in the data can suffer from coordination failure in the labor market such that an unbounded-AI steady state coexists but agents are bounding for to the some-AI steady state instead. In sum, for any economy between the blue line and the red line in Figure 2, the data we observe can not allow us to distinguish whether the economy is heading to the some-AI steady state or the unbounded steady state - both are observational equivalent in their current state. It highlights the uncertain future of AI economy.

Possibility of Local Indeterminacy and Oscillations. To illustrate the possibility of local indeterminacy, consider the following alternatives: firms have all the bargaining power, $\theta = 1$, a higher vacancy elasticity of matching $\xi = 0.765$ (as in Hall 2005), and a higher AI elasticity of productivity $\varepsilon_y = 6$. To isolate the effect from the change in the steady state, we assume the unemployed workers receive transfers financed by taxing the firm-worker pairs such that the steady-state surplus ratio is maintained at $s_\infty = s^{\mathrm{AI}}$. Based in these parameter values, the eigenvalues feature a negative real root and two complex roots with negative real parts. As such, the transition path toward the some-AI steady state features local oscillations in employment and AI capability dynamics. Note that while ε_y indicates how sensitive output is to AI, ξ highlights the role of matching frictions. When ξ is high, job finding is sensitive to matching surplus. As employment continues to drop toward the some-AI steady state level, AI learning become less effective and output gain is falling significantly when ε_y is high. Under this scenario, the drop in employment can overshoot to a level below its steady state, followed by decumulation of AI capability, then correction by raising employment and finally rebuilding

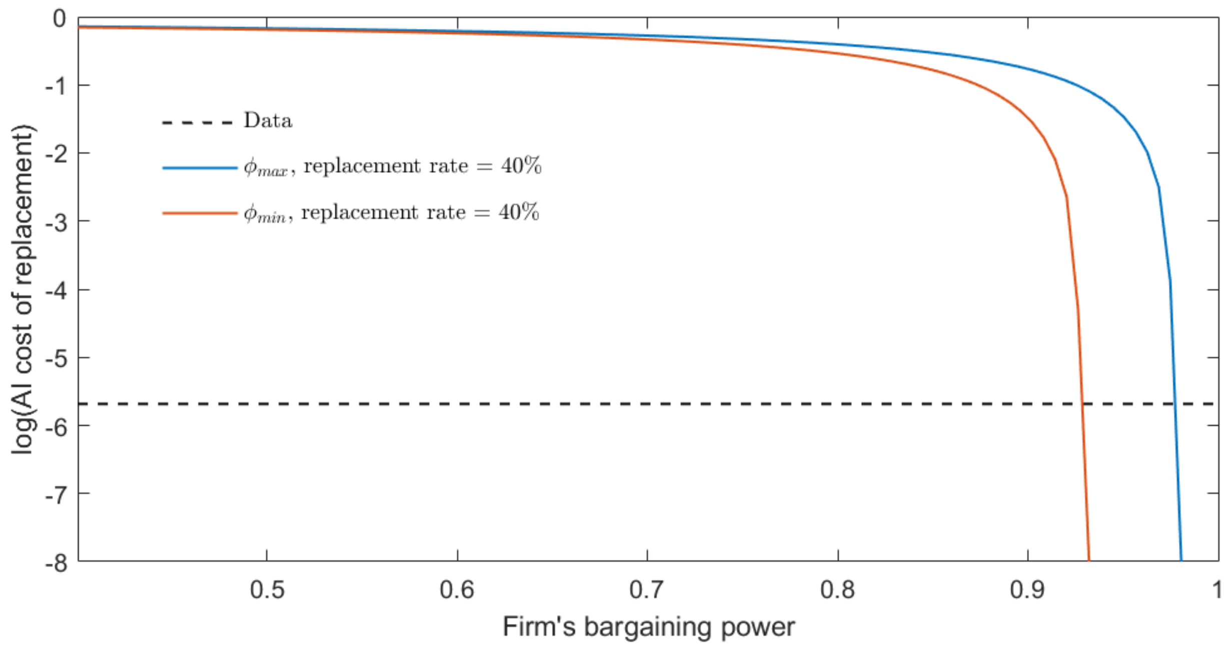

Figure 2: Parameter space of global indeterminacy.

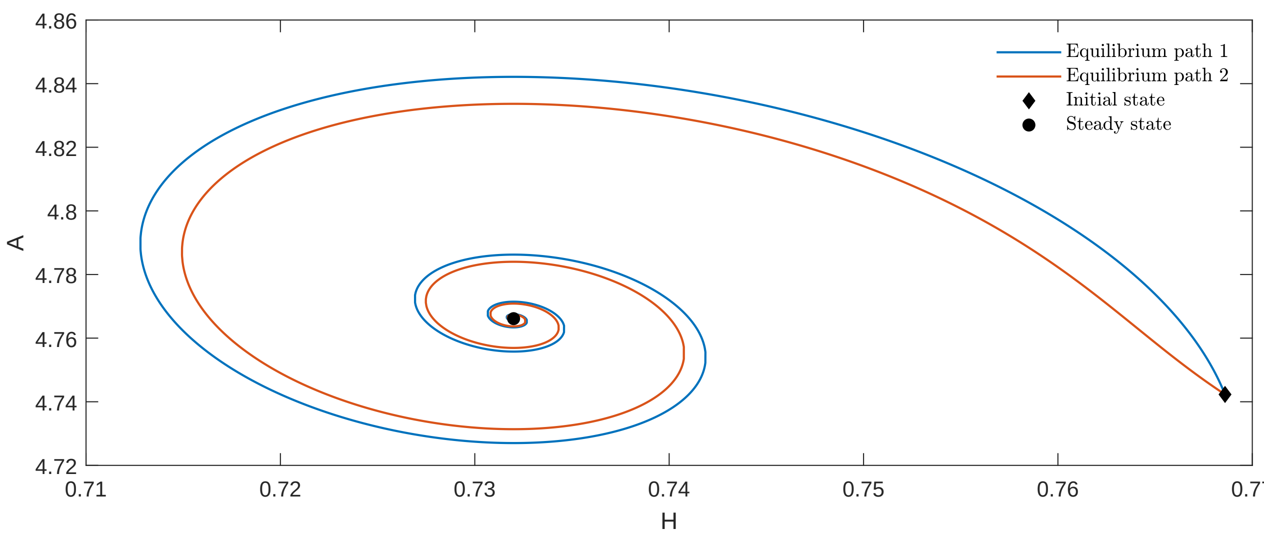

Figure 3: Spirals with local indeterminacy.

of AI capability. That is, oscillation around the some-AI steady state emerges.

Interestingly, there is also a continuum of such spirals leading to the some-AI steady state. That is, local indeterminacy arises due to self-fulfilling prophecy. Based on the above parameters, Figure 3 depicts two different spirals starting from the same initial state around the some-AI steady state. In particular, equilibrium path 1 (the blue locus) in Figure 3 starts with a higher surplus ratio than equilibrium path 2 (the red locus), leading to higher initial job-finding rate and hence higher employment level in the short run. Such a higher surplus

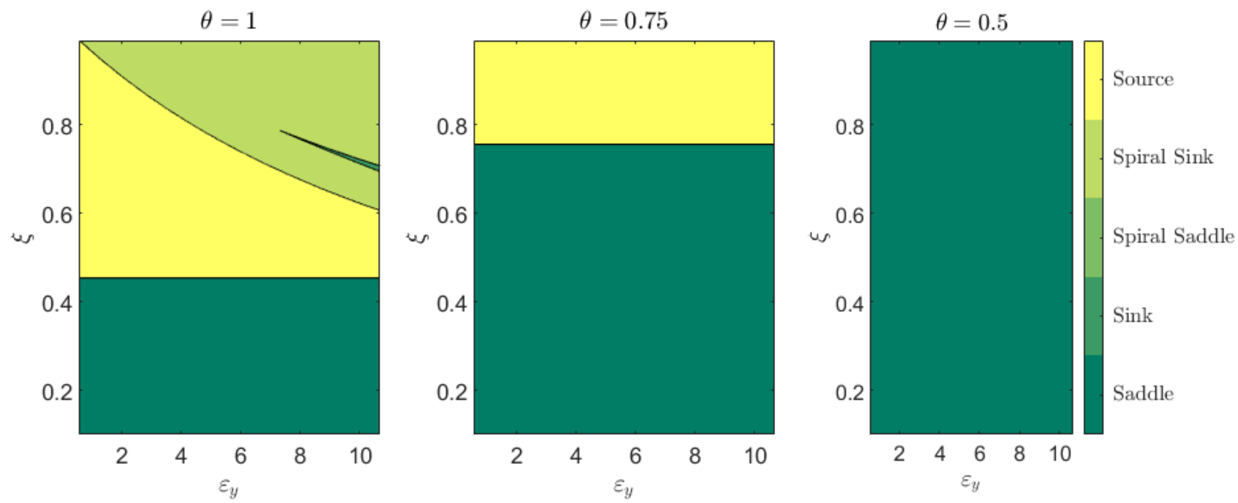

Figure 4: Equilibrium patterns of the some-AI steady state in the space of the AI elasticity of productivity and the vacancy elasticity of matching.

ratio is self-fulfilling, since agents on equilibrium path 1 expect higher AI capability in the short run, and hence they also expect higher labor productivity and faster arrival of profitable AI displacement of worker in the short run. Then firms are willing to create more vacancies, leading to higher employment, which enables faster AI learning, realizing higher AI capability they expect in the first place.

Summary of Equilibrium Patterns. It is informative to summarize all possible equilibrium outcomes in (ε_y, ξ)-space. Again, we assume transfers to maintain the same some-AI steady state $s_\infty = s^{\mathrm{AI}}$ to isolate the effect from the change in the steady state. As shown in the case of $\theta = 1$ by Figure 4, when the vacancy elasticity of matching is low (dark green region), the equilibrium is locally determinate, featuring the some-AI steady state with converging saddle. When the vacancy elasticity of matching is moderate (yellow region), the some-AI steady state becomes unstable (source) and hence the unbounded-AI steady state (which always feature converging saddle) is the only accessible one. Finally, when the AI elasticity of productivity and the vacancy elasticity of matching are both sufficiently high (light green region), we have a continuum of equilibria where spirals emerge around the some-AI steady state (spiral sink). This region of spiral sink disappears when the firm's bargaining power is lower, as shown in the cases of $\theta = 0.75$ and $\theta = 0.5$ in Figure 4, where in the latter case the equilibrium is globally and locally determinate featuring a nonoscillating saddle.

Welfare. Finally, we conclude with our quantitative analysis on the welfare. Under the calibrated parameters, adopting AI to replace workers is constrained optimal in the steady state, since the necessary condition for adoption is violated.[6] Thus, according to Proposition 10, it is optimal to subsidize the firm-worker pairs, i.e., $T_t > 0$. The intuition is that the social value

[6] Suppose not, then it is straightforward to compute that the social value of employment is $\omega^H_\infty = 0.441 < \pi^A = 1.949$, leading to contradiction.

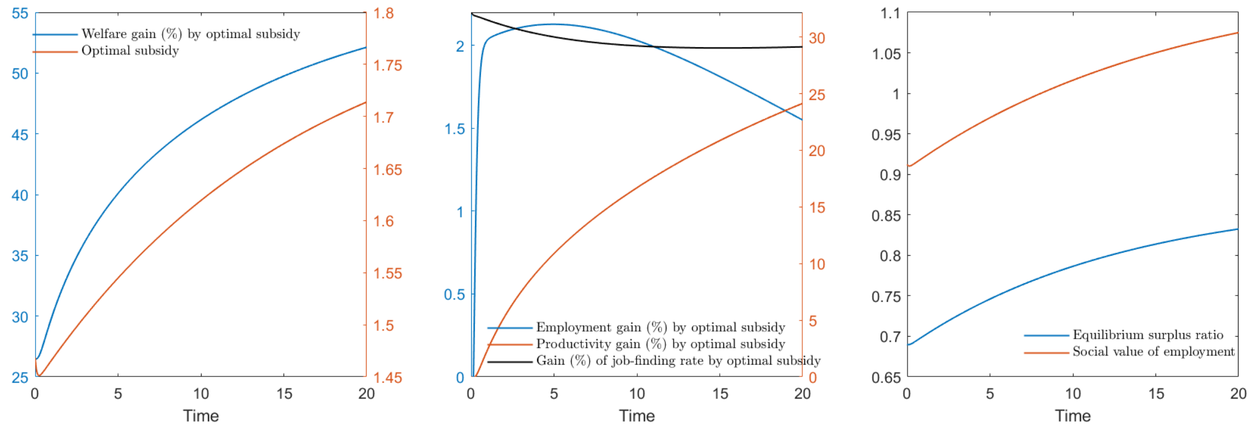

Figure 5: Welfare gain of optimal subsidy and its impact on employment, productivity and job-finding rate.

of employment is undervalued by the equilibrium match surplus ratio, illustrated by $\omega_t^H > s_t$ in the right panel of Figure 5. To raise the equilibrium match surplus ratio up to the social value of employment, it is optimal to subsidize the firm-worker pairs. The left panel of Figure 5 reports the level of optimal subsidy T_t (red line), which should be increasing over time. The optimal subsidy improves the welfare (measured by (38)) by 26.6% initially, raising to 52.1% in two decades. The mechanism of the welfare gain is mainly through higher job-finding rates and higher labor productivity, as the subsidy encourages firms to create vacancies, which raises employment level for AI to learn faster.

6. Conclusions

AI creates and destroys jobs. In this paper, we have introduced the first labor-search model of technological unemployment explicitly driven by GenAI. Unlike standard models of technical change, our model have conceptualized AI as a learning-by-using technology, emphasizing the realistic delay in worker replacement resulting from AI's need for experiential learning. Our analysis has identified the conditions under which AI either fails to take hold (no-AI steady state), takes off but stops at a limited capability (some-AI), or sustains an endogenous growth in the long run (unbounded-AI). Unlike standard labor-search frameworks, productivity improvement by AI raises job-finding rates and wages, but also the long-run unemployment in some-AI steady state – the phenomenon of technological unemployment. The presence of AI also introduces potential short-run oscillations in labor productivity, wages, and unemployment, alongside long-run multiple equilibria. These short-run fluctuations reflect the competing forces of job creation and destruction in frictional labor markets, under the feedback of AI learning mechanism. Long-run indeterminacy arises particularly when worker replacement costs and AI learning rates lie within moderate ranges. Together, these findings highlight the inherent unpredictability of AI's economic impact.

Upon fitting to micro evidence of GenAI and labor search data, we have established the some-AI equilibrium as the most likely outcome, though unbounded-AI equilibrium may emerge with reasonable adjustments in benchmark parametrization. In some-AI steady state employment loss is sizable and labor productivity is limited. In the unbounded-AI equilibrium, fast labor productivity improvement together with enhanced job finding lead to a modest employment gain and sustained endogenous growth.

We have also derived welfare criteria enabling assessment of whether firms exhibit tendencies toward excessive or insufficient AI adoption. Our findings further elucidate why the Hosios condition, a standard benchmark for optimality in search environments, generally fails to restore efficient outcomes in the labor market. Facing the risks and opportunities brought by AI, it is optimal to subsidize jobs before they are destroyed by AI. Quantitative exercises show a significant welfare gain from this policy.

Along these lines, it is natural to extend the analysis, particularly by including one or multiple dimensions of heterogeneity. One the one hand, because of the differential usage by workers of different ages, skills and occupations, one may examine the distributional effects of AI in the short and the long run. On the other, firms in different industries may also have different adoption probability of AI. This can lead to different bargaining and job sorting outcomes. Moreover, if AI can be embodied in a separate human capital accumulation process through, say, on the job learning, the additional dynamic interactions may lead to even rich equilibrium patterns and possibly different labor market consequences. While these are interesting avenues of research, they are beyond the scope of the current paper and hence left for future work.

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7. Online Appendix for Proofs

7.1. Proof of Proposition 1

Note that the dynamics of $\dot{S}_t^n$, captured by (7), is independent to n. Thus we can guess and verify that $S_t^n = S_t$ for any $n \geq 0$. Since $I_t^{n+1} = 1$ if and only if $\pi^A y_t > S_t^{n+1} = S_t$, we also have $I_t^n = I_t$ for any $n \geq 0$. **QED**

7.2. Proof of Proposition 2

Part a. Suppose a steady state featuring $A_\infty \in (0, \infty)$ exists. Then $\dot{A}_\infty = 0$ implies $H_\infty = H^{\mathrm{AI}} \equiv \frac{\delta}{\mu}$. Hence, $\dot{H}_\infty = 0$ implies $A_\infty = \frac{\alpha(\mu/\delta-1)-\sigma}{\rho\mu I_\infty}$. Thus, we must have $I_\infty = 1$ in this case.

Part b. Suppose a steady state featuring $A_\infty = \infty$ exists. Suppose $I_\infty = 1$, implies $s_\infty < \pi^A$ from the adoption decision (6).Then evaluating $\dot{s}_\infty = 0$ with A_∞, we must have $s_\infty = \pi^A$, leading to contradiction. Thus, we must have $I_\infty = 0$. In this case. $\dot{H}_\infty = 0$ implies $H_\infty = H^{\mathrm{MP}} \equiv \frac{\alpha}{\alpha+\sigma}$.

Part c. Suppose a steady state featuring $A_\infty = 0$ exists. Then $\dot{H}_\infty = 0$ implies $H_\infty = H^{\mathrm{MP}} \equiv \frac{\alpha}{\alpha+\sigma}$, independent to the value of I_∞.

Part d. We skip the proof for the comparative statics for A_∞, g_∞^A and H_∞ as they are straightforward. For the comparative statics of w_∞^0 in the some-AI steady state, note that

$$s_\infty = \frac{1 - b + \rho\mu A_\infty \pi^A}{r + \sigma + \alpha(1-\theta) + \rho\mu A_\infty}.$$

It is straightforward is show that

$$\frac{\partial s_\infty}{\partial(\rho\mu A_\infty)} = \frac{\pi^A - s_\infty}{r + \sigma + \alpha(1-\theta) + \rho\mu A_\infty} > 0$$

where the latter inequality follows from the result of $I_\infty = 1$ in the the some-AI steady state, where (6) implies that $I_\infty = 1$ iff $\pi^A > s_\infty$. Note that

$$\frac{w_\infty^0}{y} = 1 - \theta(1-b) + (1-\theta)\left(\theta\alpha s_\infty + \rho\mu A_\infty \pi^A\right) \Rightarrow \frac{\partial w_\infty^0}{\partial(\rho\mu A_\infty)} = (1-\theta)\left(\theta\alpha\frac{\partial s_\infty}{\partial(\rho\mu A_\infty)} + \pi^A\right) > 0.$$

From part a we have $\rho\mu A_\infty = \alpha(\mu/\delta - 1) + \sigma$, implies $\partial w_\infty^0/\partial\mu > 0$ and $\partial w_\infty^0/\partial\delta < 0$.

For the comparative statics of w_∞^1 in the unbounded-AI steady state, note that

$$s_\infty = \frac{1-b}{r + \sigma + \alpha(1-\theta)} \Rightarrow \frac{\partial s_\infty}{\partial\alpha} < 0 \text{ and } \frac{\partial s_\infty}{\partial\sigma} < 0.$$

Using the fact that $\frac{w^1_\infty}{y} = 1 - \theta(1-b) + (1-\theta)\left[\theta\alpha s_\infty - (r+\sigma)\pi^A\right]$, we have

$$\frac{\partial w^1_\infty}{\partial \alpha} = \frac{(1-\theta)\theta(r+\sigma)}{r+\sigma+\alpha(1-\theta)}\frac{s_\infty}{y}\frac{\partial s_\infty}{\partial \alpha} < 0 \text{ and } \frac{\partial w^1_\infty}{\partial \sigma} = (1-\theta)\left(\theta\alpha\frac{\partial s_\infty}{\partial \sigma} - \pi^A\right) < 0.$$

QED

7.3. *Proof of Proposition 3*

Note that there always exists s_∞ such that $\dot{s}_\infty = 0$, which is uniquely given by

$$s_\infty = \frac{1-b+\rho\mu I_\infty A_\infty \pi^A}{r+\sigma+(1-\theta)\alpha+\rho\mu I_\infty A_\infty}, \text{ where } I_\infty = 1 \text{ iff } s_\infty < \pi^A.$$

Substituting the above solution of s_∞, we have

$$\begin{aligned} s_\infty < \pi^A \quad &\Leftrightarrow \quad \frac{1-b+\rho\mu I_\infty A_\infty \pi^A}{r+\sigma+(1-\theta)\alpha+\rho\mu I_\infty A_\infty} < \pi^A, \\ &\Leftrightarrow \quad \frac{1-b}{r+\sigma+(1-\theta)\alpha} < \pi^A, \end{aligned}$$

which establishes part a and b after we combine with Proposition 2. In the case of $s_\infty \geq \pi^A$, i.e., $\frac{1-b}{r+\sigma+\alpha(1-\theta)} \geq \pi^A$, given the result that $I_\infty = 0$ and $H_\infty = H^{\mathrm{MP}}$ in this case, we have

$$g^A_\infty = \mu H^{\mathrm{MP}} - \delta = \frac{(\mu-\delta)\alpha - \delta\sigma}{\alpha+\sigma}.$$

Thus, if $(\mu-\delta)\alpha > \delta\sigma$, then we have $g^A_\infty > 0$ and $A_\infty = \infty$, which establishes part b-i; if $(\mu-\delta)\alpha < \delta\sigma$, then we have $g^A_\infty < 0$ and $A_\infty = 0$, which establishes part b-ii; if $(\mu-\delta)\alpha = \delta\sigma$, then we have $g^A_\infty = 0$ and $A_\infty \in (0,\infty)$, which establishes part b-iii. **QED**

7.4. *Proof of Proposition 4*

Around the unbounded-AI steady state, we have $I_\infty = 0$ from Proposition 2, thus the local dynamics of H_t and s_t is given by

$$\dot{H}_t = \alpha(1-H_t) - \sigma H_t, \tag{49}$$

$$\dot{s}_t = [r+\sigma+(1-\theta)\alpha]s_t - (1-b). \tag{50}$$

The eigenvalues of this ODE system of H_t and s_t are simply $-(\alpha+\sigma)<0$ and $r+\sigma+(1-\theta)\alpha>0$. Since H_t is a predetermined variable but s_t is a jump variable, the above ODE system of H_t and s_t is locally determinate around the unbounded-AI steady state iff there is one positive eigenvalue and one negative eigenvalue, which is the case. Finally, since $\dot{A}_t/A_t=\mu H_t-\delta$, the path of A_t is also uniquely followed the path of H_t.

Around the some-AI steady state, we have $I_\infty=1$ from Proposition 2, thus the local dynamics of A_t and H_t is given by

$$\begin{aligned}\dot{A}_t &= (\mu H_t-\delta)A_t, &(51)\\ \dot{H}_t &= \alpha(1-H_t)-\sigma H_t-\rho\mu A_t H_t. &(52)\end{aligned}$$

The Jacobian matrix evaluated at $H_\infty=\delta/\mu$ of the some-AI steady state is

$$J=\begin{pmatrix}0 & \mu A_\infty\\ -\rho\delta & -(\alpha+\sigma+\rho\mu A_\infty)\end{pmatrix}$$

Then we have

$$\begin{aligned}\det(J) &= \rho\delta\mu A_\infty>0,\\ \text{trace}(J) &= -(\alpha+\sigma+\rho\mu A_\infty)<0.\end{aligned}$$

Since in a 2-by-2 system, $\det(J)$ is the product of eigenvalues and trace(J) is the sum of eigenvalues, the fact that $\det(J)>0$ and trace$(J)<0$ implies we must have two negative eigenvalues. Since both A_t and H_t are predetermined variables, the above ODE system of A_t and H_t is locally determinate around the some-AI steady state iff there are two negative (real part if complex) eigenvalues, which is the case. The path is spiral if the eigenvalues are complex, which is the case if

$$\begin{aligned}\text{trace}(J)^2 &< 4\det(J)\Leftrightarrow\left(\frac{\alpha\mu}{\delta}\right)^2<4\left[\alpha(\mu-\delta)-\sigma\delta\right]\\ &\Leftrightarrow\ \mu\alpha<4\delta^2\left(1-\frac{\delta(\alpha+\sigma)}{\mu\alpha}\right),\end{aligned}$$

where we have substituted $A_\infty=\frac{1}{\rho\mu}\left[\alpha\left(\frac{\mu}{\delta}-1\right)-\sigma\right]$ in some-AI steady state. **QED**

7.5. *Proof of Proposition 5*

From the first order condition of (20), it is straight-forward to see $I_t^p=1$ if and only if $\pi^A>\mathcal{W}_H(H_t,A_t)/y=\omega_t^H$. Consider two scenarios. In the first scenario, suppose $\omega_t^H\leq\pi^A$ for all t. i.e., $I_t^p=0$. Then, from (21) and (22) we have $\omega_t^A=0$ and $\omega_t^H=\frac{1-b}{r+\sigma+\alpha}$. Thus, the premise $\omega_t^H \leq \pi$ is satisfied iff $\frac{1-b}{r+\sigma+\alpha} \geq \pi^A$. In the second scenario, suppose $\omega_t^H > \pi^A$ for some t. In this case, we have $I_t^p = 1$ when t is sufficiently large and close to the steady state. Finally, note that from Proposition 3 we have already known that $I_t = 0$ iff $\frac{1-b}{r+\sigma+\alpha(1-\theta)} \geq \pi^A$. Part a: If $\pi^A \leq \frac{1-b}{r+\sigma+\alpha}$, since $\frac{1-b}{r+\sigma+\alpha(1-\theta)} \geq \frac{1-b}{r+\sigma+\alpha} \geq \pi^A$, then we must have $I_t = I_t^p = 0$ in this case. Part b: if $\frac{1-b}{r+\sigma+\alpha} < \pi^A \leq \frac{1-b}{r+\sigma+\alpha(1-\theta)}$, then $I_t = 0 \leq I_t^p \in \{0, 1\}$. Part c: if $\frac{1-b}{r+\sigma+\alpha(1-\theta)} < \pi^A$, then $I_t = 1 \geq I_t^p \in \{0, 1\}$. **QED**

7.6. *Proof of Proposition 6*

In the equilibrium, AI is adopted iff $s_t < \pi^A$; in the constrained optimum, AI is adopted iff $\omega_t^H < \pi^A$. To implement the constrained optimal allocation with the equilibrium, one can set firm's bargaining power θ_t^p such that $s_t = \omega_t^H$ for all t. Recall the dynamics of s_t and ω_t^H are

$$\begin{aligned} \dot{s}_t &= \left[r + \sigma + (1 - \theta_t^p)\,\alpha\right] s_t - (1 - b) - \rho\mu A_t \left(\pi^A - s_t\right)_+, \\ \dot{\omega}_t^H &= (r + \sigma + \alpha)\,\omega_t^H - (1 - b) - \rho\mu A_t^p \left(\pi^A - \omega_t^H\right)_+ - \mu A_t^p \omega_t^A. \end{aligned}$$

Thus, to have $s_t = \omega_t^H$, it is necessary that $\theta_t^p = \mu A_t^p \omega_t^A / \omega_t^H$.

7.7. *Proof of Proposition 7*

Note that there always exists s_∞ such that $\dot{s}_\infty = 0$, which is uniquely given by

$$s_\infty = \frac{1 - b + \rho\mu I_\infty A_\infty \pi^A}{r + \sigma + (1 - \theta)\,\alpha - \varepsilon_y\left(\mu H_\infty - \delta\right) + \rho\mu I_\infty A_\infty}, \text{ where } I_\infty = 1 \text{ iff } s_\infty < \pi^A.$$

Substituting the above solution of s_∞, we have

$$\begin{aligned} s_\infty < \pi^A &\Leftrightarrow \frac{1 - b + \rho\mu I_\infty A_\infty \pi^A}{r + \sigma + (1 - \theta)\,\alpha - \varepsilon_y\left(\mu H_\infty - \delta\right) + \rho\mu I_\infty A_\infty} < \pi^A, \\ &\Leftrightarrow \frac{1 - b}{r + \sigma + (1 - \theta)\,\alpha - \varepsilon_y\left(\mu H_\infty - \delta\right)} < \pi^A. \end{aligned}$$

Part a. In the case of $s_\infty < \pi^A$, given the result that $I_\infty = 1$, $H_\infty = H^{\text{AI}} = \delta/\mu$ and $A_\infty = A^{\text{AI}}$, the premise $s_\infty < \pi^A$ is satisfied iff

$$\frac{1 - b}{r + \sigma + \alpha\,(1 - \theta)} < \pi^A.$$

Part b. In the case of $s_\infty \geq \pi^A$, given the result that $I_\infty = 0$ and $H_\infty = H^{\text{MP}}$, the premise

$s_\infty \geq \pi^A$ is satisfied iff

$$\frac{1-b}{r+\sigma+\alpha(1-\theta)-\varepsilon_y(\mu H^{\mathrm{MP}}-\delta)} \geq \pi^A.$$

In this case, the growth rate of AI capability is

$$g_\infty^A = \mu H^{\mathrm{MP}} - \delta = \frac{(\mu-\delta)\alpha - \delta\sigma}{\alpha+\sigma}.$$

Thus, if $(\mu-\delta)\alpha > \delta\sigma$, then we have $g_\infty^A > 0$ and $A_\infty = \infty$, which establishes part b-i; if $(\mu-\delta)\alpha < \delta\sigma$, then we have $g_\infty^A < 0$ and $A_\infty = 0$, which establishes part b-ii; if $(\mu-\delta)\alpha = \delta\sigma$, then we have $g_\infty^A = 0$ and $A_\infty \in (0,\infty)$, which establishes part b-iii.

Part b-iv. Consider $H_0 > H^{\mathrm{MP}}$ and $\frac{1-b}{r+\sigma+\alpha(1-\theta)-\varepsilon_y(\mu H^{\mathrm{MP}}-\delta)} \geq \pi^A$. In this case we have $s_\infty \geq \pi^A$ from the beginning of part b. We want to show there exists an equilibrium path where $\dot{s}_t \leq 0$ such that $s_t \geq \pi^A$, i.e., $I_t = 0$ for all t. Guess that $I_t = 0$ for all t. In this case, H_t admits a closed-form solution given by

$$H_t = e^{-(\alpha+\sigma)t} H_0 + \left[1 - e^{-(\alpha+\sigma)t}\right] H^{\mathrm{MP}}.$$

Given $H_0 > H^{\mathrm{MP}}$, we have $\dot{H}_t < 0$ for all t. Since H_t is a monotone path of t, we can express the ODE in H_t instead of t such that $s_t = s(H_t)$ where $s(H)$ solves the following ODE in H:

$$s'(H) = \frac{\dot{s}_t}{\dot{H}_t} = \frac{[r+\sigma+(1-\theta)\alpha - \varepsilon_y(\mu H-\delta)]\, s(H) - (1-b)}{\alpha(1-H)-\sigma H}.$$

Applying the L'Hôpital's rule at the steady state $H_\infty = H^{\mathrm{MP}}$, we have

$$\begin{aligned} s'(H_\infty) &= \frac{\left[r+\sigma+(1-\theta)\alpha - \varepsilon_y\left(\mu H^{\mathrm{MP}}-\delta\right)\right] s'(H_\infty) - \mu\varepsilon_y s_\infty}{-(\alpha+\sigma)}, \\ &= \frac{\mu\varepsilon_y s_\infty}{r+2\sigma+(2-\theta)\alpha - \varepsilon_y(\mu H^{\mathrm{MP}}-\delta)} > 0. \end{aligned}$$

To show $\dot{s}_t \leq 0$, we need to establish that $s'(H_t) \geq 0$ for all t since we already had $\dot{H}_t < 0$. Suppose not, i.e., there is τ such that $s'(H_\tau) < 0$. Since $s'(H_\infty) > 0$, there must exist H_* with $s'(H_\infty) = 0$. It implies there exists $H_2 > H_* > H_1$, such that $s(H_2) = s(H_1)$ but

$s'(H_2) \leq 0 < s'(H_1)$. Then we have

$$\begin{aligned}
0 \geq s'(H_2) &= \frac{[r+\sigma+(1-\theta)\alpha-\varepsilon_y(\mu H_2-\delta)]\,s(H_2)-(1-b)}{\alpha(1-H_2)-\sigma H_2} \\
&= \frac{[r+\sigma+(1-\theta)\alpha-\varepsilon_y(\mu H_2-\delta)]\,s(H_1)-(1-b)}{\alpha(1-H_2)-\sigma H_2} \\
&> \frac{[r+\sigma+(1-\theta)\alpha-\varepsilon_y(\mu H_1-\delta)]\,s(H_1)-(1-b)}{\alpha(1-H_2)-\sigma H_2} \\
&= s'(H_1)\,\frac{\alpha(1-H_1)-\sigma H_1}{\alpha(1-H_2)-\sigma H_2} \\
&> 0,
\end{aligned}$$

contradiction. **QED**

7.8. *Proof of Proposition 8*

In the some-AI steady state, we have $I_\infty = 1$, implying $s_\infty^{\text{AI}} < \pi$; in the unbounded-AI steady state, we have $I_\infty = 0$, implying . Since $\alpha(s) \equiv \Lambda\left(\frac{\kappa}{\theta s}\right)$ is increasing in s, we must have $\alpha\left(s^{\text{AJJ}}\right) > \alpha\left(s^{\text{AI}}\right)$, following the fact that $s_\infty^{\text{AJJ}} \geq \pi > s_\infty^{\text{AI}}$. Also, since $g_\infty^A > 0$ in the unbounded-AI steady state, we have

$$0 < g_\infty^A = \mu H^{\text{AJJ}} - \delta = \mu\left(H^{\text{AJJ}} - H^{\text{AI}}\right),$$

implying $H^{\text{AJJ}} > H^{\text{AI}}$. Finally, note that, from (12), the labor share in the some-AI steady state is

$$\frac{w_\infty^0}{y_\infty} = 1-\theta(1-b)+(1-\theta)\left[\theta\alpha\left(s^{\text{AI}}\right)s^{\text{AI}}+\rho\mu A^{\text{AI}}\pi^A\right];$$

the labor share in the unbounded-AI steady state is

$$\frac{w_\infty^1}{y_\infty} = 1-\theta(1-b)+(1-\theta)\left[\theta\alpha\left(s^{\text{AJJ}}\right)s^{\text{AJJ}}-\left[r+\sigma-\varepsilon_y\mu\left(H^{\text{AJJ}}-H^{\text{AI}}\right)\right]\pi^A\right].$$

Thus we have

$$\frac{w_\infty^0}{y_\infty}-\frac{w_\infty^1}{y_\infty} = (1-\theta)\left[\theta\left[\alpha\left(s^{\text{AI}}\right)s^{\text{AI}}-\alpha\left(s^{\text{AJJ}}\right)s^{\text{AJJ}}\right]+\left[\rho\mu A^{\text{AI}}+r+\sigma-\varepsilon_y\mu\left(H^{\text{AJJ}}-H^{\text{AI}}\right)\right]\pi^A\right],$$

which is positive when θ is close to zero. **QED**

7.9. *Proof of Proposition 9*

Note that

$$s^{\text{AJJ}} = \frac{1-b}{r+\sigma+\alpha\left(s^{\text{AJJ}}\right)(1-\theta)-\varepsilon_y\left(\mu H^{\text{AJJ}}-\delta\right)}$$

Using the fact that $H^{\text{AJJ}} = \alpha\left(s^{\text{AJJ}}\right)/\left[\alpha\left(s^{\text{AJJ}}\right)+\sigma\right]$ to rearrange terms, we have

$$\varepsilon_y g_A = r+\sigma\left(\frac{1-\theta}{1-\frac{g_A+\delta}{\mu}}+\theta\right)-\frac{1-b}{s^{\text{AJJ}}}.$$

Given s^{AJJ}, the RHS is a increasing function of g_A, which cuts the LHS from below that determines the steady-state g_A. Now suppose μ increases but g_A decreases. The latter must imply H^{AJJ} and s^{AJJ} decrease. With μ increases and s^{AJJ} decrease, the RHS as a function of g_A shifts down. But since the RHS cuts the LHS from below, the steady-state g_A increases, contradiction. The similar argument applies to prove the part of δ. **QED**

7.10. *Proof of Proposition 10*

In the equilibrium, AI is adopted iff $s_t < \pi^A$ and the job-finding rate is given by $\alpha_t = \Lambda\left(\frac{\kappa}{\theta s_t}\right)$; in the constrained optimum, AI is adopted iff $\omega_t^H < \pi^A$ and the job-finding rate is given by $\Theta'\left(\alpha_t^p\right) = \omega_t^H/\kappa$. To implement the constrained optimal allocation with the equilibrium, one can set firm's bargaining power $\theta_t^p = \xi_t$ such that $s_t = \omega_t^H$ for all t. To see it, note that

$$f_t^p = \frac{\xi_t}{\theta_t^p} f_t^p = \frac{1}{\theta_t^p \Theta'\left(\alpha_t^p\right)} = \frac{\kappa}{\theta_t^p \omega_t^H} = \frac{\kappa}{\theta_t^p s_t} = \Lambda^{-1}\left(\alpha_t\right) = f_t$$

Recall the dynamics of s_t under the transfers and ω_t^H are

$$\begin{aligned}
\dot{s}_t &= \left[r+\sigma+(1-\theta)\alpha(s_t)-\varepsilon_y\left(\mu H_t-\delta\right)\right]s_t-(1-b)-T_t-\rho\mu A_t\left(\pi^A-s_t\right)_+,\\
\dot{\omega}_t^H &= \left[r+\sigma+\alpha_t^p-\varepsilon_y\left(\mu H_t^p-\delta\right)\right]\omega_t^H-\kappa\Theta\left(\alpha_t^p\right)-(1-b)-\rho\mu A_t^p\left(\pi^A-\omega_t^H\right)_+-\mu A_t^p\omega_t^A.
\end{aligned}$$

Thus, to have $s_t = \omega_t^H$, it is necessary that $T_t = \mu A_t^p \omega_t^A$.

Finally, to prove $T_t \geq 0$, it is sufficient to show $\omega_t^A \geq 0$. Suppose not, i.e., there exists t_0 such that $\omega_{t_0}^A < 0$. Note that at the planner's steady state, we have

$$\omega_\infty^A = \frac{\rho\mu H_\infty^p\left(\pi^A-\omega_\infty^H\right)_+}{r+\delta-(1+\varepsilon_y)\left(\mu H_\infty^p-\delta\right)} \geq 0$$

Integrating by parts the social value of AI (44) from t_0 to t, we have

$$\omega_t^A = e^{\int_{t_0}^t \left[r+\delta-(1+\varepsilon_y)\left(\mu H_\tau^p-\delta\right)\right]d\tau}\omega_{t_0}^A - \rho\mu \int_{t_0}^t e^{\int_{t'}^t \left[r+\delta-(1+\varepsilon_y)\left(\mu H_\tau^p-\delta\right)\right]d\tau} H_{t'}^p \left(\pi^A - \omega_{t'}^H\right)_+ dt'.$$

Given the premise $\omega_{t_0}^A < 0$, we must have $\omega_t^A < 0$ for all $t > t_0$ and $\omega_\infty^A = -\infty$, contradicting the fact that $\omega_\infty^A \geq 0$. **QED**

7.11. Non-existence of Unbounded Steady State in the Calibration

As we have shown in the theory session, there could be global indeterminacy. To proceed our quantitative exercise, we construct and calibrate both the unbounded-AI steady state and some-AI steady states and then verify their existence.[7] Suppose there exists an unbounded-AI steady state $\left\{H^{\text{AJJ}}, A^{\text{AJJ}}, s^{\text{AJJ}}\right\}$such that its initial job-finding rate matches the data, $\Lambda(s_0) = \alpha_0$. Using the numerical algorithm detailed in the Appendix, we can compute and calibrate an equilibrium path leading to an unbounded-AI steady state in the full model with the empirical moments in Table 4, where we found the following fixed point:

$$\begin{aligned} H^{\text{AJJ}} &= \frac{\alpha^{\text{AJJ}}}{\alpha^{\text{AJJ}} + \sigma} = 0.95014, \ \alpha^{\text{AJJ}} = \left(\frac{\theta}{\kappa} s^{\text{AJJ}}\right)^{\frac{\xi}{1-\xi}} = 3.945, \\ s^{\text{AJJ}} &= \frac{1-b}{r+\sigma+\alpha^{\text{AJJ}}(1-\theta) - \varepsilon_y\left(\mu H^{\text{AJJ}} - \delta\right)} = 0.327. \end{aligned}$$

The unbounded-AI equilibrium therefore does not result in technological unemployment. Instead, due to a higher job finding rate, it features an employment gain of 0.015%. Asymptotically as time goes to infinity, AI is not adopted to replace workers.

The necessary and sufficient conditions for the existence of an unbounded-AI steady state are:

$$\begin{aligned} 0 &< g_\infty = \mu H^{\text{AJJ}} - \delta = 0.138, \\ 1.949 &= \pi^A \leq s^{\text{AJJ}} = 0.327, \end{aligned}$$

which leads to contradiction. Thus, an unbounded-AI steady state that calibrates to the empirical moments in Table 4 does not exists. We will discuss below under what circumstances the unbounded-AI equilibrium may arise.

8. Further Discussion on the AI-induced Employment Loss

Three issues regarding the AI-induced employment loss merit further discussion.

[7] Since $\delta < \mu$ in Table 4, we can rule out the existence of the no-AI steady state.

First, in the standard labor search model (Mortensen and Pissarides (1994)), the unemployment rate is pinned down by the ratio of the job-separation rate to the job-finding rate. By contrast, in the some-AI steady state of our model, technological unemployment is entirely determined by the ratio of the AI error rate to the AI learning rate. The job-finding and job-separation rates determine the steady-state level of AI capacity instead.

Second, one may wonder whether the sizable technological unemployment persists when firms have a stand-ready option to create vacancies for AI immediately without firing workers. In the online Appendix we consider this extension. When this stand-ready option is half as profitable as the fully developed option, denoted by $\pi^C = 0.5\pi^A$, the fraction of workers displaced by AI *rises* from 17% to 21%. The main reason is that the stand-ready option improves the firm's bargaining position and stimulates vacancy creation, which in turn accelerates AI learning from workers and leads to a higher AI capability in the steady state. As a result, the displacement effect is amplified. Hence, if firms can create AI jobs without firing workers immediately, the long-run employment loss would be even larger. The unemployment rate reported in our main results—absent this stand-ready option—should therefore be viewed as conservative.

Third, to better illustrate how the employment loss depends on the AI-specific features, we consider three stages of technological change: (i) Pre-computer: no productivity growth and no displacement; (ii) Computer: productivity growth driven by computers but no displacement; (iii) AI: productivity growth driven by AI learning with worker displacement. The computer stage serves as a counterfactual benchmark, allowing us to compare labor-market outcomes in the absence of productivity growth (pre-computer stage) with those arising from the AI-specific features of learning and displacement (AI stage). At the pre-computer stage, the steady-state employment is given by solving

$$\begin{aligned} s^{\text{pre}} &= \frac{1-b}{r+\sigma+\alpha^{\text{pre}}(1-\theta)}, \\ \alpha^{\text{pre}} &= \left(\frac{\theta}{\kappa} s^{\text{re}}\right)^{\frac{\xi}{1-\xi}}. \end{aligned}$$

which yields $\alpha^{\text{pre}} = 2.7239$. In the steady state, the pre-computer level of employment is thus:

$$H^{\text{pre}} = \frac{\alpha^{\text{pre}}}{\alpha^{\text{pre}}+\sigma} = \frac{2.7239}{2.7239+0.207} = 0.9294,$$

corresponding to an average unemployment rate of 7.06%. To achieve its counterfactual purposes, the computer stage is designed so that labor productivity is assumed to grow exogenously at the 10-year average rate used in our calibration. Note that we deliberately abstract from any displacement effect of computers in order to perform a conservative counterfactual for the impact of productivity growth on employment. The steady-state employment satisfies:

$$
\begin{aligned}
s^{\mathrm{com}} &= \frac{1-b}{r+\sigma+\alpha^{\mathrm{com}}(1-\theta)-g_y}, \\
\alpha^{\mathrm{com}} &= \left(\frac{\theta}{\kappa}s^{\mathrm{com}}\right)^{\frac{\xi}{1-\xi}}.
\end{aligned}
$$

yielding $\alpha^{\mathrm{com}} = 2.7866$ and the steady-state counterfactual employment:

$$
H^{\mathrm{com}} = \frac{\alpha^{\mathrm{com}}}{\alpha^{\mathrm{com}}+\sigma} = \frac{2.7866}{2.7866+0.207} = 0.9309.
$$

This corresponds to an average unemployment rate of 6.9%. Comparing the pre-computer stage with the computer stage, exogenous productivity growth alone lowers unemployment by only 0.16 percentage points (7.06% − 6.9%), consistent with the empirical observation that although the diffusion computers and the internet did not induce unemployment, its employment gain was modest. By contrast, comparing the computer stage with the AI stage, the AI-specific features generate a long-run employment loss of 93.1% − 73.2% = 19.9%.

9. Online Appendix for the Numerical Algorithm

In this appendix, we detail how we solve and compute the equilibrium path of the following ODE system of the full model:

$$
\begin{aligned}
\dot{H}_t &= \alpha(1-H_t)-\sigma H_t-\rho\mu A_t H_t I_t, \text{ where } I_t = 1 \text{ iff } s_t<\pi^A, &(53)\\
\dot{A}_t &= (\mu H_t-\delta)A_t, &(54)\\
\dot{s}_t &= [r+\sigma+(1-\theta)\alpha(s_t)-\varepsilon_y(\mu H_t-\delta)]s_t-(1-b)-\rho\mu A_t\left(\pi^A-s_t\right)_+, &(55)
\end{aligned}
$$

where $\{H_0, A_0\}$ is given but s_0 is the equilibrium object to be computed. The system is global (in)determinate if $\{H_\infty, A_\infty, s_\infty\}$ is (not) unique. The system is local (in)determinate at the steady state $\{H_\infty, A_\infty, s_\infty\}$ if the path $\{H_t, A_t, s_t\}_{t=\tau}^{\infty}$ is (not) unique when $\{H_\tau, A_\tau, s_\tau\}$ is arbitrarily close to $\{H_\infty, A_\infty, s_\infty\}$. In particular, we need to compute s_0 that leads the equilibrium path from $\{H_0, A_0, s_0\}$ to $\{H_\infty, A_\infty, s_\infty\}$. It is well-recognized in the literature of economics growth that that the presence of the forward-looking ODE of $\dot{s}_t$, or any typical HJB equation, makes the system of ODEs singular such that the ODE of $\dot{s}_t$ is sensitive to the initial condition s_0 (converging to positive or negative infinity) and the standard routine of solving the boundary value problem cannot apply. Here we adopt a numerical strategy that solves the $\{H_t, A_t\}$ forward from (H_0, A_0) given $\{s_t\}$, then solves the $\{s_t\}$ backward from s_∞ given $\{H_t, A_t\}$. We iterate this process until a tolerance of error is reached.

We begin by solving the steady states, which could be non-stationary featuring a balanced growth path. We then we outline a numerical algorithm that approximate the dynamics around a steady state $\{H_\infty, A_\infty, s_\infty\}$ computed above with a linearized model. Next, using solution $\{H_t, A_t, s_t\}_{t=0}^{\infty}$ of the linearized model above as the initial guess, we outline an iteration algorithm that computes global dynamics$\{H_t, A_t, s_t\}_{t=0}^{\infty}$. Moreover, we summarize the algorithm that is used to compute other equilibrium objects, for instance, the paths of wages and welfare. Finally, we report detailed quantitative results of the benchmark and the full models.

9.1. *Computing the global dynamics leading to the some-AI steady state*

Recall the free-entry condition under the matching function $m = u^{1-\xi}v^{\xi}$ is $\kappa y_t = f_t \Pi_t$, which is equivalent to

$$\alpha(s_t) = \left(\frac{\theta}{\kappa} s_t\right)^{\frac{\xi}{1-\xi}}.$$

Evaluating the steady-state level of job-finding rate α^{AI} at the the some-AI steady-state level of surplus ratio s^{AI}, we have

$$\alpha^{\mathrm{AI}} = \left(\frac{\theta}{\kappa} s^{\mathrm{AI}}\right)^{\frac{\xi}{1-\xi}},$$

thus, we can substitute out the unknown parameter θ/κ and express α^{AI} and s^{AI} as the parameters for the job-finding rate function such that

$$\alpha(s_t) = \alpha^{\mathrm{AI}} \left(\frac{s_t}{s^{\mathrm{AI}}}\right)^{\frac{\xi}{1-\xi}}. \tag{56}$$

Given α^{AI} at the some-AI steady state, the steady-state level of surplus ratio s^{AI} (and also H^{AI} and A^{AI}) is given by

$$\begin{aligned} s^{\mathrm{AI}} &= \frac{1 - b + \pi^A \left[\alpha^{\mathrm{AI}}\left(\frac{\mu}{\delta} - 1\right) - \sigma\right]}{r + \left(\frac{\mu}{\delta} - \theta\right)\alpha^{\mathrm{AI}}}, \qquad (57) \\ H^{\mathrm{AI}} &= \frac{\delta}{\mu}, \\ A^{\mathrm{AI}} &= \frac{1}{\rho\mu}\left[\alpha^{\mathrm{AI}}\left(\frac{\mu}{\delta} - 1\right) - \sigma\right], \end{aligned}$$

where the necessary condition of the existence of some-AI steady state is

$$s^{\mathrm{AI}} < \pi^A.$$

9.1.1. Computing a linearized model around the steady state

Linearizing the ODE system around the some-AI steady state with $I_\infty = 1$, we have

$$\begin{pmatrix} \dot{H}_t \\ \dot{A}_t \\ \dot{s}_t \end{pmatrix} = \mathbf{J}^{\mathrm{AI}} \begin{pmatrix} H_t - H^{\mathrm{AI}} \\ A_t - A^{\mathrm{AI}} \\ s_t - s^{\mathrm{AI}} \end{pmatrix},$$

where the Jacobian matrix is given by

$$\mathbf{J}^{\mathrm{AI}} = \begin{bmatrix} J_{11}^{\mathrm{AI}} & J_{12}^{\mathrm{AI}} & J_{13}^{\mathrm{AI}} \\ J_{21}^{\mathrm{AI}} & 0 & 0 \\ J_{31}^{\mathrm{AI}} & J_{32}^{\mathrm{AI}} & J_{33}^{\mathrm{AI}} \end{bmatrix},$$

with the matrix entries given by

$$\begin{aligned}
J_{11}^{\mathrm{AI}} &\equiv \left.\frac{\partial \dot{H}_t}{\partial H_t}\right|_{\{H^{\mathrm{AI}},A^{\mathrm{AI}},s^{\mathrm{AI}}\}} = -\alpha^{\mathrm{AI}}\frac{\mu}{\delta}, \\
J_{12}^{\mathrm{AI}} &\equiv \left.\frac{\partial \dot{H}_t}{\partial A_t}\right|_{\{H^{\mathrm{AI}},A^{\mathrm{AI}},s^{\mathrm{AI}}\}} = -\rho\delta, \\
J_{13}^{\mathrm{AI}} &\equiv \left.\frac{\partial \dot{H}_t}{\partial s_t}\right|_{\{H^{\mathrm{AI}},A^{\mathrm{AI}},s^{\mathrm{AI}}\}} = \alpha'\left(s^{\mathrm{AI}}\right)\left(1 - H^{\mathrm{AI}}\right) = \frac{\xi\alpha^{\mathrm{AI}}\left(1 - H^{\mathrm{AI}}\right)}{\left(1-\xi\right)s^{\mathrm{AI}}}, \\
J_{21}^{\mathrm{AI}} &\equiv \left.\frac{\partial \dot{A}_t}{\partial H_t}\right|_{\{H^{\mathrm{AI}},A^{\mathrm{AI}},s^{\mathrm{AI}}\}} = \frac{1}{\rho}\left[\alpha^{\mathrm{AI}}\left(\frac{\mu}{\delta} - 1\right) - \sigma\right]
\end{aligned}$$

$$\begin{aligned}
J_{31}^{\mathrm{AI}} &\equiv \left.\frac{\partial \dot{s}_t}{\partial H_t}\right|_{\{H^{\mathrm{AI}},A^{\mathrm{AI}},s^{\mathrm{AI}}\}} = -\varepsilon_y \mu s^{\mathrm{AI}} \\
J_{32}^{\mathrm{AI}} &\equiv \left.\frac{\partial \dot{s}_t}{\partial A_t}\right|_{\{H^{\mathrm{AI}},A^{\mathrm{AI}},s^{\mathrm{AI}}\}} = -\rho\mu\left(\pi^A - s^{\mathrm{AI}}\right) \\
J_{33}^{\mathrm{AI}} &\equiv \left.\frac{\partial \dot{s}_t}{\partial s_t}\right|_{\{H^{\mathrm{AI}},A^{\mathrm{AI}},s^{\mathrm{AI}}\}} = r + \alpha^{\mathrm{AI}}\left(\frac{\mu}{\delta} + \frac{1-\theta}{1-\xi} - 1\right)
\end{aligned}$$

Evaluating around the some-AI steady state, we obtain the general solution:

$$\begin{pmatrix} H_t \\ A_t \\ s_t \end{pmatrix} = \begin{pmatrix} H^{\mathrm{AI}} \\ A^{\mathrm{AI}} \\ s^{\mathrm{AI}} \end{pmatrix} + \tilde{C}_1 e^{\tilde{\lambda}_1 t}\tilde{\mathbf{x}}_1 + \tilde{C}_2 e^{\tilde{\lambda}_2 t}\tilde{\mathbf{x}}_2 + \tilde{C}_3 e^{\tilde{\lambda}_3 t}\tilde{\mathbf{x}}_3, \tag{58}$$

where $\tilde{\mathbf{x}}_i \equiv \left\{\tilde{x}_i^H, \tilde{x}_i^A, \tilde{x}_i^s\right\}$ is the eigenvector of the associated eigenvalue $\tilde{\lambda}_i$, where $i = 1, 2, 3$ and real $\left(\tilde{\lambda}_1\right) \leq$ real $\left(\tilde{\lambda}_2\right) \leq$ real $\left(\tilde{\lambda}_3\right)$, solving the characteristic equation:

$$\begin{aligned} 0 &= \left|\mathbf{J}^{\mathrm{AI}} - \tilde{\lambda}\mathbf{I}\right| = -\tilde{\lambda}^3 + \mathrm{tr}\left(\mathbf{J}^{\mathrm{AI}}\right)\tilde{\lambda}^2 + \left(J_{21}J_{12} + J_{31}J_{13} - J_{11}J_{33}\right)\tilde{\lambda} - \det\left(\mathbf{J}^{\mathrm{AI}}\right). \\ \mathrm{tr}\left(\mathbf{J}^{\mathrm{AI}}\right) &= J_{11} + J_{33} \\ \det\left(\mathbf{J}^{\mathrm{AI}}\right) &= -J_{21}\left(J_{13}J_{32} - J_{12}J_{33}\right) \end{aligned}$$

If the linearized model is local determinate, we have real $\left(\tilde{\lambda}_2\right) \leq 0$ and real $\left(\tilde{\lambda}_3\right) > 0$. Thus, it implies that $\tilde{C}_3 = 0$ so the system will converge the some-AI steady state. Given the initial state of the economy $\{H_0, A_0\}$, we can solve $\tilde{C}_1$ and $\tilde{C}_2$ as

$$\tilde{C}_1 = \frac{(H_0 - H^{\mathrm{AI}})\tilde{x}_2^A - (A_0 - A^{\mathrm{AI}})\tilde{x}_2^H}{\tilde{x}_1^H\tilde{x}_2^A - \tilde{x}_2^H\tilde{x}_1^A}, \tag{59}$$

$$\tilde{C}_2 = \frac{(A_0 - A^{\mathrm{AI}})\tilde{x}_1^H - (H_0 - H^{\mathrm{AI}})\tilde{x}_1^A}{\tilde{x}_1^H\tilde{x}_2^A - \tilde{x}_2^H\tilde{x}_1^A}. \tag{60}$$

Thus, the linearized model of s_t is

$$s_t = s^{\mathrm{AI}} + \tilde{C}_1 e^{\tilde{\lambda}_1 t}\tilde{x}_1^s + \tilde{C}_2 e^{\tilde{\lambda}_2 t}\tilde{x}_2^s \tag{61}$$

9.1.2. Computing the global dynamics

Suppose the equilibrium path converges to the steady state monotonically. Iterate the following process:

1. Set $j = 0$ with

$$\begin{aligned} s^0\left(A; \alpha^{\mathrm{AI}}\right) &\equiv s^{\mathrm{AI}} + \tilde{C}_1 e^{\tilde{\lambda}_1 t}\tilde{x}_1^s + \tilde{C}_2 e^{\tilde{\lambda}_2 t}\tilde{x}_2^s, \\ \text{s.t. } \exists t\ A &= A^{\mathrm{AI}} + \tilde{C}_1 e^{\tilde{\lambda}_1 t}\tilde{x}_1^A + \tilde{C}_2 e^{\tilde{\lambda}_2 t}\tilde{x}_2^A. \end{aligned} \tag{62}$$

where $\tilde{C}_1$ and $\tilde{C}_2$ are given by (59) and (60).

2. Given $s^j\left(A;\alpha^{\mathrm{AI}}\right)$, integrate the following ODE of $H^j\left(A\right)$ forward from A_0 with the boundary condition $H^j\left(A_0\right) = H_0$ to A^{AI}:

$$\frac{dH^j}{dA} = \begin{cases} \frac{\alpha(s)\left(1-H^j\right)-\sigma H^j-\rho\mu A H^j 1_{s^j<\pi^A}}{\left(\mu H^j-\delta\right)A} \text{ if } A \neq A^{\mathrm{AI}} \\ \frac{\tilde{x}_2^H}{\tilde{x}_2^A} \text{ if } A = A^{\mathrm{AI}} \end{cases} \tag{63}$$

3. Given $H^j\left(A\right)$, integrate the following ODE of $s^{j+1}\left(A;\alpha^{\mathrm{AI}}\right)$ backward from A^{AI} with the boundary condition $s^{j+1}\left(A^{\mathrm{AI}};\alpha^{\mathrm{AI}}\right) = s^{\mathrm{AI}}$ to A_0:

$$\frac{ds^{j+1}}{dA} = \begin{cases} \frac{\left\{ \begin{array}{c} \left\{r+\sigma+\left(1-\theta\right)\alpha\left(s^{j+1}\right)-\varepsilon_y\left(\mu H^j-\delta\right)\right\}s^{j+1} \\ -\left(1-b\right)-\rho\mu A_t\left\{\pi^A-s^{j+1}\right\}_+ \end{array} \right\}}{\left(\mu H^j-\delta\right)A} \text{ if } A \neq A^{\mathrm{AI}}, \\ \frac{\tilde{x}_2^s}{\tilde{x}_2^A} \text{ if } A = A^{\mathrm{AI}}. \end{cases} \tag{64}$$

4. Repeat step 2 and 3 with $j = j+1$ until $\max_A \left|H^{j+1}\left(A\right) - H^j\left(A\right)\right| < 10^{-6}$. The process results in $s\left(A;\alpha^{\mathrm{AI}}\right)$.

5. Given the initial job-finding rate α_0, find the steady-state job-finding rate α^{AI} such that $s\left(A;\alpha^{\mathrm{AI}}\right)$ resulted from steps 1 to 4 above solves

$$\alpha_0 = \alpha\left[s\left(A_0;\alpha^{\mathrm{AI}}\right)\right]. \tag{65}$$

where an initial guess is given by the fixed point of α^{AI} from the linearized model such that

$$\alpha_0 = \alpha^{\mathrm{AI}}\left(\frac{s_0}{s^{\mathrm{AI}}}\right)^{\frac{\xi}{1-\xi}}, \tag{66}$$

where s_0 is given by (61) and s^{AI} by (57).

6. Once it is computed, the corresponding vacancy-posting cost is

$$\kappa = \theta s^{\mathrm{AI}}\left(\alpha^{\mathrm{AI}}\right)^{-\frac{1-\xi}{\xi}}. \tag{67}$$

9.2. *Computing the global dynamics leading to the unbounded-AI steady state*

Evaluating the steady-state level of job-finding rate α^{AJJ} at the unbounded-AI steady-state level of surplus ratio s^{AJJ}, we have

$$\alpha^{\mathrm{AJJ}} = \left(\frac{\theta}{\kappa} s^{\mathrm{AJJ}}\right)^{\frac{\xi}{1-\xi}},$$

thus, we can substitute out the unknown parameter θ/κ and express α^{AJJ} and s^{AJJ} as the parameters for the job-finding rate function such that

$$\alpha(s_t) = \alpha^{\mathrm{AJJ}} \left(\frac{s_t}{s^{\mathrm{AJJ}}}\right)^{\frac{\xi}{1-\xi}}. \tag{68}$$

Given α^{AJJ} at the unbounded-AI steady state, the steady-state level of employment H^{AJJ} and surplus ratio s^{AJJ} are given by

$$\begin{aligned} H^{\mathrm{AJJ}} &= \frac{\alpha^{\mathrm{AJJ}}}{\alpha^{\mathrm{AJJ}} + \sigma}, \\ s^{\mathrm{AJJ}} &= \frac{1-b}{r + \sigma + (1-\theta)\,\alpha^{\mathrm{AJJ}} - \varepsilon_y \left(\mu H^{\mathrm{AJJ}} - \delta\right)}, \end{aligned}$$

where the necessary condition of the existence of unbounded-AI steady state is

$$\begin{aligned} s^{\mathrm{AJJ}} &\geq \pi^{A}, \\ H^{\mathrm{AJJ}} &> \frac{\delta}{\mu}. \end{aligned}$$

9.2.1. *Computing a linearized model around the steady state*

Linearizing the ODE system around the unbounded-AI steady state with $I_\infty = 1$, we have

$$\begin{pmatrix} \dot{H}_t \\ \dot{s}_t \end{pmatrix} = \mathbf{J}^{\mathrm{AJJ}} \begin{pmatrix} H_t - H^{\mathrm{AI}} \\ s_t - s^{\mathrm{AI}} \end{pmatrix},$$

where the Jacobian matrix is given by

$$\mathbf{J}^{\mathrm{AJJ}} = \begin{bmatrix} J_{11}^{\mathrm{AJJ}} & J_{12}^{\mathrm{AJJ}} \\ J_{21}^{\mathrm{AJJ}} & J_{22}^{\mathrm{AJJ}} \end{bmatrix},$$

with the matrix entries given by

$$\begin{aligned}
J_{11}^{\mathrm{AJJ}} &\equiv \left.\frac{\partial \dot{H}_t}{\partial H_t}\right|_{\{H^{\mathrm{AJJ}},A^{\mathrm{AJJ}},s^{\mathrm{AJJ}}\}} = -\alpha^{\mathrm{AJJ}} - \sigma, \\
J_{12}^{\mathrm{AJJ}} &\equiv \left.\frac{\partial \dot{H}_t}{\partial s_t}\right|_{\{H^{\mathrm{AJJ}},A^{\mathrm{AJJ}},s^{\mathrm{AJJ}}\}} = \frac{\xi\alpha^{\mathrm{AJJ}}\left(1-H^{\mathrm{AJJ}}\right)}{(1-\xi)\,s^{\mathrm{AJJ}}},
\end{aligned}$$

$$\begin{aligned}
J_{21}^{\mathrm{AJJ}} &\equiv \left.\frac{\partial \dot{s}_t}{\partial H_t}\right|_{\{H^{\mathrm{AJJ}},A^{\mathrm{AJJ}},s^{\mathrm{AJJ}}\}} = -\varepsilon_y \mu s^{\mathrm{AJJ}} \\
J_{22}^{\mathrm{AI}} &\equiv \left.\frac{\partial \dot{s}_t}{\partial s_t}\right|_{\{H^{\mathrm{AJJ}},A^{\mathrm{AJJ}},s^{\mathrm{AJJ}}\}} = r + \sigma + \alpha^{\mathrm{AI}}\frac{1-\theta}{1-\xi} - \varepsilon_y\left(\mu H^{\mathrm{AJJ}} - \delta\right)
\end{aligned}$$

Evaluating around the some-AI steady state, we obtain the general solution:

$$\begin{pmatrix} H_t \\ s_t \end{pmatrix} = \begin{pmatrix} H^{\mathrm{AJJ}} \\ s^{\mathrm{AJJ}} \end{pmatrix} + \tilde{C}_1 e^{\tilde{\lambda}_1 t}\tilde{\mathbf{x}}_1 + \tilde{C}_2 e^{\tilde{\lambda}_2 t}\tilde{\mathbf{x}}_2, \tag{69}$$

where $\tilde{\mathbf{x}}_i \equiv \left\{\tilde{x}_i^H, \tilde{x}_i^s\right\}$ is the eigenvector of the associated eigenvalue $\tilde{\lambda}_i$, where $i = 1, 2$ and real $\left(\tilde{\lambda}_1\right) \leq$ real $\left(\tilde{\lambda}_2\right)$, solving the characteristic equation:

$$\begin{aligned}
0 &= \left|\mathbf{J} - \tilde{\lambda}\mathbf{I}\right| = \tilde{\lambda}^2 - \mathrm{tr}\left(\mathbf{J}^{\mathrm{AJJ}}\right)\tilde{\lambda} + \det\left(\mathbf{J}^{\mathrm{AJJ}}\right), \\
\mathrm{tr}\left(\mathbf{J}^{\mathrm{AJJ}}\right) &= J_{11}^{\mathrm{AJJ}} + J_{22}^{\mathrm{AJJ}}, \\
\det\left(\mathbf{J}^{\mathrm{AJJ}}\right) &= J_{11}^{\mathrm{AJJ}} J_{22}^{\mathrm{AJJ}} - J_{12}^{\mathrm{AJJ}} J_{21}^{\mathrm{AJJ}},
\end{aligned}$$

If the linearized model is local determinate, we have real $\left(\tilde{\lambda}_1\right) \leq 0$ and real $\left(\tilde{\lambda}_2\right) > 0$. Thus, it implies that $\tilde{C}_2 = 0$ so the system will converge the unbounded-AI steady state. Given the initial state of the economy H_0, we can solve $\tilde{C}_1$ as

$$\tilde{C}_1 = \frac{H_0 - H^{\mathrm{AJJ}}}{\tilde{x}_1^H},$$

Thus, the linearized model is

$$H_t = H^{\text{AJJ}} + \tilde{C}_1 e^{\tilde{\lambda}_1 t} \tilde{x}_1^H \quad (70)$$

$$s_t = s^{\text{AJJ}} + \tilde{C}_1 e^{\tilde{\lambda}_1 t} \tilde{x}_1^s \quad (71)$$

$$A_t = A_0 \exp\left\{ \left(\mu H^{\text{AJJ}} - \delta\right) t + \frac{\mu \tilde{C}_1 \tilde{x}_1^H}{\tilde{\lambda}_1} \left(e^{\tilde{\lambda}_1 t} - 1\right) \right\} \quad (72)$$

9.2.2. Computing the global dynamics

Suppose the equilibrium path converges to the steady state monotonically. Iterate the following process:

1. Integrate the following ODE backward from H^{AJJ} with the boundary condition $s\left(H^{\text{AJJ}}\right) = s^{\text{AJJ}}$:

$$\frac{ds}{dH} = \begin{cases} \frac{\{r+\sigma+(1-\theta)\alpha(s)-\varepsilon_y(\mu H-\delta)\}s-(1-b)}{\alpha(s)(1-H)-\sigma H} \text{ if } H \neq H^{\text{AJJ}} \\ \frac{\tilde{x}_1^s}{\tilde{x}_1^H} \text{ if } H = H^{\text{AJJ}} \end{cases} \quad (73)$$

   Stop integrating whenever $H = H_0$ or $s\left(H_\tau\right) = \pi^A$ at some $H_\tau \neq H_0$.

2. If (73) stops at $H = H_0$ with $s\left(H_t\right) < \pi^A$ for all t, then set $s_0 = s\left(H_0\right)$.

3. If (73) stops at $s\left(H_\tau\right) = \pi^A$ at some $H_\tau \neq H_0$, then given A_τ, continue integrating the following system of ODEs from H_τ with the boundary condition $s^A\left(H_\tau\right) = s\left(H_\tau\right)$ and $A\left(H_\tau\right) = A_\tau$:

$$\frac{ds^A}{dH} = \frac{\left\{ \begin{array}{c} \left\{r + \sigma + (1-\theta)\alpha\left(s^A\right) - \varepsilon_y\left(\mu H - \delta\right)\right\} s^A \\ -(1-b) - \rho\mu A \left\{\pi^A - s^A\right\}_+ \end{array} \right\}}{\alpha\left(s^A\right)(1-H) - \sigma H - \rho\mu H A 1_{s^A < \pi^A}} \quad (74)$$

$$\frac{dA}{dH} = \frac{\left(\mu H - \delta\right) A}{\alpha\left(s^A\right)(1-H) - \sigma H - \rho\mu H A 1_{s^A < \pi^A}} \quad (75)$$

   Stop integrating when $H = H_0$. Solve A_τ such that $A\left(H_0\right) = A_0$, where an initial guess is

$$A_\tau = A_0 \exp\left\{ \left(\mu H^{\text{AJJ}} - \delta\right) \tau + \frac{\mu \tilde{C}_1 \tilde{x}_1^H}{\tilde{\lambda}_1} \left(e^{\tilde{\lambda}_1 \tau} - 1\right) \right\}$$

   Set $s_0 = s^A\left(H_0\right)$

4. Given s_0 from either step 4 or step 5, set the error as

$$error = \alpha_0 - \alpha\left(s_0\right). \quad (76)$$

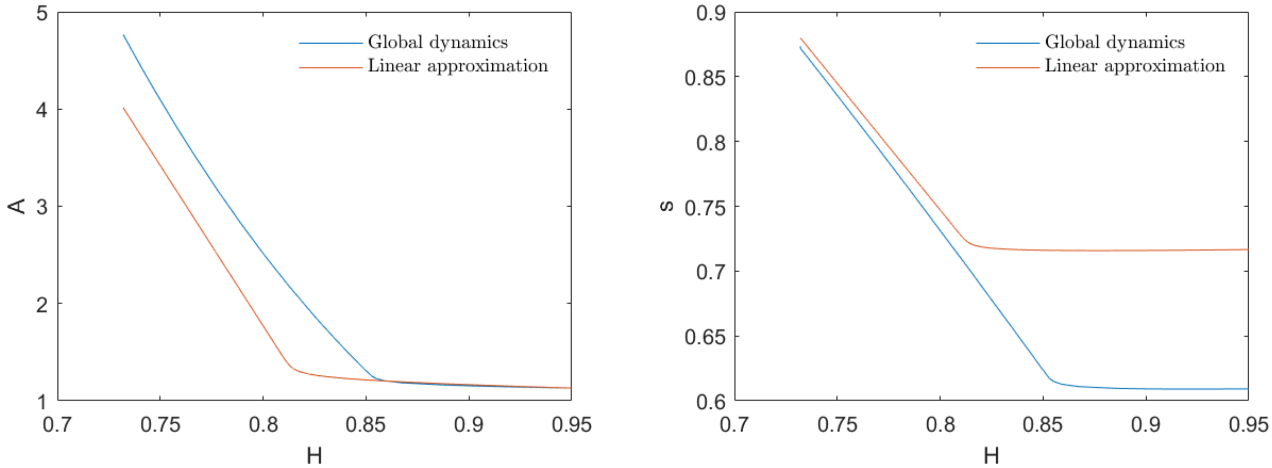

Figure 6: Global dynamics vs. linearized approximation.

Solve the steady-state job-finding rate α^{AI} such that $error = 0$.

9.3. *Computing the global dynamics leading to the no-AI steady state*

The algorithm is the same as to compute the unbounded-AI steady state, except that necessary condition becomes

$$H^{\mathrm{MP}} < \frac{\delta}{\mu},$$

instead of $H^{\mathrm{AJJ}} > \delta/\mu$.

9.4. *Global Solution v.s. Linear Approximation*

Note that we compute the entire transitional dynamics without linearizing around the steady state, a typical approximation in the literature. In the full model, the approximation error by the linearization around the steady state can be sizable for two reasons. First, the linearized dynamics away from the steady state could be different. Second, the levels of steady state implied by the linearized approximation can also be very different. It is because in order to calibrate the initial job-finding rate to its target, the different path generated by the linearized approximation requires different parameters and hence implies different steady state level. The contrast between global and local dynamics is illustrated in the Figure 6, where the left panel depicts the projection of the transition path in (H_t, A_t)-space and the right panel the transition path of s_t. It turns out the second factor dominates and leads to the significant approximate error. Recall the second factor arises because we need to calibrate to the initial job-finding rate, without fixing the long-run impact of AI. If the job-finding rate is exogenous, as in the benchmark model, the linearized dynamics does provide a reasonably good approximation.

10. Online Appendix for the Extension of Stand-ready Option

We further allow for an extra stand-ready option, under which the firm can adopt AI immediately to replace a worker. This yields a net present value of profits equal to $\pi^C y_t$, where $\pi^C < \pi^A$. The inequality ensures that the stand-ready option is strictly less profitable or less well-developed than the benchmark option.

Formally, this extension is equivalent to imposing $W_t^n - U_t = (1-\theta)\left(S_t - \pi^C y_t\right)$ for all n, so that the firm's profit becomes $\Pi_t = \theta\left(S_t - \pi^C y_t\right) + \pi^C y_t$. In the equilibrium, the joint surplus of the job never falls below π^C; otherwise, the firm would exercise the stand-ready option and replace all workers immediately with AI.

The joint surplus of the job becomes

$$\dot{s}_t = (r+\sigma-g_t)\, s_t - (1-b) - \rho\mu A_t \left(\pi^A - s_t\right)_+ + \alpha_t (1-\theta)\left(s_t - \pi^C\right), \tag{77}$$

where

$$s_t \geq \pi^C.$$

Define $\pi_t \equiv \theta s_t + (1-\theta)\,\pi^C$ as the profit per output, and $\bar{\pi}^A \equiv \theta\pi^A + (1-\theta)\,\pi^C$. Using the ODE of s_t, we have

$$\begin{aligned}\dot{\pi}_t &= \left[r+\sigma+\alpha_t(1-\theta)-g_t\right]\pi_t - \theta(1-b) - \rho\mu A_t\left(\bar{\pi}^A - \pi_t\right)_+ \\ &\quad - (r+\sigma+\alpha_t-g_t)(1-\theta)\,\pi^C\end{aligned}$$

Accordingly, vacancy creation is governed by the modified free-entry condition. Denote π^{AI} as the profit (per output) in the some-AI steady state. The fraction of workers displaced by AI is thus

$$L = \frac{\rho\mu A^{\mathrm{AI}}}{\sigma + \rho\mu A^{\mathrm{AI}}}\left(1 - H^{\mathrm{AI}}\right)$$

The job-finding rate is

$$\alpha^{\mathrm{AI}} = \alpha\left(\pi^{\mathrm{AI}}\right) \equiv \Lambda\left(\frac{\kappa}{\pi^{\mathrm{AI}}}\right)$$

Since there is no census on the value of π^C, we compare the fraction of workers displaced by AI against a range of π^C. Our model in the main text is the special case $\pi^C = 0$. Figure 7 reports the findings.

11. Online Appendix for Robustness Checks

We check the robustness of alternative parametrization. First and most importantly, we check firm's bargaining power θ, due to its wide range in the literature, varying from a high value of 0.765 in Hall (2005) to a low value of 0.28 in Shimer (2005). At $\theta = 0.765$, by

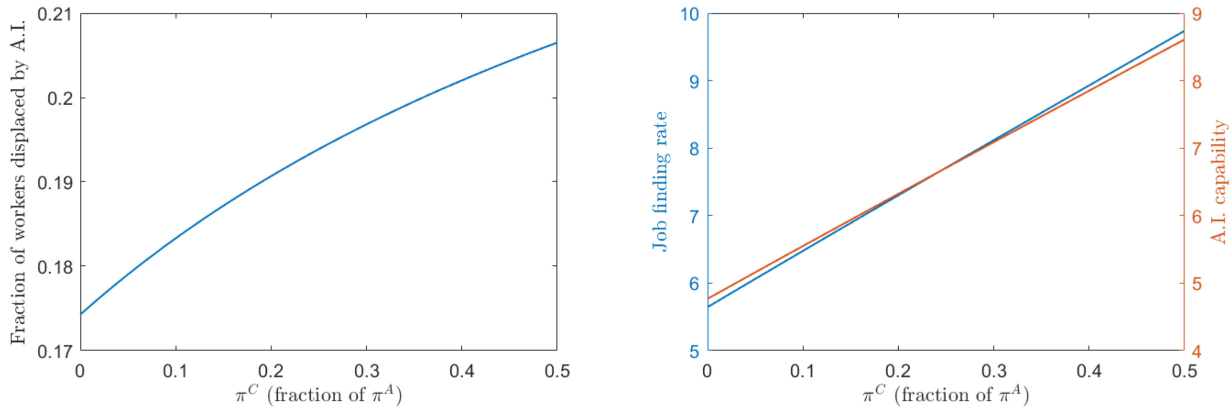

Figure 7: Extension with Stand-ready Option.

recalibrating to fit α_0 (requires κ to be recalibrated), the some-AI steady state remains as the unique equilibrium that features a monotone converging saddle. The employment loss is still at 23%, while AI capability and output growth are moderately lower. By taking $\theta = 0.28$ (and adjusting κ), we have the same equilibrium pattern and employment loss, with moderately higher AI capability and output growth.

Second, as discussed, the replacement rate in some studies could be much higher, so we check what happens with $b = 0.54$ (with κ recalibrated). We again have the same equilibrium pattern, but with moderately higher AI and output growth. Third, the benchmark worker replacement cost might appear to be low, so we raise it to $\phi = 0.1$ (with κ recalibrated) and continue obtaining the same equilibrium pattern and employment loss with moderately lower AI and output growth.

Finally, one may argue AI decay may not follow the Moore's Law exactly. We thus perturb the AI capability decay half life by ± 1 month. With 19 month half life (lower decay to $\delta = 0.438$ and recalibrating κ), we have the same equilibrium pattern but the employment loss is widened to 27.1% in conjunction with a significantly faster AI and output growth. With 17 month half life ($\delta = 0.489$, with κ recalibrated), the reverse is true with less employment loss at 18.5% and much slower AI and output growth. Among all, the steady-state equilibrium values appears to be most sensitive to AI capability decay half life, though equilibrium pattern stays unchanged.