Diversification Benefits and International Financial Contagion: The Impact of Hidden Asymptotic Dependence

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Abstract

We slightly extend the Poon et al. (2004) methodology for the estimation of bivariate extremal dependence into a multi-asset setting. This is achieved by first splitting the portfolio into two sub-portfolios and then estimating the extremal dependence structure between them. This approach provides important insights for better understanding international financial contagion and systemic risk. For example, it is possible to find sub-portfolios that exhibit asymptotic tail dependence even if all possible bivariate asset pairs within the portfolio are asymptotically independent of each other. We refer to this situation as "hidden asymptotic dependence" since it cannot be detected with the standard pairwise dependence analysis. Then, we investigate its importance for portfolio selection by measuring the diversification benefits for all possible three-asset portfolios of S&P/IFCI emerging market stock indices. We found that the diversification benefits are significantly reduced for the portfolios where hidden asymptotic dependence is observed. Motivated with this result, we propose new measures of international financial contagion by extending the existing measures from the literature in a way to incorporate hidden asymptotic dependencies. Results show that incorporation of hidden dependencies reveal a much severe picture of potential for financial contagion.

1 Introduction

Tail dependence or extreme value dependence recently became an important statistical subject that has interesting financial applications. The catastrophic market crashes of recent decades proved to be extremely contagious as the markets of different asset classes became more and more integrated, not only nationally but also globally. During earlier episodes of global distresses, both regulators and academics considered the

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sudden increases in price co-movements or correlations as a breakdown of transmission mechanisms among financial markets, or contagion. However, changes in correlation coefficients do not necessarily indicate contagion, as they do not always imply a structural change in the data generating process. Boyer, Gibson, and Loretan (1997) and Forbes and Rigobon (2002) argue that unlike markets in normal times, stronger interdependence between different asset classes is observed during market crashes as a continuation of same data generating process. This observation raised concerns about the usefulness of Pearson's correlation coefficient in diagnosing the international financial contagion. Also, Pearson's correlation has many pitfalls as a measure of dependence. For example, Pearson's correlation can summarize all the information on the dependence of two assets returns only if they are multivariate normally distributed. However, it is well documented that asset returns are skewed and have fatter tails than the normal distribution. Also, Pearson's correlation measures linear dependence only, omitting any potential nonlinear dependencies. More importantly, analysis based on Pearson's correlation likely underestimates the probability of simultaneous crashes in multiple markets. This is because it gives equal weight to all observations resulting in an average dependence measure which is dominated by the observations from normal times, however the dependence during financial turmoil is stronger than what is implied by this average dependence[1].

As a result, more researchers have emphasized the need to develop and use complementary dependence metrics beyond Pearson's correlation coefficient when studying international financial contagion. In particular, tail dependence has become an important area of study due to the importance of accurate modeling of catastrophic events in financial markets. Chan-Lau et al. (2004) developed measures of financial contagion using extreme value dependence metrics χ and $\bar{\chi}$ which are borrowed from multivariate extreme value theory literature. Technical details of these measures and their estimation methods are discussed in later sections of our study.

Basically, Ledford and Tawn (1996, 1997 and 1998) developed a joint distribution specification that differentiates between asymptotically dependent and asymptotically independent tail structures. The extremal dependence measure χ is defined as:

$$\chi = \lim_{q \to 1} \Pr(L_1 > L_{1,q} | L_2 > L_{2,q}), \tag{1}$$

where $L_{1,q}$ and $L_{2,q}$ stands for the qth quantile of the marginal distribution of the loss variables L_1 and L_2, respectively. If $\chi = 0$, then L_1 and L_2 are said to be asymptotically independent. If $\chi = c > 0$, then L_1 and L_2 are said to be asymptotically dependent and the value of χ determines the strength of asymptotic dependence. Since χ captures the dependence in the limiting extremes and is equal to zero for all asymptotically independent variables, Coles et al.(1999) developed a complementary tail dependence measure, $\bar{\chi}$, that captures the dependence at the finite levels of the extremes. It is defined as:

$$\bar{\chi} = \lim_{q \to 1} \frac{2 \log \Pr(L_2 > L_{2,q})}{\log \Pr(L_1 > L_{1,q},\ L_2 > L_{2,q})} - 1. \tag{2}$$

Under asymptotic independence, higher $\bar{\chi}$ implies slower convergence to zero in the limiting conditional probability given in (1).

[1]For a comprehensive discussion on the drawbacks of correlation see Embrechts et al. (1999)

Besides international financial contagion, researchers found interesting new venues for empirical applications of these new extreme value dependence measures. For example, Poon et al. (2004) investigate the tail dependence among international stock markets and also provide examples for applications of tail dependence analysis in sharp ratio targeting, hedging, and option valuation. Balla et al. (2014) used them to quantify the systemic risk in U.S. depository firms; and showed that systemic risk measures developed from tail dependence metrics can capture the historical downturns in the banking industry very well. Burkart (2006) used them to quantify the co-movement of default risk for large reinsurance firms and found that they convey information above and beyond what is contained in Pearsons correlation coefficient. Kellner and Gatzert (2013) used them to quantify the basis risk of index-linked hedging strategies and found that incorporation of tail dependence increases the accuracy of basis risk estimates. De Carvalho and Rua (2013) used tail dependence in quantifying the interdependence between the output growths in OECD countries and found that cross-country tail dependence in output growth is much stronger than that implied by correlation analysis.

In addition to risk quantification, several studies investigated the implications of tail dependence for portfolio selection. For example, DiTraglia and Gerlach (2013) quantify the tail dependence between the Dow Jones Industrial Average (DJIA) index and the individual component stocks in it. They conclude that tail dependence is priced by the investors, i.e., investors require a premium return for those stocks that exhibit stronger tail dependence with the DJIA index. Chollete et al. (2012) studies international diversification and finds that the correlation coefficient does not vary with returns, but extreme dependence vary monotonically with returns. This suggests that investors require some compensation in good times for the joint downside risk. Bradley and Taqqu (2004) studied the impact of extremal dependence on the weight given to the riskier asset in a two-asset portfolio optimization problem for risk averse investors. Ergen (2014) studied the two-asset portfolio optimization problem for stocks in emerging markets and found that tail dependence is one of the key drivers of diversification benefits. Also, tail dependence has better explanatory power for the variation in diversification benefits compared to the correlation coefficient.

The advantages of tail dependence analysis with respect to mainstream correlation analysis are investigated thoroughly by all of the aforementioned studies. However, both approaches share a common shortcoming in that they only measure the dependence between two individual assets. With a multi-asset portfolio, the analysis is performed for all asset pairs to construct a dependence matrix. However, a dependence matrix doesn't capture the complicated multivariate dependence relationships fully unless we believe in the multivariate normality of asset returns. In this paper, while we are interested in empirical analysis of diversification benefits and financial contagion among emerging market stocks, we first attack this problem by slightly extending the extremal dependence estimation methodology proposed by Poon et al. (2004) to a multi-asset setting. We accomplish this by first splitting the portfolio into two mutually exclusive and collectively exhaustive sub-portfolios and then estimating the tail dependence structure between these sub-portfolios. This approach produces an interesting result that has important implications for better understanding of systemic risk and financial contagion. Basically, we identified some portfolios where the two sub-portfolios are asymptotically dependent despite the fact that all possible asset

pairs are asymptotically independent. In other words, the tail dependence between the sub-portfolios can be stronger than the tail dependence between any of the asset pairs in a portfolio. We refer to this as hidden asymptotic dependence since it can be ignored under the standard pairwise dependence analysis. We interpret this finding as indicative of stronger contagion effect when multiple markets crash compared to when an isolated individual market crashes. Therefore, we investigate its implications for diversification benefits and financial contagion.

To explore the impact on diversification benefits, we analyze all possible three-asset portfolios in our sample of 15 emerging stock indices. Among these three-asset portfolios, we identify the ones that exhibit hidden asymptotic dependence after controlling for the standard pairwise asymptotic dependence. We also measure the diversification benefits for each of these three-asset portfolios with the reduction in tail risk measures, VaR and ES, resulting from investing in the optimal three-asset portfolio rather than investing in the optimal two-asset portfolio. We find that the existence of hidden asymptotic dependence significantly reduces diversification benefits but not as much as the asymptotic dependence between the pairs of assets. We believe that these results make intuitive sense and provide valuable information to portfolio managers. Portfolios can not be regarded as resilient to crisis simply based on bi-variate dependence analysis due the existence of hidden dependencies resulting from complicated underlying relationships.

We also explore the implications of hidden asymptotic dependence on financial contagion. Chan-Lau et al. (2004) proposed using the ratio of asymptotically dependent country pairs to all possible country pairs as an indicator of international financial contagion. This indicator can be loosely interpreted as the average probability of an isolated crisis in one country to spread to some other country. However, it doesn't account for the probability of crisis spreading when multiple countries are in crisis. This is because it focuses on tail dependencies among individual countries and therefore doesn't account for the hidden asymptotic dependencies thereby resulting in underestimation of the level of financial contagion. We propose a new metric as a measure of financial contagion to improve this existing measure. We extend it to incorporate hidden asymptotic dependencies by analyzing the tail dependence between the individual countries and the composite emerging market index. Incorporation of hidden asymptotic dependence results in a significant increase to the indicators of international financial contagion because it accounts for the possibility of more widespread distresses in the emerging markets in a broader sense. Our analysis demonstrates a much severe picture of potential for financial contagion.

The remainder of this paper is organized as follows. In section 2, we introduce the tail risk measures, VaR and ES; and the application of the extreme value theory (EVT) used in tail risk estimation. In section 3, we introduce the extreme value dependence measures used in this study and the estimation methodology developed by Poon et al. (2004). In section 4 , we introduce our data set on emerging market equity indices and present descriptive statistics. In section 5, we describe the concept of hidden asymptotic dependence after extending the Poon et al. (2004) methodology for the estimation of tail dependence to a multi-asset setting. Then, we go on to present our empirical methodology to identify the portfolios that exhibit hidden asymptotic dependence and analyze its impact on diversification benefits. In section 6, we extend the existing indicators of international financial contagion to incorporate the hidden

asymptotic dependencies. Section 7 concludes.

2 Estimation of Tail Risk Measures

In this section, we describe the empirical methodology used in producing the tail risk measures such as Value-at-Risk (VaR) and Expected Shortfall (ES). This background review is needed because later in section 5.2, diversification benefits of portfolios are calculated based on the potential reduction in these tail risk measures.

VaR is the level of loss that is not expected to be breached at a certain confidence level. More formally, it is an extreme quantile in the tail of a loss distribution and can be written as:

$$VaR_q = inf\{l \in \Re : F_L(l) \geq q\}, \tag{3}$$

where L denotes the loss, F_L is the distribution function of losses, and q is the confidence level or quantile at which VaR is calculated. VaR is heavily criticized based on both theoretical and empirical grounds for not capturing the tail risk appropriately. For example, Artzner et al. (1997) define a unified framework for market risks in which coherent measures of risk satisfy four basic properties: translation invariance, sub-additivity, positive homogeneity and monotonicity. VaR fails to satisfy the conditions required to be considered a coherent risk measure. Also, VaR simply provides an extreme loss level; it fails to summarize the potential severity of losses above that level.

Acerbi and Tasche (2002) show that the Expected Shortfall (ES) is a coherent risk measure which resolves the deficiencies of Value-at-Risk. Besides having nice theoretical properties, ES also provides a more detailed picture of risk as it summarizes more information regarding potential losses beyond VaR. Expected Shortfall can be intuitively defined as the expectation of loss given that the loss has already breached the VaR_q level. More formally, it is given by:

$$ES_q = \frac{1}{1-q}\int_q^1 VaR_u(L)du \tag{4}$$

In this paper, both tail risk measures are calculated using EVT methods. McNeil (1999) found that EVT methods provide more accurate tail estimation with respect to conventional distributional approaches. This finding is confirmed by many other studies[2]. EVT models the extreme tail of the data as a separate distribution instead of imposing a single distribution on the entire sample. According to EVT, the distribution of losses above a high threshold uniformly approaches the Generalized Pareto Distribution (GPD) as the threshold is increased. More formally,

$$\lim_{u \to l_o} \Pr(L - u \leq y | L > u) = G_{\xi,\beta(u)} \qquad \text{for} \qquad 0 < y < l_o - u \ . \tag{5}$$

where L is the loss, u is a high threshold, ℓ_o is the right end point of the support of L, ξ is the shape, and $\beta > 0$ is the scale parameter of the limiting GPD. This result holds regardless of the original distribution of the loss data. Therefore, EVT can be considered as a central limit theorem for extremes.

[2]See Ergen (2014,a), Gencay and Selcuk (2004), Bali(2003 and 2007), Furiò and Climent (2013) and Rossignolo (2013).

Implementation of the EVT method in practice requires the selection of a high threshold and fitting the GPD to the exceedance data above this threshold. However, if the threshold is selected too low, this will result in bias since the exceedance data will be far away from the limiting GPD. Within a simulation framework using fat-tailed distributions, McNeil and Frey (2000) found that selecting the threshold at the 90th percentile of the data results in quantile estimates with relatively low bias. Since the sample size in this study is quite large we will select the threshold at the 95th quantile; exceeding their minimum recommendation. Once GPD is fit to the tail data, it is straightforward to show that VaR and ES at quantiles higher than the threshold can be calculated as below[3]:

$$VaR_q = u + \frac{\hat{\beta}}{\hat{\xi}} \left((\frac{1-q}{1-F_\ell(u)})^{-\xi} - 1 \right), ^4 \tag{6}$$

$$ES_q = \frac{VaR_q + \beta - \xi u}{1-\xi} \tag{7}$$

3 Extremal Dependence Measures

Multivariate extreme value theory (MEVT) has been a useful tool in the development of new tail dependence measures. Although the standard implementation of MEVT assumes asymptotic dependence by default, Ledford and Tawn (1996, 1997, 1998) made an important contribution by differentiating between two different tail dependence structures: asymptotically dependent and asymptotically independent. This distinction is very important because it implies either the existence of or lack thereof dependence in the limiting joint extremes. It is possible that such extremes have never been observed in the data; however dependence at those levels is what matters most for risk management against catastrophic losses. Formally, the tail dependence measure χ is defined as:

$$\chi = \lim_{q \to 1} \Pr(L_1 > L_{1,q} | L_2 > L_{2,q}); \ \chi \in [0,1]. \tag{8}$$

where $L_{1,q}$ and $L_{2,q}$ denotes the qth quantile of the marginal distribution of the loss variables L_1 and L_2, respectively. More intuitively, χ can be thought of as the limiting probability of an extreme crash in an asset conditional on an extreme crash in some other asset. If $\chi = 0$, then the two loss variables L_1 and L_2 are called asymptotically independent. If $\chi = c > 0$, then the two loss variables are called asymptotically dependent. It is important to note that asymptotic independence doesn't imply exact independence. It only implies that the two loss variables L_1 and L_2 become independent in their limiting joint extremes. However, at finite levels of extremes, they can still exhibit strong tail dependence. Since χ captures the limiting dependence and it is equal to zero for all asymptotically independent variables, Coles et al. (1999) developed another dependence measure, $\bar{\chi}$, in order to measure the strength of tail dependence within the asymptotically independent family. It basically captures the tail dependence

[3]For more detailed description of the Extreme Value Theory and the calculation of tail risk measures see McNeil et al. (2005), chapter 7.

[4]$F_\ell(u)$ in 6 can be estimated non-parametrically as 0.95 since we select the threshold, u, at the 95th quantile of the data.

at the finite levels of extremes and is given by:

$$\bar{\chi} = \lim_{q \to 1} \frac{2 \log \Pr(L_2 > L_{2,q})}{\log \Pr(L_1 > L_{1,q},\ L_2 > L_{2,q})} - 1;\ \bar{\chi} \in [-1, 1]. \tag{9}$$

In the case of asymptotic independence, $\bar{\chi}$ measures how slow the limiting probability in (8) converges to zero. Therefore, a higher $\bar{\chi}$ implies stronger tail dependence at finite levels of extremes for asymptotically independent variables. Coles et al (1999) also showed that in case of asymptotic dependence, *chibar* hits its upper boundary, i.e., $\bar{\chi} = 1$. Therefore, we have:

$$\begin{aligned} \bar{\chi} = 1 \quad iff \quad \chi = c > 0 \\ \bar{\chi} < 1 \quad iff \quad \chi = 0. \end{aligned} \tag{10}$$

Using these theoretical results, Poon et al. (2004) developed a hypothesis testing framework for the existence of asymptotic dependence which essentially tests the null hypothesis of $\bar{\chi} = 1$. The methodology works in three steps. In the first step, the loss variables, (L_1, L_2), are transformed into unit Frechet distributed variables by using probability integral transformations:

$$Z_1 = \frac{-1}{\log F_{L_1}(l_1)} \text{ and } Z_2 = \frac{-1}{\log F_{L_2}(l_2)}, \tag{11}$$

where F_{L_1} and F_{L_2} are the marginal distribution functions of L_1 and L_2, respectively[5]. In our implementation, we use the empirical distribution up to the 95th quantile of the data and a GPD fit for the tail beyond the 95th quantile in place of $F_{L_1}(l_1)$ and $F_{L_2}(l_2)$. It is important to note that (11) is a monotonic transformation which does not change the order of the data and therefore does not have any impact on the extremal dependence measures. In other words, $\chi_{L_1,L_2} = \chi_{Z_1,Z_2}$ and $\bar{\chi}_{L_1,L_2} = \bar{\chi}_{Z_1,Z_2}$.

In the second step, Poon et. al (2004) show that for two unit-Frechet distributed variables Z_1 and Z_2, we have $\bar{\chi}_{Z_1,Z_2} = 2\eta_T - 1$ where η_T is the tail index for the univariate variable $T = min(Z_1, Z_2)$. Basically, the tail index is a parameter that governs the decay rate in the tails of sub-exponential univariate distributions. Its properties are well studied in the literature and Hill's estimation became the standard methodology for its estimation. Therefore, the problem of tail dependence estimation reduces to a tail index estimation for the univariate variable $T = min(Z_1, Z_2)$ [6]. To perform that, we calculate $T = \min(Z_1, Z_2)$ in the second step of the procedure and estimate its tail index using Hill's (1975) estimator in the third and final step. We will skip the technical details for Hill's estimator for the purpose of conciseness. As shown in Poon et al. (2004) one can eventually obtain:

$$\bar{\chi} = \frac{2}{N_u} \sum_{i=1}^{i=N_u} log\left(\frac{t_i}{u}\right) \ - \ 1 \tag{12}$$

where u is a high threshold for the univariate variable t and N_u is the number of observations above u[7]. Inference can be performed using the maximum likelihood properties of Hill's estimator.

[5]The inverse distribution function (quantile function) for unit Frechet distribution is given by $-1/log(x)$

[6]Assumingly, this is the motivation for Poon et al. (2004) to convert marginal variables to unit Frechet in the first step of the procedure because they know that a solution to the problem exists in the unit Frechet world.

[7]In this paper we used the 95th percentile of the variable T as the high threshold.

4 Data and Descriptive Statistics

We use daily equity index data from 15 emerging markets: Brazil, Chile, Mexico, Peru, China, India, Indonesia, Malaysia, Philippines, South Korea, Taiwan, Thailand, Turkey, Check Republic and Hungary. The source of the data is Standard & Poors (S&P) and we downloaded it from Bloomberg. The International Financial Corporation (IFC) of the World Bank started to collect these index data on June 30, 1995, but was later acquired by S&P. It is the most reliable emerging market equity index data for both academic literature and industry practitioners, and it is known as the S&P/IFCI equity price index. The indices are market capitalization weighted averages of the prices for equities that are available for trade by international investors and they are dollar denominated. Therefore, returns calculated from these index data reflect the perspective of a U.S. investor. Our time series starts from June 30, 1995 when these index data were starting to be collected and runs to May 3, 2012 for all countries. Therefore, it is a balanced panel containing 4,351 observations for each country. We present the descriptive statistics of daily log-return data in Table 1. The minimum and maximum daily returns reflect the extremely volatile nature of emerging market returns series along with the standard deviation. The mean and median returns have generally been positive in our sample period. The excess kurtosis is very erratic for some countries which is a reflection of the extremely fat-tails of portfolio return distributions.

Country	nobs	Minimum	Maximum	Mean	Median	Stdev	Skewness	Kurtosis
Brazil	4351	−16.85	16.40	0.06	0.11	2.33	−0.32	6.84
Chile	4351	−11.64	14.78	0.03	0.02	1.29	−0.34	10.82
Mexico	4351	−15.03	16.13	0.05	0.08	1.80	−0.08	7.09
Peru	4351	−11.78	11.34	0.06	0.05	1.61	−0.34	6.44
China	4351	−15.34	13.94	0.03	0.02	2.08	−0.12	5.97
India	4351	−12.72	19.04	0.03	0.05	1.76	−0.13	6.79
Indonesia	4351	−40.88	25.55	0.02	0.03	2.84	−0.97	25.09
Malaysia	4351	−23.68	22.99	0.01	0.00	1.66	0.65	34.64
Philippines	4351	−13.86	21.33	0.00	0.00	1.72	0.44	12.44
South.Korea	4351	−21.56	26.82	0.02	0.02	2.52	0.14	12.42
Taiwan	4351	−11.07	8.24	0.01	0.00	1.73	−0.06	2.40
Thailand	4351	−17.09	16.54	0.00	0.00	2.07	0.20	7.97
Turkey	4351	−27.02	22.67	0.04	0.01	3.10	−0.08	6.21
Czech.Republic	4351	−14.77	20.82	0.05	0.05	1.81	−0.08	9.50
Hungary	4351	−20.19	20.33	0.04	0.07	2.32	−0.25	8.35

Table 1: Summary Statistics

The skewness and excess kurtosis observed in the return data justifies our approach to estimate risk measures VaR and ES using EVT methods. These tail risk measures are presented in Table 2 for all countries in our sample at several extreme quantiles. Indonesia stands out as the most risky stock market among its peers for this sample period. Turkey, Hungary, South Korea and Brazil follows it with a large margin.

	VaR			ES		
Country	99	99.9	99.99	99	99.9	99.99
Brazil	6.67	13.42	24.96	9.56	18.36	33.42
Chile	3.53	7.92	17.23	5.43	11.95	25.76
Mexico	4.97	9.97	18.18	7.11	13.48	23.95
Peru	4.72	9.36	16.13	6.71	12.26	20.34
China	5.92	11.01	18.05	8.10	14.02	22.23
India	5.17	8.73	12.47	6.71	10.35	14.17
Indonesia	8.38	20.79	46.73	13.75	32.00	70.16
Malaysia	4.78	10.23	18.72	7.11	13.86	24.39
Philippines	4.93	8.62	12.90	6.52	10.46	15.03
South.Korea	7.33	14.37	24.47	10.34	18.69	30.66
Taiwan	4.78	7.25	9.47	5.86	8.22	10.34
Thailand	5.69	10.67	17.90	7.81	13.76	22.40
Turkey	8.56	16.27	27.81	11.86	21.20	35.21
Czech.Republic	5.04	9.97	17.99	7.15	13.40	23.56
Hungary	6.62	13.50	24.90	9.56	18.38	32.96

Table 2: VaR and ES estimates

Next, we follow the Poon et al. (2004) methodology to determine the asymptotic dependence structure for pairs of emerging markets. Point estimates of $\bar{\chi}$ and their associated probability values for the null hypothesis of $\bar{\chi} = 1$ are presented in Table 3.

The pairs with probability values above 5% are shown in bold font. Out of the 105 emerging market stock index pairs, 91 of them are asymptotically independent of each other and there are only 14 country pairs asymptotically dependent to each other. The point estimates of χ for these 14 country pairs and their standard errors are given in Table 4. We observe a pattern for asymptotically dependent pairs. Out of the 14 asymptotically dependent pairs, 11 of them are in geographic proximity in the sense that they are either both South East Asian or both Latin American countries.

5 Hidden Asymptotic Dependence and Diversification Benefits

In this section, we extend the Poon et al. (2004) methodology for the estimation of bivariate extremal dependence into a multi-asset setting. This is performed by splitting the portfolio into two sub-portfolios that are mutually exclusive and collectively exhaustive and then analyzing the extremal dependence structure between them. Essentially, we are implementing the Poon et al. (2004) methodology after reducing the dimensionality of the problem to two. Although this is a direct extension of Poon et al. (2004) method, the empirical results and their implications provide important insights and help to develop a deeper understanding of financial contagion and systemic risk phenomena. For simplicity, we considered only three-asset portfolios in this study, but our empirical approach is generic and can easily be extended to portfolios of higher

	Bra	Chl	Mex	Per	Chn	Ind	Idns	Mal	Phil	Kor	Twn	Thai	Tur	Chk	Hun
Bra															
Chl	**0.91** (0.507)														
Mex	**0.96** (0.735)	**1** (1)													
Per	**0.98** (0.897)	**0.89** (0.402)	**0.93** (0.582)												
Chn	0.57 (0)	0.66 (0.003)	0.66 (0.003)	0.47 (0)											
Ind	0.57 (0)	0.66 (0.002)	0.66 (0.002)	0.71 (0.012)	0.68 (0.005)										
Idns	0.48 (0)	0.45 (0)	0.4 (0)	0.34 (0)	0.7 (0.01)	0.63 (0.001)									
Mal	0.46 (0)	0.48 (0)	0.47 (0)	0.35 (0)	0.51 (0)	0.45 (0)	**0.91** (0.492)								
Phil	0.53 (0)	0.52 (0)	0.48 (0)	0.38 (0)	0.63 (0.001)	0.59 (0)	**0.78** (0.071)	0.7 (0.01)							
Kor	0.56 (0)	0.57 (0)	0.52 (0)	0.4 (0)	0.7 (0.009)	0.6 (0)	0.69 (0.006)	0.69 (0.007)	0.7 (0.008)						
Twn	0.53 (0)	0.49 (0)	0.42 (0)	0.42 (0)	0.73 (0.022)	0.56 (0)	0.51 (0)	0.43 (0)	0.6 (0)	0.67 (0.004)					
Thai	0.43 (0)	0.46 (0)	0.47 (0)	0.35 (0)	0.68 (0.005)	0.52 (0)	**0.86** (0.25)	0.67 (0.003)	**0.83** (0.172)	**0.79** (0.09)	0.5 (0)				
Tur	0.65 (0.002)	0.69 (0.007)	**0.77** (0.054)	0.68 (0.005)	0.55 (0)	0.58 (0)	0.36 (0)	0.48 (0)	0.3 (0)	0.51 (0)	0.49 (0)	0.37 (0)			
Chk	0.71 (0.012)	**0.87** (0.293)	**0.91** (0.472)	0.71 (0.011)	0.72 (0.015)	0.75 (0.032)	0.6 (0)	0.43 (0)	0.46 (0)	0.51 (0)	0.57 (0)	0.46 (0)	0.57 (0)		
Hun	0.72 (0.016)	0.72 (0.018)	0.74 (0.026)	0.71 (0.012)	0.7 (0.01)	0.68 (0.005)	0.52 (0)	0.48 (0)	0.48 (0)	0.52 (0)	0.65 (0.002)	0.52 (0)	0.58 (0)	0.74 (0.026)	

Table 3: $\bar{\chi}$ Estimates and Probability Values for Daily S&P/IFCI Losses

Country Pair	χ	Prob.Value
Bra-Chl	0.44	0.03
Bra-Mex	0.52	0.03
Bra-Per	0.38	0.02
Chl-Mex	0.42	0.03
Chl-Per	0.36	0.02
Chl-Chk	0.31	0.02
Mex-Per	0.36	0.02
Mex-Tur	0.31	0.02
Mex-Chk	0.33	0.02
Inds-Mal	0.38	0.03
Inds-Phil	0.36	0.02
Inds-Thai	0.33	0.02
Phil-Thai	0.33	0.02
Kor-Thai	0.33	0.02

Table 4: χ Estimates and Standard Errors for Daily S&P/IFCI Losses

dimensions. In a three-asset setting, our approach requires the estimation of the extremal dependence structure between the sub-portfolios generated by two of the assets given by:

$$w_i Z_i + w_j Z_j,$$

and the third asset Z_k. We present an example in Figure 1 that illustrates the concept of hidden asymptotic dependence. In this example, we use a portfolio that consists of Turkey, Indonesia and Mexico. First, the portfolio is split into two sub-portfolios. The first sub-portfolio consists of a combination of Turkey and Indonesia, and the second sub-portfolio consists of only Mexico. The horizontal axis designates the weight of Turkey, w_{Turkey}, in the first sub-portfolio, and the vertical axis designates the estimate of $\bar{\chi}$ between the two sub-portfolios. The solid line shows the point estimate of $\bar{\chi}$ whereas the dashed lines are the 95 percent confidence intervals for the estimate of $\bar{\chi}$. On the very left end of the curve where $w_{Turkey} = 0$, we estimate $\bar{\chi}$ between Mexico and Indonesia as 0.4, and the asymptotic dependence hypothesis ($H_0 : \bar{\chi}_{Mexico-Indonesia} = 1$) is rejected because $\bar{\chi} = 1$ point falls outside of the confidence interval designated with dashed lines. On the very right end of the curve where $w_{Turkey} = 1$, we estimate $\bar{\chi}$ between Mexico and Turkey as 0.64, and the null hypothesis of asymptotic dependence ($H_0 : \bar{\chi}_{Mexico-Turkey} = 1$) is rejected again since $\bar{\chi} = 1$ point lies above the 95 percent confidence interval. On another note, Turkey and Indonesia are also estimated to be asymptotically independent (not observable on this plot). Therefore, all possible asset pairs are asymptotically independent from each other in this example. However, there are some sub-portfolio weights for which Mexico is asymptotically dependent with the sub-portfolio generated by Turkey and Indonesia ($0.6 \leq w_{Turkey} \leq 0.8$).

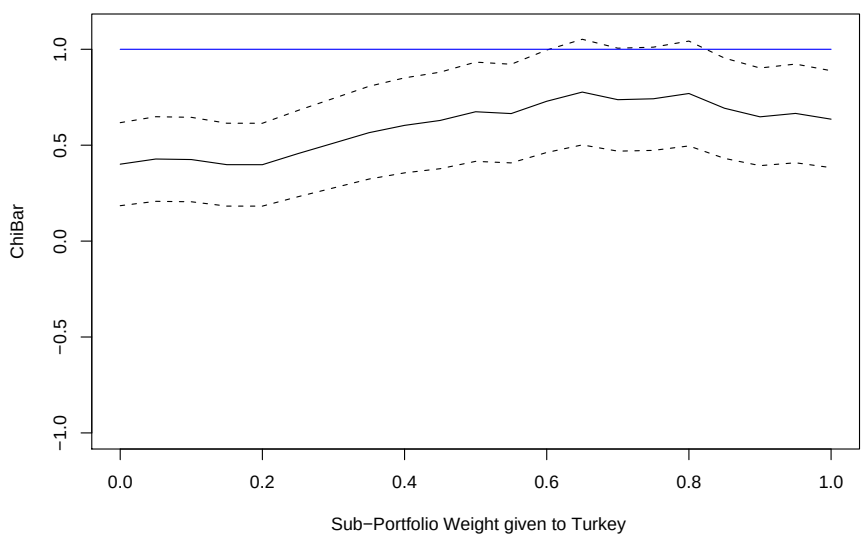

Figure 1: Portfolio Level Hidden Asymptotic Dependence

This finding seems counter-intuitive at first since more diversification is expected to reduce risk. However, we believe that it is consistent with the idea that there are limits to diversification due to the existence of systemic risk and contagion effects. More intuitively, our finding implies that a crash in a portfolio composed of Turkey and Indonesia is more likely to spread to Mexico as compared to an isolated crash in either Turkey or Indonesia. We believe that this intuition makes sense since a crisis in multiple emerging markets should be more likely to spread compared to an isolated crisis in a single country. Basically, at times of severe stress in global financial markets, diversification benefits are lost. Unfortunately, these are also the times when investors need diversification benefits most.

In the rest of the paper, we will refer to portfolios as in the above example as "hidden asymptotic dependent" because the conventional pairwise dependence analysis cannot identify it. Portfolio managers analyzing the dependence between every possible asset pair in their portfolio may feel secure if they observe no asymptotic dependence. However, such analysis may prove insufficient if hidden dependencies exist in the portfolio that are not explicit at the surface. We conjecture that the existence of hidden asymptotic dependence can be important and can have significant implications for portfolio selection. Next, we will investigate this conjecture.

5.1 Identification of Hidden Asymptotic Dependence

The methodology to identify the existence of hidden asymptotic dependence is described above with a visual aid presented in Figure 1. In this section, we describe our algorithmic implementation more formally in detail. We investigate every possible three-asset portfolio from our emerging market stock index data. Triples of countries can be chosen in 455 different ways from our pool of 15 countries. We classify each one of these country-triples into one of three groups according to their tail dependence structure. First, the conventional bivariate dependence analysis is administered between all pairs of countries. If any two countries in the triple are asymptotically dependent with each other, we place such triples into the first group and label them as "asymptotically dependent" triples. This group has 158 country-triples. For the remaining 297 triples, we perform an analysis similar to what is presented in Figure 1. We construct two sub-portfolios where the first sub-portfolio consists of the combination of first two assets $w_iL_i + w_jL_j$ and the second sub-portfolio is the third asset L_k alone. For the first sub-portfolio, we use all possible weights from 0 to 1 using a grid of 0.01 increments. Therefore there are 101 different ways to construct the first sub-portfolio upon fixing the country to be the single asset second sub-portfolio. Then, we estimate the $\bar{\chi}$ between the two sub-portfolios. This procedure gives us 101 point estimates of $\bar{\chi}$ which can potentially be used to make a plot similar to Figure 1.

Then, we rotate the third asset, i.e. we also analyze the tail dependence structure between sub-portfolios L_i and $w_jL_j + w_kL_k$; and the tail dependence between sub-portfolios L_j and $w_iL_i + w_kL_j$. For the example presented in figure 1, this amounts to using Indonesia alone as the second sub-portfolio and then Turkey alone as the second sub-portfolio. As a result, there are 303 different ways of constructing sub-portfolios. If there is at least one case out of these 303 different constructions where the sub-portfolios are asymptotically dependent with each other, we categorize these triples into the second group and label them as "hidden asymptotic dependent". We found that 190 country-triples exhibit hidden asymptotic dependence. Finally, the remaining 107 triples exhibit neither bivariate asymptotic dependence nor the hidden asymptotic dependence.

5.2 Calculation of Diversification Benefits

In order to investigate the impact of hidden asymptotic dependence on portfolio selection, we need to develop a measure of diversification benefits. Some finance practitioners argue that the primary purpose of diversification is to achieve risk reduction. Elton and Gruber (1995) didn't find any evidence for international CAPM, and therefore argued that the risk averse investors who cannot forecast expected returns would prefer

to simply minimize the risk. Following this idea, Li, Sarkar and Wang (2003) measured the benefits of international diversification by the reduction in portfolio variance. Ergen (2014) argued that strongly risk averse investors would minimize VaR or ES rather than variance because their motive is to guard against extreme losses. This also makes sense from a regulatory perspective since diversification benefit is primarily measured by the reduction in risk by regulators. Therefore, in this study we define diversification benefits simply as the reduction in tail risk measures. In a two-asset portfolio selection problem, this can be defined as the percentage reduction in VaR and ES achieved by the optimal two-asset portfolio as compared to the optimal single-asset portfolio as in Ergen (2014). In other words, it is the additional benefit from diversifying into a second asset. Here, in a three-asset setting, we measure it by the reduction in VaR and ES achieved by the optimal three-asset portfolio compared to the optimal two-asset portfolio. More intuitively, our measure can be thought of as the additional benefit from diversifying into the third asset. In order to calculate the diversification benefits as such, we construct three-asset linear portfolios as:

$$Z(w) = w_1 Z_1 + w_2 Z_2 + w_3 Z_3, \tag{13}$$

where Z_i is the percentage loss of individual assets, and $w \in G = \{w : w_1 + w_2 + w_3 = 1,\ 0 \leq w_i \leq 1,\ w_i = 0.01k,\ k \in \mathbb{Z}\}$. Therefore, portfolio weights are constructed using a grid with equal weight increments of 0.01. This grid generates 5,050 possible portfolio weights. Figure 2 provides a visual aid for better understanding of the construction of portfolio weights. The weights assigned to first and second assets are shown on the x and y axes, respectively. The weight given to the third asset is determined by $1 - w_1 - w_2$. The three corners of the triangle correspond to three single-asset portfolios, and the three sides correspond to two-asset portfolios. [8]

Then, the VaR an ES for all 5,050 portfolios are calculated following the EVT methodology described in Section 2. Then diversification benefits for an country-triple are calculated as:

$$DB_{VaR} = 100 \left(1 - \frac{\min_{w \in B} VaR_\alpha(Z(w))}{\min_{w \in G} VaR_\alpha(Z(w))} \right), \tag{14}$$

$$DB_{ES} = 100 \left(1 - \frac{\min_{w \in B} ES_\alpha(Z(w))}{\min_{w \in G} ES_\alpha(Z(w))} \right), \tag{15}$$

where $B = \{w : w_1 + w_2 + w_3 = 1;\ 0 \leq w_i \leq 1;\ w_i = 0.01k;\ k \in \mathbb{Z};\ \exists\ i\ w_i = 0\}$ is the boundary set which is shown with the bullet circles on the sides and in the corners of the triangle in Figure 2. On the other hand, the global set G includes all the points in the figure. As such, set B restricts the investor to invest in at most two assets whereas the set G doesn't impose any restrictions. Therefore, the diversification benefits calculated using these formulae quantifies the reduction in VaR and ES that result from diversifying into a third asset from a two-asset optimal portfolio.

[8] In this figure, we present the portfolio weights with 0.05 increments for illustrative purposes to make it visually clear, but in our analysis we use increments of 0.01.

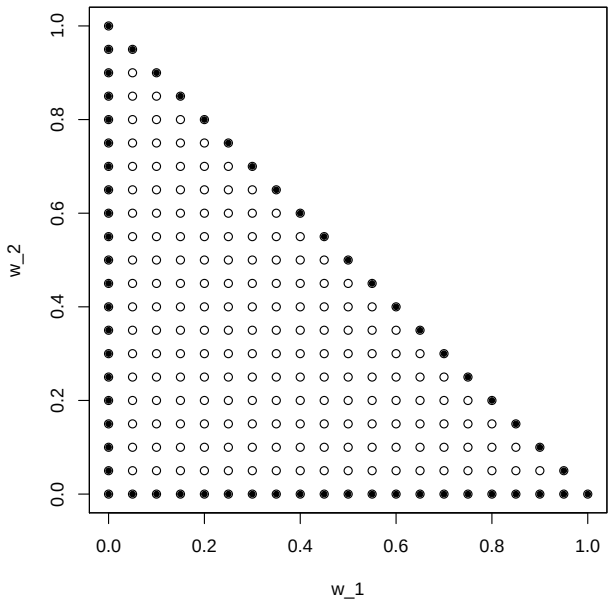

Figure 2: Weights Used in the Three Asset Portfolio Problem

5.3 Empirical Results

We classify each three-asset-portfolio into one of three groups as described in section 5.1: asymptotically dependent, hidden asymptotic dependent, and asymptotically independent. We also calculate the diversification benefits for each of the three-country portfolios as described in section 5.2. In order to make comparisons among the three groups of asset-triples, we use the mean and median diversification benefits. The results based on VaR at several extreme quantiles are presented for all three groups in Table 5.

		Asymptotically Dependent	Hidden Asymptotic Dependent	Asymptotically Independent
Mean	0.975	1.50	2.65	2.91
	0.99	1.46	2.58	3.00
	0.999	1.39	2.35	3.52
Median	0.975	0.36	1.97	2.22
	0.99	0.37	1.84	2.58
	0.999	0.57	1.46	2.29

Table 5: Mean and Median Diversification Benefits based on VaR for Different Dependence Structures

At the $99.9th$ quantile, the average diversification benefit for the 158 triples that exhibit pairwise asymptotic dependence is 1.39 percent. For the 190 triples that exhibit hidden asymptotic dependence, the average diversification benefit is 2.35 percent. Finally, the average diversification benefit for the 107 triples that do not exhibit neither pairwise nor hidden asymptotic dependence is 3.52 percent. The same pattern

is observed for $99th$ and $97.5th$ quantiles as well. The median diversification benefit for the asymptotically dependent portfolio group is 0.57 percent. It is 1.46 percent for portfolios exhibiting hidden asymptotic dependence and 2.29 percent for the asymptotically independent portfolio group. The same increasing pattern for diversification benefits is observed for other quantiles as well.

The average and median diversification benefit based on ES at several extreme quantiles are presented for all three groups in Table 5.3. The pattern of average diversification benefits for the three groups stays the same. They are lowest for those triples with pairwise asymptotic dependence and highest when there is no asymptotic dependence at all. Triples with hidden asymptotic dependence provide diversification benefits in-between the other two.

		Asymptotically Dependent	Hidden Asymptotic Dependent	Asymptotically Independent
Mean	0.975	1.15	2.24	2.87
	0.99	1.13	2.11	3.01
	0.999	2.26	3.44	4.71
Median	0.975	0.17	1.59	2.45
	0.99	0.23	1.46	2.28
	0.999	1.55	2.58	3.16

Table 6: Mean and Median Diversification Benefits based on ES for Different Dependence Structures

These results imply that the existence of hidden asymptotic dependence reduces the diversification benefits, but not as severely as the pairwise asymptotic dependence. We think that these results are intuitive and support our earlier conjecture that hidden asymptotic dependence has significant implications for portfolio selection.

6 International Financial Contagion

Chan Lau et al. (2004) rightly argued that financial contagion to be defined as the probability of simultaneous large loss observations across different financial markets rather than increases in correlations. Therefore, they used the extremal dependence coefficients to develop new measures of international financial contagion for a number of mature and emerging equity markets. Using stock price indices of these countries, they estimated $\bar{\chi}$ for each country pair and then calculate the ratio of country pairs that are asymptotically dependent to the total number of country pairs given by

$$IFC = \frac{\sum_i \sum_{j \neq i} AsympDep_{i,j}}{N \times (N-1)} \tag{16}$$

where $AsympDep_{i,j}$ is a binary variable taking value of 1 if the countries i and j are asymptotically dependent and 0 otherwise. They found that this new indicator captures the downturns in the emerging markets quite well. For example, during the Mexican peso crisis in 1995 and during the Asian crisis of 1997, the ratio of asymptotically dependent countries went up significantly. This measure can be loosely interpreted as

the probability of an extreme crash in one country to spread to another country. However, this metric suffers from the pairwise nature of the analysis. Since the dependence is estimated between the pairs of individual countries, it cannot identify and simply ignore the hidden asymptotic dependencies. In other words, it doesn't provide information about the probability of further propagation of a crisis when markets in multiple countries crash simultaneously. This leads to underestimation of the chances of crisis propagation to new countries and therefore, it doesn't provide a complete picture of the real severity of financial contagion.

In this section, we extend the financial contagion metric proposed by Chan Lau et al. (2004) to incorporate hidden asymptotic dependencies. In the previous section, we found that a portfolio of Turkey and Indonesia to be asymptotically dependent with Mexico although the individual countries are asymptotically independent of each other (See Figure 1). We argued that this is because it is more likely for a crash in a portfolio of Turkey and Indonesia to spread to Mexico as compared to an isolated crash in either Turkey or Indonesia. Expanding on this idea, it should be more likely for a widespread systemic stress in emerging markets to spread to one of the unaffected countries compared to an isolated crash in only one emerging market. Therefore, if there was a systemic composite index of emerging market stock prices, it is expected to have significant asymptotic dependence with the individual countries. Since the composite index would be a portfolio of all countries in the sample, its tail dependence with individual countries would implicitly incorporate the hidden asymptotic dependencies. Fortunately, such systemic composite index data is readily available from our data source. S&P/IFC has been collecting stock index data not only for the individual emerging markets, but also they calculate an emerging markets composite index from the country level data. The EM composite index is the market capitalization weighted average of all emerging market stocks[9]. We propose that tail dependence analysis to be performed between the countries and the EM composite index and financial contagion be measured with the ratio of asymptotically dependent countries to the total number of countries. More formally, we propose to measure the financial contagion with

$$IFC_{new} = \frac{\sum AsympDep_{i,comp.index}}{N} \tag{17}$$

where $AsympDep_{i,comp.index}$ is a binary variable that takes a value of 1 if the country i is asymptotically dependent with the emerging markets composite index and 0 otherwise. We use a 6 year rolling window sample of daily returns to calculate both our proposed measure and the chan-Lau et al.(2004) measure given in (16). We slide our sample by one quarter in each iteration dropping the earliest quarter from the sample and adding one more quarter to the end. For example the data from 1995Q3-2001Q2 is used to calculate the measures for 2001Q2; the data from 1995Q4-2001Q3 is used to calculate the measures for 2001Q3 and so on. The time series plot of both measures is presented in Figure 3. As we conjectured earlier, the results confirm that the degree of international financial contagion is in fact much severe than implied by the pairwise dependence analysis.

Poon et al. (2004) demonstrates that a great deal of asymptotic dependence is driven by the volatility clustering in multiple markets. Their finding is confirmed by multiple studies subsequently. Therefore, they propose to filter out this volatility

[9]It includes more countries than in our sample.

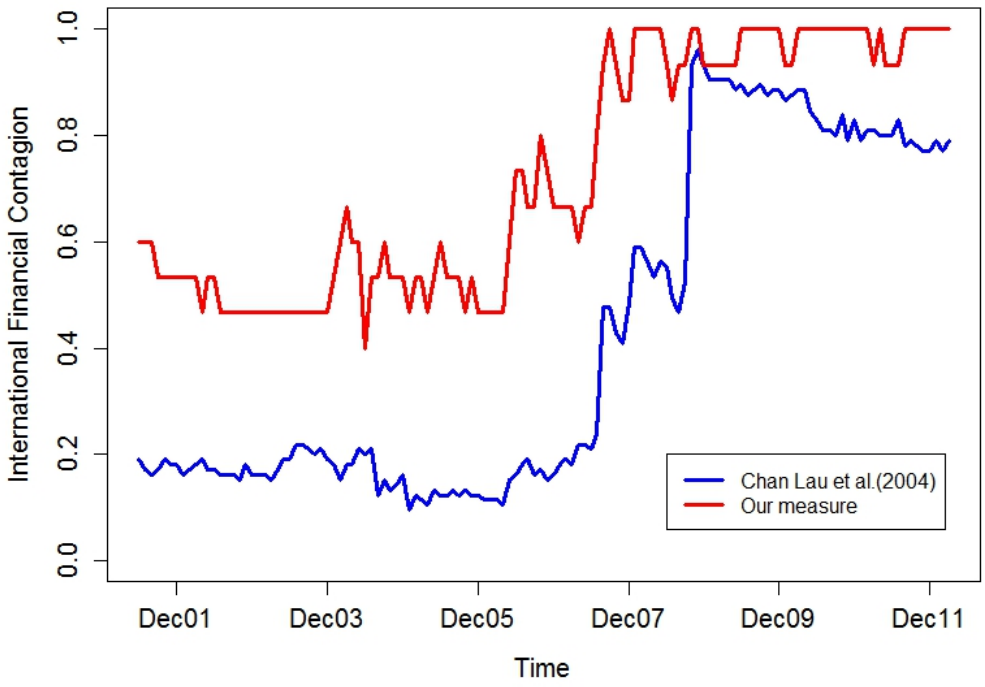

Figure 3: IFC Plot

impact by using Generalized Autoragressive Conditonal Heteroscedasticity (GARCH) type models first and then implementing the extremal dependence analysis to the filtered returns. To check the robustness of our results, we also follow this approach by fitting univariate GARCH(1,1) models to the individual country returns as well as the composite emerging market index returns. We extract the standardized residuals from these fits and estimate asymptotic dependence between the residuals. Both the GARCH(1,1) fits and the analysis of extremal dependence on standardized residuals are performed using 6 year rolling window samples as explained above. Time series plot of both contagion measures is presented in Figure 4.

As suggested by earlier studies, the number of cases with asymptotic dependence are reduced significantly due to the filtering of returns but our main conclusion remains well supported and actually even more pronounced in this case. Incorporation of hidden asymptotic dependence provides a more complete picture of the severity of financial contagion. The plots presented in both Figures 3 and 4 indicate that the severity of financial contagion is significantly stronger than implied by existing measures that rely on pairwise extreme value dependence analysis.

7 Conclusion

In this study, we extended the Poon et al. (2004) methodology for estimation of extremal dependence to a multi-asset setting by analyzing the tail dependence structure between the sub-portfolios within a multi-asset portfolio. This lead us to the discovery of hidden asymptotic tail dependencies that cannot be identified with the pairwise tail

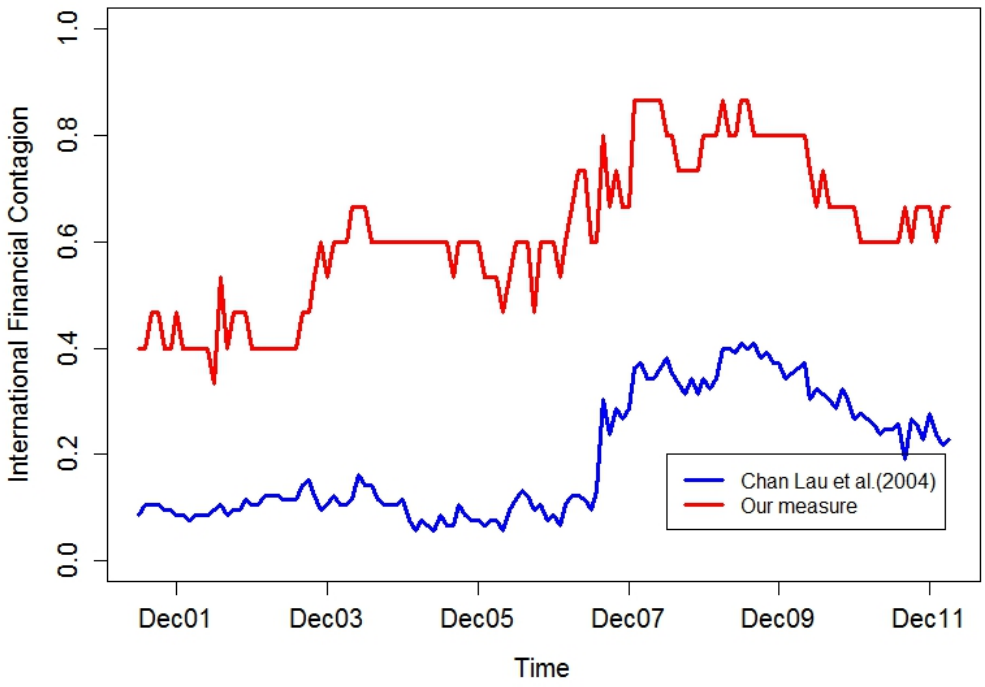

Figure 4: IFC Plot with GARCH filtering

dependence analysis between the individual assets. simply put, it is a situation where all possible asset pairs within the portfolio are asymptotically independent of each other but it is possible to split the portfolio into two sub-portfolios that are asymptotically dependent. This finding implies that there are limits to diversification due to systemic risk and contagion effects.

We investigated the impact of hidden asymptotic dependence on diversification benefits formally. We assumed that the investors guarding against extreme losses would seek to minimize tail risk measures such as VaR and ES. For all possible three-asset portfolios, we measured the diversification benefits with the reduction in these tail risk measures as a result of investing in the optimal three-asset portfolio rather than investing in the optimal two-asset portfolio. We found that the existence of hidden asymptotic dependence significantly reduces the benefits from diversification but not as much as the asymptotic dependence between the pairs of assets. We believe that these results make intuitive sense and offer valuable information to portfolio managers. Portfolio managers may believe that they built a safe portfolio when all asset pairs in their portfolio are asymptotically independent of each other. However, pairwise dependence analysis may not be sufficient if hidden dependencies exist that are not explicit at the surface.

More intuitively, our results indicate that a crisis in multiple emerging markets is more likely to spread to other countries compared to an isolated crisis in a single country. However, pairwise dependence analysis cannot capture this effect, leading to underestimation of chances for simultaneous crashes in multiple markets. Motivated with our results and these ideas, we proposed a new measure of international financial

contagion that incorporates the hidden asymptotic dependencies. Our results indicate that the severity of financial contagion is significantly stronger than implied by existing measures that rely on extreme value dependence analysis of country pairs.

This research can be extended in several ways. As discussed in the introduction, the tail dependence measures have been used in many financial applications such as hedging, options pricing, Sharpe ratio targeting, measurement of the systemic risk in the banking industry and financial contagion in international markets. In this paper, we documented the hidden asymptotic dependence and its implications for diversification benefits and the measurement of financial contagion. We believe that similar extensions can be made in other financial applications particularly in the field of systemic risk.

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