Accounting for Post-Crisis Inflation: A Retro Analysis*

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VERY PRELMINARY
COMMENTS WELCOME

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Abstract

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1 Introduction

There is a time-honored tradition in economics to construct new models in light of new events, discarding previously successful approaches. This is particularly so in light of the 2008 financial crisis. Much has been written about the failures of the pre-crisis models to incorporate financial frictions and much has happened in the literature since then. While we applaud (and participate) in these ongoing efforts, there also is the risk that the latest models fit well to the latest events because they have been designed this way and not because one might have constructed these models ex ante: the danger of overfitting is always present. For that reason, it may be good to take pause once in a while and use a prominent pre-crisis model to study a prominent post-crisis question. Let us call this a "retro analysis".

This is what we do in this paper. We examine the evolution of post-crisis inflation which has puzzled many observers. Often, it is not inflation per se, which is of interest, but — per Phillips curve reasoning — the behavior of labor markets. As a subsidiary question, we examine the evolution of employment and seek to understand how it is related to the evolution of inflation.

Inflation behavior has been on the minds of many. On December 18th, 2013, the Wall Street Journal opened with the headline "Low Inflation Tests World's Central Banks — Subdued Prices Persist Despite Years of Easy Money; Deflation Still a Threat." A number of academic researchers have raised the issue of the connection between monetary policy, fiscal policy and aggregate economic conditions, and whether or not we should expect inflation to be higher or lower than currently observed. Discussion of the zero lower bound, anchoring medium-term expectations as well as the tools of unconventional monetary policy feature prominent in many of these debates, as do arguments regarding the fiscal theory of the price level. Many more issues

such as the renewed attention to the financial sector in modeling these connections come into play as well. For example, Hall (2011) argues that "the inflation rate hardly responded to conditions in product and labor markets, else deflation might have occurred". In his Presidential Address, he points out that all theories based on the concept of non-accelerating NAIRU fail to explain the dynamics of inflation during the recent crisis. He suggests that, in a liquidity trap, inflation is close to exogenous since it is pinned down by the interest rate fixed at the zero lower bound. As a consequence, the real interest rate cannot fully adjust in response to a decrease in demand and this should lead to low inflation or even deflation and high unemployment rate. Indeed, the fear of deflation was not merely an academic concern. Using option prices, Matthias Fleckenstein and Lustig (2013) estimate that the 1-year ahead inflation expected by the market dropped at the end of 2008 to - 6%.

The absence of deflation has been interpreted as a failure of the Phillips curve and the theories linking economic slackness to deflationary pressure, but also as a serious challenge to New Keynesian theories, linking the stimulus effects of government spending to producing future inflation and thereby lower unemployment, see Dupor and Li (2014) . Other explanations have been attempted. Gordon (2013) estimates the Phillips curve and concludes that, when we correctly take into account the supply shocks, the relation holds. In his discussion to Ball and Mazumber (2011) 's paper, Stock argues that the missing deflation puzzle is actually a puzzle regarding only the CPI inflation. The puzzle disappears when we consider PCE-XFE, headline CPI, headline PCE and median CPI inflation. By contrast, Coibion and Gorodnichenko (2013) argue that the missing deflation puzzle can be explained by a rise in inflation expectations. Building on the fiscal theory of the price level, see Cochrane (1998) , Cochrane (2011) , Leeper and Zhou (2013) , Leeper (2013) argue that the expansion of government debt may have led

to higher inflation.

We instead proceed by accounting for the post-crisis movements in inflation and employment, using a benchmark DSGE model. To do so, we shall use the well-known pre-crisis Smets-Wouters (2007) model. As a prominent version of a New Keynesian model, the Smets-Wouters (2007) has a Phillips Curve built in by construction, thereby featuring a key tradeoff at the heart of the recent debates. We use it to decompose the movements in inflation and employment generally, as well as specifically for the events of 2008 and beyond. Are different types of shocks dominant post 2007, for example? Is the behavior of inflation and employment or is their relationship after 2007 unusual, compared to what we have learned from the rest of the sample? Answers to these questions may be useful in judging how far a theory update may need to go in order to correct previous deficiencies, or where to look more closely. We shall use pre-crisis estimation and solution techniques too, effectively pretending, for example, that agents are unaware that the nominal rate cannot sink below zero and may therefore be continuously surprised by positive shocks to the policy rate, when the rate is zero rather than what the Taylor rule in this model would have predicted.

Obviously and given recent advances in the literature, quite a bit is wrong with modeling monetary policy and the impact of the zero lower bound in this manner, or with disregarding the changes in key financial frictions. The zero lower bound is a key focus in, say, Eggertsson (2006) , Werning (2011) or Eggertsson and Woodford (2005) . Financial frictions are key in many papers surveyed by Brunnermeier et al. (2011) . But this is precisely the point of this paper: how far do we get by employing this "retro" model rather than imposing recent advances (and there are many, pulling in different directions)? Are there odd shocks that this model needs to impose in order to explain the observed movements in inflation, that would point to a major deficit in employing this model, for example? What is the "smoking gun"

and the tell-tale sign that this model is wrong to begin with? For example, if our pseudo[1] monetary policy shocks explain little for the series under investigation, it may then well be that the zero lower bound constraint was not of major importance there. One can read this as an exercise in "wedge accounting" in the spirit of Chari et al. (2007) , except that we are using the Smets-Wouters (2007) model as our measuring stick.

Our exercise delivers several interesting insights. First, wage and price markup shocks are key for understanding fluctuations in inflation before the crisis. This is in line with the results by King and Watson (2012) who found that inflation dynamics is primarily due to shocks to expected inflation rather than changes in the underlying marginal cost. This pattern holds for pretty much the entire pre-crisis sample, from 1948 to 2007. By contrast, a substantial gap between actual inflation and inflation explained by markup shocks arises after 2007: based on these two shocks alone, inflation should have been more than one percent higher. We examine why this is so and then examine the shocks that account for the gap. We find that monetary policy shocks are perhaps of greatest importance here (recall the "ignorance" regarding the ZLB!), accumulating from the middle of 2008 to about 2010, but not beyond: afterwards, the low interest rate was roughly consistent with predictions of the model-specific Taylor rule, confirming Cogan et al. (2010) 's view on the length of the liquidity trap. Government expenditure shocks play hardly any role at all. Instead, investment adjustment costs, preference shocks and (in 2009) technology shocks account for the remaining gap. We juxtapose this with the movements in employment and likewise find that monetary shocks, investment adjustment costs, preference shocks and price markup shocks account for the bulk of the movements, while government expenditures played a rather minor role.

The absence of deflation in the data has been interpreted as a failure of

[1] We shall leave out the moniker "pseudo" for the remainder of the paper.

the Phillips curve and the theories linking economic slackness to deflationary pressure, but also as a serious challenge to New Keynesian theories, linking the stimulus effects of government spending to producing future inflation and thereby lower unemployment, see Dupor and Li (2014) . We therefore view our results as broadly consistent with their perspective. Likewise, the modest effect of government spending on employment without a corresponding effect on inflation is broadly consistent with the view of Conley and Dupor (2013) , that the ARRA was largely a government jobs program.

In section 2, we describe our approach. Section 3, shows the results for inflation and employment for the entire sample from 1948 to 2013. In section 4, we "zoom in" on the crisis and post-crisis episode, starting in 2007 and provide some counterfactuals. Some robustness checks are in section F. Section 7 offers some discussion and conclusion.

2 Our approach

The Smets-Wouters (2007) model assumes a representative household, whose utility function has habit formation and is separable in consumption and leisure. Households can save by investing in capital or by buying government bonds. In addition, households choose the level of capital and of capital utilization. Investment in capital is costly. Labor unions and firms set respectively wages and prices according to a Calvo pricing mechanism. Both wages and prices are partially indexed to inflation. Finally, the interest rate is set according to a Taylor rule with weights on both inflation and output gap and the government expenditure is assumed to be exogenous. There are 7 shocks in the model: shocks to the technological process, to the consumer's preferences, to the government expenditure, to the investment adjustment cost, a price and a wage markup disturbance and a monetary policy shock. Further details can be found in Smets and Wouters (2007) as well as in the

technical appendix to this paper.

We solve and re-estimate the model using Dynare and a sample for the period 1948Q1-2013Q1, rather than ending in 2004Q4, as in their paper. Otherwise, we use the same Bayesian approach and prior as well as the same (extended) time series as in Smets and Wouters (2007) , i.e. the seven US time series on the log difference of real GDP, real consumption, real investment, the real wage, log hours worked , the log difference of the GDP deflator and the federal funds rate. While some differences in the estimates arise, none are particularly remarkable. Details are in the appendix.

Next, we use the estimated model to calculate a (Wold) shock decomposition. This is a standard method in the literature, see Fernandez-Villaverde et al. (2007) , and a brief reminder shall suffice. After log-linearization, the model can be written as a system of linear equations:

$$\begin{aligned} x_t &= Ax_{t-1} + Bu_t \\ y_t &= Cx_{t-1} + Du_t \end{aligned}$$

where x_t is the vector of model variables, y_t is the vector of observed variables (possibly overlapping with x_t), u_t is the vector of economic shocks, and A, B, C and D are matrices and functions of the underlying parameters. Dynare provides us with estimates of these matrices: we use the posterior mean. With seven observables and seven shocks, D is square (and invertible). We set $x_0 = 0$ and then recursively calculate

$$u_t = D^{-1}(y_t - Cx_{t-1})$$

With this, we can now decompose the movements of inflation and employment into the sequence of present and past shocks.

The decomposition above also affords us to calculate a variance decom-

position for the variances of the four-quarter-ahead forecast error as well as the unconditional variance (or infinite-horizon forecast error[2]). We compute them at the posterior mean for the coefficient matrices and the posterior mean for the shock variances. To compare the contributions of the shocks across subsamples, we use the estimated variances for u_t for these subsamples as well.

3 Accounting for inflation: full sample

Figures 1 and 2 report the historical decomposition of inflation into individual shocks. In each panel, we compare observed inflation (solid blue line) and the inflation we would have observed if only one shock was non zero for the whole period of time (dashed red line). The dashed red line close to the horizontal axis reveals that the corresponding shock plays little role in explaining the inflation dynamics.

A few conclusions can be drawn from our exercise. First, the contribution of the monetary shock over the entire sample period is important but small. There is a bit of a positive push from monetary policy shocks in the late 70's, followed by a negative push in the early 80's, in line with the standard narrative of fairly loose monetary policy during the 70's, followed by the Volcker disinflation. There also is a bit of a negative push at the end, after 2007, presumably due to the zero lower bound constraint. We shall discuss this more below, but what should be noted here is that these movements are not unusual by full-sample standards. Second, government expenditure had a remarkably negligible effect on inflation.

Third and by contrast, the shocks to the wage and the price markups are particularly relevant: when combined as in Figure (3), the two shocks

[2]For practical reasons, we calculate a variance decomposition of the 200-period ahead forecast error.

explain the great majority of the observed variation in inflation in the sample. Visually, that graph tells the story that not much would have been missed in thinking about inflation, if one had only concentrated on these two shocks alone. There is one glaring discrepancy, however, and that is the crisis and post-crisis episode of 2007 and beyond.

4 Accounting for inflation after 2008

While price and wage markup shocks nearly suffice to account for the inflation movements prior to the crisis of 2008, a gap opens after that date. Indeed, by themselves they would have predicted a considerably higher inflation than what has been observed in the data. This answers a key question of this paper: there is no deflation and, actually, some inflation for the same reason that there was inflation at other times, namely price- and wage-markup shocks. It remains to understand why inflation was actually lower than what these shocks would have predicted by themselves. What accounts for this gap? And how different is the evolution of all other variables, due to the presence of these shocks? Figures 4, 6 and 7 provide the answer, by showing the decomposition of inflation for the last part of the sample. Figures 8 and 9 compare the actual data for other variables to the movement in these variables, if only price and wage markup shocks had been present.

In order to provide a quantitative measure of the relative importance of the other shocks to explain the gap between actual inflation and inflation explained by wage and price markup, we compute the integral of the area between the two time series and attribute the difference to the remaining shocks. The remaining shocks all, essentially, point in a negative direction, as figure 5 shows. We calculate the contribution of each shock to the total area: results are reported in table 1. The percentages reported are defined as the undiscounted fraction of the area between actual inflation and inflation

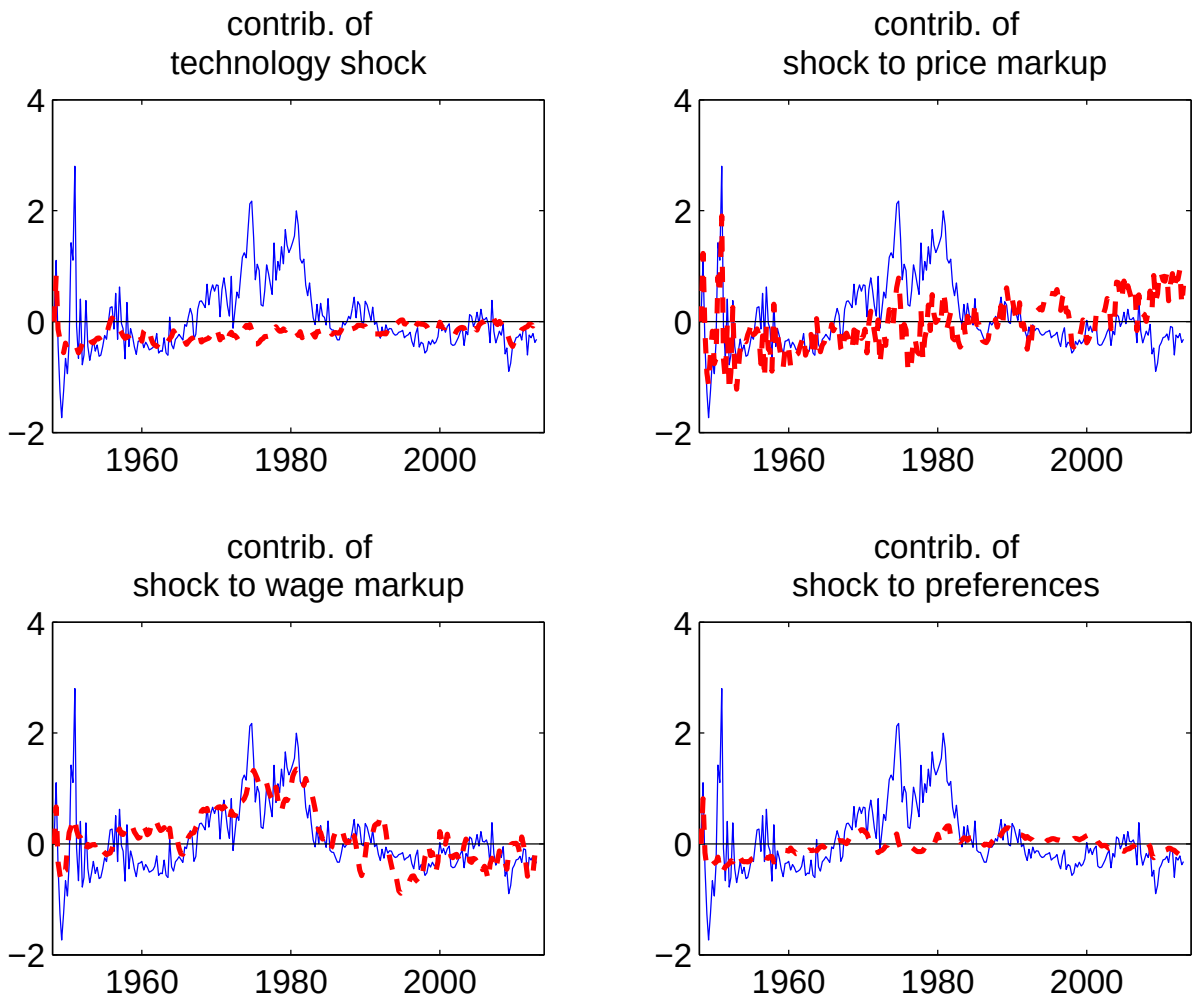

Figure 1: Historical shocks decomposition of inflation (relative to long-run constant) for the period 1948Q1-2013Q1. Solid line is actual inflation. The dashed line is inflation predicted by each shock individually.

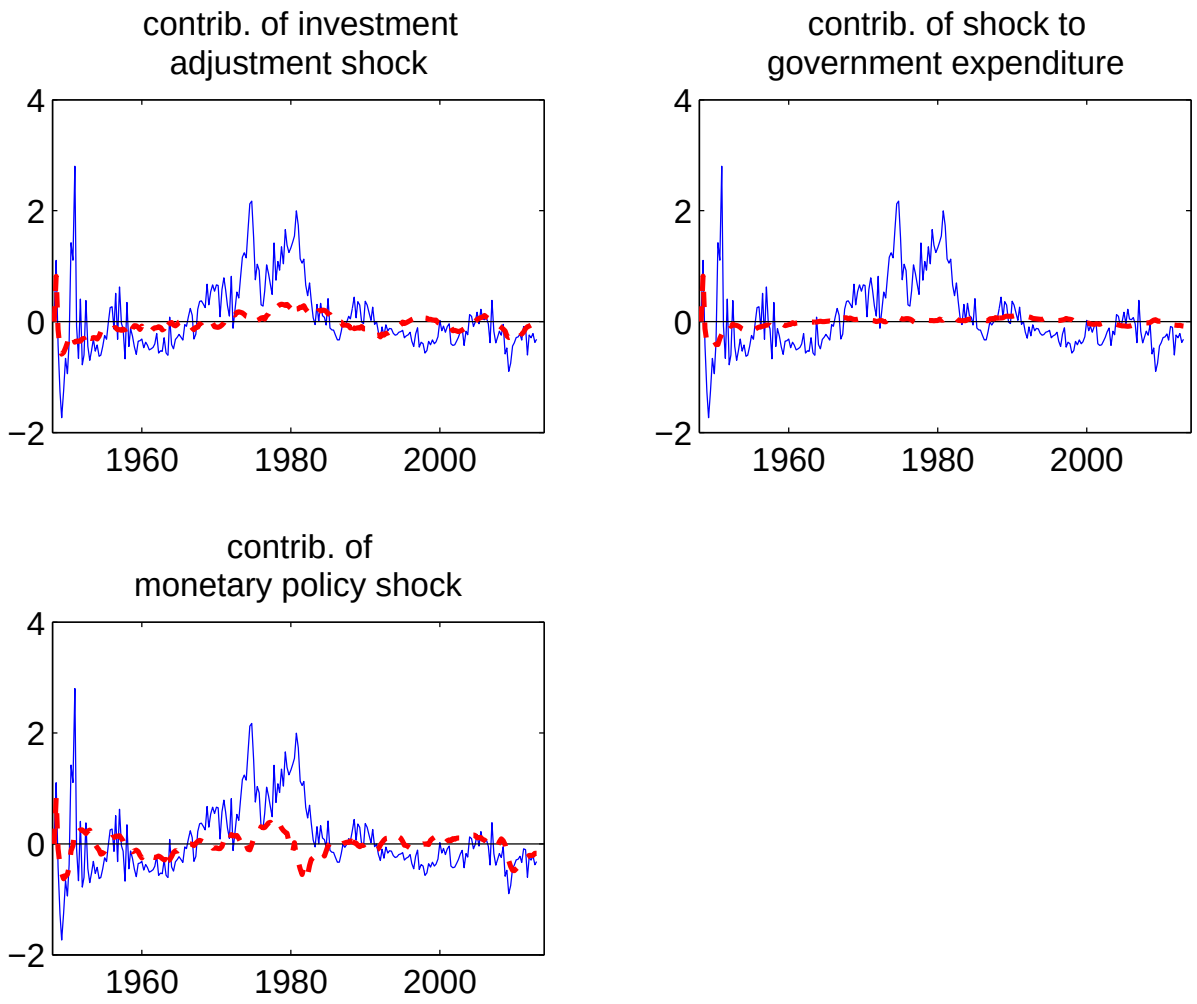

Figure 2: Historical shocks decomposition of inflation (relative to long-run constant) for the period 1948Q1-2013Q1. Solid line is actual inflation. The dashed line is inflation predicted by each shock individually.

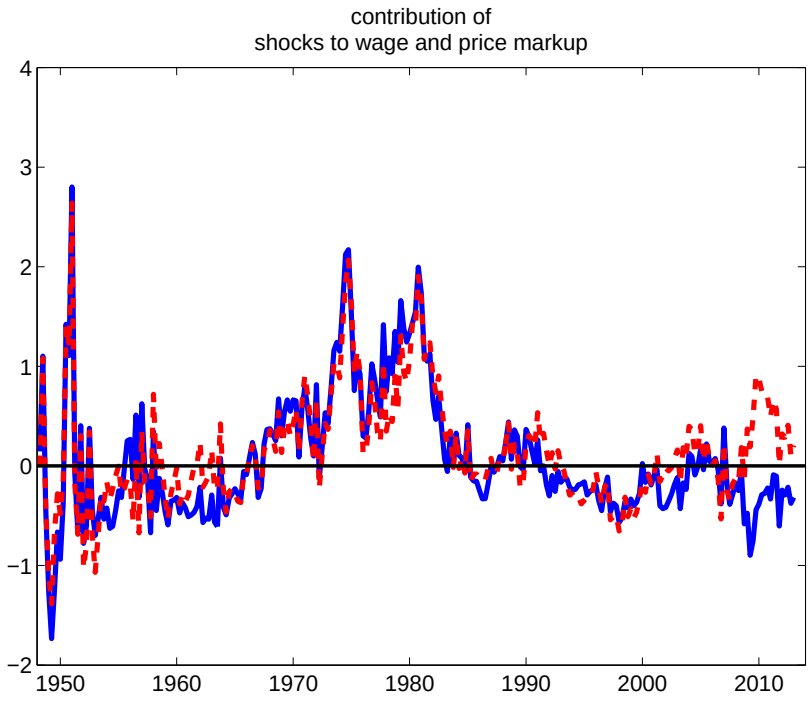

Figure 3: Historical shocks decomposition of inflation (relative to long-run constant) for the period 2005Q1-2013Q1. Solid line is actual inflation. The dashed line is inflation predicted by price and wage markup shocks.

Table 1: Contribution of the other shocks to the inflation not explained by price and wage markup

technology	21%
preferences	21%
investment adjustment cost	19%
fiscal policy	8%
monetary policy	31%

Percentages are defined as the fraction of the area between actual inflation and inflation predicted by wage markup and price markup between 2008 to 2013 that is explained by the contribution of the remaining shocks. Contributions in different time periods are not discounted.

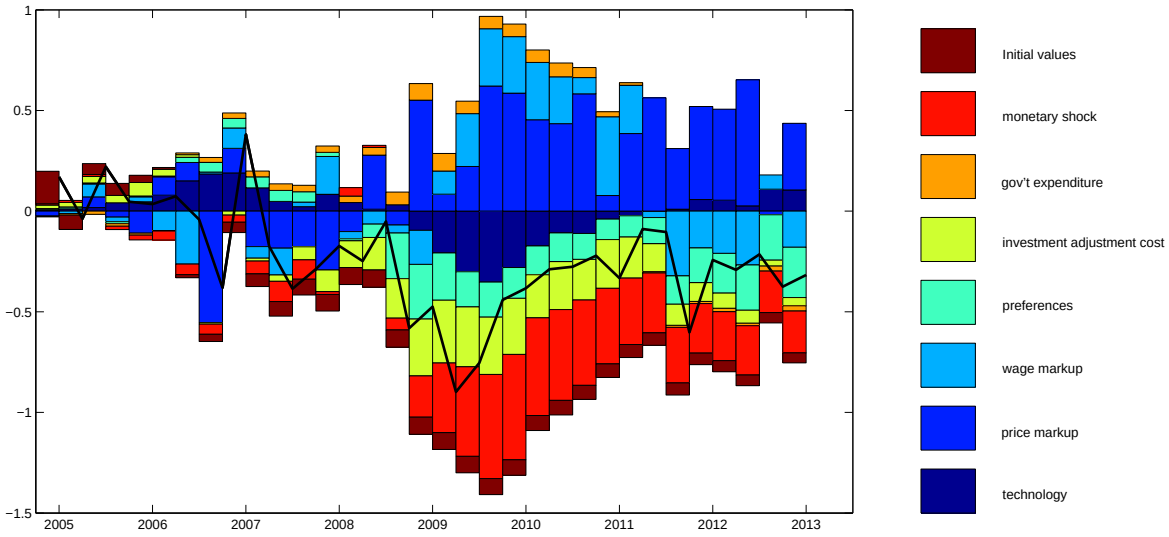

Figure 4: Historical shock decomposition of inflation since 2005

Table 2: Standard Deviation of the Estimated Shocks

	1948-2007	2008-2013
Technology	0.50	0.52
Price Markup	0.21	0.16
Wage Markup	0.26	0.44
Preferences	0.13	0.10
Monetary Policy	0.50	0.43
Gov't Exp	0.65	0.48
Inv.Adj.Cost	0.22	0.19

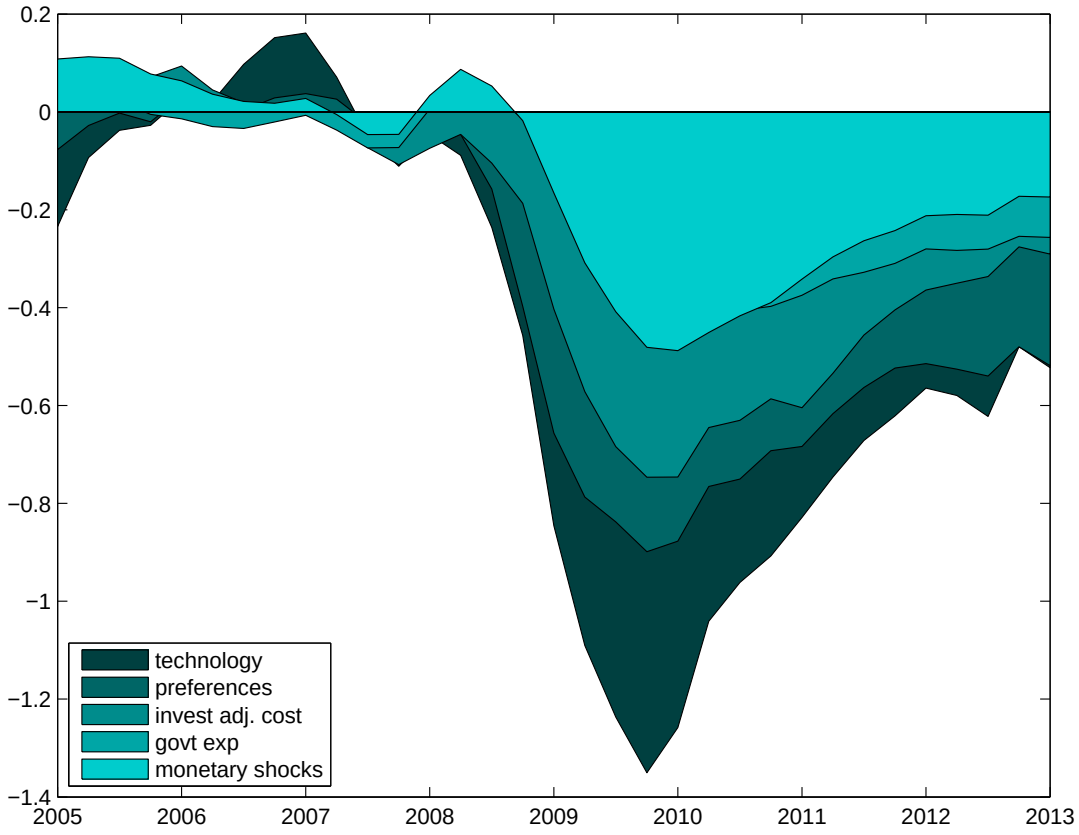

Figure 5: Inflation not explained by price and wage markup shocks

Table 3: 4-periods ahead Variance Decomposition of Inflation

	1948-2013	1948-2007	2008-2013
Technology	3.71	3.78	3.53
Price Markup	56.43	58.73	29.76
Wage Markup	26.80	23.99	58.63
Preferences	3.85	3.98	2.16
Monetary Policy	2.87	2.98	1.88
Gov't Exp	0.48	0.51	0.24
Inv.Adj.Cost	5.85	6.03	3.79

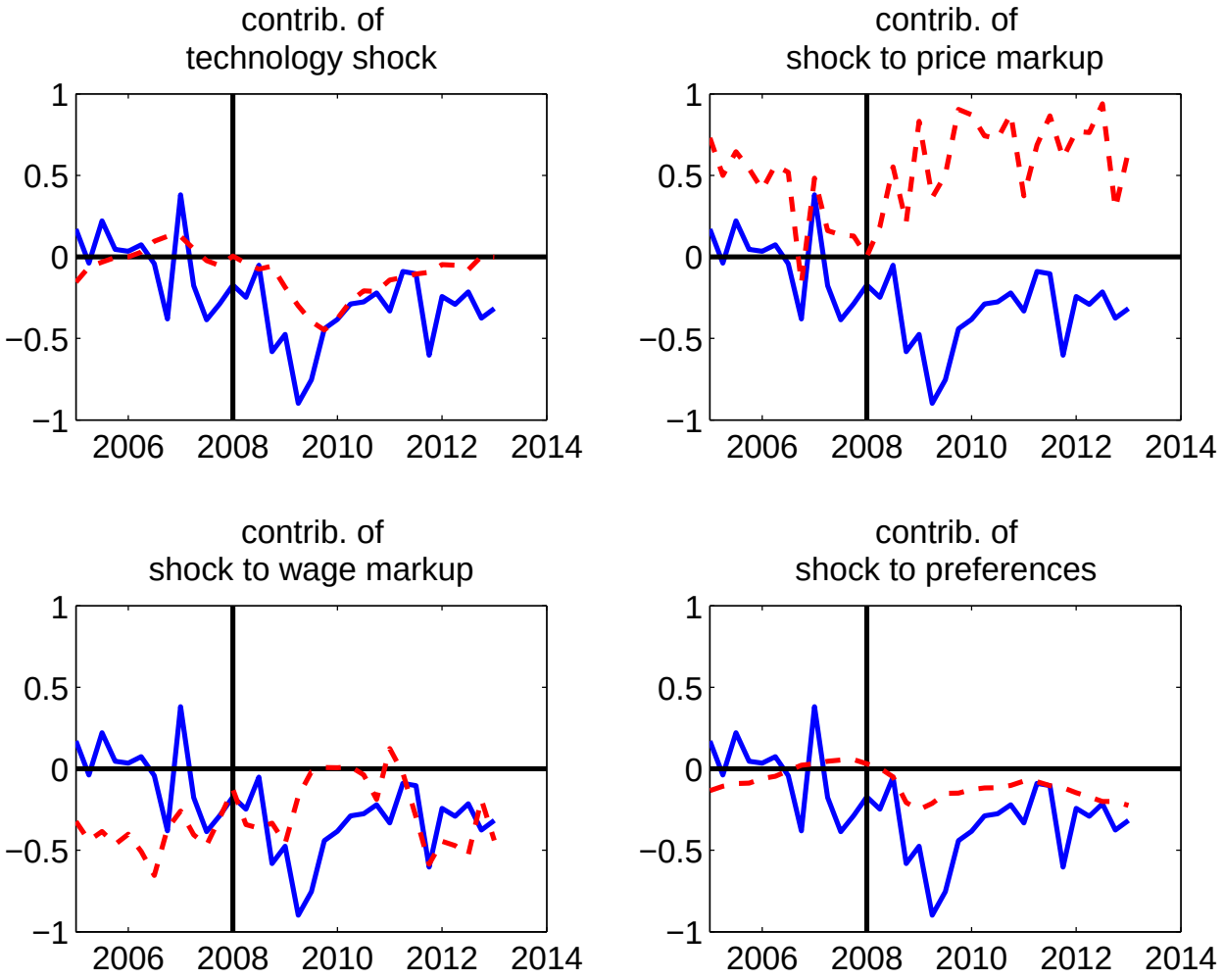

Figure 6: Historical shock decomposition of inflation (relative to long-run constant) for the period 2007Q4-2013Q1. Solid line is actual inflation. The dashed line is inflation predicted by each shock individually.

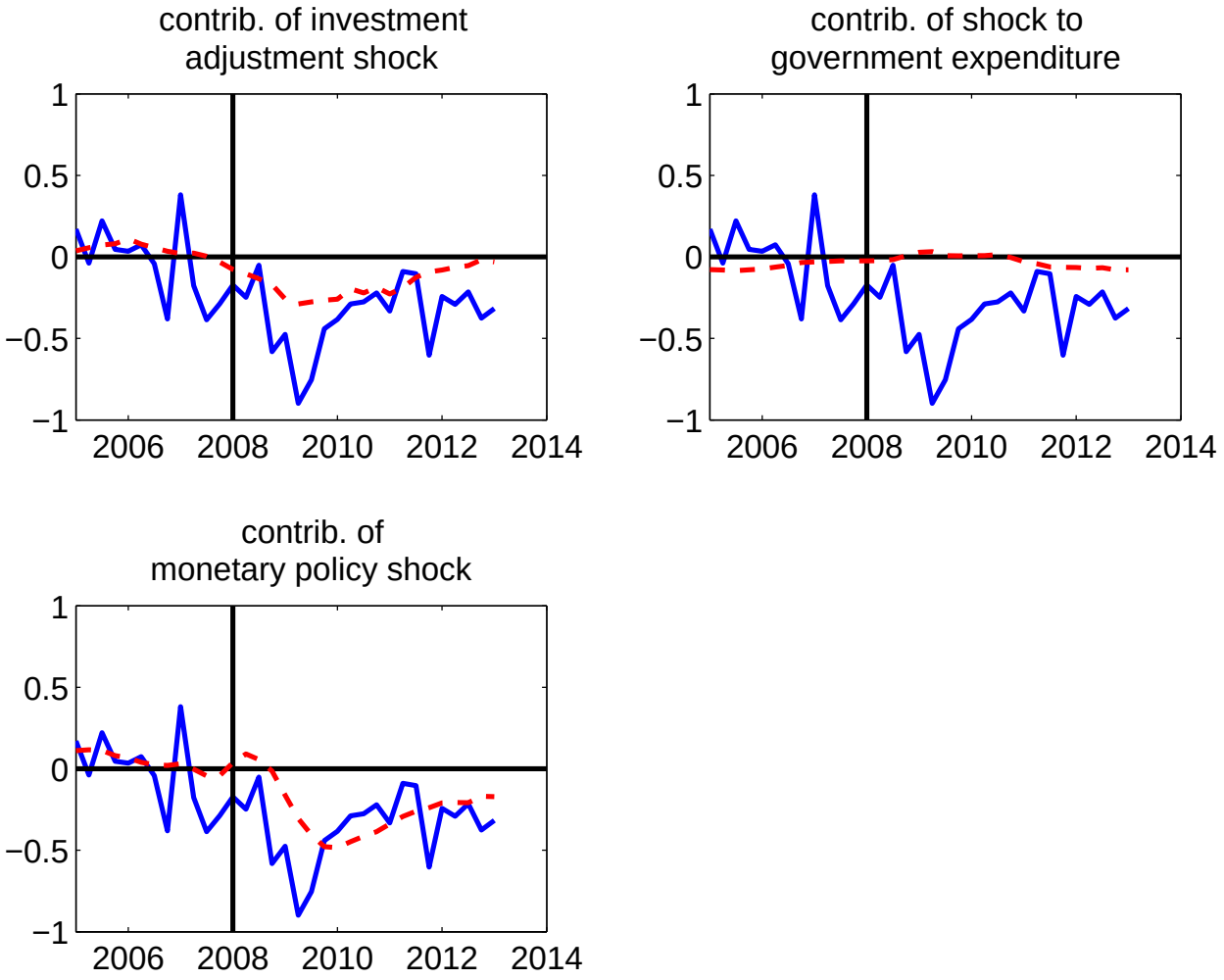

Figure 7: Historical shock decomposition of inflation (relative to long-run constant) for the period 2007Q4-2013Q1. Solid line is actual inflation. The dashed line is inflation predicted by each shock individually.

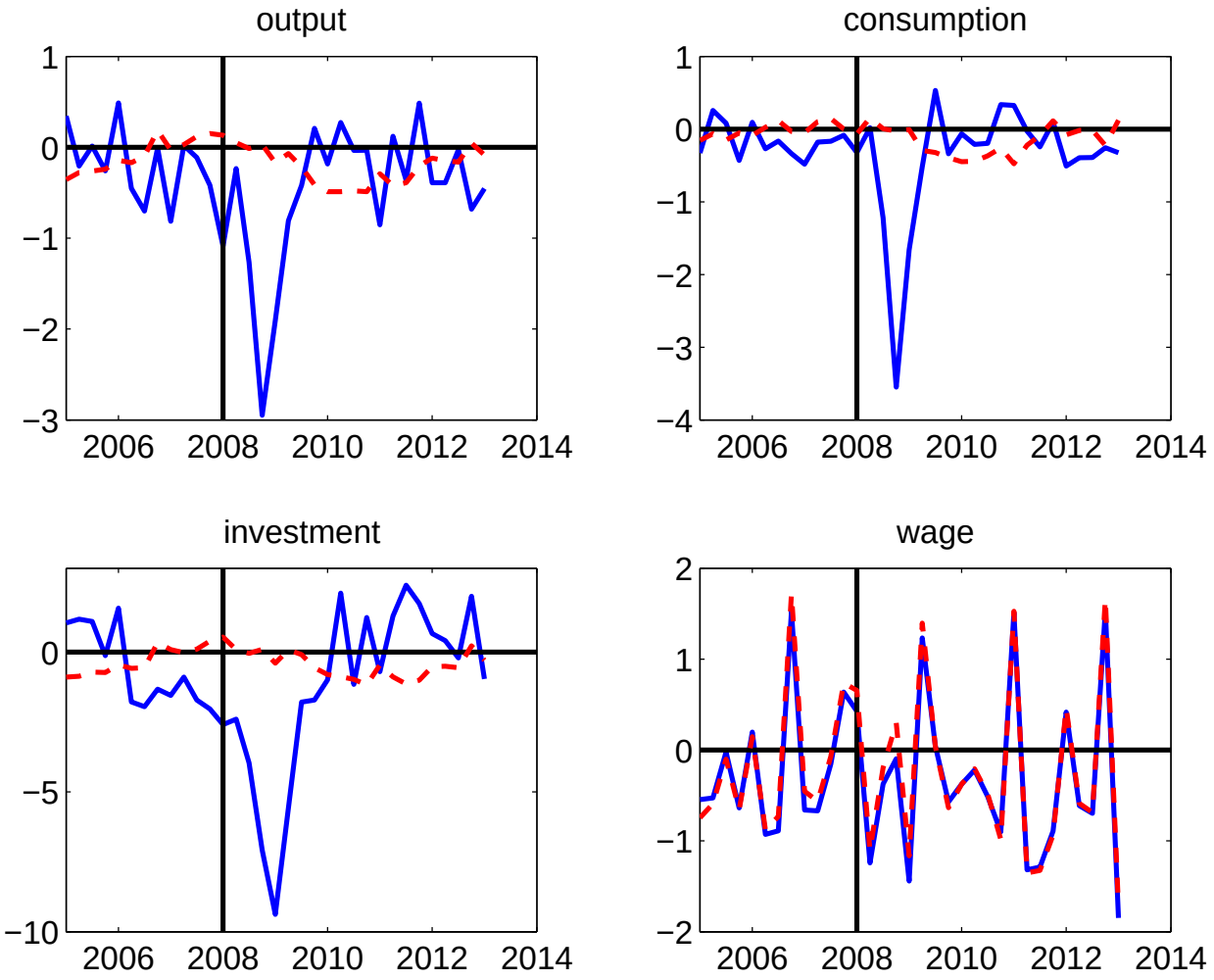

Figure 8: Contribution of shocks to price and wage markup for the period 1948Q1-2013Q1. Solid line is data. The dashed line is the part that is accounted for by price and wage markup shocks only.

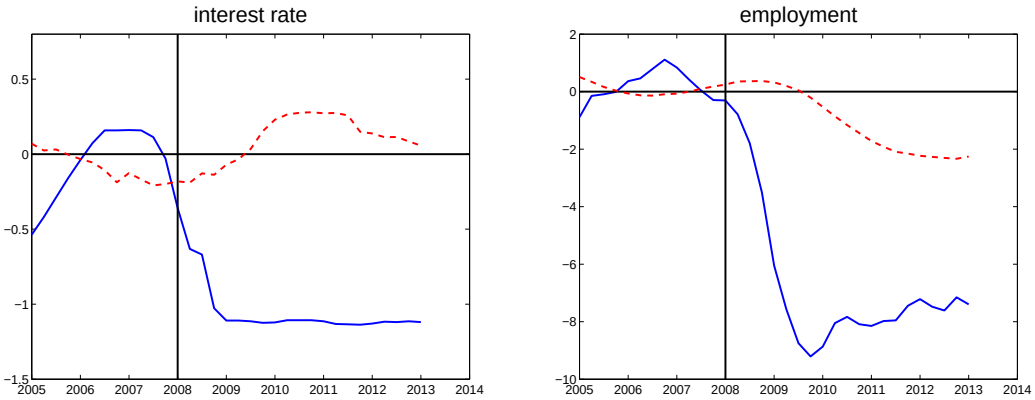

Figure 9: Contribution of shocks to price and wage markup for the period 1948Q1-2013Q1. Solid line is data. The dashed line is the part that is accounted for by price and wage markup shocks only.

Table 4: Asymptotic Variance Decomposition of Inflation

	1948-2013	1948-2007	2008-2013
Technology	3.79	3.92	3.08
Price Markup	43.43	45.87	19.57
Wage Markup	36.95	33.56	69.05
Preferences	3.84	4.02	1.84
Monetary Policy	3.36	3.55	1.88
Gov't Exp	1.56	1.68	0.67
Inv.Adj.Cost	7.07	7.39	3.91

Notes: 200-periods ahead

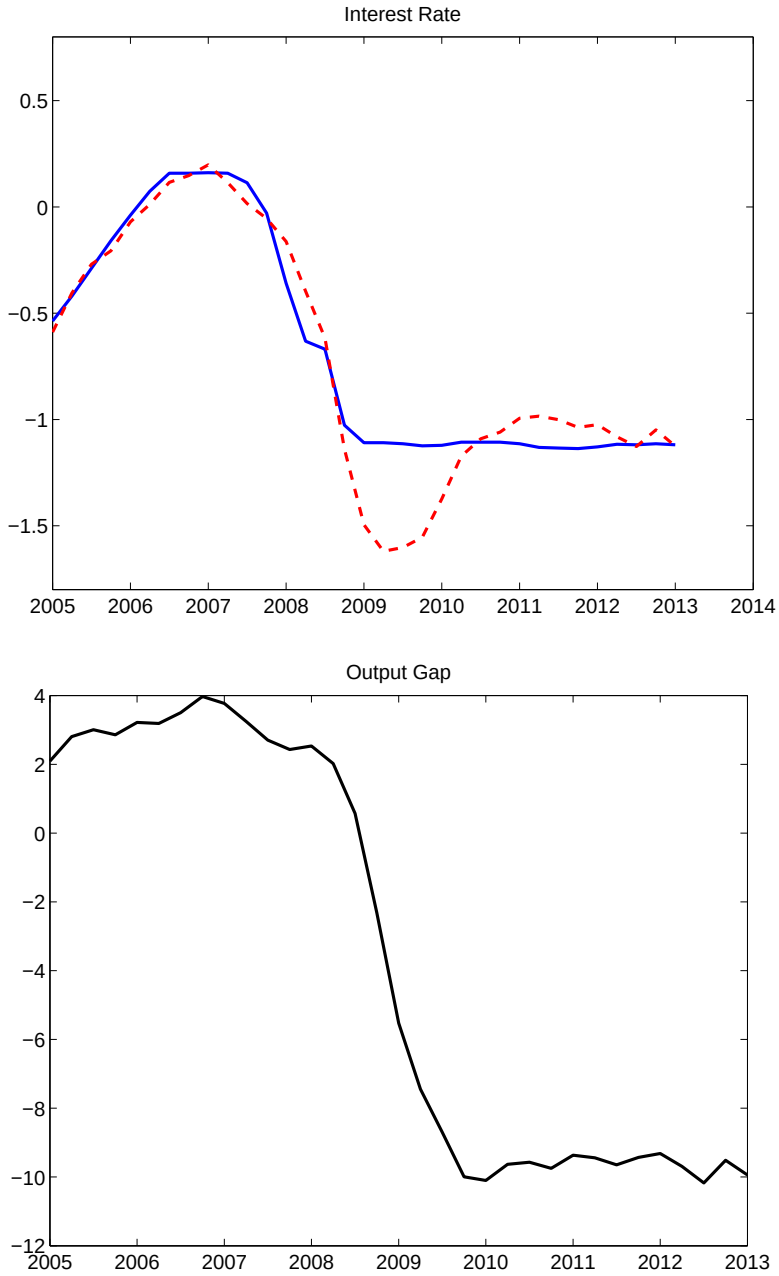

Figure 10: A comparison between interest rate and the output gap. The first figure presents the real interest rate (solid line) and the estimated interest rate in absence of monetary shocks.

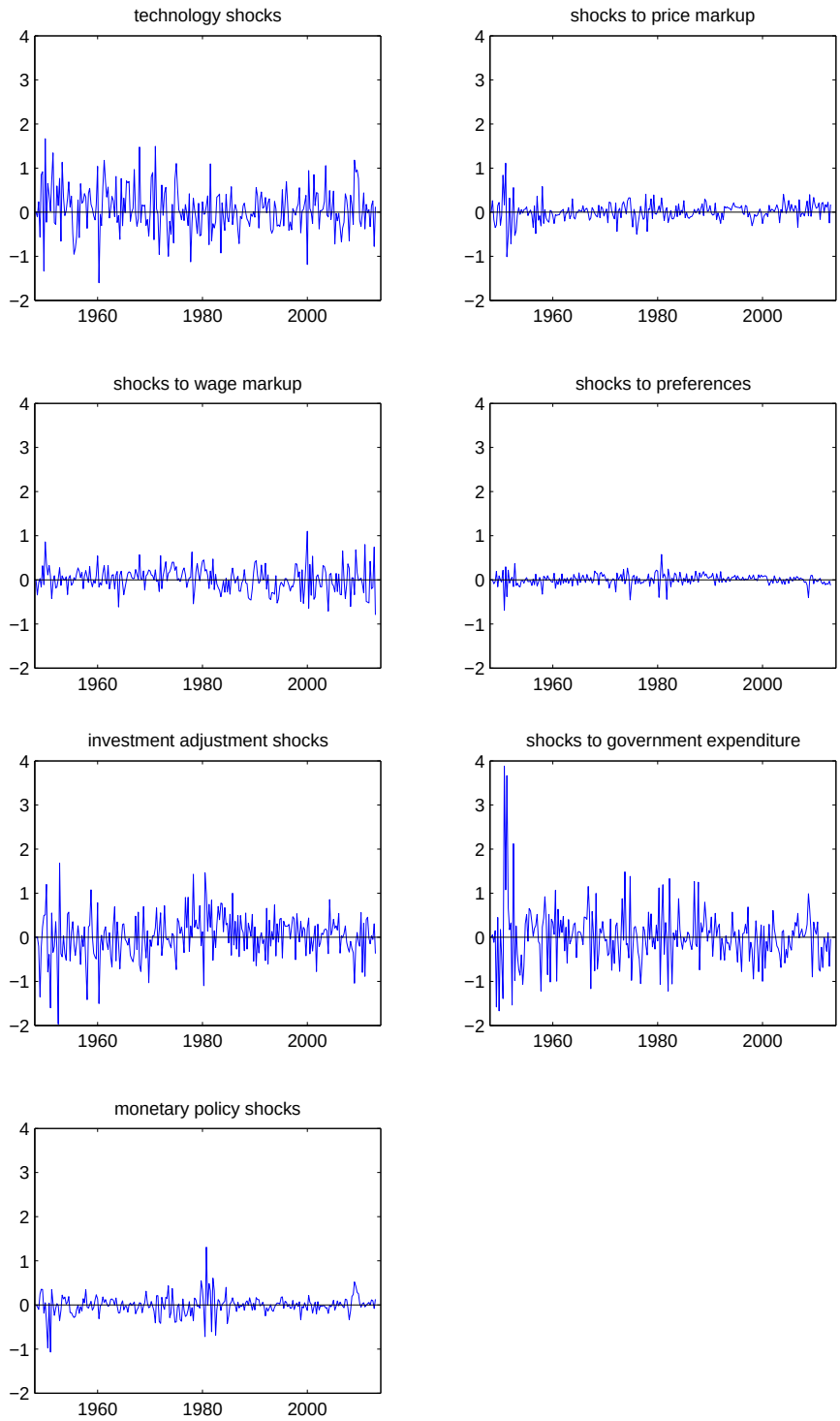

Figure 11: Time series of the estimated shocks

predicted by wage markup and price markup between 2008 to 2013 that is explained by the contribution of the remaining shocks. Monetary shocks play a fairly substantial role, shocks to technology, to preferences and to the investment adjustment cost are important as well. By contrast, the contribution from government spending shocks are negligible.

Presumably, the substantial role of monetary policy shocks for explaining that gap is due to the zero lower bound. The model is log-linearized and effectively assumes that the central bank can lower the nominal rate below zero: when observations of zero nominal rates come in, the agents in the model treat these as a surprising tightening. It should therefore not surprise, that these pseudo-shocks to monetary policy had the effect of reducing inflation.

The model does not contradict the consensus in the literature on the presence of a great degree of slackness in the economy after 2007. Indeed, figure 10 confirms that the output gap was large and negative in those years. Likewise, we have seen that the zero lower bound resulted in a tighter monetary policy from 2008 to 2010 than what would have been predicted by an unconstrained Taylor rule, pushing inflation down.

It should be noted, however, that the magnitude of this effect is rather modest. The model predicts that the inflation drop due to the liquidity trap, were all the other shocks absent, would have been only 0.5% below zero. Furthermore, the duration of the zero lower bound constraint is fairly short, according to the model. The zero lower bound was hit in 2008, but after 2010 the interest rate suggested by the Taylor rule was actually higher than the one implemented by the Fed by one percent until the second part of 2012 (Figure 10). According the model then, the surprise tightening from 20087 to 2010 was followed by a surprisingly loose monetary policy afterwards, though that counterswing was more modest in size.

Interestingly, one obtains an even smaller role of monetary policy shocks

when examining a variance decomposition only. To calculate it for the post-2007 episode and to compare it to the full sample, we estimated the variance of the shocks for the post-2007 episode as well as for the full sample, see table 2, and then proceeded to calculate a 4-step ahead and a 200-step ahead variance decomposition in the usual manner, effectively assuming that these variances then stay constant. The results are in tables 3 and 4. The full sample results confirm once again that price and wage markup shocks explain the bulk of the movements in inflation. According to these tables, the contribution of monetary policy shocks was practically neglible: remarkably, this is also true for the post-2007 sample, in contrast to the insights from figure 4 or table 1. The explanation for this difference emerges from visually inspecting the time series for the shocks in figure 11. Unsurprisingly, during the zero lower bound episode from 2008 to 2010, there are a bunch of positive monetary policy shocks, all pointing in the direction of a surprise tightening. While the standard deviation of these shocks is not remarkable in historical comparison, their effects pile up due to being virtually all of the same sign. For that reason, figure 4 and table 1 provide better insight into the actual evolution of inflation than the variance decompositions in tables 3 and 4. Nonetheless, they show that perhaps these zero lower bound monetary policy shocks do not amount to much, while instead the volatility of wage markup shocks increased markedly. It may well be worth to understand this latter phenomenon better in future research.

The second and third panels of Figure 6, reporting the inflation rate explained by the shocks to the price markup and wage markup, provide an answer to the missing deflation puzzle: the positive contribution of the price and wage markup shocks was the cause of the observed positive inflation rate, despite the negative contribution of other factors. If those two shocks were absent, the model predicts that inflation would have been negative.

5 Accounting for Employment

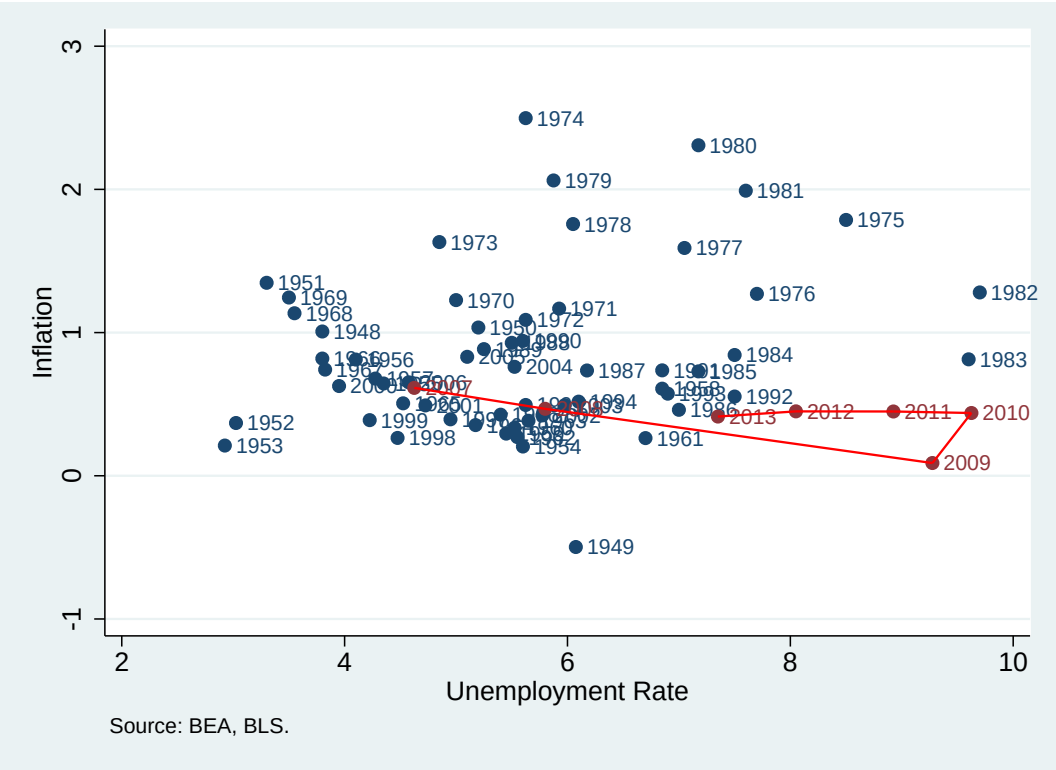

Figure 12: Phillips Curve using unemployment rate as a measure of slackness.

Inflation is perhaps not of the central concern in these debates: rather, it is the relationship of inflation to employment (or unemployment), which is the reason why many observers have been worried about deflation. This Phillips Curve relationship is examined in figures 13, 14 and 12. The relationship with inflation is certainly not a tight one, and it is therefore imperative to account for the movements in employment and contrast them with those in inflation.

The shock-by-shock decomposition for the entire sample is provided in figures 15 and 16. The overall secular movements seem to be well explained by the labor markup shock alone: however, a substantial gap opens for 2007

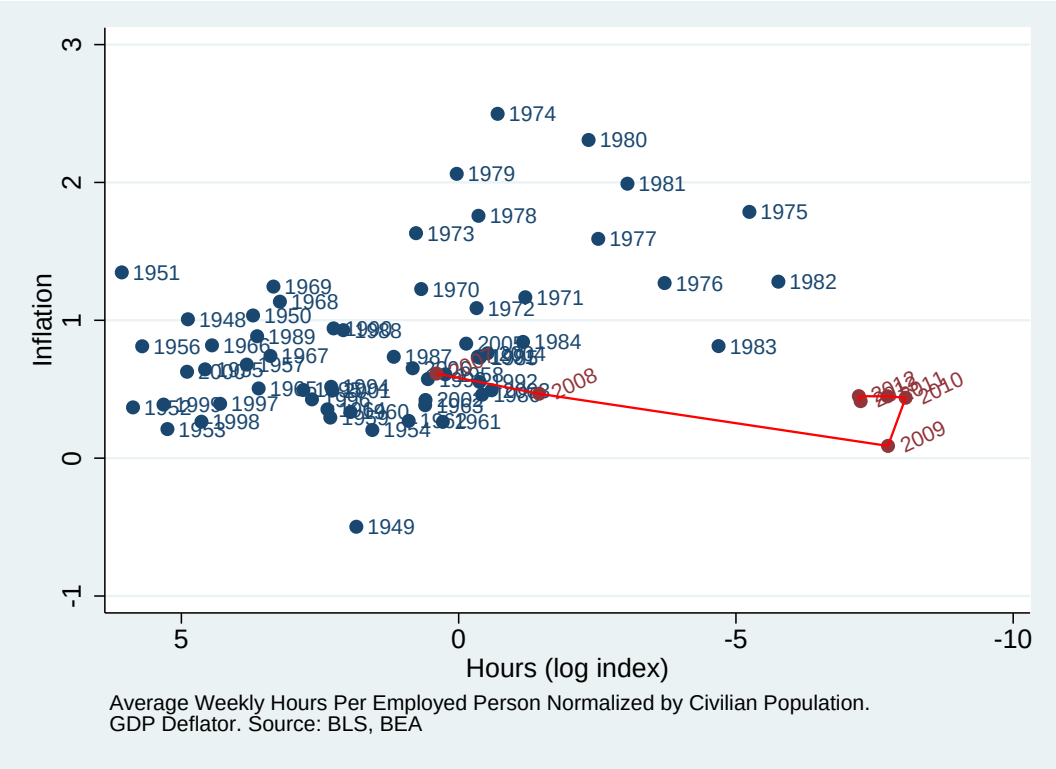

Figure 13: Phillips Curve using employment rate as a measure of slackness.

Table 5: 4-periods ahead Variance Decomposition of Employment

	1948-2013	1948-2007	2008-2013
Technology	1.48	1.45	2.25
Price Markup	3.76	3.77	3.17
Wage Markup	1.49	1.28	5.21
Preferences	32.79	32.70	29.45
Monetary Policy	27.98	28.05	29.32
Gov't Exp	13.39	13.71	10.75
Inv.Adj.Cost	19.12	19.03	19.84

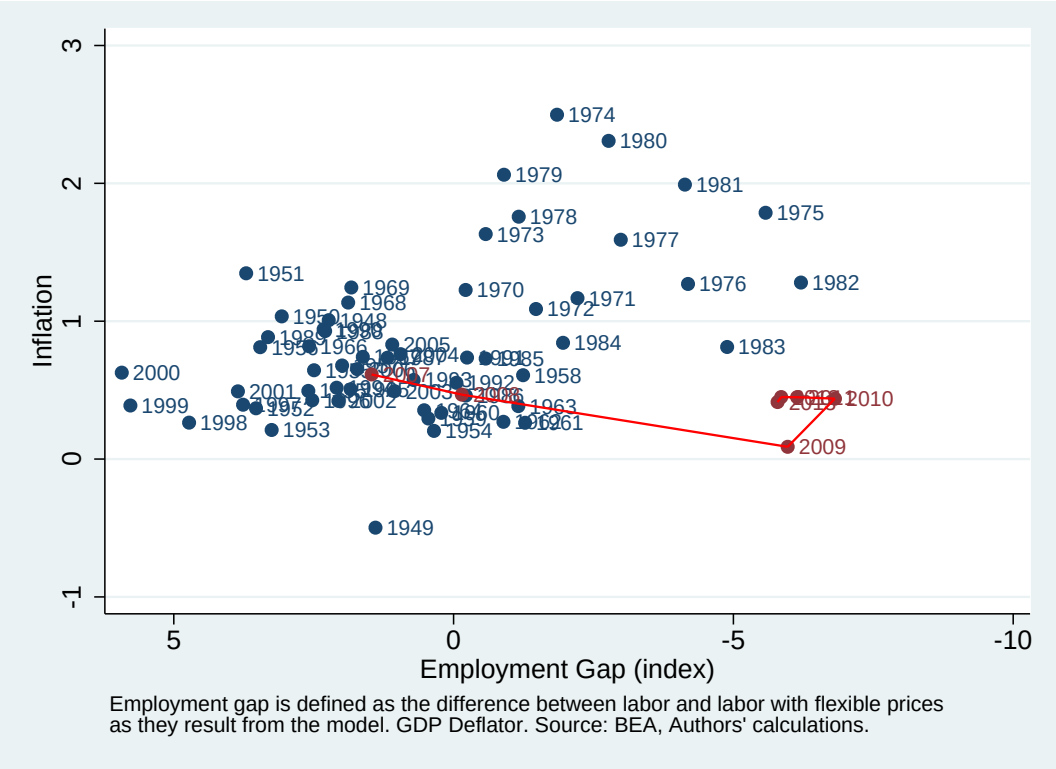

Figure 14: Phillips Curve using employment gap as a measure of slackness.

Table 6: Asymptotic Variance Decomposition of Employment

	1948-2013	1948-2007	2008-2013
Technology	3.13	3.19	2.82
Price Markup	16.18	16.87	8.10
Wage Markup	28.82	25.84	59.83
Preferences	12.56	13.00	6.69
Monetary Policy	17.02	17.71	10.58
Gov't Exp	12.46	13.24	5.93
Inv.Adj.Cost	9.84	10.16	6.05

Notes: 200-periods ahead.

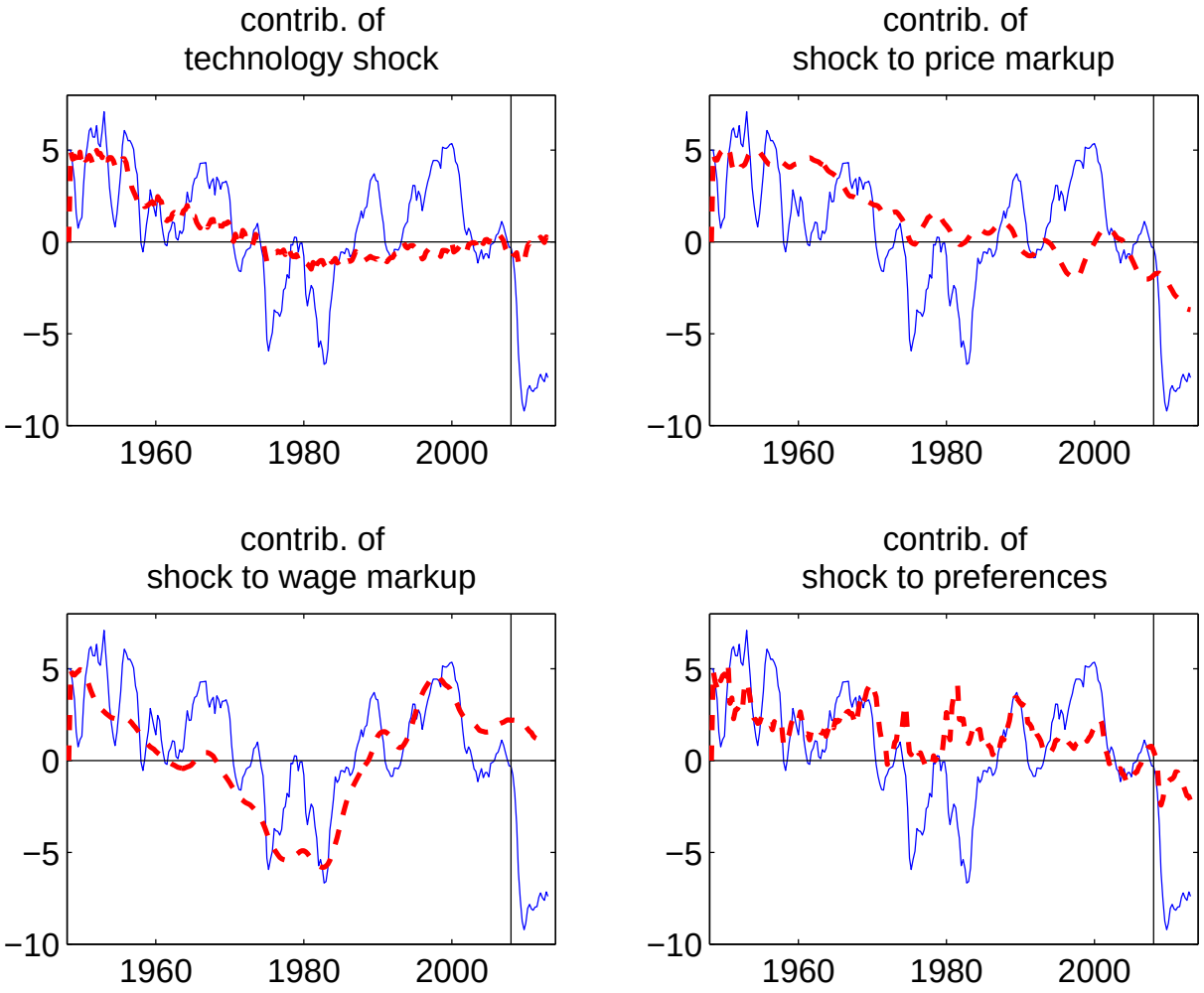

Figure 15: Historical shock decomposition of employment for the period 1948-2013. Solid line is actual employment. The dashed line is employment predicted by each shock individually.

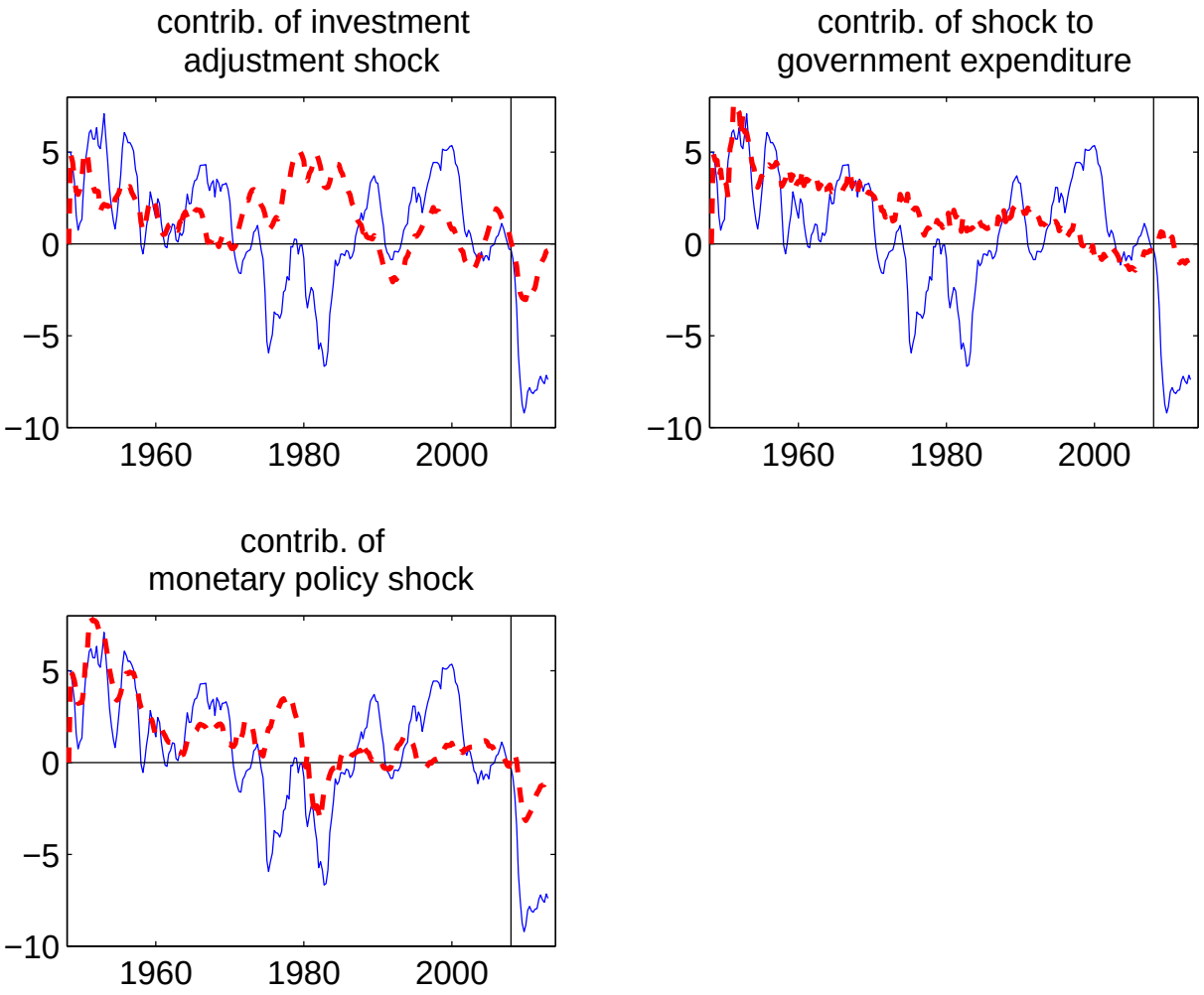

Figure 16: Historical shock decomposition of employment for the period 1948-2013. Solid line is actual employment. The dashed line is employment predicted by each shock individually.

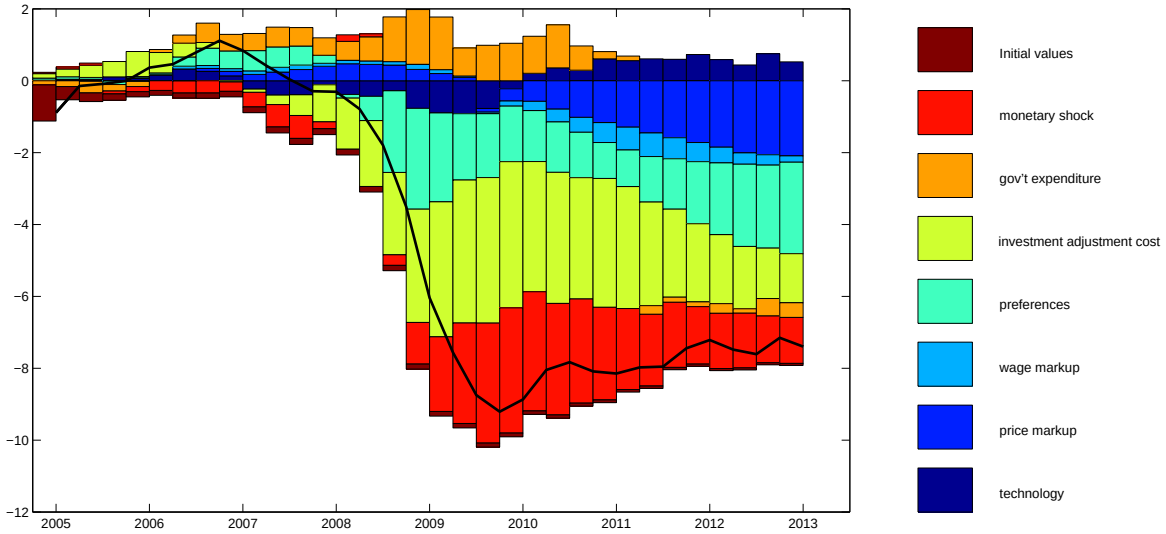

Figure 17: Historical shock decomposition of the labor

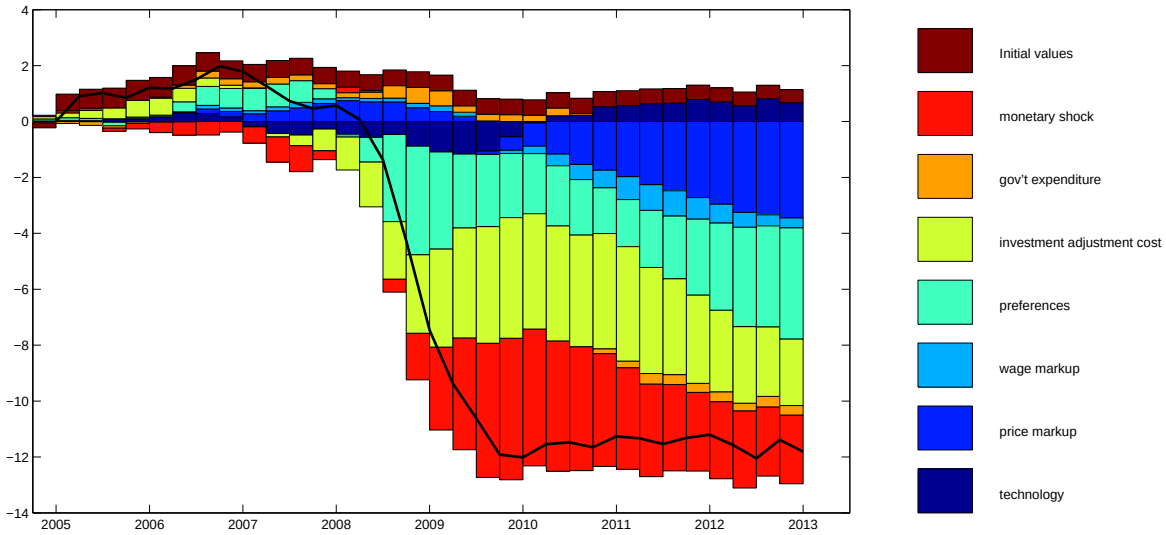

Figure 18: Historical shock decomposition of the output gap

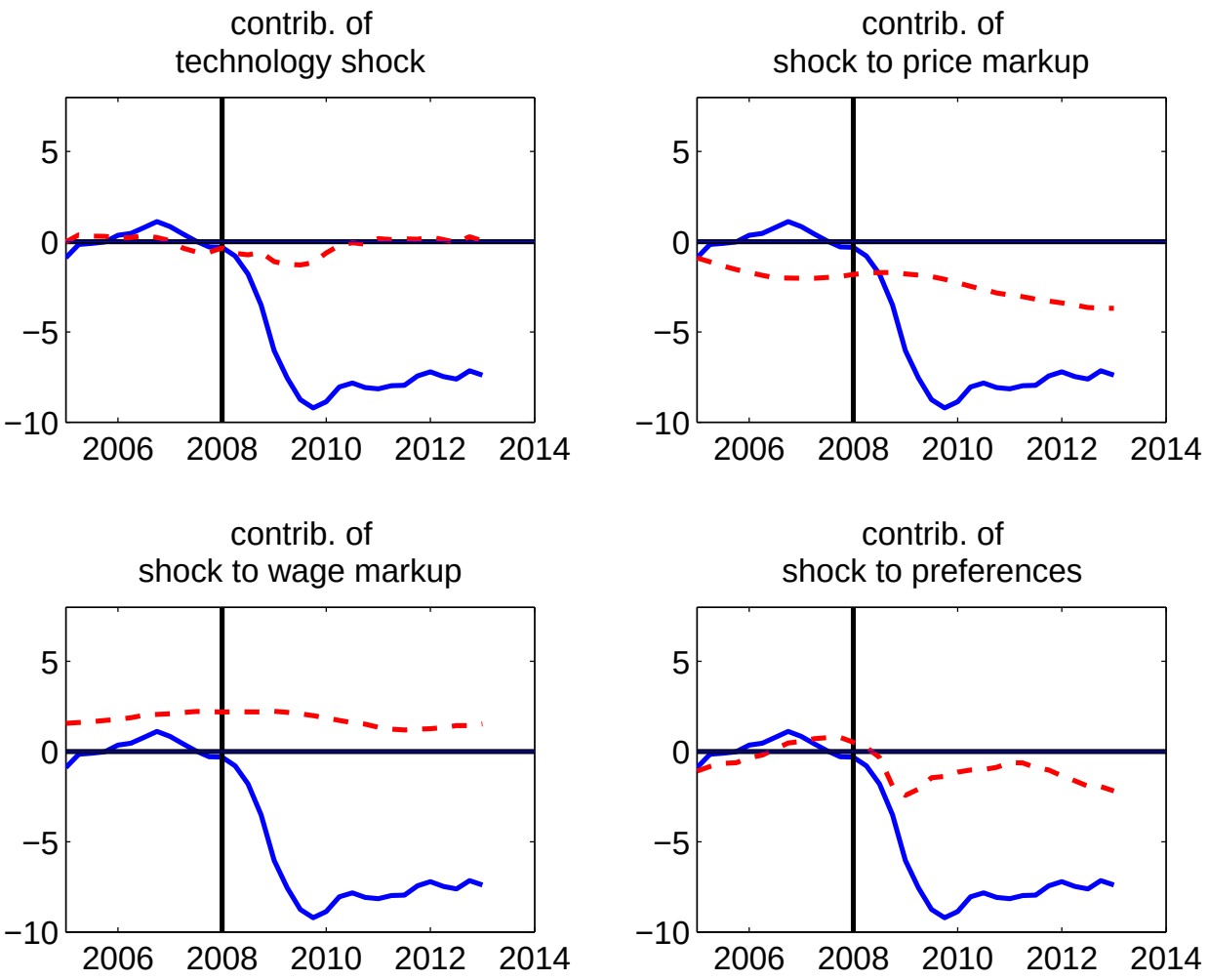

Figure 19: Historical shock decomposition of employment for the period 2005-2013. Solid line is actual employment. The dashed line is employment predicted by each shock individually.

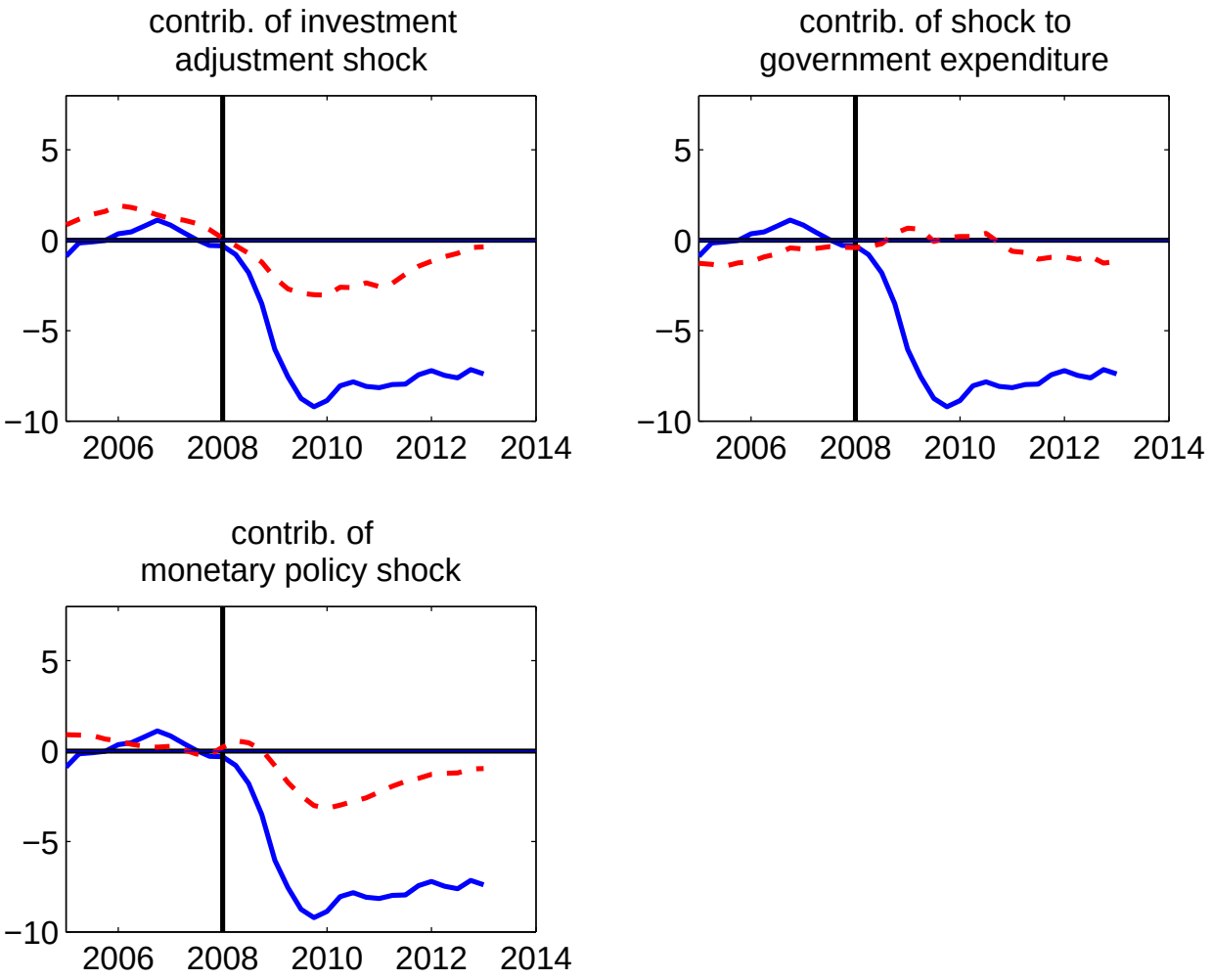

Figure 20: Historical shock decomposition of employment for the period 2005-2013. Solid line is actual employment. The dashed line is employment predicted by each shock individually.

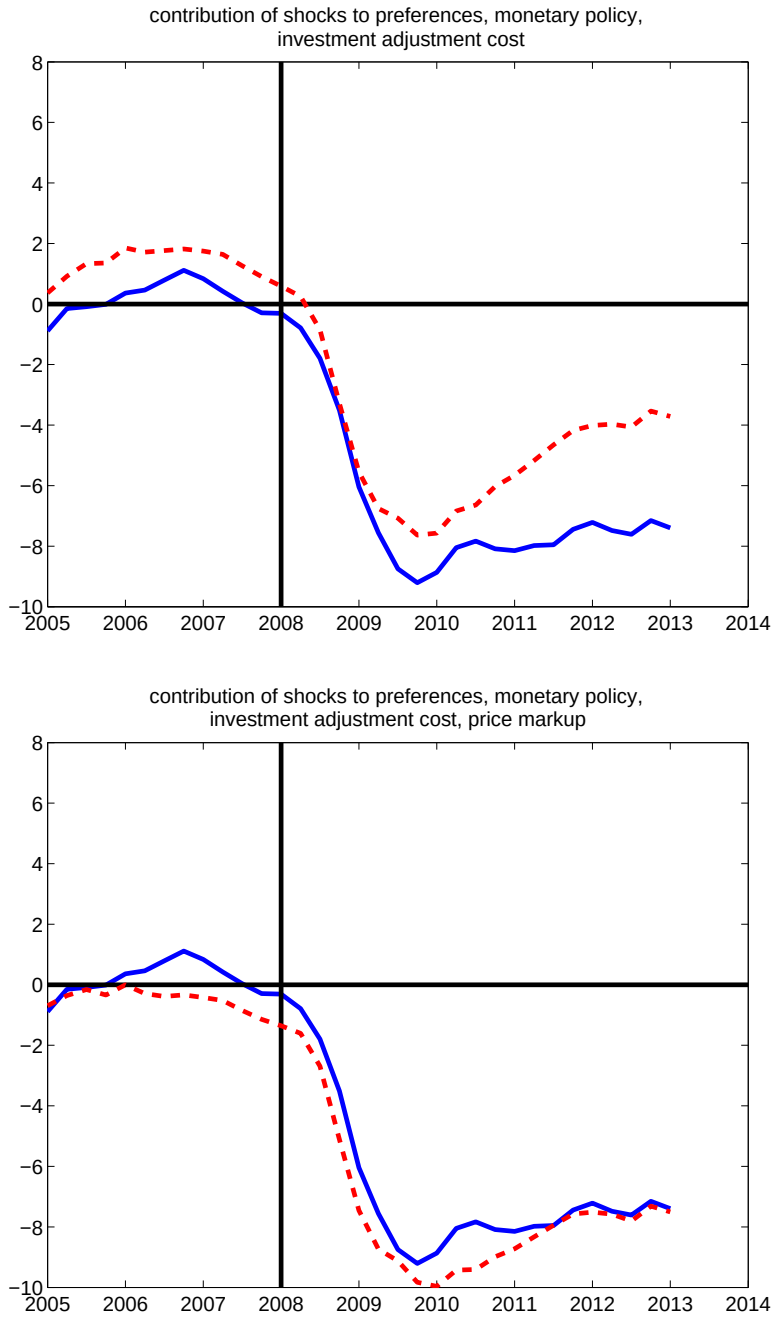

Figure 21: Historical shock decomposition of employment for the period 2005-2013. Solid line is actual employment. The dashed line is employment predicted by each shock individually.

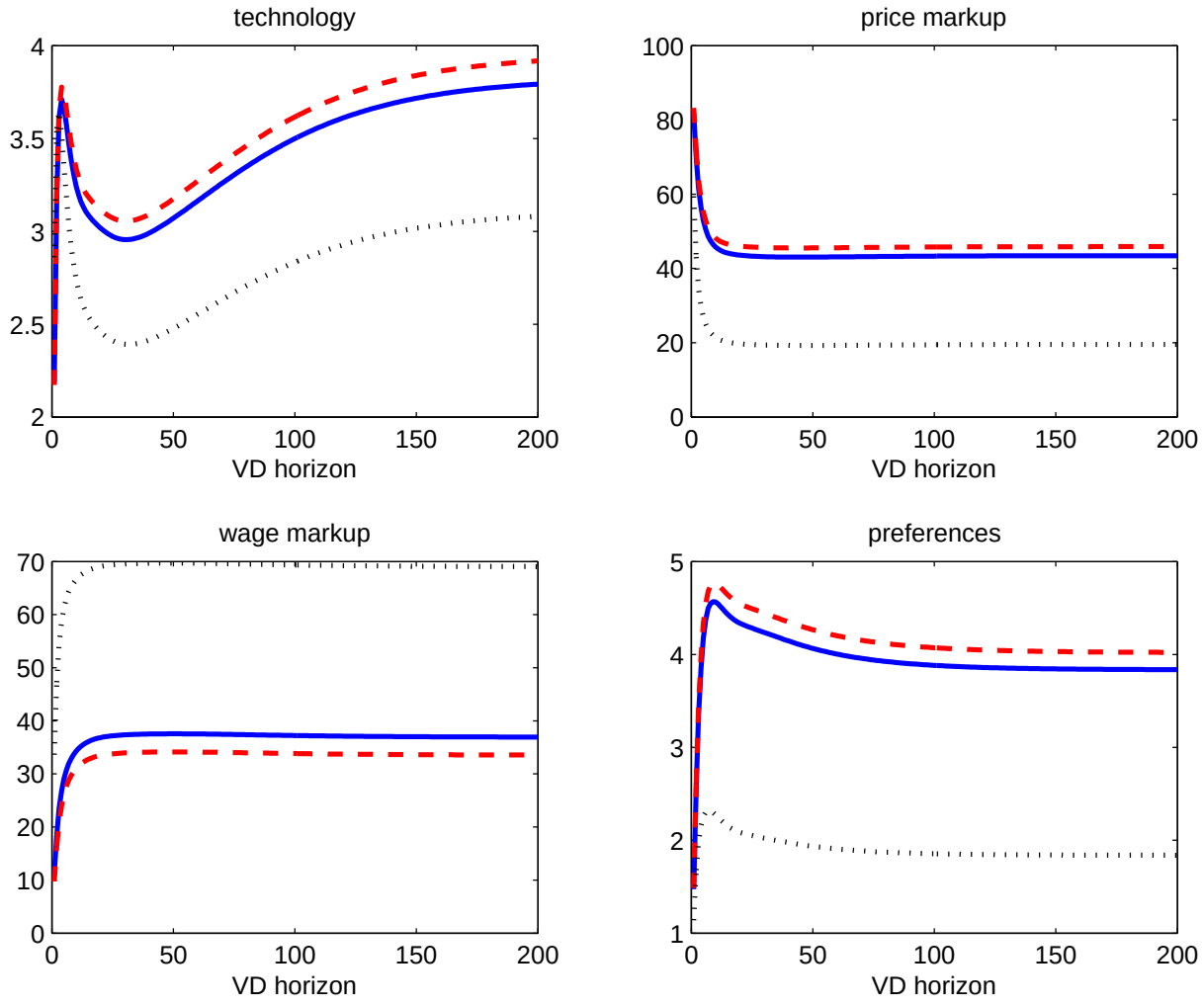

Figure 22: Convergence of the Variance Decomposition coefficients for Inflation. Blue solid line: full sample. Red dashed line: sample until 2007. Black dotted line: sample from 2008.

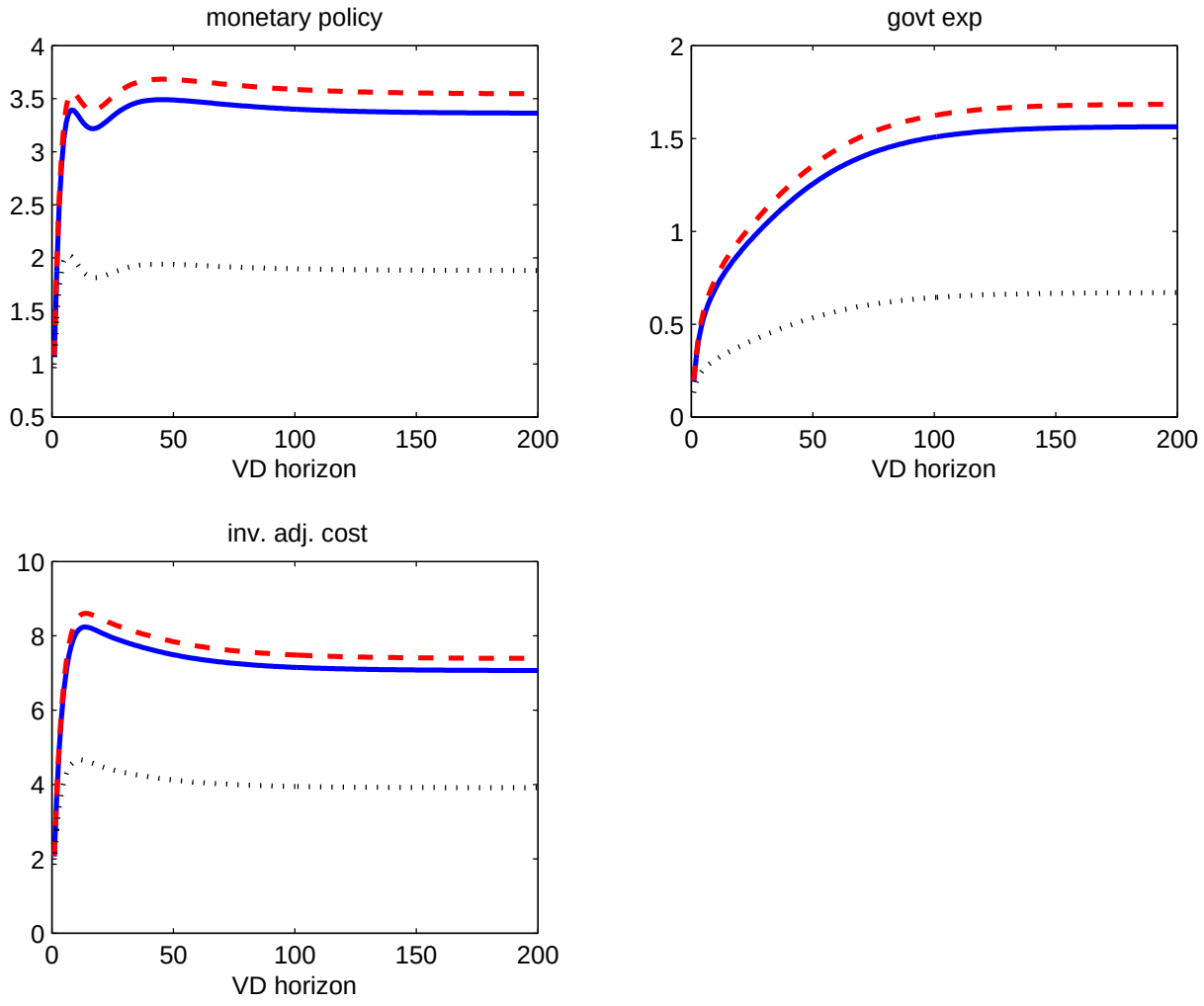

Figure 23: Convergence of the Variance Decomposition coefficients for Inflation. Blue solid line: full sample. Red dashed line: sample until 2007. Black dotted line: sample from 2008.

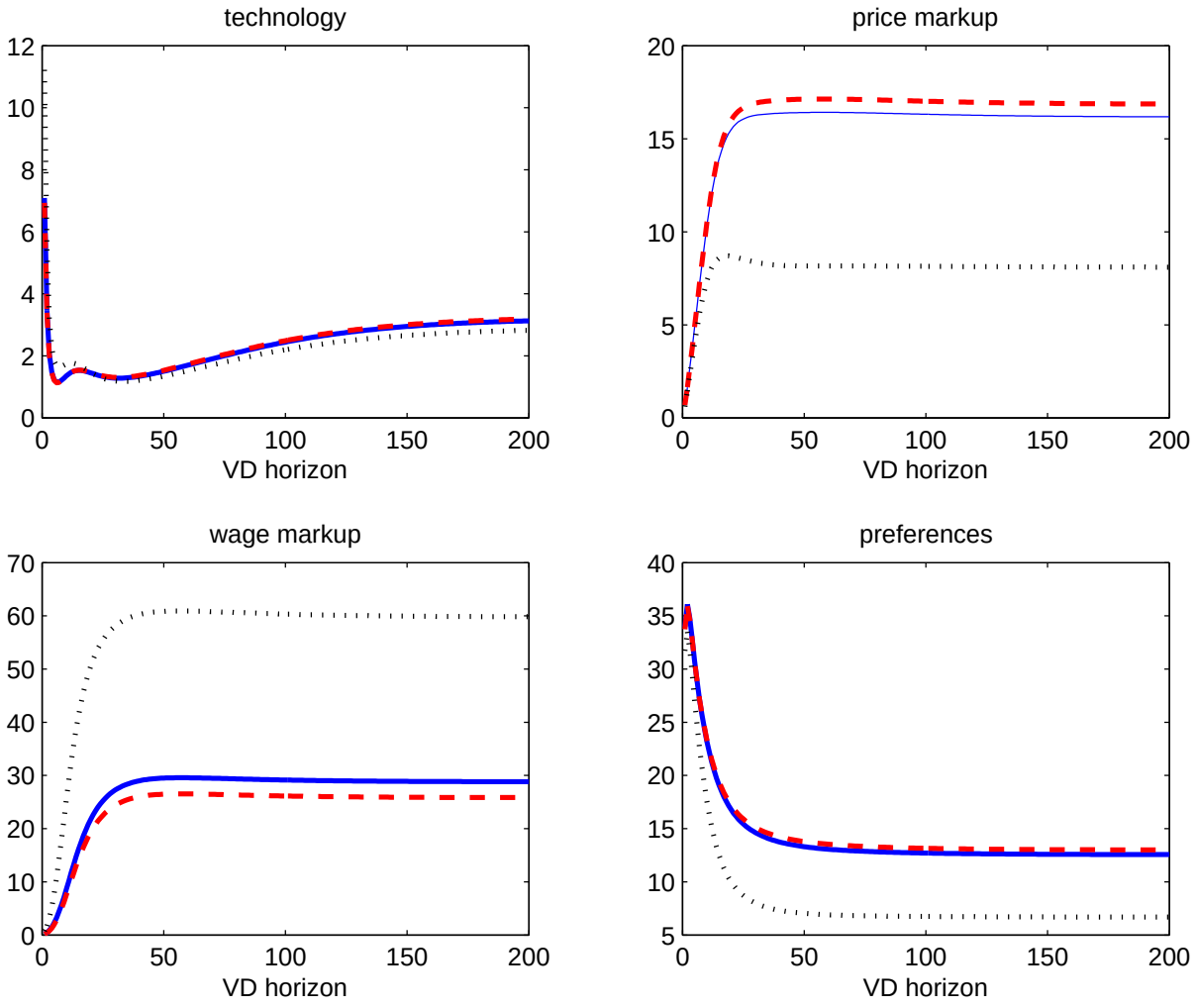

Figure 24: Convergence of the Variance Decomposition coefficients for Employment. Blue solid line: full sample. Red dashed line: sample until 2007. Black dotted line: sample from 2008.

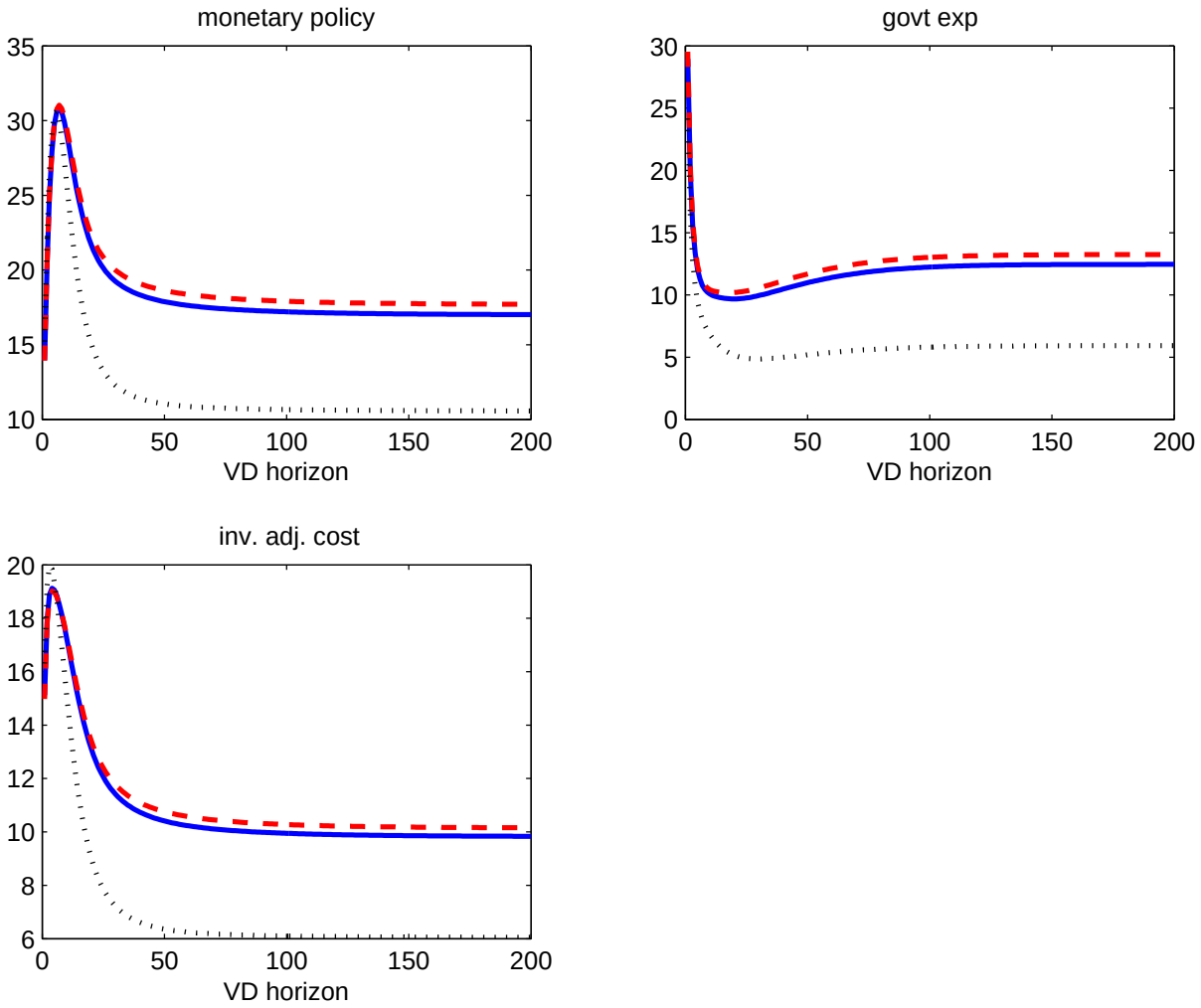

Figure 25: Convergence of the Variance Decomposition coefficients for Employment. Blue solid line: full sample. Red dashed line: sample until 2007. Black dotted line: sample from 2008.

and beyond (actually starting earlier than that): the labor markup shock would have actually predicted much higher employment.

Thus, figures 17, 19, 20 and 21 provide a decomposition of the labor movements for 2007 and beyond. A combination of preference shocks, monetary policy shocks and investment adjustment shocks seem to largely account for the decline: adding price markup shocks as well provides a nearly complete account. By contrast, government expenditure shocks as well as technology shocks played only a modest role at best.

This also emerges from the variance decompositions in tables 5 and 6. It should be noted that the contribution of monetary policy shocks for the post-2005 sample is not particularly different from that of the full sample for the 4-period ahead variance decomposition and actually declines for the asymptotic variance decomposition. The same is true for preference shocks and investment adjustment shocks. Wage markup shocks, by contrast, explain a lot, and even more so asymptotically for the post-2005 sample. Given that these shocks imply higher employment than currently observed in the data, this may be good news for those who are worried about high unemployment rates. There is a difference here for all these numbers in terms of their contribution to the four-quarter ahead variance of the forecast error versus the unconditional (or 200-period-ahead) variance. This is due to some shocks having a larger impact at high frequencies, while other shocks account more for the low frequencies. Figure **??** shows visually, how the variance shares change, as the horizon is changed.

6 Zero Lower Bound

One obvious objection to our approach is related to the fact that the model does not take explicitly into account the zero lower bound. As a result, it may be hard to judge whether we are overestimating or underestimating the

effects of a tightened monetary policy on inflation and output during the crisis years.

We use a very simple model to gain some insight on the issue. Although the following exercise does not provide a quantitative answer, it allows us to qualitatively assess the effect of misrepresenting the zero lower bound as a sequence of positive monetary shocks.

The following equations describe the evolution of inflation and the output gap in a stylized New-Keynesian model:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{1}$$

$$x_t = -\frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^e) + E x_{t+1} + \mathbb{1}_{t=0} \xi_x \epsilon_t^d \tag{2}$$

where the natural interest rate is given by $r_t^e = \rho + \sigma E_t \Delta x_{t+1}$[3]

To close the model we need an equation for the interest rate. We want to compare two interest rate paths. In the first one (Case A), the agents recognize the existence of the zero lower bound.

- Case A: $i_t = \max\{0, i_t^{\#}\}$

In the second case (Case B), the zero lower bound is modeled as an unexpected exogenous positive shock.

- Case B: $i_t = i_t^{\#} + \epsilon_t^m$

where $i_t^{\#}$ is the interest rate consistent with the Taylor rule.

$$i_t^{\#} = \rho + \alpha \pi_t + \gamma x_t \tag{3}$$

We assume that the length of the liquidity trap is exogenous and it is known to agents under Case A. The qualitative results are not affected by the introduction of this simplification.

[3]The natural interest rate depends ultimately on technological shocks.

Consider a sufficiently large demand shock. As a result of it, the economy is in a liquidity trap from period t = 0 to $t = \tau$. The behavior of the interest rate is described in the left panel of Figure 26. The interest rate becomes zero in period zero and it remains at the zero lower bound from period zero to period τ. Afterwards it stays positive forever.

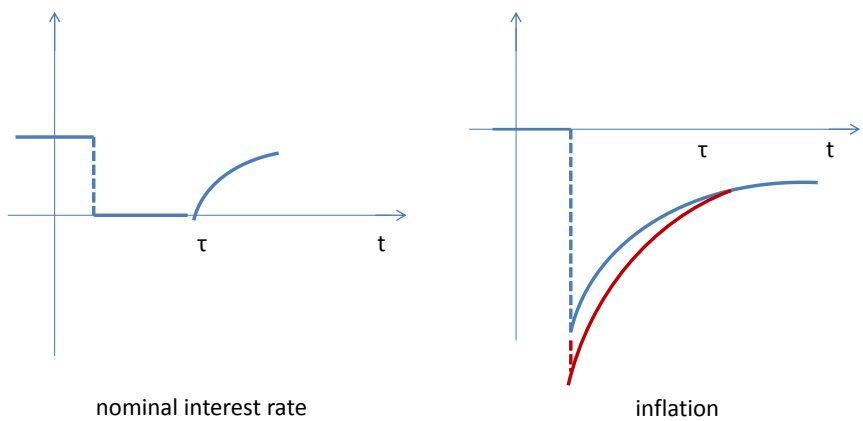

Figure 26: Inflation path in the case in which the agents internalize the zero lower bound (Case A = red) and in the case in which they do not (Case B = blue)

We compare the behavior of the economy under the two cases described above. In the first case, the agents fully internalize the fact that the economy is stuck in a liquidity trap. In the latter, the liquidity trap is a shock of decreasing intensity occurring each period.

From $\tau + 1$ onwards, the two cases do not differ. Therefore, output gap and inflation are the same. At τ, the economy is in a liquidity trap, so the

nominal interest rate is zero.

$$x_\tau^A = -\frac{1}{\sigma}(0 - E_\tau \pi_{\tau+1}^A - r_\tau^{e,A}) + E_\tau x_{\tau+1} \tag{4}$$

$$x_\tau^B = -\frac{1}{\sigma}(0 - E_\tau \pi_{\tau+1}^B - r_\tau^{e,B}) + E_\tau x_{\tau+1} \tag{5}$$

Hence, also at τ, output gap and therefore inflation are the same. The two cases have different predictions for the expected inflation rate and expected output gap. However, since the economy under the two scenarios is the same since $t = \tau + 1$, the expectations over the future are the same and therefore in period τ, the resulting realization of the output gap is the same.

However, expectations over the output gap in the previous period differ:

$$E_{\tau-1} x_\tau^A = -\frac{1}{\sigma}(0 - E_{\tau-1} \pi_{\tau+1}^A - E_{\tau-1} r_\tau^{e,A}) + E_{\tau-1} x_{\tau+1} \tag{6}$$

$$E_{\tau-1} x_\tau^B = -\frac{1}{\sigma}(i_\tau^{\#} - E_{\tau-1} \pi_{\tau+1}^B - E_{\tau-1} r_\tau^{e,B}) + E_{\tau-1} x_{\tau+1} \tag{7}$$

It follows that $E_{\tau-1} x_\tau^B > E_{\tau-1} x_\tau^A$, because $i_\tau^{\#} < 0$. Therefore, at $\tau - 1$ both output gap and inflation are higher in Case B than in Case A.

Recursively, we can prove it for any period from $t = 2$ to $t = \tau - 1$. Since the realization for the periods since $t = \tau$ are the same under the two scenarios and in Case A inflation is always lower than inflation under Case B, it follows that, on impact, it has to be that output gap and inflation drop more in Case A than in Case B. Moreover, the output gap when agents fully internalize the zero lower bound grows at a faster pace after a recession, but the recession itself is deeper.

This little exercise shows that our analysis might be underestimating the effect of the zero lower bound on inflation.

On the other hand, Eggertsson and Woodford (2005) have argued that some version of price path targeting can help, when in a liquidity trap, essentially promising to keep interest rates low after exiting the zero lower bound,

in case there was too much deflation. Indeed, one can read our figure 10 that this is what the Federal Reserve Bank has been doing, as it exited the constraint around mid-2010, according the model. The analysis in their paper implies that there then shouldn't be a deflation in the first place, as agents anticipate the correspondingly looser policy upon exit, if deflation should materialize. From that perspective, our analysis might be overstating the effect of the zero lower bound on inflation.

It should be clear from this discussion that a full resolution requires subtle choices regarding the expectations of agents regarding the path of future monetary policy variables at the onset of the 2008 crisis. Morevoer, a full shock decomposition would require inverting a nonlinear representation, when taking into account the zero lower bound constraints. Both are beyond the scope of this paper.

7 Conclusion

In spite of the widespread fears, the US did not experience a period of deflation during the recession following the events of 2008. During these years, inflation has been low but positive and stable, despite the slackness in the economy. It is therefore of interest to understand why this is so.

We used a pre-crisis "retro" model by Smets and Wouters (2007) to account for the movements in inflation. The model allows for a "risk premium" shock to move the value of capital, but does not otherwise incorporate financial frictions. Moreover, the log-linearized solution and equations effectively mean, that agents consider zero nominal interest rates as a surprisingly tight monetary policy stance, if the unconstrained Taylor rule would have implied negative rates instead.

While price and wage markup shocks nearly suffice to account for the inflation movements prior to the crisis of 2008, a gap opens after that date.

Indeed, by themselves they would have predicted a considerably higher inflation than what has been observed in the data. This answers a key question of this paper: there is no deflation and, actually, some inflation for the same reason that there was inflation at other times, namely price- and wage-markup shocks.

The gap is explained by the other shocks in the system, notably monetary policy shocks. The unconstrained Taylor rule of the model predicts that interest rates should have been somewhat negative from 2008 to 2010, while the zero lower bound constrained them from that: this is the effect many observers feared and pointed to. It should be noted, however, that the magnitude of this effect is rather modest. The model predicts that the inflation drop due to the liquidity trap, were all the other shocks absent, would have been only 0.5% below zero. Furthermore, the duration of the zero lower bound constraint is fairly short, according to the model. The zero lower bound was hit in 2008, but after 2010 the interest rate suggested by the Taylor rule was actually higher than the one implemented by the Fed by one percent until the second part of 2012 (Figure 10). According the model then, the surprise tightening from 2008 to 2010 was followed by a surprisingly loose monetary policy afterwards, though that counterswing was more modest in size.

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Table 7: Summary statistics

	1948Q1 - 2004Q4		1948Q1 - 2013Q1	
	mean	std dev	mean	std dev
consumption	0.535	0.847	0.485	0.839
investment	0.560	2.383	0.426	2.434
output	0.514	1.005	0.458	0.978
labor	1.348	2.900	0.663	3.523
inflation	0.8402	0.6540	0.796	0.628
wage rate	0.483	0.603	0.442	0.647
interest rate	1.325	0.867	1.214	0.877

Growth rates of consumption, investment, output and real wage. Consumption, investment, output and hours worked are net of labor force growth, normalized to 1 in 2005Q1. Variables are deflated using the GDP deflator.

Appendix

A Re-Estimating the Smets-Wouters model

Following Smets and Wouters, we adopt a Bayesian approach and we estimate the posterior distribution using a MH algorithm with 250,000 simulations. Priors are the same as the ones assumed in Smets-Wouters. Like them, we use 7 time series to estimate the model: output, consumption, investment, inflation, hours worked, real wages, and the (Federal Fund) interest rate. We compute consumption, investment and hours worked per-capita using a population index normalized to 1 in 1992Q4. We use the GDP deflator as a measure of inflation. Consumption, investment, output and wage are at constant prices. Variables are net of the labor force growth, normalized to 1 in 2005Q1. For the interest rate, we use the Federal Funds Rate.

We use data from U.S. Bureau of Economic Analysis and U.S. Bureau of Labor Statistics for the period 1948Q1-2013Q1. Table 7 collects some sum-

mary statistics for the the sample used by Smets and Wouter (left column) and our sample (right column). In general, the mean is smaller and unsurprisingly the standard deviation is larger when we consider the extended sample. The average of labor hours relative to the hours worked in 2005 Q2 (2005Q2=0) decreases considerably, from 1.348 to 0.663 with the inclusion of the observations from 2005Q1 to 2013Q1. Contrary to the original paper, our sample starts from 1948Q1 instead of 1947Q1 because these data points are not available for the most recent vintages of the time series.

Table 8: Posterior distributions (Metropolis Hasting 250000 simulations). Comparison between our sample and the sample used by Smets and Wouters.

		1948Q1 - 2013Q1			1948Q1 - 2004Q4		
		mean	HDP inf	HPD sup	mean	HDP inf	HPD sup
$\sigma(e_Z)$	technology	0.503	0.458	0.547	0.522	0.467	0.579
$\sigma(e_{b^2})$	preferences	0.136	0.099	0.172	0.112	0.069	0.153
$\sigma(e_g)$	gov't exp	0.647	0.602	0.695	0.734	0.671	0.794
$\sigma(e_\mu)$		0.488	0.421	0.554	0.435	0.333	0.529
$\sigma(e_{ms})$	monetary shock	0.226	0.206	0.247	0.236	0.212	0.258
$\sigma(e_p)$	price shock	0.208	0.180	0.235	0.216	0.184	0.247
$\sigma(e_w)$	wage shock	0.278	0.249	0.307	0.250	0.219	0.281
ρ_Z		0.985	0.978	0.992	0.974	0.958	0.991
ρ_{b^2}		0.765	0.658	0.871	0.852	0.769	0.934
ρ_g		0.977	0.965	0.988	0.978	0.964	0.991
ρ_μ		0.780	0.694	0.873	0.673	0.565	0.784
ρ_{ms}		0.167	0.076	0.256	0.137	0.049	0.220
ρ_p		0.975	0.957	0.996	0.979	0.961	0.997

Continued on next page

Table 8 – continued from previous page

		1948Q1 - 2013Q1			1948Q1 - 2004Q4		
		mean	HDP inf	HPD sup	mean	HDP inf	HPD sup
ρ_w		0.960	0.936	0.986	0.965	0.944	0.988
θ_p	price MA	0.882	0.823	0.944	0.872	0.805	0.941
θ_w	wage MA	0.922	0.880	0.965	0.890	0.838	0.943
$S''(\gamma)$	inv. adj cost	4.063	2.572	5.514	6.747	5.094	8.391
σ_c	intertemp elast	1.984	1.528	2.443	1.450	1.082	1.817
h	habits	0.384	0.287	0.483	0.506	0.421	0.585
ζ_w	wage stickiness	0.803	0.739	0.866	0.749	0.679	0.816
ν_L	labor	1.638	0.862	2.391	1.971	1.135	2.770
ζ_p	price stickiness	0.675	0.584	0.761	0.608	0.531	0.680
ι_w	wage index	0.489	0.319	0.656	0.523	0.359	0.682
ι_p	price index	0.290	0.162	0.415	0.311	0.168	0.451
$czcap$	capital utiliz.	0.690	0.528	0.854	0.453	0.316	0.596
Φ	fixed cost	1.609	1.495	1.721	1.441	1.318	1.569
ψ_1	monetary p.	1.969	1.705	2.226	1.982	1.691	2.266
ρ_R	monetary p.	0.881	0.853	0.909	0.884	0.858	0.910
ψ_2	monetary p.	0.084	0.043	0.123	0.084	0.015	0.146
ψ_3	monetary p.	0.187	0.152	0.221	0.223	0.178	0.269
$const_\pi$	measurem eq	0.740	0.612	0.868	0.725	0.574	0.871
$constebeta$	measurem eq	0.248	0.091	0.398	0.251	0.094	0.403
$const_L$	measurem eq	-0.021	-2.097	2.128	0.558	-1.719	2.750
$ctrend$	growth rate	0.469	0.440	0.499	0.469	0.440	0.499
ρ_{gZ}	gov't exp	0.629	0.495	0.761	0.629	0.495	0.761
α	$\frac{k_*}{y_*}$	0.192	0.129	0.252	0.192	0.129	0.252

There are some changes in the estimated posteriors once we extend the time period, but they appear to be minor and not particularly remarkable. A comparison is in table **??**. There, HPD inf and HPD low are respectively the lower bound and the upper bound of a 90% HPD interval. In particular, the intertemporal elasticity of substitution σ_c falls a bit, and the habit level as well as the adjustment costs rises. Robustness checks and estimation results for different subsamples and specifications of the model are reported in a technical appendix.

B Impulse Response Functions

Figures 27 to 33 contain the impulse responses for the parameters at the mean of the full sample posterior.

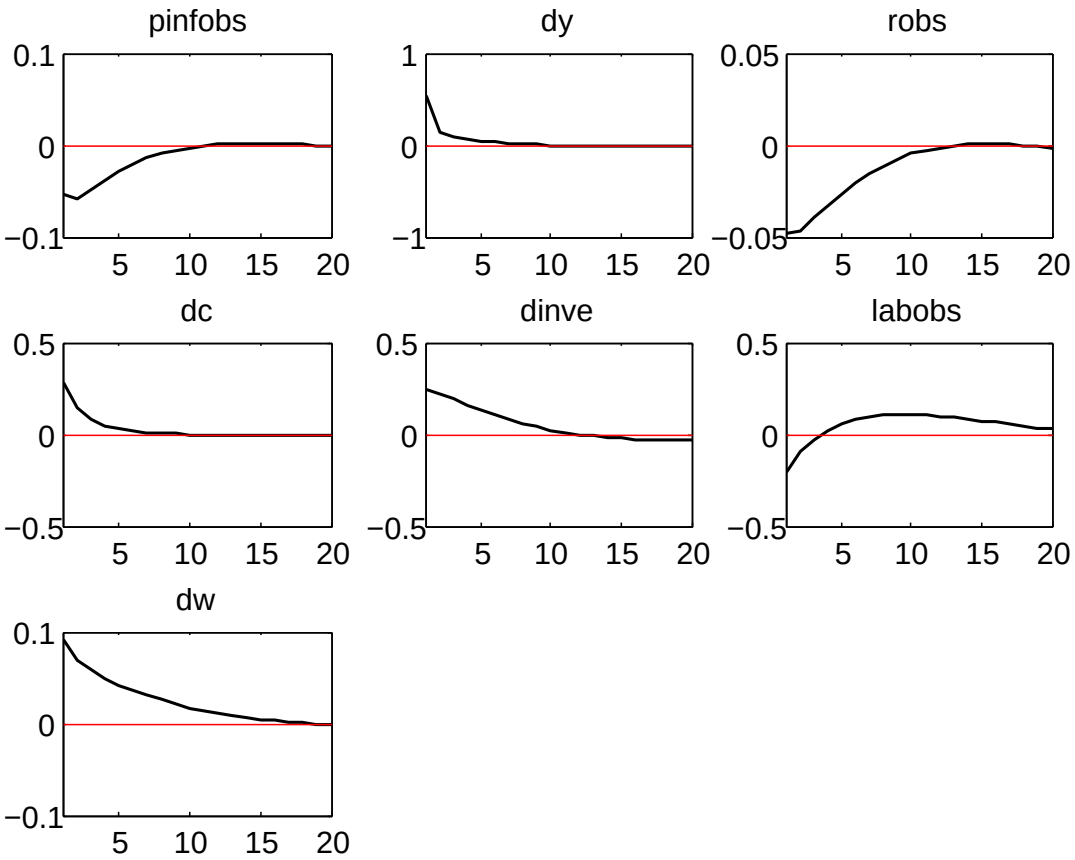

Figure 27: Impulse Response Functions to a shock to technology

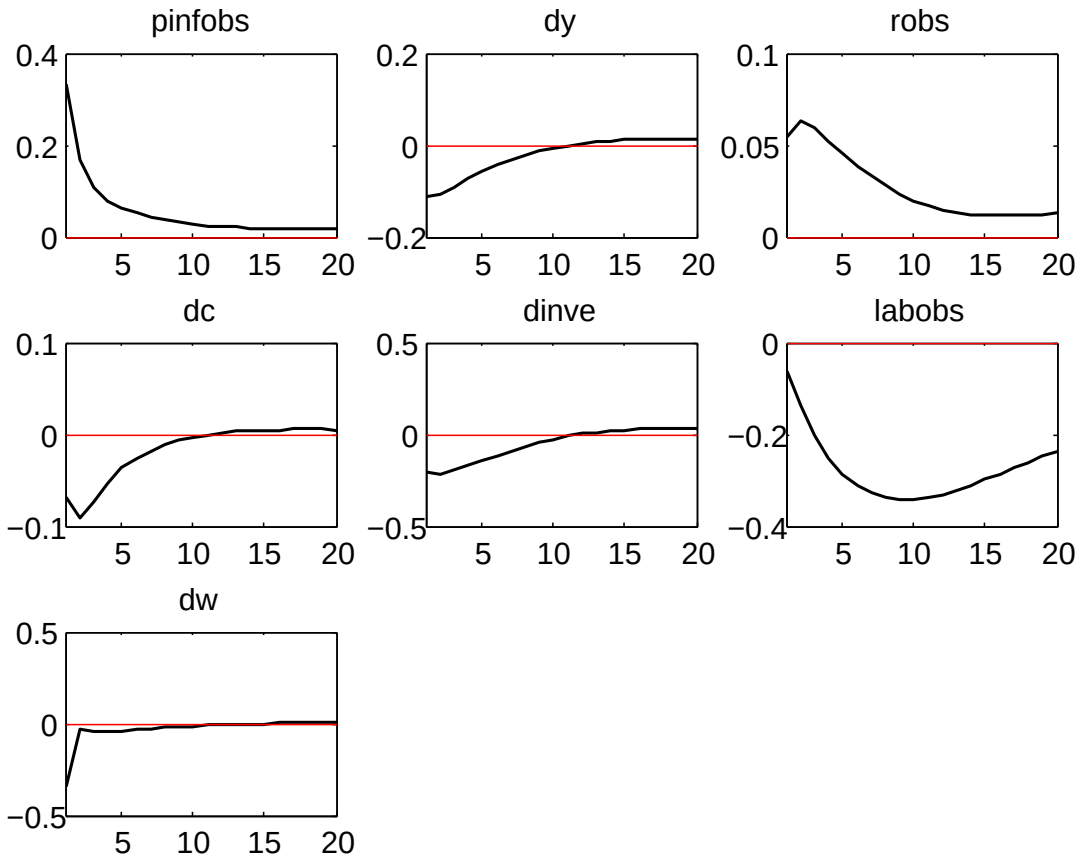

Figure 28: Impulse Response Functions to a shock to price markup

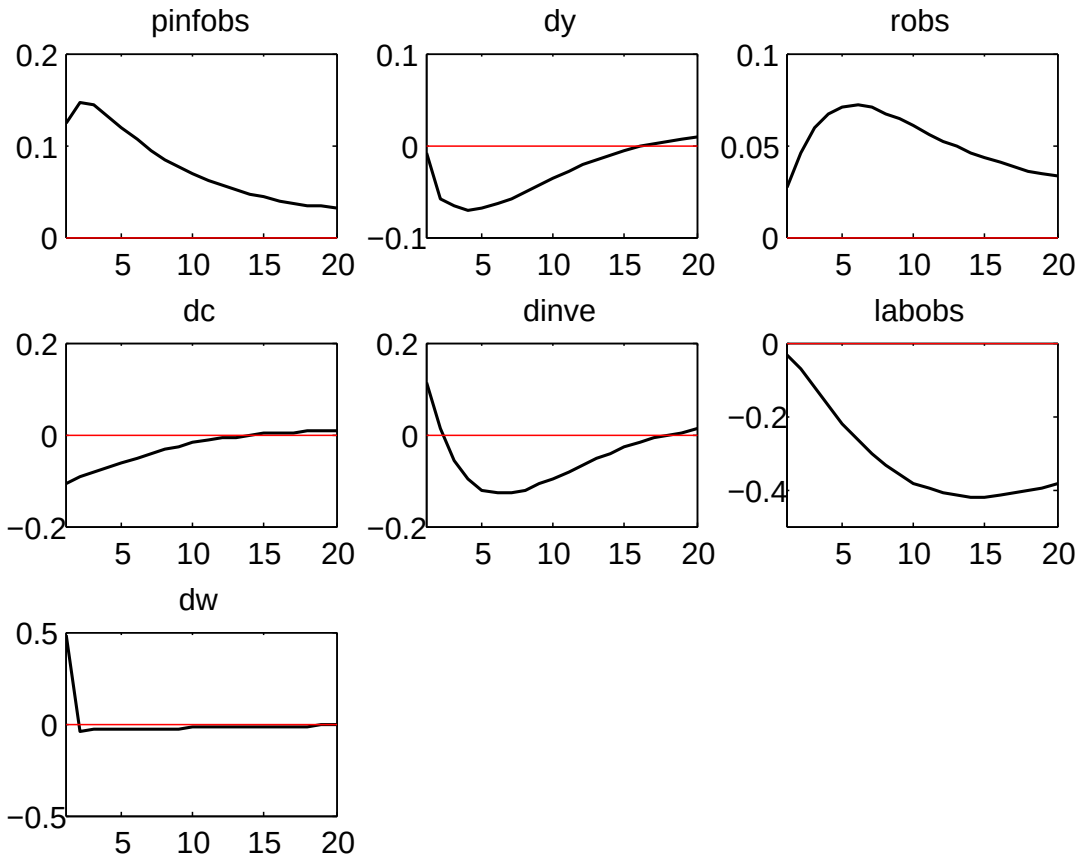

Figure 29: Impulse Response Functions to a shock to wage markup

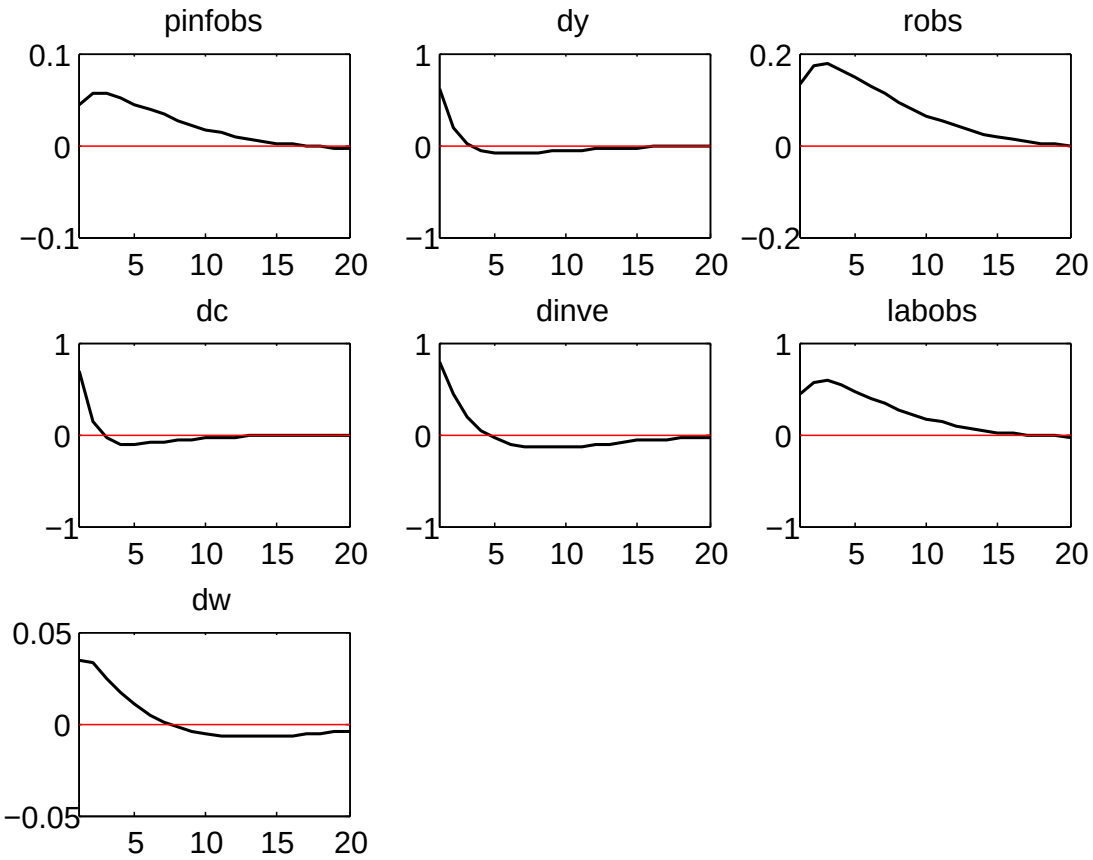

Figure 30: Impulse Response Functions to a shock to preferences

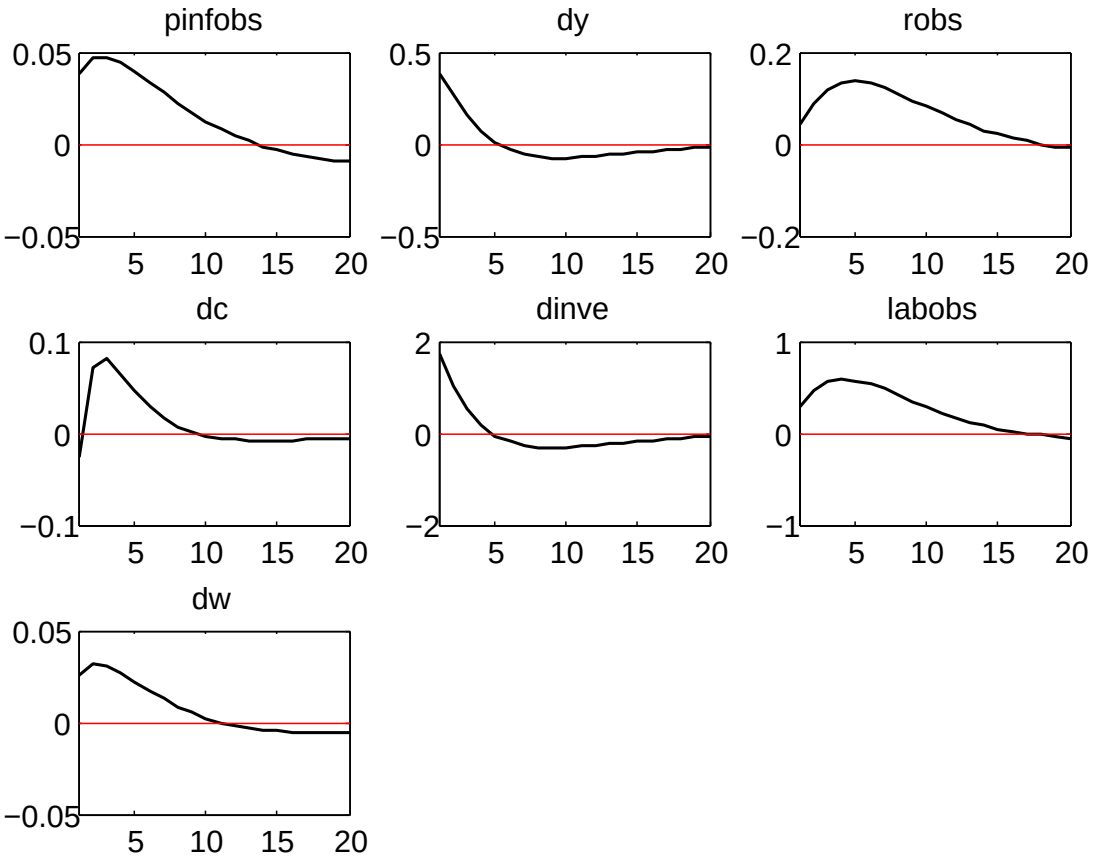

Figure 31: Impulse Response Functions to a shock to investment adjustment cost

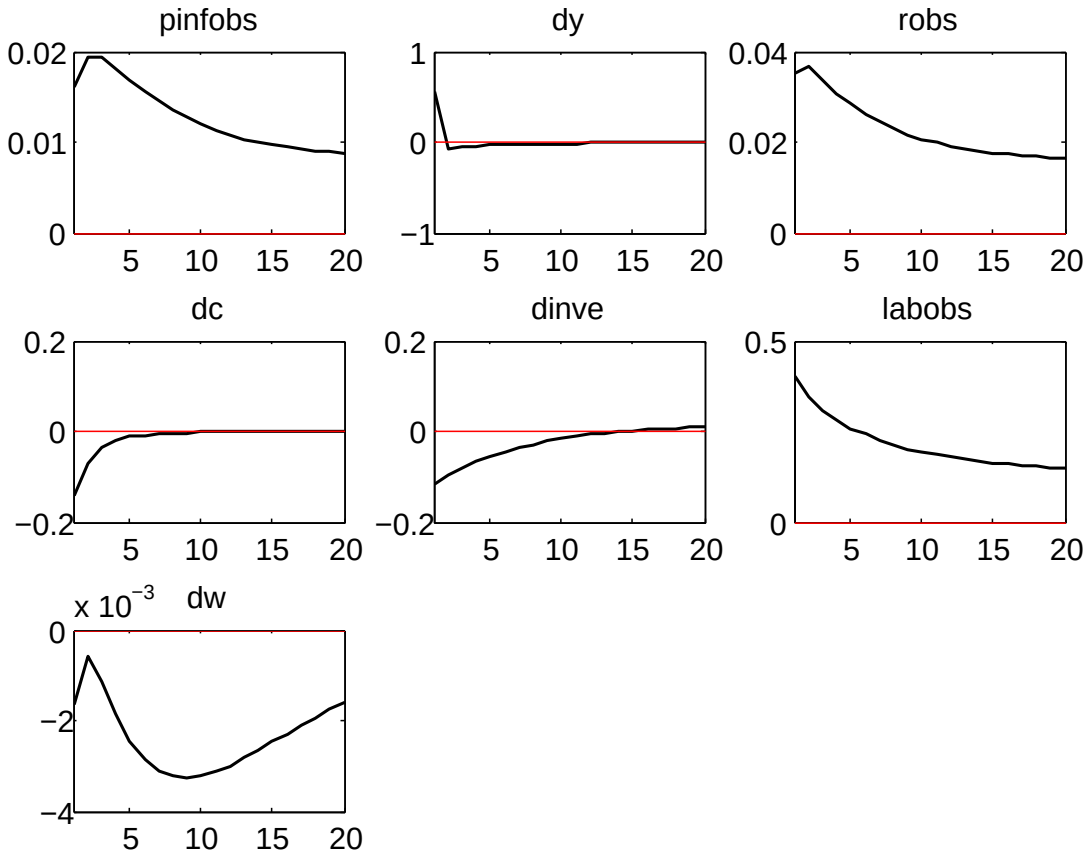

Figure 32: Impulse Response Functions to a shock to government expenditure

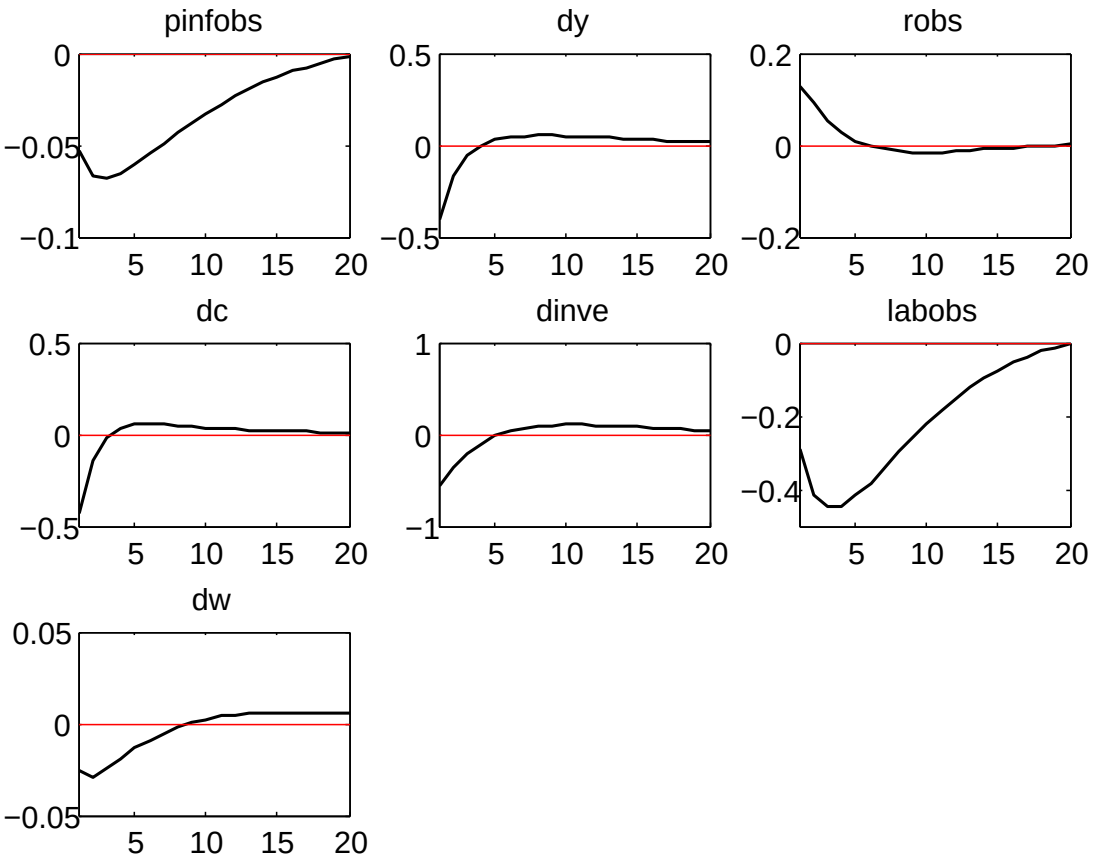

Figure 33: Impulse Response Functions to a monetary policy shock

Technical Appendix — Not for Publication

C The model

The model is described in detail in Smets and Wouters (2007) . This is a review, for convenience. Households maximize the sum of the discounted flow of per-period utility with respect to a consumption good, labor, investment, effective capital and capital utilization. The utility function has external habits. Households can transfer resources in the future by investing in physical capital or by buying government bonds. Capital adjustments come at a cost that depends on the investment growth.

Final good producers use a continuum with mass 1 of imperfectly substitutable intermediate goods to produce a final consumption good. They solve a static problem in which they maximize profits subject to a production function and a zero-profit condition. They take the price of the final good and of the intermediate goods as given. Intermediate firms maximize the sum of the discounted flow of profits assuming a Calvo price setting with partial indexation to inflation subject to the demand of intermediate goods by the final good producers. They produce intermediate goods using capital and labor.

Households provide labor to the labor unions. Labor unions sell labor to labor packers that resell the labor to the intermediate firms. Labor packers operate in a competitive market. They buy labor from unions and repackage it according to a Dixit-Stiglitz aggregator. They maximize profits taking as given the intermediate wage and the final wage and subject to a zero profit condition. Labor unions maximize the sum of the discounted flow of profits assuming a Calvo price setting with partial indexation to inflation subject to the demand of labor by the labor packers.

Finally, a Taylor rule and a stochastic process for government expenditure