The Fiscal Multiplier*

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Abstract

We measure the size of the fiscal multiplier using a heterogeneous agents model with incomplete markets, capital and rigid prices and wages. This environment captures all elements that are considered essential for a quantitative analysis. First, output is (partially) demand determined due to pricing frictions in product and labor markets, so that a fiscal stimulus increases aggregate demand. Second, incomplete markets deliver a realistic distribution of the marginal propensity to consume across the population, whereas all households counterfactually behave according to the permanent income hypothesis if markets are complete. Here, poor households feature high MPCs and thus tend to spend a large fraction of the additional income that arises as a result of a fiscal stimulus, assigning a quantitatively important role to the standard textbook Keynesian cross logic. Interestingly, and unlike conventional wisdom would suggest, our dynamic forward looking model reinforces this channel significantly. Third, the model features a realistic wealth to income ratio since we allow two assets, government bonds and capital.

We find that market incompleteness plays the key role in determining the size of the fiscal multiplier, which is about 1.35 if deficit financed and about 0.6 if tax financed. Surprisingly, the size of fiscal multiplier remains similar in the Great Recession where the economy was in a liquidity trap. Finally, we elucidate the differences between our heterogeneous-agent incomplete-markets model to those featuring complete markets or hand-to-mouth consumers.

Keywords: Fiscal Multiplier, Incomplete Markets, Sticky Prices, Liquidity Trap

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1 Introduction

In an attempt to stabilize the economy during the Great Recession monetary authorities lowered nominal interest rates to nearly zero and fixed them at that level for a long time. Having reached the limit of traditional monetary policy, U.S. legislators stepped in with the largest fiscal stimulus since the 1930s. Almost a trillion dollars was to be spent by the government, much of it early on, but also with significant spending budgeted to future years up to 2019.

Although attempts to stabilize the economy through fiscal spending occur in virtually every recession, the questions of how much and through which channels an increase in government spending affects output, employment and investment are classic, but the answers are far from being settled. The traditional logic describing the effects of these policies is well known. A government spending stimulus increases aggregate demand which leads to higher labor demand and thus more employment and higher wages. Higher labor income then stimulates consumption, in particular of poor households, which leads to even higher aggregate demand, and thus higher employment, higher labor income, more consumption and so on. The equilibrium impact of an initial government spending of $1 on output - the fiscal multiplier - is then the sum of the initial increase in government spending and the induced private consumption response.

This simple argument is based on two essential elements which ensure that the stimulus has a *direct* impact on output and employment as well as an *indirect* multiplier effect on private consumption. The first element is that output is demand determined, which ensures that the increase in government spending stimulates aggregate demand. The typical underlying assumption is that prices are rigid so that firms adjust quantities and not only prices as a response to increased government demand. Firms increase production to satisfy this demand by raising employment and wages, which leads to higher household income. We name this demand and associated output stimulus through an increase in government spending the direct effect. It differs from the full equilibrium effect in that it keeps prices and taxes unchanged, and, most importantly, does not take into account indirect multiplier effects which arise from higher private consumption.

The second element is a significant deviation from the permanent income hypothesis, such that households have a high marginal propensity to consume (MPC) out of the transitory increase in income induced by the stimulus, generating a nontrivial indirect effect. Higher private consumption due to the direct effect then leads to more labor demand, higher labor income and again more consumption and so on.

While this simple logic may be intuitively appealing, a quantitative assessment of a stimulus policy requires both elements to be disciplined by the empirical behavior of households and firms to determine the size of the direct or indirect effects. This requires a model that, first, features the right amount of nominal rigidities, so that the aggregate demand channel is as in the data. And, second, it requires incorporating observed marginal propensities to consume which imply a substantial deviation from the permanent income hypothesis.

In this paper we measure the size of the fiscal multiplier in a dynamic equilibrium model featuring these two elements disciplined by the observed behavior in micro data. Specifically, we extend the standard Bewley-Imrohoroglu-Huggett-Aiyagari model to include New-Keynesian style nominal price and wage rigidities. Introducing incomplete markets allows the model to match the rich joint distribution of income, earnings and wealth. Such heterogeneity is crucial in generating a realistic distribution of MPCs and, more generally, for assessing the effects of policies that induce redistribution. The nominal rigidities allow for the model to have a meaningful demand channel operating.

Clearly, "the fiscal multiplier" is not a single number – its size crucially depends on how it is financed (debt, distortionary taxation, reduction of transfers), how persistent fiscal policy is, what households and firms expect about future policy changes, and whether spending is increased or transfers are directed to low-income households. These important details can be incorporated in the model but are difficult to control for in empirical studies. Perhaps it is due to these difficulties that, despite the importance of this research question, no consensus on the size of the multiplier has been reached and findings come with substantial uncertainty (see Ramey, 2011, for a survey).[1] Of course, we are not the first to attempt to sidestep these difficulties faced in empirical work by relying on a more theoretical approach. Instead, our contribution is to assess the fiscal multiplier using a model that simultaneously features a

[1] Most of the empirical studies use aggregate data to measure the strength of the fiscal multiplier, which range from around 0.6 to 1.8, although "reasonable people can argue, however, that the data do not reject 0.5 or 2.0" (see Ramey, 2011). Another more recent branch of the literature looks at cross-state evidence and typically finds larger multipliers. However, as Ramey (2011) and Farhi and Werning (2013) have pointed out, the size of the local multipliers found in those studies may not be very informative about the magnitude of aggregate multipliers. For example, the local multiplier could be 1.5 whereas the aggregate multiplier is 0.

demand channel and a realistic consumption response to changes in income.[2]

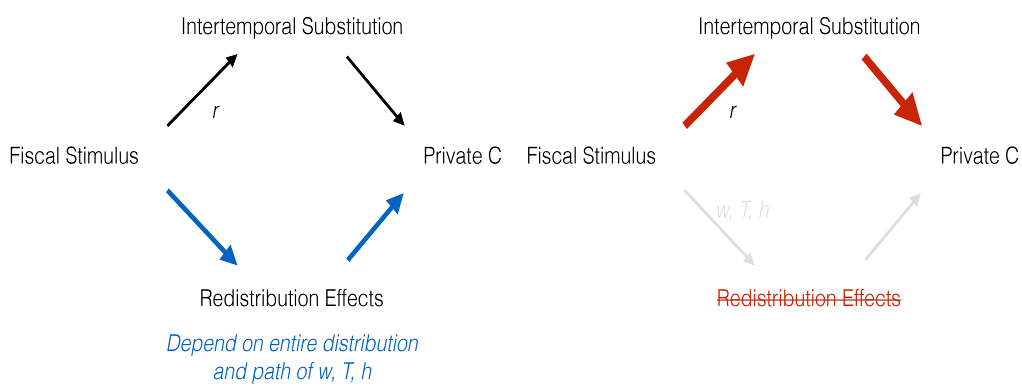
One strand of the existing literature assumes flexible prices and thus eliminates the demand channel. An early example is Baxter and King (1993) who used a representative agent model. Later contributions with heterogeneous agents and incomplete markets include Heathcote (2005) and Brinca et al. (2016). This framework is limited in its ability to provide a full assessment as only the supply but not the demand channel is operative, that is the first essential element is not present.

Another strand of the literature uses New Keynesian models with sticky prices and wages to compute the fiscal multiplier, e.g. Christiano et al. (2011). Nominal rigidities provide a role for the demand channel but now the second essential element is missing because in existing models used for the analysis of fiscal stimulus households are assumed to be representative agents. Such households behave exactly like permanent-income ones and there is no heterogeneity in the marginal propensity to consume. Further, the MPC in response to a temporary shock is small, which stands in the face of the findings of a large empirical literature that has documented substantial MPC heterogeneity and large consumption responses to transitory income and transfer payments. More generally, the consumption block embedded in the Representative Agent New Keynesian (RANK) model focuses on intertemporal substitution of consumption only, whereas the data assign only a small role to such considerations (Kaplan and Violante, 2014; Kaplan et al., 2016).

In our model the fiscal multiplier operates through two channels — intertemporal substitution and redistribution — with interesting interactions. The intertemporal substitution channel describes how government spending changes real interest rates and how this changes private consumption. The strength of this channels depends first on the magnitude of the response of real interest rates and second on how this change in real interest rates affects private consumption. The redistribution channel describes the distributional consequences of changes in prices, income, taxes etc. induced by government spending. The strength of this channel depends on the magnitude of the changes in response to spending, and on how that

[2]There is a growing literature which incorporates nominal rigidities into incomplete markets models, for example Oh and Reis (2012), Guerrieri and Lorenzoni (2015), Gornemann et al. (2012), Kaplan et al. (2016), Auclert (2016) and Lütticke (2015), McKay and Reis (2016), McKay et al. (2015), Bayer et al. (2015), Ravn and Sterk (2013) and Den Haan et al. (2015), but we are not aware of any contribution in this literature which considers fiscal multipliers.

redistribution affects private consumption. Here it is important that the response of labor earnings is in line with the data for at least two reasons. First, for asset poor workers income moves basically one-to-one with earning **WAGES !!!** Second, the profits of firms move roughly inversely with wages in response to a demand stimulus. Introducing wage rigidities to match the empirical properties of wages bounds the volatility of profits in the model, which is crucial for policy evaluation as the distributional effects arising from the distribution of profits have first order implications. That is why we extend previous work on Heterogeneous Agents New Keynesian (HANK)-type models (that feature incomplete markets and price rigidities, but flexible wages), and allow wages to be as rigid as observed in the data. In addition, these two channels do not operate independently of each other in general equilibrium, but may reinforce or attenuate each other as changes in real interest rate have distributional effects, and redistribution itself affects the equilibrium real interest rate. Panel a) of Figure 1 illustrates the two channels in incomplete markets. In contrast Panel b) of Figure 1 shows the mechanism in complete markets. In this special but standard case only the intertemporal channel is operative.



(a) Incomplete Markets (b) Complete Markets

Figure 1: Channels of Fiscal Stimulus

Our paper is the first to quantify the size of those channels in a model where both are present in a meaningful way and to compare the results to the standard model with complete markets. On the one hand, the theoretical findings in Hagedorn (2016, 2018) imply that the response of inflation and real interest rates to changes in government spending are smaller in incomplete markets than in complete market models, suggesting that the multiplier is smaller here. This is a consequence of the result that incomplete markets combined with fiscal policy specified partially in nominal terms delivers a globally determined price level independently of how monetary policy is specified. Kaplan et al. (2016) show that a given change in real interest rate has smaller effects in incomplete market models than in complete market models. Both arguments together - a smaller response and a smaller impact of real interest rates - imply that the intertemporal channel is weaker here than if markets were complete. On the other hand, the redistributional channel is larger in incomplete market models (as it is absent in complete markets), suggesting that the multiplier is larger here than in complete market models.

Our quantitative analysis combines both channels and their interaction in equilibrium. We find that the impact multiplier of an increase in government spending when the nominal interest rate is pegged is equal to 0.6 if spending is tax financed and 1.35 if it is deficit financed. We then apply our model to assess the size of the fiscal multiplier in a liquidity trap, a question that has received renewed interest in the aftermath of the Great Recession. We therefore engineer a liquidity trap where the natural real interest rate falls below zero and is consistent with salient aggregate dynamics during the Great Recession. The results from the benchmark analysis are relatively little changed. The impact multiplier is now about 0.66 for tax financed and 1.29 for deficit financed spending, implying that the multiplier is not state-dependent in our non-linear model. While this lack of state dependence is also a feature of standard RANK models, two stark differences to the complete markets case require an explanation: the size if the multiplier depends on how it is financed, and the multiplier is smaller and within a reasonable range (Ramey, 2011). The dependence on the type of financing and specifically deficit spending being more effective in stimulating the economy than tax financing is not surprising in models where Ricardian equivalence is violated. Increasing spending and taxes at the same time first stimulates demand but then offsets it through raising taxes which also affects high MPC households. In contrast, with deficit financing, the newly issued debt is mainly bought by low MPC households whereas high MPC households consume a large fraction of their additional income. Deficit financing thus implicitly redistributes from asset-rich households with low MPC who finance their consumption more from asset income to low-asset households with high MPC who rely more on labor income so that the aggregate MPC increases.

The reason why we find smaller multipliers in a liquidity trap is that the response of the real interest rate is much smaller in our model than in complete markets models. As a result the magnitude of the intertemporal channel is quite small in our model whereas it is much larger if markets are complete. Our preferred decomposition of the strength of the two channels shows that the intertemporal channel contributes $0.84(0.9)$ and the redistributional channel contributes $0.51(-0.3)$ to the multiplier of $1.35(0.6)$ if deficit (tax) financed with a similar decomposition in a liquidity trap. The multiplier is high on impact but dies out quite quickly so that the cumulative multiplier, which is the discounted average multiplier over time, falls to 0.5 if spending is tax financed and 1.2 if it is deficit financed.

We also investigate whether some of the multiplier puzzles that have been documented for RANK models carry over to our model. In RANK models, the multiplier increases if prices become more flexible and is unbounded when price rigidities vanish implying a discontinuity at fully flexible prices where the multiplier is smaller than one. The reason is that the inflation response is larger when prices are more flexible and that the private consumption response is one-to-one related to the inflation rate since only the intertemporal substitution channel is operating in RANK models. In contrast, we find that the multiplier in a liquidity trap gets smaller if prices become more flexible and that the discontinuity at fully flexible prices disappears. Again the muted intertemporal substitution channel in combination with different responses of inflation and real interest rates explains our findings. In particular we do not find large deflations as we see in liquidity traps in RANK models.

Farhi and Werning (2016) show that in complete markets New Keynesian models the further the spending is in the future the larger is the impact, suggesting that “forward-spending” is an effective fiscal policy tool. Again the muted response of inflation and real interest rates in our model leads to a different conclusion. When considering the effect of a pre-announced anticipated spending increase, we find this “forward-spending” to be less effective than an unexpected stimulus. For example, the cumulative multiplier for spending pre-announced four quarters in advance is 0.45 if it is tax financed and 1.3 if it is deficit financed. The contemporaneous stimulus is even more effective than the anticipated one since firms raise prices immediately in anticipation of future higher demand which leads to output losses before the actual policy is implemented.

Our results also indicate that one should be cautious about proposals that for government spending to be effective it has to be large (). Interpreting higher effectiveness as a higher multiplier, we find that scaling up the stimulus decreases its effectiveness. The multiplier is decreasing in the size of the spending stimulus.

The benchmark analysis isolates the effects of fiscal policy by assuming a nominal interest fixed at zero. When we deviate from this assumption and assume that monetary policy is described by an interest rate feedback rule instead, the multiplier drops from 1.35 to 0.62 for deficit financing. The interest rate rule translates the output and price increases into higher nominal and real interest rates, which contract demand through the intertemporal substitution channel. In addition, higher real rates redistribute towards asset-rich households which are the low MPC households, implying a further contraction in aggregate demand and a smaller multiplier than in complete markets models.

Our incomplete markets model also allows us to conduct a meaningful analysis of transfer multipliers, which is an important objective as many stimulus policies take the form of transfers and not an increase in spending. We find that a 1$ deficit-financed increase in lump-sum transfers increases output on impact by 71 cents. We also use the theoretical model to compute the welfare consequences of temporary increases in government spending and in transfer payments. This exercise is more interesting than in a complete markets environment since the welfare gains of high MPC households may outweigh the losses of low MPC (rich) households.

Finally, we compare our findings to those from a Two-Agent New Keynesian (TANK) model where one fraction of households is hand-to-mouth and the other fraction behaves according to the permanent income hypothesis Bilbiie (2008, 2017). A TANK model delivers a much smaller multiplier even for the same MPC - relating current income to current consumption - than our HANK model. The reason for this stark difference is the different consumption response both to current and future income increases in both models. Hand-to-mouth consumers spend the full increase in current income but do not respond to increases in future income.

In contrast the consumption response is less extreme in our model. Households respond to both current and future income changes, albeit the latter response is smaller. As a result, the logic underlying the size of the multiplier is dynamic. An increase in fiscal spending leads to higher income which leads to higher private consumption demand not only today but also in all future periods. This higher path of private spending leads to a higher income path which again increases spending in all periods. In particular we find that today's consumption responds mainly because future income increases and not because of an increase in current income. Indeed, the increase in private consumption due to the increase in current income is similar to the small consumption increase in the TANK model since both models feature the same impact MPC.

We also show that our incomplete markets models with nominal fiscal policy overcomes a Catch-22 of HANK models with fiscal policy fully specified in real terms (Bilbiie, 2018). TANK models generate large multipliers but at the same time feature the same puzzles as RANK models. As Bilbiie (2018) shows, HANK models can resolve the puzzles but then the multiplier is small. Other calibrations can generate large multipliers but then the puzzles are even aggravated, a Catch-22. Different form Bilbiie, government bonds are nominal in our HANK model, which implies price level determinacy and that the puzzles are resolved and at the same time multipliers can be, depending on the distributional channel and the type of financing, large.

The paper is organized as follows. Section 2 presents our incomplete markets model with price and wage rigidities. In Section 3 we study the size of the government-spending multiplier both for an interest rate peg and when monetary policy is described by a Taylor rule. Section 4 concludes.

2 Model

The model is a standard New Keynesian model with one important modification: Markets are incomplete as in Aiyagari (1994, 1995). Price setting faces some constraints as price adjustments are costly as in Rotemberg (1982) leading to price rigidities. As is standard in the New Keynesian literature, final output is produced in several intermediate steps. Final good producers combine the intermediate goods to produce and sell their output in a competitive goods market. Intermediate goods producers are monopolistically competitive. They set a price they charge to the final good producer to maximize profits taking into account the price adjustment costs they face. The intermediate goods producer rent inputs, capital and a composite of differentiated labor, in competitive factor markets. We also allow for sticky wages and assume that differentiated labor is monopolistically supplied as well.

2.1 Households

The economy consists of a continuum of agents normalized to measure 1 who are ex-ante heterogenous with respect to their subjective discount factors, have CRRA preferences over consumption and additively separable preferences for leisure:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t),$$

where

$$u(c,h) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} - g(h) & \text{if } \sigma \neq 1 \\ \log(c) - g(h) & \text{if } \sigma = 1, \end{cases}$$

$\beta \in (0,1)$ is the household-specific subjective discount factor and $g(h)$ is the disutility of labor. Agents' labor productivity $\{s_t\}_{t=0}^{\infty}$ is stochastic and is characterized by an N-state Markov chain that can take on values $s_t \in \mathcal{S} = \{s_1, \cdots, s_N\}$ with transition probability characterized by $p(s_{t+1}|s_t)$ and $\int s = 1$. Agents rent their labor services, $h_t s_t$, to firms for a real wage w_t and their nominal assets a_t to the capital market for a nominal rent i_t^a and a real return $(1+r_t^a) = \frac{1+i_t^a}{1+\pi_t}$, where $1+\pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate (P_t is the price of the final good) . The nominal return on bonds is i_t with a real return $(1+r_t) = \frac{1+i_t}{1+\pi_t}$. There are two classes of assets, bonds and capital with potentially different returns, but households can invest in one asset A, which the mutual fund (described below) collects and allocates to bonds and capital.

To allow for sticky wages we follow the literature and assume that each household i provides differentiated labor services, h_{it}. These differentiated labor services are transformed by a representative, competitive labor recruiting firm into an aggregate effective labor input, H_t, using the following technology:

$$H_t = \left(\int_0^1 s_{it} (h_{it})^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{1}$$

where ϵ_w is the elasticity of substitution across labor services.

A middleman firm (e.g. a union) sells households labor services to the labor recruiter, which given aggregate labor demand H_t by the intermediate goods sector, minimizes costs

$$\int_0^1 W_{it} s_{it} h_{it} di, \tag{2}$$

implying a demand for the labor services of household i:

$$h_{it} = h(W_{it}; W_t, H_t) = \left(\frac{W_{it}}{W_t}\right)^{-\epsilon_w} H_t, \tag{3}$$

where W_t is the (equilibrium) nominal wage which can be expressed as

$$W_t = \left(\int_0^1 s_{it} W_{it}^{1-\epsilon_w} di\right)^{\frac{1}{1-\epsilon_w}}.$$

The union sets a nominal wage $\hat{W}_t$ for an effective unit of labor (so that $W_{it} = \hat{W}_t$) to maximize profits subject to wage adjustment costs modeled similarly to the price adjustment costs in Rotemberg (1982). These adjustment costs are proportional to idiosyncratic productivity s_{it}, are measured in units of aggregate output, and are given by a quadratic function of the change in wages above and beyond steady state wage inflation $\overline{\Pi}^w$,

$$\Theta\left(s_{it}, W_{it} = \hat{W}_t, W_{it-1} = \hat{W}_{t-1}; H_t\right) = s_{it}\frac{\theta_w}{2}\left(\frac{W_{it}}{W_{it-1}} - \overline{\Pi}^w\right)^2 H_t = s_{it}\frac{\theta_w}{2}\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w\right)^2 H_t.$$

The union's wage setting problem is to maximize[3]

$$\begin{aligned} V_t^w\left(\hat{W}_{t-1}\right) &\equiv \max_{\hat{W}_t} \int \left(\frac{s_{it}(1-\tau_t)\hat{W}_t}{P_t} h(\hat{W}_t; W_t, H_t) - \frac{g(h(\hat{W}_t; W_t, H_t))}{u'(C_t)}\right) di \\ &- \int s_{it}\frac{\theta_w}{2}\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w\right)^2 H_t di + \frac{1}{1+r_t} V_{t+1}^w\left(\hat{W}_t\right), \end{aligned} \tag{4}$$

where C_t is aggregate consumption and τ_t is a proportional tax on labor income.

Some algebra (see the appendix) yields, using $h_{it} = H_t$ and $\hat{W}_t = W_t$ and defining the real wage $w_t = \frac{W_t}{P_t}$, the wage inflation equation

$$\theta_w\left(\pi_t^w - \overline{\Pi}^w\right)\pi_t^w = (1-\tau_t)(1-\epsilon_w)w_t + \epsilon_w \frac{g'(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} + \frac{1}{1+r_t}\theta_w\left(\pi_{t+1}^w - \overline{\Pi}^w\right)\pi_{t+1}^w \frac{H_{t+1}}{H_t}. \tag{5}$$

The wage adjustment process does not involve actual costs but is as-if those costs were actually present. We make this assumption to avoid significant movements of these adjustment costs in response to e.g. a fiscal stimulus or in a liquidity trap. Such swings would matter in our

[3] Equivalently one can think of a continuum of unions each setting the wage for a representative part of the population with $\int s = 1$ at all times.

incomplete markets model and might yield quite different implications from price setting à la Calvo.

Thus, at time t an agent faces the following budget constraint:

$$P_t c_t + a_{t+1} = (1 + i_t^a) a_t + (1 - \tau_t) P_t w_t h_t s_t + T_t$$

where T_t is a nominal lump sum transfer. Households take prices, wages and hours h from the middleman's wage setting problem as given. Thus, we can rewrite the agent's problem recursively as follows:

$$\begin{aligned} V(a, s, \beta; \Omega) = \max_{c \geq 0, a' \geq 0} & u(c, h) + \beta \sum_{s \in \mathcal{S}} p(s'|s) V(a', s', \beta; \Omega') \\ \text{subj. to} \quad & Pc + a' = (1 + i^a) a + P(1 - \tau) w h s + T \\ & \Omega' = \Upsilon(\Omega) \end{aligned} \tag{6}$$

where $\Omega(a, s, \beta)$ is the distribution on the space $X = A \times S \times \mathcal{B}$, agents asset holdings $a \in A$, labor productivity $s \in S$ and discount factor $\beta \in \mathcal{B}$, across the population, which will together with the policy variables determine the equilibrium prices. Let $\mathbb{B}(X) = \mathbb{A} \times \mathcal{P}(S) \times \mathcal{P}(\mathcal{B})$ be the σ-algebra over X, defined as the cartesian product over the Borel σ-algebra on A and the power sets of S and $\mathcal{B}$. Define our space $M = (X, \mathbb{B}(X))$, and let $\mathcal{M}$ be the set of probability measures over M. Υ is an equilibrium object that specifies the evolution of the distribution Ω.

2.2 Production

2.2.1 Intermediate-Goods Firms

Each intermediate good j is produced by a monopolistically competitive producer using the technology:

$$Y_{jt} = \begin{cases} K_{jt}^{\alpha} H_{jt}^{1-\alpha} - F & \text{if} \geq 0 \\ 0 & \text{otherwise} \end{cases}, \tag{7}$$

where $0 < \alpha < 1$, K_{jt} is capital services rented, H_{jt} is labor services rented and the fixed cost of production are denoted $F > 0$.

Intermediate-goods firms rent capital and labor in perfectly competitive factor markets.

Profits are fully taxed by the government. A firm's real marginal cost is $mc_{jt} = \partial S_t(Y_{jt})/\partial Y_{jt}$, where

$$S_t(Y_{jt}) = \min_{K_{jt},H_{jt}} r_t^k K_{jt} + w_t H_{jt}, \text{ where } Y_{jt} \text{ is given by (7)} \tag{8}$$

Given our functional forms, we have

$$mc_t = \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} (r_t^k)^{\alpha}(w_t)^{1-\alpha} \tag{9}$$

and

$$\frac{K_{jt}}{H_{jt}} = \frac{\alpha w_t}{(1-\alpha) r_t^k} \tag{10}$$

Prices are sticky as intermediate-goods firms face Rotemberg (1982) price adjustment costs. Given last period's individual price p_{jt-1} and the aggregate state $(P_t, Y_t, Z_t, w_t, r_t)$, the firm chooses this period's price p_{jt} to maximize the present discounted value of future profits, satisfying all demand. The intermediate goods firm's pricing problem is

$$V_t^{IGF}(p_{jt-1}) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t} y\left(p_{jt}; P_t, Y_t\right) - S(y(p_{jt}; P_t, Y_t)) - \frac{\theta}{2}\left(\frac{p_{jt}}{p_{jt-1}} - \overline{\Pi}\right)^2 Y_t - F + \frac{1}{1+r_t} V_{t+1}^{IGF}(p_{jt}),$$

where F are fixed operating costs.

Some algebra (in the appendix) yields the New Keynesian Phillips Curve

$$(1-\epsilon) + \epsilon mc_t - \theta\left(\pi_t - \overline{\Pi}\right)\pi_t + \frac{1}{1+r_t}\theta\left(\pi_{t+1} - \overline{\Pi}\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t} = 0.$$

The equilibrium real profit of each intermediate goods firm is then

$$d_t = Y_t - F - S(Y_t).$$

2.2.2 Final Good Producer

A competitive representative final goods producer aggregates a continuum of intermediate goods indexed by $j \in [0,1]$ and with prices p_{jt}:

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}.$$

where ϵ is the elasticity of substitution across goods. Given a level of aggregate demand Y_t, cost minimization for the final goods producer implies that the demand for the intermediate good j is given by

$$y_{jt} = y(p_{jt}; P_t, Y_t) = \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} Y_t, \tag{11}$$

where P is the (equilibrium) price of the final good and can be expressed as

$$P_t = \left(\int_0^1 p_{jt}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}.$$

2.2.3 Mutual Fund

The mutual fund collects households savings A_{t+1}/P_{t+1} and pays a real return $\tilde{r}_t^a$ and invests them in real bonds B_{t+1}/P_{t+1} and capital K_{t+1}. It maximizes

$$V^{MF}(K_t) \equiv \max_{K_{t+1}, \frac{B_{t+1}}{P_{t+1}}} (1 + r_{t+1}^k - \delta)K_{t+1} + (1 + r_{t+1})B_{t+1}/P_{t+1} - (1 + \tilde{r}_{t+1}^a)(A_{t+1}/P_{t+1}) + \frac{V^{MF}(K_{t+1})}{(1 + \tilde{r}_{t+2}^a)}$$

such that $A_{t+1}/P_{t+1} = K_{t+1} + B_{t+1}/P_{t+1} + \Phi(K_{t+1}, K_t)$ and for adjustment costs $\Phi(K_{t+1}, K_t)$, taking K_t and K_{t+2} as given. The equilibrium first-order conditions are

$$\begin{aligned}
r_{t+1} &= \tilde{r}_{t+1}^a, \\
1 + r_{t+1}^k - \delta &= (1 + \tilde{r}_{t+1}^a)(1 + \Phi_1(K_{t+1}, K_t)) + \Phi_2(K_{t+2}, K_{t+1}), \\
A_{t+1}/P_{t+1} &= K_{t+1} + B_{t+1}/P_{t+1} + \Phi(K_{t+1}, K_t).
\end{aligned}$$

The total dividends of the fund are

$$D_{t+1}^{MF} = (1 + r_{t+1}^k - \delta)K_{t+1} + (1 + r_{t+1})B_{t+1}/P_{t+1} - (1 + \tilde{r}_{t+1}^a)(A_{t+1}/P_{t+1}),$$

and after-tax per unit of investment are $d_{t+1}^{MF} = (1-\tau_k)D_{t+1}^{MF}/(A_{t+1}/P_{t+1})$. Households therefore receive (or have to pay) $d_{t+1}^{MF} A_{t+1}/P_{t+1}$ in period $t+1$ per unit invested such that households' real return equals

$$1 + r_{t+1}^a = 1 + \tilde{r}_{t+1}^a + d_{t+1}^{MF}$$

and corresponding nominal return

$$1 + i^a_{t+1} = (1 + r^a_{t+1})\frac{P_{t+1}}{P_t}.$$

2.3 Government

The government obtains revenue from taxing labor income, issuing bonds and taxing profits and dividends. Household labor income wsh is taxed progressively with a nominal lump-sum transfer T_t and a proportional tax τ:

$$\tilde{T}(wsh) = -T + \tau P wsh.$$

The government issues nominal bonds denoted by B^g, with negative values denoting government asset holdings and fully taxes profits of intermediate goods firms away, obtaining nominal revenue Pd. The government also taxes dividend income at the rate τ_k. The government uses the revenue to finance exogenous nominal government expenditures, G_t, interest payments on bonds and transfers to households. The government budget constraint is therefore given by:

$$B^g_{t+1} = (1 + i_t)B^g_t + G_t - P_t d_t - \tau_k P_t D^{MF}_t - \int \tilde{T}_t(w_t s_t h_t) d\Omega. \tag{12}$$

2.4 Equilibrium

Market clearing requires that the labor demanded by the firm is equal to the aggregate labor supplied by households, that the demand for bonds issued by the government and capital equal their supplies and that the amount of assets provided by households equals their demand by the mutual fund:

$$K_{t+1} + B_{t+1}/P_{t+1} + \Phi(K_{t+1}, K_t) = A_{t+1}/P_{t+1} = \int \frac{a_{t+1}(a_t, s_t, \beta_t)}{P_{t+1}} d\Omega_t \tag{13}$$

$$B_t = B^g_t \tag{14}$$

$$K_t = \int K_{jt} dj \tag{15}$$

$$H_t = \int H_{jt} dj = H_{jt} = \int h_{it} di = h_{it} \tag{16}$$

where $a_{t+1}(a_t, s_t, \beta_t)$ is the asset choice of an agent with asset level a_t, labor productivity s_t and discount factor β_t.

Definition: A monetary competitive equilibrium is a sequence of tax rates τ_t and τ_k, nominal transfers T_t, nominal government spending G_t, supply of government bonds B_t^g, value functions $v_t : X \times \mathcal{M} \to \mathcal{R}$ with policy functions $a_t : X \times \mathcal{M} \to \mathcal{R}_+$ and $c_t : X \times \mathcal{M} \to \mathcal{R}_+$, hours choices $H_t, H_{jt}, h_{it} : \mathcal{M} \to \mathcal{R}_+$, capital decisions $K_t, K_{jt} : \mathcal{M} \to \mathcal{R}_+$, bond choices $B_t : \mathcal{M} \to \mathcal{R}_+$, price levels $P_t : \mathcal{M} \to \mathcal{R}_+$, pricing functions $r_t, r_t^a, r_t^k, \tilde{r}_t^a : \mathcal{M} \to \mathcal{R}$ and $w_t : \mathcal{M} \to \mathcal{R}_+$, and a law of motion $\Upsilon : \mathcal{M} \to \mathcal{M}$, such that:

1. v_t satisfies the Bellman equation with corresponding policy functions a_t and c_t given price sequences $r_t^a(), w_t()$ and hours h_t.

2. Firms maximize profits taking prices P_t, r_t^k, w_t as given.

3. Wages are set optimally by middlemen.

4. The mutual fund maximizes profits taking prices as given.

5. For all $\Omega \in \mathcal{M}$:

$$
\begin{aligned}
K_{t+1} + B_{t+1}/P_{t+1} + \Phi(K_{t+1}, K_t) &= \int \frac{a_{t+1}(a_t, s_t, \beta_t)}{P_{t+1}} d\Omega_t, \\
B_t &= B_t^g \\
K_t &= \int K_{jt} dj \\
H_t &= \int H_{jt} dj = H_{jt} = \int h_{it} di = h_{it} \\
Y_t = K_t^\alpha H_t^{1-\alpha} &= \int c_t(a_t, s_t, \beta_t) d\Omega_t + \frac{G_t}{P_t} + F + K_{t+1} \\
&\quad -(1-\delta)K_t + \Phi(K_{t+1}, K_t).
\end{aligned}
$$

6. Aggregate law of motion Υ generated by a' and p.

3 The Fiscal Multiplier

In this Section we calculate the fiscal multiplier in our model with incomplete markets, conducting the following experiment. Assume that the economy is in steady state with nominal bonds B_{ss}, government spending G_{ss}, transfers T_{ss} and a tax rate τ_{ss} and where the price level is P_{ss}. The real value of bonds is then B_{ss}/P_{ss}, the real value government expenditure is G_{ss}/P_{ss} and so on. We then consider an M.I.T. (unexpected and never-again-occurring) shock to government expenditures and compute the impulse response to this persistent innovation in G. Eventually the economy will reach the new steady state characterized by government bonds $B_{ss}^{new} = B_{ss}$, government spending $G_{ss}^{new} = G_{ss}$, transfers $T_{ss}^{new} = T_{ss}$, a tax rate $\tau_{ss}^{new} = \tau_{ss}$ and the price level is P_{ss}^{new}.

3.1 The Fiscal Multiplier in Incomplete Market Models

In our model the fiscal multiplier operates through two channels — intertemporal substitution and redistribution — with interesting interactions. The intertemporal substitution channel describes how government spending changes real interest rates and how this changes private consumption. The distributional channel describes how government spending changes prices, income, taxes etc., the redistributional consequences of these changes and the resulting impact on private consumption. In addition to capturing the relevant transmission mechanisms for fiscal policy, our HANK framework also delivers price level determinacy (Hagedorn, 2016, 2018). This allows us to study arbitrary combinations of monetary and fiscal policy, in particular a constant nominal interest rate as prevails at the zero lower bound. We do not face the indeterminacy issues with the representative-agent New Keynesian model at the ZLB raised by Cochrane (2015) and have a uniquely determined fiscal multiplier at the ZLB, one of the cases where we're most interested in knowing its size.

We now explain the role of these two channels in determining the fiscal multiplier and how they interact with price level determinacy in our model, what determines their strength and explain the differences to complete markets.

3.1.1 Intertemporal Substitution Channel

To understand the workings of the intertemporal substitution channel in our model it is instructive to start with the complete markets case where this is the only channel operating. We then move to incomplete markets to explain and understand the differences.

The size of the multiplier m is determined by the response of the real interest rates only. The Consumption Euler equation for our utility function, $\frac{C^{1-\sigma}}{1-\sigma} + ...$, is

$$C_t^{-\sigma} = \beta(1 + r_{t+1})C_{t+1}^{-\sigma}. \tag{17}$$

Iterating this equation and assuming that consumption is back to the steady-state level at time T, $C_T = C_{ss}$, we obtain for consumption at time $t = 1$ when spending is increased,

$$C_1^{-\sigma} = \Big(\prod_{t=1}^{T-1} \big(\beta(1 + r_{t+1})\Big)C_T^{-\sigma}, \tag{18}$$

so that the initial percentage increase in consumption equals

$$\frac{C_1}{C_{ss}} = \Big(\prod_{t=1}^{T-1} \beta(1 + r_{t+1})\Big)^{\frac{-1}{\sigma}} = \Big(\prod_{t=1}^{T-1} \frac{1 + r_{t+1}}{1 + r_{ss}}\Big)^{\frac{-1}{\sigma}} \tag{19}$$

where we have used that $\beta(1+r_{ss}) = 1$ in a complete markets steady state. The fiscal multiplier m - the dollar change in output for each dollar increase in g - is one-to-one related to the percentage change in private consumption

$$m = 1 + \Big(\frac{C_1}{C_{ss}} - 1\Big)\frac{C_{ss}}{\Delta g} \tag{20}$$

and is thus one-to-one related to the accumulated response of real interest rate which is induced by the fiscal stimulus,

$$m = 1 + \frac{C_{ss}}{\Delta g}\Bigg(\Big(\prod_{t=1}^{T-1} \frac{1 + r_{t+1}}{1 + r_{ss}}\Big)^{\frac{-1}{\sigma}} - 1\Bigg). \tag{21}$$

The log of the multiplier is the proportional to

$$\log(m) \approx \underbrace{\frac{1}{\sigma}}_{\text{Intertemporal Substitution}} \sum_{t=1}^{T-1} \underbrace{(\log(1+r_{ss}) - \log(1+r_{t+1})}_{\text{Change of real interest rates}} -1, \tag{22}$$

which can be decomposed in the change in the real interest rate, $\approx r_t - r_{ss}$, and the effect of this change on consumption, whose strength is governed by the IES, $\frac{1}{\sigma}$.

Both components of the intertemporal substitution channel are weaker in incomplete markets models. The effect of the real interest rate on consumption is smaller since some households are credit constrained and thus not on their Euler equation, breaking the tight link between consumption and real interest rates. Also the change in real interest rates is smaller. To understand the difference assume for simplicity that the nominal interest rate is fixed at i_{ss} and that the steady state is reached after T periods such that

$$\prod_{t=1}^{T-1}(1+r_{t+1}) = \prod_{t=1}^{T-1}\left(\frac{1+i_{ss}}{1+\pi_{t+1}}\right) = \prod_{t=1}^{T-1}(1+i_{ss})\frac{P_T}{P_1} = \prod_{t=1}^{T-1}(1+i_{ss})\frac{P_{ss}^{new}}{P_1}, \tag{23}$$

so that the response of $\prod_{t=1}^{T-1}(1+r_{t+1})$ is one-to-one related to the response of P_{ss}^{new}/P_1. This response is quite large in complete market models (Christiano et al., 2011) but small here.

Both results are a consequence of the result that incomplete markets combined with fiscal policy specified partially in nominal terms delivers a globally determined price level independently of how monetary policy is specified (Hagedorn, 2016, 2018). We refer the reader to these papers for details and only provide the intuition for the key result that $P_{ss}^{new} = P_{ss}$. Define households' real steady-state asset demand as S. The asset demand S, the real stock of capital, K_{ss}, and the amount of nominal bonds is the same, $B_{ss}^{new} = B_{ss}$, in both steady states and therefore both price levels solve the same asset market clearing condition[4]

$$S(1+r_{ss}, ...) = K_{ss} + \frac{B_{ss}}{P_{ss}} = K_{ss} + \frac{B_{ss}^{new}}{P_{ss}} = K_{ss} + \frac{B_{ss}^{new}}{P_{ss}^{new}}. \tag{24}$$

Together these arguments imply that the intertemporal substitution channel is weaker in our incomplete markets model than in the corresponding complete markets model, where

[4] Price level determinacy is a consequence of only two empirically well-grounded assumptions, incomplete markets and nominal government bonds, and in particular does not require any further selection criteria, such as a price or inflation targeting rule.

$P_{ss}^{new} > P_{ss}$. Note that since P_1 typically increases in response to a stimulus, $P_{ss}^{new}/P_1 < 1$, and thus $\prod_{t=1}^{T-1}(1 + r_{t+1}) < \prod_{t=1}^{T-1}(1 + i_{ss})$ that is the intertemporal substitution channel by itself implies a multiplier smaller than one, $m < 1$. The multiplier here is smaller than one whereas it is equal to one in Woodford (2011) since the real interest rate is constant in Woodford (2011) but not here.[5]

3.1.2 Distributional Consequences of a Stimulus

An increase in spending, the necessary adjustments in taxes and transfers and the resulting responses of prices and hours operate through various distributional channels. Changes in the tax code naturally deliver winners and losers. An increase in the price level and of labor income leads to a redistribution from households who finance their consumption more from asset income to households who rely more on labor income. Changes in interest rates also redistribute between debtors and lenders.

These redistributions matter due to the endogenous heterogeneity in the MPCs in the data and replicated in our incomplete markets model. This heterogeneity together with the redistribution determines the aggregate consumption response, and since output is demand determined due to price rigidities, also determines output. Individual household consumption c_t depends on transfers T, tax rates τ, labor income wh, prices P and nominal interest rates i, so that aggregate private consumption

$$C_t\big(\{T_t, \tau_t, w_t h_t, P_t, i_t^a\}_{t\geq 0}\big) = \int c_t(a, s; \{T_t, \tau_t, w_t h_t, P_t, i_t^a\}_{t\geq 0}) d\Omega_t. \tag{25}$$

In our model hours are a household choice variable but demand determined as well. Of course, consumption and hours worked are jointly determined in equilibrium but to understand the demand response of the fiscal stimulus it turns out to be useful to consider wh as exogenous for consumption decisions here. In particular it allows us to distinguish between the initial impact,"first round", demand impulse due to the policy change and "second, third ... round"

[5]We also overcome a indeterminacy problem in Woodford (2011). He assumes that after a fiscal stimulus consumption converges back to its pre-stimulus level, ruling out other belief driven steady states. Since the real interest rate is constant this implies that consumption is equal to its steady-state level in all periods, implying a multiplier equal to one. If instead households believe income to change permanently by $x\%$, then consumption demand increases by $x\%$ as well, confirming the initial belief as an equilibrium outcome. There is a steady state for each $x \neq 0$, each assumed away in Woodford (2011) and who instead focuses on the $x = 0$ equilibrium. Such assumptions to ensure determinacy are not needed here.

due to equilibrium responses. Those arise in our model since an initial policy-induced demand stimulus leads to more employment by firms, and so higher labor income which in turn implies more consumption demand, which again leads to more employment and so on until an equilibrium is reached where all variables are mutually consistent. Denoting pre stimulus variables by a bar, we can now decompose the aggregate consumption response,

$$(\Delta C)_t = C_t\big(\{T_t, \tau_t, w_t h_t, P_t, i^a_t\}_{t\geq 0}\big) - C_t\big(\{\bar{T}, \bar{\tau}, \bar{w}\bar{h}, \bar{P}, \bar{i}^a\}_{t\geq 0}\big), \tag{26}$$

into its different channels:

$$\begin{aligned}
(\Delta C)_t \quad &= \quad \underbrace{C_t\big(\{T_t, \tau_t, \bar{w}\bar{h}, \bar{P}, \bar{i}^a\}_{t\geq 0}\big) - C_t\big(\{\bar{T}, \bar{\tau}, \bar{w}\bar{h}, \bar{P}, \bar{i}^a\}_{t\geq 0}\big)}_{\text{Direct Impact of Transfers and Taxes}} && (27)\\
&+ \quad \underbrace{C_t\big(\{T_t, \tau_t, w_t h_t, \bar{P}, \bar{i}^a\}_{t\geq 0}\big) - C_t\big(\{T_t, \tau_t, \bar{w}\bar{h}, \bar{P}, \bar{i}^a\}_{t\geq 0}\big)}_{\text{Indirect Equilibrium Effect: Labor Income}} && (28)\\
&+ \quad \underbrace{C_t\big(\{T_t, \tau_t, w_t h_t, P_t, i^a_t\}_{t\geq 0}\big) - C_t\big(\{T_t, \tau_t, w_t h_t, \bar{P}, \bar{i}^a\}_{t\geq 0}\big)}_{\text{Price and Interest rate Adjustment}} && (29)
\end{aligned}$$

Total demand is the sum of private consumption demand C and real government consumption $g = G/P$, which both determine output. The private consumption response does not directly depend on G/P but it does indirectly. First, transfers T and taxes τ have to adjust to balance the intertemporal government budget constraint. Second, increases in G/P translate one-for-one into increases in demand. On impact an increase by Δg increases demand by Δg and thus hours worked from $\bar{h}$ to $\bar{h} + \Delta h^g$, where Δh^g is the amount of hours needed to produce Δg while keeping the capital stock unchanged. As before, this increase in labor income stimulates private demand which in turn leads to higher employment, then again higher consumption and so on until convergence. We therefore decompose the total demand effect ΔD of an increase in government spending by Δg as

$$
\begin{aligned}
(\Delta D)_t &= \underbrace{(\Delta g)_t}_{\text{Direct Govt' Spending Response}} + \underbrace{(\Delta C)_t}_{\text{Private Consumption Response}} && (30)\\
&= (\mathbf{\Delta g})_{\mathbf{t}} + \underbrace{C_t\big(\{\bar{T}, \bar{\tau}, \bar{w}(\bar{h} + \mathbf{\Delta h^g}), \bar{P}, \bar{i}^a\}_{t\geq 0}\big) - C_t\big(\{\bar{T}, \bar{\tau}, \bar{w}\bar{h}, \bar{P}, \bar{i}^a\}_{t\geq 0}\big)}_{\text{Direct Impact on Private Consumption}} && (31)\\
&+ \underbrace{C_t\big(\{T_t, \tau_t, w_t h_t, P_t, i^a_t\}_{t\geq 0}\big) - C_t\big(\{\bar{T}, \bar{\tau}, \bar{w}(\bar{h} + \mathbf{\Delta h^g}), \bar{P}, \bar{i}^a\}_{t\geq 0}\big)}_{\text{Indirect Private Consumption Response}} && (32)
\end{aligned}
$$

A fiscal stimulus, in addition to the immediate impact on government demand, also leads to higher employment and labor income and thus stimulates private consumption, the *Direct Impact on Private Consumption*. The remainder of the private consumption is as above the sum of the direct impact of transfers and taxes, the indirect equilibrium effects of labor income and price and interest rate adjustment, such that the full decomposition of the total demand effect $(\Delta D)_t$ is

$$
\begin{aligned}
(\Delta D)_t &= \underbrace{(\Delta g)_t}_{\text{Direct G Impact}} && (33)\\
&+ \underbrace{C_t\big(\{\bar{T}, \bar{\tau}, \bar{w}(\bar{h} + \Delta h^g), \bar{P}, \bar{i}^a\}_{t\geq 0}\big) - C_t\big(\{\bar{T}, \bar{\tau}, \bar{w}\bar{h}, \bar{P}, \bar{i}^a\}_{t\geq 0}\big)}_{\text{Direct G Impact on C}} && (34)\\
&+ \underbrace{C_t\big(\{T_t, \tau_t, \bar{w}(\bar{h} + \Delta h^g), \bar{P}, \bar{i}^a\}_{t\geq 0}\big) - C_t\big(\{\bar{T}, \bar{\tau}, \bar{w}(\bar{h} + \Delta h^g), \bar{P}, \bar{i}^a\}_{t\geq 0}\big)}_{\text{Indirect Tax/Transfer Impact}} && (35)\\
&+ \underbrace{C_t\big(\{T_t, \tau_t, w_t h_t, \bar{P}, \bar{i}^a\}_{t\geq 0}\big) - C_t\big(\{T_t, \tau_t, \bar{w}(\bar{h} + \Delta h^g), \bar{P}, \bar{i}^a\}_{t\geq 0}\big)}_{\text{Indirect Labor Income Impact}} && (36)\\
&+ \underbrace{C_t\big(\{T_t, \tau_t, w_t h_t, P_t, i^a_t\}_{t\geq 0}\big) - C_t\big(\{T_t, \tau_t, w_t h_t, \bar{P}, \bar{i}^a\}_{t\geq 0}\big)}_{\text{Indirect Price and Interest Impact}}. && (37)
\end{aligned}
$$

3.1.3 Investment Channel

Investment demand is another component of aggregate demand and the strength of this channel depends both on the cost of investment - the real interest rate r^k_t - and the demand for intermediate goods. Intermediate goods firms set prices subject to Rotemberg adjustment costs and have to satisfy the resulting demand Y_{jt} for their product through hiring labor H_{jt}

or buying capital goods K_{jt}, which leads to the cost minimization condition

$$\frac{K_{jt}}{H_{jt}} = \frac{\alpha w_t}{(1-\alpha) r_t^k}. \tag{38}$$

Given prices w_t and r_t^k, a higher demand Y_{jt} leads to an increase both in capital and employment, the demand channel of investment. Higher capital costs r_t^k dampen investment demand but only if they increase more than wages. Since firm have to satisfy demand, the relative costs of the input factors matter. As an example suppose that r_t^k increases by 10% but that wages increase by 20%, so that $\frac{\alpha w_t}{(1-\alpha) r_t^k}$ increases. In this case, firms would substitute from labor to capital although capital costs have increased just because the costs of the other input factor, wages, has increased even more. Two features of our calibration make this scenario less likely. High capital adjustment costs imply a strong response of r_t^k to capital changes and wage rigidities dampen the movement of wages in response to demand fluctuations. In addition to these partial equilibrium considerations, general equilibrium requires asset market clearing, that is the real interest rate received by households has to be such that they are willing to absorb all assets supplied, bonds and capital. If the stimulus is financed by increasing government debt that could lead to higher real interest rates, since Ricardian equivalence doesn't hold here, and thus to crowding out of investment. The same redistributive forces that affect consumption behavior in turn affect the savings behavior of households. If this leads to a fall in savings, higher real interest rates are required in equilibrium, implying a drop in investment.

3.1.4 Multiplier: Definition

As we can now be sure that the fiscal multiplier is well defined in our economy, we now follow Farhi and Werning (2013) in computing the response of the economy to a fiscal stimulus.

Concretely, we compute the response of the economy to an unexpected increase in the path of nominal government spending to $G_0, G_1, G_2, \ldots, G_t, \ldots, G_{ss}$, where G_{ss} is the steady-state nominal spending level and $G_t \geq G_{ss}$.

We summarize the effects of spending on output in several ways. First, we compute the path of dynamic multipliers as the sequence of

$$m_t^{DYN} = \frac{\frac{Y_t}{Y_{ss}} - 1}{\frac{G_0 P_{ss}}{P_0 G_{ss}} - 1} \frac{Y_{ss}}{G_{ss}/P_{ss}}, \tag{39}$$

and the present value multipliers as

$$m_t^{PV} = \frac{\sum_{k=0}^{t} \beta^k (\frac{Y_k}{Y_{ss}} - 1)}{\sum_{k=0}^{t} \beta^k (\frac{G_k P_{ss}}{P_k G_{ss}} - 1)} \frac{Y_{ss}}{G_{ss}/P_{ss}}, \tag{40}$$

where the two statistics coincide at $t = 0$ and represent the impact multiplier. A useful statistic is then the long-run present value multiplier, which represents the discounted percentage change in real output relative to the discounted percentage change in real government spending for any path of government spending:

$$\overline{M} = m_\infty^{PV} = \frac{\sum_{t=0}^{\infty} \beta^t (\frac{Y_t}{Y_{ss}} - 1)}{\sum_{t=0}^{\infty} \beta^t (\frac{G_t P_{ss}}{P_t G_{ss}} - 1)} \frac{Y_{ss}}{G_{ss}/P_{ss}}, \tag{41}$$

where P_{ss}, G_{ss}, Y_{ss} are the steady state price level, nominal spending and real output respectively and $\frac{G_t}{P_t}$ is real government spending.

For comparison with the complete markets case we also compute the as-if dynamic complete markets multiplier, m_t^{CM}, using the price path we obtain from our incomplete markets model. Knowing the sequence of prices implies the sequence of real interest rates which is sufficient to derive the full consumption path in a model with complete markets since intertemporal substitution is the only channel operating. Iterating the consumption Euler equation yields the as-if percentage response of aggregate consumption,

$$\frac{C_t^{CM}}{C_{ss}} - 1 = \prod_{s=t}^{\infty} (1 + \pi_{t+1}) - 1 = \frac{P_{ss}^{new}}{P_t} - 1.$$

Since the multiplier is in terms of units of consumption and not in percentages, adjusting for the magnitudes of steady state consumption, output and government spending,

$$\begin{aligned} m_t^{CM} &= \frac{\frac{C_t^{CM} - C_{ss}}{C_{ss}} \frac{C_{ss}}{Y_{ss}} + \frac{G_t/P_t - G_{ss}/P_{ss}}{Y_{ss}}}{\frac{G_0 P_{ss}}{P_0 G_{ss}} - 1} \frac{Y_{ss}}{G_{ss}/P_{ss}} \\ &= \frac{\frac{C_t^{CM} - C_{ss}}{C_{ss}}}{\frac{G_0 P_{ss}}{P_0 G_{ss}} - 1} \frac{C_{ss}}{G_{ss}/P_{ss}} + \frac{G_t/P_t - G_{ss}/P_{ss}}{G_0/P_0 - G_{ss}/P_{ss}} \\ &= \frac{\frac{\mathbf{P_{ss}^{new}} - \mathbf{P_t}}{\mathbf{P_t}}}{\frac{G_0 P_{ss}}{P_0 G_{ss}} - 1} \frac{C_{ss}}{G_{ss}/P_{ss}} + \frac{G_t/P_t - G_{ss}/P_{ss}}{G_0/P_0 - G_{ss}/P_{ss}}. \end{aligned}$$

The as-if complete markets multiplier also allows us to decompose the total multiplier into its two channels, the intertemporal substitution and the distributional one. The total multiplier we report combines both channels while the as-if multiplier captures only the first channel but shuts down the second one. This allows us to interpret the difference between these two multipliers as the contribution of the distributional channel,

$$m_t^{Distribution} = m_t^{DYN} - m_t^{CM}, \tag{42}$$

and the as-if multiplier as the contribution of the intertemporal substitution channel.

We proceed similarly for comparison with a Two Agent New Keynesian model (TANK), that combines a permanent income household with a hand to mouth household. The permanent income household behaves like a representative agent so that the full consumption path can be computed as above using the consumption Euler equation. The consumption of hand to mouth households equals $(1-\tau)w_t h_t + T_t$, where we import w_t and T_t from our incomplete markets model. We assume that the demand h_t is the same for all households and solve it using the goods market clearing condition. Combining consumption of both groups yields an aggregate consumption path which we use to compute the asif TANK multiplier path, m_t^{TANK}.

3.2 Calibration

To quantitatively assess the size of the fiscal multiplier we now calibrate the model.

Preferences Households have separable preferences over labor and constant relative risk aversion preferences for consumption. We set the risk-aversion parameter, σ, equal to 1. Following Krueger et al. (2016), we assume permanent discount factor heterogeneity across agents. We allow for two values of the discount factor, which we choose along with the relative proportions to match the Gini of net worth net of home equity, the ratio of median and 30th percentile of networth net of home equity in the 2013 SCF, and aggregate savings to quarterly GDP of 11.46.[6] We assume the functional form for g:

$$g(h) = \psi \frac{h^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \tag{43}$$

[6]We calibrate to a capital to quarterly output ratio of 10.26, and government debt to quarterly GDP ratio of 1.2.

We set the Frisch elasticity, $\varphi = 0.5$, following micro estimates. We choose $\psi = 0.6$ such that in steady state $h = 1/3$.

Productivity Process We follow Krueger et al. (2016) who use data from the Panel Survey of Income Dynamics to estimate a stochastic process for labor productivity. They estimate that log income consists of a persistent and transitory component. They estimate that the persistent shock has an annual persistence of 0.9695 and variance of innovations of 0.0384. The transitory shock is estimated to have variance 0.0522. We follow Krueger et al. (2016) in converting these annual estimates into a quarterly process. We discretize the persistent shock into a seven state Markov chain using the Rouwenhorst method and integrate over the transitory shock using Gauss-Hermite quadrature with three nodes.

Production Technology We set the capital share $\alpha = 0.36$. We choose the quarterly depreciation rate $\delta = 0.032$ to generate a zero real return on capital net of depreciation when the capital output ratio is 10.26. We assume the function form for Φ:

$$\Phi(K', K) = \frac{\phi_k}{2} \left(\frac{K' - K}{K} \right)^2 K, \tag{44}$$

and set $\phi_k = 17$ to match estimates of the elasticity of investment to Tobin's q from Eberly et al. (2008). We choose the elasticity of substitution between intermediate goods, $\epsilon = 10$, to match an average markup of 10%. The adjustment cost parameter on prices and wages is set to $\theta = \theta_w = 300$ to match the slope of the NK Philips curve, $\epsilon/\theta = 0.03$, in Christiano et al. (2011). We set the firm operating cost F equal to the steady state markup such that steady state profits equal 0 (Basu and Fernald, 1997). These profits are fully taxed and are distributed to households as lump-sum transfers in the benchmark.

Government We set the proportional labor income tax, τ equal to 25%, and the dividend tax, τ_k equal to 36%. We set nominal government spending, G in steady state equal to 6% of output (Brinca et al., 2016). The value of of lump-sum transfers T is set to 8.55% of output such that roughly 40% of households receive a net transfer from the government (Kaplan et al., 2016).

Table I: Calibrated Parameters

Parameter	Interpretation	Internally Calibrated	Value
σ	Risk-aversion	N	1
β_i	Discount Factors	Y	(0.9994,0.9929)
φ	Frisch Elasticity	N	0.5
ψ	Labor disutility	Y	0.6
ϵ	Elas. substitution intermediates	N	10
ϵ_w	Elas. substitution labor	N	10
θ_p	Price adjustment	N	300
θ_w	Wage adjustment	N	300
Φ	Firm Fixed Cost/GDP	Y	0.1
τ	Labor tax	N	25%
τ_k	Dividend tax	N	36%
T	Transfer/GDP	Y	8.55%

Monetary Policy For the benchmark specification we assume that the monetary authority operates a constant interest rate peg of $i = 0$. Note that the results in Hagedorn (2016) imply that there is a unique response of prices, output, consumption and employment although monetary policy is not following an active interest rate rule where nominal interest are increased in response to inflation. For purposes of comparison, we will also a specification where we assume that the monetary authority follows a Taylor rule, which sets the nominal interest rate according to:

$$i_{t+1} = \max(X_{t+1}, 0) \tag{45}$$

where

$$X_{t+1} = \left(\frac{1}{\zeta}\right)\left(\frac{P_t}{P_{ss}}\right)^{\phi_1(1-\rho_R)}\left(\frac{Y_t}{Y_{ss}}\right)^{\phi_2(1-\rho_R)}[\zeta(1+i_t)]^{\rho_R} - 1.$$

We follow the literature in setting $\rho_R = 0.8$, $\phi_1 = 0.5$, $\phi_2 = 0.25$ and $\zeta = 1/(1+r_{ss})$. Fiscal monetary coordination will be carried out under various schemes listed in the next section.

Steady State Model Fit In the model 3% of agents have 0 wealth, and 10% of agents less than \$1000. The annual MPC[7] out of transitory income equals 0.4, which is in the middle

[7] We compute the annual MPC using the quarterly MPC via the formula: $MPC_a = 1 - (1 - MPC_q)^4$. The quarterly MPC in the model is 0.12

range of empirical estimates 0.2-0.6 (e.g. Johnson et al., 2006).

Before moving to the full general equilibrium analysis, we provide several partial equilibrium consumption results to build intuition for our general equilibrium findings and to show that our incomplete markets model is closer to the empirical findings in the micro consumption literature than standard frameworks. In all of the following experiments, we consider standard consumption/savings household problems for prices fixed at their steady-state values. To illustrate the properties of the MPC in our model, the experiments differ in the timing and the amount of transfers households receive.

Each graph of the four panels in Figure 2 has four separate experiments, where each line corresponds to the aggregate consumption path in response to finding out at date 0 that all households will receive a transfer either at date 0, or date 4, or date 8, or date 12 without an obligation to ever repay. The four panels differ in the size of the transfer received, either \$10, \$100, \$1000 or \$10000.

Consider first the experiment of giving a household \$10 today. A permanent income household would save basically all of the money and consume a small fraction. In our model households face idiosyncratic income risk, inducing a income smoothing desire but also credit constraints, preventing perfect smoothing. A borrowing constrained household would consume the full \$10. Unconstrained, but low asset households will also consume a large fraction of the transfer because it relaxes precautionary savings motives. These arguments together imply an initial MPC significantly larger than in complete markets models. The fraction of the transfers not spent in the initial period is spent in the following periods at a decaying rate. If households receive larger transfers the initial MPC falls, mainly because larger transfers are more likely to relax the credit constraints. For example, a \$10,000 transfer is more than enough to relax all households' credit constraints, so that even the borrowing constrained household will not consume the full transfer. Now, suppose households do not receive the transfer today but only learn today that they will receive a transfer in a future period, at date 4,8, or 12. The credit constrained households canot respond until the transfer arrives. The unconstrained households are able to smooth consumption, but their MPC is lower, so the initial rise in consumption is smaller than before.

These model properties are consistent with the data but inconsistent with a complete markets RANK model. We now show that using a simple deviation from the RANK model with two types of agents, a Two Agent New Keynesian model (TANK), that combines a permanent income household with a hand to mouth household, does not deliver either. The TANK model can match high MPCs and is theoretically tractable. We now replicate the same experiment in a TANK model shown in Figure 3. Instead of tent-shaped impulse responses we obtain in our HANK model, the TANK model delivers spiky responses. The TANK model also misses out on the sensitivity to the size of the transfer and all of the dynamic anticipation and propagation effects because the response of permanent income households is minimal and consumption of hand-to-mouth households responds to current income only. To understand why this is important we need to think about how the general equilibrium Keynesian cross multiplier logic works.

As we saw, if a household receives a transfer in the first period, it's spent over all the future periods. Now, in general equilibrium, that would mean an increase in aggregate demand and also in income not only today but also in all future periods. Consumption today increases not only because of the increase in income today but also because of the increases in future income by relaxing precautionary savings motives. To illustrate this point we now combine the four previous experiments, so that households will receive a transfers at all dates 0, 4, 8 and 12. Figure 4 shows that now the impact response of consumption is nearly twice as high, even though in period 0 the same income transfer is received. This is all in partial equilibrium still, but in general equilibrium the demand and income increases at different times reinforce each other, generating what Auclert et al. (2018) have coined as an intertemporal keynesian cross. If contrasted with the TANK economy, again, because the anticipation and propagation effects are basically zero, we would miss out on the dynamic multiplier path.

3.3 Results

We can now compute the equilibrium response of prices, employment, output and consumption to a persistent increase in nominal government consumption by one percent where spending follows an AR(1) process with parameter $\rho_g = 0.9$ after the initial innovation. Balancing the government budget when government spending is increased requires to adjust taxes or debt or both. As Ricardian equivalence does not hold in our model different assumptions on the path of taxes and debt will have different implications for the path of aggregate consumption

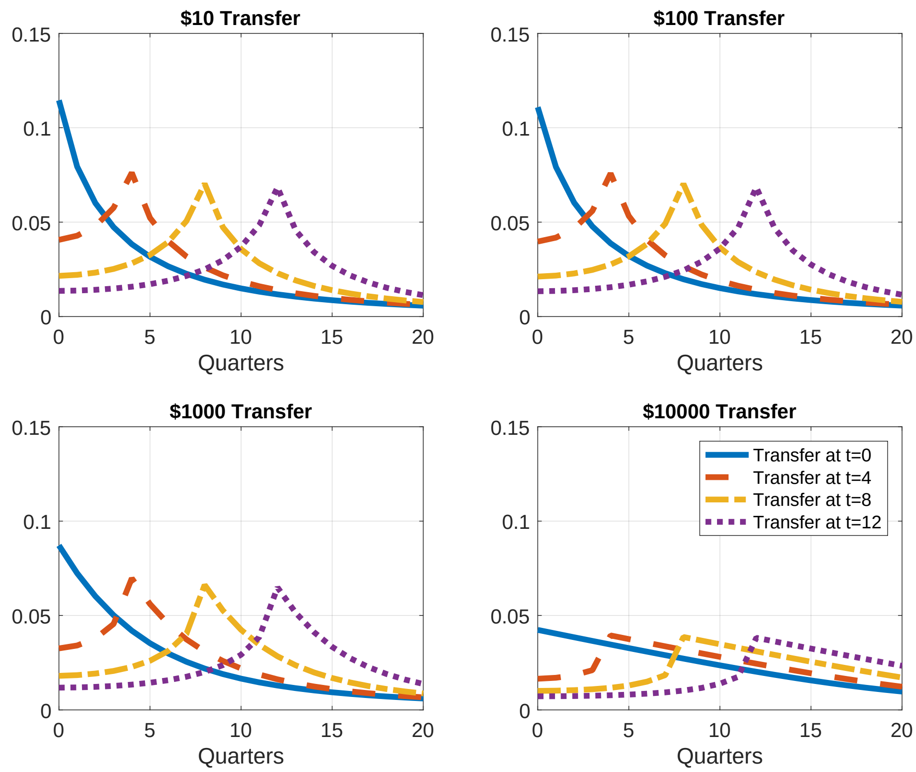


Figure 2: Propensity to consume for Transfers of Size 10\$, 100\$ 1000\$ and 10000\$.

and therefore prices and the change in output. We consider three scenarios:

1. Transfer are adjusted period by period to keep nominal debt constant.
2. Deficit financing and delayed cuts in gobernment expenditure after 40 quarters.
3. Deficit financing and delayed transfers to pay back debt after 40 quarters.

For each of the three scenarios we report the dynamic response of hours, consumption, output, prices, tax revenue and debt as well as the of the path of dynamic and static multipliers m_t^D, m_t^S and of the as-if complete markets multiplier m_t^{CM} and the as-if TANK multiplier m_t^{TANK} and the summary multiplier $\overline{M}$.

3.3.1 Tax Financing: Constant nominal debt

Under the first financing scheme, we assume that the government adjusts lump-sum transfers period by period to keep the level of nominal debt constant. The four panels of Figure 5 show

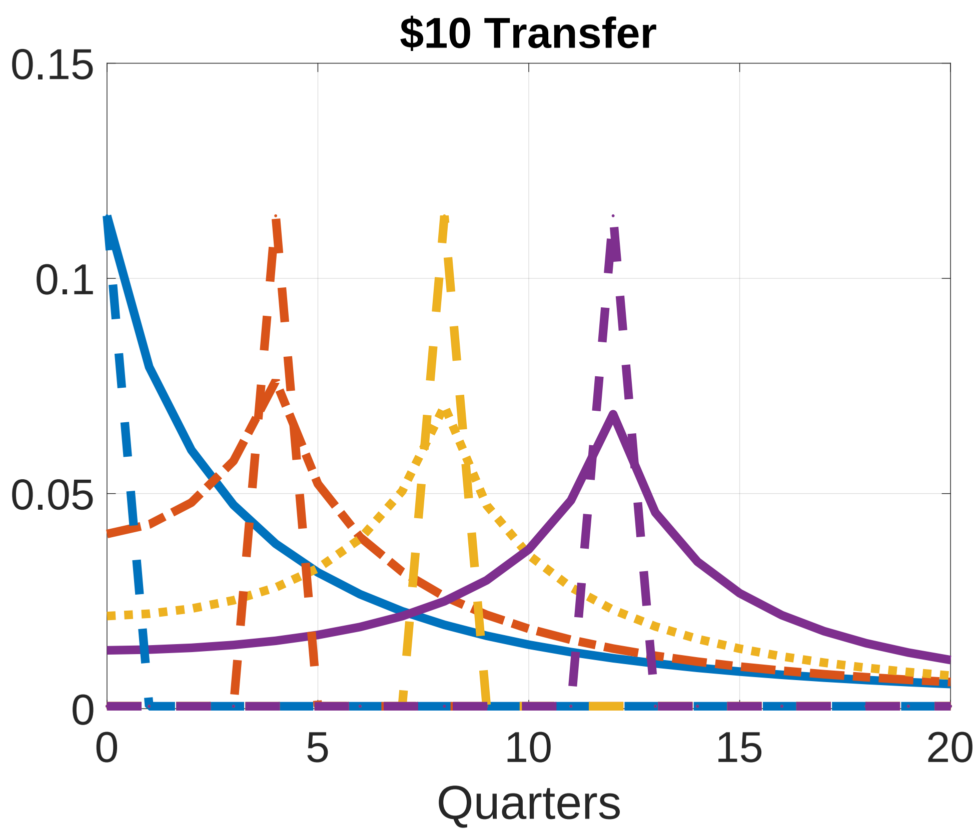


Figure 3: Propensity to consume for Transfers of Size 10$, for HANK and TANK.

the results for the aggregate consumption and output response, the different multipliers, the decomposition of aggregate consumption, and government bonds.

The level of nominal government bonds is unchanged since the stimulus is tax-financed. On impact G increases by 1% (0.06% of output) and consumption decreases by 0.027% of output leading to an impact multiplier of 0.60. The dynamic multiplier converges to zero since the consumption response although negative slowly dies out and becomes small relative to initial government spending increase. The decomposition of the total consumption response reveals the quantitative importance of the direct, the indirect and the price effects. The stimulus of 0.15% directly increases households labor supply by the same amount, leading to a aggregate consumption response of 0.012% of output. (equation 34). The contemporaneous cut in transfers lowers aggregate consumption by 0.031% on impact (equation 35) , implying a total initial negative effect of -0.018%. This effect is negative since the government spending increases households income proportional to their productivity and thus benefits high income

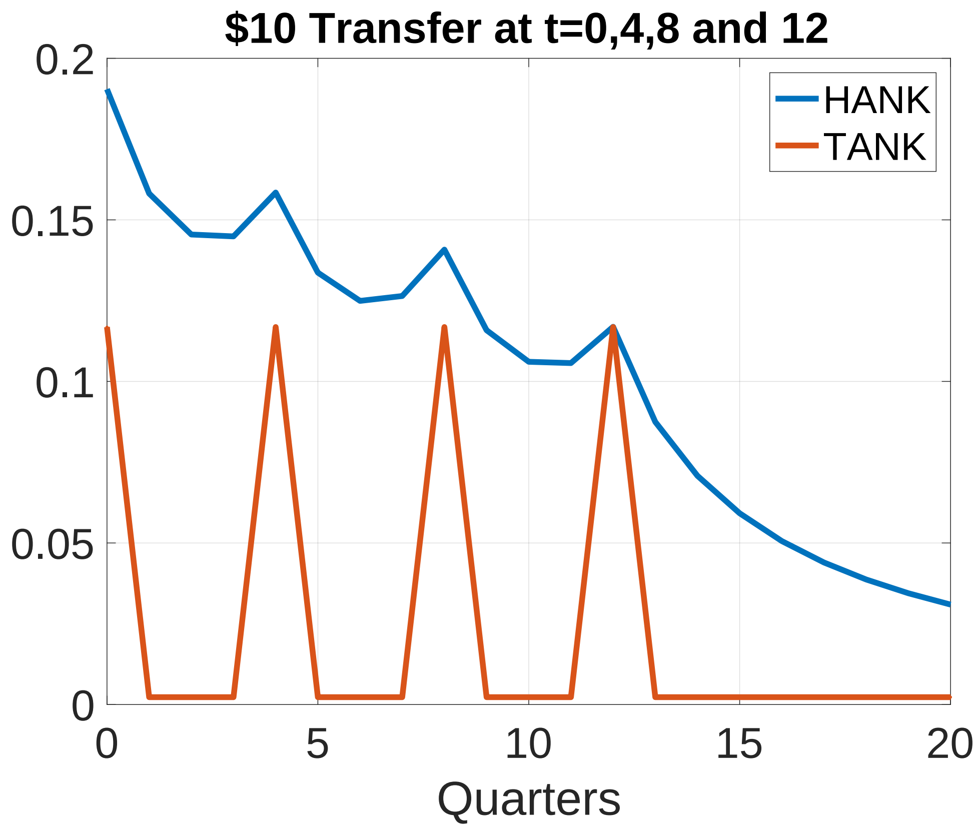


Figure 4: Propensity to consume for Transfers of Size 10$ that are repaid in periods 0, 4, 8 and 12.

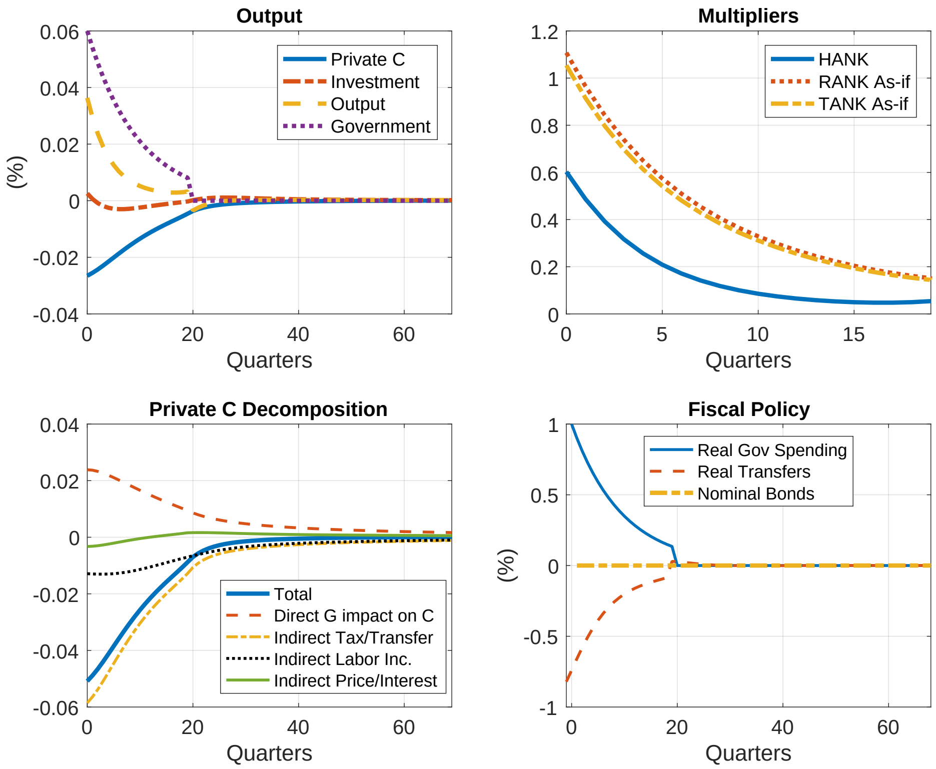
households more where the transfer cut is uniformly across all income groups and thus negatively affects high MPH households. This decrease leads to lower consumption demand, which in turn leads to lower labor demand, lower labor income and again lower consumption demand until an equilibrium is reached. These indirect multiplier effects sum up to -0.007% (equation 36) further lowering the consumption response. Finally, the decomposition shows that the price increase (and the unchanged interest rate) effects are small (equation 37).

The impulse response of the remaining variables to a 1% innovation in government spending are plotted in Figure A-7 in the appendix. The cumulated multiplier, reported in Table II, is only 0.43.

3.3.2 Deficit financing

Under this scenario we assume that real transfers are kept constant during the first 40 quarters after the innovation to government spending. Then, the government is assumed to adjust

Figure 5: Fiscal Multiplier and Aggregate Consumption: Tax Financing

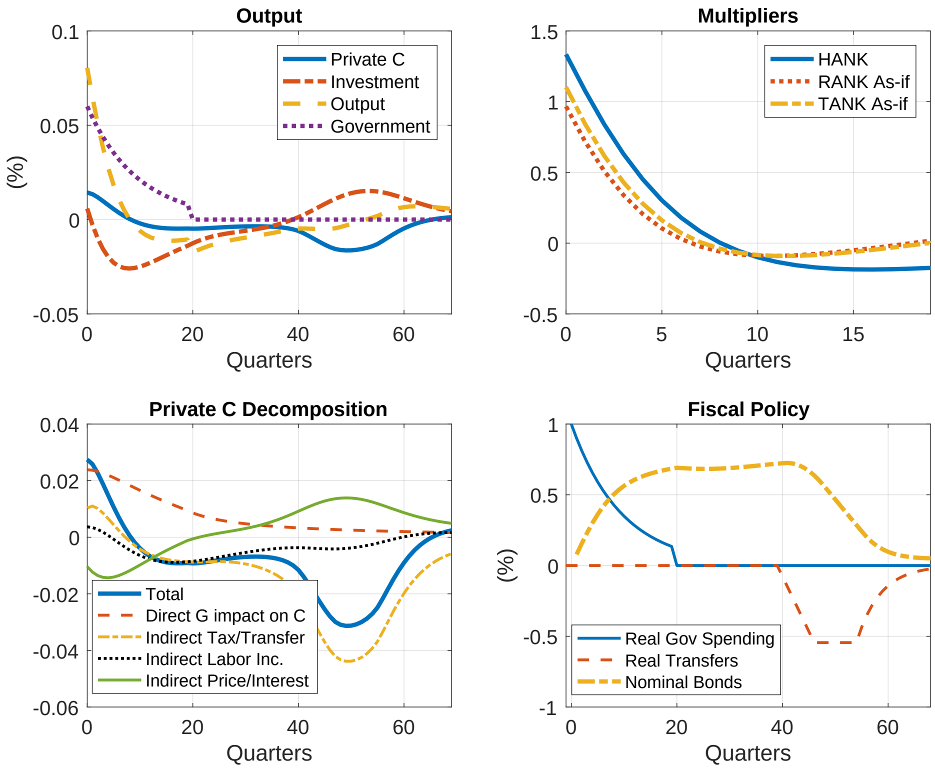


transfers linearly over eight quarters, keep them constant for eight quarters, and then allow transfers to revert back to the real steady state level with an autocorrelation parameter of 0.8. Thus, under this timing scheme, the government chooses only the level of adjustment to transfers to guarantee that nominal government debt returns to its original level.

The four panels of Figure 6 summarize the main results for the aggregate consumption and output response, different multipliers, the decomposition of aggregate consumption, and government bonds.

Deficit instead of tax financing increases the initial multiplier from 0.847 to 1.156 and the initial aggregate consumption response from -0.023% to 0.024%. The decomposition of the consumption responds makes clear why. The direct impact of the spending stimulus is basically identical (0.020%) but now there is no initial offsetting effect from contemporaneously higher taxes. The total initial effect thus equals 0.0197 (-0.018% before), almost identical to the direct spending impact, leading to a larger increase in labor demand and households income.

Figure 6: Fiscal Multiplier and Aggregate Consumption: Deficit Financing



The indirect multiplier effects now accumulate to 0.003% The deficit financing leads to an increase in government bonds and the consumption response becomes negative only from period 9 onwards. However, the increase in government spending is ultimately financed through a future reduction in transfers, which results in a contraction in future output. Thus, despite the cumulated discounted multiplier is 1.081, slightly smaller than the impact multiplier. The impulse responses of the remaining variables are plotted in Figure A-8 in the appendix.

3.4 Further Analysis

We now extend the analysis in various directions. First we use a Taylor interest rate rule to describe monetary policy instead of a nominal interest rate fixed at the ZLB in Section 3.4.1. We then ask how the size of the fiscal multiplier depends on the timing of spending ("forward spending") and on the persistence of the stimulus in Sections 3.4.2 and 3.4.3. So far we have focused - as does the literature - on the effects of an increase in government spending. Another

Table II: Cumulated Multiplier $\overline{M}$

Tax Financing	Deficit Financing
0.43	0.55

stimulus policy is to increase transfers and we consider such policies in Section 3.4.4. Finally, we consider spending and transfer policies in a liquidity trap in Sections 3.4.6, 3.4.7 and 3.4.8. We also investigate how the size of the multiplier depends on the scale of the stimulus and on the degree of price and wage rigidities.

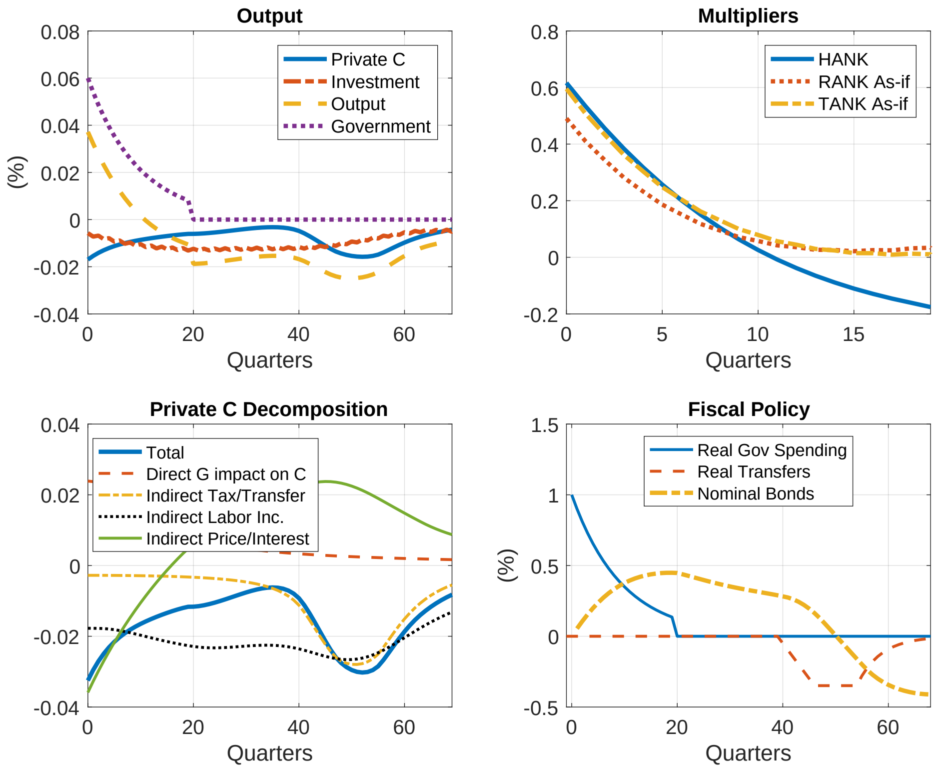
3.4.1 Taylor Rule

We find similar results if instead of an interest rate peg, the monetary authority follows a Taylor rule. This is not surprising since the prices respond only very little when the interest is pegged at zero. The four panels of Figure 6 summarize the main results. The impulse response are plotted in Figure A-3.The same conclusion is reached for tax instead of deficit financing as the impulse responses in Figure A-4 show.

3.4.2 Forward Spending

The multiplier gets smaller if the spending is pre-announced to occur at a future date, 4 quarters from now. The additional spending is deficit financed. The price level now increases gradually in anticipation of the future increase in government spending such that Initially output falls before it increases at the time of the spending increase 4 quarters in the future. However, the increase in consumption as well as the multiplier at that time are smaller than the corresponding multiplier in the case when the stimulus occurs immediately and is deficit financed. The impulse responses to a spending increase 8 quarters in the future are plotted in Figure A-5 in the appendix.

Figure 7: Fiscal Multiplier and Aggregate Consumption: Taylor Rule and Deficit Financing



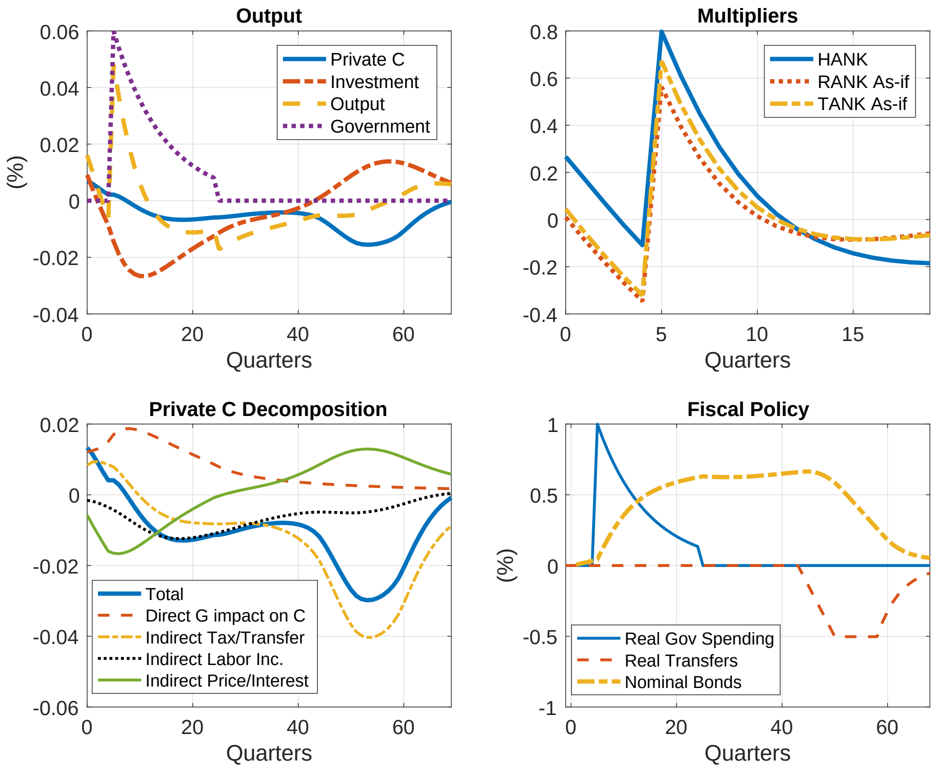
3.4.3 More Persistent Spending

We now again compute the response of prices, employment, output and consumption to a persistent increase in nominal government consumption by one percent where spending follows an AR(1) process but now with varying degrees of persistence $\rho_g \in [0, 1]$. Figure A-6 in the appendix shows the impulse responses and Figure 9 the dynamic and the cumulative multiplier for various degrees of persistence

3.4.4 Transfer Multiplier

In this section we consider the multiplier in response to a one percent increase in government transfers. We assume that nominal government spending adjusts to keep real government spending constant in response to the innovation in transfers. We allow the government to finance the increase in transfers by first increasing government debt, but by increasing future transfers as in the previous section to pay back the debt.

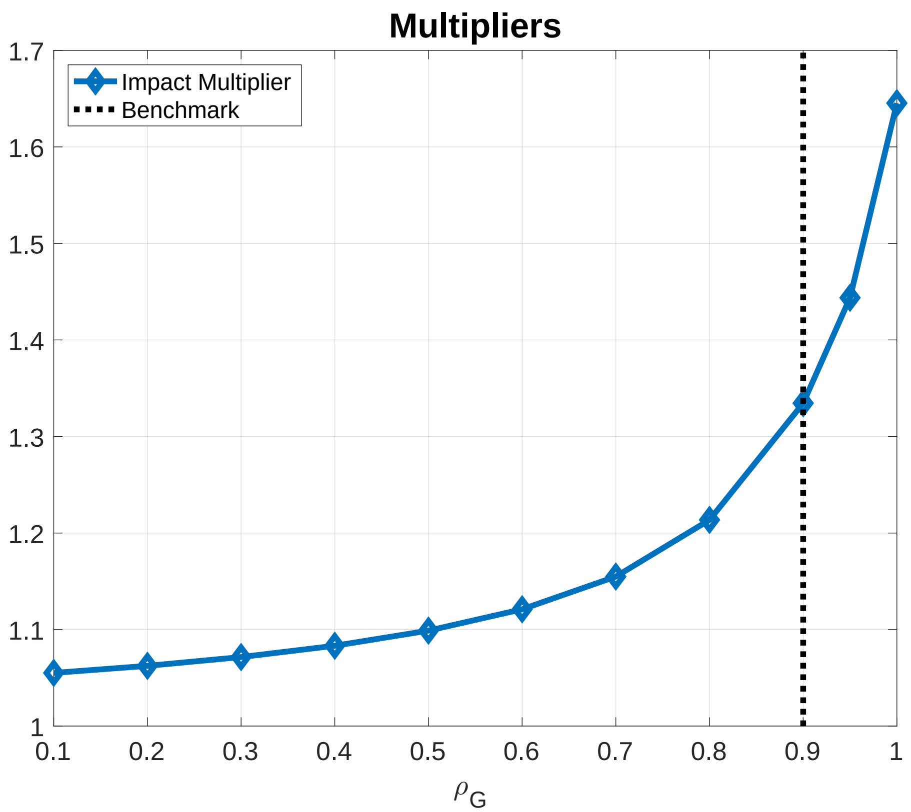
Figure 8: Future (+8 quarters) spending: Deficit Financing



The impulse response is plotted in Figure A-9. The impulse response is qualitatively and quantitatively similar to the impulse response to an increase in government spending with delayed repayment. Output rises more, however, when transfers increase than when spending increases. This can be understood because, in addition to an increase in spending coming from an increase in the price level and a decline in the real rate, the heterogeneity in marginal propensities to consume means that some households will increase their spending by even more than would be implied from the fall in the real rate in a representative agent model. However, the cumulative multiplier ends up being around -0.1. As the future decrease in transfers needed to return nominal government debt to its steady state level are sufficiently contractionary to offset the contemporaneous gains.

Figure 10 shows the results.

Figure 9: Multipliers: Persistence ρ_G of spending



3.4.5 Repayment through G

In this section we consider the multiplier in the deficit-financed case, but instead of cutting transfers in order to bring nominal debt back to its steady-state value, we instead cut government spending G following the same scheme used for transfers.

The impulse response is plotted in Figure 11. The impulse response is qualitatively and quantitatively similar to the impulse response to an increase in government spending with delayed repayment. Output rises more, however, when spending is used to repay the debt than when transfers are used.

3.4.6 Liquidity Trap

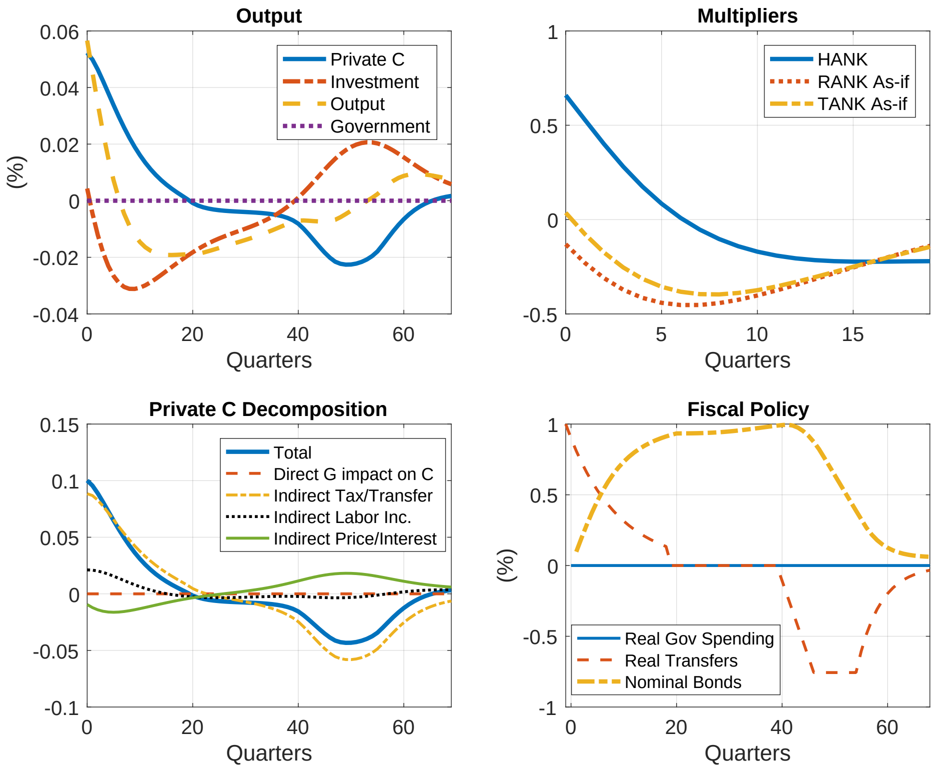
In this section we explore the extent to which the size of the multiplier may vary with other shocks hitting the economy. In particular, we consider what the government multiplier is after a demand shock. We therefore first have to generate a liquidity trap in the model, where the

Table III: Main Results Consumption and Multipliers

	Baseline		Forward	Taylor Rule	G-Finaced	Transfer
	Tax-Finance	Deficit	Deficit	Deficit	Deficit	Deficit
	(1)	(2)	(3)	(4)	(5)	(6)
Impact Mult.	0.60	1.33	0.80		1.29	0.66
Cumul	0.43	0.55	0.17		0.30	-0.3
$100 \times \Delta C_0$	-2.7	1.4	0.2		1.5	5.2
Decomposition of Consumption						
Direct G on C	1.2	1.2	0.9		1.1	0.0
Tax/Transfers	-3.1	0.5	0.4		0.7	4.6
Indirect Income	-0.7	0.2	-0.2		0.2	1.1
Prices	-0.1	-0.5	-0.9		-0.6	-0.5

Note - The table contains the impact and the cumulated multiplier $\overline{M}$ as well as the initial consumption response ΔC_0 (as a % of output). The last four rows show the decomposition of the initial aggregate consumption response into the direct G impact on C (equation 34), the effect of taxes/transfers (equation 35), indirect income effects (equation 36) and the price and interest rate effects (equation 37).

Figure 10: Transfer Multipliers (Deficit Financed)

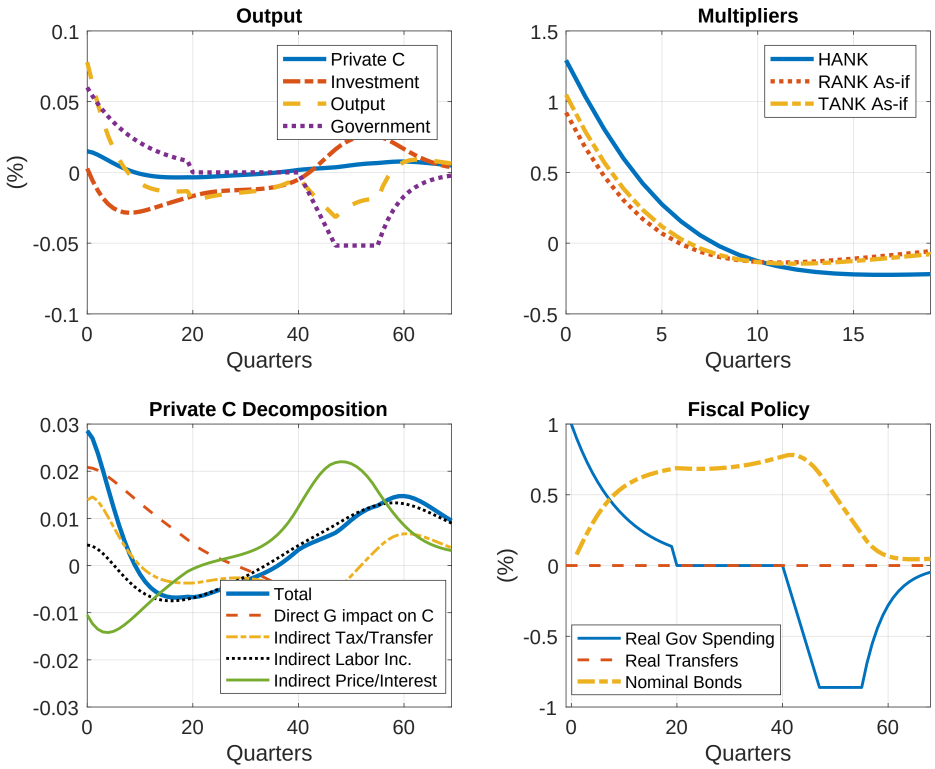


ZLB on nominal interest rates is binding. In doing so we follow Cochrane (2015) and construct a series of discount factors $\{\beta_t\}_{(t=1,2,...)}$ such that the natural real rate of interest - the real interest interest rate in a world with flexible prices and wages - equals -2% for 5 years and then returns to zero afterwards. All other parameters are unchanged.

We then feed the series of discount factors $\{\beta_t\}_{(t=1,2,...)}$ into our model with price and wage rigidities and calculate the response of the economy, which is shown in Figure 12. The resulting recession is quite large as output initially drops by about 5 percent. We solve for the impulse response to the demand shock under two scenarios previously considered, one - tax financing - where real government debt is kept constant and the other - deficit financing - where we adjust transfers in the future to return back to steady state nominal debt.

Under these two scenarios, we also compute the effect of a simultaneous (at the same time as the liquidity trap starts) 1% increase in nominal government spending. Thus, we can compute the fiscal multiplier as the percent increase in output under this scenario, relative

Figure 11: Spending Multipliers (Deficit repayed using G)



to the benchmark with no increase in spending, divided by the relative percent differences in government spending. The multipliers are plotted in the left panel of Figure 13. The right panel of Figure 13 shows the transfer multiplier, where again only deficit financing is meaningful.

3.4.7 Scale Effects

We consider the size of the multiplier in the liquidity trap described above and how it depends on the scale of the government spending and transfer stimulus. The left panel of Figure 14 shows the government spending multiplier for a $1\%, 2\%, 5\%, 10\%$ increase. The right panel of Figure 14 shows the same for the transfer multiplier again for $1\%, 2\%, 5\%, 10\%$ increases.

3.4.8 The degree of price and wage rigidities

In New Keynesian complete markets models, the size of the multiplier increases if prices become more flexible. There is however a discontinuity at fully flexible prices. The multiplier

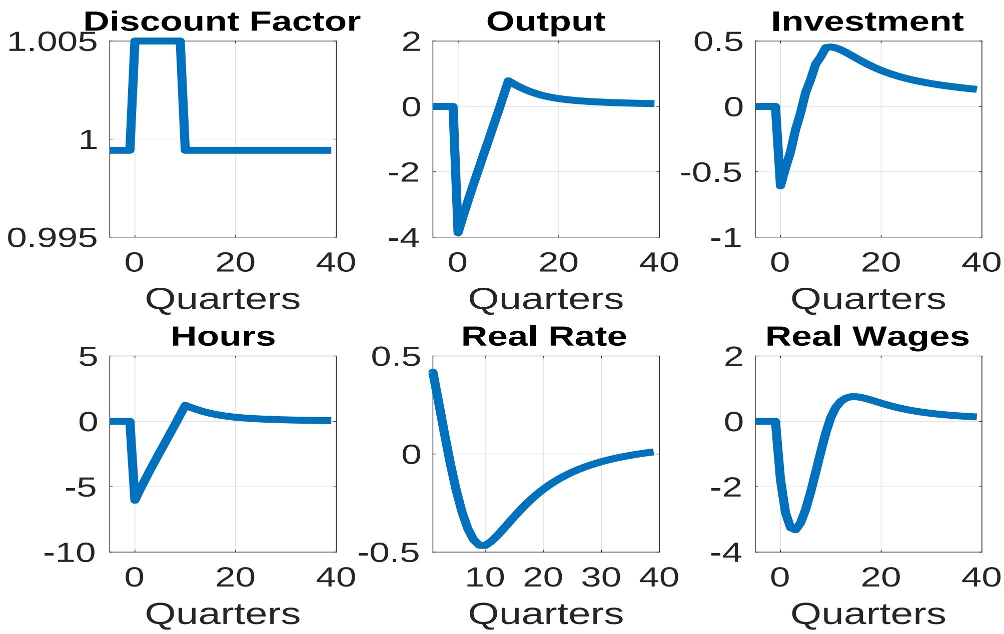
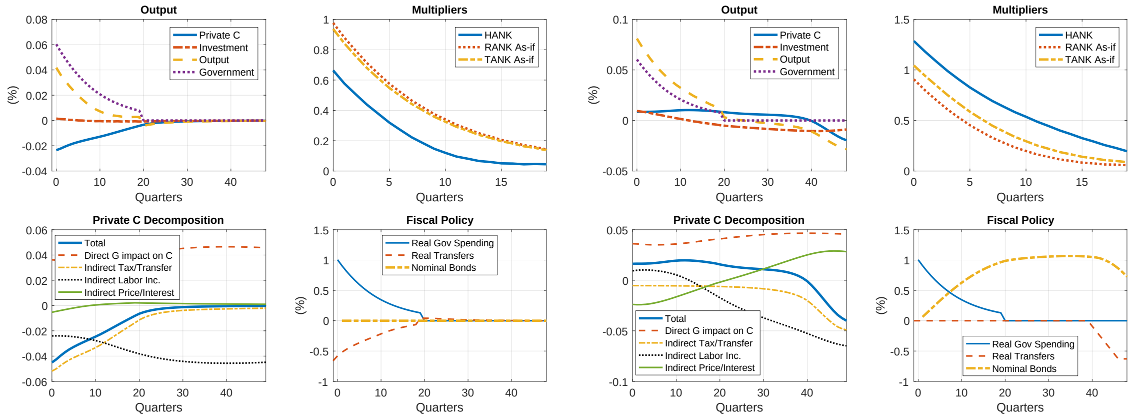


Figure 12: Economy in a Liquidity Trap

is smaller than one if prices are fully flexible, but is arbitrarily large if the degree of rigidity is close to but not equal to zero. The reason is again the response of inflation and thus real interest rates. The more flexible prices are, the larger is the deflation in a liquidity trap and the larger is the inflation increase in response to the stimulus. Since the inflation response is one-to-one related to the output gain, the multiplier is decreasing in $\theta_p = \theta_w$.

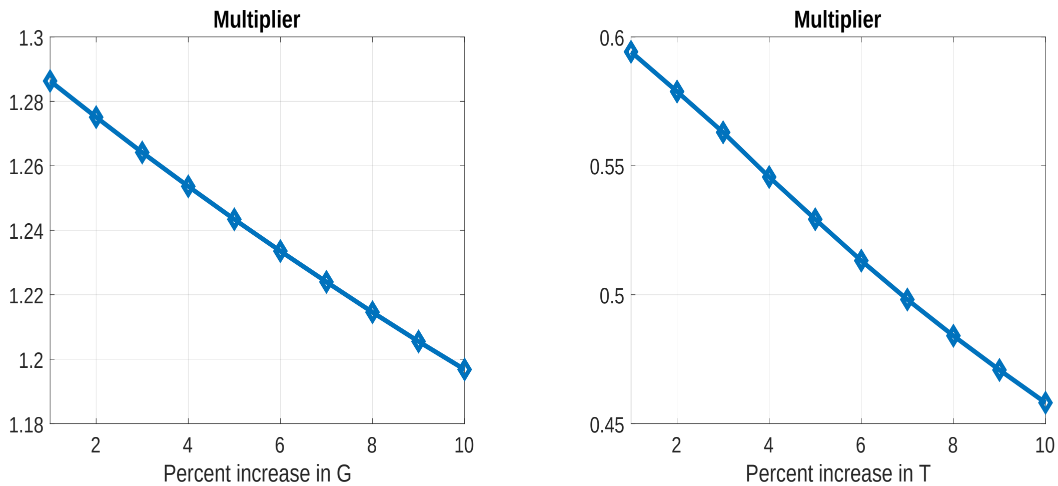
Figure 15 shows that this result is overturned in our incomplete markets model. The multiplier falls if $\theta_p = \theta_w$ converges to zero and the discontinuity disappears. If prices become more flexible - $\theta_p = \theta_w$ fall from 300 to 0 - the complete market effects, just described, initially dominate for $\theta_p > 30$ and inflation falls more but eventually the flexible price effect dominate and the inflation rate increases in a liquidity trap. If prices are fully flexible an increase in the discount factor stimulates savings and thus implies a fall in the real interest rate or equivalently an increase in inflation. A fiscal stimulus raises the real interest rate in a fully flexible price economy that is the inflation rate falls and the multiplier is smaller than one. For higher degrees of rigidities the stimulus increases inflation and the multiplier is larger than one.

price flexibility diminishes the effectiveness of spending, instead of increasing it.



(a) Spending Multiplier (Tax Financed) (b) Spending Multiplier (Deficit Financed) r

Figure 13: Fiscal Multipliers in a Liquidity Trap



(a) Scale of Spending (b) Scale of Transfers

Figure 14: Multiplier in a Liquidity Trap and Scale of Spending/Transfers (Deficit Financing)

4 Conclusions

We construct a Heterogeneous Agent New Keynesian (HANK) model with several key ingredients needed to quantify the effects of a fiscal stimulus. Ricardian equivalence does not hold so that policy has distributional consequences. Market incompleteness implies MPC heterogeneity across households as observed in the data, such that the textbook Keynesian cross logic applies. A demand stimulus leads to higher household incomes which are then spent according to their MPCs, creating more demand and again more income and so on. In constrast to static textbook models, ours is a full dynamic model so that demand is also governed by intertem-

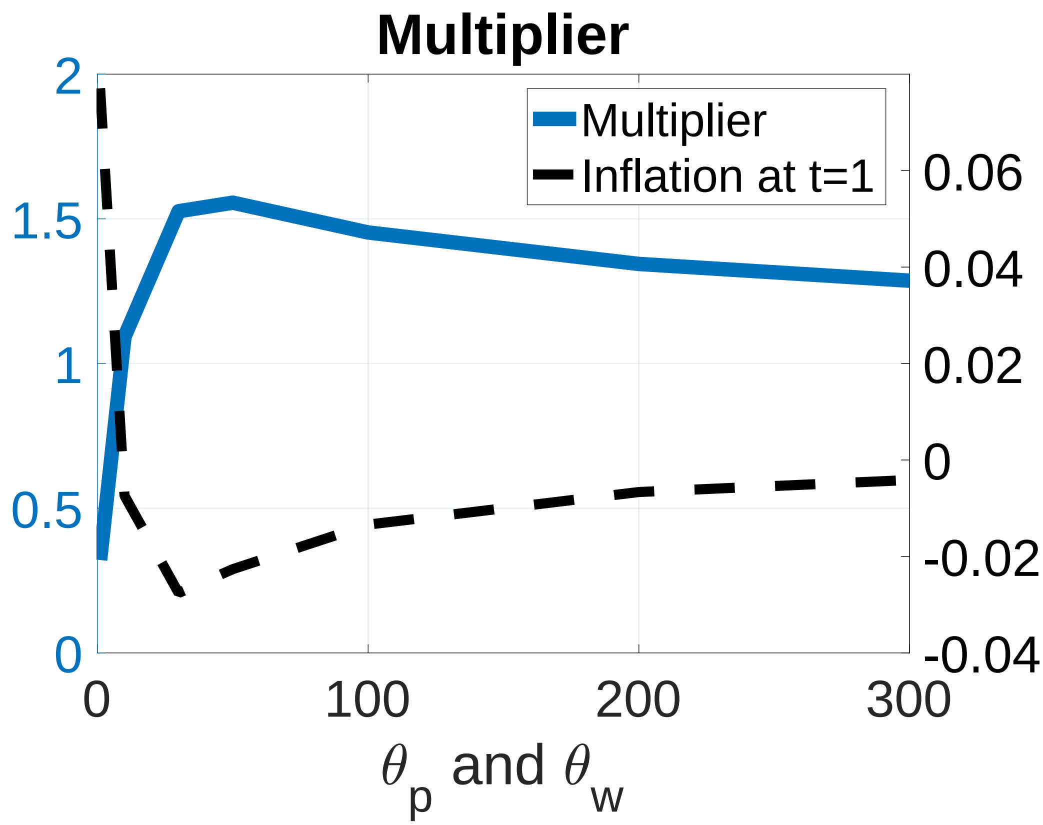


Figure 15: Multiplier and Inflation in initial period $t = 1$ in a Liquidity Trap and Degree of Rigidities $\theta_p = \theta_w$

poral motives. In particular, households take into account that a deficit-financed spending stimulus leads to either higher taxes or lower government expenditure in the future. This demand channel is complemented by an investment channel, which follows a similar logic. A stimulus increases demand for both employment and investment. A higher investment demand stimulates the production of investment goods which leads to more demand for employment and investment and so on until prices adjust and in equilibrium is reached. The equilibrium increase in investment depends on the elasticity of interest rates and wages and those prices determine firms substitution between labor and capital. In addition, deficit-financing crowds out investment. Our model delivers a uniquely determined price level since government bonds are nominal, implying that we obtain a unique fiscal multiplier for arbitrary combinations of monetary and fiscal policies including an interest rate peg.

Our main quantitative results are that the deficit financing multiplier is 1.35 and significantly larger than the tax financing multiplier, 0.6. The liquidity trap multipliers are similar

(1.29/0.66), that is we have no strong state-dependence in our model. Distributional effects account for 0.51 of the 1.35 multiplier when deficit financed and contribute a negative number, -0.3, for tax financing. We also show that several liquidity trap puzzles that have been documented in New Keynesian models, disappear.

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I Figures

I.1 Impulse Responses: Main Results (Section 3.3)

Figure A-1: Impulse response to a 1% increase in nominal government spending: **Tax Financing** (Constant Nominal Debt).

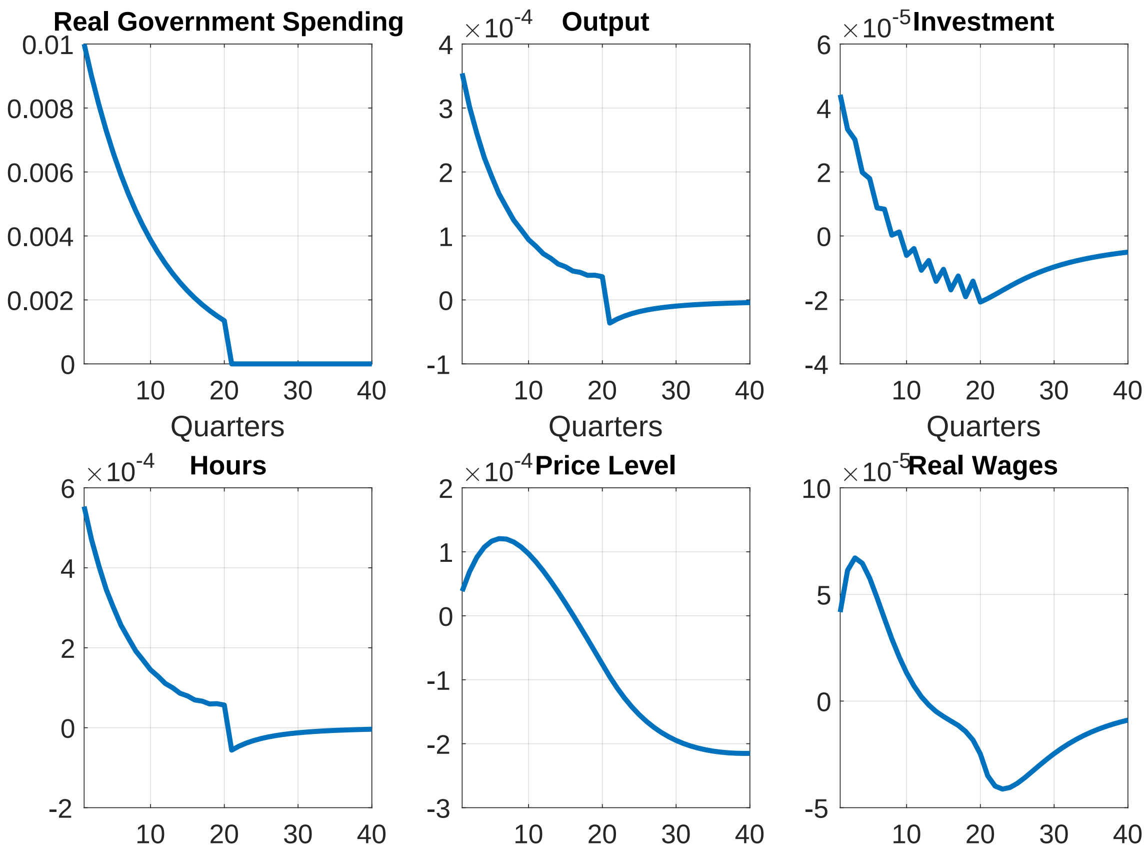
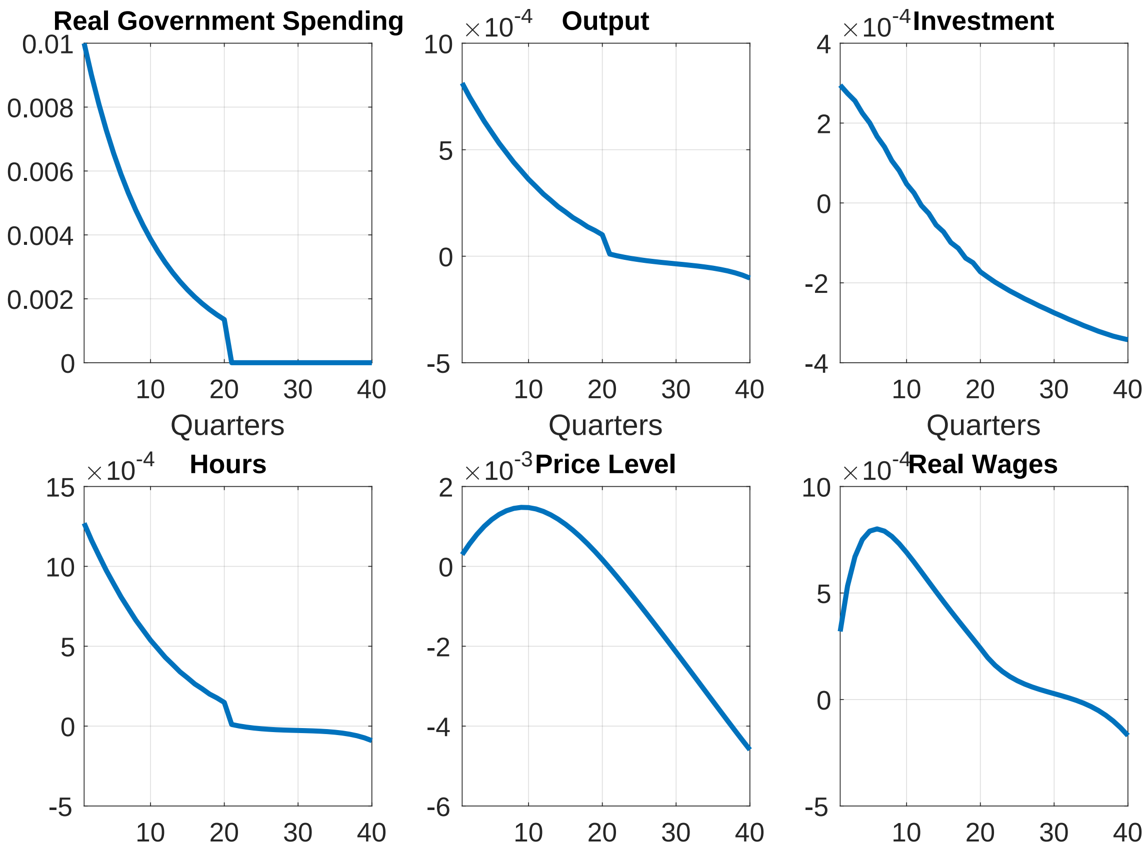


Figure A-2: Impulse response to a 1% increase in nominal government spending: **Deficit Financing**



I.2 Impulse Responses: Taylor Rule

Figure A-3: Impulse response to a 1% increase in nominal government spending: **Deficit Financing, Taylor Rule**

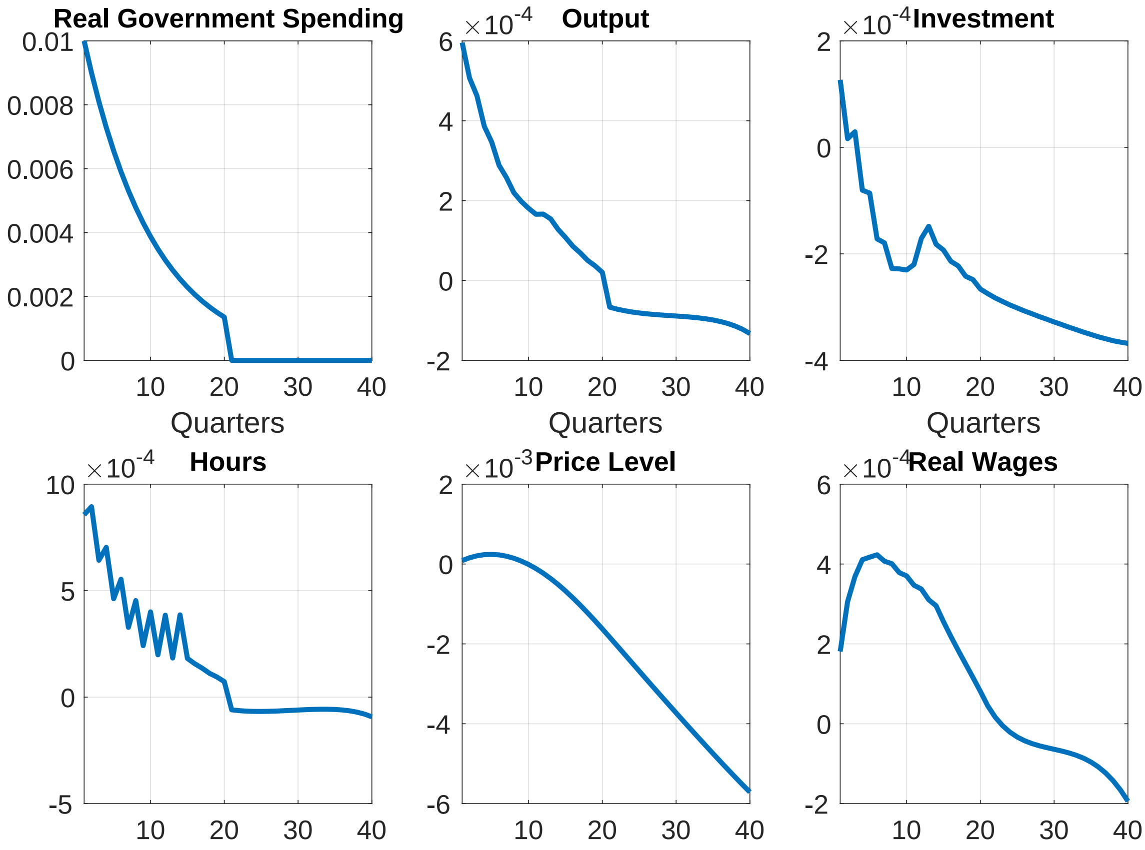
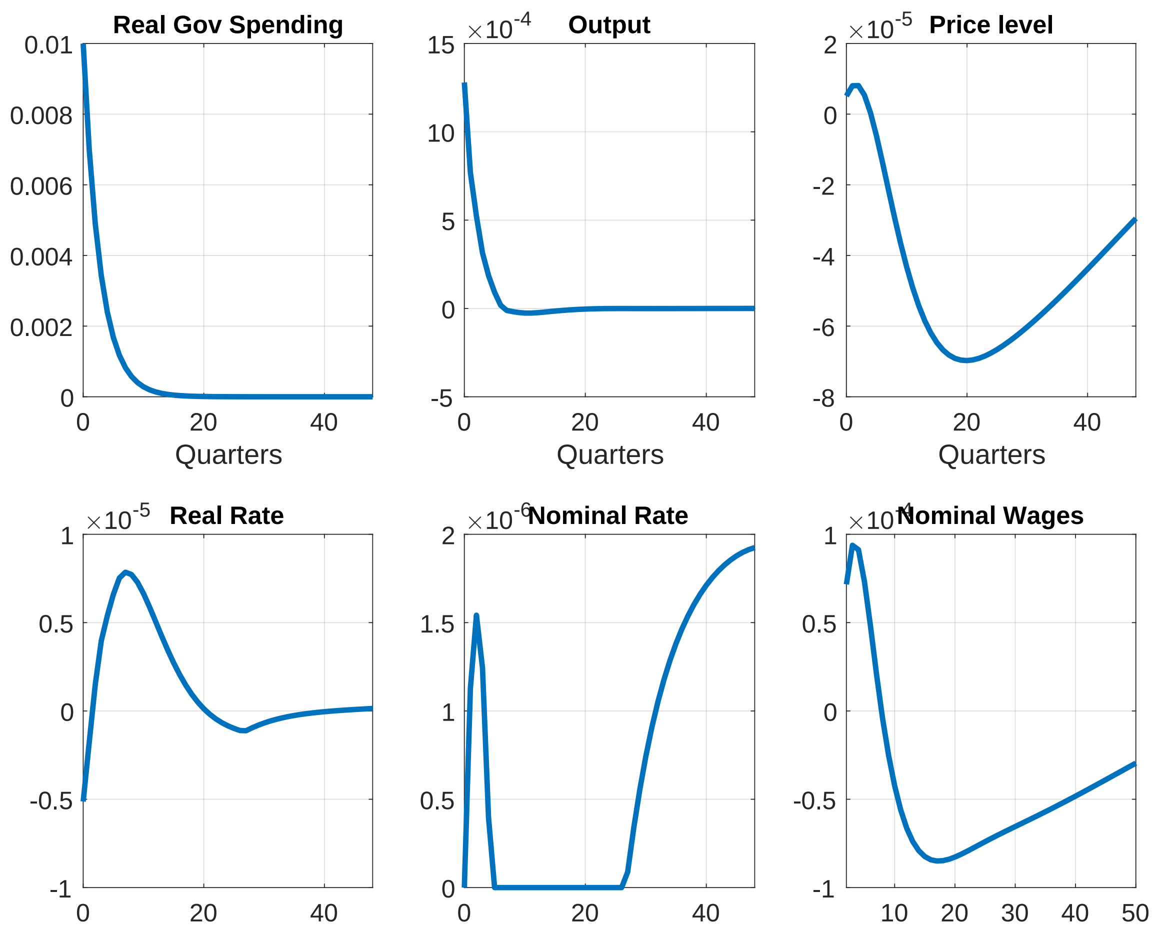
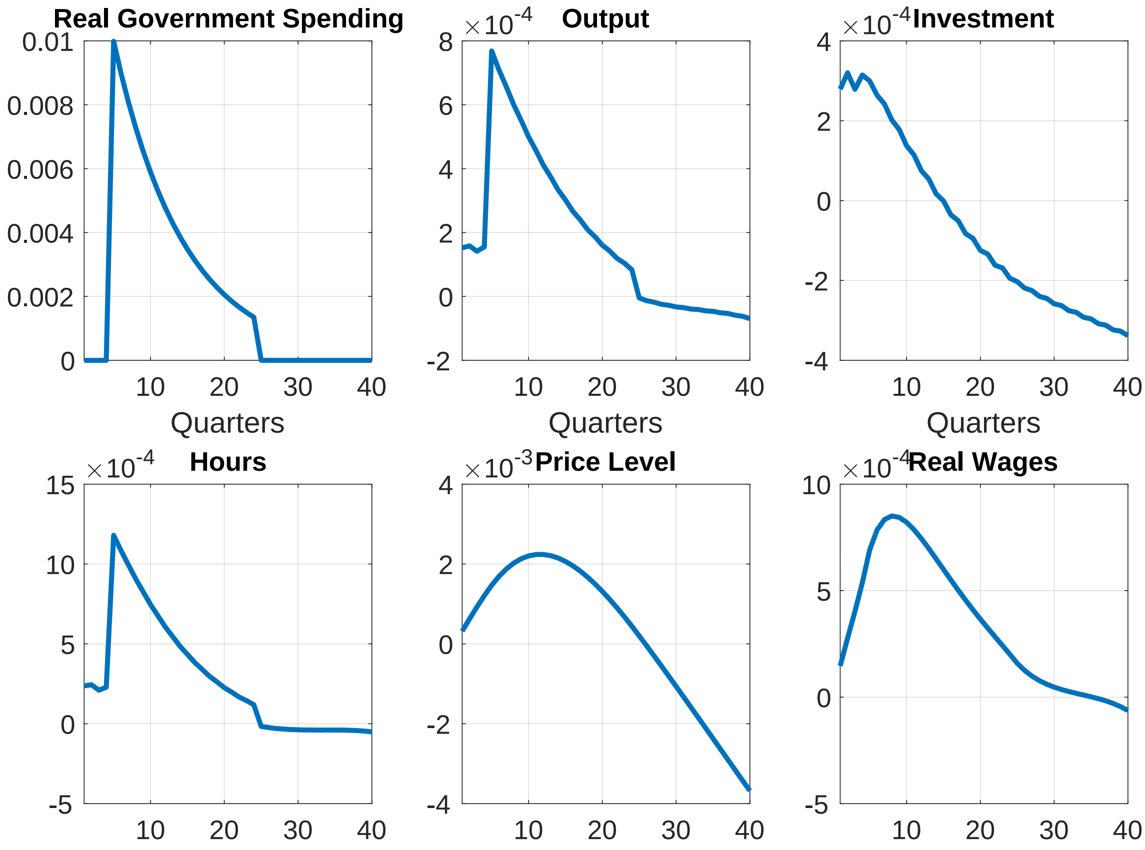


Figure A-4: Impulse response to a 1% increase in nominal government spending: **Tax Financing, Taylor Rule**



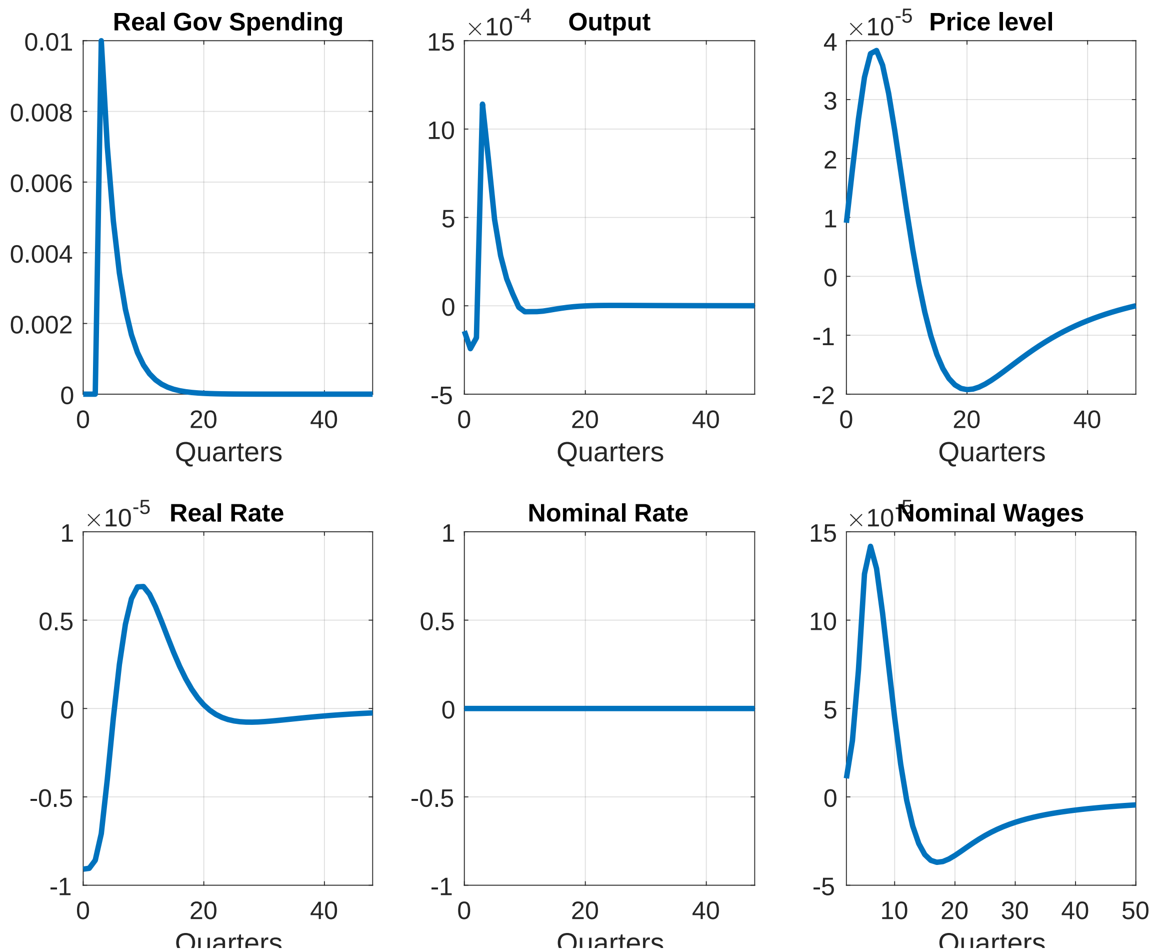
I.3 Impulse Responses: Forward Spending

Figure A-5: Impulse response to a **future** (+4 quarters) 1% increase in nominal government spending: **Deficit Financing**



I.4 Impulse Responses: Higher persistence

Figure A-6: NOT UPDATED Impulse response to a 1% increase in nominal government spending **(Persistence 0.9)): Deficit Financing**



I.5 Impulse Responses: Liquidity Trap

Figure A-7: Impulse response to a 1% increase in nominal government spending: **Tax Financing** (Constant Nominal Debt).

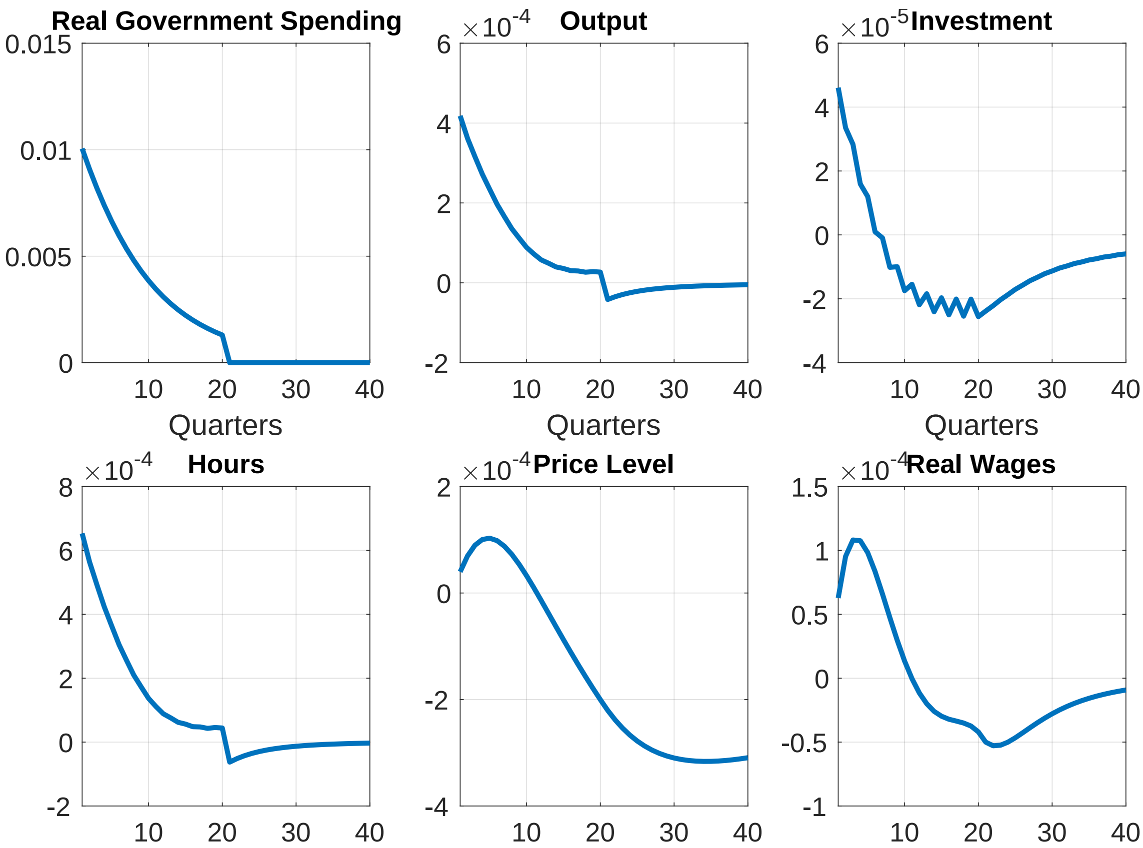
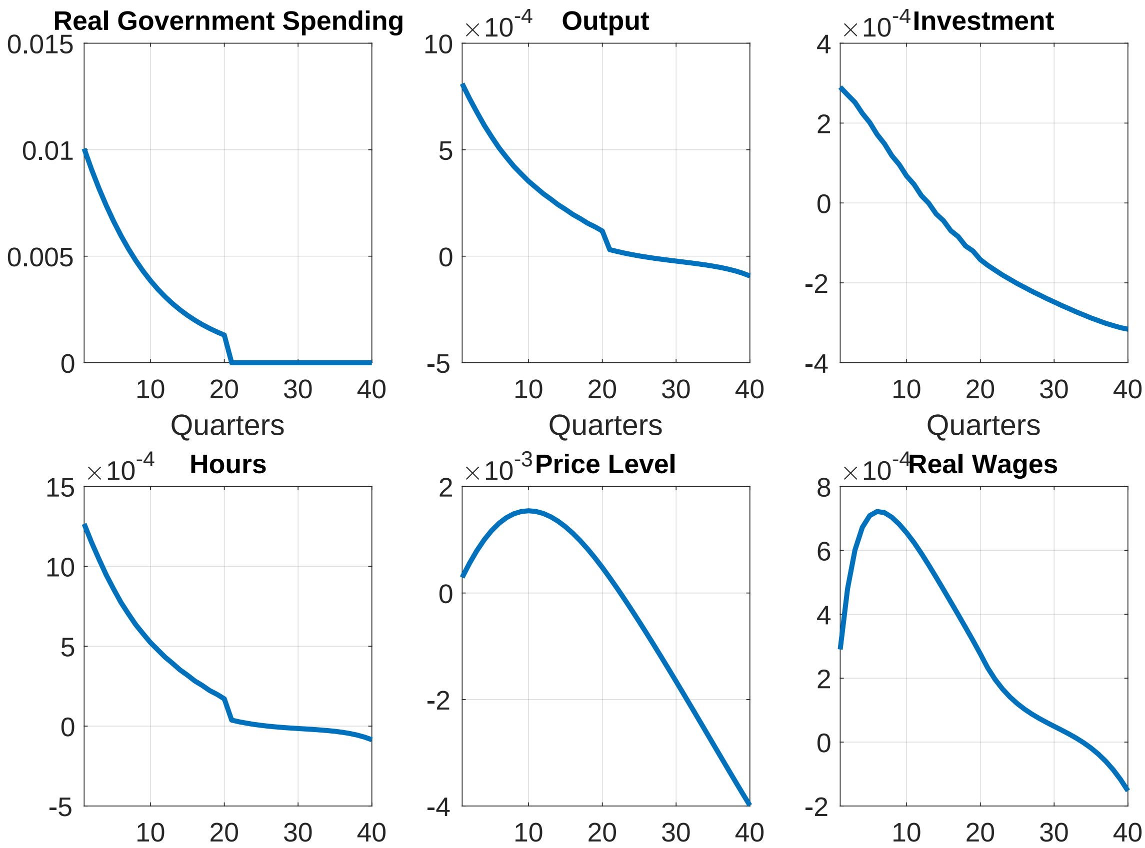
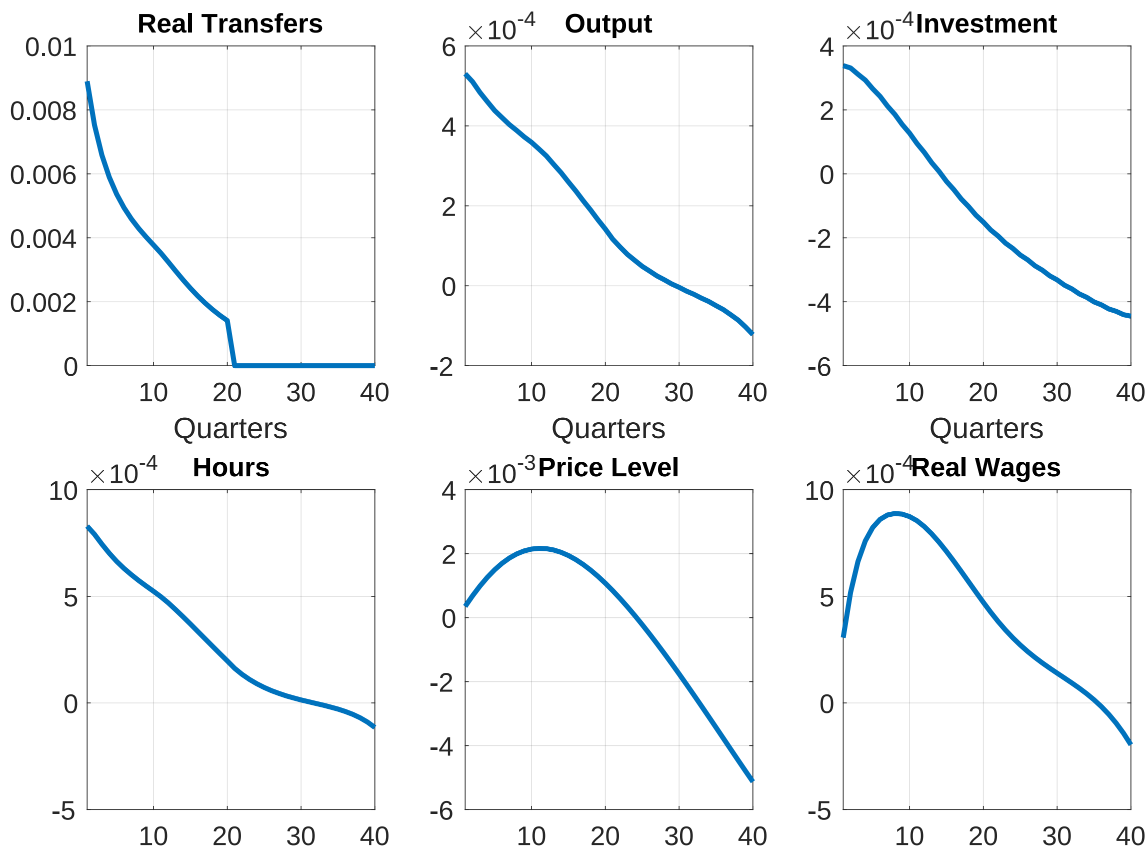


Figure A-8: Impulse response to a 1% increase in nominal government spending: **Deficit Financing**



I.6 Impulse Responses: Transfer Multiplier

Figure A-9: Impulse response to a 1% increase in nominal government **Transfers**: **Deficit Financing, Liquidity Trap**



II Derivations and Proofs

II.1 Derivation Pricing Equation

The firm's pricing problem is

$$V_t\left(p_{jt-1}\right) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t} y\left(p_{jt}; P_t, Y_t\right) - S_t(Y_{jt}) - \frac{\theta}{2}\left(\frac{p_{jt}}{p_{jt-1}} - \overline{\Pi}\right)^2 Y_t + \frac{1}{1+r_t} V_{t+1}\left(p_{jt}\right),$$

subject to the constraints $y_{jt} = Z_t K_{jt}^{\alpha} H_{jt}^{1-\alpha}$ and $y(p_{jt}; P_t, Y_t) = \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} Y_t$.
Equivalently,

$$V_t\left(p_{jt-1}\right) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t}\left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} Y_t - S_t(Y_{jt}) - \frac{\theta}{2}\left(\frac{p_{jt}}{p_{jt-1}} - \overline{\Pi}\right)^2 Y_t + \frac{1}{1+r_t} V_{t+1}\left(p_{jt}\right).$$

The FOC w.r.t p_{jt} is

$$(1-\epsilon)\left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} \frac{Y_t}{P_t} + \epsilon mc_{jt} - \theta\left(\frac{p_{jt}}{p_{jt-1}} - \overline{\Pi}\right) \frac{Y_t}{p_{jt-1}} + \frac{1}{1+r_t} V'_{t+1}(p_{jt}) = 0 \tag{A1}$$

and the envelope condition is

$$V'_{t+1} = \theta\left(\frac{p_{jt+1}}{p_{jt}} - \overline{\Pi}\right) \frac{p_{jt+1}}{p_{jt}} \frac{Y_{t+1}}{p_{jt}}. \tag{A2}$$

Combining the FOC and and the envelope condition,

$$\begin{aligned} & (1-\epsilon)\left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} \frac{Y_t}{P_t} + \epsilon mc_{jt} \\ - \quad & \theta\left(\frac{p_{jt}}{p_{jt-1}} - \overline{\Pi}\right) \frac{Y_t}{p_{jt-1}} + \frac{1}{1+r_t} \theta\left(\frac{p_{jt+1}}{p_{jt}} - \overline{\Pi}\right) \frac{p_{jt+1}}{p_{jt}} \frac{Y_{t+1}}{p_{jt}} = 0. \end{aligned} \tag{A3}$$

Using that all firms choose the same price in equilibrium,

$$\begin{aligned} & (1-\epsilon) + \epsilon mc_t \\ - \quad & \theta\left(\pi_t - \overline{\Pi}\right) \pi_t + \frac{1}{1+r_t} \theta\left(\pi_{t+1} - \overline{\Pi}\right) \pi_{t+1} \frac{Y_{t+1}}{Y_t} = 0. \end{aligned} \tag{A4}$$

II.2 Derivation Wage Equation

$$\Theta\left(s_{it}, W_{it}, W_{it-1}; Y_t\right) = s_{it} \frac{\theta_w}{2}\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w\right)^2 H_t.$$

The middleman's wage setting problem is to maximize

$$
\begin{aligned}
& V_t^w\left(\hat{W}_{t-1}\right) \\
\equiv\ & \max_{\hat{W}_t} \int \Big(\frac{s_{it}(1-\tau_t)\hat{W}_t}{P_t} h(\hat{W}_t; W_t, H_t) - \frac{g(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} di - \int s_{it} \frac{\theta_w}{2}\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w\right)^2 H_t di \\
+\ & \frac{1}{1+r_t} V_{t+1}^w\left(\hat{W}_t\right),
\end{aligned}
\tag{A5}
$$

where $h_{it} = h(W_{it}; W_t, H_t) = \left(\frac{W_{it}}{W_t}\right)^{-\epsilon_w} H_t$.

The FOC w.r.t $\hat{W}_t$

$$
(1-\tau_t)(1-\epsilon_w)\left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w} \frac{H_t}{P_t} + \epsilon_w \frac{g'(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} \left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w - 1} \frac{H_t}{W_t} \tag{A6}
$$

$$
-\theta_w\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi^w}\right)\frac{H_t}{\hat{W}_{t-1}} + \frac{1}{1+r_t} V'_{t+1}(\hat{W}_t) = 0 \tag{A7}
$$

and the envelope condition

$$
V'_{t+1} \quad = \quad \theta_w\left(\frac{\hat{W}_{t+1}}{\hat{W}_t} - \overline{\Pi^w}\right)\frac{\hat{W}_{t+1}}{\hat{W}_t}\frac{H_{t+1}}{\hat{W}_t}, \tag{A8}
$$

where we have used that $\int s = 1$.

Combining the FOC and and the envelope condition

$$
\begin{aligned}
& (1-\tau_t)(1-\epsilon_w)\left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w} \frac{H_t}{P_t} + \epsilon_w \frac{g'(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} \left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w - 1} \frac{H_t}{W_t} \\
-\ & \theta_w\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi^w}\right)\frac{H_t}{\hat{W}_{t-1}} + \frac{1}{1+r_t}\theta_w\left(\frac{\hat{W}_{t+1}}{\hat{W}_t} - \overline{\Pi^w}\right)\frac{\hat{W}_{t+1}}{\hat{W}_t}\frac{H_{t+1}}{\hat{W}_t} = 0
\end{aligned}
\tag{A9}
$$

Using that $\hat{W}_t = W_t$, $\pi_t^w = \frac{W_t}{W_{t-1}} = \frac{\hat{W}_t}{\hat{W}_{t-1}}$ and $h_{it} = H_t$:

$$
\begin{aligned}
& (1-\tau_t)(1-\epsilon_w)\frac{W_t}{P_t} + \epsilon_w \frac{g'(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} \\
-\ & \theta_w\left(\pi_t^w - \overline{\Pi^w}\right)\pi_t^w + \frac{1}{1+r_t}\theta_w\left(\pi_{t+1}^w - \overline{\Pi^w}\right)\pi_{t+1}^w \frac{H_{t+1}}{H_t} = 0
\end{aligned}
\tag{A10}
$$