HALL, “USING EMPIRICAL MARGINAL COST TO MEASURE MARKET POWER IN THE US ECONOMY”

Distribution of true value across industries
**Market power**

Lerner index:

\[ \mathcal{L} = \frac{p - \frac{\partial c}{\partial y}}{p} = \frac{1}{\epsilon} \]
Market power

Lerner index:

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Ratio of price to marginal cost,

\[ \mu = \frac{p}{\partial c/\partial y} = \frac{1}{1 - \mathcal{L}} = \frac{\epsilon}{\epsilon - 1} \]

which maps the Lerner index from \( \mathcal{L} \in [0, 1] \) to \( \mu \in [1, \infty] \).
Measuring $\mathcal{L}$

*Demand side:* Measure residual demand elasticity $\epsilon$ by some IV strategy based on an oligopoly model.
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*Profit margin:* Use accounting data to compare total revenue to total cost or to variable cost.
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*Marginal revenue product:* Estimate the production function and compare the output elasticity of a factor to its revenue share.
**Empirical partial derivative**

Numerator is the change in cost not associated with changes in factor prices and the denominator is the change in output not associated with the change in Hicks-neutral productivity.
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Change in cost is

\[ dc = \sum_i x_i \, dw_i + \sum_i w_i \, dx_i \]
Empirical partial derivative

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Cost is

\[ c = \sum_i w_i x_i \]

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The first summation is the component associated with changes in factor prices, while the second is the desired component purged of effects from changing factor prices:

\[ \sum_i w_i dx_i \]
**ADJUSTED CHANGE IN OUTPUT**

The technology is

\[ y = A f(x) \]

so output growth is

\[ dy = Adf(x) + f(x)dA = Adf(x) + y \frac{dA}{A} \]
**Adjusted change in output**

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\[ dy = Adf(x) + f(x) dA = Adf(x) + \frac{y dA}{A} \]

The desired component purged of effects from changing productivity is

\[ Adf(x) = dy - y \frac{dA}{A} \]
EMPIRICAL MARGINAL COST

Marginal cost is the ratio of adjusted cost change to adjusted output change,

\[ m = \frac{\sum_i w_i \, dx_i}{dy - y \, dA/A} \]
Empirical marginal cost

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\[ m = \frac{\sum_i w_i dx_i}{dy - y \frac{dA}{A}} \]

The Lerner index is

\[ L = \frac{p - m}{p} = 1 - \frac{\sum_i w_i dx_i}{p(dy - y \frac{dA}{A})}. \]

so

\[ 1 - L = \frac{\sum_i w_i dx_i}{p(dy - y \frac{dA}{A})}. \]
Connect to the Solow residual

Let

\[ \alpha_i = \frac{w_i x_i}{p y}, \]

the share of factor \( i \) in revenue, \( p y \)
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The equation can then be written

\[
(1 - \mathcal{L}) \left( dy - y \frac{dA}{A} \right) = y \sum_i \alpha_i \frac{dx_i}{x_i}.
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\[ (1 - L) \left( dy - y \frac{dA}{A} \right) = y \sum_i \alpha_i \frac{dx_i}{x_i}. \]

Dividing by \( y \) and rearranging yields a useful result,

\[ \frac{dy}{y} - \sum_i \alpha_i \frac{dx_i}{x_i} = L \frac{dy}{y} + (1 - L) \frac{dA}{A}. \]
Relation to TFP data

With discrete time,

\[ \Delta \log y - \sum_i \alpha_i \Delta \log x_i = \mathcal{L} \Delta \log y + (1 - \mathcal{L}) \Delta \log A \]
Relation to TFP data

With discrete time,

$$\Delta \log y - \sum_i \alpha_i \Delta \log x_i = \mathcal{L} \Delta \log y + (1 - \mathcal{L})\Delta \log A$$

This formulation is useful because the left-hand side is the Solow residual, calculated meticulously in productivity accounts.
If $\mathcal{L} > 0$, the Solow residual does not measure actual technical progress, because it does not adjust for market power.
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This derivation of the measurement of $\mathcal{L} > 0$ does not assume anything about optimal choice by the firm, apart from remaining on its production function. The firm is not necessarily satisfying its first-order conditions in the output market or any input market. The Lerner index does not necessarily describe the residual demand function facing the firm, effects of market power by sellers of inputs including labor unions, or monopsony power of the firm in those input markets.
The adjusted growth rate of productivity, $a = (1 - \mathcal{L}) \Delta \log A$, is a statistical residual in the equation. It can only be measured with knowledge of the Lerner index.
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The most basic approach is to treat $\mathcal{L}$ as a parameter to be estimated in time-series or panel data, with suitable instrumental variables. Eligible instruments are variables that are uncorrelated with productivity growth but are correlated with output and inputs. The residual based on the estimated value of $\mathcal{L}$ is the estimated rate of true productivity growth, adjusted for market power.
Assume that the firm is a price-taker in all of its input markets, and the firm equates the marginal revenue product of a factor to its price.
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Then the approach yields values of the true Lerner index.
The assumption that the firm is a price taker in its input markets does not mean that those markets are competitive. That property is sufficient but not necessary.
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The price-taking assumption would apply if a labor union or dominant seller of another input chose to exercise its market power by sticking to a fixed non-negotiable price quote.
Returns to scale

Notice that the assumptions do not include constant returns to scale
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But the second-order condition for profit maximization requires that the Lerner index exceed $1 - 1/\gamma$, where $\gamma$ is the returns-to-scale index of the production function, the elasticity of $f(\theta x)$ with respect to $\theta$, at $\theta = 1$. 
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A firm with strong increasing returns and weaker market power will not satisfy the second-order condition.
Monopsony in input markets

Suppose the elasticity of the wage with respect to the firm’s level of employment is \( \lambda \). Then the observed labor share is depressed by the fact that the average wage understates the marginal wage:

\[
\alpha = \frac{w n}{p y} = (1 - L) \frac{\gamma}{1 + \lambda}
\]
Monopsony in input markets

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$$\alpha = \frac{wn}{py} = (1 - \mathcal{L}) \frac{\gamma}{1 + \lambda}$$

This propagates through the rest of the math to the conclusion,

$$\frac{dy}{y} - \alpha \frac{dn}{n} = \frac{\mathcal{L} - \lambda}{1 + \lambda} \frac{dy}{y}$$
MONOPSONY IN INPUT MARKETS

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\frac{dy}{y} - \alpha \frac{dn}{n} = \mathcal{L} - \lambda \frac{dy}{1 + \lambda} y
\]

Thus the coefficient on the right side of the equation is \( \frac{\mathcal{L} - \lambda}{1 + \lambda} \), which is less than \( \mathcal{L} \) for any positive value of the monopsony parameter \( \lambda \).
CONCLUSIONS ABOUT APPLICABILITY

*Increasing returns to scale*. The approach is robust to increasing returns.
Conclusions about applicability

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**Decreasing returns to scale.** This occurs when factors, notably capital, involve delays, adjustment costs, or permanent restrictions. The approach is robust to decreasing returns, which will be accompanied by profit in excess of factor costs.
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*Market power held by a seller of an input.* If a seller of an input, such as a labor union, exercises its market power by setting a higher price, the approach takes account of the true marginal cost associated with that input, and the calculation uncovers the true Lerner index of the firm.
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Monopsony power in an input market. The average price paid for the input understates the effective marginal price. The employment share is understated and the estimate of $L$ is correspondingly understated.
KLEMS data in the Solow productivity framework
Data

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Annual starting in 1987; 60 distinct non-overlapping industries
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Annual starting in 1987; 60 distinct non-overlapping industries

Advantages of the data relative to data in earlier work on production-side measurement of market power

- Rigorous adherence to proper measurement of output—no reliance on value added
- Uniform use of the modern NAICS industry definitions
- Breakdown of inputs into 5 categories: capital, labor, energy, materials, and services
- Aggregation of capital and labor inputs from detailed underlying data using appropriate methods
- Use of Tørnqvist’s refinement of the weights applied to log-changes in factor inputs
Instrumental variables—log differences

- Military purchases of equipment
- Military purchases of ships
- Military purchases of software
- Military expenditure on research and development
- The oil price
Sampling variation

30 percent of the industries have negative values of the estimated Lerner index, \( L_i \) even though the true value of \( \mathcal{L} \) cannot be negative
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The statistical model is

$$L = \mathcal{L} + \eta$$

where $\mathcal{L}$ is distributed as beta($\nu, \beta$), with density proportional to $\mathcal{L}^{\nu-1}(1 - \mathcal{L})^{\beta-1}$.
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where $\mathcal{L}$ is distributed as beta($\nu, \beta$), with density proportional to $\mathcal{L}^{\nu-1}(1 - \mathcal{L})^{\beta-1}$

The measurement error $\eta_i$ accounts for the residual distribution of the measured index.
Four assumptions identify the model:

1. The true value of the Lerner index obeys the beta distribution, so it is between zero and one
2. The second shape parameter of the beta distribution of the true Lerner index is $\beta = 8$, a reasonable family
3. The two components are statistically independent, a standard assumption
4. The mean of the measurement error $\eta$ is zero, another standard assumption
The Family of Beta Distributions with Second Shape Parameter = 8
The desired untangling is possible

Identification Theorem: The mean of the measured Lerner index identifies the first shape parameter of the beta distribution of the true Lerner index; the distribution of the measurement error $\eta$ is identified by solving a convolution problem.
Moments of the Distribution of the Estimated Lerner Index, and Inferred Properties of the Distributions of the True Index and the Error in Measurement

<table>
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<tr>
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<th>Mean</th>
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<tr>
<td>Moments of estimated</td>
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<tr>
<td>Lerner indexes across</td>
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<tr>
<td>industries</td>
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<td>Skewness</td>
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<td>Shape parameter of true</td>
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<td>Lerner index</td>
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<td>Moments of true Lerner</td>
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<td>indexes across industries</td>
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<td>errors</td>
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Inferred Distributions of True Lerner Index across Industries
Actual Cumulative Frequencies of Estimates and Calculated Cumulative Distribution Functions from the Statistical Model
The change in the Lerner index over time

Extend the specification to include an industry-specific linear time trend:

$$\Delta \log y_t - \sum_i \alpha_{i,t} \Delta x_{i,t} = (\phi_i + \psi_i t) \Delta \log y_t - a_t$$
Evidence about the Statistical Reliability of the Finding of an Upward Trend in the Markup Ratio

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<table>
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<tbody>
<tr>
<td>Weighted average of estimate of trend $\psi$</td>
<td>0.0061</td>
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<tr>
<td>Standard error</td>
<td>0.0051</td>
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<tr>
<td>$t$-statistic for hypothesis $\psi = 0$</td>
<td>1.20</td>
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<tr>
<td>$p$-value, one-tailed</td>
<td>0.11</td>
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Implied Values of the Lerner Index by Year
**Conclusions**

- **Empirical partial derivative method—Hall (2018)**
  - **Strengths**
    - Uses excellent data
    - Avoids challenging estimation of production functions
    - Robust to increasing or decreasing returns to scale
  - **Weakness**
    - Relies on industry aggregates and misses within-industry heterogeneity

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  - **Strength**
    - Uses data on individual firms and handles firm-level heterogeneity
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Conclusions, continued

  - Strengths
    - Uses excellent data
    - Avoids challenging estimation of production functions
    - Helps measure returns to scale
  - Weakness
    - Measures market power only on condition of constant returns