The Rise and Fall of Labor Force Growth: Implications for Firm Demographics and Aggregate Trends*

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Abstract

The US economy has undergone a number of puzzling changes since the 1970s. The business startup rate has declined from 14 to 8 percent, the aggregate firm exit rate has declined from 9 to 7 percent, the aggregate labor share has declined by 6 percentage points, and economic activity is increasingly concentrated in large firms. This paper shows that the rise and fall of labor force growth provides a unified quantitative explanation for these apparently disparate trends. We find that accounting for changes in firm demographics is crucial. The changes in labor force growth lead to changes in firm demographics, which affect aggregate trends. Quantitatively, the model accounts for most of the changes in aggregate trends.

J.E.L. Codes: J11, E13, E20, L16, L26
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1 Introduction

The US economy has undergone a number of puzzling changes in recent decades. Since the 1970s, the business startup rate has declined from 14 to 8 percent, the rate of firm exit has declined from 9 to 7 percent, the (corporate) labor share has declined by 6 percentage points, and economic activity is increasingly concentrated in large firms. This paper provides a unified quantitative explanation for these apparently disparate trends.

The startup rate is related to the exit rate, the labor force growth rate, and the growth rate in average firm size by an accounting identity,

$$\underbrace{\lambda}_{\text{Startup Rate}} = \underbrace{\xi}_{\text{Exit Rate}} + \underbrace{\dot{N}}_{\text{LF Growth}} - \underbrace{\dot{e}}_{\text{AFS Growth}} \tag{1}$$

This identity comes from the definition of average firm size, $e \equiv N/M$, where N is the number of workers and M is the number of firms. It follows that the growth rate in the number of firms is labor force growth minus the growth rate of average firm size, $\dot{M} = \dot{N} - \dot{e}$. The change in the number of firms equals the startup rate minus the exit rate, $\dot{M} = \lambda - \xi$. Combining these two equations leads to accounting identity (1).

In the stationary equilibrium of a standard firm dynamics model, or in models with no firm and age dynamics, average firm size and exit rate are constant. Therefore, changes in labor force growth become a natural candidate to study changes in the startup rate. Can a change in labor force growth, by itself, explain the drop in startup rates? Qualitatively, yes. Labor force growth has declined by roughly 2 percentage points since the 1970s. Quantitatively, no. The accounting identity (1) implies that the drop in labor force growth by itself can only account for about a third of the 6 percentage point drop in the startup rate.

The remaining 4 percentage point decline in the startup rate is accounted for by the exit rate (2pp) and average firm size (2pp). Therefore, accounting for the change in the exit rate and the average firm size is crucial for explaining the full extent of the startup rate decline. Why have exit and average size changed? We find that firm demographics are largely responsible. Since the 1980s, the age distribution has shifted towards older firms. Because older firms are bigger and exit at lower rates, an aging distribution naturally generates a lower aggregate exit rate and a higher average firm size.

We show that changes in population demographics lead to changes in firm demographics. A change in the growth rate of the labor force affects the startup rate contemporaneously. As a result, the firm age distribution changes, leading to a change in both the aggregate exit rate and average firm size in the next period. By the accounting identity (1), these changes feed back into the startup rate next period. Because of the feedback effect, the history of past entry matters for firm demographics.

Figure 1 shows the civilian labor force growth rate in the US for each decade starting in the 1940s. The rapid rise in labor force growth from the 1940s to the 1970s amplifies the effects of the subsequent decline in the labor force. The rapid increase in labor force growth generates a rapid increase in startup rates and in the number of firms. The subsequent aging of these earlier startups results in a larger pool of incumbents, leaving less room for potential startups to fill when labor force growth declines. Therefore both the rise and fall of labor force growth are important to capture the impact of population demographics on firm demographics. We show that a standard general equilibrium firm dynamics model fed with the rise and fall in labor force since the 1940s is able to account for the decline in the startup rate, the decline of the exit rate, and the increase in average firm size.

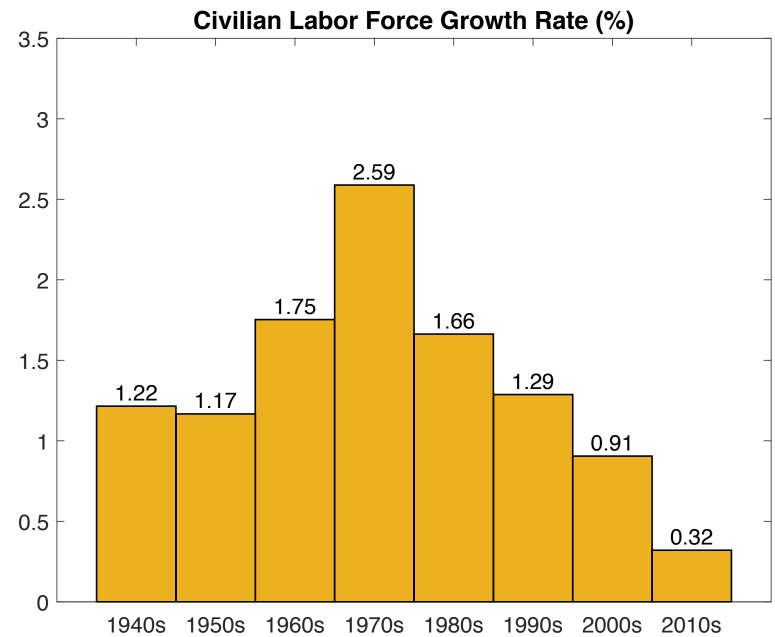


Figure 1: Civilian Labor Force Growth Rate

In addition, the calibrated model generates an increased concentration of economic activity in large firms. Specifically, the employment share of firms with 250 employees or more increases in the model by a similar amount as in the data. Since firm age and size are correlated then firm aging shows up as concentration by size. Similar to the model, we find that firm aging in the data is primarily responsible for the observed increase in employment concentration at large firms. We look within age-size categories and find that it is age that matters for concentration, not size. For example, the employment share of young large firms in the data has declined by 11 percentage points while the employment share of mature small firms increased increased by 4.5 percentage points. Our model matches these changes in employment share by age-size categories.

Finally, firm aging combined with overhead labor can generate the decline in the aggregate labor share. In the presence of fixed overhead labor, a firm's labor share is decreasing with size as the overhead labor represents a smaller fraction of revenues as firms grow. Therefore, an aging firm distribution gives higher weights to firms with lower labor share, causing the aggregate labor share to decline. Overhead labor also generates a dispersion of labor productivity across firms, a point made by Bartelsman, Haltiwanger and Scarpetta (2013). Reassuringly, we find that our model implied measures of labor productivity dispersion and overhead labor are close to those reported in previous studies.

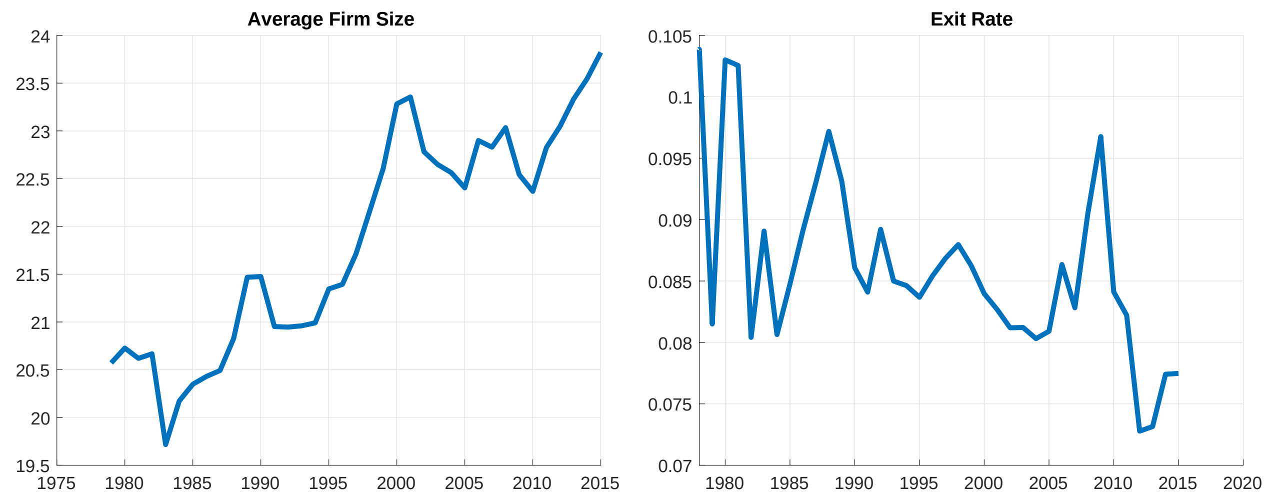
We then test the ability of the model to explain movements in the startup rate in earlier decades. Using novel firm data from 1940 to 1962, we find that changes in the labor force growth rate, when fed through the model, are able to match the size of the fluctuations in the startup rate for that time period. Specifically, the large fluctuations in labor force growth associated with World War II lead to equally large fluctuations in the startup rate, which we observe in the data.

Related Literature. Our paper builds on a wealth of recent empirical evidence from disconnected strands of the literature. Pugsley and Şahin (2018) document that the decline in the startup rate is widespread across sectors and regions and is linked to the aging of the firm distribution. Karabarbounis and Neiman (2014) find that the decline in the labor share is primarily a within-industry rather than a cross-industry phenomenon. Grullon, Larkin and Michaely (2017) document increased concentration across most U.S. industries, whereas Barkai (2017) documents a positive correlation between industry concentration and the decline in the labor share. Lastly, Autor, Dorn, Katz, Patterson and Van Reenen (2017) document a negative relationship between firm size and firm labor share. Our paper incorporates all of these empirical findings into one unified explanation.

We are not the first paper to propose changes in labor force growth as a driver of aggregate trends. Karahan, Pugsley and Şahin (2018) find a positive correlation between the decline in the startup rate and the decline in labor force growth across states, and explore the relationship in a Hopenhayn (1992a)-style model. Our study is broader in scope and it emphasizes a different mechanism, firm demographics and the history of past entry, which we find to be more quantitatively important than the direct effect of labor force changes on the startup rate.

2 Data

We obtain data on firms from the Business Dynamics Statistics (BDS) produced by the US Census Bureau. The BDS dataset has near universal coverage of private sector firms with



(a) Average firm size (b) Exit rate

Figure 2

paid employees in the US from 1977-2014.

We start by looking at the time series of the aggregate exit rate and average firm size in US data. Figure 2a shows the average firm size in the US has increased steadily from 19 employees to 23.5 employees. Figure 2b shows the aggregate exit rate has declined steadily from 9 percentage points to about 7 percentage points in the data.

What is driving the aggregate trends in average firm size and exit rates? Figure 3 shows average firm size and exit rates by firm age over time. The figure shows that, conditional on age, firms are not getting larger. For example, a typical five year old firm has same size in 1980 and 2014, with no discernible trend. The same pattern holds for exit rates: conditional on age, firms exit rates do not exhibit a trend over the 1977-2014 time period. Therefore, Figure 3 suggests that the aggregate trends in average firm size and exit rates are not driven by changes in the corresponding variables at the individual firm level.

Table 1 shows the time-series mean of average firm size and exit rate by firm age over the 1977-2014 period. The table highlights how average firm size and exit rates evolve as the typical firm ages. Average firm size increases with firm age. The typical startup contains about 6 employees, whereas the typical 26+ year old firm contains about 38 employees. Firm exit rates follow the opposite pattern: young firms exit at higher rates than older firms. About 22 percent of zero year old firms exit by age one. In contrast, the exit rate for 26+ year old firms is about 5 percent.

What about changes in the age composition of US firms? Firms in the US are getting older. Figure 4 shows the share of firms aged 11+ in US data over time. The share of 11+ firms has risen steadily from 32 percent in 1986 to 49 percent in 2015. This fact suggests that the composition of firms by ages might be an important driver of aggregate trends.

Figure 3: Average firm size and exit rate by age in the data

Table 1: Average firm size and exit rate by age

Age	Average firm size	Exit rate (%)
0	6.05	–
1	7.73	21.85
2	8.46	15.86
3	9.14	13.43
4	9.77	11.68
5	10.36	10.48
6-10	11.98	8.34
11-15	15.13	6.44
16-20	19.11	5.54
21-25	24.48	5.04
26+	38.34	4.73

Because older firms are bigger, see table 1, the aging of firm population might be driving the increase in average firm size in the data. Similarly, because older firms exit at lower rates, firm aging might be driving the decline in aggregate exit rates observed in the data.

Can firm aging account for concentration? As noted in the introduction, firm concentration is measured as share of employment by firms of size 250+. Concentration in the data has gone up by about 7 percentange points, from 51% in 1977 to 58% in 2015. We shed new light on this fact by documenting that firm aging plays an important role in driving the

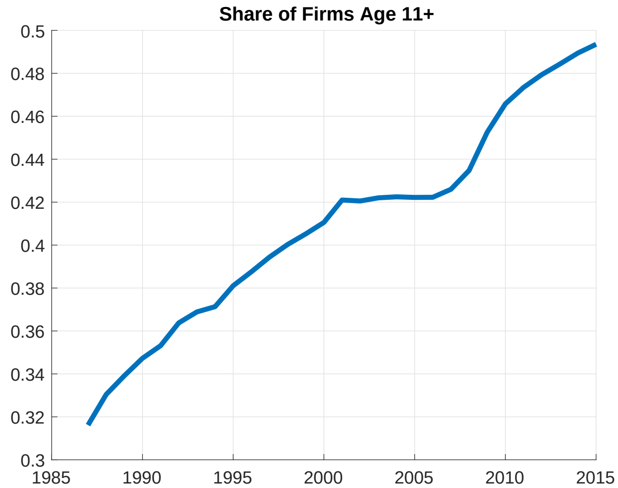


Figure 4: Share of firms age 11+ in the data

increase in firm concentration. Table 2 shows the share of employment by firms aged 11+ has increased by 14 percentage points over roughly the same time period. Firm size and firm age are correlated in the data, so it is not surprising that the employment share of both categories increases over the same time period. To isolate the role of firm size and age, we decompose firm concentration by age-size categories. We divide firms by size into small ($<$ 250 employees) and large (250+ employees), and by age into young (age $<$ 11) and mature (age 11+). Table 2 shows changes in the employment share of each age-size category over the 1986-2014 period. The table shows that age is an important driver of the increase in concentration in the data. Regardless of size, the employment share of young firms in the data has declined. Similarly, the employment share of mature firms has increased, for both small and large sizes. If we condition on firm size, the employment share of young firms has declined, while the share of older firms has increased. These facts suggest that firm aging shows up as increased concentration in the data.

Regressions. While the evidence in Figures 3 and 4 suggests firm aging plays an important role in driving the aggregate trends in average firm size and firm exit rates, it is not conclusive. It might be the case that the trend is due to other forces. For example, the US economy has undergone structural transformation over the sample period. The share of manufacturing in GDP has declined, while the share of services has increased. If firms in the service sector are larger, changes in sectoral composition might be driving the aggregate.

To separate the effect of firm aging, we calculate the trend in average firm size and aggregate exit rate after controlling for firm age, sector and age-sector interactions. We

Table 2: Change in employment share by age-size categories

Category	Data (pp.)
Large	7.00
Mature	14.00
Young	
Small	-11.06
Large	-3.09
Mature	
Small	4.51
Large	9.63

Notes. Data is from 1987 to 2014. Small firms have less than 250 employees, and large firms have 250+ employees. Young firms have age less than 11 years, and mature firms have age 11+.

regress

$$y_{ajt} = \beta_y \text{year} + \sum_a \beta_a \text{age} + \sum_j \beta_j \text{sector} + \sum_a \sum_j \beta_{aj} (\text{age} \times \text{sector}) + \varepsilon_{ajt}$$

where y_{ajt} equals log average firm size or firm exit rates. We start with a specification that features the year variable with an intercept term. We then add age, sector, age-sector dummies in successive specifications.

Table 3 presents our findings for log average firm size. In the first specification, we find that the coefficient on year is positive and statistically significant. This specification indicates that log average firm size grows at of 0.6% a year, averaged across age groups and sectors. Once we control for firm age, however, the year coefficient becomes negative. After controlling for age, log average firm size declines at 0.5% per year. The negative trend in average firm size is robust to the inclusion of sector and age-sector interactions. Table 3 shows that the coefficient on each age category is statistically different from zero, at the 1% level, and increasing in age. This is exactly the pattern one would expect if firm aging was driving the increase in average firm size over time. As firms age, they get larger. Therefore an increase in the share of older firms would increase average firm size in the aggregate.

Table 4 presents our findings for firm exit rates. The patterns we found for average firm size repeat here. The first specification indicates that firm exit rates decline 0.15 percentage points every year. The addition of firm-age controls brings the year-coefficient close to zero. The inclusion of sector and age-sector controls does not change the trend. The table shows that older firms exit at lower rates. Therefore, an increase in the share of older firms will lead to a decline in the aggregate exit rate.

Table 3: Regression of log average firm size on year

Variable	Specification			
	(1)	(2)	(3)	(4)
Year	0.006***	-0.005***	-0.005***	-0.005***
	(0.001)	(0.001)	(0.000)	(0.000)
AGE:				
Age 0		1.839***	1.435***	1.441***
		(0.023)	(0.015)	(0.026)
Age 1		2.080***	1.676***	1.717***
		(0.023)	(0.015)	(0.026)
Age 2		2.171***	1.767***	1.806***
		(0.023)	(0.015)	(0.026)
Age 3		2.247***	1.843***	1.868***
		(0.024)	(0.015)	(0.026)
Age 4		2.319***	1.915***	1.941***
		(0.024)	(0.015)	(0.026)
Age 5		2.378***	1.974***	2.002***
		(0.024)	(0.015)	(0.027)
Age 6 to 10		2.526***	2.122***	2.159***
		(0.027)	(0.016)	(0.029)
Age 11 to 15		2.748***	2.344***	2.323***
		(0.029)	(0.017)	(0.032)
Age 16 to 20		2.977***	2.573***	2.472***
		(0.032)	(0.018)	(0.035)
Age 21 to 25		3.251***	2.847***	2.579***
		(0.035)	(0.019)	(0.039)
SECTOR CONTROLS	No	No	Yes	Yes
SECTOR×AGE CONTROLS	No	No	No	Yes
R^2	0.015	0.978	0.995	0.996
Observations	2,682	2,682	2,682	2,682

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 4: Regression of exit rate on year

Variable	Specification			
	(1)	(2)	(3)	(4)
Year	-0.151***	-0.011*	-0.011**	-0.011**
	(0.012)	(0.006)	(0.005)	(0.005)
AGE:				
Age 1		21.780***	19.381***	19.036***
		(0.178)	(0.188)	(0.342)
Age 2		16.143***	13.744***	12.702***
		(0.178)	(0.188)	(0.342)
Age 3		13.673***	11.274***	10.765***
		(0.181)	(0.190)	(0.347)
Age 4		12.029***	9.629***	9.380***
		(0.185)	(0.192)	(0.352)
Age 5		10.753***	8.354***	8.331***
		(0.189)	(0.194)	(0.358)
Age 6 to 10		8.647***	6.247***	6.695***
		(0.208)	(0.206)	(0.390)
Age 11 to 15		6.711***	4.312***	5.160***
		(0.225)	(0.218)	(0.421)
Age 16 to 20		5.901***	3.501***	4.582***
		(0.246)	(0.232)	(0.461)
Age 21 to 25		5.416***	3.017***	4.420***
		(0.271)	(0.250)	(0.514)
SECTOR CONTROLS	No	No	Yes	Yes
SECTOR×AGE CONTROLS	No	No	No	Yes
R^2	0.065	0.962	0.976	0.978
Observations	2,358	2,358	2,358	2,358

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

3 The Model

There is a single homogenous good and a fixed endowment of a resource (labor) N_t inelastically supplied that is also the numeraire. A production unit has production function $s_{it} f(n_{it})$ where s_{it} corresponds to a productivity shock, that follows a Markov process with conditional distribution $F(s_{t+1}|s_t)$, independent across firms. In addition, production units have a fixed cost c_f denominated in units of labor. which can include an entrepreneur and other labor overhead. If a production unit is shut down, its residual value is normalized to zero. All agents in the economy have common discount factor β.

Assumption 1. *The conditional distribution F is continuous and strictly decreasing in s_t.*

Let $c(s,q)$ and $\pi(s,p)$ denote cost and profit functions derived from the above technology, where q is output and p the price of the good in terms of labor and let $n(s,p)$ denote labor demand. Assume the last two functions are continuous and strictly increasing in s and p. The value of a production unit is given by the Bellman equation:

$$v_t(s,\mathbf{p}) = \max\left\{0, \pi(s,p_t) + \beta E v\left(s',\mathbf{p}|s\right)\right\}$$

when confronted with a deterministic path of prices $\mathbf{p}=\{p_\tau\}_{\tau\geq t}$. Here zero is the value of exit while the right hand side under the maximization is the continuation value of this productive unit. As shown in Hopenhayn (1992b), the value function is strictly increasing in s and p_t when nonzero. Letting

$$s_t^* = \inf\left\{s|\pi(s,p_t) + \beta E v\left(s',\mathbf{p}|s\right) > 0\right\}, \tag{2}$$

a production unit is shut down iff $s \leq s_t^*$.

The technology for entry of a new productive unit is as follows. Upon paying a cost of entry of c_e units of labor, the initial productivity is drawn from distribution G, independently across entrants and time. Prior to entry, the expected value of an entrant net of the entry cost

$$v_t^e(\mathbf{p}) = \int v_t(s,\mathbf{p})\, dG(s) - c_e. \tag{3}$$

Given an initial measure μ_0, the exit thresholds s_t^* together with entry flows m_t determine uniquely the measures sequence of measures $\{\mu_t\}$ as follows. For any set of productivities A, define recursively

$$\mu_{t+1}(A) = m_{t+1}\left(\int_{s\epsilon A, s\geq s_{t+1}^*} dG(s)\right) + \int\int_{s\epsilon A, s\geq s_{t+1}^*} dF(s|x)\, d\mu(x) \tag{4}$$

The first term in the right hand side corresponds to entrants, excluding those that exit immediately while the second term includes incumbents after the realization of new productivities, excluding those that exit.

3.1 Equilibrium

Let μ_t denote the measure of productive units operating at time t, where for a fixed set A of firm types, $\mu_t(A)$ measures the magnitude of firms that at time t have $s_{it} \epsilon A$. Let $M_t = \int d\mu_t(s)$ denote the total mass of firms. Denote by m_t the mass of entrants at time t. Labor market clearing requires that:

$$\int n(s, p_t)\, d\mu_t(s) + M_t c_f + m_t c_e = N_t. \tag{5}$$

The first term is productive labor demand, the second term overhead and the third labor utilized for entry, e.g. entrepreneurs in startups. The right hand side represents total labor inelastically supplied.

An equilibrium for a given sequence $\{N_t\}$ and given initial measure μ_0 is given by shutdown thresholds $\{s_t^*\}$, mass of entrants $\{m_t\}$, measures of production units $\{\mu_t\}$ and prices $\mathbf{p} = \{p_t\}$ such that:

1. Shutdown thresholds are given by equation (2);
2. No rents for entrants: $v_t^e(\mathbf{p}) \leq 0$ and $v_t^e(\mathbf{p})\, m_t = 0$;
3. Market clearing condition (5) holds.
4. The sequence μ_t is generated recursively by equation (4) given the initial measure μ_0.

Following (reference to my paper), there exists a unique equilibrium. It can be easily characterized when entry is strictly positive in every period, that is in reality the relevant case. Let p^* be a constant price such that $v_t^e(p^*) = 0$ for all t. Under the above assumptions this price is unique, corresponding to the stationary equilibrium price in Hopenhayn (1992b). Let $s_t^* = s^*$ be the corresponding shutdown threshold. Given the initial distribution μ_0 we derive recursively a sequence m_t^* so that the market clearing condition is satisfied in every period as follows.

Let S_n denote the probability that an entrant survives at least n periods, i.e. that the state $s_{i\tau} \geq s^*$ for ages τ from 0 to n. Let $\tilde{\mu}_n$ denote the cross-sectional probability distribution of productivities for production units in the age n cohort. These can be obtained recursively as follows:

1. Let $S_0 = (1 - G(s^*))$ and $\tilde{\mu}_0(ds) = G(ds)/S_0$, that is the distribution for entrants draws G conditional on $s_0 \geq s^*$.

2. Let $S_n = S_{n-1} \int P(s_n \geq s^* | s_{n-1})\, d\tilde{\mu}_{n-1}(s_{n-1})$, where the term under the integral is the probability that a productive in cohort $n-1$ is not shutdown in the next period, and let
$$\tilde{\mu}_n(ds) = \frac{\int P(ds_n | s_{n-1})\, d\tilde{\mu}_{n-1}(s_{n-1})}{S_n / S_{n-1}}.$$

Let $e_n = \int n(s, p^*)\, d\tilde{\mu}_n$, the average employment of a productive in the cohort of age n. Total employment of that cohort at time t depends on the original number of entrants in that cohort and the survival rate, namely

$$E_{tn} = m_{t-n} S_n e_n$$

Thus at time t employment from incumbents (i.e. excluding new entrants) is given by $E_t^I = \sum_1^t E_{tn}$. Adding the employment of entrants $m_t(S_0 e_0 + c_e)$ we get the market clearing condition:
$$N_t = m_t(S_0 e_0 + c_e) + E_T^I. \tag{6}$$

This determines implicitly m_t, the only unknown in the above equation at time t. Provided that $E_t^I < N_t$, entry will be strictly positive in every period and all equilibrium conditions are satisfied. Assuming N_t is an increasing sequence, a sufficient condition for positive entry every period is that $N_t - E_{t+1}^I > 0$. Note that

$$\begin{aligned} N_t &= m_t c_e + m_t S_0 e_0 + m_{t-1} S_1 e_1 + \ldots + m_0 S_t e_t \\ E_{t+1}^I &= m_t S_1 e_1 + m_{t-1} S_2 e_2 + \ldots + m_0 S_{t+1} e_{t+1}, \end{aligned}$$

so E_{t+1}^I is the inner product of the same vector of the mass of entrants with a forward shift in the corresponding terms $S_n e_n$ and excluding entry costs. Thus a sufficient condition for entry to be positive every period is that the term $S_n e_n$ decreases with n. This condition says that, adjusting for differences in entry rates, the total employment of a cohort is decreasing in age. Survival rates are decreasing in n by definition but average size of a cohort, if properly calibrated to the data, is increasing. Thus shutdown rates must be sufficiently high to offset the latter growth. In the model, this is a property that depends on the stochastic process for the shocks s_{it} and the threshold s^*. But given these parameters its easy to verify.[1]

[1] Models that assume permanent productivity shocks and exogenous exit trivially satisfy this condition as well as the technology in Mortensen and Pissarides (reference) where productivity shocks are redrawn with some probability from the same distribution as entrants.

Proposition 1. *Suppose that N_t is a nondecreasing sequence and $S_n e_n$ is non-increasing. Then the unique equilibrium has constant price $p_t = p^*$ and exit thresholds $s_t^* = s^*$.*

Corollary 1. *Under the Assumptions of Proposition 1, exit rates and average firm size, by cohorts are time invariant.*

This Corollary implies that changes in aggregate entry and exit rates as well as average firm size will be driven only by changes in firm demographics, the age distribution of firms and the rate of increase in population.

3.2 The Turnover of Production Units

In this section we examine the determinants of aggregate rates of entry and exit and in particular the role of firm demographics, i.e. the age distribution of production units.

Total exit ξ_t at time t is the sum of exit rates of different age cohorts. Exit of productive units of age n equals the difference in survival rates between $S_{n-1} - S_n$ multiplied by the size of that cohort, that is the entry mass at the time that cohort entered, m_{t-n}. We follow here the convention that the age at which a unit is shut down corresponds to the age at which this unit was last productive. As for the new entrants, while the model allows for immediate exit we will consider that $m_t S_0$ is in practice the measure of entry in this economy and thus exclude $m_t \left(1 - S_0\right)$ from total exits. It follows that

$$\xi_t = \sum_{n=1}^{t} m_{t-n} \left(S_{n-1} - S_n\right)$$

The number of firms at $t - 1$ is given by:

$$M_{t-1} = \sum_{n=1}^{t} m_{t-n} S_{n-1}$$

Letting α_{tn} denote the share of units of age n in the total population of productive units at time t and $h_{n-1} = (S_{n-1} - S_n)/S_{n-1}$ the hazard rate of exit of units of age $n - 1$, the aggregate exit rate ξ_t / M_{t-1} can be expressed as the weighted average of different cohort hazard rates of exit

$$\xi_t / M_{t-1} = \sum_{n=1}^{t} \alpha_{t-1,n-1} h_{n-1}.$$

Taking these hazard rates as fixed, this is only a function of the age distribution of productive units, which in turn is determined by past entry rates. The exception of course is when exit rates are the same for all cohorts in which case firm demographics plays no role.

Consider now entry rates.[2] Let $e_t = N_t/M_t$. It follows that the rate of growth in the number of firms

$$\frac{M_t}{M_{t-1}} = \frac{N_t}{N_{t-1}} \frac{e_{t-1}}{e_t}. \tag{7}$$

Now the mass of productive units can be decomposed in incumbents that survived plus entrants. Letting S_t denote the average survival rate from $t-1$ to t, it follows that

$$M_t = S_t M_{t-1} + m_t.$$

Substituting from M_t in (7) gives the following expression for the entry rate:

$$\lambda_t = \frac{m_t}{M_{t-1}} = \frac{N_t}{N_{t-1}} \frac{e_{t-1}}{e_t} - S_t \tag{8}$$

In the special case where average employment is constant, this equation reduces to

$$\lambda_t = \frac{N_t - N_{t-1}}{N_t} + \xi_t$$

so the entry rate equals the sum of the population growth rate and the exit rate; as average firm size is constant, the total mass of firms needs to grow at the rate of population growth to clear the market. Entry rate must be enough to replace the exiting units and in addition create this extra employment. This justifies the above formula.

In a balanced growth path where population grows at a constant rate, cohort entry weights decay as a function of age at this rate and average firm size is constant. More precisely, if $N_{t+1}/N_t = \gamma$, then

$$N_t = m_t \sum_{n=0}^{\infty} \gamma^{-n} S_n e_n$$

the total number of firms

$$M_t = m_t \sum_{n=0}^{\infty} \gamma^{-n} S_n$$

[2] We are defining as $m_t S_0$ as "measured entry". Had we assumed that all entrants must remain at least one period in the market, then $S_0 = 1$ so m_t would then be measured entry.

Exit at time t is $m_t (1 - S_0) + m_t \sum_{n=1}^{\infty} \gamma^{-n} (S_{n-1} - S_n)$ so that the exit rate

$$\begin{aligned} \xi_t &= \frac{m_t (1 - S_0) + m_t \sum_{n=1}^{\infty} \gamma^{-n} (S_{n-1} - S_n)}{M_{t-1}} \\ &= \frac{m_t (1 - S_0) + m_t \sum_{n=1}^{\infty} \gamma^{-n} (S_{n-1} - S_n)}{\gamma^{-1} M_t} \\ &= \frac{(1 - S_0) + \sum_{n=1}^{\infty} \gamma^{-n} (S_{n-1} - S_n)}{\sum_{n=0}^{\infty} \gamma^{-n} S_n}. \end{aligned}$$

Which is independent of t and so is the entry rate. The same holds in a model where productivity shocks are fully persistent or randomly redrawn from the same distribution as the one faced by entrants (as in Mortensen and Pissarides), average firm size is constant so the above formula applies. In particular this means that the rate of entry is independent of history and only depends on current population growth. If exit rates are not age dependent, the same will also be true for exit.

More generally, exit rates will depend on the age distribution of firms and thus the history of past entry. As survival rates are increasing in age, a larger share of young firms will be associated to higher exit rates and consequently higher entry. In addition, changes in average firm size will impact entry rates. A rise in entry rates will increase the share of younger productive units which tend to be smaller. This should lower average firm size, and from equation (8) increase the rate of entry. Thus a rise in population growth will lead to increased entry rates over and above those needed to accommodate the increase in the labor force. This multiplier effect will operate similarly in the opposite direction when facing a decrease in the rate of growth in the labor force. Our main result is stated below.

Theorem 1. *Assume survival rates S_n and average firm size e_n are increasing in age. An increase (decrease) in the rate of growth of the labor force will result in an increase (decrease) of entry rates over and beyond the rate of increase (decrease) in the labor force.*

3.3 Temporary Increase in Population Growth

The results described in the previous section can be illustrated in the face of a temporary increase in the growth rate of the labor force. Suppose initially the labor force is stationary. It then grows at a constant rate γ for T periods to then return indefinitely to its initial zero level. The rate of exit is exogenous, so $S_n = (1 - \delta)^n$ and average firm size e_n increases with the age of the cohort at rate g, so $e_n = (1 + g)^n$ where $0 < g < \delta$. At the original steady state there is a mass M_0 if firms, so $m = \delta M_0$ and average firm size is $\bar{e}_0 = \delta e_0 / (\delta - g)$. For long T, the rate of entry converges to $\lambda_T = \delta + \gamma$, the relative share of firms of the cohort of age n to $\left(\frac{\delta+\gamma}{1+\gamma}\right)\left(\frac{1-\delta}{1+\gamma}\right)^n$ and average firm size to

$$\left(\frac{\delta+\gamma}{1+\gamma}\right)\sum\left(\frac{(1-\delta)(1+g)}{1+\gamma}\right)^n=\left(\frac{\delta+\gamma}{1+\gamma-(1-\delta)(1+g)}\right)$$

4 Quantitative Analysis

The quantitative exercise is is as follows. We assume the US economy was in a stationary equilibrium before the 1940s. We then feed the time series of the civilian labor force growth rate in the data, starting in 1940, through the model economy. We calibrate the model such that the model-implied time series match the data in 1979, the first year for which we have firm-level data. This calibration strategy allows us to capture the effects of the rise and the fall in the labor force on firm demographics, without requiring firm-level data from the 1940s.

Functional forms. The production function of a firm is $f(s,n)=sn^{\alpha}$, where $0<\alpha<1$ captures decreasing returns to scale at the firm level. Firm productivity follows an AR(1) process:

$$\log(s_{t+1})=\mu_s+\rho\log(s_t)+\varepsilon_{t+1};\qquad \varepsilon_{t+1}\sim\mathcal{N}(0,\sigma_{\varepsilon}^2) \tag{9}$$

with ρ as the persistence, μ_s as the drift and σ_{ε}^2 as the variance of shocks. The distribution of startup productivities G is normal with mean s_0 and variance $\sigma_{\varepsilon}^2/(1-\rho^2)$.

Calibration. The model period is set to one year. The time discount factor is $\beta=0.96$, which implies an annual interest rate of 4%. The returns to scale parameter is $\alpha=0.64$, set to match the labor share in the data. The steady-state labor force growth rate g is set to the standard value of one percent.

The parameters c_e, c_f, μ_s, ρ, σ_{ε} and s_0 need to be calibrated. The entry cost parameter c_e is set such that $p^*=1$. As discusssed in Hopenhayn and Rogerson (1993), normalizing the equilibrium price gets around the identification problem that arises because price and the idiosyncratic shock enter the firm's objective function multiplicatively. The remaining parameters are calibrated to match the startup rate in 1979, average startup size in the sample, average firm size in 1979, unconditional 5-year firm exit rates and conditional 5-year firm growth rates.

We target the startup rate in 1979 in order to have a common starting point for the time series of the startup rate in the model and the data. We can then evaluate model performance by comparing how the two series evolve over the subsequent years. Average startup size in the data has changed little over time, so we target its sample average. Average startup size in the model is constant over time. It is determined primarily by the mean of the startup distribution, s_0. The dispersion of the productivity process affects the weight on productivity gridpoints where firms exit, so it primarily targets the 5-year exit rate. As with the startup rate, the average firm size in 1979 is targeted because we use 1979 as the common starting point for the model. The parameter c_f enters into the firm size calculation and plays an important role in determining average firm size. The persistence parameter ρ determines how quickly firms grow, so we use it to target the 5-year growth rate of firms. Table 5 summarizes the parameter values.

Table 5: Calibration parameters and corresponding targets in the data and model

	Assigned		
	Value	Definition	Basis
β	0.96	Discount factor	Annual real interest rate of 4%
α	0.64	Returns to scale	Labor share
g	0.01	Labor force growth rate (SS)	Standard

	Calibrated				
	Value	Definition	Target	Data	Model
c_e	0.084	Entry cost	$p^* = 1$	—	—
c_f	4.45	Operating cost	Average firm size in 1979	19.50	19.60
s_0	-2.49	Mean of G	Average startup size	6.05	6.02
μ_s	-0.007	Drift in AR(1)	Startup rate in 1979	13.83%	13.51%
ρ	0.965	Persistence of AR(1)	5-year growth rate	72.00%	72.88%
σ_ε^2	0.068	Variance of shocks	5-year exit rate	51.61 %	52.86%

Results. Table 5 shows how well the model matches the calibration targets. The parsimonious model does a good job of matching the targets.

Figure 5 shows how the model generated time series for the startup rate compares to the startup rate in the data. The figure includes the growth rate of the civilian labor force in the data that is fed through the model. The rise and fall of labor force growth is evident in the inverse U-shape observed in the data, with the peak around 1979. The startup rate follows the same inverse-U pattern. We are calibrating the model to 1979 only, so there is no apriori reason why the startup rate in the data for other years should match the model-implied series. As the figure shows, the model does an excellent job of matching key features of the data. The model generates the steady decline in the startup rate observed since the 1980s. The startup rate in the data declined from 13.83% to 8.11%, whereas the startup rate in the model declined from 13.51% to 7.86%. Therefore the model generates almost all of the decline observed in the data.

The recent literature on the business startup rate focuses on the steady decline observed in the data since 1979. As Figure 5 shows, the decline was preceded by an increase in the startup rate. The data exhibit a steady increase in the startup rate from 1950 to 1979.[3] The model generates the increase in the startup rate observed in the data. The civilian labor force growth rate spiked during World War II. Figure 5 shows that there was a corresponding spike in startup rate in both the model and the data. The ability of the calibrated model to match both the long term trends and short term fluctuations indicates that the central role of changes in the labor force growth rate in the evolution of the startup rate.

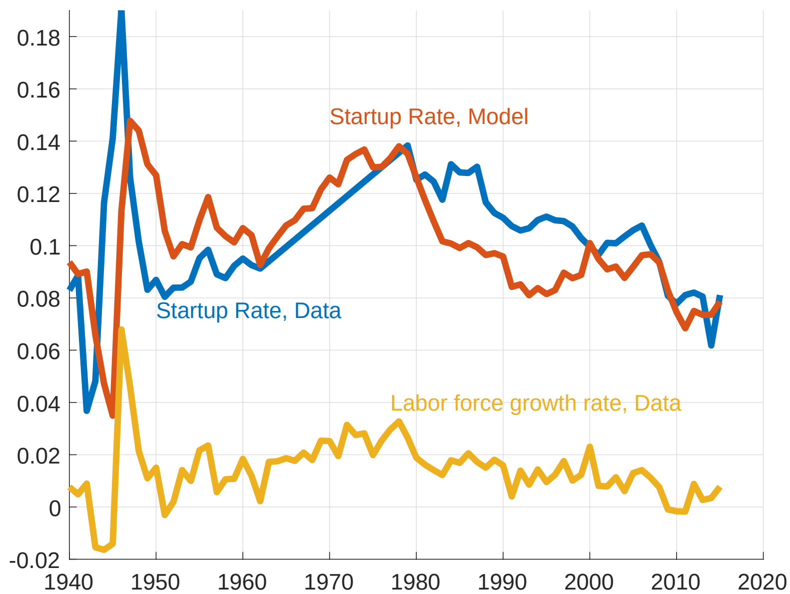
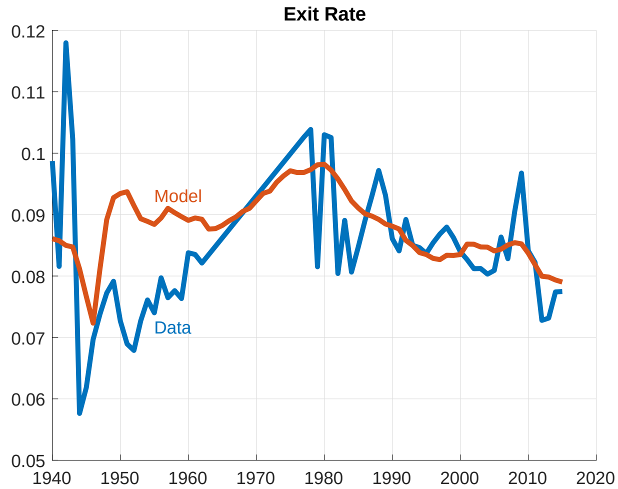


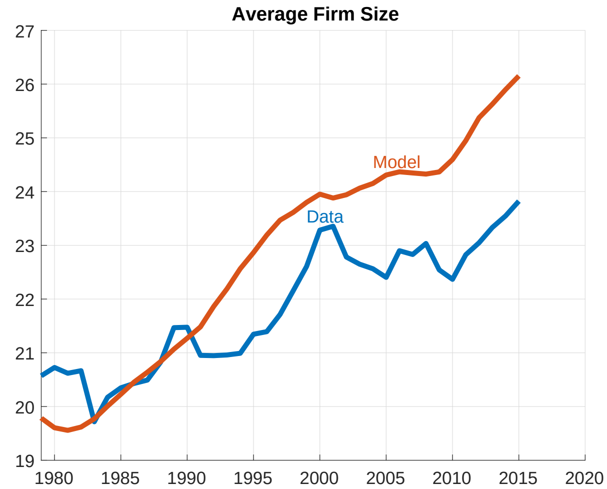
Figure 5: Startup rate in the model and the data

The startup rate declined by 6 percentage points from its peak in 1979. However, the labor force growth rate in the data declined by roughly 2 percentage points over the same time period. By the accounting identity (1), the remaining 4 percentage points must be accounted-for by the aggregate exit rate and by changes in average firm size. How do the model generated time series for these variables compare to the data? Figures 6a and 6b compare the model generated time series to the data. Figure 6a shows that the model does an excellent job of matching the steady decline in exit rates observed in the data since 1979.

[3]The startup rate from 1940 to 1962 is obtained from the U.S. Department of Commerce's Survey of Current Business. The startup rate is 'New Businesses' divided by the 'Operating Businesses'. The 1963 edition was the last one to report a 'Business Population and Turnover' section. From 1963, the Survey of Current Business reports instead 'Business Incorporations', which only include stock corporations. We linearly interpolate the data from 1962-1979. Karahan, Pugsley and Şahin (2018) estimate the startup rate using establishment level data and find a similar increase for 1965 to 1979.



(a) Exit rate



(b) Average firm size

Figure 6

The exit rate in the model declines by 2 percentage points from 1979-2014, same as in the data. Given that the model matches the startup rate and the exit rate well, the accounting identity (1) implies that the model will match the increase in the average firm size observed in the data. Figure 6b confirms this prediction.

What is the underlying mechanism that allows the model to match the time series? Firm demographics. Table 6 presents exit rate and average firm size by firm age. Overall the model does an excellent job matching average exit rate by firm age. This is perhaps not very surprising, given that we are calibrating to the 5-year unconditional exit rate. The model, however, overshoots average firm size by age for older firms. Firms in the model grow at a faster rate than firms in the data. This high growth rate comes from matching the low average startup size and high average firm size. In the data, firm growth exhibits a discontinuity: the growth rate of 0-year old firms is almost twice that of 1-year olds, but the growth rate of 1-year olds is only slightly higher than 2-year olds. This discontinuity is absent in the model, so the model overshoots firm growth rates.

Given that the model does a good job of matching firm exit rates and average firm size by age, the ability of the model to match the aggregate time series of these variables depends on how well it matches the age distribution of firms. Figure 7 shows the share of firms aged 11+ in the model and the data over time. The model does an excellent job of capturing firm aging observed in the data. The share of 11+ firms in the data increased by 18 percentage points in the data and by 15 percentage points in the model. The aging of the firms in the model is driven by the change in labor force growth: as the rate of labor force growth declines, the startup rate declines and the firm-age distribution shifts towards older firms.

Table 6: Exit rate and average firm size by age in the data and model

Age	**Exit rate**		**Average firm size**	
	Data(%)	Model(%)	Data	Model
0	–	–	6.05	6.02
1	21.85	22.09	7.73	6.73
2	15.86	15.29	8.46	7.51
3	13.43	12.21	9.14	8.37
4	11.68	10.40	9.77	9.34
5	10.48	9.19	10.36	10.41
6-10	8.34	7.32	11.98	14.06
11-15	6.44	5.71	15.13	21.88
16-20	5.54	5.00	19.11	30.53
21-25	5.04	4.61	24.48	38.80
26+	4.73	4.15	38.34	56.29

Counterfactual. To evaluate the contribution of firm demographics towards generating the time series for the startup rate, we shut down that channel and recalibrate the model. We do so by assuming that there is a representative firm in the economy. The firm productivity process in the modified economy follows a process with $\mu_s = 0$, $\rho = 1$ and $\sigma_\varepsilon = 0$. The resulting transition matrix is the identity matrix, $F = \mathbf{I}$, implying that the productivity of a firm equals its productivity drawn at birth. Firms do not grow or shrink in this economy. Therefore there is no exogenous exit. We generate exit by assuming that all firms exit at a constant rate δ.

We recalibrate the modified economy. We calibrate δ and mean of startup productivity distribution s_0 to match the startup rate and the average firm size in 1979. All other parameters, except c_e, equal their benchmark values. As in the benchmark, the entry cost c_e is set such that $p^* = 1$. The calibrated parameter values are $\delta = 11.07\%$, $s_0 = -2.84$ and $c_e = 0.006$.

Figure 8 shows the model-implied startup rate when we shutdown the effect due to firm demographics. In that case the decline in the startup rate from its peak in 1979 to the end of the sample is much smaller than the benchmark. The startup rate declines by 6 percentage points in the benchmark case, while the decline in the counterfactual is 2 percentage points. If we look earlier in the sample, the increase in the startup rate in the modified economy is much smaller than the benchmark. Because it features a representative firm, the exit rate and average firm size are constant in the modified economy. Therefore it cannot match the time-series evolution of either the exit rate or average firm size. The counterfactual exercise demonstrates the importance of firm demographics in driving aggregate trends in the startup rate, exit rate and average firm size.

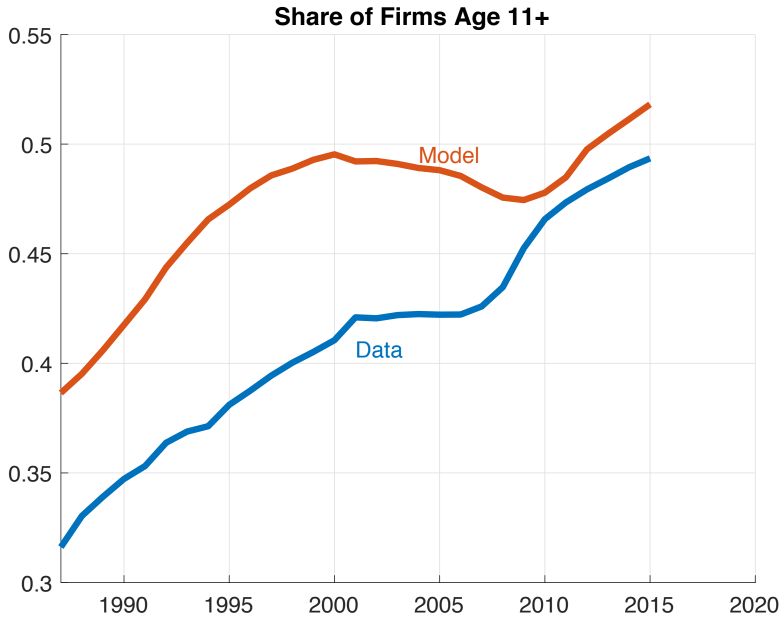


Figure 7: Share of firms aged 11+ in the model and the data

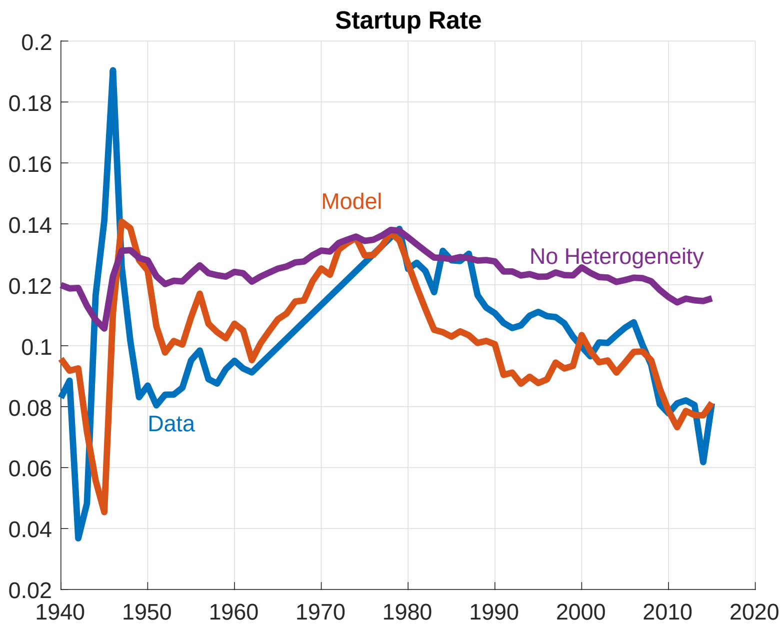
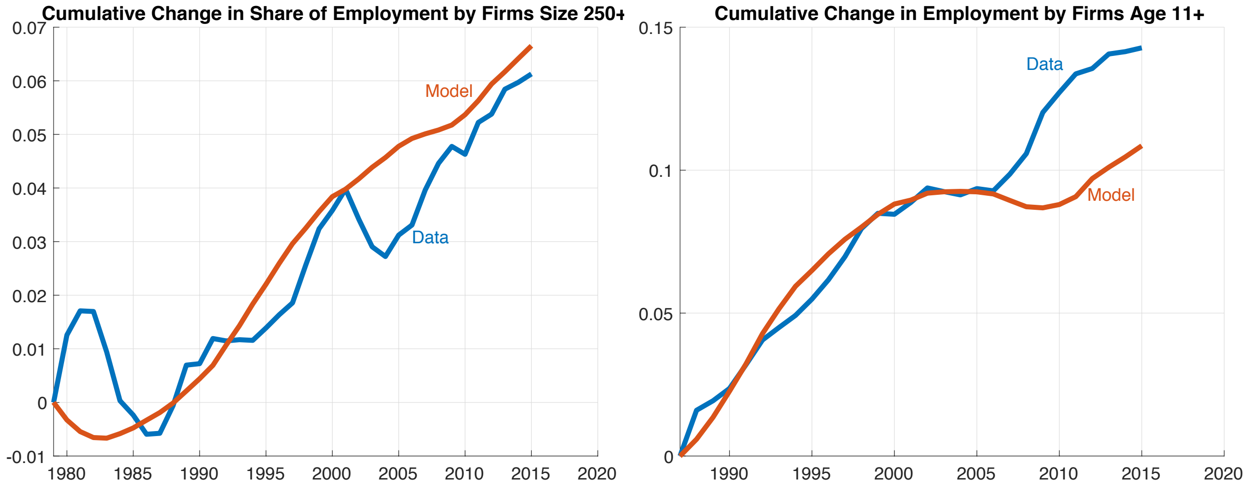


Figure 8: Startup rate counterfactual



(a) Concentration by size

(b) Concentration by age

Figure 9: Firm concentration by size and age in the model and the data

4.1 Non-targeted moments

Concentration. We documented in Section 2 that firm aging is an important driver of the recent increase in firm concentration observed in the US. Because the calibrated model replicates the pattern of firm aging in the data, we ask whether the model can also generate an increase in firm concentration. We use two different measures of firm concentration. *Concentration by size* is the share of employment in firms with 250+ employees. *Concentration by age* is the share of employment in firms aged 11+. Because age and size are correlated, we expect the two measures to exhibit similar patterns. Figures 9a and 9b show the cumulative change in concentration by size and age in the model and the data. The data on concentration by age starts in 1987 because we cannot identify firms aged 11+ in the data before that year. As the figures show, the model does an excellent job of matching the increase in concentration by both size and age.

As we did for the data in Section 2, we decompose the cumulative change in concentration in the model into age-size categories. Table 7 compares the model to the data. The model matches a key feature of the data: concentration is primarily about firm age. Young firms have a lower employment share in both the model and the data, regardless of size. Similarly, mature firms have a higher employment share regardless of size. The model does an excellent job of matching the magnitudes of the decline in the employment share by age-size categories.

Aggregate labor share. A recent literature documents that the aggregate labor in the US has declined since the 1980s; see e.g. Karabarbounis and Neiman (2014). Autor, Dorn, Katz, Patterson and Van Reenen (2017) use Census data to document that firm-level labor

Table 7: Change in employment share by age-size categories from 1987 to 2015

Category	Data (pp.)	Model (pp.)
Young		
Small	-11.06	-10.52
Large	-3.09	-1.01
Mature		
Small	4.51	3.25
Large	9.63	7.66

Notes. Small firms have less than 250 employees. Large firms have 250+ employees. Young firms have age less than 11 years. Mature firms have age 11+.

shares are negatively related to firm size. Our model generates this negative relationship because of overhead labor. The ratio of overhead labor to total employment is greater for smaller firms in our model. Therefore, firm-level labor shares decline with firm size. Because size and age are positively correlated in the data, firm aging might generate a decline in the aggregate labor share if it shifts the firm-size distribution towards larger firms. Figure 10 plots the cumulative change in the aggregate labor share in the model and the data. The aggregate labor share in the model is measured as the sum of firm-level labor shares, weighted by the value-added weight of the firm. The model does an excellent job of matching the decline in the aggregate labor share. The labor share in the model declines by 6 percentage points, while that in the model declines by 5 percentage points.

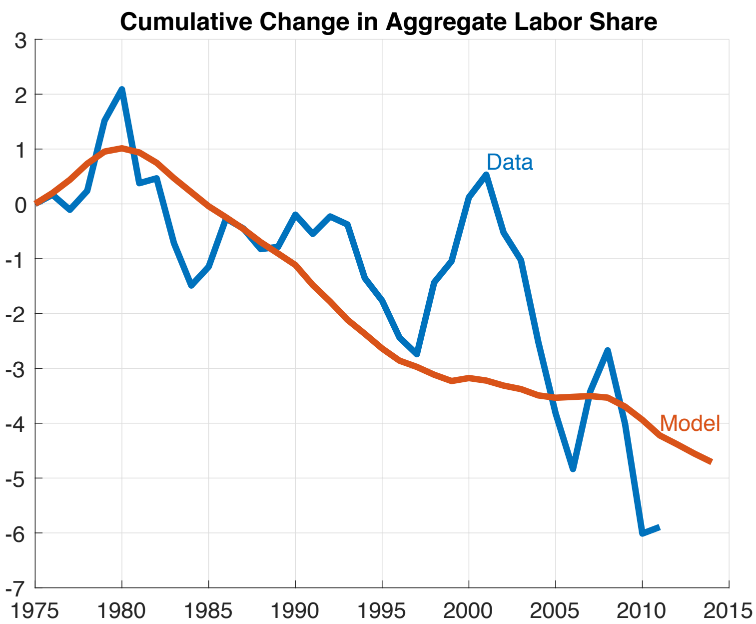


Figure 10: Cumulative change in aggregate labor share

Because overhead labor is central to generating the negative relationship between firm-level labor shares and firm size, we evaluate model performance along other non-targeted moments related to overhead labor. Table 8 summarizes our findings. Discussing manufacturing data, Ramey (1991) proposes that a reasonable value for the percentage of total employment that goes towards overhead labor is 20 percent. The corresponding number in the calibrated model is 18 percent. Bartelsman, Haltiwanger and Scarpetta (2013) emphasize the central role of overheard labor in generating dispersion in labor productivity in a frictionless model of firm dynamics. They report the standard deviation and OP covariance of log labor productivity in the US. Our model matches the standard deviation of 0.58 in the US data almost perfectly. The model underestimates the OP covariance, 0.82 in the data versus 0.51 in the model. As Bartelsman, Haltiwanger and Scarpetta (2013) note, it is not possible to match both the standard deviation and the OP covariance in standard models of firm dynamics. To quantify the negative relationship between the two variables, we regress labor share on firm size on simulated data from the calibrated model. We find a regression coefficient of -0.32, which is consistent with the range of coefficients provided by Autor, Dorn, Katz, Patterson and Van Reenen (2017).

Table 8: Non-targeted moments related to overhead labor

	Model	Estimate	Source	Notes
Overhead Labor/Total Employment	0.18	≈ 0.20	Ramey	Manufacturing Data
Std. Dev. Labor Productivity	0.57	0.58	BHS	Aggregate Data
OP Covariance Labor Productivity	0.82	0.51	BHS	Aggregate Data
Reg. Coeff. Labor Share on Firm Size	-0.32	[-2.37, 0]	ADKPV	Various Sectors

Ramey: Ramey (1991)
BHS: Bartelsman, Haltiwanger and Scarpetta (2013)
ADKPV: Autor, Dorn, Katz, Patterson and Van Reenen (2017)

5 Conclusion

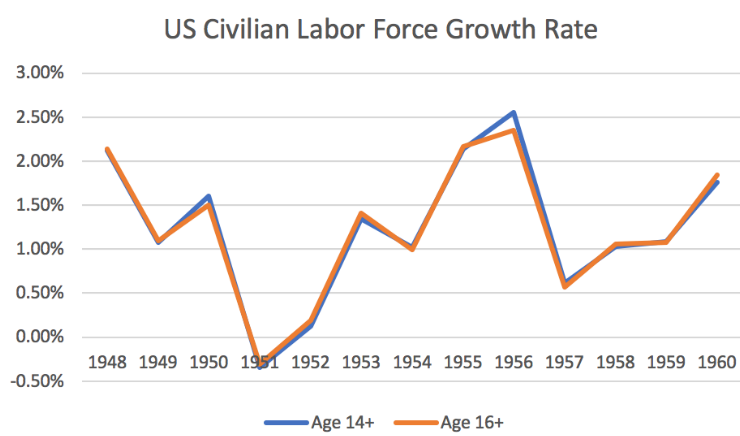
Recent decades have witnessed a decrease in startup rates, firm exit rates and an increase in average firm size in the US. We show that the interplay of population and firm demographics can account for much of these trends. Taking labor force growth as the driving force and feeding it through a calibrated general equilibrium firm dynamics model replicates the patterns well. We emphasize the role of firm demographics and the history of past entry. In the process, we uncover the key role played by firm aging in driving concentration and the decline in the labor share. Overall, our paper provides a unified quantitative explanation for a set of apparently disparate trends.

Appendix A Data Appendix

Firm-level data 1979-2015. Data to calculate firm-level data comes from the Business Dynamics Statistics (BDS) produced by the U.S. Census Bureau. The BDS dataset has near universal coverage of private sector firms with paid employees. BDS data starts in 1977, but best practice suggests dropping the years 1977 and 1978 due to suspected measurement error (e.g. Moscarini and Postel-Vinay, 2012). We follow the same practice.

Startup Rates 1940-1962. The startup rate from 1940 to 1962 is obtained from the U.S. Department of Commerce's Survey of Current Business. The startup rate is 'New Businesses' divided by the 'Operating Businesses'. The 1963 edition was the last one to report a 'Business Population and Turnover' section. From 1963, the Survey of Current Business reports instead 'Business Incorporations', which only include stock corporations. All non-farm businesses are included, regardless of size.

Civilian Labor Force Growth Rate 1940-2014. We obtain civilian labor force data from the Bureau of Labor Statistics (BLS) for the years 1947 to 2014, and from Lebergott (1964) from 1940 to 1947. The civilian labor force definition in BLS includes persons 16 years of age and over while in Lebergott the definition includes persons 14 years of age and over. Lebergott's measure using the 14+ definition covers the years 1900 to 1960, so we can compare the difference in growth rates using both rates. For the 1947-1960 period, the growth rates of both definitions of civilian labor force are nearly identical.



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