Annual Review of Economics

Idea Flows and Economic Growth

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- This paper was an invitation by the Annual Review of Economics. We offered a variety of theoretical models that share the common idea that people learn from interactions with other people

- Our interest began with ideas from Kortum and Eaton, using Frechet distributions on (1997, 1999, 2000). Developed by Alvarez, Buera, Lucas (2009, 2014). Many others involved as we will show

- I will sketch an early example of the kind of learning we have in mind. Then consider related work by Perla and Tonetti (2014) and Lucas and Moll (2014). Then Paco will take over to discuss some more recent work

- To begin, then, think of a closed economy where all agents produce single good with value $z \in (0, \infty)$ at date t. Law of motion is $F(z,t)$

- Agent z meets z' at rate α. Leaves with productivity $\max(z, z')$

$$\frac{\partial F(z,t)}{\partial t} = -\alpha F(z,t)\left[1 - F(z,t)\right]$$

$$\frac{\partial f(z,t)}{\partial t} = \alpha F(z,t) f(z,t) - \alpha f(z,t)\left[1 - F(z,t)\right]$$

- Riccati equation is

$$F(z,t) = \left\{1 + \left[\frac{1 - F(z,0)}{F(z,0)}\right] e^{\alpha t}\right\}^{-1}$$

- Fate of economy completely set at $t = 0$!

- Consider Balanced Growth Path (BGP), with CDF $\Phi(z)$ (density $\phi(z)$)

- Growth rate $\nu > 0$ such that

$$F(e^{\nu t}z, t) = \Phi(z) \quad \text{for all} \quad t \geq 0$$

- Turns out that possible BGP must be two parameter family

$$\Phi(z) = \frac{1}{1 + \lambda z^{-1/\theta}}\ , \quad \theta, \lambda > 0$$

- θ is a constant measure of variance of log earnings; α a constant; ν a constant growth rate

- Sometimes convenient to begin with "continuous arrival":

$$\frac{\partial \log F(z,t)}{\partial t} = \alpha \log F(z,t)$$

- With continuous arrival the solution is

$$\log F(z,t) = \log F(z,0) e^{\alpha t}$$

- Possible BGP requires

$$\log \Phi(z) = -\lambda z^{-1/\theta}, \;\; \theta, \lambda \;\; \text{for all} \;\; z > 0$$

  for some $\lambda > 0$

- CDF-growth rate pair (Φ, ν) is a BGP iff $\theta = \nu/\alpha$

- If we are thinking of each $F(z,t)$ as an individual agent (as I have been doing) we want to involve people of all ages, from birth to death or retirement

- If so then for economy-wide BGP it must be the case that given ν the collection of people of all ages Φ is constant over time

- For US census data at 10 year intervals from 1940-2010 this stands up pretty well (cf Jacob Mincer)

- None of these models so far involves people making choices. Let's look at two that do: Perla and Tonetti (2014) and Lucas and Moll (2014)

- Begin with familiar

$$\frac{\partial F(z,t)}{\partial t} = -\alpha F(z,t)\left[1 - F(z,t)\right]$$

- Now assume worker's time divided into fraction $\tilde{s}$ searching for ideas and $1 - \tilde{s}$ for producing goods (see Ben-Porath (1967), many others). Then

$$\frac{\partial F(z,t)}{\partial t} = -\left[1 - F(z,t)\right]\int_0^s \alpha\left[\tilde{s}(y,t)\right] f(y,t)dy$$

- Assume agents are risk neutral, expected present value maximizers

- Then

$$V(z,t) = \mathbf{E}_t \left\{ \int_t^\infty e^{-\rho(\tilde{s}-t)} \left[1 - \tilde{s}\left(\tilde{z}(\tau), \tau\right)\right] \tilde{z}(\tau) d\tau \mid \tilde{z}(\tau) = z \right\}$$

- Associated Bellman equation

$$\rho V(z,t) =$$

$$= \max_{\tilde{s}=[0,1]} \left\{ (1-\tilde{s})\, z + \frac{\partial V(z,t)}{\partial t} + \alpha(\tilde{s}) \int_z^\infty \left[V(y,t) - V(z,t)\right] f(y,t) dy \right\}$$

- BGP is a number γ (the same as $\nu = \theta\alpha$) such that

$$F(z,t) = \Phi(ze^{-\gamma t}) \quad \text{and} \quad V(z,t) = e^{\gamma t}\nu\left(ze^{-\gamma t}\right)$$

- Let $x = ze^{-\gamma t}$ to get

$$\phi(x)x\gamma = 1 - \Phi(x)\int_0^x \alpha\left[s(y)\right]\phi(y)dy$$

- In same way

$$(\rho - \gamma)\,\nu\,(x) + v'(x)x\gamma =$$

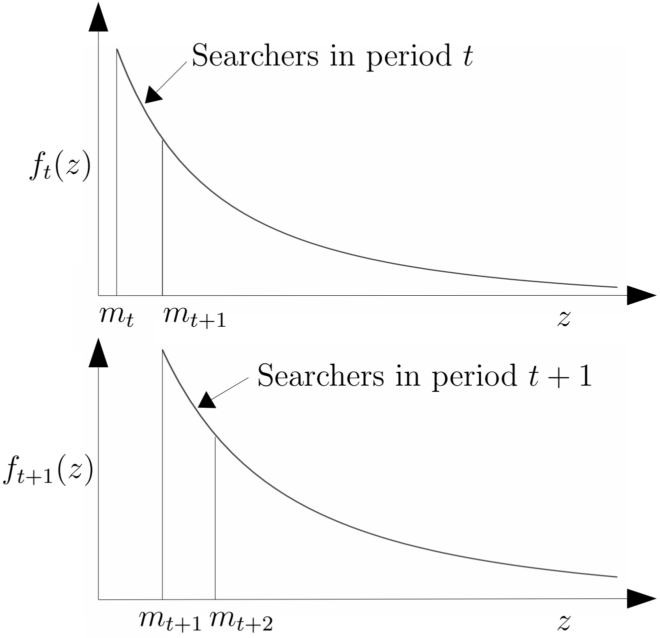
$$\max_{s=[0,1]}\left\{(1-\sigma)\,x + \alpha(s)\int_z^\infty\left[\nu\,(y) - \nu\,(x)\right]\phi(y)dy\right\}$$

- Good place to begin is the Perla and Tonetti model in which $\alpha(s)$ is linear: $\alpha(s) = \alpha s$. In this case $\alpha(s)$ will be either 0 or α. Agents choosing $s = 0$ will spend full time searching and produce nothing. Agents choosing $s = 1$ will produce with whatever skill z they have and maintain this level z into the immediate future.

- Perla and Tonetti assume that searchers draw from the conditional distribution

$$F(z, t \mid z > \zeta(t)) = \mathbb{P}\mathsf{r}\left\{\tilde{z} \leq z \mid \tilde{z} > \zeta(t)\right\}$$

- Agents below $\zeta(t)$ do not produce anything and we may as well think of them as a mass point π (say) at cutoff the $\zeta(t)$



Searchers in period t
$f_t(z)$
m_t
m_{t+1}
z
Searchers in period $t+1$
$f_{t+1}(z)$
m_{t+1}
m_{t+2}
z

- In our paper, we work with a continuous time version of the Perla and Tonetti model and specialize to a BGP. This lets us compare Perla and Tonetti with the model of Lucas and Moll that considers a full range of technology functions $\alpha(s)$

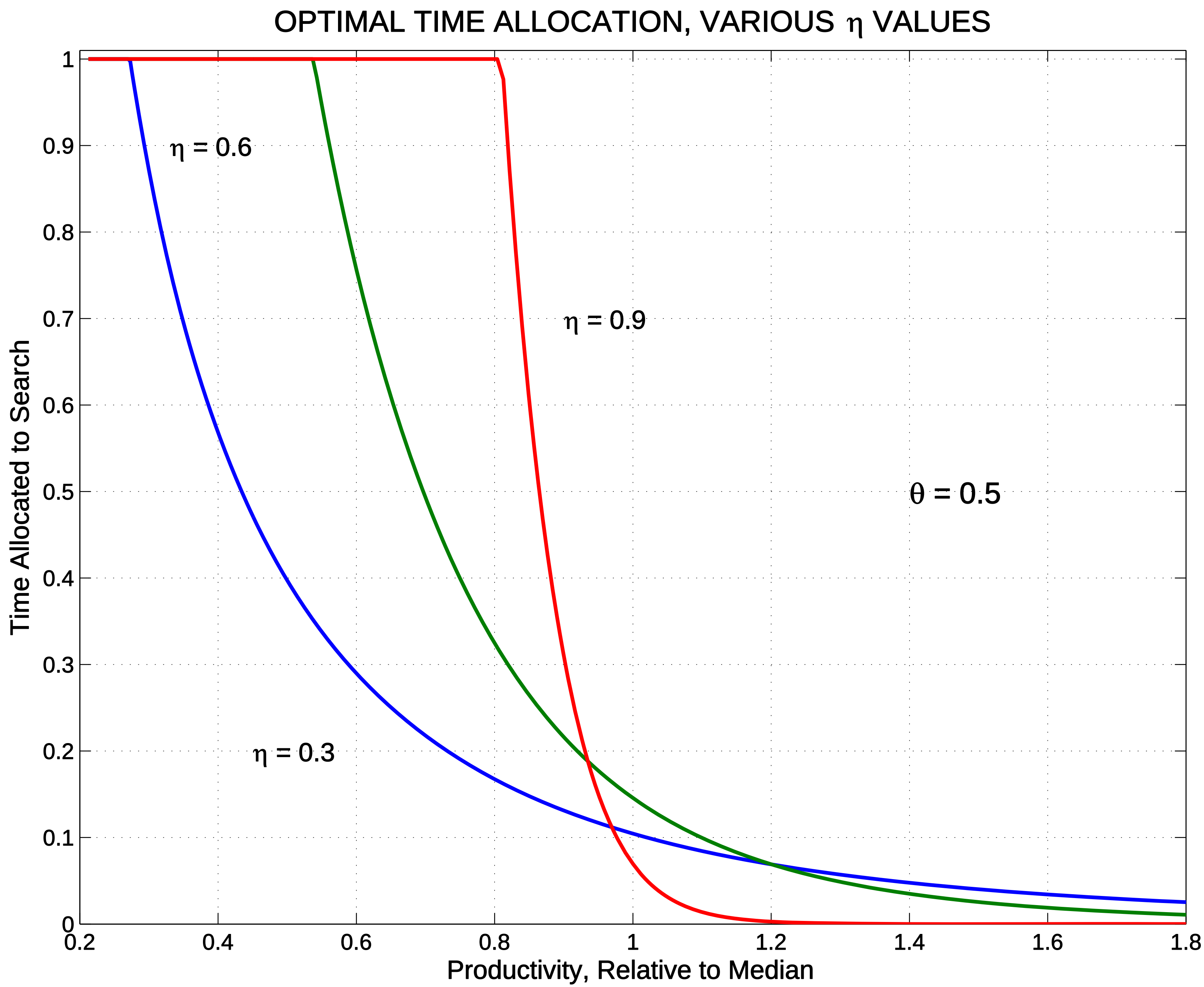
- In particular, we choose the function

$$\alpha(s) = \alpha_0 s^\eta$$

  where η ranges from 0 to 1.

- Then $\eta = 1$ fits the Perla and Tonetti model.

- $\eta < 1$ works for different levels of the Ben-Porath model

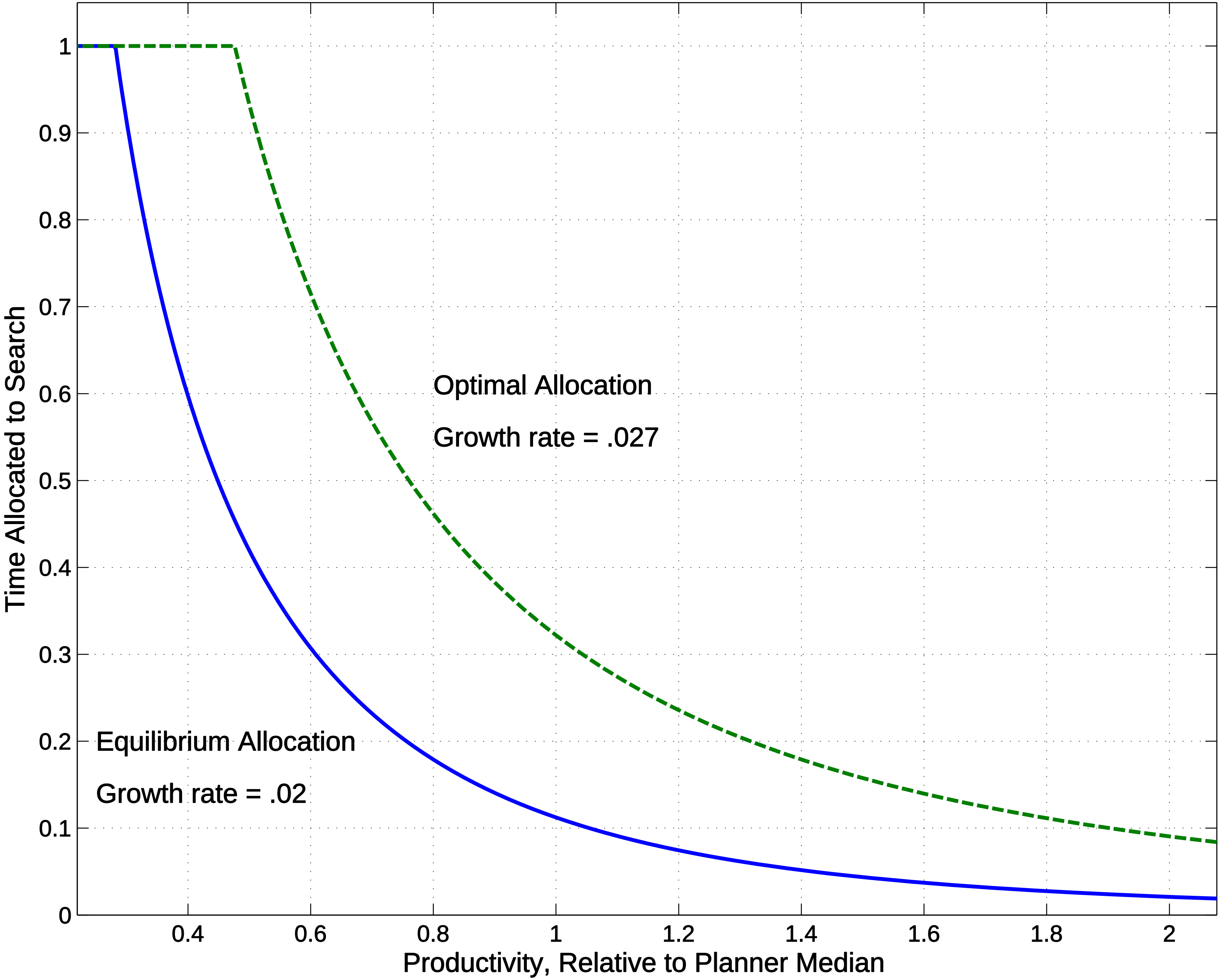


OPTIMAL TIME ALLOCATION, VARIOUS η VALUES
Time Allocated to Search
Productivity, Relative to Median
η = 0.6
η = 0.9
η = 0.3
θ = 0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

- Both these models have the feature that individual agents will not produce optimal results. We gain from meeting those who know more than we do but they do not charge us for this

- Of course all the successful economies support individual knowledge

- But are we getting it right?



Time Allocated to Search
Productivity, Relative to Planner Median
Optimal Allocation
Growth rate = .027
Equilibrium Allocation
Growth rate = .02
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Plan for the Remaining of Our Presentation

- Briefly discuss applications

- Analyze idea flows in finite economies, a response to skeptics

- Briefly discuss applications
  - Evolution of individual earnings in the US: Lucas (2009)
  - Evolution of the size of firms: Caicedo et al. (2019)
  - Trade and idea flows: Alvarez et al. (2014), Buera & Oberfield (2016), Perla et al. (2015), Sampson (2015)
  - Idea flows within organizations/firms: de la Croix (2016), Herkenhoff et al. (2018), Jarosh (2018), Kirker (2019)
- Analyze idea flows in finite economies, a response to skeptics

Trade and Idea Flows: Alvarez et al. (2014)

- Ricardian model of trade, countries $i = 1, ..., n$, wages $w_i(t)$, transportation costs κ_{ij}, preferences $U = \left[\int c(s)^{1-\frac{1}{\eta}} ds \right]^{\frac{\eta}{\eta-1}}.$

- Evolution of country i's distribution of productivity

$$\frac{\partial \log F_i(z,t)}{\partial t} = -\alpha_i \log G_i(z,t)$$

where

$$\begin{aligned} G_i(z,t) &= \Pr\{\text{sellers active in } i \text{ at } t \text{ has productivity} \leq z\} \\ &= \sum_{j=1}^{n} \int_0^z f_j(y,t) \prod_{k \neq j} F_k\left(\frac{w_k(t)\,\kappa_{ij}}{w_j(t)\,\kappa_{ik}} y, t\right) dy. \end{aligned}$$

- Common growth rate

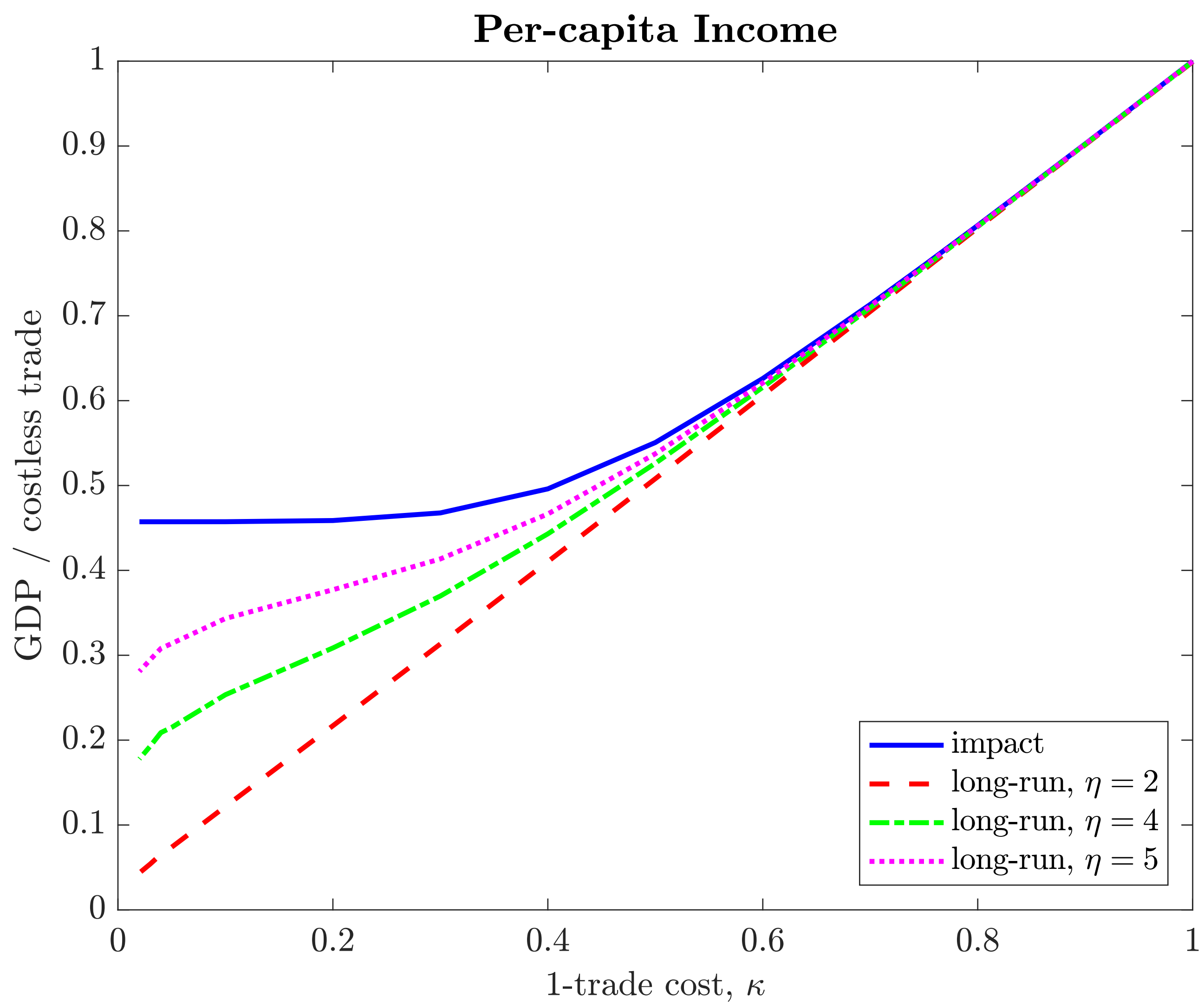
$$v = \theta \sum_{i=1}^{n} \alpha_i$$

- Proportional Pareto tails, independent of trade costs,

$$\lim_{z \to \infty} \left[1 - \Phi_i\left(z\right)\right] \sim \frac{\alpha_i}{\sum_{j=1}^{n} \alpha_j} \lambda z^{-1/\theta}, \quad i = 1, ..., n.$$

- Trade costs affect features of the distribution beyond the right tail

- Therefore, trade costs affect per-capita income when goods are poor substitute, low η



Per-capita Income
GDP / costless trade
1-trade cost, κ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
impact
long-run, $\eta = 2$
long-run, $\eta = 4$
long-run, $\eta = 5$

Trade and Idea Flows: Buera & Oberfield (2016)

- New idea $z = \epsilon \cdot (\tilde{z})^{\beta}$, original component $\epsilon \sim H(\epsilon) = 1 - \epsilon^{1/\theta}$, from others $\tilde{z} \sim G(\tilde{z}, t)$

- Evolution of the distribution of productivity

$$\frac{\partial \log F(z,t)}{\partial t} = -\alpha(t) \int_0^{\infty} \left[1 - H\left(\frac{z}{(\tilde{z})^{\beta}}\right)\right] dG(\tilde{z}, t)$$

$\Rightarrow$

$$F(z,t) = e^{\lambda(t) z^{1/\theta}}, \text{ where } \frac{\partial \lambda(t)}{\partial t} = \alpha(t) \int_o^{\infty} \tilde{z}^{\beta/\theta} dG(\tilde{z}, t).$$

- Long-run growth if $\dot{\alpha}(t)/\alpha(t) = \gamma > 0$, e.g., due to population growth,

$$\nu = \gamma\theta/(1-\beta).$$

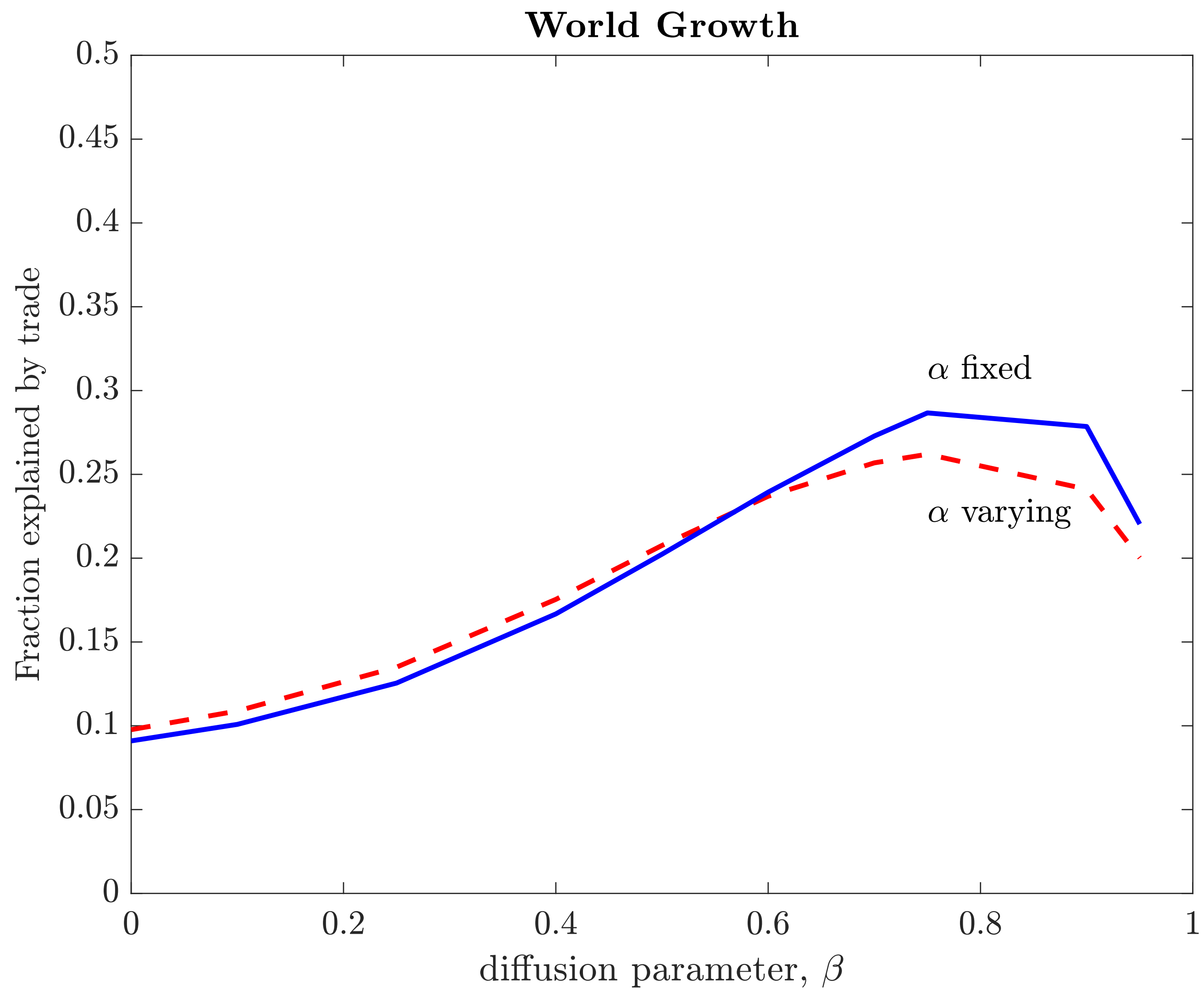
- If $G(\tilde{z}, t)$ as in Alvarez et al. (2014), then

$$\frac{\partial \lambda_i(t)}{\partial t} \propto \sum_{j=1}^{n} \left(\pi_{ij}(t)\right)^{1-\beta} \left(\lambda_j(t)\right)^{\beta}$$

where $\pi_{ij}(t)$ denotes the share of country i's expenditure coming from country j.

- Trade frictions affect the right tail of the distributions of productivity.

- Very tractable model that can be appended to Ricardian quantitative trade model.


World Growth
Fraction explained by trade
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
diffusion parameter, β
α fixed
α varying

Trade and Idea Flows: Perla et al. (2105) & Sampson (2015)

- Alternative $G(z,t)$: learning from domestic producers

- Different trade model: Melitz style, differentiated goods, adoption/entry & exit margins

- Lower trade costs lead to better set of domestic producers, more adoption/entry & exit

- Lower trade costs lead to higher long-run growth

$$\nu = \theta \left[\frac{\eta - 1}{\frac{1}{\theta} + 1 - \eta} \frac{f}{f_e} \frac{1}{\pi_{ii}} - \rho \right].$$

- Do these model provide a plausible theory of growth?

- Do they rely too heavily on the tails, continuum populations?

- Again, the basic equation of idea flows is

$$\frac{\partial F(z,t)}{\partial t} = -\alpha \left[1 - G(z,t)\right] F(z,t)$$

where $G(z,t)$ is the source distribution.

- This equation provides a theory of long-run growth

$$\nu = \alpha\theta$$

provided

$$\lim_{z \to \infty} \left[1 - G(z,t)\right] z^{1/\theta} = k, \text{ for some constant} k > 0.$$

- What's the interpretation of the tail condition?

- If $1 - G(z,t) = 1 - F(z,t)$, growth requires that there are enough arbitrarily good ideas to be learned!

- If $1 - G(z,t)$ includes new, original ideas, e.g., Kortum (1997),

$$\frac{\partial F(z,t)}{\partial t} = -\left\{\alpha\left[1 - F(z,t)\right] + \beta\left[1 - H(z)\right]\right\} F(z,t)$$

and

$$\nu = \alpha\theta$$

provided $\lim_{z\to\infty}\left[1 - H(z)\right] z^{1/\theta} = k$, i.e., growth requires that there are enough arbitrarily good ideas to be discovered!

- Is long-run growth possible in a finite population of size n?

- Assume $H(z) = 1 - z^{1/\theta}$, ideas diffuse instantaneously, i.e., α is very large relative to β, implying an effective arrival $n\beta$.

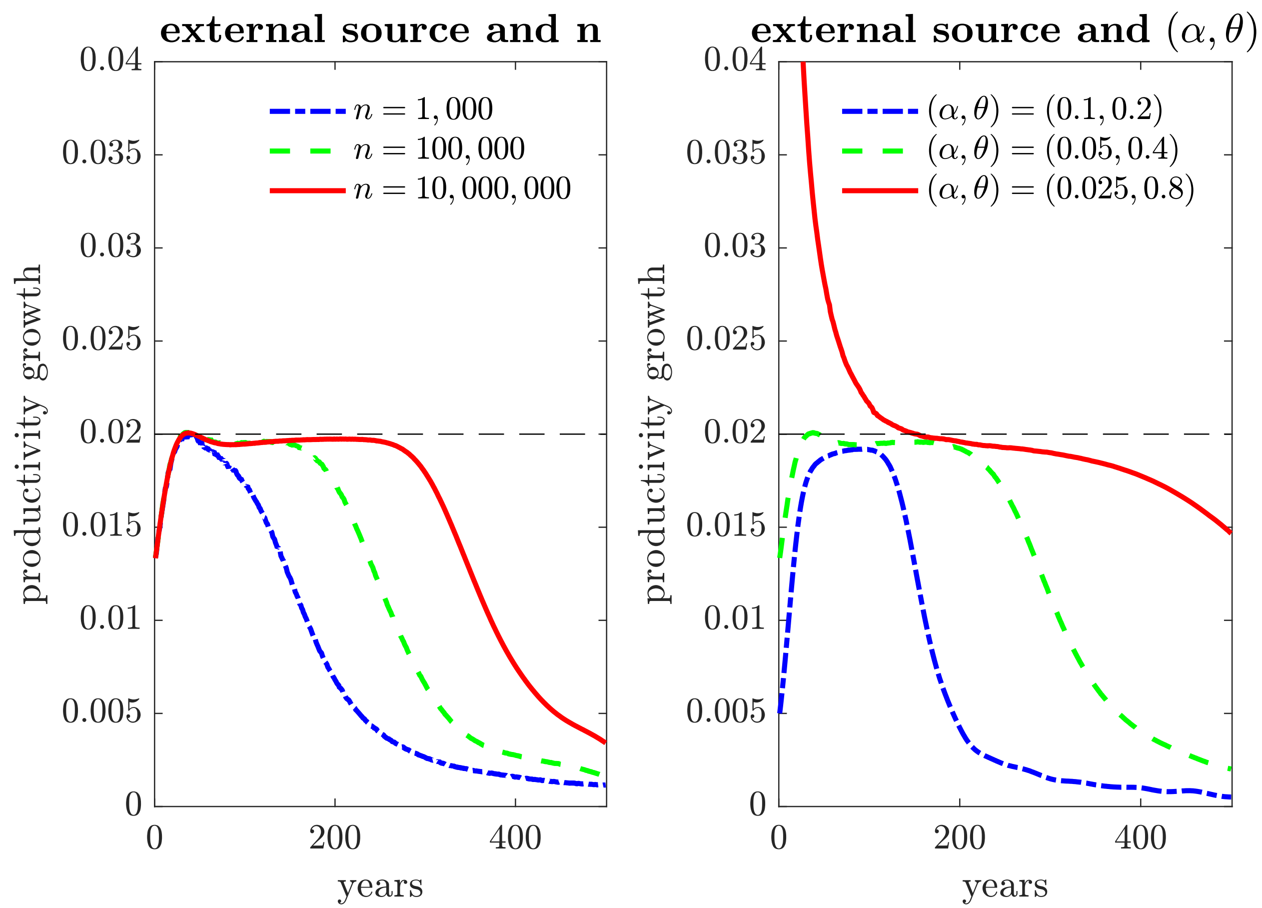
- Assuming $F(z, 0) = 1$, $z \geq 1$, the evolution of the distribution of productivity equals

$$F(z, t) = e^{n\beta t z^{1/\theta}}$$

and the growth rate of expected productivity is

$$\frac{1}{\mathbb{E}_t[z]} \frac{\partial \mathbb{E}_t[z]}{\partial t} = \frac{\theta}{t} \rightarrow 0 \text{ as } t \rightarrow \infty!$$

(This example was suggested to us by Chad Jones)



Simulation of Finite Population Examples: Internal & External Source

- An alternative avenue to have long-run growth w/ finite populations

$$\frac{\partial F(z,t)}{\partial t} = -\alpha\left[1 - F(z,t)\right]F(z,t) + \frac{\sigma^2}{2}\frac{\partial^2 F(z,t)}{\partial z^2}$$

where we now work with the log of productivity $\tilde{z}$, $z = \log \tilde{z}$.

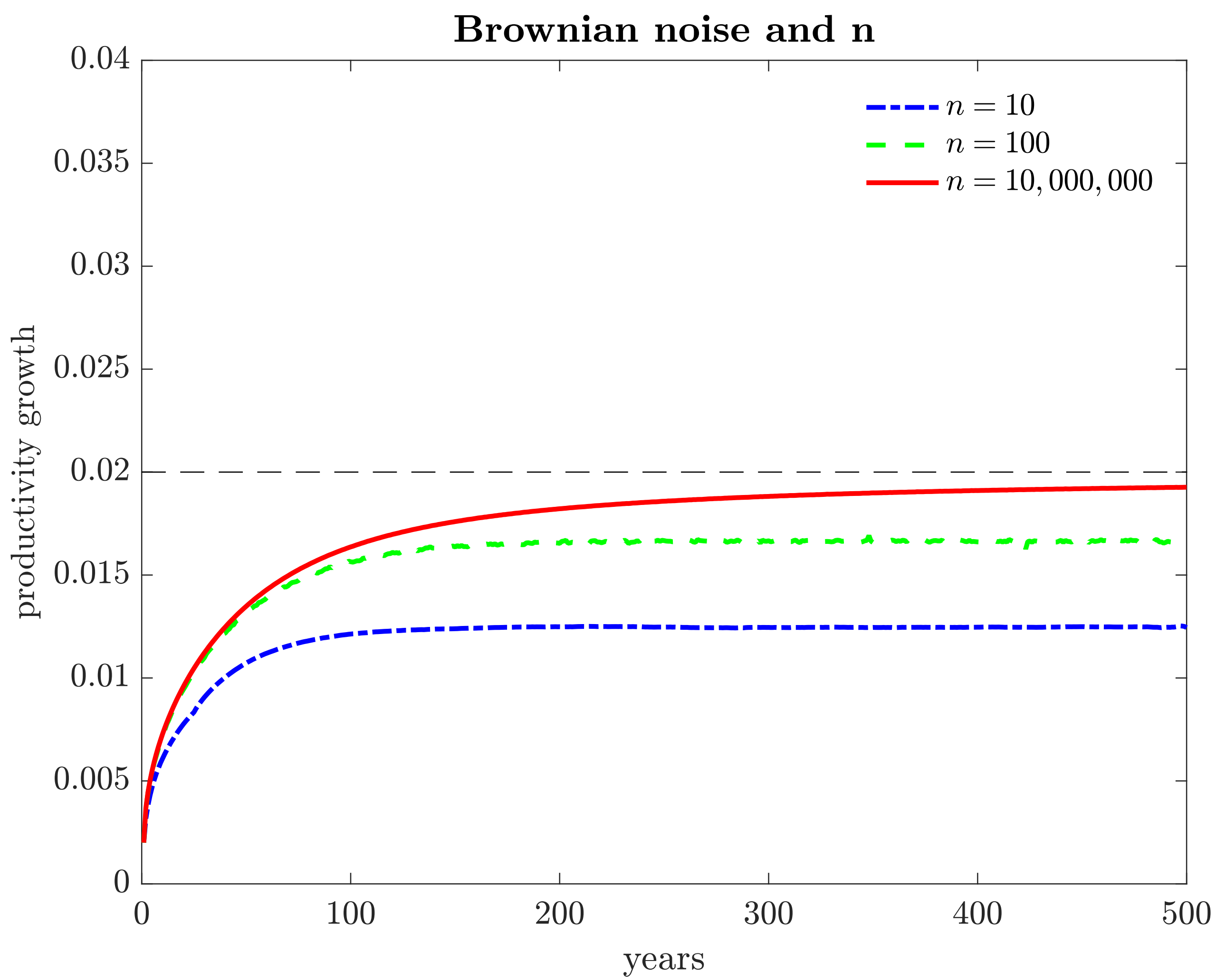
- Long-run growth and the right tail of the stationary distribution

$$\nu = \sigma\sqrt{2\alpha}$$

and

$$\lim_{t\to\infty}\left[1 - F(z+\nu t, t)\right] \sim \lambda z e^{-\theta z},\ \ \theta = \sigma/\sqrt{2\alpha}.$$

- Fisher (1937), Kolmogorov et al (1937),..., Luttmer (2012), Staley (2011)



Finite Population with Brownian Noise/ Experimentation