Innovation, Knowledge Diffusion, and Selection

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Motivation

- Background: Neo-Schumpeterian models of creative destruction:
  - Elegantly account for spillover (+) & displacement (-) externalities
  - Allow normative analyses in settings with firm heterogeneity

    Lentz & Mortensen (2016); Acemoglu et al. (2018); Aghion et al. (2017); ...
- Observation: Standard models of industry dynamics & investment:

  Hopenhayn (1992); Ericson & Pakes (1995); ...
  - Can account for creative destruction through selection
  - Allow for productivity shocks and rich demand & market structures
- Question: does heterogeneity matter for innovation policy?
  - Benchmark Neo-Schumpeterian models of creative destruction: No!

    Klette & Kortum (2004); Atkeson & Burstein (2019)
  - A selection-driven theory of creative destruction: Yes!

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Overview of the Paper

- Model: Industry dynamics & investment + Knowledge diffusion

  Atkeson & Burstein '10 + (Luttmer '07, '12; Lucas & Moll '14; Sampson, '16; ...)

  - Monop. competitive firms innovate to improve own productivity
  - Entrants (imperfectly) adopt ideas of incumbents
  - Innovation + adoption + shocks $\Rightarrow$ reallocation, selection, growth

- Theoretical Results:
  - Conditions to generate unique Stationary Constant Growth Path
  - Socially optimal allocations and their implementation
  - Novel source of innovation misallocation across heterogeneous firms

- Calibration Results:
  - Quantitatively, heterogeneity first-order for optimal innovation policy

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Related Literature

- Neo-Schumpeterian theories of firm innovation and growth

  Klette & Kortum (2004); Lentz & Mortensen (2008, 2014); Akcigit & Kerr (2010); Peters (2011); Acemoglu et al. (2013); Atkeson & Burstein (2015)

  New: Implications of selection + Match facts on firm growth

- Theories of knowledge diffusion and growth

  Luttmer (2007, 2012); Alvarez et al. (2008); Poschke (2009); Lucas & Moll (2011); Perla & Tonetti (2014); Benhabib et al. (2014); Stokey (2014); Perla et al. (2015); Buera & Oberfield (2015); Sampson (2016)

  New: Process innovation

- Firm heterogeneity, allocative distortions, and aggregate productivity

  Hopenhayn & Rogerson (1993); Ericson & Pakes (1995); Restuccia & Rogerson (2008); Hsieh & Klenow (2009); Atkeson & Burstein (2010); Hsieh & Klenow (2014); Hopenhayn (2014); & many others ...

  New: Growth + Innovation misallocations

Outline

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Demand and Firms

- Mass $N_t = N_0 e^{g_N t}$ of consumers inelastically supply unit of labor:
  - Dynastic intertemporal utility: $\int_0^\infty N_t e^{-rt} \log q_t \, dt$
  - CES($1+\rho$) over firm products: $q_t^{\frac{\rho}{1+\rho}} = \int q_t(\omega)^{\frac{\rho}{1+\rho}} d\omega$

- Mass M_t of monop. competitive firms with ideas $Z \sim F(\cdot; t)$:
  - Idea Z_t of a firm ω evolves according to:

$$dZ_t = \Gamma_t \, dt + \sigma Z_t d\mathscr{W}_t$$

  - Produce Γ_t new ideas and Q_t units of good
  - Hire variable l production & i innovation workers (+ fixed ψ_f)

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Production Function

- Goods and ideas production function:

$$\begin{pmatrix} Q \\ \Gamma \end{pmatrix} = \boldsymbol{G}(l,i;Z) = Z \times \begin{pmatrix} G^q(l,i) \\ G^\gamma(l,i) \end{pmatrix}$$

Assumption.

1. $G^q : \mathbb{R}^2_+ \to \mathbb{R}_+$ *contin. & homog of deg 1 (CRS) s.t.* $\partial_l G^q > \partial_i G^q$
2. $G^\gamma : \mathbb{R}^2_+ \to [0,\overline{\gamma}]$ *concave, contin., & homog of deg* $\beta \geq 0$ *s.t.*

$$\partial_i G^\gamma > 0, \qquad \forall (l,i) : G^\gamma(l,i) < \overline{\gamma}$$

Details

- For example:

$$G^q(l,i) = l \qquad G^\gamma(l,i) = l^\beta \times G^\gamma\left(1, \frac{i}{l}\right)$$

Incumbent Problem

- HJB Equation for the firm value $v(Z,t)$ for $Z > Z_o(t)$:

$$rv - \frac{\partial v}{\partial t} = \max_{Q,\Gamma,l,i} \underbrace{N_t^{\frac{1}{1+\rho}} P_t^{\frac{\rho}{1+\rho}} Q^{\frac{\rho}{1+\rho}} - w_t\left(l + i + \psi_f\right)}_{\text{flow profits}} + \underbrace{\Gamma \frac{\partial v}{\partial Z} + \frac{\sigma^2}{2} Z^2 \frac{\partial^2 v}{\partial Z^2}}_{\text{growth and diffusion}}$$

$$\begin{pmatrix} Q \\ \Gamma \end{pmatrix} \leq Z \times \begin{pmatrix} G^q(l,i) \\ G^\gamma(l,i) \end{pmatrix}$$

  - With boundary conditions $v(Z_o(t),t) = \frac{\partial v}{\partial Z}(Z_o(t),t) = 0$

- Assume along equilibrium, continuous solution exits:

$$\begin{pmatrix} Q(Z;t) \\ \Gamma(Z;t) \end{pmatrix} \Leftrightarrow \begin{pmatrix} l(Z;t) \\ i(Z;t) \end{pmatrix} \Leftrightarrow \begin{pmatrix} l_v(Z;t) = l + i \\ x(Z;t) \equiv i/l \end{pmatrix}$$

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Generalized Gibrat Equilibrium

Definition.

Equilibria that generate very large firms ($\lim_{Z\to\infty} l(Z,t) = \infty$) s.t.:

1. Constant expected employment growth (original Gibrat's law):

$$\lim_{Z\to\infty} \mathbb{E}_t\left[g_l(Z;t)\right] \equiv \lim_{Z\to\infty} \mathbb{E}_t\left[\log \frac{l(Z_{t+1},t+1)}{l(Z_t,t)} \middle| Z_t = Z\right] = g_{l,t}^{**}$$

2. Finite innovation intensity:

$$\lim_{Z\to\infty} x(Z;t) = \lim_{Z\to\infty} \frac{i(Z,t)}{l(Z,t)} = x_t^{**} > 0$$

Proposition.

A necessary condition for the existence of GGE is $\beta = 0$.

- Intuition:

$$\lim_{Z\to\infty} \frac{\Gamma(Z,t)}{Z} = \gamma_t^{**} = \lim_{Z\to\infty} l(Z,t)^{\beta} \times G^{\gamma}(1, x_t^{**})$$

Firm Static Decisions with $\beta = 0$

Corollary.

Decreasing function $J(\cdot)$ exists s.t. profit flow of firm choosing $\gamma \equiv \Gamma / Z$:

$$\Pi(Z, \gamma, t) = \max_{l_v} N_t^{\frac{1}{1+\rho}} P_t^{\frac{\rho}{1+\rho}} \left[Z \times J(\gamma) \times l_v \right]^{\frac{\rho}{1+\rho}} - w_t \left(l_v + \psi_f \right)$$

- Profit maximization: constant markup pricing

$$P(Z, \gamma; t) = \left(1 + \frac{1}{\rho}\right) \frac{w_t}{Z \times J(\gamma)}$$

  - Innovation costs $\Rightarrow$ (effective) productivity $Z \times J(\gamma)$

- Model nests prior specificartions compatible with long-run growth

  Luttmer (2010); Atkeson & Burstein (2010, 2019); Stokey (2014); Benhabib et al. (2018)

Prior Specifications

Static Allocations

- Aggregate productivity with distribution $F(Z;t)$ at time t:

$$\overline{Z}_t \equiv \left[\int (ZJ(\gamma(Z,t)))^{\rho} \, dF(Z;t) \right]^{\frac{1}{\rho}}$$

- Sales of firm with productivity Z and ideas growth γ:

$$s(Z,\gamma,t) = \underbrace{\frac{N_t}{M_t}}_{\text{average sales}} \times \underbrace{\left(\frac{Z}{\overline{\overline{Z}}_t} \right)^{\rho}}_{\text{relative productivity}} \times \underbrace{J(\gamma)^{\rho}}_{\text{Innovation costs}}$$

- Profits and employment:

$$\pi(Z,\gamma,t) = \frac{1}{1+\rho} s(Z,\gamma,t) - \psi_f w_t$$

$$\left(l_v(Z,\gamma,t) + \psi_f\right) w_t = \frac{\rho}{1+\rho} s(Z,\gamma,t) + \psi_f w_t$$

Adoption and Entry Process

- Potential entrants hire ψ_e workers per unit flow of entry
- Get to adopt ideas from current distribution $F(Z;t)$

Nests: Eekhout & Jovanovic '02; Luttmer '07, '12; Atkeson & Burstein '10; Sampson '16

1. Whose strategies are they likely to adopt?
   - Draw rank (between 0-1) from distribution F_a
   - Adopt idea Z_a s.t. rank $= F(Z_a;t)$
2. How much of the productivity is transferred through adoption?
   - Entry productivity
   $$Z_e = e^{\gamma_e} \widetilde{Z}_t^{\eta} Z_a^{1-\eta}$$
   - Average productivity $\widetilde{Z}_t \equiv \int Z dF(Z;t)$

Stationary Constant Growth Paths

Definition.

Equilibrium paths such that asymptotically:

1. Mass and average productivity grow at constant rates

$$M_t = M^* e^{g_M t} \qquad \overline{Z}_t = Z^* e^{g_Z t}$$

2. Distribution of firm size stationary with finite mean
3. There is nonzero flow of entrants

Formal Definition of Equilibrium

$\Rightarrow$ Mass of firms grows at the same rate as the market size

$$\text{average sales} = \frac{N_t}{M_t} \to \text{const.} \Rightarrow g_M = g_N$$

Profitability States

- Transformation of productivity states $\mathsf{z}(\cdot,t) : Z \mapsto z$

$$\exp\left(\mathsf{z}(\cdot,t)\right) \equiv \frac{1}{\rho+1} \times \frac{1}{\psi_f w_t} \times \frac{N_t}{M_t}\left(\frac{Z}{\overline{\overline{Z}}_t}\right)^{\rho}$$

  - Profits-to-fixed costs ratio of firm with productivity Z at time t:

$$\frac{\pi(Z,\gamma,t)}{\psi_f w_t} = e^{\mathsf{z}(Z,t)} \times J(\gamma)^{\rho} - 1$$

- Evolution of profitability $z_t \equiv \mathsf{z}(Z_t,t)$ along a SCGP:

$$dz_t = \Big(\rho\gamma - \underbrace{\rho\left(g_Z + \frac{1}{2}\sigma^2\right)}_{\text{Profit erosion}}\Big)\,dt + \rho\sigma\, d\mathscr{W}_t$$

SCGP Value Function

Proposition 1.

Assume $J(\cdot)^{\rho}$ is concave & g_Z satisfies:

$$g_Z > \overline{\gamma} + (\rho - 1)\frac{1}{2}\sigma^2 - \frac{1}{\rho}\left[r + \lim_{\gamma \to \overline{\gamma}} \frac{J(\gamma)}{J'(\gamma)}\right].$$

1. *Unique, continuous, increasing function $V(\cdot)$:*

$$\frac{v(Z,t)}{w_t \Psi_f} = V(\mathrm{z}(Z,t))$$

2. *Unique, continuous, increasing function $\gamma^*(\cdot)$ and threshold z_o:*

$$\gamma(Z,t) \equiv \gamma^*(\mathrm{z}(Z,t)) \qquad z_o = \mathrm{z}(Z_o(t),t)$$

3. *Asymptotically:* $\lim_{z\to\infty} V(z) = u^* e^z$ & $\lim_{z\to\infty} \gamma^*(z) = \gamma^{**}$ *s.t.:*

$$u^* = \max_{\gamma} \frac{J(\gamma)^{-\rho}}{r + \rho\left(g_Z - \frac{1}{2}(\rho - 1)\sigma^2 - \gamma\right)}.$$

Concavity of $J(\cdot)$

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Stationary Distribution

Proposition.

Assume mass of firms grows with positive rate s.t.:

$$g_M = g_N > \rho\left(\gamma^{**} - g_Z\right) + \frac{1}{2}\rho\left(\rho - 1\right)\sigma^2$$

and that one of the two conditions hold:

1. *Likelihood of frontier adoption approaches zero* $f_a(1) = 0$
2. *Adoption is imperfect* $\eta > 0$

Then, there exists a stationary distribution $H(\cdot)$ *with Pareto tail:*

$$\zeta = \frac{1}{\rho}\left[\sqrt{\left(\frac{g_Z - \gamma^{**}}{\sigma^2} + 1\right)^2 + \frac{2g_M}{\sigma^2}} + \frac{g_Z - \gamma^{**}}{\sigma^2} + 1\right]$$

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Adoption Process and Hysteresis

- Perfect adoption ($\eta = 0$) from frontier ($f_a(1) > 0$):
  - Potential for multiplicity of equilibria and hysteresis

    Luttmer (2012); Benhabib, Perla & Tonetti (2018)

    - Continuum of equilibria with Pareto tails indexed by entry rate λ_e
    - Long-run growth rate depends on initial distribution
  - Intuition: Entry $\uparrow$ $\Rightarrow$ Tail thickness $\uparrow$ $\Rightarrow$ Adoption incentives $\uparrow$
- Imperfect adoption ($\eta > 0$) or without frontier adoption ($f_a(1) \to 0$):
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Decomposition of Productivity Growth

- Free entry pins down g_Z

Lemma.

We can decompose the rate of productivity growth as

$$g_Z \approx \underbrace{\mathbb{E}_S\left[\gamma^*(z)\right] + \frac{1}{2}(\rho-1)\sigma^2}_{\text{Incumbents}} + \underbrace{\lambda_o \cdot \left(\frac{S_e - S_o}{\rho}\right)}_{\substack{\text{Selection} \\ \text{(Creative Destruction)}}} - \underbrace{g_M \cdot \left(\frac{1-S_e}{\rho}\right)}_{\text{Net Entry}}$$

Definition of market shares

$S(z)$	: Distribution of market shares	$\gamma^*(z)$	: Innovation of incumbents
S_e	: Market share of entrants	S_o	: Market share of exiting firms
λ_o	: Rate of exit	g_M	: Rate of growth of mass of firms
σ	: Productivity volatility	ρ	: Substitutability parameter

Outline

Outline

Overview of the Insights

- Selection implies differences in firm expected lifetimes:

  firm productivity $\rightarrow$ expected lifetime $\rightarrow$ externalities (+ & -)

  1. Stronger **knowledge spillovers** from innovations of productive firms

     productive ideas $\rightarrow$ live longer $\rightarrow$ diffuse further (+)

  2. Stronger **dynamic competition** from innovations of productive firms

     productive ideas $\rightarrow$ shorten other lives $\rightarrow$ crowd out innovation (-)

- Markets generically misallocate innovation across heterog. firms

Social Firm Value Function $V_s(z)$

Flow prod. value — Expected growth — Volatility

$$rV_s(z) = \max_{\gamma} \quad e^z J(\gamma)^{-\rho} - 1 \quad + \rho\left(\gamma - \left(g_{Z,s} + \frac{\sigma^2}{2\rho}\right)\right) V_s'(z) \quad + \quad \frac{\rho^2\sigma^2}{2} V_s''(z)$$

- Boundary conditions $V(z_{o,s}) = V'(z_{o,s}) = 0$ & for very large z

$\gamma_s(z)$: Optimal firm innovation

$z_{o,s}$: Optimal efficiency cutoff for exit

$g_{Z,s}$: Optimal productivity growth

J : Effective innovation cost

Social Firm Value Function $V_s(z)$

$$rV_s(z) = \max_{\gamma} \quad \overbrace{e^z J(\gamma)^{-\rho} - 1}^{\text{Flow prod. value}} \quad + \overbrace{\rho\left(\gamma - \left(g_{Z,s} + \frac{\sigma^2}{2\rho}\right)\right) V_s'(z)}^{\text{Expected growth}} \quad + \overbrace{\frac{\rho^2\sigma^2}{2} V_s''(z)}^{\text{Volatility}}$$

$$+ \quad \overbrace{\underbrace{\lambda_{e,s}}_{\text{Rate of entry}} \quad \times \quad \underbrace{(U_s(z) - \Psi_s)}_{\text{Expected spillover value to entrants}}}^{\text{Flow social spillover value}}$$

- Spillover function U_s: monotonically increasing & $U_s(z_{o,s}) = 0$

$$\lim_{z\to\infty} U_s(z) \propto f_a(1) \times e^{(1-\eta)z}$$

- Social costs $\Psi_s > 0$

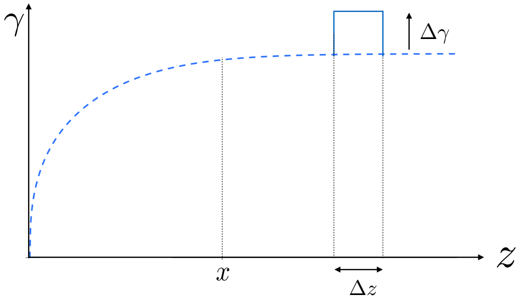
Details

- Boundary conditions $V(z_{o,s}) = V'(z_{o,s}) = 0$ & for very large z

$\gamma_s(z)$: Optimal firm innovation
$z_{o,s}$: Optimal efficiency cutoff for exit
$g_{Z,s}$: Optimal productivity growth
J : Effective innovation cost

Dynamic Competition

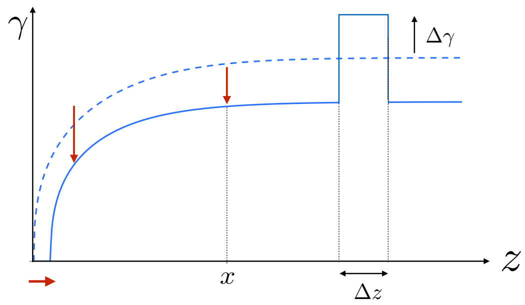
- Firms in neighborhood Δz around z raise innovation by $\Delta\gamma$
- Response of firm at efficiency x, if no one else responds?



Dynamic Competition

- Firms in neighborhood Δz around z raise innovation by $\Delta\gamma$
- Response of firm at efficiency x, if no one else responds?

$$\Delta g_Z \propto S(z)\,\Delta z \Delta\gamma$$



Dynamic Competition

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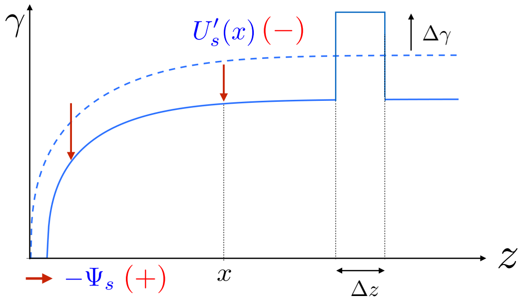
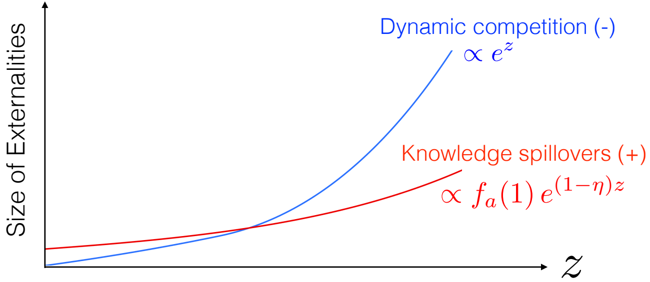


Two Externalities

- How do knowledge spillovers and dynamic competition vary with z?



- Social/private gap in innovation returns **not** equalized across firms

Benchmark Model of Creative Destruction

- No selection, no market size growth, head-to-head competition

  Hybrid model à al Garcia-Macia et al. (2016)

$$\eta = 0 \qquad f_a \equiv 1 \qquad \gamma_e > 0$$

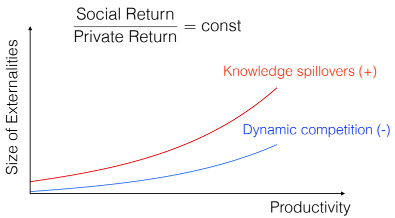
- Equilibrium:
  - Likelihood of exit independent of productivity $\Rightarrow \gamma^* (z) = \gamma^\dagger$
  - Productivity growth decomposition

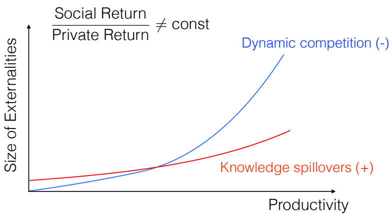
$$g_Z = \underbrace{\gamma^\dagger + \frac{1}{2} (\rho - 1) \sigma^2}_{\text{Incumbents}} + \underbrace{\lambda_e \gamma_e}_{\text{Creative Destruction}}$$

- Normative Implications:
  - All incumbents underinvest in innovation under the market
  - Social/private gap in returns equalized across firms

Two Views of Creative Destruction

Random Head-Head Competition → Selection among Differentiated Products





- Inefficiency in aggregate innovation

  Atkeson & Burstein (2015)

- Inefficiency in the distribution of innovation

Implementation

1. Output reward:
   - Profitability-dependent cash $U_s(\cdot)$
   - Lump-sum operation tax Ψ_s

2. Input subsidy:
   - Innovation input subsidies at profitability-dependent rate $\tau_i(z)$
   - Entry cost subsidies at rate τ_e
   - Lump-sum operation tax τ_o
   - Production-labor input taxes at rate $\tau_l(z) = -\tau_i(z)\varphi(\gamma_s(z))$

Details of the Optimal Input Subsidy Scheme

Outline

Outline

Preview of Quantitative Results

- Calibrate to moments of firm life-cycle dynamics in the US:
  - Business Dynamics Statistics (1987-2007) for manufacturing & retail
- Market vs. optimal allocations in both sectors:
  - Too much incumbent innovation, too little entry, too concentrated
  - Optimal subsidy rate inverted U-shape in firm productivity
- In retail trade:
  - Higher volatility, adoption less perfect
  - Weaker initial rise in subsidies, swift decline among top firms
- Optimal policy spends considerably less than uniform subsidies

Calibration (1/3): Specification

- Costs of process innovation:

$$\varphi(\gamma) \equiv \frac{\chi \gamma^{1+\kappa}}{1+\kappa}, \qquad 0 \le \gamma \le \bar{\gamma} \equiv \left(\frac{\kappa}{\chi} \frac{1+\kappa}{\rho(1+\kappa)+1} \right)^{1/(1+\kappa)}$$

- Adoption distribution:

$$F_a(x) = x^{\mu},\ x \in [0,1] \qquad f_a(1) = \mu$$

  - $\mu = 1$: uniform adoption
  - $\mu > 1$: bias toward productive ideas
  - $\mu < 1$: bias toward unproductive ideas

Calibration (2/3): Parameters

1. Innovation:
   - Volatility σ
   - Cost (scale) χ
   - Cost (curvature) κ
2. Entry & Adoption:
   - Directedness of adoption μ
   - Decreasing returns in transfer η
   - Entry efficiency shifter γ_e
   - Costs of entry ψ
3. Demand:
   - Growth of demand g_N
   - Substitution elasticity $1+\rho$
   - Discount rate r

More Details

Calibration (2/3): Parameters

1. Innovation:
   - Volatility σ
   - Cost (scale) χ
   - Cost (curvature) κ
2. Entry & Adoption:
   - Directedness of adoption μ
   - Decreasing returns in transfer η
   - Entry efficiency shifter γ_e
   - Costs of entry ψ
3. Demand:
   - Growth of demand g_N
   - Substitution elasticity $1+\rho$ $\leftarrow$ 3 (common value)
   - Discount rate r $\leftarrow$ 0.05 (common vlaue)

More Details

Calibration (2/3): Parameters

1. Innovation:
   - Volatility σ
   - Cost (scale) χ
   - Cost (curvature) κ $\leftarrow$ cost elasticity of R&D (prior work)
2. Entry & Adoption:
   - Directedness of adoption μ
   - Decreasing returns in transfer η
   - Entry efficiency shifter γ_e
   - Costs of entry ψ $\leftarrow$ rate of productivity growth g_Z (data)
3. Demand:
   - Growth of demand g_N $\leftarrow$ rate of employment growth (data)
   - Substitution elasticity $1+\rho$
   - Discount rate r

More Details

Calibration (2/3): Parameters

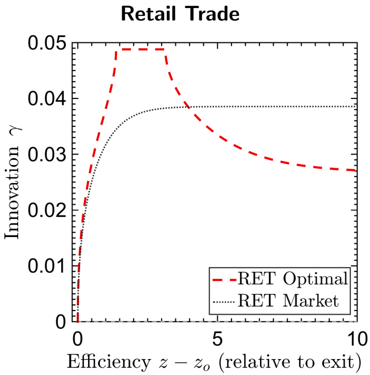
1. Innovation:
    - Volatility σ $\leftarrow$ rate of emp reallocation of old firms (data)
    - Cost (scale) χ $\leftarrow$ rate of emp growth of old firms (data)
    - Cost (curvature) κ
2. Entry & Adoption:
    - Directedness of adoption μ $\leftarrow$ rate of exit & age (data)
    - Decreasing returns in transfer η $\leftarrow$ rate of exit & age (data)
    - Entry efficiency shifter γ_e $\leftarrow$ rate of exit & age (data)
    - Costs of entry ψ
3. Demand:
    - Growth of demand g_N
    - Substitution elasticity $1+\rho$
    - Discount rate r

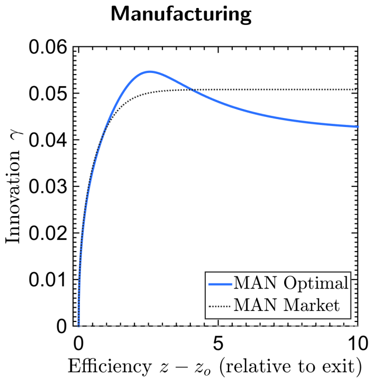
More Details

Calibration (3/3): Moments & Calibrated Parameters

Targeted Moment		**Retail Trade**		**Manufacturing**	
		Model	**Data**	**Model**	**Data**
Rate of productivity growth	g_Z	0.032	0.032	0.038	0.038
Rate of emp growth	g_N	0.017	0.017	0.000	-0.012
Cost elasticity of investment		1.0	1.0	1.0	1.0
Rate of entry		0.117	0.119	0.079	0.079
Rate of emp growth of old (15-20 yr)		-0.015	-0.007	-0.024	-0.017
Rate of reallocation of old (15-20 yr)		0.26	0.23	0.20	0.20
Rate of exit of mature (10-15 yr)		0.075	0.083	0.060	0.060
Rate of exit of old (15-20 yr)		0.065	0.071	0.054	0.052
Parameter					
Volatility	σ	0.19		0.15	
Innovation costs	χ	13360		8103	
Curvature of costs	κ	2.2		2.3	
Entry costs	ψ	6.2		11.3	
Directedness of adoption	μ	3		1	
Decreasing returns of adoption	η	0.6		0.3	
Entry Productivity Shifter	γ_e	-0.5		-0.5	

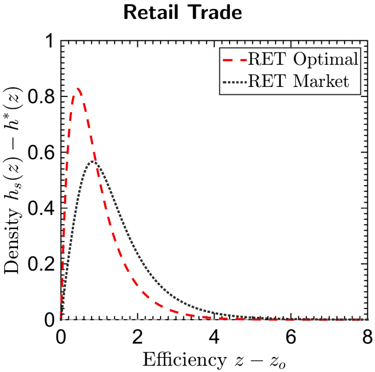
Normative Analysis (1/4): Optimal vs Market Innovation



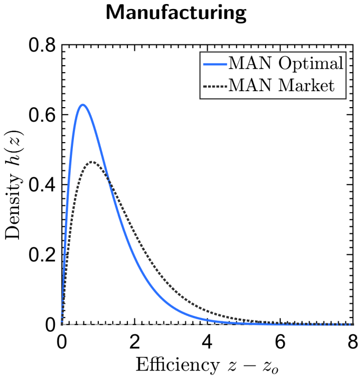


- Market underinvestment among **medium**-efficiency firms

Normative Analysis (2/4): Optimal vs Market Distribution



Pareto tail index: 1.17 → 1.77

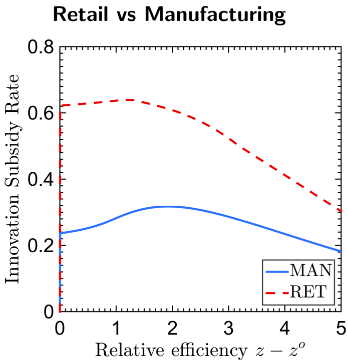


Pareto tail index: 1.20 → 1.77

- Market equilibrium **too concentrated**

Concentration & Pareto Tail Index

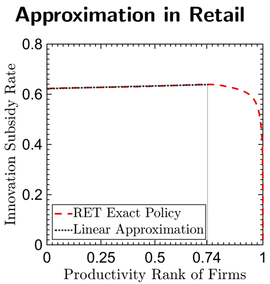
Normative Analysis (3/4): Innovation Subsidies

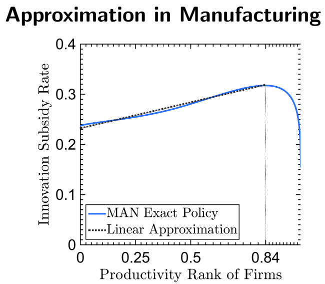


		Retail trade	Manufacturing
Optimal Rate of Growth	$g_{Z,s}$	0.052	0.047
Entrant Subsidy	$-\tau_{e,s}$	0.45	0.18
Operation Tax	$\tau_{o,s}$	4.90	2.95
Average Incumbent Subsidy	$-\mathbb{E}_H[\tau_{i,s}]$	0.62	0.28

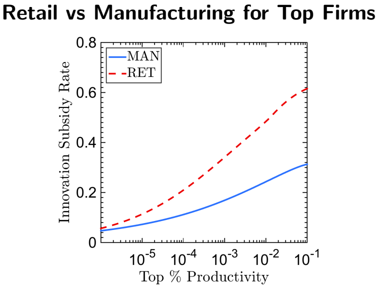
Implementation of Optimal Policy

Normative Analysis (4/4): Innovation Subsidies

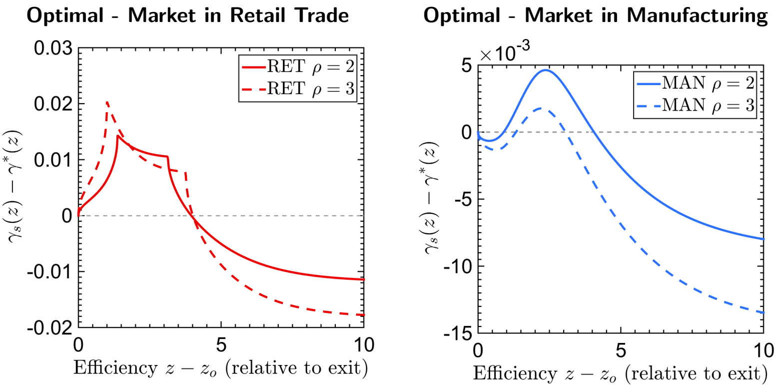




	RET	MAN
Optimal Rate of Growth	0.052	0.047
Baseline Approx. Subsidy Rate	0.62	0.23
Slope of Approx. Subsidy Rate	0.02	0.10
Optimal Schedule Spending	0.32	0.20
Best Uniform Spending	2.57	1.96

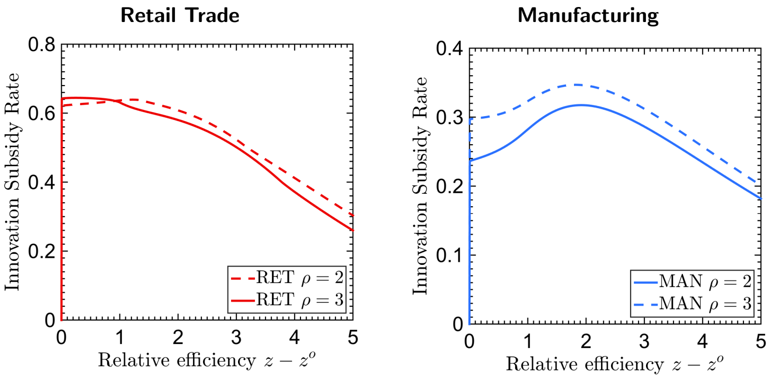


Competition & Efficiency (1/2): Innovation Investments



- **Experiment:**
  - Raise substitutability parameter ρ
  - Adjust cost scale χ $\rightarrow$ maintain rate of emp growth of large firms

Competition & Efficiency (2/2): Optimal Subsidies



- Peak of subsidy rate shifts to the left $\leftarrow$ dynamic competition $\uparrow$

Summary of Contributions

- Model unifies theories of firm selection and creative destruction:
  - Derived conditions that ensure uniqueness of stationary equilibria
  - Showed market equilibria misallocate innovation across firms
- Key Implications:
  - Innovation policy has to account for heterogeneity
  - Effective policies likely to vary depending on:
    1. Volatility of firm outcomes
    2. Strength of product market rivalry
    3. Relative spillovers of large to small firms

Assumptions on the Production Function

Assumption.

Production function $\boldsymbol{G}(\cdot,\cdot)$ *has the following properties:*

- $G^q(\cdot,\cdot):\mathbb{R}^2_{\geq 0}\to\mathbb{R}_{\geq 0}$
  - $G^q(l,i)>0$ *for all* $l>0$ *and* $i\geq 0$ *(WLOG* $G^q(1,0)=1$*)*
  - *Continuous & homog. deg. 1 (CRS)*
  - *MP of "production" labor* $\partial G^q(l,i)/\partial l>0$ *& greater than MP of "innovation" labor* $\partial G^q(l,i)/\partial i$ *everywhere.*

- $G^\gamma(\cdot,\cdot):\mathbb{R}^2_{\geq 0}\to[0,\overline{\gamma}]$, *with* $\overline{\gamma}\in\mathbb{R}_{\geq 0}\cup\{\infty\}$
  - $G^\gamma(l,0)=G^\gamma(0,i)=0$ *for all* l *and* i
  - *Continuous & homogeneous of degree* $\beta\geq 0$
  - *MP of "innovation" labor* $\partial G^\gamma(l,i)/\partial i\geq 0$ *everywhere*
  - $\forall l$*: there exists some* $\bar{i}(l)\in\mathbb{R}_{\geq 0}\cup\{\infty\}$ *s.t.* $\partial G^\gamma(l,i)/\partial i>0$, G^γ *strictly concave for* $\forall i\in[0,\bar{i}(l))$ *&* $G^\gamma(l,i)=\overline{\gamma}$ *for all* $i\geq\bar{i}(l)$.

Back

Alternative Specifications

- Luttmer (2010):

$$\begin{pmatrix} Q \\ \Gamma \end{pmatrix} = \boldsymbol{G}(l,i;Z) = Z \times \begin{pmatrix} l \times \mathsf{C}(1-\tau) \\ \mathsf{G}(\tau) \end{pmatrix}$$

  - Isomorphic to the model here for the choice with $J(\gamma) = \mathsf{C}\left(1 - \mathsf{G}^{-1}(\gamma)\right)$
  - No notion of innovation investments

- Atkeson & Burstein (2010):

$$\begin{pmatrix} Q \\ \Gamma \end{pmatrix} = \boldsymbol{G}(l,i;Z) = Z \times \begin{pmatrix} l \\ \mathsf{G}\left(\frac{i}{Z^{\rho}}\right) \end{pmatrix}$$

  - Does not allow sustained innovtion and productivity growth

- Stokey (2014); Atkeson & Burstein (2019); Benhabib et al. (2015):

$$\begin{pmatrix} Q \\ \Gamma \end{pmatrix} = \boldsymbol{G}(l,i;Z) = Z \times \begin{pmatrix} l \\ \mathsf{G}\left(\frac{i}{s(Z,t)}\right) \end{pmatrix}$$

  - Isomorphic to the model here with $G^{\gamma}(l,i) = \mathsf{G}\left(\frac{\rho}{1+\rho}\frac{i}{l+i}\right)$
  - No notion of primitive production function for ideas

Back

Dynamic Industry Equilibrium

Definition.

Given an initial industry measure M_0, a time path of wages $[w_t]_{t=0}^{\infty}$, interest rate r, and a time path of industry aggregate sales $[N_t]_{t=0}^{\infty}$, an industry tuple $\mathscr{I} \equiv [F(\cdot;t), \gamma(\cdot,t), Z_o(t), \lambda_e(t)]_{t=0}^{\infty}$ characterizes a dynamic industry equilibrium if

1. The policy function $\boldsymbol{\varsigma} \equiv [\gamma_t^*(\cdot), Z_o(t)]_{t=0}^{\infty}$ constitutes a solution to the firm's HJB equation, Firm HJB
2. The measure $F(\cdot;t)$ satisfies the KFE with the corresponding boundary conditions. Industry KFE
3. Rate of entry $\lambda_e(t)$ satisfies the free entry condition,

Back

General Equilibrium

Definition.

Consider an industry tuple $\mathscr{I}$ and a path of population $[N_t]_{t=0}^{\infty}$ such that $N_t/M_t > \psi_f + \psi_e \lambda_e(t)$, for all t. A path of wages, consumption, and per-capita assets $[w_t, q_t, a_t]_{t=0}^{\infty}$ and the industry $\mathscr{I}$ together form a dynamic general equilibrium if we have

1. Given the path of wages $[w_t]_{t=0}^{\infty}$, interest rate $r_t = r$, and aggregate sales (and population) N_t, the industry constitutes a dynamic industry equilibrium,
2. The paths of wages, per-capita consumption, and per-capita assets satisfy

$$Q_t = \frac{\rho}{\rho+1} \frac{M_t^{1/\rho} \overline{Z}_t}{w_t}, \qquad w_t = \frac{\rho}{\rho+1} \frac{N_t}{N_t - M_t \left(\psi_f + \psi_e \lambda_e(t)\right)},$$

$$a_t = \frac{M_t}{N_t} \int \left[v(Z,t) + \Psi_e w_t \lambda_e(t) \right] dF(Z;t),$$

for all t, where $\overline{Z}_t$ is aggregate productivity, and a_t satisfies a transversality condition.

Back

Concavity of $J(\cdot)$

Lemma.

Consider the case of:

$$G^q(l,i) = l$$
$$G^\gamma(l,i) = \varphi^{-1}\left(\frac{i}{l}\right)$$

with $\bar{\gamma}$ *and* $\bar{x}$ *s.t.* $\lim_{\gamma\uparrow\bar{\gamma}}\varphi(\gamma) = \bar{x}$ *and* $\varphi^{-1}(x) = \bar{\gamma}$ *for* $x > \bar{x}$*. Then*

$$J(\gamma) \equiv (1+\varphi(\gamma))^{-1}$$

and a sufficient condition for strict concavity of $J(\cdot)$ *for* $\gamma < \bar{\gamma}$*:*

$$\frac{\varepsilon_{\varphi'}(\gamma)}{\varepsilon_{\varphi}(\gamma)} > (1+\rho)\frac{\bar{x}}{1+\bar{x}}$$

where $\varepsilon_{\varphi}(\gamma) \equiv \gamma\varphi'(\gamma)/\varphi(\gamma)$ *is elasticity of* φ*.*

Market Shares

- Market shares of incumbents:

$$S(z) \equiv \frac{h(z)\, e^{z} J(\gamma^{*}(z))^{\rho}}{\mathbb{E}_{H}\left[h(z)\, e^{z} J(\gamma^{*}(z))^{\rho}\right]}$$

- Market share of entrants

$$S_e \equiv \frac{\mathbb{E}_{H_e}\left[h(z)\, e^{z} J(\gamma^{*}(z))^{\rho}\right]}{\mathbb{E}_{H}\left[h(z)\, e^{z} J(\gamma^{*}(z))^{\rho}\right]}$$

- Market share of exiting firms

$$S_o \equiv \frac{e^{z_o}}{\mathbb{E}_{H}\left[h(z)\, e^{z} J(\gamma^{*}(z))^{\rho}\right]}$$

H_e : Distribution of entrant efficiency
$\gamma^{*}(z)$: Innovation of incumbents

H : Distribution of incumbent efficiency
z_o : Exit cutoff

Back

Knowledge Spillovers & Diffusion

Example.

Uniform adoption distribution $f_a(x) = 1$.

- Remember: efficiency of entrants adopting z:

$$z_e(z) \equiv \eta \tilde{z} + (1-\eta) z + \rho \gamma_e$$

- Social spillover function: $U_s^{Diff}(z) = V\big(z_e(z)\big) - V\big(z_e(z_o)\big)$
- Social costs: $\Psi_s^{Diff} = \mathbb{E}_H\left[V(z_e(z))\right] - V\big(z_e(z_o)\big)$

Full Characterization in the General Case Back

H : Distribution of efficiencies
γ_e : Entry productivity shifter
$\tilde{z}$: Efficiency of stock of ideas
η : Decreasing returns to adoption
ρ : Substitutability parameter
z_o : Efficiency cutoff

Socially Optimal Allocations: Main Result

- Spillover function and social cost:

$$U_s(z) = \eta \left(e^{\frac{z-z_O}{\rho}} - 1\right) \tilde{u}_s + (1-\eta)\, u_s(z),$$
$$\Psi_s = \eta \left(e^{\frac{\tilde{z}-z_O}{\rho}} - 1\right) \tilde{u}_s + (1-\eta)\, \psi_s,$$

where the knowledge diffusion terms are given by

$$\psi_s \equiv \int_{z_{O,s}}^{\infty} V_s'(\eta\tilde{z} + (1-\eta)x + \rho\gamma_e)\, f_a(H_s(x))\,(1-H_s(x))\, dx, \quad (1)$$
$$u_s(z) \equiv \int_{z_{O,s}}^{z} V_s'(\eta\tilde{z} + (1-\eta)x + \rho\gamma_e)\, f_a(H_s(x))\, dx, \quad (2)$$

and the term corresponding to the contribution to the industry-wide knowledge stock is given by

$$\tilde{u}_s \equiv \rho e^{-\frac{\tilde{z}-z_O}{\rho}} \int_{z_{O,s}}^{\infty} V_s'(\eta\tilde{z} + (1-\eta)x + \rho\gamma_e)\, f_a(H_s(x))\, h_s(x)\, dx. \quad (3)$$

Back

Pigouvian Implementation of the Optimal Policy

Corollary.

Let $\widehat{V}_s(\cdot)$ be the value function under the market equilibrium featuring the additional taxes and subsidies. This value function and the operation tax τ_o satisfy

$$r\widehat{V}_s(z) = e^z J(\gamma_s(z))^\rho - (1+\tau_o) + \rho\left(\gamma_s(z) - g_{Z,s} - \frac{1}{2}\sigma^2\right)\widehat{V}_s'(z) + \frac{\rho^2\sigma^2}{2}\widehat{V}_s''(z),$$

subject to the boundary conditions $\widehat{V}_s'(z_{o,s}) = \widehat{V}_s(z_{o,s}) = 0$. Given the value function $\widehat{V}_s(\cdot)$ and the social value function V_s satisfying social HJB, the taxes are given by

$$\tau_{i,s}(z) = \frac{\widehat{V}_s'(z)}{V_s'(z)} - 1, \qquad \tau_{e,s} = \frac{\mathbb{E}_{F_s}\left[\widehat{V}_s'(z)\right]}{\mathbb{E}_{F_s}\left[V_s(z)\right]} - 1,$$

and τ_o is such that the cutoff matches $z_{o,s}$.

Back

Calibration: Adoption Process & Firm Lifetime

- Conditional on the primitives innovation & demand:

$$\text{Incumbents profitability } H(z) \quad \xrightarrow{\text{Adoption}} \quad H_e(z) \text{ Entrant profitability}$$

- Indirect approach:

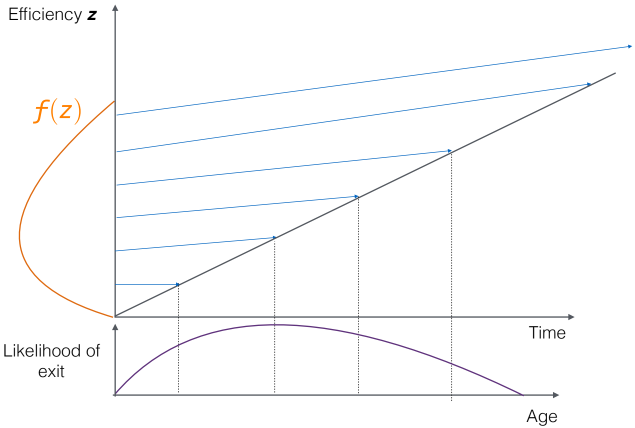
$$\text{Entrant profitability } H_e(z) \quad \xrightarrow{\text{Selection}} \quad \text{Distribution of firm lifetimes}$$

- Distribution of firm lifetimes $\equiv$ Relation of age vs. hazard of exit

Back More Details

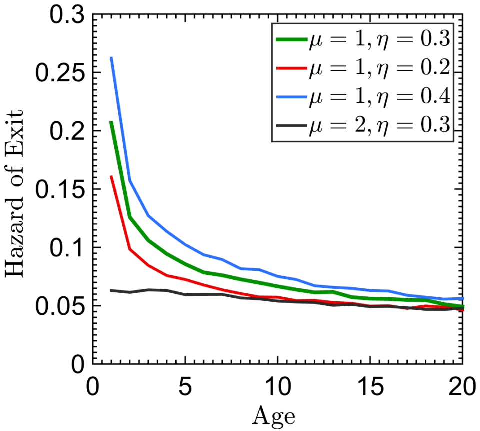
Identification of Adoption Process (1/2): Varying Calibrated Values

- Constraint based on the model: $H_e(z) = H\left(\frac{z-\gamma_e-\eta\tilde{z}}{1-\eta}\right)^{\mu}$



Identification of Adoption Process (2/2): Varying Calibrated Values

- Changing values of (μ, η) around calibrated values $(1, 0.3)$ in manufacturing.



Back