Declining Search Frictions, Unemployment and Growth

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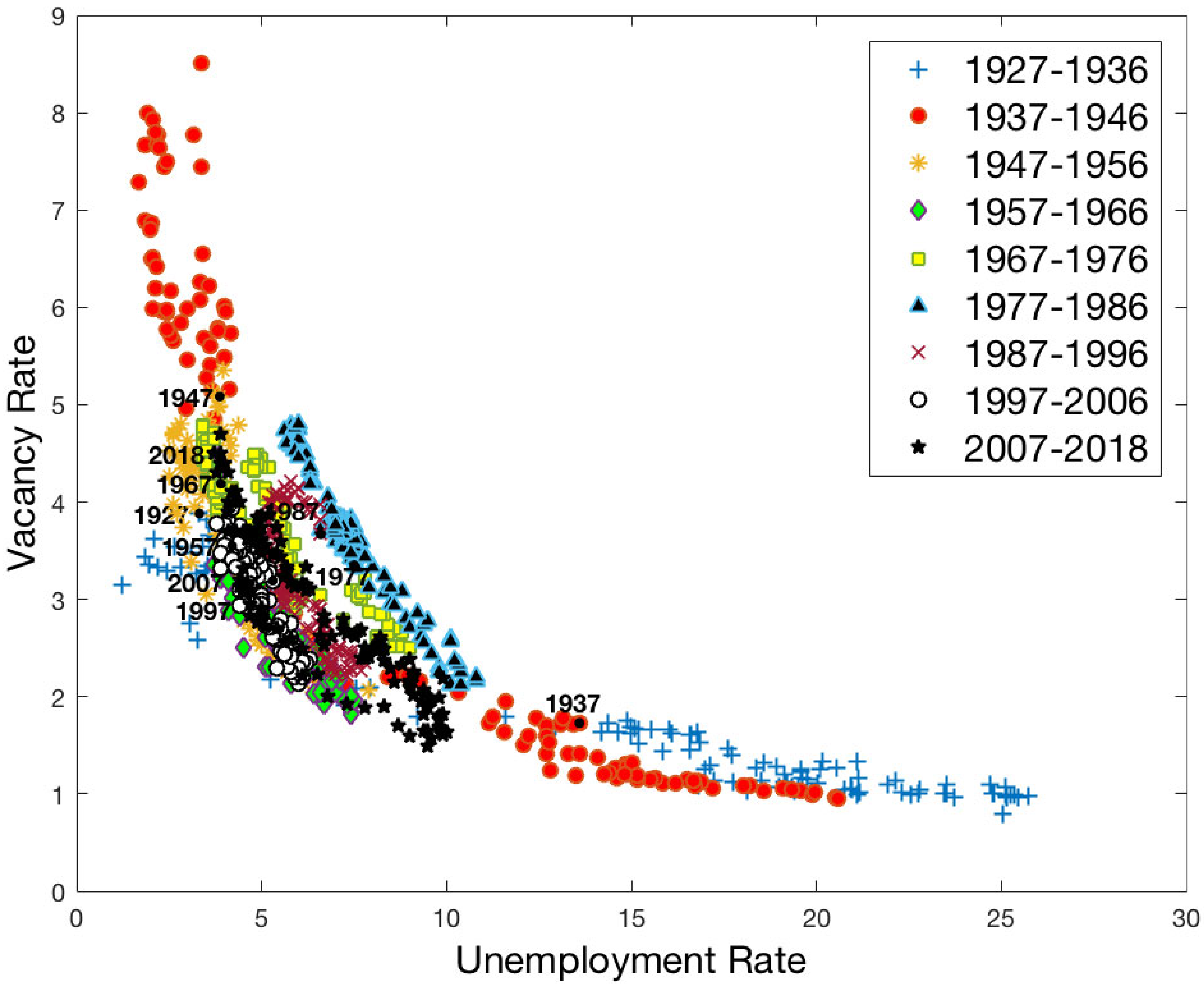
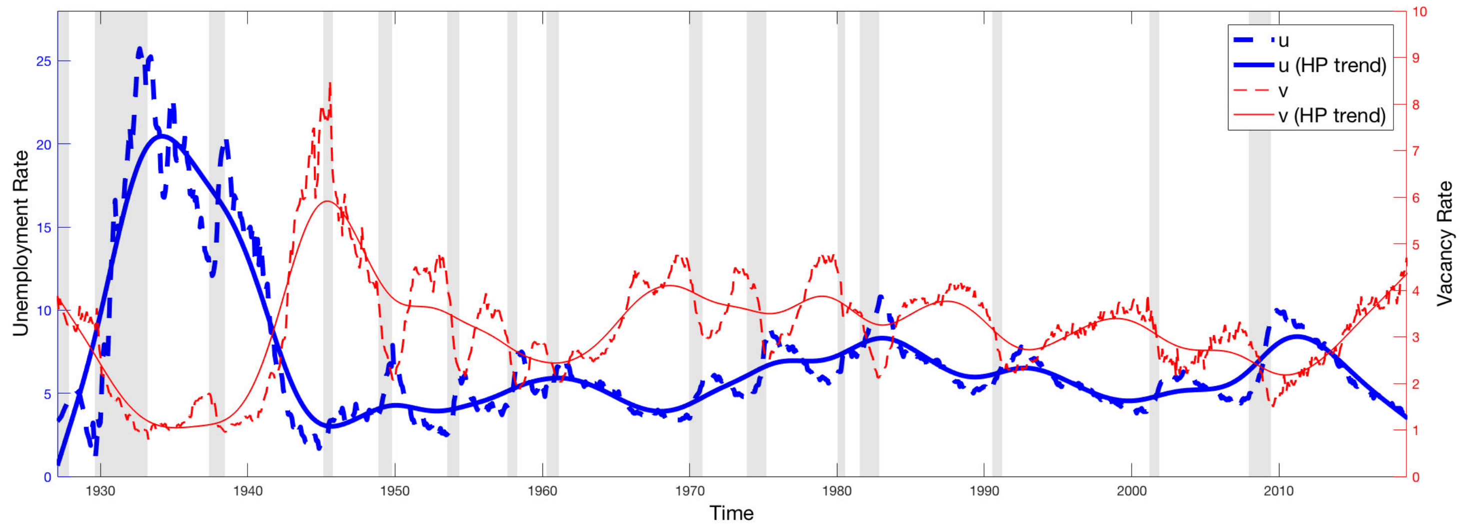
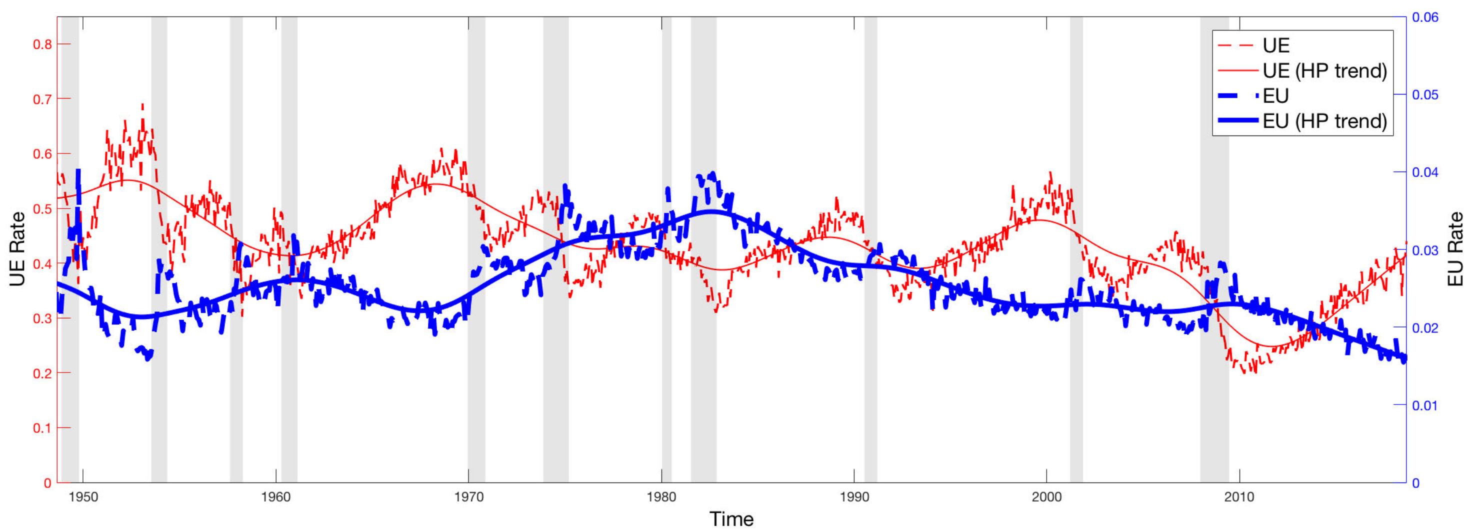


Vacancy Rate
Unemployment Rate
0
1
2
3
4
5
6
7
8
9
0
5
10
15
20
25
30
1927-1936
1937-1946
1947-1956
1957-1966
1967-1976
1977-1986
1987-1996
1997-2006
2007-2018
1947
2018
1967
1927
1957
2007
1997
1987
1977
1937


Unemployment Rate
Vacancy Rate
Time
u
u (HP trend)
v
v (HP trend)


UE Rate
EU Rate
Time
UE
UE (HP trend)
EU
EU (HP trend)

Introduction

Stylized long-run facts:

1. No secular movement in the Beveridge curve

$$u = \frac{\delta_t}{\delta_t + A_t p(v/u)}.$$

2. No secular movement along the Beveridge curve

$$u_t,\ v_t,\ v_t/u_t \text{ stationary.}$$

3. No secular trend in UE and EU rates.

From the perspective of search theory, the above observations imply that the efficiency A_t of the search technology has not improved from 1926 to 2017. Telephone? Fax? Mobile? Internet? Smart phone? All irrelevant!

Introduction

Modify standard search model to distinguish between meetings and matches:

1. Identify conditions for a **Balanced Growth Path** in which Beveridge Curve, u, v, UE and EU rates are constant despite improving search technology.

   Under these conditions, improvements in search technology show up in labor productivity growth not u.

2. Under the same conditions, u, v, UE and EU rates are constant over time and across markets despite increasing or decreasing returns to scale in search.

3. Develop a strategy to measure improvements in search technology, returns to scale in search and their contribution to labor productivity growth.

Related Literature

1. **Labor search**: Pissarides (1985), Mortensen and Pissarides (1994).

   Sanity check: Theory argues that u and v are due to information frictions. Basic check of the theory is to look at u and v across environments with different information frictions.

2. **Retail and IT**: Baye and Morgan (2005), Ellison and Ellison (2018).

   Similar puzzle: Why doesn't IT lower price dispersion and price level?

3. **Knowledge diffusion**: Lucas (2006), Lucas and Moll (2014), Perla and Tonetti (2014), Buera and Oberfield (2018), Kortum (1997).

   Connection: Random search models with growth

   Difference: Exogenous vs. endogenous growth. Market vs. no market.

Environment

Workers:

Firms:

Labor market:

Exogenous processes:

Environment

Workers:

- *population*: measure 1;
- *objective*: max pv of income $\{b_t, w_t\}$ discounted at rate r.

Firms:

Labor market:

Exogenous processes:

Environment

Workers:

Firms:

- *population*: positive measure;
- *technology*:

  maintain vacancies v_t at flow unit cost k_t;

  CRTS technology: 1 unit of labor –> $y_t z$ units of output;
- *objective*: max pv of income $y_t z - w_t$ discounted at rate r.

Labor market:

Exogenous processes:

Environment

Workers:

Firms:

Labor market:

- u_t and v_t come together through a matching fn $A_t M(u_t, v_t)$;
- u meets v at rate $A_t p(\theta_t)$, where $\theta_t = v_t/u_t$ and $p(\theta_t) = M(1, \theta_t)$;
- upon meeting u and v observe quality $z \sim F$, decide whether to match and bargain over terms of trade.
- **Matches are inspection goods**.

Exogenous processes:

Environment

Workers:

Firms:

Labor market:

Exogenous processes:

- search efficiency A_t grows at the rate g_A;
- production efficiency y_t grows at the rate g_y;
- unemp. benefit b_t grows at the rate g_b;
- vacancy cost k_t grows at the rate g_k.

Balanced Growth Path

Initial State: a measure of unemployed workers u_0, and a distribution of employed workers across match qualities G_0

Rational Expectation Equilibrium: time-path for value and policy functions, unemployment u_t and employment G_t, that satisfy optimality, market-clearing, and consistency conditions given (u_0, G_0).

Balanced Growth Path: Initial State and a Rational Expectation Equilibrium such that some variables are constant over time (u, $h_{ue,}$ h_{eu}, θ) and others grow at a constant rate (G_t).

Definition of a BGP

A BGP is a $\{R_t, S_t, G_t\}$ and a $\{\theta_t, h_{ue}, h_{eu}, u_t, g_z\}$ s.t.

1. Reservation quality R_t:

$$y_t R_t = b_t + A_t p(\theta_t)\gamma \int_{R_t} S_t(z) dF(z)$$

2. Surplus of a match S_t:

$$rS_t(z) = y_t(z - R_t) + \dot{S}_t(z)$$

3. Market tightness θ_t:

$$k_t = A_t \frac{p(\theta_t)}{\theta_t}(1-\gamma)\int_{R_t} S_t(z) dF(z)$$

Definition of a BGP

A BGP is a $\{R_t, S_t, G_t\}$ and a $\{\theta_t, h_{ue}, h_{eu}, u_t, g_z\}$ s.t.

4. Stationarity of UE, EU, u and θ_t:

$$\begin{aligned} A_t p(\theta)(1 - F(R_t)) &= h_{ue}, \\ G_t'(R_t)\dot{R}_t &= h_{eu}, \\ (1 - u_t)h_{eu} &= u_t h_{ue}, \\ \theta_t &= \theta. \end{aligned}$$

5. Distribution G_t of workers across z such that every quantile $z_t(x)$ grows at some constant rate g_z:

$$(1 - u_t)G_t'(z_t(x))z_t(x)g_z + u_t A_t p(\theta_t)[F(z_t(x)) - F(R_t)] = (1 - u_t)G_t'(R_t)R_t g_z.$$

Necessary Conditions for a BGP

N1 A BGP may exist only if F is Pareto with some coefficient α.

Sketch: The stationarity condition for the UE rate is:

$$A_t p(\theta)(1 - F(R_t)) = h_{ue}, \ \forall t \geq 0.$$

Differentiating with respect to t, we obtain

$$g_A = \frac{F'(R_t)}{1 - F(R_t)} R_t g_z$$

The differential equation for F has the unique solution

$$F(z) = 1 - \left(\frac{z_\ell}{z}\right)^\alpha.$$

Necessary Conditions for a BGP

N2 A BGP may exist only if g_b and g_k are equal to $g_y + g_z$.

Sketch: Combining the equilibrium conditions for R_t and θ, we obtain:

$$y_t R_t = b_t + \frac{\gamma}{1-\gamma}\theta k_t, \; \forall t \geq 0.$$

The condition above can only be satisfied if

$$g_b, g_k = g_y + g_z$$

Existence and Uniqueness of a BGP

The condition for the surplus can be written as:

$$S_t(z) = \int_t^{t+d} e^{-r(\tau-t)}[y_\tau z - y_\tau R_\tau]d\tau.$$

Solving the integral gives:

$$S_t(z) = y_t\left\{\frac{z}{r-g_y}\left[1-\left(\frac{R_t}{z}\right)^{\frac{r-gy}{gz}}\right]-\frac{R_t}{r-g_y-g_z}\left[1-\left(\frac{R_t}{z}\right)^{\frac{r-gy-gz}{gz}}\right]\right\}.$$

Using the expression above and the fact that F is Pareto, we can solve for the expected surplus of a meeting between a firm and a worker

$$\int_{R_t} S_t(z)dF(z) = \Phi y_t R_t^{-(\alpha-1)}.$$

Existence and Uniqueness of a BGP

1. Reservation quality R_t grows at the constant rate $g_z = g_A/\alpha$:

$$y_t R_t = b_t + A_t p(\theta)\gamma \underbrace{\int_{R_t} S_t(z)\,dF(z)}_{\Phi y_t R_t^{-(\alpha-1)}}$$

2. Market tightness θ_t is constant

$$k_t = A_t \frac{p(\theta)}{\theta}(1-\gamma) \underbrace{\int_{R_t} S_t(z)\,dF(z)}_{\Phi y_t R_t^{-(\alpha-1)}}$$

Existence and Uniqueness of a BGP

3. Quality distribution $G_t(z)$ grows at constant rate $g_z = g_A/\alpha$ and starts at

$$G_0(z) = 1 - \left(\frac{R_0}{z}\right)^{\alpha}.$$

4. Unemployment u_t is constant and starts at

$$u_0 = \frac{g_A}{g_A + A_0 p(\theta)[1 - F(R_0)]}.$$

5. UE and EU rates are both constant.

Existence and Uniqueness of a BGP

We have established the following.

Proposition 1: Take arbitrary growth rates $g_y > 0$ and $g_A > 0$.

A BGP exists if and only if:

(**a**) F is Pareto with coefficient α;

(**b**) g_b and g_k are equal to $g_y + g_A/\alpha$.

If a BGP exists, it is unique and such that:

(**i**) u, θ, h_{ue}, h_{eu} are constant over time;

(**ii**) G_t is Pareto truncated at R_t growing at rate $g_z = g_A/\alpha$;

(**iii**) labor productivity grows at rate $g_y + g_A/\alpha$.

Existence and Uniqueness of a BGP

Comments to Proposition 1:

1. The conditions that b and k grow at the same rate as labor productivity obtain endogenously if unemployment benefits are proportional to wages and if vacancy costs are labor costs.

2. The condition that F is Pareto does not mean that there is a large variance in the quality of different matches. Indeed, for α large, the variance of F is low.

3. The condition that F is Pareto means that the support of match quality is unbounded. This can be relaxed. The BGP holds between 1929 to 2019, as long as F is Pareto over the interval $[R_{1929}, R_{2019}]$ and the conditional mean of F for $z > R_{2019}$ is the same mean as for a Pareto.

4. Isomorphic model: differentiated workers and vacancies located on a circle, output depends on distance $z(d) = \pi^{1/\alpha} d^{-1/\alpha}$ (Boyan transform)

Search on the Job

The baseline model assumes that workers only search when unemployed. We consider a more general environment, in which workers search for jobs both when unemployed and when employed. We let $\rho \in [0,1)$ denote the relative search intensity of employed workers.

Search on the Job

A BGP is a $\{R_t, S_t, G_t\}$ and a $\{\theta_t, h_{ue}, h_{eu}, u_t, g_z\}$ s.t.

1. Reservation quality R_t:

$$y_t R_t = b_t + A_t p(\theta)(1-\rho)\gamma \int_{R_t} S_t(z)dF(z)$$

2. Surplus of a match S_t:

$$rS_t(z) = y_t(z - R_t) - A_t p(\theta)\rho\gamma\left[S_t(z)(1-F(z)) + \int_{R_t}^{z} S_t(\hat{z})dF(\hat{z})\right] + \mathring{S}_t(z).$$

3. Market tightness θ_t:

$$k_t = A_t \frac{p(\theta_t)}{\theta_t}\frac{u}{u+\rho(1-u)}(1-\gamma)\int_{R_t} S_t(\hat{z})dF(\hat{z})$$
$$+A_t \frac{p(\theta_t)}{\theta_t}\frac{\rho(1-u)}{u+\rho(1-u)}(1-\gamma)\int_{R_t}\left[\int_{z}(S_t(\hat{z}) - S_t(z))dF(\hat{z})\right]dG_t(z).$$

Search on the Job

A BGP is a $\{R_t, S_t, G_t\}$ and a $\{\theta_t, h_{ue}, h_{eu}, u_t, g_z\}$ s.t.

4. Stationarity of UE, EU, u_t and θ_t:

$$\begin{aligned} A_t p(\theta_t)(1 - F(R_t)) &= h_{ue}, \\ G_t'(R_t)\dot{R}_t &= h_{eu}, \\ (1 - u_t)h_{eu} &= u_t h_{ue}. \end{aligned}$$

5. Distribution G_t of workers across z such that every quantile $z_t(x)$ grows at some constant rate g_z:

$$\begin{aligned} & (1 - u_t)[G_t(z_t(x)e^{g_z dt}) - G_t(z_t(x))] + u_t A_t p(\theta_t)[F(z_t(x)e^{g_z dt}) - F(R_t e^{g_z dt})]dt \\ = \quad & (1 - u_t)[G_t(R_t e^{g_z dt}) - G_t(R_t)] + (1 - u_t)\rho A_t p(\theta_t)[1 - F(z_t(x))]G_t(z_t(x)). \end{aligned}$$

Search on the Job

N1 A BGP may exist only if F is Pareto with some coefficient α.

N2 A BGP may exist only if g_b and g_k are equal to $g_y + g_z$.

Search on the Job

The condition for the surplus is:

$$rS_t(z) = y_t(z - R_t) - A_t p(\theta)\rho\gamma\left[S_t(z)(1 - F(z)) + \int_{R_t}^{z} S_t(\hat{z})dF(\hat{z}) \right] + \mathring{S}_t(z).$$

Solving for $S_t(z)$ seems hopeless....

Search on the Job

We guess and we verify that

$$S_t(ze^{g_z t}) = S_0(z)e^{(g_y+g_z)t}.$$

This allows us to rewrite the condition for $S_t(z)$ as:

$$rS_t(z) = y_t(z - R_t) - A_t p(\theta)\rho\gamma\left[S_t(z)(1 - F(z)) + \int_{R_t}^{z} S_t(\hat{z})dF(\hat{z}) \right]$$

$$+ (g_y + g_z)S_t(z) - zg_z S_t'(z).$$

Then, we show that:

- the expected surplus $\bar{S}_{u,t}$ of a meeting between a firm and an unemployed worker grows at the constant rate $g_y - (\alpha - 1)g_z$;
- the expected surplus $\bar{S}_{e,t}$ of a meeting between a firm and an employed worker grows at the constant rate $g_y - (\alpha - 1)g_z$.

Search on the Job

1. Reservation quality R_t grows at the constant rate $g_z = g_A/\alpha$:

$$y_t R_t = b_t + A_t p(\theta)(1-\rho)\gamma \underbrace{\bar{S}_{u,t}}_{g_y - (\alpha - 1)g_z}$$

2. Market tightness θ_t is constant

$$k_t = A_t \frac{p(\theta)}{\theta}(1-\gamma) \underbrace{\left\{ \frac{u}{u + \rho(1-u)} \bar{S}_{u,t} + \frac{\rho(1-u)}{u + \rho(1-u)} \bar{S}_{e,t} \right\}}_{g_y - (\alpha - 1)g_z}$$

Search on the Job

3. Quality distribution $G_t(z)$ grows at constant rate g_z and starts at

$$G_0(z) = \frac{\exp(-A_0 p(\theta)\rho\overline{F}(z)/g_A) - \exp(-A_0 p(\theta)\rho\overline{F}(R_0)/g_A)}{1 - \exp(-A_0 p(\theta)\rho\overline{F}(R_0)/g_A)}.$$

4. Unemployment u_t is constant and starts at

$$u_0 = \frac{\rho\exp(-A_0 p(\theta)\rho\overline{F}(R_0)/g_A)}{1 - (1-\rho)\exp(-A_0 p(\theta)\rho\overline{F}(R_0)/g_A)}.$$

5. UE and EU rates are both constant.

Search on the Job

Workers search on the job with relative intensity $\rho \in [0,1]$.

Proposition 2: Take arbitrary growth rates $g_y > 0$ and $g_A > 0$.

A BGP exists if and only if:

(**a**) F is Pareto with coefficient α;

(**b**) g_b and g_k are equal to $g_y + g_A/\alpha$.

Any BGP is such that:

(**i**) u, θ, h_{ue}, h_{eu} are constant over time;

(**ii**) G_t is Fréchet truncated at R_t and grows at rate $g_z = g_A/\alpha$;

(**iii**) labor productivity grows at rate $g_y + g_A/\alpha$.

Population Growth

Baseline model assumes constant population. Assumption is w.l.o.g. as long as the matching function has constant returns to scale.

We consider a more general environment in which the population might grow and the matching function may have non-constant returns to scale.

- Population:

$$N_t = N_0 \exp(g_N t);$$

- Matching function:

$$A_t N_t^{\beta} \cdot [N_t M(u_t, v_t)].$$

* The overall efficiency of the search process is

$$\hat{A}_t = N_t^{\beta} A_t = \hat{A}_0 e^{(\beta g_N + g_A)t}.$$

Population Growth

Population grows at rate g_N and matching fn is $A_t N_t^{\beta+1} M(u_t, v_t)$.

Proposition 3: Take arbitrary growth rates $g_y > 0$, $g_A > 0$, $g_N > 0$ such that the overall search efficiency improves over time, i.e. $g_A + \beta g_N > 0$.

A BGP exists if and only if:

(**a**) F is Pareto with coefficient α;

(**b**) g_b and g_k are equal to $g_y + (g_A + \beta g_N)/\alpha$.

Any BGP is such that:

(**i**) u, θ, h_{ue}, h_{eu} are constant over time;

(**ii**) G_t is Pareto truncated at R_t and grows at rate $g_z = (g_A + \beta g_N)/\alpha$;

(**iii**) labor productivity grows at rate $g_y + (g_A + \beta g_N)/\alpha$.

Back-of-the-Envelope Calculations

Assuming a BGP, some quantitative questions arise:

1. Can’t infer growth of search efficiency from time trends of u, θ, h_{ue}, h_{eu}.

   a. Measure technological improvements in search process?

   b. Measure contribution to economic growth of improvements in search process?

2. Can’t infer returns to scale in search from time trends or cross-sections of u, θ, h_{ue}, h_{eu}

   a. Measure returns to scale in search process?

   b. Measure the contribution to economic growth of returns to scale in search process?

Back-of-the-Envelope Calculations

Average number of applications per vacancy are

$$A_t N_t^{\beta} q(\theta_t).$$

- In the model, applications per vacancy grow at rate $\beta g_N + g_A$.
- In the data, applications per vacancy were 24 in 1981 (EOPP) and 45 in 2010 (Career Builder, SnagAJob).
- These observations suggest

$$\beta g_N + g_A = 2.2\%$$

Back-of-the-Envelope Calculations

Relative number of applications per vacancy in two markets of sizes N_1 and N_2 with the same search technology is

$$\frac{A_{1,t} N_{1,t}^{\beta} q(\theta_{1,t})}{A_{2,t} N_{2,t}^{\beta} q(\theta_{2,t})} = \left(\frac{N_{1,t}}{N_{2,t}}\right)^{\beta}.$$

- In the model, elasticity of applications per vacancy wrt size is β.
- In the data, elasticity of applications per vacancy wrt size is 0.52.
- These observations suggest $\beta = 0.52$ and

$$\beta g_N = 0.52 \cdot 1.1\% = 0.6\%$$

$$g_A = \underbrace{g_A + \beta g_N}_{2.2\%} - \underbrace{\beta g_N}_{0.6\%} = 1.6\%$$

Back-of-the-Envelope Calculations

Contribution of Declining Search Frictions

	Pareto coefficient		
1981-2010	$\alpha = 4$	$\alpha = 8$	$\alpha = 16$
labor productivity growth		1.9%	
cont. of search technology	0.4%	0.2%	0.1%
cont. of IRS in search	0.15%	0.07%	0.04%
cont. of declining search frictions	0.55%	0.27%	0.14%

Back-of-the-Envelope Calculations

Returns to Scale and Productivity across Cities

	$\alpha = 4$	$\alpha = 8$	$\alpha = 16$
0.5 million workers	0.91	0.95	0.98
1 million workers	1	1	1
10 million workers	1.34	1.16	1.08