Self-Fulfilling Debt Crises, Revisited

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Sovereign Debt Crises

- Fundamentals **and** beliefs appear to be important drivers of crises
  - Crisis countries tend to have weak fundamentals
  - But timing of a crisis can be mysterious — comes without warning and fundamentals are often no worse

What We Do

- Develop a framework that gives both fundamentals and beliefs a role in debt crises and . . .
- . . . generates different types of crises in a quantitatively realistic way
  - Outright default
  - No default but a spike up in spreads on new borrowing ("borrowing into high spreads")
  - No default but heavy capital outflows ("sudden stops")

Small Open-Economy Environment

- Stochastic endowment $Y_t \sim \Gamma(Y, Y_{t-1}), Y_t \in \mathbb{Y}$
- Preferences of government:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t),$$

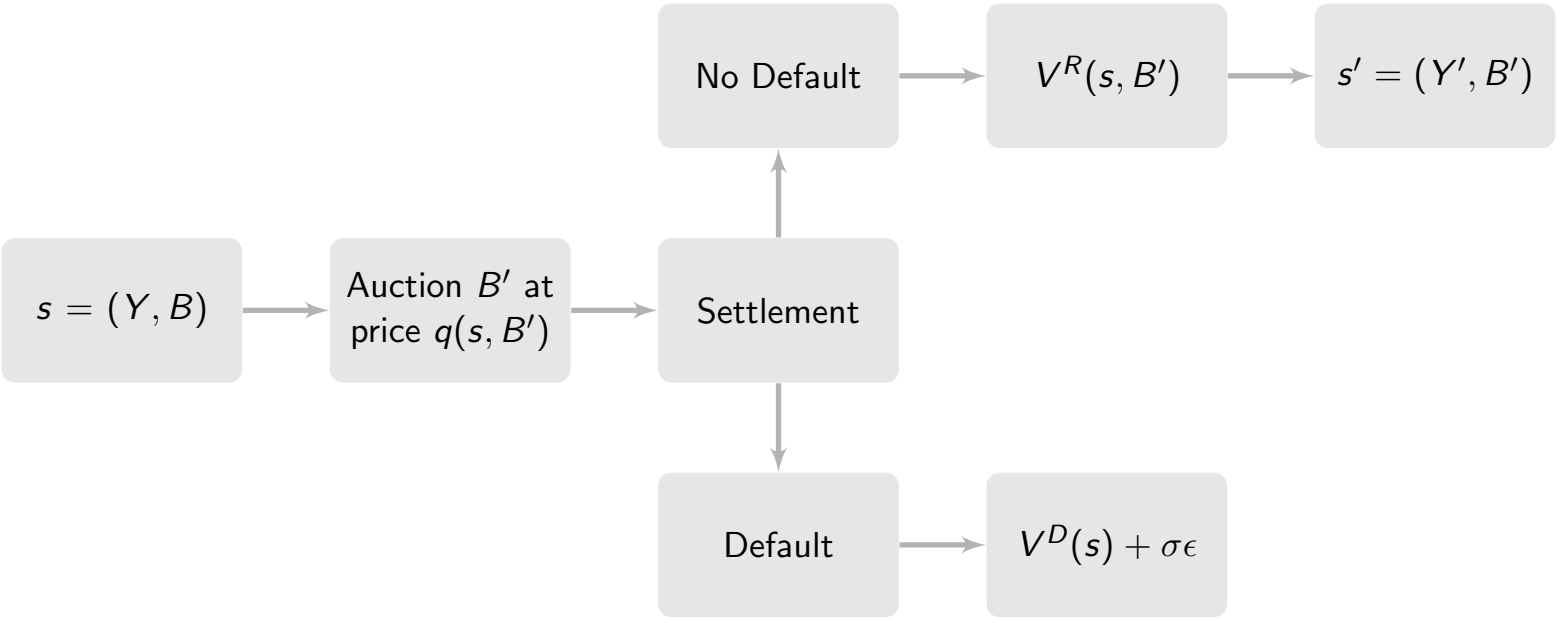
- Risk-neutral lenders discount at $R^{-1} = (1 + r^*)^{-1}$
- One-period bonds*: $B \in [\overline{B}, \infty), \quad \overline{B} < 0$
- Default leads to permanent autarky*

Timing and Interim Uncertainty

- Enter with (Y, B)
- Auction B' and get $q(Y, B, B')B'$
- An additive shock to default payoff is realized
  - Modeled as $\sigma\epsilon$, where $\epsilon \sim F$, $\epsilon \in [0, 1]$
- Choose to default or repay B

Timing

In a picture . . .



Bellmans*

For exposition, assumes Y is i.i.d.

- Value of repayment at settlement:

$$V^R(s, B') = \begin{cases} u(Y + q(s, B')B' - B) + \beta \mathbb{E}\left[V(s')|B' \in s'\right] \\ -\infty \text{ if } B' \text{ is infeasible} \end{cases}$$

- Value of default at settlement (net of shock):

$$V^D(s, B') \equiv u(Y + q(s, B')B') + \beta \mathbb{E}\left[\frac{u(Y)}{1-\beta}\right]$$

- Value at s

$$V(s) = \max_{B'} \left\{ \max \left\langle V^R(s, B'), V^D(s) + \sigma\epsilon \right\rangle \right\}$$

Strategic Default

$$\max \left\langle V^R(s, B'), V^D(s) + \sigma\epsilon \right\rangle$$

- Sovereign repays if and only if:

$$\sigma\epsilon \leq \underbrace{\left[V^R(s, B') - V^D(s, B')\right]}_{\equiv \Delta(s, B')}$$

- Prob of repayment at settlement is $F(\frac{1}{\sigma} \cdot \Delta(s, B'))$
- If $\sigma^{-1} \cdot \Delta(s, B') \in (0, 1)$, prob of def at settlement is **positive**

Pricing of Debt

- Lenders are risk-neutral, so price must get the risk-free rate in expectation
- Price of B', given Y and B

$$q(s, B') = R^{-1} F(\sigma^{-1} \cdot \Delta(s, B')) \times \underbrace{\mathbb{E}\left[F(\sigma^{-1} \cdot \Delta(s', \mathcal{B}(s', B')))|B'\right]}_{q_{EG}(B')}$$

- Lenders can be defaulted on in $t+1$ (as in Eaton-Gersovitz) **and** t (as in Cole-Kehoe)
- Holding fixed $q_{EG}(B')$, the possibility of current period default supports **multiple equilibrium prices at** t

(Static) Multiplicity

Keep in mind $\Delta(s, B'|q) = V^R(s, B'|q) - V^D(s, B'|q)$

- **Given** Y, B, B' and $q_{EG}(B')$, a candidate equilibrium price $\tilde{q}$ for B' must satisfy

$$\tilde{q} = F(\sigma^{-1} \cdot \Delta(s, B'|\tilde{q})) \times q_{EG}(B')$$

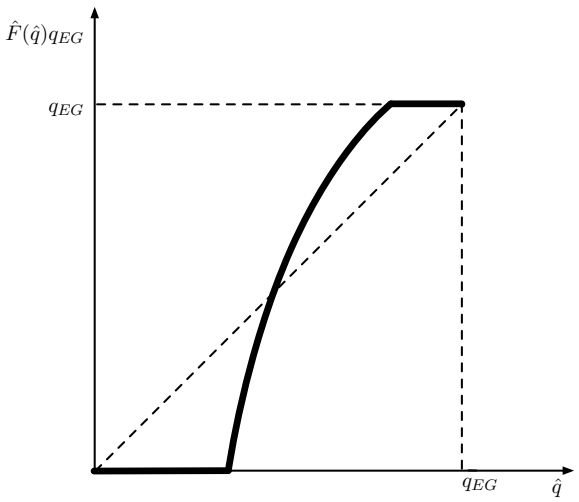
$$\tilde{q} = F\left(\sigma^{-1} \cdot \left[\begin{array}{c} u(Y - B + \tilde{q}B') - u(Y + \tilde{q}B') \\ +\beta\mathbb{E}\left[V(s')|B'\right] - \beta\underline{V}^D \end{array}\right]\right) q_{EG}(B')$$

$$= \tilde{F}(\tilde{q}|\cdot)q_{EG}(B'), \text{ where}$$

$$\tilde{F}(\tilde{q}|\cdot) : [0, q_{EG}(B')] \to [0, 1]$$

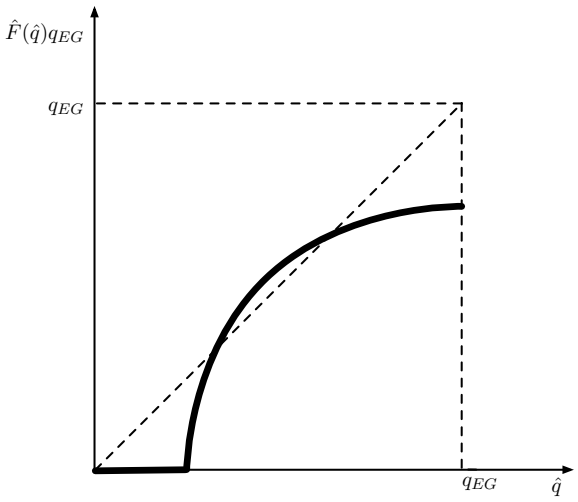
- $B > 0$ and u concave $\Rightarrow \Delta(s, B'|\tilde{q}) \uparrow$ in $\tilde{q} \Rightarrow \tilde{F} \uparrow$ in $\tilde{q}$

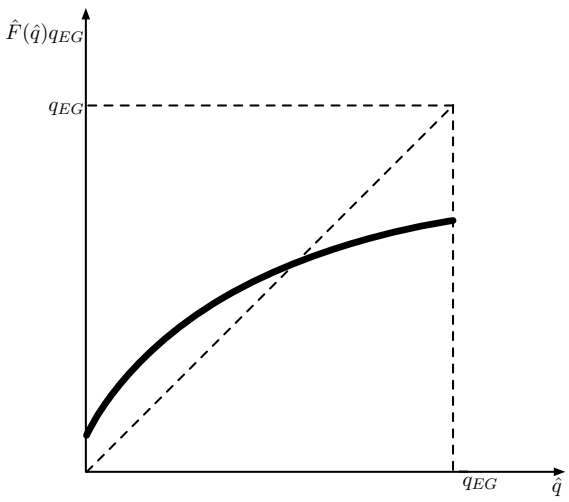
Static Multiplicity



- Consider some (Y, B) for which the sovereign strictly prefers repayment at q_{EG} and default at 0
- Then there is a **positive** equilibrium price $< q_{EG}$ for which default at settlement can happen with positive probability

Other Possibilities*





- Always-default & always-repay $\tilde{F}(\tilde{q}|\cdot)q_{EG}(B')$ functions are also possible

Static Multiplicity

Number and Types of Equilibrium Prices

Given Y, B and B'

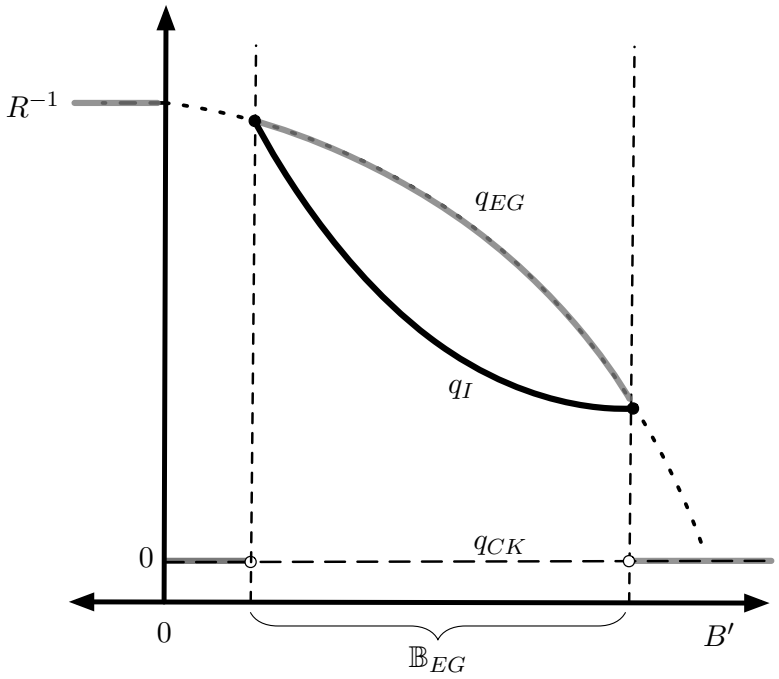
- If σ sufficiently small, at most 3 equilibria
- If F uniform, $\tilde{F}(\tilde{q})q_{EG}$ is concave, so at most 3 equilibria regardless of magnitude of σ

From Multplicity to Belief Regimes

- ρ indexes lender beliefs and, now, $s = (Y, \rho, B)$
- We assume $\rho \in \{O, P, C\}$
  - "O" denotes optimistic and $q(Y, O, B, B')$ is the highest eq price possible
  - "P" denotes pessimistic and $q(Y, P, B, B')$ is lowest eq price possible
  - "C" denotes "concerned" and $q(Y, C, B, B')$ is an interior price if one exists, otherwise it is the optimistic price
- The value of ρ is selected at random each period from a known 3-point distribution

Example Price Schedules

Grey is "ρ=O", Dash is "ρ=P" and Black is "ρ=C"



Payoff Figures

Example Price Schedules

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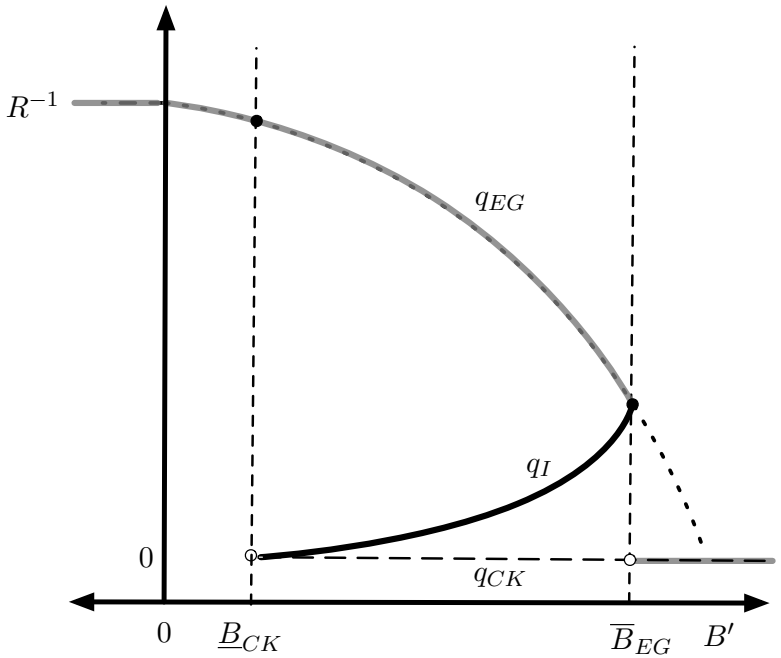
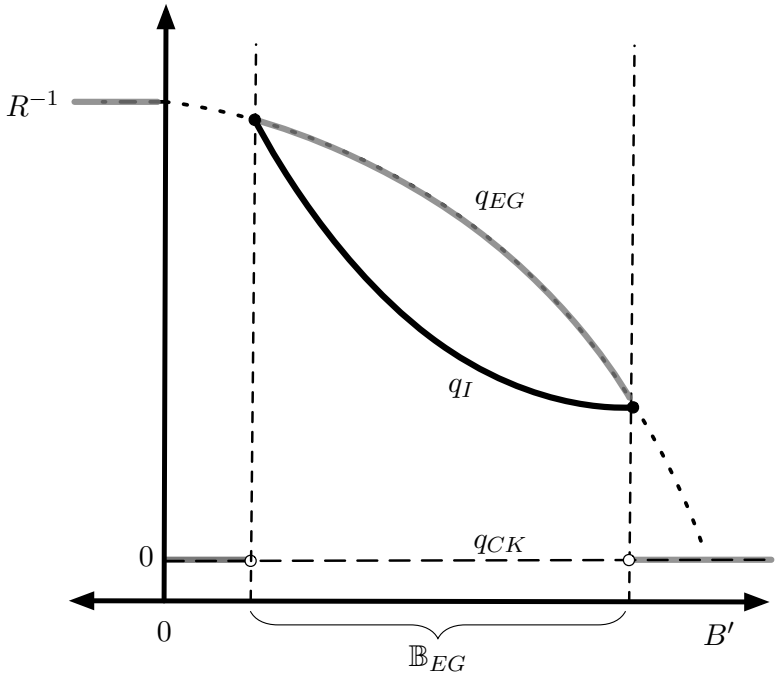


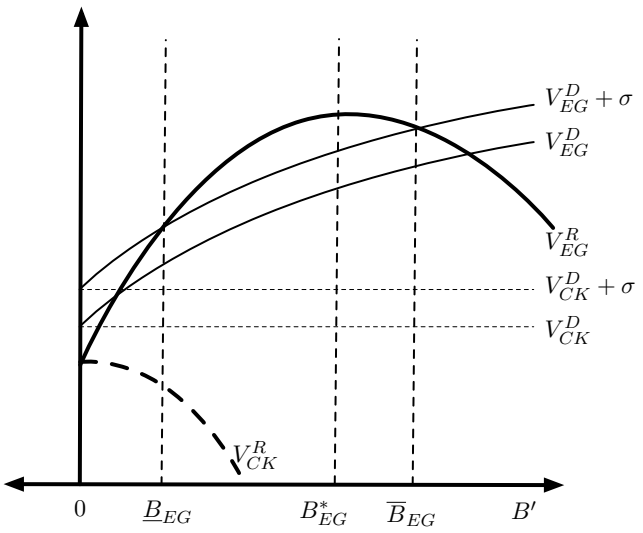
Payoff Figures

Choice of B' When $\rho = \mathbf{C}$

B' with an interior price maybe a dominated choice!



What Is The Interim Uncertainty Doing?



- If $\rho = O$, the sovereign chooses B_{EG}^{*}
- If $\rho = C$, the interim risk generates **"borrowing into high spreads"** by inducing the sovereign to choose $\overline{B}_{EG} > B_{EG}^{*}$

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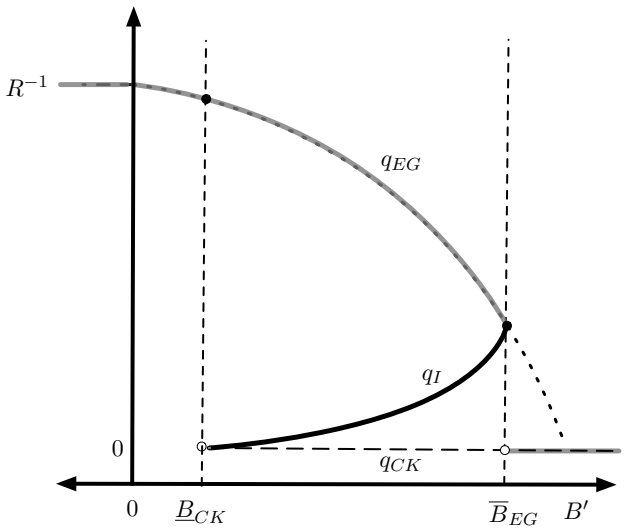


- If $\rho = C$, the interim risk could generate a **"sudden stop"** by inducing the sovereign to choose $\underline{B}_{CK} < \overline{B}^*_{EG}$

Quantification

Mexico, Argentina

- Drop i.i.d. Y assumption and instead assume that growth rate of Y follows an AR1 process
- Drop the assumption that default leads to permanent autarky and instead allow reentry with some constant probability
- Drop the assumption that default is costless – output is lower by a proportion d_0 during default and exclusion

Mexico*

- $g_t = (1-\mu)\bar{g} + \mu g_{t-1} + \varepsilon_t + [\nu_t - \nu_{t-1}]$
- $\rho \in \{O, C\}$
- $\beta = 0.80$, CRRA $= 2$
- Re-entry prob $= 0.125$, $r = 0.01$, $\epsilon \sim U$ and $\sigma = 0.0001$
- $d_0 = 0.176$ and $\pi_C = 0.0225$ set to match avg debt-to-GDP and s.d. of spreads

Moments*

	BM	Alt I	Alt II
Avg Debt-to-Qtrly-GDP	65.6*%	66.1 %	30.2 %
Default Frequency	0.50 %	0.10 %	0.0 %
Spread Volatility	2.5* %	0.10 %	0.0 %
σ_c/σ_y	1.20	1.15	1.05

Simulated Moments

Alt I : $\rho \in \{O\}$

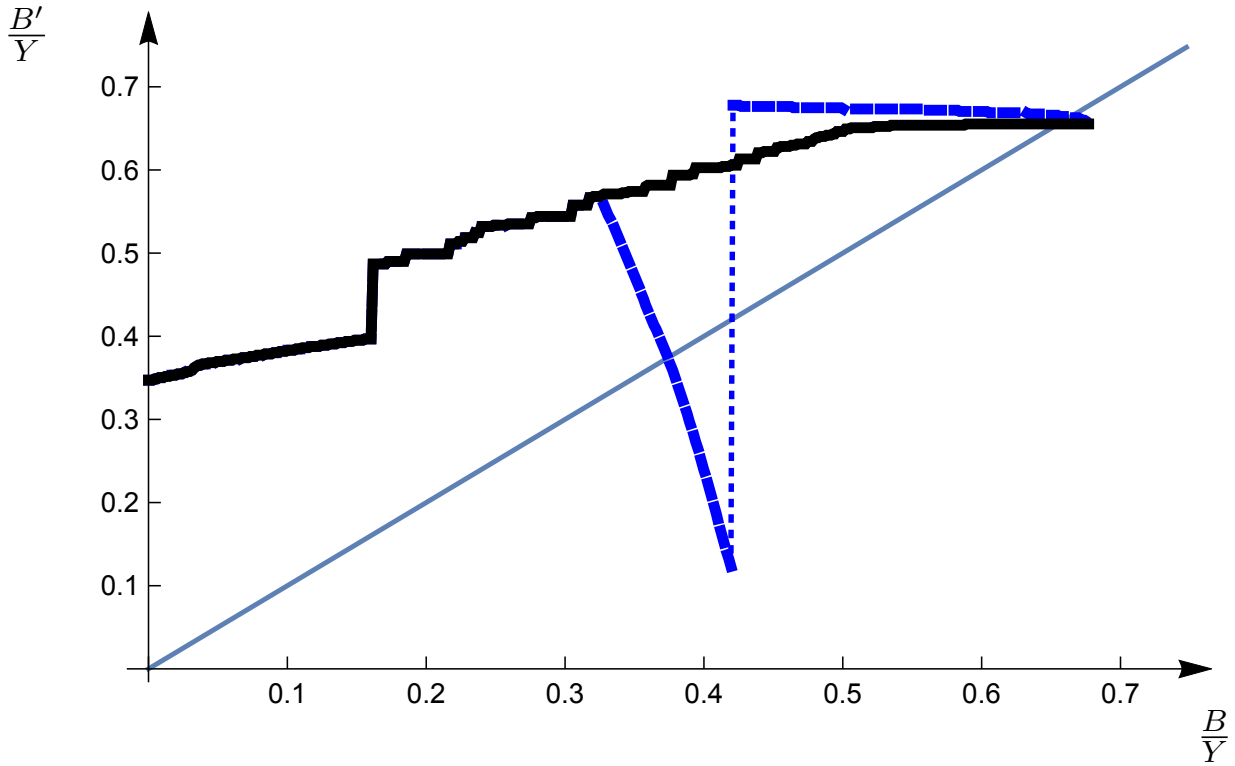
Alt II : $\rho \in \{O, P\}$; O with prob 0.975 (as in BM)

- **The C regime is key for the volatility of spreads**
- Replacing C with P makes the sovereign lower debt levels to the point where **all** defaults and **all** spread volatility is eliminated !

What Sorts of Crises Do We Get in Simulations ?

- Defaults happen once every 200 years (calibration target)
- Faced with $\rho = C$, **borrowing into high spreads** without default is relatively common
- But **sudden stops** while possible (almost) never happen

Policy Function in the Base Model*

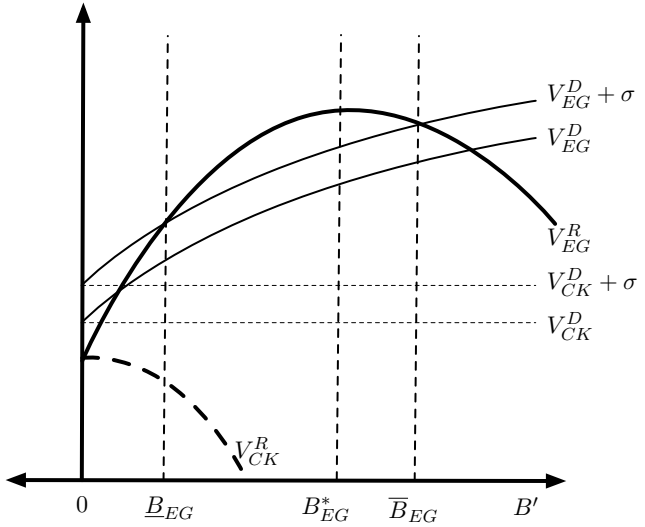


Summary

- Allowed sovereign to default following auction of new bonds to permit **multiple equilibrium price schedules**
- Explored a **new type of equilibria** in which a crisis is a shift to a **low but positive** price schedule
- Faced with this (new type of) crisis price schedule, **the sovereign does not default but might either borrow more or de-leverage severely**
- This framework might shed light on how and why "whatever it takes" type of announcements can be effective in reducing spreads or restoring the flow of credit

B' and Equilibrium Possibilities

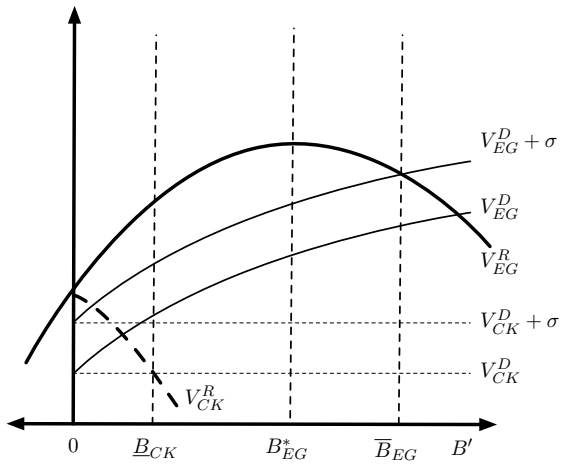
Canonical Crisis Zone



- 0 is an equilibrium price for all $B' \geq 0$
- The EG price is an equilibrium price for $B' \in [\underline{B}_{EG}, \overline{B}_{EG}]$
- Faced with a zero price schedule, the sovereign will default

B' and Equilibrium Possibilities

Extended Crisis Zone and Sudden Stops



- 0 is an equilibrium price for all $B' \geq \underline{B}_{CK}$
- The EG price is an equilibrium price for $B' \in [0, \overline{B}_{EG}]$
- Faced with prices that are q_{EG} for $B' \leq \underline{B}_{CK}$ but zero for $B' > B_{CK}$, the sovereign may choose to **reduce B' rather than default**

Price Schedules