Rare Disasters, Financial Development, and Sovereign Debt

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Introduction

- Emerging economies pay high, volatile risk premia on their sovereign debt, despite having much lower debt-to-GDP ratios than developed countries.
  - Reinhart, Rogoff and Savastano (2003) call this phenomenon "debt intolerance"

- Debt intolerance of emerging markets is inconsistent with classic Eaton and Gersovitz (1981) sovereign debt model.
  - Key cost of defaulting in model is loss of access to capital markets.
  - Since real GDP growth is generally more volatile in emerging markets than in developed countries, the loss of market access is more costly for emerging markets.
  - Other things equal, emerging markets should be less likely to default, pay lower credit spreads on their sovereign debt, and have higher debt capacity.

Financial development

- We propose a model of sovereign debt where countries have different levels of financial development.
- By financial development, we mean the extent to which countries can hedge shocks to their economy in international capital markets.
  - In practice, developed countries have greater ability to issue stocks as well as debt that has long maturities and is denominated in local currency.
  - To simplify, we model financial development as greater ability to hedge jump shocks.

Financial development

"The economies and financial markets of emerging market economies (EMEs) tend (with some exceptions) to be more volatile than those of advanced economies. This is true whether one looks at output growth, exchange rates, interest rates or capital flows. Given this volatility, one would expect hedging markets in EMEs to be well developed. But this does not seem to be the case. EMEs make up about one third of the global economy when measured at market exchange rates and just under one half when measured at purchasing power parity. Their share in global trade is 36 percent. Still, derivatives referencing their currencies or interest rates account for only 10 percent of the global turnover of such contracts, despite notable growth in some cases in recent years."

BIS Quarterly Review, 2016.

Financial development and optimal debt management

- How does the ability to hedge affect debt capacity and optimal debt management?
- We find that low levels of financial development generate debt intolerance.
  - Low debt capacity, high credit spreads, and limited ability to smooth consumption.
- We find that hedging and financial wealth are complements.

Model

- Continuous time
  - Solved analytically up to an ordinary differential equation with economically intuitive boundary conditions.

- Preferences
  - Recursive utility proposed by Duffie and Epstein (continuous time version of Epstein-Zin).
  - Separation of risk aversion and intertemporal substitution important in quantitative analysis.
  - Allows model to generate plausible debt-output ratios without the large discount rates used in literature.
  - We use a conventional discount rate and risk aversion combined with low EIS, which we interpret as reflecting expenditure commitments that are difficult to change.

Model

- Output
  - We adopt the jump-diffusion process used in the rare-disaster literature (Barro and Jin (2011)).
  - Diffusion shocks are useful in generating empirically plausible paths for output
  - Jumps shocks are essential to generate default in equilibrium.

Model

- Default costs
  - Decline in output (quantitatively important)
  - Temporary loss of access to international capital markets. Access regained with constant probability (quantitatively unimportant).
  - Model generates plausible debt-output ratios without non-linear default costs used in literature.
  - Linear default costs consistent with recent evidence by Hebert and Schreger (2016).

- Financial markets (outside default state)
  - Country can invest in risk-free international bond.
  - Issue non-contingent debt that can be defaulted upon.
  - Hedge diffusion shocks.
  - Potentially hedge jump shocks

Model

- Diffusion shocks do not generate default in our model because these shocks are hedgeable and our debt is short term.

- Only uninsurable jump shocks that are large enough generate default.
  - For insurable jump shocks, it is more efficient to hedge than to default.
  - For uninsurable jump shocks of moderate sizes, it is more efficient to preserve the option to default in the future against larger losses than to default.

- Default in other continuous-time models
  - Nuno and Thomas (2015), Tourre (2017), and DeMarzo, He, and Tourre (2018) generate default in models with diffusion shocks by assuming that countries issue term debt with exogenous maturity and that debt issuance is locally deterministic.
  - Bornstein (2017) generates default by assuming that output follows a Poisson process in a continuous-time version of Arellano (2008).

Three versions of the model

1. First best: perfect commitment and full spanning.
   - Country hedges fully.
   - Debt capacity is present value of output.

2. Model with limited commitment and full spanning.
   - No default in equilibrium, so credit spreads are zero.
   - As in Kehoe and Levine (1993), debt capacity is reduced so that country does not succumb to the temptation to default.
   - It is optimal to hedge all risks in international financial markets.
   - Volatility of output is eliminated.
   - Relatively high debt capacity.

Three versions of the model

3. Benchmark model: limited commitment and limited spanning.
   - Defaultable debt is a partial hedge against risks that cannot be insured.
   - *It is not optimal to fully hedge risks that can be hedged!*
   - Hedging instruments are used to increase debt capacity by ensuring that small shocks do not trigger default.
   - Output is volatile.
   - Credit spreads are high and volatile.
   - Debt capacity is low.

Endowment process

- Output, Y_t, follows jump-diffusion process.

$$\frac{dY_t}{Y_{t-}} = \mu dt + \sigma d\mathcal{B}_t - (1 - Z) d\mathcal{J}_t \,, \quad Y_0 > 0 \,,$$

- $\mathcal{B}_t$ is a standard Brownian motion.
- $\mathcal{J}$ is a pure jump process:
  - $d\mathcal{J}_t = 1$ if a jump occurs at t and $d\mathcal{J}_t = 0$ otherwise
  - Poisson process with a constant arrival rate, λ
  - If a jump occurs at time t, Y_t goes from Y_{t-} to $Y_t = ZY_{t-}$
  - Z has pdf $G(Z)$ with $Z > 0$.
- Expected growth rate of output

$$g = \mu - \lambda[1 - \mathbb{E}(Z)]$$

Endowment process in discrete time

$$\ln Y_{t+\Delta} - \ln Y_t = \left(\mu - \frac{\sigma^2}{2}\right)\Delta + \sigma\sqrt{\Delta}\,\epsilon_{t+\Delta} - (1 - Z)\nu_{t+\Delta}$$

- time-t conditional distribution of $\epsilon_{t+\Delta}$ is a standard normal;
- $\nu_{t+\Delta} = 1$ with probability $\lambda\Delta$ and zero with probability $(1 - \lambda\Delta)$;
- $\sigma^2/2$ is the Jensen-inequality correction associated with the diffusion shock.

Preferences

- Recursive utility with normalized aggregator $f(C, V)$:

$$V_t = \mathbb{E}_t \left[\int_t^\infty f(C_u, V_u) du \right],$$

- Duffie-Epstein's continuous-time version of Epstein-Zin.

$$f(C, V) = \frac{\rho}{1 - \psi^{-1}} \frac{C^{1-\psi^{-1}} - [(1-\gamma)V]^\chi}{[(1-\gamma)V]^{\chi-1}}$$

where $\chi = \frac{1-\psi^{-1}}{1-\gamma}$

- Time-separable CRRA is special case

Epstein-Zin preferences in discrete time

$$V_t = \left[(1-\beta)c_t^{1-1/\psi} + \beta(EV_{t+1}^{1-\gamma})^{(1-1/\psi)/(1-\gamma)}\right]^{1/(1-1/\psi)}$$

- β is the discount factor,
- γ is the relative-risk-aversion coefficient for static gambles,
- ψ is the elasticity of substitution with respect to deterministic income changes.

- What is the role of risk aversion and intertemporal substitution in determining debt capacity and optimal debt management?

Financial assets and market structure

- The country has four investment and financing opportunities
  - it can insure its diffusion risk through hedging contracts
  - it can buy insurance against certain jumps
  - it can borrow in the sovereign debt market at an interest rate that is the sum of the risk-free rate, r, and an endogenous credit spread, π_t
  - it can save at the risk-free rate, r

- Upon default on its sovereign debt, the country enters autarky and loses access to all four investment and financing opportunities

Diffusion risk hedging contracts

- Diffusive shocks are idiosyncratic and markets for contracts that hedge these shocks are perfectly competitive
- No upfront payment for holding this security and no risk premium for bearing idiosyncratic risk
- Investor's instantaneous gain or loss at time $t + dt$: $\sigma d\mathcal{B}_t = \sigma \left(\mathcal{B}_{t+dt} - \mathcal{B}_t\right)$
- Normalize the volatility as: σ
- The holdings of diffusion risk contracts at time $t-$: Θ_{t-}

Jump insurance contracts and premia

- Jump shocks are idiosyncratic.
- Insurance contracts for all intervals $(Z, Z + dZ)$ for $Z \geq Z^*$ are available for trading and are competitively priced.
- Buyer of $X_{t-}(Z)$ units of jump insurance at time $t-$ makes continuous insurance premium payments equal to $X_{t-}(Z)\lambda\, dG(Z)$.
- Receives a lump-sum payment $X_{t-}(Z)$ if the recovery fraction falls in interval $(Z, Z + dZ)$
- Total jump insurance premium per period:

$$\Phi_{t-} = \lambda \int_{Z^*}^{1} X_{t-}(Z) dG(Z) \equiv \lambda\, \mathbb{E}\left[X_{t-}(Z)\, \mathcal{I}_{Z \geq Z^*}\right]$$

Sovereign debt

- Denote the country's financial wealth by W_t
- Sovereign debt is continuously repaid and reissued
- Sovereign debt is held and priced in competitive markets by well-diversified foreign investors
- The maximal amount of sovereign debt is stochastic and endogenously determined in equilibrium

Sovereign default

- Country has option to default at any time on its sovereign debt
- Two costs of defaulting
  - Output drops permanently from Y_t to αY_t, $\alpha \in (0, 1)$.
  - Country goes into autarky where it has no access to international financial markets.
- Country re-gains full access to financial markets with probability ξ per unit of time.
- Upon exiting autarky, country starts afresh with no debt ($W_t = 0$).

Credit spread

- Debt is risky because it is not collateralized and the borrower has option to default.
- Use D_t to denote the level of sovereign debt: $D_t = -W_t > 0$ when $W_t < 0$.
- When country does not default, it pays principal and interest to creditors, $D_t(1 + (r + \pi_t)dt)$, and borrows D_{t+dt} at time $t + dt$.
- When the country defaults it pays nothing to creditors at time $t + dt$.

Optimality

- The country maximizes utility of the representative agent by choosing
  - consumption,
  - diffusion and jump risk hedging demands,
  - sovereign debt issue,
  - default timing,

  for given
  - output process,
  - equilibrium pricing of sovereign debt,
  - insurance contracts for diffusion and jump shocks.

Model solution

- $V(W_t, Y_t)$ is the representative agent's value function for the normal regime.
- $\widehat{V}(Y_t)$ the value function for the autarky regime.
- Closed form ordinary differential equation + economically intuitive boundary conditions.
- Three steps
  - Analyze the decision problem in the normal regime,
  - Characterize the boundary conditions for endogenous default,
  - Convert the two-dimensional optimization problem into a one-dimensional problem.

Value function

- The value function takes the form:

$$V(W,Y) = \frac{[bP(W,Y)]^{1-\gamma}}{1-\gamma}\,,$$

$$b = \rho\left[\frac{r+\psi(\rho-r)}{\rho}\right]^{\frac{1}{1-\psi}}$$

- $P(W,Y)$ is certainty equivalent wealth

- $P(W,Y)$ is the total wealth that makes the agent indifferent between the status quo and having a wealth level $P(W,Y)$ and no output:

$$V(W,Y) = V(P(W,Y),0)\,.$$

Autarky regime

- The financial wealth is always zero in autarky: $W_t \equiv 0$
- Value function $\widehat{V}(Y)$ satisfies HJB equation:

$$0 = f(Y, \widehat{V}) + \mu Y \widehat{V}'(Y) + \frac{\sigma^2 Y^2}{2} \widehat{V}''(Y) + \lambda \mathbb{E}\left[\widehat{V}(ZY) - \widehat{V}(Y)\right] + \xi \left[V(0, Y) - \widehat{V}(Y)\right]$$

- The value function takes the form:

$$\widehat{V}(Y) = \frac{(b\,\widehat{p}\,Y)^{1-\gamma}}{1-\gamma}$$

Connecting the normal regime with autarky

- Denote $-\underline{W_t}$ by debt capacity, i.e the maximum debt that the country can issue

- It's indifferent between repaying its debt and defaulting:

$$V(\underline{W_t}, Y_t) = \widehat{V}(\alpha Y_t)$$

- This value-matching condition implies: $\underline{W_t} = \underline{W}(Y_t)$

- The implied borrowing constraint:

$$W_t \geq \underline{W}(Y_t)$$

Homogeneity property

- Homogeneity property: reduce number of state variables from 2 to 1
- Scaled financial wealth is the model's effective state variable:

$$w_t = \frac{W_t}{Y_t}$$

- The scaled debt capacity: $\underline{w} = \underline{W_t}/Y_t$
- The scaled decisions:

$$c_t = C_t/Y_t, \theta_t = \Theta_t/Y_t, x_t = X_t/Y_t$$

- The scaled certainty equivalent wealth: $p(w_t) = P(W_t, Y_t)/Y_t$

Endogenous default threshold

- Only uninsurable large jump shocks trigger default.
- For insurable output shocks, it is more efficient to hedge than to default.
- For uninsurable downward jump shocks of moderate sizes, it is more efficient to preserve the option to default in the future against larger losses than to default.

Equilibrium credit spread

- The competitive-market zero-profit condition implies

$$-W_{t-}(1 + rdt) = -W_{t-}(1 + (r + \pi_{t-})dt)\,[1 - \lambda\, G(\underline{Z}(w_{t-}))dt] + \lambda\, G(\underline{Z}(w_{t-}))dt \times 0$$

- It implies

$$\pi(w_{t-}) = \lambda\, G(\underline{Z}(w_{t-}))$$

Determining scaled debt capacity

- It is not optimal to default in response to diffusive shock
- Country sets controlled volatility process to zero at $\underline{w}$:

$$\sigma_w(\underline{w}) = 0$$

- It implies

$$\theta(\underline{w}) = \underline{w}$$

- To ensure $w_t \geq \underline{w}$ for all $t \geq 0$, $\mu_w(\underline{w}) \geq 0$ is required

First best: full spanning and full commitment (FB)

- Country fully hedges diffusion and jump risk
- Full spanning means that $Z^* = 0$
- As in Friedman (1957) and Hall (1978), the present value of output is

$$H_t = \mathbb{E}_t \left(\int_t^\infty e^{-r(u-t)} Y_u du \right)$$

- The scaled non-financial wealth is

$$h = \frac{1}{r - g}$$

- To ensure that non-financial wealth is finite, we require that $r > g$
- To ensure that utility is finite, we require

$$\rho > (1 - \psi^{-1}) r$$

Model solution for FB

- Scaled total wealth:

$$p^{FB}(w) = P^{FB}(W, Y)/Y = (W + H)/Y = w + h$$

- The scaled endogenous debt capacity: $-\underline{w}^{FB} = h$
- The optimal consumption-output ratio:
  $c^{FB}(w) = m\, p^{FB}(w) = m(w + h)$ where $m = r + \psi\,(\rho - r)$
- The optimal hedging demand for diffusive shocks: $\theta^{FB}(w) = -h$
- The optimal hedging demand for jump risk: $x^{FB}(w, Z) = (1 - Z)h$
- The scaled jump insurance premium: $\phi^{FB}(w) = \lambda(1 - \mathbb{E}(Z))h$
- The default trigger for Z: $\underline{Z}(w) = Z^* = 0$

Calibration

- The size distribution of jumps is governed by a power law

$$G(Z) = Z^{\beta}$$

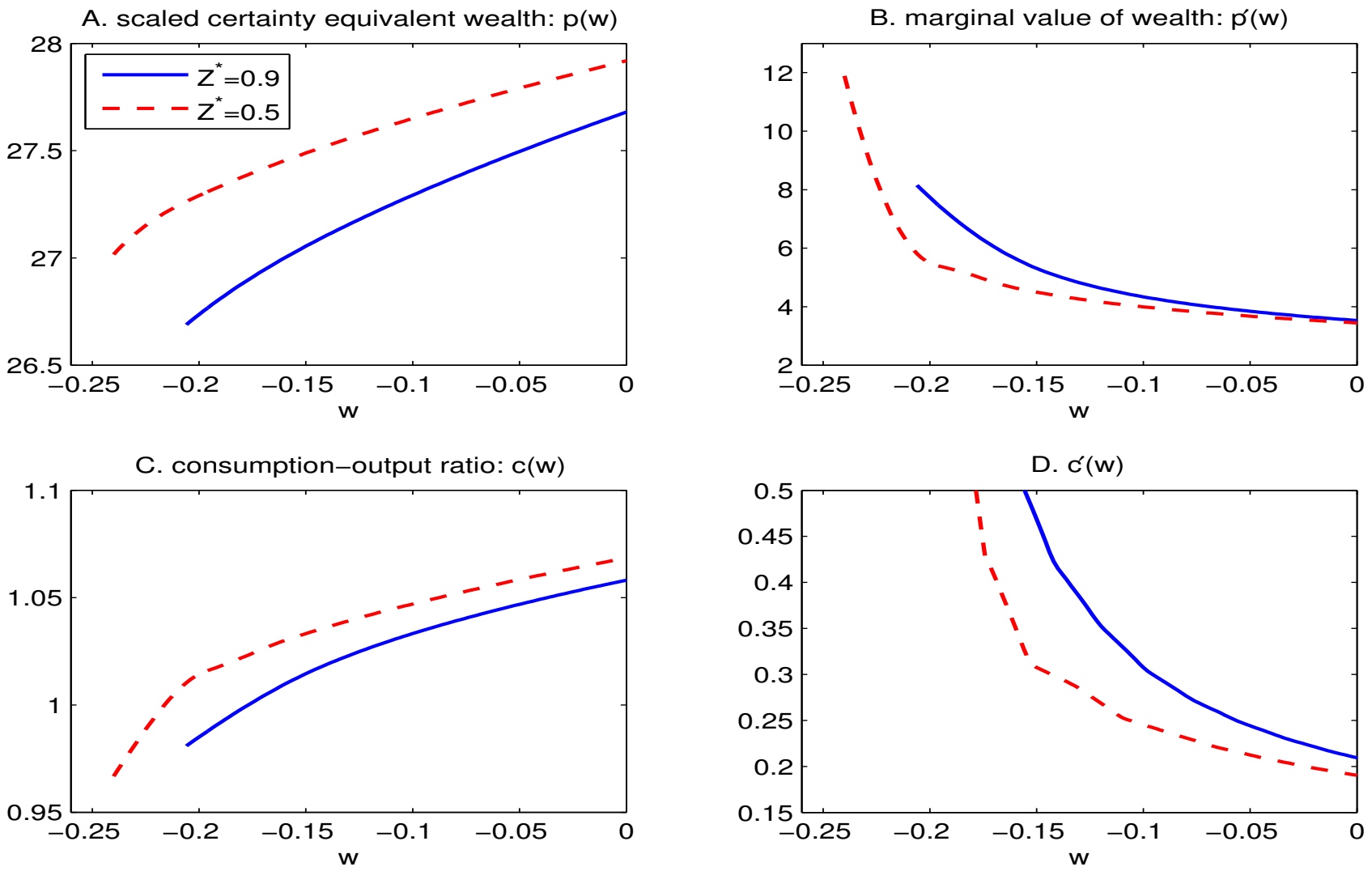
- Parameters taken from the literature
  - Following Aguiar and Gopinath (2006), we set $\gamma = 2$ and $r = 4\%$
  - Following Gelos, Sahay, and Sandleris (2011), we set $\xi = 0.25$
  - Following Barro (2009), we set $\rho = 5.2$
  - Following Barro and Jin (2011), we set $\beta = 6.3$, $\lambda = 0.073$ and $Z^* = 0.9$ to match the 3.8 percent disaster probability
- Parameters calibrated to Argentina data
  - Four parameters: elasticity, $\psi = 0.047$, drift parameter in the absence of jumps, $\mu = 2.7\%$, diffusion volatility $\sigma = 4.5\%$, and the default distress cost $\alpha = 0.975$
  - Four moments:
    - ★ Average growth rate of output of 1.7 percent per annum
    - ★ standard deviation of the growth rate of output of 6.66 percent
    - ★ an average debt-to-GDP ratio of 15 percent
    - ★ unconditional default probability of 3 percent

Table: Parameter values

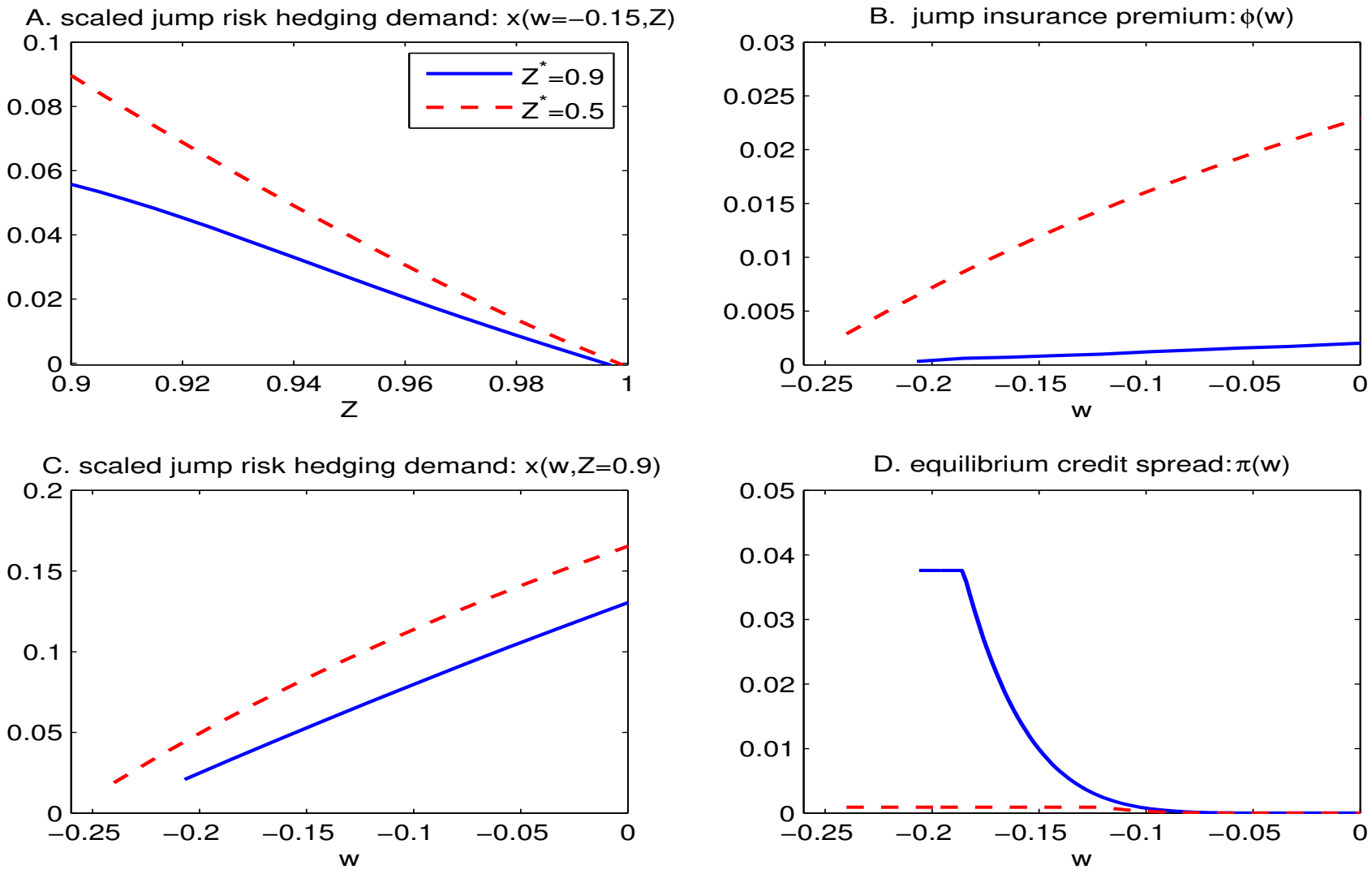
Parameters	Symbol	Value
risk aversion	γ	2
elasticity of intertemporal substitution	ψ	0.047
subjective discount rate	ρ	5.2%
risk-free rate	r	4%
financial development parameter	Z^*	0.9
output drift (in the absence of jumps)	μ	2.7%
output diffusion volatility	σ	4.5%
jump arrival rate	λ	0.073
power law parameter	β	6.3
default distress cost	α	97.5%
autarky exit rate	ξ	0.25
Targeted observables		
average output growth rate	g	1.7%
output growth volatility		6.66%
average debt-output ratio		15%
unconditional default probability		3%

All parameter values are continuously compounded and annualized

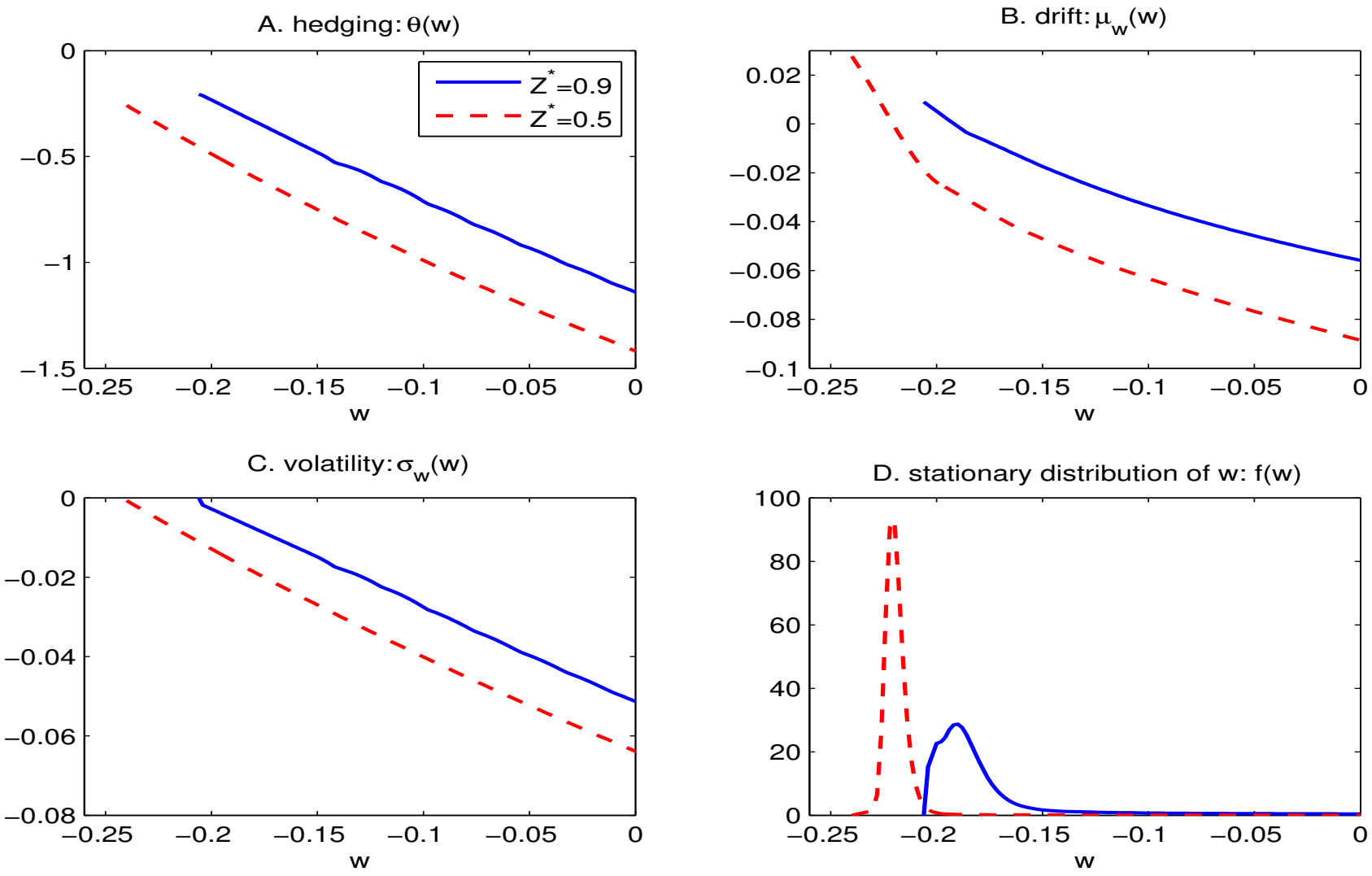
Certainty equivalent wealth, consumption, and MPC



Jump risk hedging demand, jump insurance premium, and equilibrium credit spread



Diffusion risk hedging demand, drift, volatility, and stationary distribution



Financial development and debt intolerance

Z^*	Average debt-output ratio	Default probability	Debt capacity $\lvert\underline{w}\rvert$
1	14.7%	4.0%	20.5%
0.9	**15%**	**3.1%**	**20.7%**
0.5	20.7%	0.1%	24%
0	18.9%	0	25%
FB	4,348%	0	$h = 4,348\%$

Table: $Z^* = 1$ corresponds to no jump hedging, and $Z^* = 0$ to full spanning.

Financial development and debt intolerance

- The more limited is the spanning of assets at a country's disposal, the more severe is its debt intolerance.
- When spanning is limited, it is not optimal to fully hedge risks that can be hedged. The country uses the available hedging instruments to increase its debt capacity by ensuring that default is not triggered by shocks that can be hedged.
- Countries with more limited spanning hedge less and endure more volatility in consumption.
- These countries are also more likely to default, so lenders charge them a higher credit spread to cover the expected default losses.

The effects of EIS (ψ)

ψ	debt-output ratio	default probability	debt capacity $\lvert\underline{w}\rvert$
0	15.3%	3.26%	19.8%
0.047	**15%**	**3.09%**	**20.7%**
0.25	17.9%	1.10%	34.2%
0.5	5.5%	0.65%	42.2%

The effects of EIS (ψ)

- Capital markets are more willing to lend to countries with higher intertemporal substitution, since it is less costly (in terms of utility) for these countries to cut consumption in response to adverse shocks to service their debt.

- The average debt-output ratio is non-monotonic in ψ.
  - When ψ is low when the country is close debt capacity it is costly to move away. So for low values of ψ the average debt-output ratio inherits the positive relation between debt capacity and ψ.
  - For sufficiently high values of ψ, as ψ rises the average debt-output ratio falls even though debt capacity is expanding. In this case, the country has strong incentives to save away from the debt region where credit spreads are relatively high. This behavior results in a low average debt-output ratio.

The effects of risk aversion (γ)

γ	debt-output ratio	default probability	debt capacity $\lvert\underline{w}\rvert$
1	15.6%	3.25%	19.6%
2	**15%**	**3.09%**	**20.7%**
3	15.4%	2.51%	23%

The effects of risk aversion (γ)

- Increasing γ raises the cost of default since it is more costly to bear consumption volatility in the autarky regime.
- With default more costly, the country defaults less often and debt capacity is higher.
- The effect of γ on the average debt-output ratio is quite small.

Recursive utility versus expected utility

- Our calibration uses a low value of the EIS.
- Why doesn't this calibration work with expected utility?
- With expected utility low EIS implies high risk aversion ($\gamma = \psi^{-1}$).
- A high γ generates a large debt capacity because the utility cost of defaulting and bearing the consumption volatility associated with autarky is high.
- But, this high default cost induces the country to avoid borrowing, so the average debt-output ratio is low or even negative.

The effects of distress costs $(1-\alpha)$

$(1-\alpha)$	debt-output ratio	default probability	debt capacity $\lvert\underline{w}\rvert$
5%	36.9%	2.96%	49%
2.5%	**15%**	**3.09%**	**20.7%**
1%	5.5%	3.18%	7.2%
0%	0.1%	3.63%	0.2%

- Since the cost of autarky is modest, when the distress cost is low, debt capacity is very low, close to the zero limit emphasized by Bulow and Roggoff (1989).

The effects of ξ

ξ	debt-output ratio	default probability	debt capacity $\lvert\underline{w}\rvert$
0	15.8%	3.06%	21.0%
0.25	**15%**	**3.09%**	**20.7%**
0.5	14.7%	3.13%	18.2%
1	13.2%	3.15%	16.0%

- Bulow and Rogoff (1989) critique: autarky might be difficult to sustain because creditors cannot commit ex-ante to exclude the defaulting borrower from ex-post risk-sharing arrangements.
- Distress costs are sufficient to sustain the existence of sovereign debt.
- In this sense, our model is immune to the Bulow-Rogoff critique.

Conclusion

- Tractable model of sovereign debt that features a jump-diffusion process for output used in the rare-disasters literature, recursive preferences that separate the role of intertemporal substitution and risk aversion, and partial insurance against jump risk.
  - Low levels of financial development generate debt intolerance, i.e., low debt levels that are associated with high credit spreads, i.e., low debt levels that are associated with high credit spreads.
  - Generates plausible debt-output ratios with conventional discount rates and linear default costs.

- Possible extensions
  - Risk premium demanded by foreign investors to compensate their exposures to the systematic components of sovereign default risk.
  - Moral hazard problem associated with insurance.
  - Sudden stops and debt rollover risk.