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Calculating the Natural Rate of Interest: A Comparison of Two Alternative Approaches

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The natural rate of interest is a key concept in monetary economics because its level relative to the real rate of interest allows economists to assess the stance of monetary policy. However, the natural rate cannot be observed; it must be calculated using identifying assumptions. This *Economic Brief* compares the popular Laubach-Williams approach to calculating the natural rate with an alternative method that imposes fewer theoretical restrictions. Both approaches indicate that the natural rate has been above the real rate for a long time.

The natural rate of interest is one of the key concepts for understanding and interpreting macroeconomic relationships and the effects of monetary policy. Its modern usage dates back to the Swedish economist Knut Wicksell, who in 1898 defined it as the interest rate that is compatible with a stable price level.[1] An increase in the interest rate above its natural rate contracts economic activity and leads to lower prices, while a decline relative to the natural rate has the opposite effect. In Wicksell's view, equality of a market interest rate with its natural counterpart therefore guarantees price and economic stability.

A century later, Columbia University economist Michael Woodford brought renewed attention to the concept of the natural rate and connected it with modern macroeconomic thought.[2] He demonstrated how a modern New Keynesian framework, with intertemporally optimizing and forward-looking consumers and firms that constantly react to economic shocks, gives rise to a natural rate of interest akin to Wicksell's original concept. Woodford's innovation was to show how the natural rate relates to economic fundamentals such as productivity shocks or changes in consumers' preferences. Moreover, an inflation-targeting central bank can steer the economy toward the natural rate and price stability by conducting policy through the application of a Taylor rule, which links the policy rate to measures of economic activity and prices.

Naturally, monetary policymakers should have a deep interest in the level of the natural interest rate because it presents a guidepost as to whether policy is too tight or too loose, just as in Wicksell's original view. The problem is that the natural rate is fundamentally unobservable. It is a hypothetical construct that cannot be measured directly. Instead, economists have developed various empirical methods that attempt to derive the natural rate from actual data.[3]

The Unobserved Component Model of Laubach and Williams

The most commonly used approach to compute the natural rate was developed by Federal Reserve economists Thomas Laubach and John Williams in a widely cited paper from 2003.[4] Their approach uses a "state-space" model to calculate the fundamentally unobservable natural rate from actual data by specifying a simple theoretical relationship that links interest rates to measures of economic activity. This relationship has an economic foundation in a traditional Keynesian model, where movements in the real interest rate affect consumption and investment decisions. For example, an increase in the real rate due to a hike in the federal funds rate would, when prices are sticky, reduce consumption and investment. A similar relationship can be derived from a more modern forward-looking framework where real rate movements affect intertemporal household decisions on savings and portfolio allocation.

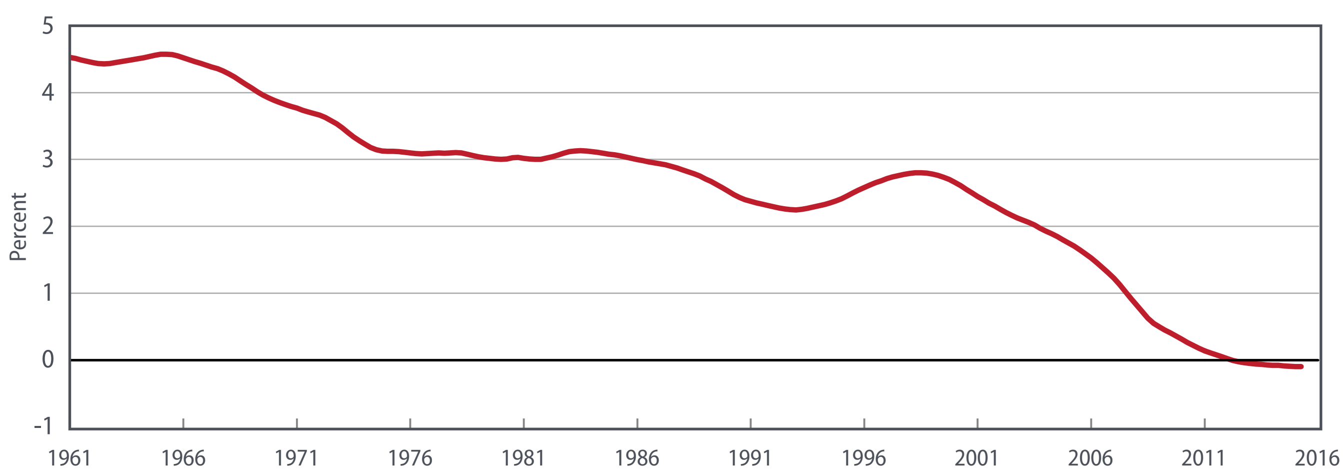
The insight that Laubach and Williams brought to this discussion is Wicksell's idea that what matters for economic activity in the short run—that is, for the business cycle and economic stabilization—is the level of the interest rate relative to its natural rate. Moreover, they recognize that it is the real rate of interest—that is, the nominal rate net of the effects of inflation—that underlies economic decisions. Laubach and Williams then develop a small-scale economic model that connects these relationships in a manner that is informed by theoretical reasoning but allows enough flexibility to capture the data well.

In this empirical framework, the real natural rate is an unobserved component in that it has to be inferred from observable data, such as the inflation rate, the growth rate of gross domestic product, and the federal funds rate. This can be accomplished by means of the Kalman filter, which allows the researcher to predict the values of unknown variables through the restrictions implied by the empirical model. The natural rate of interest is identified in their paper as the variable in a regression of the output gap on past output gaps that best helps to fit this theoretical relationship.

Figure 1 shows the natural rate computed with the Laubach-Williams framework for the data range from 1961 through the second quarter of 2015.[5] The estimate shows a secular decline of the rate from a high of 4.5 percent at the beginning of the sample toward negative territory in 2011. According to their results, the natural interest rate has thus been negative for more than three years.

In line with the argument above, namely that the natural rate of interest can be used as a measure of the stance of monetary policy, the findings by Laubach and Williams suggest that policymakers could consider extending the current policy of a federal

Figure 1: Natural Rate of Interest from Laubach-Williams



Sources: Laubach and Williams (2003) with updated estimates from the San Francisco Fed funds rate close to zero and an enlarged balance sheet due to a policy of quantitative easing. However, what matters for this interpretation is not the *absolute* level of the natural rate, but its level *relative* to the corresponding real rate. Figure 2 therefore shows the real interest rate computed as the difference between the federal funds rate and the expected personal consumption expenditures (PCE) inflation rate. As can be seen, the real rate is lower than the natural rate by a full percentage point and has been that low or lower since 2009. Based on this metric, this finding suggests that policy is not tight enough—and has not been for a while.[6] This impression also is supported by the accompanying estimate of the output gap from the Laubach-Williams framework, which has been positive since the middle of 2014. This implies that economic output is running above its potential, indicating that any inflationary pressures could be reined in by a higher federal funds rate.

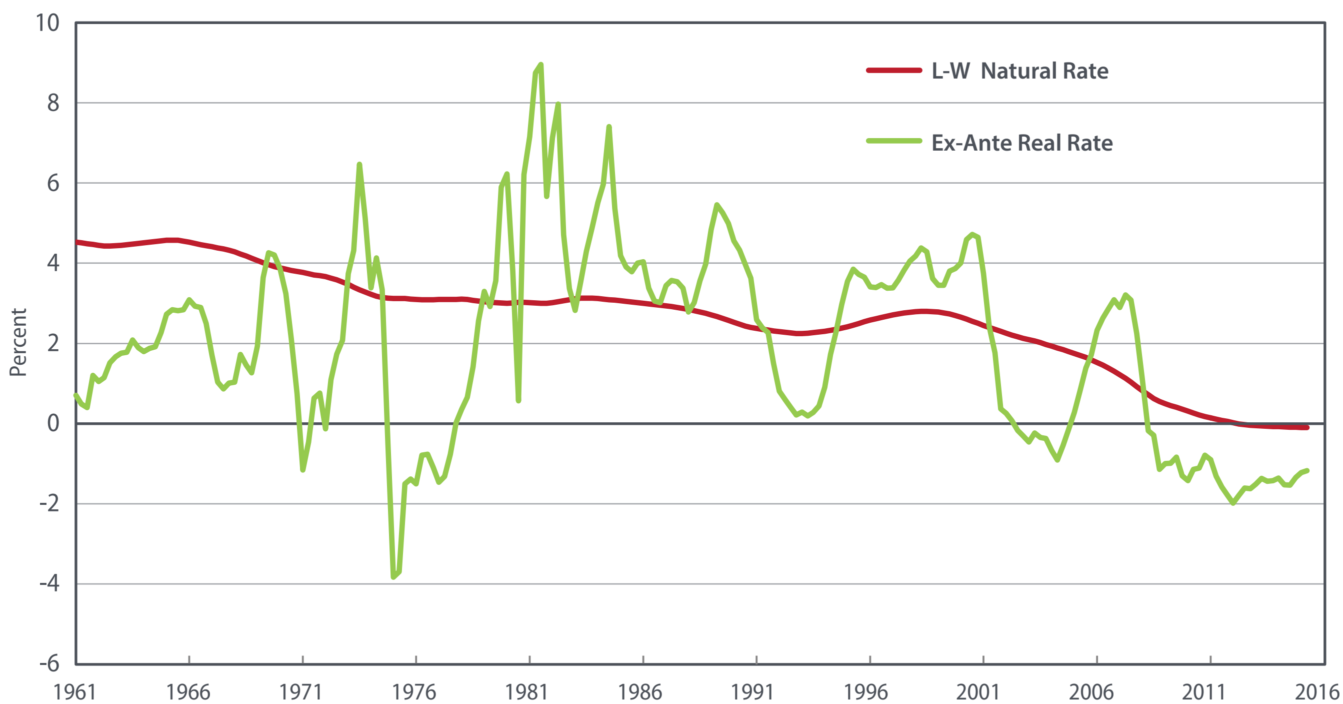
In addition, the discussion above raises questions about the robustness and precision of the Laubach-Williams natural rate estimates. In their paper, they consider a number of alternative specifications for their empirical framework. The overall impression from their robustness checks is that natural rate estimates can vary considerably.[7] Moreover, the estimates come with considerable standard errors to the effect that the level of the natural rate cannot be ascertained with a high degree of confidence.

The approach by Laubach and Williams relies on an empirical model that has some underpinnings in theoretical models of the economy. One alternative would be to impose more theoretical rigor.[8] Moving in the opposite direction, another alternative would be to take a less structural approach.

An Alternative Approach: TVP-VARs

A time-varying parameter vector autoregressive (TVP-VAR) model is a flexible framework for studying the complex relationships among macroeconomic data.[9] It is a time-series model that explains the evolution of economic variables as a function of their own lagged values and random shocks. What distinguishes a TVP-VAR from the more standard VAR approach is that

Figure 2: Natural Rate from Laubach-Williams vs. the Ex-Ante Real Rate



Sources: Laubach and Williams (2003) with updated estimates from the San Francisco Fed

Notes: The ex-ante real rate was computed as the difference between the federal funds rate and the expected PCE inflation rate. Interest rates are reported as annual rates, while the inflation rate is annualized quarter over quarter.

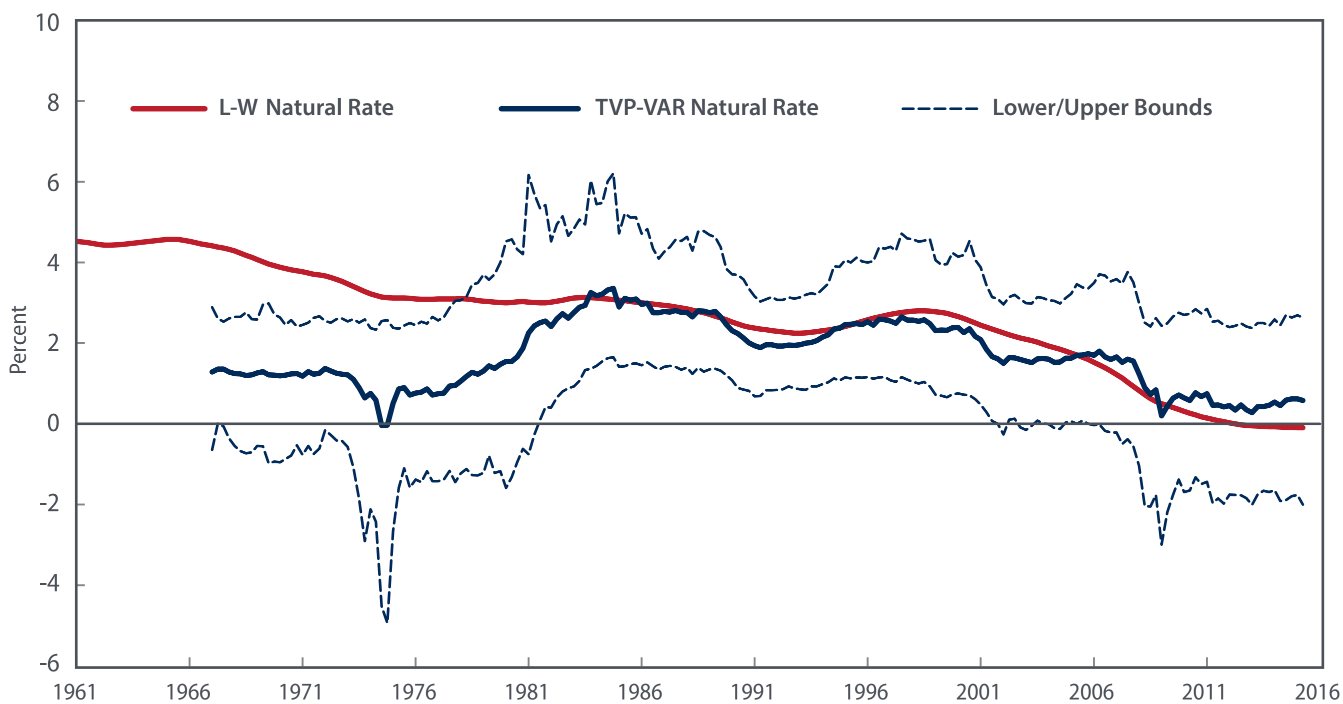
the parameters of the model, namely the lag coefficients and the variances of the economic shocks, are allowed to vary over time. This framework is thus capable of capturing a variety of nonlinear behaviors that are apparent in macroeconomic time series, such as asymmetric movements of variables over the course of the business cycle or the overall decline in macroeconomic volatility since the mid-1980s.

This latter issue is especially pertinent to the question of the behavior of the natural rate of interest. As the figures above show, there appears to be a secular decline in the real rate and its natural counterpart, as estimated by Laubach and Williams, as well as changes in the former's volatility over the sample period. Moreover, the existence of the zero lower bound on the nominal interest rate introduces nonlinearity in macroeconomic relationships by itself. These considerations thus render a TVP-VAR an attractive framework for capturing the natural rate.

What distinguishes this approach from Laubach and Williams' method is that the TVP-VAR imposes much less of an economic structure. Whereas Laubach and Williams posited economic relationships between the key macroeconomic variables that may or may not be supported, a TVP-VAR is largely agnostic on this dimension. It simply captures the co-movement between these variables in a flexible manner. The identification of the natural rate in Laubach and Williams critically rests on the assumptions governing the underlying economic model.

How then could a TVP-VAR be used to extract the natural rate from the data? Essentially by going back to Wicksell's original vision. Wicksell regarded the natural rate of interest as the rate at which an economy is in a stable price equilibrium.[10] This is not to say that the natural rate necessarily remains constant at that point since, as Wicksell recognized, it is affected by real economic disturbances. Moreover, in his framework, it is a feature of the real interest rate that it eventually moves toward and converges with its natural counterpart. This insight can thus be used to provide an estimate of the natural rate.

Figure 3: Natural Rate Calculations from Laubach-Williams vs. a TVP-VAR Model



Sources: Authors' calculations using a time-varying parameter vector autoregressive (TVP-VAR) model; Laubach and Williams (2003) with updated estimates from the San Francisco Fed

Notes: The solid blue line marks the median posterior estimate, and the dashed blue lines indicate the upper and lower bounds of the 90 percent confidence region.

We estimate a simple TVP-VAR for three variables—the growth rate of real gross domestic product, the PCE inflation rate, and the measure of the real interest rate used by Laubach and Williams—over the sample period 1961 through the second quarter of 2015. As discussed above, the TVP-VAR approach is well-suited to capture both the secular and business-cycle characteristics of this data range. We propose as a measure of the natural rate of interest the conditional long-horizon forecast of the observed real rate. Our chosen time horizon is five years, and the forecast is computed for each data point since 1967. In contrast to stationary fixed-coefficient VARs, the forecasts do not revert to the sample mean because the coefficients of the TVP-VAR follow random walks. The results from this exercise are reported in Figure 3.

Our estimate also shows a secular decline in the natural rate from about 3.5 percent in the early 1980s to 0.5 percent in the second quarter of 2015. The estimated path also exhibits sharp drops during the recession of 2001 and the onset of the Great Recession in 2007. Interestingly, the large movements in the real rate throughout the 1970s up until the middle of the 1980s do not feed through to the path of the natural rate as much as one might expect. Since we allow for stochastic volatility in the TVP-VAR, these fluctuations are largely attributed to changes in the shock variances.

The most notable finding, however, is that our estimate of the natural interest rate never turns negative. In addition, the natural rate has been above the measured real rate throughout the post-2009 recovery, which suggests that monetary policy has been too loose in the Wicksellian sense. This finding is qualitatively in line with Laubach and Williams, who also find a positive gap between the two rates, albeit a smaller one on account of their lower natural rate estimate.

Figure 3 also contains an estimate of the uncertainty surrounding the natural rate path. We show the bounds of the 90 percent confidence region, which indicates the probability that the natural rate path is contained within these bounds. The error bounds are roughly 2 percentage points above and below the estimated natural rate. Moreover, the Laubach-Williams estimate is well within the confidence region from our approach since the early 1980s—implying that the estimates are statistically indistinguishable from each other.

Conclusion

The natural rate of interest is a theoretically sound concept for understanding business cycles and assessing the stance of monetary policy. Computing the natural rate is a different matter, however. Since it is a fundamentally unobservable variable, it has to be extracted from data using identifying assumptions. The actual estimate thus rests almost entirely on the assumptions going into the economic model the researcher decides to impose on the data. Two alternative approaches to computing the natural rate, the more restricted model of Laubach and Williams and a much less restricted TVP-VAR, yield similar estimates since the early 1980s. In addition, the uncertainty surrounding the estimates is large enough to make the estimates statistically indistinct for the past three decades. Nevertheless, both point estimates concur that the real rate of interest has been below its natural counterpart for an extended period. ■

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Endnotes

1. See Knut Wicksell, *Interest and Prices: A Study of the Causes Regulating the Value of Money*, 1898, English translation, London: Macmillan and Company, 1936, p. 102.
2. In an obvious nod to Wicksell's work, Woodford titled his book "Interest and Prices." See Michael Woodford, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton, N.J.: Princeton University Press, 2003.
3. Based on the New Keynesian framework developed by Woodford, economists also have developed dynamic stochastic general equilibrium (DSGE) models that can be formally estimated and from which natural rates can be calculated by linking them to unobservable economic shocks. A recent example of this approach is Robert Barsky, Alejandro Justiniano, and Leonardo Melosi, "The Natural Rate of Interest and Its Usefulness for Monetary Policy," *American Economic Review*, May 2014, vol. 104, no. 5, pp. 37–43. Implications and pitfalls of this methodology have been discussed in previous articles of

the *Economic Brief* series by Thomas A. Lubik and Stephen Slivinski, "Is the Output Gap a Faulty Gauge for Monetary Policy?" January 2010, No. 10-01, and by Thomas A. Lubik and Jessie Romero, "Monetary Policy with Unknown Natural Rates," July 2011, No. 11-07. In the current article, we will focus instead on two alternative methods.

[4] See Thomas Laubach and John C. Williams, "Measuring the Natural Rate of Interest," *Review of Economics and Statistics*, November 2003, vol. 85, no. 4, pp. 1063–1070. This paper has received renewed attention now that Williams is president of the Federal Reserve Bank of San Francisco and Laubach is director of monetary affairs at the Board of Governors.

[5] The estimates are available from the San Francisco Fed at: www.frbsf.org/economic-research/economists/john-williams/Laubach_Williams_updated_estimates.xlsx.

[6] The expected inflation rate is computed as the forecast from a simple time-series model that has been fitted to the PCE inflation rate data. This impression remains unchanged when other measures of the real rate are used, such as ex-post interest rates.

[7] Figure 3 in their paper shows that a 70 percent confidence interval for the path of the natural rate can extend up to 3 percentage points above and below the point estimate. This finding of very wide error bands is also reinforced by the related paper by Silvia Fabiani and Ricardo Mestre, "A System Approach for Measuring the Euro Area NAIRU," *Empirical Economics*, May 2004, vol. 29, no. 2, pp. 311–341.

[8] See the references in footnote 3 for further discussion.

[9] TVP-VARs were introduced to the macroeconomic literature by Timothy Cogley and Thomas J. Sargent, "Evolving Post-World War II U.S. Inflation Dynamics," *NBER Macroeconomics Annual 2001*, vol. 16, pp. 331–388, and by Giorgio E. Primiceri, "Time Varying Structural Vector Autoregressions and Monetary Policy," *Review of Economic Studies*, July 2005, vol. 72, no. 3, pp. 821–852. Thomas A. Lubik and Christian Matthes, "Time-Varying Parameter Vector Autoregressions: Theory, Uses, and Applications," Forthcoming, Federal Reserve Bank of Richmond *Economic Quarterly*, provides an introduction to the methodology and an overview of the existing literature and its limitations.

[10] This notion coincides with the concept of a flexible-price equilibrium in New Keynesian monetary policy models. The flexible-price equilibrium is a hypothetical counterpart to an actual sticky-price economy and represents the best outcome that a monetary policymaker can achieve. This concept is further discussed and assessed in Lubik and Slivinski (2010).