

Corporate Capital Structure: The Control Roles of Bank and Public Debt with Taxes and Costly Bankruptcy

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Corporate finance theory studies the way that firms choose to raise funds. Traditionally, this theory focused on the effect of capital structure on income tax payments and exogenously specified administrative costs of bankruptcy. More recently, this theory has emphasized the effect of capital structure on the control of subsequent investment decisions of the firm, in settings where managers' and investors' incentives are not perfectly aligned. Both the tax-oriented approach and the control-oriented approach capture important aspects of the decision that firms make when they choose a method of finance. To date, however, the insights from the two theories have not been integrated. Tax-oriented theories typically ignore issues of corporate control, while control-oriented theories typically ignore taxes. In addition, tax-oriented theories consider only a firm's choice between debt and equity, while some of the control-oriented theories study the importance of the source of debt finance: the choice between bank loans (privately placed debt) and bonds (publicly issued debt).

This article combines traditional tax-based capital structure theory with an analysis of the control and incentive effects of debt. It presents a model of both the firm's choice of the amount of debt and equity and its choice between bank loans and publicly traded debt. Following the traditional approach, capital structure choice is framed as a trade-off between tax savings of debt and costs of bankruptcy. Accounting for the control roles of bank loans and public debt

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emphasized in more recent work then allows for the endogenous determination of bankruptcy costs. The model shows how the costs of bankruptcy can sometimes be negative (so bankruptcy becomes a net benefit), when bankruptcy allows claim holders to prevent a borrower from undertaking an unprofitable investment.

Endogenous bankruptcy costs depend on the type of debt used and the characteristics of the borrower. One relevant borrower characteristic is the correlation between the return from past investments and the profitability of new investment. If this correlation is high, then the borrower will be unable to refinance debt only when its old and new investments are both unprofitable, so inability to refinance indicates that new investment is unprofitable and bankruptcy desirable. If the correlation is low, then the inability to refinance is not a clear indicator of poor prospects for new investment, and bankruptcy due to the inability to refinance will sometimes be quite costly.

Modigliani and Miller (1958) established the framework for studying capital structure by finding apparently reasonable conditions rendering a firm's capital structure irrelevant to its value. The earliest generalization was Modigliani and Miller (1963), which viewed capital structure as an attempt to reduce taxes. They studied the implications of a tax advantage to debt over equity that still exists in the United States. Corporate taxes are avoided for interest payments but not for dividends. If there are no other advantages to equity over debt, the conclusion is that firms should issue no equity and should issue debt with face value equal to the highest possible future value of the firm. Such all-debt firms would almost always default on their debt. Modigliani and Miller assumed that there was no cost associated with frequent default.

The next generalization in the literature assumes that there is an exogenous cost of default—a bankruptcy cost. This bankruptcy cost is a disadvantage of issuing too much debt that is traded off against the taxes saved. This forms the basis for the traditional “trade-off” approach to capital structure in Robichek and Myers (1965) and Kraus and Litzenberger (1973). This approach does not analyze the source of such bankruptcy costs or allow any non-tax benefits of debt. It identifies volatility of firm value as a force that limits debt. It predicts that firms with high-variance cash flow distributions will choose less debt and more equity than those with low variance. It also predicts that firms will be financed only with equity when there is no corporate income tax advantage to debt. Little empirical support for these implications exists, however.

Recent approaches to capital structure view the capital structure as influencing the investment decisions of the firm, either by providing incentives to management (see Jensen and Meckling [1976], Townsend [1979], Diamond [1984], and Gale and Hellwig [1985]) or by allocating some control of the firm to someone other than the management. When capital structure serves to transfer control from management to bondholders, one obtains a theory of the debt-to-equity ratio, as in Aghion and Bolton (1992), Bolton and Scharfstein

(1993), Diamond (1991a, 1993b), Hart and Moore (1989, 1990, 1991), Jensen (1986, 1989), Stulz (1990), and Titman (1984). Other research studies how financial contracts allocate control between management and bank lenders, as in Diamond (1984, 1991b, 1993a). Such work provides a theory of the characteristics of firms that use bank finance instead of issuing securities directly to the public. These recent approaches often ignore taxes and bankruptcy costs, however, because capital structure has important effects even without taxes or costs of bankruptcy. Since taxes and bankruptcy costs do exist, it is important to see how they interact with the phenomena described more recently.

This article integrates the bondholder control and bank control views into the tax savings versus bankruptcy cost approach to optimal capital structure. I allow for the effects of a debt default on the transfer of control of firm operating decisions. Default has different effects for publicly issued debt and bank debt. The costs of financial distress are specified as three separate components: the costs of restructuring defaulted public debt, the cost of ceasing a firm's operations, and the cost of lost going-concern value if a firm enters bankruptcy and then reorganizes. An optimal capital structure is determined by the interaction of these costs with the tax and restructuring advantages of equity, public debt, and bank debt. I use the term *bank debt* as a shorthand for privately placed debt, including debt held by insurance companies and other financial intermediaries.

The balance of this article is organized as follows. Section 1 describes both the tax savings from issuing debt rather than equity and the cost differences between bank debt and debt issued directly to the public. Section 2 outlines a model of capital structure choice. It begins by using the model to illustrate the results of traditional capital structure theory based on a trade-off of tax savings versus fixed bankruptcy costs. It describes the component costs of default on debt. The costs of defaulting on public debt and on private debt are analyzed in the two subsections under Section 2. Section 3 shows how the correlation between the cash from existing investments and the profitability of new investment influences the amount of debt and the type of debt a firm will choose to issue. Section 4 discusses the conclusions and implications that one can draw from the model.

1. THE TRADITIONAL THEORY

The older capital structure theories frame capital structure as a choice that balances the tax savings from debt against the exogenous bankruptcy costs incurred when there is default on debt. The model in this article is framed within this trade-off, in order to learn how the insights from the traditional approach interact with the newer, control-oriented approach. Before showing how to frame the newer approach in the context of the traditional approach, a simple capital structure model without control elements is presented.

Tax Savings Due to Debt

The tax advantage of debt over equity is due to the deductibility of interest payments from corporate income tax. Dividends and retained earnings are not deductible. If the firm's investors are not subject to different personal taxes for debt and equity, the corporate tax savings is the only tax effect of capital structure.¹ I assume that corporate taxes are a fraction t of corporate profits and that there are no personal taxes. A one dollar payment to equity costs the firm one dollar, and is worth one dollar to the investor. A one dollar payment of interest to a public debt holder costs the firm $1 - t$ dollars, because it reduces taxable income by one dollar. The interest payment is worth one dollar to the investor. Thus, there is an increase in the firm's after-tax profit of t when one dollar of payments to equity is replaced by one dollar of payments to debt. This increased profit makes debt a lower-cost form of capital than equity.

The model considers two types of debt: bank loans and public debt. Payments to the holders of either are deductible from corporate income. There are cost-of-capital differences, however, because the bank incurs operating costs and corporate taxes of its own. In addition, banks are subject to expenses that are equivalent to taxes, such as reserve requirements and Federal Deposit Insurance Corporation (FDIC) premiums in excess of the value of deposit insurance. Reserve requirements are a tax because no interest is paid on reserves, and FDIC premiums in excess of the value of deposit insurance increases a bank's cost of funding itself with deposits. Let the sum of the bank's added costs and taxes be denoted by z , per dollar of its income. A one dollar payment of bank interest saves t in corporate tax for the firm, but incurs bank taxes and costs of $z \geq 0$. The net savings from replacing a one dollar payment to corporate equity with a one dollar payment on a bank loan is then $t - z$. Bank loans are more costly than public debt, but have a lower cost of default, which is described later. Bank debt is, on balance, less costly than equity: I assume that $t > z$.

To keep the notation simple, I will overstate the tax advantage of debt by assuming that principal as well as interest payments are deductible from corporate tax. No qualitative results depend on this simplification.

The Model

On date 0, the firm chooses a capital structure. On date 1, several events occur. The cash flows from the firm's previous investments arrive. The firm faces new

¹ If investors are subject to differential individual taxes, there is a tax advantage to debt if the sum of individual and corporate tax is lowest for debt payments (Miller 1977). The personal tax advantages of equity are due to low taxation of capital gains and deferral of unrealized capital gains. I will formally introduce only corporate tax savings and assume that the investors are tax-exempt, but the corporate tax rate can be interpreted as the net corporate and personal tax saving of payments to debt over payments to equity.

investment opportunities and chooses a new investment. Finally, both public and bank debt contracts mature. The firm can pay its debts with the cash from its investments and from the proceeds obtained from issuing new securities. If the firm continues operations after date 1, it is liquidated at date 2, with residual claims going to equity owners in proportion to their ownership.

The firm chooses a date-0 capital structure to maximize its market value. The firm can issue either public or bank debt. Let the face value of public debt be R . Public debt must be fully repaid or there will be bankruptcy. The United States Federal Trust Indenture Act makes it difficult to restructure out of court because a vote to forgive or extend the debt requires unanimous consent (see Roe [1987] and Gertner and Scharfstein [1991]). While there are methods of restructuring public debt to avoid a default, these are costly and sometimes unsuccessful.

Instead of public debt, the firm can issue bank debt (get a bank loan), with face value denoted by r . Bank debt can be renegotiated, with the possibility of avoiding bankruptcy. I do not allow combinations of the two types of debt. Focusing on the choice between the two types of debt simplifies the analysis without producing misleading results. A bank's incentive to extend maturity and restructure debt is removed when combined with a large amount of public debt (see Bulow and Shoven [1979], Gertner and Scharfstein [1991], and Diamond [1993a, 1993b]). In addition to either type of debt, the firm can issue equity, a claim that requires no fixed date-1 payment. Equity is a proportional claim on any and all dividends the firm may declare, but the firm has no legal obligation to pay dividends in any period that it is not being liquidated. I assume that the firm will not be liquidated until date 2, absent outside intervention. The date-2 value depends on the firm's manager's decisions on date 1, as well as on past decisions.

The market value on date 0 of a date-1 cash flow is its discounted present value. I assume, for simplicity, that all investors are risk-neutral and that interest rates are zero, implying that the discounted present value is just the expected value of the cash flow distribution.²

The next two subsections review traditional tax-oriented capital structure theory where the control role of debt is absent. To illustrate the added implications of the control role of debt, I will review traditional capital structure theory, which allows no control role. This will provide a framework for understanding the control role of debt.

² Alternatively I could assume that there are complete Arrow-Debreu markets, implying that there is a market price today for every risk. This allows market prices to provide appropriate discount rates for any risk. In this case one replaces the probability of a given cash flow with the market price of one dollar delivered in the situation in which the cash flow is equal to that amount.

Review of Traditional Capital Structure Theory Without Bankruptcy Costs

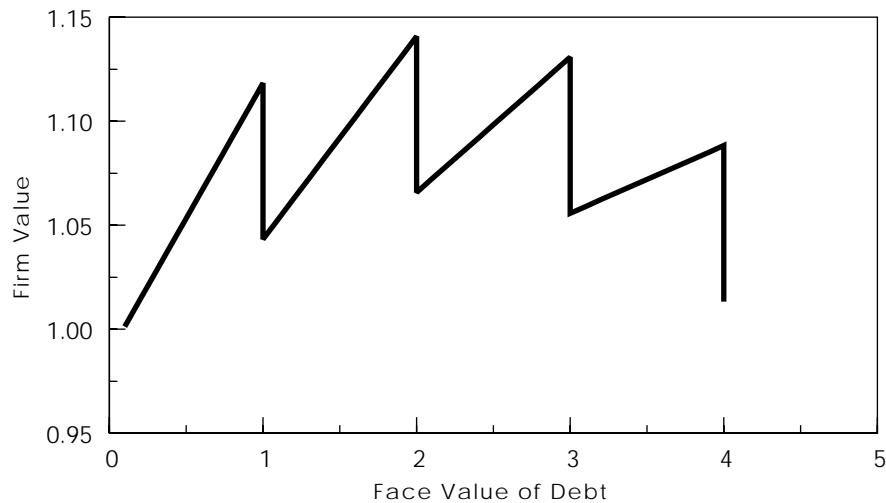
The traditional approach to capital structure abstracts from issues related to the control of the firm's future investment decisions. Thus, consider the simple case in which the firm is liquidated at date 1 because it has no new investment opportunities. Assume that all debt is public debt and bankruptcy has no cost. The only role for capital structure is to minimize taxes.

The date-0 value of the firm if unlevered (all equity) is then the discounted value of the after-tax profits. The pre-tax value of the firm on date 1, c , has possible realizations $c \in \{1, 2, 3, 4\}$. Each realization has equal probability, $P = 1/4$. The market value of the unlevered firm is $(P \cdot 4 + P \cdot 3 + P \cdot 2 + P \cdot 1)(1 - t) = (2.5)(1 - t) \equiv V^u$. With debt of face value $R \leq 1$, the firm can always deduct the payment from its corporate taxes, saving Rt , and firm value is $V^u + Rt$. Define τ_R as the date-0 present value of tax savings from increasing debt to R from the largest integer value less than R . This means that $\tau_1 \equiv t$ is the value of taxes saved with debt equal to one. Further increasing debt to a face value $R \in (1, 2]$, increases the value to $V^u + \tau_1 + (R - 1)(3Pt)$. The added taxes are saved only when the firm is worth more than one, because only payments made are tax-deductible. Therefore, increasing debt from one to two saves $3Pt = 3/4t \equiv \tau_2$. Similarly, $\tau_3 = t/2$ and $\tau_4 = t/4$. The increase in value from a unit increase in R decreases for higher values of R . Further increases in R save more taxes until the firm's value is maximized with $R = 4$ and firm value is 2.5.

Fixed Bankruptcy Costs

Suppose that there is a fixed cost ϕ that is incurred whenever the firm cannot fully repay its public debt (see Robichek and Myers [1965] and Kraus and Litzenberger [1973]). Think of ϕ as an unavoidable legal fee. The cost of bankruptcy trades off against tax savings to determine the value-maximizing capital structure. There is no risk of bankruptcy for debt with face value $R \leq 1$. Value increases to $V^u + \tau_1$ with $R = 1$. Further increasing the face value from $R = 1$ to $R = 2$ increases date-0 firm value by $\tau_2 - P\phi$. Increasing leverage beyond one decreases firm value if the present value of tax savings is less than that of bankruptcy costs. Because taxes are only saved for payments actually made, the marginal value of tax saving per unit of debt is reduced as debt climbs ($\tau_4 < \tau_3 < \tau_2 < \tau_1$). If $\tau_4 < P\phi$, then eventually tax savings are smaller than bankruptcy costs, and there is a limit to desired leverage. Figure 1 shows the effect of leverage on firm value under the traditional capital structure theory.³ The firm value drops by the present value of bankruptcy costs at each positive integer value. Bankruptcy costs are sufficiently large in Figure 1 to imply that the optimal value of public debt is $R = 2$.

³ The example assumes that $t = .13$ and $\phi = .3$.

Figure 1 Traditional Capital Structure Theory

Note: Firm value drops by the bankruptcy cost at each positive integer value. Bankruptcy costs are sufficiently large to imply that the optimal value of public debt is $R = 2$.

If bankruptcy costs are nontrivial, traditional capital structure theory implies that firms with high variance of value will have low leverage. Without corporate tax, the model predicts that there will be no debt issued. The crucial assumptions are that there are no effects of capital structure on the firm's decisions and that the cost of bankruptcy is the same for all bankruptcies. In what follows, future decisions are introduced by allowing the firm an investment choice at date 1. Profitable investment is a source of firm value in addition to its cash from previous investments. The firm will be in default only when the sum of the cash from old operations and the net present value of new investment is less than the amount of debt to be repaid. Before providing these details, the next section describes the costs and benefits of bankruptcy.

2. CONTROL AND THE BENEFITS OF DEBT

There are conflicting interests between the management of the firm and its outside investors. The management derives more benefits than do outsiders from the firm's growth and its continued operations. Some reasons for this conflict include the costs of a manager's immediate lost reputation if operations are closed and the increase in the manager's incremental value to the company once a project is undertaken (the manager's information is needed to most

profitably continue the project, even if the ex-ante net present value is negative). These control benefits imply that management will continue to invest even if investment prospects are bleak. The prospects of future investments cannot be costlessly observed by a court, but the prospects are observed by investors at date 1; the manager has no private information. A management incentive contract that required a court to determine the profitability of each investment would be expensive to enforce. Because outside investors observe profitability, they can prevent the manager from making a bad investment if, and only if, they have control of the firm. Investors have control only if the firm defaults on its debt. Default on public debt will require the use of bankruptcy court, but default on bank debt need not. Equity contracts have no terms that can trigger a transfer of control (I assume that a takeover is not a possibility). If the firm is financed exclusively with equity, outsiders never have control and the firm will always invest. If the firm cannot fully repay its debt obligation, then the firm cannot avoid a default and the owners of the debt can take control of the firm. The details of this process are described in the next two subsections.

The firm's net present value of new investment at date 1, N , will be one of two possible values: $N = N_G > 0$, a good investment, or $N = N_B < 0$, a bad investment. Management will prefer to invest in either case. There is a gross benefit of $-N_B$ from defaulting on debt and preventing investment decision when investment is unprofitable and $N = N_B$. The firm ought to be liquidated when $N = N_B$, but this can only be done in bankruptcy. There are no gross benefits of defaulting on debt and controlling investment when $N = N_G$. There are also costs of using bankruptcy court, described below.

The net costs of using bankruptcy court depend on the type of reorganization that is needed and the type of debt that the firm has. The administrative costs are as follows:

1. Entering into formal bankruptcy proceedings reduces the going-concern value of the firm's future investments. These are lost reputation and physical costs. These costs are only relevant if the firm reorganizes after filing for bankruptcy. This cost is denoted by γ and is incurred under bank debt or public debt.
2. There are costs of closing and quitting operations. These costs must be incurred if the firm ceases to operate and do not depend on the type of financial contracts the firm has. These are the costs of breaking other contracts, such as leases, if the firm ceases to operate. This quitting cost is denoted by q and is incurred under bank debt or public debt.
3. There are legal costs of restructuring or renegotiating public debt issues. These costs are incurred if the firm gets into formal bankruptcy proceedings without fully repaying its public debt. The costs also can be interpreted as costs of restructuring public debt outside formal

bankruptcy. The magnitude of the cost can depend on whether the firm reorganizes or quits operations; the costs are denoted by k_g and k_q , respectively. No such costs are incurred in restructuring bank debt.

The Costs and Implications of Bankruptcy Initiated by Default on Public Debt

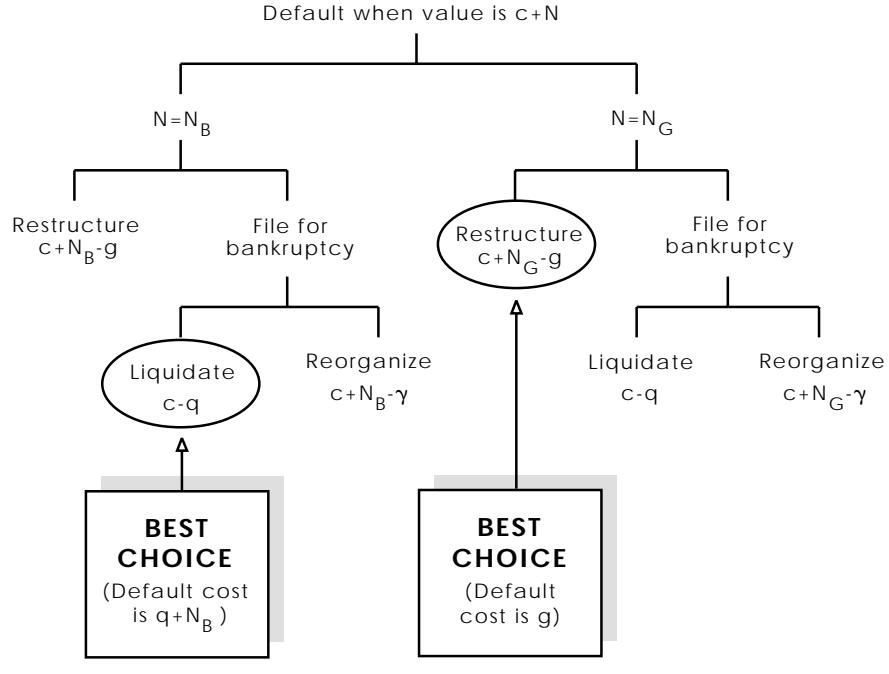
A default on public debt incurs administrative bankruptcy costs of $\gamma + k_g$ if the firm continues as a going concern and $q + k_q$ if there is liquidation. Liquidation then yields $c - q - k_q$, whereas reorganizing as a going concern yields $c + N - \gamma - k_g$. The U.S. Bankruptcy Law requires a vote of the lenders to choose the reorganization plan, suggesting that the more valuable option will be selected. I assume that the bad investment is sufficiently unprofitable that it is worth incurring bankruptcy costs to avoid it: $N_B < -(q + k_q)$. This implies that net bankruptcy costs of public debt ($q + k_q + N_B \equiv B$) are negative when $N = N_B$, on account of the control role of debt. Net bankruptcy costs are $\gamma + k_g \equiv G$ when the firm is reorganized and continues operations. I assume that the good investment is sufficiently profitable that it pays to reorganize to undertake it, i.e., $N_G > G$, and that the firm will restructure. Unlike public debt, bank debt can be restructured outside bankruptcy. The restructuring of bank debt is analyzed next.

Default on Bank Debt: Bankruptcy Versus Renegotiation

If a default is on bank debt, the bank can choose to renegotiate rather than force bankruptcy. The bank will renegotiate rather than force bankruptcy when its payoff from renegotiating exceeds what it will get in bankruptcy. When the firm is worth more as a going concern because $N = N_G$, the bank will renegotiate and save the costs of bankruptcy. When liquidation is desired because $N = N_B$, the bank will initiate bankruptcy and liquidate. A decision tree that illustrates this choice by the bank is given in Figure 2. It illustrates the bank's decision process which is described in the next two paragraphs.

The bank's payoff in bankruptcy is as follows. The value of the firm if it is reorganized as a going concern is $c + N - \gamma$ (saving k_g compared to public debt). If the firm is liquidated, the value is $c - q$ (saving k_q compared to public debt). In bankruptcy, the bank chooses the option with the largest value. Therefore, bankruptcy yields the bank the larger of $c + N - \gamma$ and $c - q$. Given a firm in bankruptcy, the bank reorganizes the firm if and only if $N \geq \gamma - q$. This implies that the bank reorganizes a bankrupt firm when it has good investments, $N = N_G$, achieving a payoff of $c + N_G - \gamma$. When a bankrupt firm has unprofitable investments, $N = N_B$, the bank liquidates, achieving a payoff of $c - q$.

I assume that the bank has substantial bargaining power. The bank can make a take-it-or-leave-it offer to the borrower to reschedule outside bankruptcy when the borrower does not pay in full. I assume that the borrower

Figure 2 The Cost of Default on Bank Debt

gets nothing in bankruptcy and will accept any offer that deters the bank from forcing bankruptcy when it otherwise would choose to file.⁴ It is possible that the bank's rescheduling is costly; let g denote this cost. I assume that $g < \gamma$ so that it is cheaper to reschedule a going concern outside bankruptcy. The rescheduling cost includes the cost of rewriting and renegotiating contracts. The bank's payoff if it reschedules the debt is $c + N - g$. It reschedules if this payoff exceeds the larger of $c + N - \gamma$ and $c - q$. Since $c + N - g$ exceeds $c + N - \gamma$, this implies that the bank will reschedule whenever $N > g - q$. The bank will reschedule when $N = N_G$, but will force bankruptcy when $N = N_B$.⁵ The savings from having bank debt instead of public debt in the event of a potential default are then k_q when $N = N_B$ (because the bank also

⁴ The results of the model are robust to giving the borrower some bargaining power and thus a positive payoff in bankruptcy. If the borrower gets a payoff of Δ in bankruptcy, the bank's take-it-or-leave-it offer must provide that borrower a payoff of Δ outside bankruptcy. One can then reinterpret N as net of the claim Δ that the borrower can appropriate.

⁵ This follows from the assumption in the previous subsection that $N_B < -(q + k_q) < g - q$ and $N_G > \gamma + k_g > g - q$.

uses bankruptcy) and $G - g$ when $N = N_G$ (because the bank then avoids bankruptcy). One expects that the major savings are due to avoiding bankruptcy for a going concern and that bank rescheduling costs are low.

One can easily extend the model to cases where there are more general managerial incentive problems in the firm. Suppose that instead of just continuing to invest when only poor investments are available, when $N = N_B$, management's objectives differ from outsiders in other ways. Management might choose an investment that is not the most profitable (absent outside intervention). The model can be reinterpreted in such a way that the transfer of control from a default leads to a change in the chosen investment instead of a liquidation.

When the bank reschedules with $N = N_G$, bank debt serves a role in avoiding bankruptcy costs that is similar to equity (which has no fixed claim that can lead to a default). When the bank forces bankruptcy with $N = N_B$, it removes cash from management's control, similar to the role of public debt described in Townsend (1979), Diamond (1984), Gale and Hellwig (1985), Lacker and Weinberg (1989), and Jensen (1986, 1989), at lower ex-post cost than does public debt. Both types of debt have the advantage over equity of blocking undesirable investment by the firm.

3. THE LINK BETWEEN CASH FLOW AND THE NET PRESENT VALUE OF NEW INVESTMENT

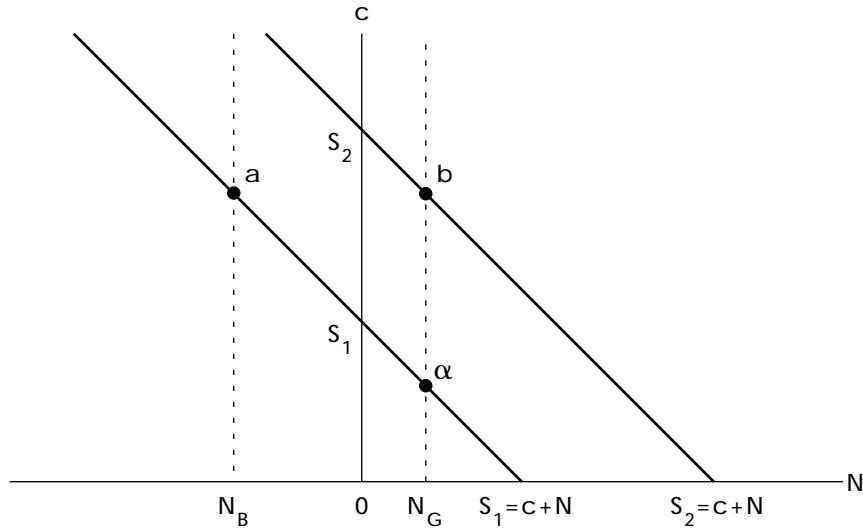
I now consider a more general model in which at date 1 the firm will have cash of c and will face new investment opportunities with net present value of N . The sum of these, $c + N \equiv S$, is the total date-1 value of the firm if it continues operations. Note that the firm is able to borrow to finance its future investments and, if these are sufficiently profitable, use the proceeds to pay off old debt. Therefore, S is the maximum that the firm is able to pay to claimants on date 1. If the firm cannot raise enough to pay off old debt, then lenders have control and can block the firm from continuing to invest. The firm will be able to fully repay its debt when S equals or exceeds the face value of the maturing debt. If S is less than the amount of debt due, there will be bankruptcy if the debt is public. If, instead, the debt is a bank loan, there will be bankruptcy if investment prospects are bad ($N = N_B$) and restructuring outside bankruptcy if investment prospects are good ($N = N_G$).

Default on public debt is desirable (the net bankruptcy cost, B , is negative) when investment prospects are bad. Public debt leads to only desirable defaults if the firm can choose a debt level such that there is default if and only if the prospects of future investment are bad. Default if and only if prospects are bad requires that total firm value, S , falls below the face value of debt, R , if and only if investment prospects are bad. Such a public debt level exists if the correlation between N and $S = N + c$ is perfect. For example, if N varies, but not c , the correlation between value, S , and prospects, N , is perfect and the

ability to refinance reveals just investment prospects. In this case one can choose an amount of debt below $c + N_G$ but above $c + N_B$ that will control investment decisions without generating any defaults with $N = N_G$. This relationship is illustrated in Figure 3. A line $c + N = S$ shows those combinations of c and N that imply the ability to refinance debt of face value S . Suppose that only N varies and the possible realizations of c and N are denoted by two horizontally aligned points such as the points marked a and b . An amount of public debt less than $S_1 = c + N_B$ never leads to default and thus fails to prevent bad investment. An amount of public debt exceeding $S_2 = c + N_G$ leads to costly defaults when $N = N_G$. Therefore, public debt exceeding $S_2 = c + N_G$ would be selected only for its tax advantages.

In the general case where cash flow is random, it might be impossible to avoid undesirable defaults when $N = N_G$ without some failure to default when $N = N_B$. Referring to Figure 3, if the point marked α is a possible realization (along with points a and b), then a face value of public debt greater than S_1 results in desirable default for realization a but undesirable default for realization α . Any face value less than or equal to S_1 implies failure to trigger default for realization a , allowing the firm to make bad investments. In general,

Figure 3 The Ability to Refinance Depends on S ($S = c + N$)



Note: The ability to avoid default on debt depends on S , the sum of cash (c) and net present value of future investment (N). The correlation between S and N determines how much information about N is revealed by a default. If all three points a , b , and α are possible, then there exists no amount of debt such that there is default if and only if $N = N_B$. If only a and b are possible and the point marked α is impossible, then for debt with face value between S_1 and S_2 , a default reveals a low value of N .

it will be impossible to find an amount of public debt that results in default if and only if the firm's investment prospect has a negative net present value. The cost of using public debt to control the firm's investment choice depends on the correlation between cash flow from previous investments and the value of future investments. Public debt is a very good control device if prospects, N , and cash from old investments, c , are both stochastic but N is much more variable than c and they have a nonnegative correlation. A related condition that makes public debt a low-cost control device is if both c and N are variable and if they are sufficiently positively correlated. For sufficiently high correlation between c and N , one can choose a face value of debt, R , such that $S \geq R$ if the probability that $N \geq N_B$ is arbitrarily high, and if $S < R$ the probability that $N = N_G$ is arbitrarily low. On the other hand, public debt is an expensive control device if c is quite variable and c and N are uncorrelated or c is negatively correlated with N . Under either condition, there is a low correlation between S and N , and many low realizations of S imply good investment prospects ($N = N_G$) while many high realizations of S imply bad investment prospects. A low correlation between S and N implies that a high probability of costly bankruptcy is required to obtain a high probability of beneficial bankruptcy that controls unprofitable investment. Referring back to Figure 3, it will be more expensive to use default on public debt to stop investment when $N = N_B$ if the point marked α is a possible realization along with a and b .

To examine the effects of the correlation between the total firm value, $S = c + N$, and the profitability of new investment, N , suppose that there are four possible date-1 realizations of S . The values of S are one, two, three, and four. Each realization has equal probability, $P = 1/4$. There is a positive, but possibly imperfect, correlation between S and N . Table 1 describes the conditional distribution of total firm value, S , given the net present value of new investment, N . When firm value is very low ($S = 1$), investment prospects are bad ($N = N_B$). When firm value is very high ($S = 4$), investment prospects are good ($N = N_G$). For intermediate values of S , either value of N is possible. A correlation parameter, u , a number between zero and one, describes how uncorrelated are S and N . Increased values of u reduce the correlation between S and N . The probability that new investment is profitable ($N = N_G$) when firm value is somewhat low ($S = 2$) is u . The probability that new investment is profitable when firm value is somewhat high ($S = 3$) is $1 - u$. When $u = 0$, S and N are perfectly correlated.⁶

⁶ The discussion in the text, combined with the definition $S = c + N$, implies the following about the value of c given each value of S . When $S = 1$, $N = N_B$ and $c = 1 - N_B$. When $S = 4$, $N = N_G$ and $c = 4 - N_G$. When $S = 2$ or $S = 3$, either value of N is possible. When $S = 2$, the pair $N = N_G$, $c = 2 - N_G$ occurs with probability u , and the pair $N = N_B$, $c = 2 - N_B$ occurs with probability $1 - u$. When $S = 3$, the pair $N = N_G$, $c = 3 - N_G$ occurs with probability $1 - u$, and the pair $N = N_B$, $c = 3 - N_B$ occurs with probability u .

Table 1 The Conditional Distribution of N Given $S \equiv c + N$

	$u=0$	$u \in (0,1)$	$u=1$
$S = 1$	$N = N_B$	$N = N_B$	$N = N_B$
$S = 2$	$N = N_B$	$N = N_B$ with probability $= 1 - u$ $N = N_G$ with probability $= u$	$N = N_G$
$S = 3$	$N = N_G$	$N = N_B$ with probability $= u$ $N = N_G$ with probability $= 1 - u$	$N = N_B$
$S = 4$	$N = N_G$	$N = N_G$	$N = N_G$

The date-0 value of the firm's cash flows is independent of u , but the correlation between S and N is decreasing in u . Increasing u decreases the correlation between cash flow and the profitability of new investment (because reducing the correlation between $S = c + N$ implies reduced correlation between c and N). Many of the implications of changing the level of the correlation parameter, u , can be seen by comparing the case of $u = 1$ with $u = 0$. The next subsection explores these implications in the case in which the firm makes use of public debt.

The Optimal Quantity of Public Debt

The value of the firm with public debt R depends on the net bankruptcy costs and tax savings of the chosen capital structure. The possible values of total firm value, $S = N + c$, are denoted by i , and $i \in \{1, 2, 3, 4\}$. Let X_i denote the net, non-tax bankruptcy cost from defaulting on public debt when $S = i$. This is a real cost from debt with face value R exceeding i . Recall that the (negative) cost of bankruptcy when investment prospects are bad ($N = N_B$) is $N_B + q + k_q \equiv B$. The (positive) cost of bankruptcy when investment prospects are good ($N = N_G$) is $\gamma + k_g = G$. The probability distribution of N given S described in Table 1 implies that the bankruptcy costs for each value of S are as follows: $X_1 = B$, $X_2 = uG + (1 - u)B$, $X_3 = uB + (1 - u)G$, $X_4 = G$.

Let $\Pi(R)$ denote the total date-0 value of a firm with public debt of face value R . The date-0 firm value, Π , depends on the value of tax savings from debt with face value R and the (possibly negative) net costs of bankruptcy, X_i . Since bankruptcy costs are incurred only if R exceeds one, $\Pi(1) = V^u + \tau_1$. Increasing R from one to two garners an incremental tax benefit of $\tau_2 = \frac{3}{4}t$ and incurs a (negative) bankruptcy cost of $\frac{1}{4}X_1$. Thus $\Pi(2) = \Pi(1) + \tau_2 - \frac{1}{4}X_1$.⁷

⁷ Similarly, $\Pi(3) = \Pi(2) + \tau_3 - \frac{1}{4}X_2$ and $\Pi(4) = \Pi(3) + \tau_4 - \frac{1}{4}X_3$.

On account of the tax savings from debt, firm value $\Pi(R)$ is increasing in R whenever incremental bankruptcy costs are non-positive ($X_R \leq 0$). Because bankruptcy is desirable when $S = 1$ ($X_1 < 0$), the optimal value of R exceeds one and the minimum optimal value of R is two (because there is no effect on the probability of bankruptcy of increasing debt between one and two). The optimal face value of public debt is equal to either two, three, or four, because increasing R in between these values saves taxes and has no effect on bankruptcy costs. Proposition 1 characterizes the optimal amount of public debt, the amount that maximizes the date-0 value of the firm.

Proposition 1 The value-maximizing face value of public debt, R^* , is given as follows:

$$R^* = 2 \text{ if } t < \min\left\{\frac{G+B}{3}, \frac{uG+(1-u)B}{2}\right\}.$$

$$R^* = 3 \text{ if } t > \frac{uG+(1-u)B}{2} \text{ and } t < uB+(1-u)G.$$

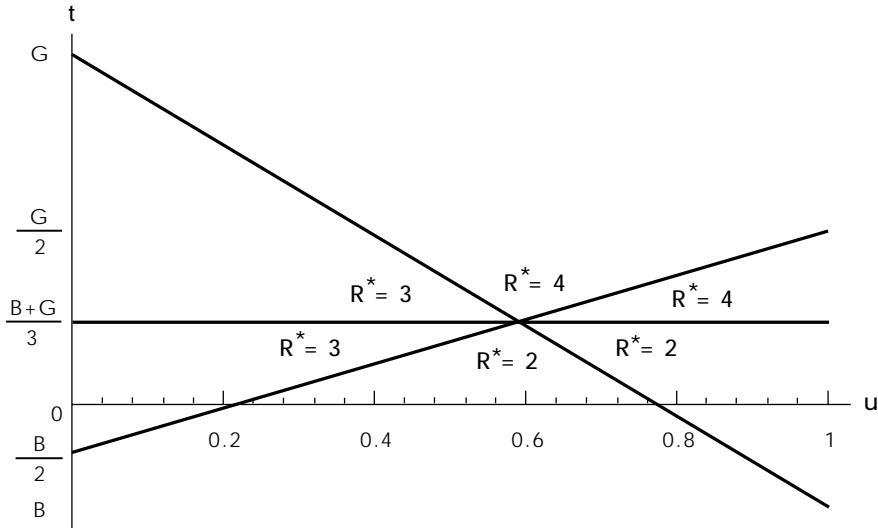
$$R^* = 4 \text{ if } t > \max\left\{\frac{G+B}{3}, uB+(1-u)G\right\}.$$

Proof: See Appendix.

Figure 4 shows an example of the results of Proposition 1 by giving the optimal face value of public debt for the possible values of the tax saving from debt, t , and the correlation parameter, u .⁸

A way to give a simple interpretation of Proposition 1 is to consider the characterization of the optimal level of public debt when tax effects dominate and when they do not dominate. One way to describe capital structure choice when taxes do not dominate is to examine the debt quantities that are selected when tax savings from debt are absent ($t = 0$). In this case, Proposition 1 implies that there is a critical value of the correlation parameter, $u = u'$, that determines whether debt with face value $R = 3$ is optimal. Debt with face value three is optimal if $u \leq u'$, and another value is best for $u > u'$. If $G + B > 0$ (default costs given good prospects are bigger than the benefits of default when prospects are bad), then the level of debt when $u > u'$ is $R = 2$ and the critical value, u' , is given by $u' = -B/(G - B)$. If, instead, $G + B < 0$ (default costs given good prospects are less than the benefits of default when prospects are bad), then the best level of debt for $u > u'$ is $R = 4$ and $u' = G/(G - B)$. The value of the firm is weakly decreasing in u in either case, because firm value given debt of $R = 3$ is decreasing in u and is independent of u for other values of R .

⁸ The example assumes that $G = .65$ and $B = -.2$.

Figure 4 Optimal Face Value of Public Debt, R^* 

Note: t is the tax rate on corporate profits. The parameter u describes how uncorrelated are total firm value, S , and the net present value of new investment, N . Increased values of u reduce the correlation between S and N . See Table 1.

If taxes are sufficiently large, $t > G$, then high leverage ($R = 4$) dominates for all values of u .⁹ The tax savings then dominate default costs regardless of the correlation structure of value and investment prospects. I assume that $t < G$, which implies that the magnitude of the correlation between total firm value, S , and the net present value of new investment, N , influences the optimal amount of public debt and the cost of using public debt as a control device.

The higher cost of using public debt as a control device when u is high and the correlation of S and N is low is illustrated in Figure 5. Figure 5 plots date-0 firm value, Π , as a function of R (the face value of public debt) for the cases of $u = 0$ (high correlation) and $u = 1$ (low correlation). The example in the figure assumes a high cost of going bankrupt when investment prospects are good, relative to tax savings of debt ($\frac{1}{3}[G + B] > t$), so that a capital structure of all public debt, $R = 4$, is not the optimum.¹⁰

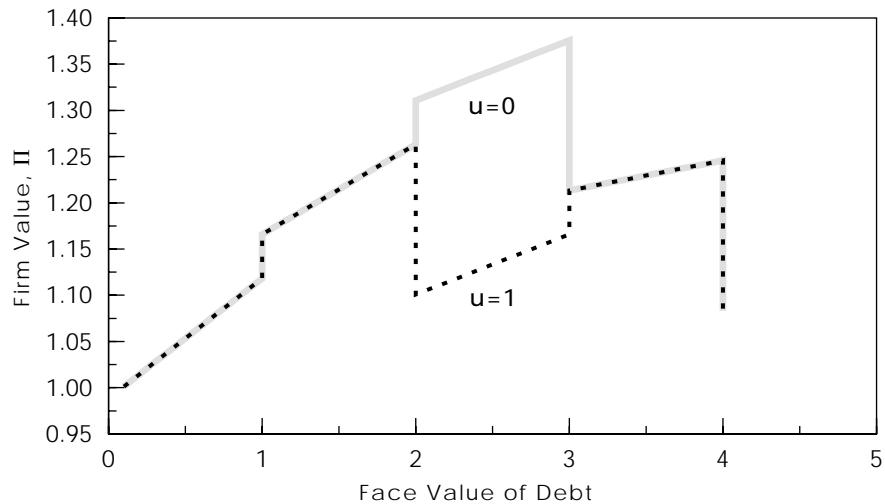
⁹ Note that because $G > 0 > B$, $[uB + (1 - u)G] \leq G$ and $(G + B)/3 \leq G$. Thus $t > G$ implies $R^* = 4$: tax savings from increasing debt from three to four are greater than the maximum bankruptcy cost, G .

¹⁰ The example assumes that $t = .13$, $\gamma = .65$, $k_g = 0$ (implying $G = .65$), $q = .2$, $k_q = 0.01$, $N_B = -.4$ (implying $B = -.2$), and $N_G = .8$.

Figure 5 shows that when $u = 0$ (high correlation between S and N), date-0 firm value is maximized with debt of three. With public debt having a face value of $R = 3$, the firm defaults if its date-1 value, S , is one or two. Defaulting when S is equal to one or two is beneficial because investment prospects are bad ($N = N_B$) in either case, and the control benefit of stopping a bad investment exceeds the administrative costs of using bankruptcy court. Default is avoided when S is equal to three or four, and for both values of S investment prospects are good ($N = N_G$).

Figure 5 also shows that when $u = 1$ (low correlation between S and N), date-0 value is maximized with debt of face value two. When $S = 2$, the firm has good investment prospects ($N = N_G$) and bankruptcy is costly. When $S = 3$, the firm has bad investment prospects and bankruptcy is beneficial. However, the costs of bankruptcy when $S = 2$ are sufficiently large that they outweigh the benefits of bankruptcy when $S = 3$. Debt with face value two avoids bankruptcy when S is either two or three. This results in higher date-0 firm value than debt with face value of four (which would result in bankruptcy both for $S = 2$ and $S = 3$).

Figure 5 Firm Value Given Public Debt When $u = 0$ and $u = 1$



Note: When $u = 0$ (high correlation between S and N), date-0 firm value is maximized with debt of three. This leads to default only if the firm has bad future investments; in addition, it saves taxes. When $u = 1$ (low correlation between S and N), date-0 value is maximized with debt of face value two. When the firm can repay exactly two, it has good investment prospects ($N = N_G$) and bankruptcy would be costly. This cost exceeds the benefits of debt with face value exceeding three (which would lead to bankruptcy when the firm can repay exactly three and has bad investment prospects).

To control the investment decision with debt implying default when $S = 3$, the debt must also be in default for all lower values of S . When the correlation between S and N is high (u is nearly zero), low firm value (S below three) implies the need to control investment and high firm value (S above three) implies no need for control. When the correlation between S and N is lower (u is nearly one), then there is need to control investment for the relatively high firm value, $S = 3$, but little or no need to control investment decisions for the relatively low firm value, $S = 2$. Low correlation implies that it is costly to default when $S = 2$, but default when $S = 2$ is necessary in order to induce a beneficial default when $S = 3$. Decreased correlation (an increase in u) between firm value, S , and the net present value of new investment, N , increases the cost of using public debt to control investment when $S = 3$ and decreases the benefits.

When $u = 0$ (high correlation between S and N), the optimal amount of public debt is $R = 3$. When $u = 1$ (low correlation between S and N), the optimal amount of public debt is $R = 2$ (because the example assumes $t < [G + B]/3$). This implies that there exists a value of u , denoted by $\hat{u} = (2t - B)/(G - B) \in (0, 1)$, such that $R = 3$ is optimal for all $u < \hat{u}$, and $R = 2$ for all $u > \hat{u}$.

Increasing the correlation between cash flow from old investment and the profitability of new investment will generally increase the optimal amount of public debt and will increase the date-0 value of the firm.¹¹ When the correlation is low, public debt is an expensive control device. If the tax benefits of debt are not extremely high, firms with low correlation will choose low debt when given a choice between public debt and equity. The next subsection examines the cost of the alternative of bank debt.

The Optimal Quantity of Bank Debt

Resolving default is less costly with bank debt than with public debt. The discussion in Section 2 entitled “Default on Bank Debt: Bankruptcy Versus Renegotiation” establishes that the total cost of resolving a default when investment prospects are good ($N = N_G$) is g for bank debt, a saving of $G - g$ over the resolution cost given public debt. This is a large saving because the bank avoids bankruptcy court when the firm is worth more as a going concern. The total cost of resolving a default when investment prospects are bad (and $N = N_B$) is $N_B + q$ for bank debt, because a bad investment with net present value of N_B is avoided, but unavoidable administrative costs of q are incurred.

¹¹ Cases in which decreasing u (increasing the correlation between S and N) decreases debt occur as described immediately after Proposition 1. This requires that control aspects of debt are very valuable ($B \ll 0$) relative to the cost of bankruptcy when investment prospects are good, and $B + G < 3t$. The debt decrease from four to three for low values of u occurs because there is then little need for control when $S = 3$.

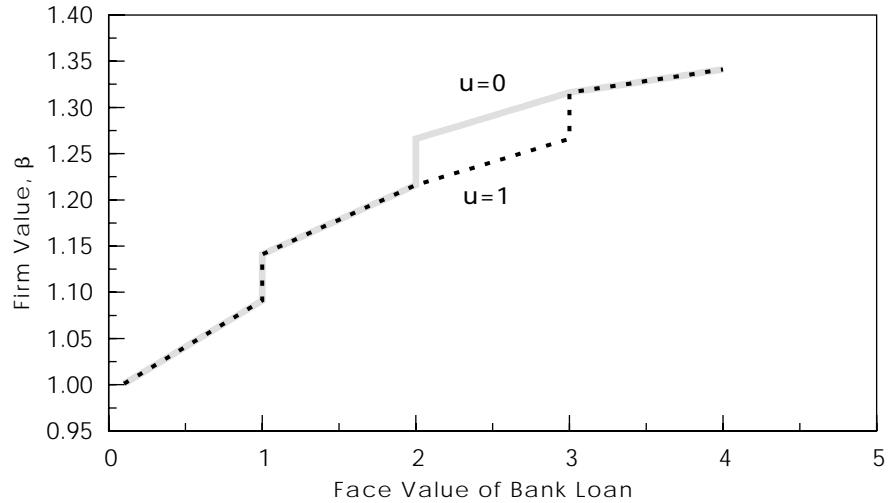
Define this cost of resolving default on bank debt when investment prospects are bad as $b \equiv N_B + q$. A default on bank debt when investment prospects are bad saves $B - b = k_q$ compared to public debt. The saving, k_q , represents the bank's comparative advantage in bankruptcy court. This cost saving is probably smaller than the saving when investment prospects are good, because bankruptcy court is not avoided.

The analysis of the optimal quantity of bank debt is similar to that of the optimal quantity of public debt, except the costs of default are those described in the previous paragraph and the bank has a higher cost of capital, as discussed in Section 1. Let x_i denote the net cost of defaulting on bank debt when date-1 firm value, S , is equal to i . Because investment prospects are bad when $S = 1$ ($N = N_B$), the cost of default on bank debt when $S = 1$, x_1 , is equal to b . Because investment prospects are good when $S = 4$ ($N = N_G$), the cost of default on bank debt when $S = 4$, x_4 , is equal to g . Recall that given total date-1 firm value, S , equal to two, the probability that $N = N_G$ is u (and the probability that $N = N_B$ is $1 - u$). The cost of default when $S = 2$ is then $x_2 = u \cdot g + (1 - u)b$. Given total date-1 firm value, S , equal to three, the probability that $N = N_G$ is $1 - u$ (and the probability that $N = N_B$ is u). The cost of default when $S = 3$ is then $x_3 = u \cdot b + (1 - u)g$. The results of Proposition 1, which describes the optimal amount of public debt, can be used to determine the optimal amount of bank debt. Substitute for the bankruptcy costs X_i the bank's costs x_i , for $i = 1, 2, 3, 4$, and instead of tax savings t , use $t - z$ to take account of the bank operating costs.

The optimal level of bank debt is less sensitive than is public debt to the correlation between firm value and the prospects for new investment. Banks have default cost advantages over public debt when investment prospects are good, which implies that undesirable defaults have a smaller effect on the value of the firm than with public debt. Figure 6 shows an example where the cost of reorganizing bank debt when prospects are good is less than the tax savings (net of bank costs) from added debt, or $g < t - z$. This assumption implies that optimal bank leverage is $r = 4$, independent of u . In this case, the optimal level of bank debt is $r = 4$ both for $u = 1$ and $u = 0$.¹² Similarly, $r = 4$ is the optimal bank debt level for all values of correlation between total date-1 firm value and net present value of new date-1 investment (all u between zero and one).

Let the value of the firm with bank debt, as a function of the amount of debt, r , be given by the function $\beta(r)$. Because I make the simplifying assumption of sufficiently low costs of reorganizing bank debt when investment prospects are good, the optimal value of bank debt is $r = 4$, and the value of the firm if it chooses the optimal bank debt is $\beta(4)$. The choice between bank and public

¹² The figure assumes $g = 0$, $z = .1$, and all of the parameters defined in footnote 10.

Figure 6 Firm Value Given Bank Debt When $u = 0$ and $u = 1$ 

Note: The optimal level of bank debt is $r = 4$ both for $u = 0$ and $u = 1$ because the costs of reorganizing a default on bank debt are less than the tax savings from additional debt. (The figure, but not the analysis in the article, assumes that there is no cost to reorganizing a default on bank debt.)

debt is equivalent to comparing this firm value, $\beta(4)$, to the date-0 firm value with the optimal level of public debt and choosing the form of debt leading to higher firm value. This comparison is discussed in the next subsection.

Bank Debt Versus Public Debt

If bank operating costs are too high, then public debt will dominate even if bank debt has default cost advantages. Similarly, if banks' default cost advantages are large, then bank debt will dominate even for rather large operating costs. For moderate levels of bank operating costs and bank debt default cost advantages, the optimal choice will depend on the characteristics of the borrower. In particular, the choice can depend on the correlation between future firm value, S , and future investment prospects, N .

Proposition 2 gives conditions where one type of debt dominates the other for all values of the correlation between S and N and then characterizes debt choice in the intermediate case where neither type of debt dominates the other. In this case, the choice depends on the parameter u , the degree to which S and N are uncorrelated.

Proposition 2 Bank debt is preferred to public debt if and only if $\theta < \min\{3t - B, t + u(G - B), G\}$, where θ is the value of bank operating costs minus the savings in default costs of bank debt of $r = 4$ versus public debt of $R = 4$ (θ is given by $\theta = 10z + g - 2k_q$).

If $\theta < \min\{t, G\}$, then bank debt is best for all values of u . If instead $\theta > \min\{3t - B, G\}$, then public debt is preferred for all values of u .

The choice between public and bank debt depends on u if $\theta < \min\{G, 3t - B\}$ and θ satisfies $t + G - B > \theta > t$ (this last condition is equivalent to $[\theta - t]/[G - B] \in (0, 1)$). In this case, bank debt is preferred if $u \geq u^*$ and public debt preferred for $u < u^*$, where the value of u^* is given by

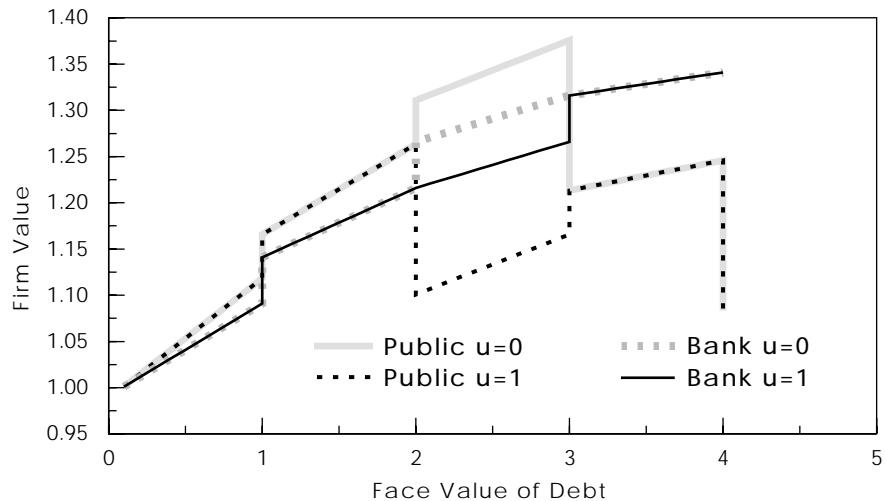
$$u^* = \frac{\theta - t}{G - B} = \frac{10z + g - 2k_q - t}{G - B}.$$

Proof: See Appendix.

Public debt dominates bank debt for all u and t if the added operating cost of bank funding minus the reorganization cost savings over those with public debt of face value $R = 4$ ($\theta = 10z + g - 2k_q$) exceeds G , the cost of a public debt default when investment prospects are good. If the cost of defaulting on public debt when prospects are good is sufficiently high, then bank debt is preferred when t is high and there are large tax savings of debt. For high tax savings, borrowers are driven to choose a debt structure where all types of default are likely, and banks' reorganization cost savings dominate independent of the level of u . At these high tax rates, the optimal level of public debt would be $R = 4$. If tax savings are lower, then bank debt is preferred if controlling investment when prospects are bad is important (B is low) and the prospects of new investment are not too correlated with total firm value (u is high). In this case, it is important to have high debt to control investment when prospects are bad, but it is not possible to do this with public debt in a way that avoids a significant chance of bankruptcy when investment prospects are good. Figure 7 superimposes Figures 5 and 6 to show an example where public debt is best when $u = 0$ but bank debt is best when $u = 1$.

Bank debt is more expensive, per unit, than public debt, because of bank costs and bank taxes. However, the borrowers that rely on bank debt are those that use large quantities of debt. This is because the non-tax cost disadvantages of public debt are most pronounced for very high levels of debt. If only low fractions of capital are raised with bank debt, it has few control advantages over public debt because bankruptcies that occur after very poor performance are not, on balance, costly.

**Figure 7 Firm Value for Both Bank and Public Debt
When $u = 0$ and $u = 1$**



Note: Figure 7 superimposes Figures 5 and 6 to show an example where public debt is best when $u = 0$ but bank debt is best when $u = 1$.

4. IMPLICATIONS AND CONCLUSIONS

Traditional capital structure theory obtains strong results by framing the choice as a trade-off between tax savings and exogenous costs of bankruptcy. When there are no costs of bankruptcy, an all-debt firm is optimal; when there are bankruptcy costs but no tax savings, an all-equity firm is optimal. This article begins by reviewing these results and illustrates the intermediate case where the trade-off yields a capital structure containing both debt and equity. In this case, firms with more variable cash flows choose less debt.

The more recent control-based theories of capital structure have not been framed as representing a trade-off of tax savings against bankruptcy cost. In some cases both taxes and bankruptcy costs have been ignored for simplicity. This article draws on elements of this control-based theory to determine the costs of bankruptcy endogenously. This reveals that the costs of bankruptcy can sometimes be negative; there are situations when bankruptcy is beneficial to prevent management from initiating a bad investment project. Interpreting bankruptcy costs as sometimes including this control benefit of debt allows integration of many of the ideas in control-based theories and the ideas in the traditional theories.

Considering the control role of debt allows a comparison of bank debt and publicly issued debt. Relative to publicly issued debt, bank debt is more expensive because banks must cover many variable operating costs. But banks enjoy an offsetting cost advantage: they can restructure outside bankruptcy those firms that default but have access to viable investment projects. In light of this second cost trade-off, the correlation between cash flow and the net present value of future investment becomes another key determinant of optimal capital structure. If this correlation is low, a firm will often default on its debt when it has viable future investments, which leads bank debt to have a net cost advantage over public debt. If this correlation is high, then a firm will only rarely default on debt when it has good new investments because these two components that determine its ability to refinance its debt move together. In this case, public debt has a cost advantage over bank debt.

If there are large tax advantages of debt over equity, then firms will be induced to issue mainly debt. A firm that issues very large amounts of debt will default on its debt even when its cash flow is fairly high and its new investments are reasonably good. In that situation, the ability to restructure bank debt outside bankruptcy is beneficial. This implies that large tax advantages of debt lead firms to substitute toward bank debt and away from both equity and public debt.

In countries that have small tax advantages to debt finance, the model predicts that those that use bank debt will have a lower correlation between total firm value and the profitability of new investment than those that use public debt. Equivalently, firms that use bank debt will be those with a lower correlation between cash flow from previous investment and the profitability of new investment. This will show up empirically as a lower correlation between the cash flows from old investment and the amount on new investment they undertake, as compared with firms that rely on public debt. Hoshi, Kashyap, and Scharfstein (1990) find exactly this correlation structure in Japanese data comparing bank borrowers with firms that rely on public debt. They explain the higher correlation between cash from old investments and the level of new investment among firms that rely on public debt as evidence that firms are sometimes unable to raise funds when they have good prospects, forcing them to rely on internal funds. Their explanation is not inconsistent with the model in this article (firms with good prospects but low total value experience financial distress). However, firms choose between the two sources of finance based on the correlation between cash flow and the optimal amount of new investment. The correlation observed in the data might be generated not only by the financing constraints of those who rely on public debt, but also by the more informative signal that lagged cash flow provides about the profitability of new investment for firms that choose public debt.

Increasing the tax advantage of debt makes more borrowers prefer bank debt. Firms with higher correlation between total value and prospects for new

investment are induced to choose high leverage with bank debt where they would have chosen low leverage with public debt at lower tax advantages. One implication of this result is that in countries with large tax advantages to debt, bank lending will be pervasive. If all firms face high costs of reorganization with public debt, banks will attract customers who need debt for control but want to save reorganization costs, plus others who do not need debt for control purposes but just for its tax savings. In these countries where banks are predicted to dominate the debt market, a bank's average customer will have a stronger correlation between cash flow and the quantity of new investment, because the firms for which cash flow is strongly correlated with the profitability of new investment opportunities are included in the set of bank customers. I am not aware of empirical evidence on these implications. This type of implication shows the importance of simultaneously considering the tax, bankruptcy, and control roles of debt. Studying the interaction of the various roles of debt yields fresh interpretations of existing empirical evidence as well as entirely new implications.

APPENDIX

Proof of Proposition 1:

Define the function τ_R as the marginal value of taxes saved by increasing debt to face value R from a debt equal to the largest integer $i < R$. For example, if the face value R is less than one, τ_R is just the total tax saving. Similarly, if the face value R is between one and two, τ_R is the total tax saving minus τ_1 . Let $I(R)$ denote the greatest integer less than or equal to R . The function τ_R is given by $\tau_R \equiv \sum_{i \geq R} P_i \{[R - I(R)] \cdot t\}$. The total value of tax benefits from debt with face R is then $\tau_R + \sum_{i < R} \tau_i$.

The date-0 value of a levered firm with public debt level R is the value of the unlevered firm, V^u , plus the tax savings, minus the bankruptcy costs. Let $\Pi(R)$ denote the total date-0 value of a firm with public debt of face value R . Recall that $P_i = P = 1/4$. Firm value is given by $\Pi(R) = \tau_R + \sum_{i < R} [\tau_i - (1/4X_i)] + V^u$, where $G = \gamma + k_g > 0$, $B = N_B + q + k_q < 0$, $X_1 = B$, $X_2 = (u \cdot G) + (1 - u) \cdot B$, $X_3 = [(1 - u)G] + (u) \cdot B$, $\tau_1 = t$, $\tau_2 = 3/4t$, $\tau_3 = 1/2t$, and $\tau_4 = 1/4t$.

The optimal face value is at least two, because $t > 0$ and $B < 0$ imply that $\Pi(R)$ is strictly increasing up to $R = 2$. The optimal value, $R^* \in \{2, 3, 4\}$, because $t > 0$ implies that $\Pi(R)$ is strictly increasing for $R \in (2, 3]$ and $R \in (3, 4]$. Finding the optimal value then involves comparing date-0 firm value, $\Pi(R)$, at these three values. The comparisons are as follows:

$\Pi(2) \geq \Pi(4)$ iff $G \geq 3t - B$, or $t \leq (G + B)/3$.

$\Pi(2) \geq \Pi(3)$ iff $t \leq \frac{1}{2}[u \cdot G + (1 - u) \cdot B]$.

$\Pi(3) \geq \Pi(4)$ iff $t \leq u \cdot B + (1 - u) \cdot G$.

The optimal value is $R^* = 2$ if $\Pi(2) \geq \Pi(4)$ and $\Pi(2) \geq \Pi(3)$, or
 $t \leq \min\{(G + B)/3, \frac{1}{2}[u \cdot G + (1 - u) \cdot B]\}$.

The optimal value is $R^* = 3$ if $\Pi(2) \leq \Pi(3)$ and $\Pi(3) \geq \Pi(4)$, or
 $t \geq \frac{1}{2}[u \cdot G + (1 - u) \cdot B]$ and $t \leq u \cdot B + (1 - u) \cdot G$.

The optimal value is $R^* = 4$ if $\Pi(2) \leq \Pi(4)$ and $\Pi(3) \leq \Pi(4)$, or
 $t \geq \max\{(G + B)/3, u \cdot B + (1 - u) \cdot G\}$.

Q.E.D.

Proof of Proposition 2:

Because the optimal value of bank debt is $r = 4$, public debt results in higher firm value if firm value, Π , with public debt of two, three, or four exceeds $\beta(4)$. Note that $\beta(r) = \tau_r - z_r + \sum_{i < r} [\tau_i - z_i - (P_i x_i)] + V^u$, where the τ_r functions are given in the proof of Proposition 1 and the other terms are as follows: $z_1 = z$, $z_2 = \frac{3}{4}z$, $z_3 = \frac{1}{2}z$, $z_4 = \frac{1}{4}z$, $b = N_B + q$, $x_1 = b$, $x_4 = g$, $x_2 = u \cdot g + (1 - u) \cdot b$, and $x_3 = u \cdot b + (1 - u)g$.

Firm value given bank debt is $\beta(4)$, given by: $\beta(4) = V^u + \frac{1}{4}[10(t - z) - 2(N_B + q) - g] = V^u + \frac{1}{4}[10(t - z) - 2(B - k_q) - g]$. Define $\theta = 10z + g - 2k_q$. The condition for $\Pi(2) \leq \beta(4)$ is $\theta < 3t - B$. The condition for $\Pi(4) \leq \beta(4)$ is $\theta < G$, which is independent of u or t . The condition for $\Pi(3) \leq \beta(4)$ is $\theta < t + u(G - B)$. Bank debt is preferred if and only if all three of these conditions are true, or $\theta < \min\{3t - B, t + u(G - B), G\}$. Bank loans are thus preferred for all $u \in [0, 1]$ if and only if this condition is true for $u = 0$, implying $\theta < \min\{t, G\}$, because $3t - B > t$. Public debt is preferred for all $u \in [0, 1]$ if and only if it is preferred for $u = 1$, implying public debt dominates if $\theta > \min\{3t - B, G\}$, because $t + G - B > G$. If neither of the two inequalities hold for θ , then the choice of lender depends on u ; this requires that θ satisfy $t + G - B > \theta > t$, because $t < G$. This condition is equivalent to $1 > (\theta - t)/(G - B) > 0$. The critical value of $u = u^*$ satisfies $\theta = t + u^*(G - B)$, or $u^* = (\theta - t)/(G - B)$.

Q.E.D.

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