The Rational Expectations Hypothesis of the Term Structure, Monetary Policy, and Time-Varying Term Premia

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M ost empirical studies of the rational expectations hypothesis of the term structure (REHTS) generally find that the data offer little support for the theory.[1] In many cases this large body of empirical work indicates that the theory does not even provide a close approximation of market behavior. This feature has led some investigators to search for alternative "irrational" theories of behavior in order to explain the data. We, on the other hand, believe that the rejections are so striking that the large amount of irrationality implied by the data is too implausible for this avenue to be treated seriously. Since the rejection of rational expectations in these studies generally involves the rejection of more complicated joint hypotheses, we choose to focus our energies on exploring a broader class of models that are consistent with REHTS.

In particular, we examine a model that incorporates Federal Reserve behavior along with a reasonable parameterization of term premia to revise the theory. The consideration of Fed behavior was first suggested by Mankiw and Miron (1986), who found that REHTS was more consistent with the data prior to the founding of the Fed. Even stronger evidence is presented in Choi and

[1] For an extensive set of results, see Campbell and Shiller (1991). Cook and Hahn (1990) and Rudebusch (1993) also give excellent surveys.

Wohar (1991), who cannot reject REHTS over the sample period of 1910–14. Cook and Hahn (1990) and Goodfriend (1991) argue persuasively that the Federal Reserve's use of a funds rate instrument, and, in particular, the way in which that instrument is employed, is partly responsible for the apparent failure of REHTS.

Recently Rudebusch (1994), in a study very much in the spirit of ours, provides some empirical support for the Cook and Hahn (1990) and Goodfriend (1991) hypothesis. Further, McCallum (1994) shows the theoretical linkage between the Fed's policy rule and the regression estimates in various tests of REHTS when the Fed responds to the behavior of longer-term interest rates.

While Fed behavior represents a potentially important component for explaining the empirical results of tests of REHTS, any explanation of these results that also maintains rational expectations must include time-varying term premia. Without time-varying term premia, tests of REHTS will not be rejected. This fact is pointed out in Mankiw and Miron (1986) and Campbell and Shiller (1991). Further, Campbell and Shiller indicate that white-noise term premia are insufficient to reconcile theory with data. We find this to be the case as well. Thus, we examine a more elaborate model of term premia coupled with Fed behavior in an attempt to explain some of the empirical results on REHTS.

Before developing a theory of Fed behavior and linking it to empirical work on REHTS, we present, in Section 1, a brief overview of the rational expectations hypothesis of the term structure. Then, in Section 2, we construct a model of Fed behavior that emodies the key elements described in Goodfriend (1991). We use the resulting model along with REHTS to generate returns on bonds of maturities ranging from one to six months. In Section 3 we focus, in essence, on the empirical regularities documented by Roberds, Runkle, and Whiteman (1993). We show that Fed behavior is not enough to reproduce their findings. Then, in Section 4, we turn our attention to incorporating a more realistic behavior of term premia. Combining these term premia with rational investor and Fed behavior generates data that is roughly consistent with the Roberds, Runkle, and Whiteman results. Section 5 concludes.

1. THE RATIONAL EXPECTATIONS THEORY OF THE TERM STRUCTURE

Tests and descriptions of the rational expectations theory of the term structure constitute a voluminous literature. An excellent survey can be found in Cook and Hahn (1990), and an exhaustive treatment is contained in Campbell and Shiller (1991). The basic idea is that with the exception of a term premium, there should be no expected difference in the returns from holding a long-term bond or rolling over a sequence of short-term bonds. As a result, the long-term interest rate should be an average of future expected short-term interest

rates plus a term premium. Specifically, the interest rate on a long-term bond of maturity n, $r_t(n)$, will obey

$$r_t(n) = \frac{1}{k} \sum_{i=0}^{k-1} E_t r_{t+mi}(m) + \phi_t(n, m), \tag{1}$$

where $r_{t+mi}(m)$ is the m period bond rate at date $t + mi$, E_t is the conditonal expectations operator over time t information, and $\phi_t(n, m)$ is the term premia between the n and m period bonds.[2] In equation (1), $k = n/m$ and is restricted to be an integer.

The rational expectations hypothesis implies that $r_{t+mi}(m) = E_t r_{t+mi}(m) + e_{t+mi}(m)$, where $e_{t+mi}(m)$ has mean zero and is uncorrelated with time t information. Using this implication, one can rearrange equation (1) to yield the following relationship:

$$\frac{1}{k} \sum_{i=1}^{k-1} [r_{t+mi}(m) - r_t(m)] = \alpha + r_t(n) - r_t(m) + v_t(n, m), \tag{2}$$

where $v_t(n, m) = \frac{1}{k} \sum_{i=1}^{k-1} e_{t+mi}(m) - [\phi_t(n, m) - \alpha]$ and α is the non-time-varying part of the term premium. Thus, future interest rate differentials on the shorter-term bond are related to the current interest rate spread between the long- and short-term bond.

Equation (2) forms the basis of the tests of the term structure that we focus on in this article. This involves running the regression

$$\frac{1}{k} \sum_{i=1}^{k-1} [r_{t+mi}(m) - r_t(m)] = \alpha + \beta[r_t(n) - r_t(m)] + v_t(n, m) \tag{3}$$

and testing if $\beta = 1$. We shall focus our attention on $n = 2$, 3, 4, 5, and 6 months and $m = 1$ and 3 months. For $m = 3$ and $n = 6$ (implying $k = 2$), the appropriate regression would be

$$^1/_2 \, [r_{t+3}(3) - r_t(3)] = \alpha + \beta[r_t(6) - r_t(3)] + v_t(6, 3). \tag{3'}$$

That is, the change in the three-month interest rate three months from now should be reflected in the difference between the current six-month and three-month rates because the pricing of the six-month bill should reflect any expected future changes in the rate paid on the three-month bill.

In the absence of time-varying term premia, the coefficient β should equal one. In practice, however, that has not been the case. For example, Table 1

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[2] Term premia arise naturally in consumption-based asset pricing models and involve the covariance of terms containing the ratio of future price-deflated expected marginal utilities of consumption to the current price-deflated marginal utility of consumption, the price of the long-term bond, and future prices of the short-term bond. See Labadie (1994).

reports some estimates obtained by Roberds, Runkle, and Whiteman (1993) and Campbell and Shiller (1991). Not only is $\beta < 1$, but the degree to which β deviates from one increases as k increases. Also, the coefficient in the regression when $n = 6$ and $m = 3$ is of the wrong sign and insignificantly different from zero.

This latter result is in stark contrast to estimates obtained by Mankiw and Miron (1986) and Choi and Wohar (1991), who find that prior to the advent of the Fed, the theory fared much better. These two sets of results, which primarily involve $r(6) - r(3)$, imply a number of possibilities among which are the following: (1) REHTS once held but no longer does (perhaps because investors have become irrational), (2) the nature of term premia has changed, or (3) Federal Reserve policy has in some way affected the nature of the empirical tests.

In analyzing these possibilities, we first note that the term premia must be time-varying for plim $\hat{\beta} \neq 1$ (i.e., the predicted value of β to be something other than one). To show this, we report the probability limit of $\hat{\beta}$ in (3′), which is adopted from the derivation in Mankiw and Miron (1986):

$$\text{plim } \hat{\beta} = \frac{\sigma^2[E_t\Delta r_{t+1}(3)] + 2\rho\sigma[E_t\Delta r_{t+1}(3)]\sigma[\phi_t(6,3)]}{\sigma^2[E_t\Delta r_{t+1}(3)] + 4\sigma^2[\phi_t(6,3)] + 4\rho\sigma[E_t\Delta r_{t+1}(3)]\sigma[\phi_t(6,3)]}, \quad (4)$$

where $\sigma^2[E_t\Delta r_{t+1}(3)]$ is the variance of the expected change in the three-month interest rate, ρ is the correlation between $E_t\Delta r_{t+1}(3)$ and $\phi_t(6,3)$, and $\sigma^2[\phi_t(6,3)]$ is the variance of the term premium.[3]

Expression (4) is informative for our purposes. Notice that for nonstochastic term premia, plim $\hat{\beta} = 1$. Hence stochastic term premia are required for plim $\hat{\beta} \neq 1$. Also observe that as $\sigma^2[\phi_t(6,3)]$ increases, plim $\hat{\beta}$ decreases. Further, note that plim $\hat{\beta}$ is a complicated function of $\sigma^2[E_t\Delta r_{t+1}(3)]$, but as this term gets fairly large, plim $\hat{\beta}$ goes to one. More generally, $\hat{\beta}$'s deviation from a value of one will depend on the ratio of the variance of the term premium to the variance of the expected change in interest rates.

It is this latter variance that Fed behavior may influence. In this regard, Mankiw and Miron (1986) document the variation over time in this variable and show that $\sigma^2[E_t\Delta r_{t+1}(3)]$ was much larger prior to the creation of the Federal Reserve System. Mankiw and Miron attribute this finding to the Fed's concern for interest rate smoothing.

As Cook and Hahn (1990) point out, however, rate smoothing cannot be the total story since the regression coefficient on $[r_t(2) - r_t(1)]$ is highly significant and close to one; moreover, for longer-term bonds the term structure does help predict future changes in interest rates. Regarding the short end of the yield curve, Cook and Hahn postulate that one must consider the discontinuous

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[3] Rudebusch (1993) derives a similar expression with $\rho = 0$.

Table 1 Coefficient Estimates from Literature

Source	Short (*m*) Long (*n*)	1 period 2 period	1 period 3 period	1 period 4 period	1 period 6 period	2 period 4 period	3 period 6 period
Campbell & Shiller Table 2, T-bills, 1952–87	coefficient standard error	0.5010 0.1190	0.4460 0.1990	0.4360 0.2380	0.2370 0.1670	0.1950 0.2810	−0.1470 0.2000
Roberds, Runkle & Whiteman, Table 6 F Fund, 1984–91	coefficient standard error	0.5925 0.0983	0.3935 0.1437	na na	0.2121 0.2822	na na	−0.1411 0.6079
Roberds, Runkle & Whiteman, Table 9 F Fund, 1984–91, SW*	coefficient standard error	0.7596 0.1359	0.2953 0.1399	na na	0.1557 0.1861	na na	−0.2971 0.3675
Roberds, Runkle & Whiteman, Table 11 F Fund, 1984–91, FOMC†	coefficient standard error	0.7119 0.1720	0.4104 0.1688	na na	0.0869 0.1878	na na	−0.3149 0.4553

* Settlement Wednesday.
† FOMC meeting date.

Note: Roberds, Runkle, and Whiteman use daily data in their regressions.

and infrequent changes in policy. Therefore, economic information that will affect future policy is often known prior to actual policy reactions. This factor implies that movements in the short end of the term structure will anticipate policy and hence have predictive content. In terms of equation (4), the variance of $\Delta E_t r_{t+1}(1)$ is likely to be greater than the variance of $\Delta E_t r_{t+1}(3)$.

Additional arguments supporting the relevance of monetary policy for tests of REHTS can be found in Goodfriend (1991) and McCallum (1994). McCallum shows that if the Fed reacts to movements in the term structure, then the strength of that reaction will influence estimates of β in tests of REHTS.

Taken together, these papers indicate that capturing Fed behavior is potentially important for understanding the term structure. We now attempt such an exercise.

2. A MODEL OF FED BEHAVIOR

Our model of Fed behavior is designed to capture the basic characteristics described by Goodfriend's (1991) analysis of Federal Reserve policy. In particular, we model the Federal Reserve's adjustment of its funds rate target as occurring at intervals and only in relatively small steps. Also, funds rate changes are often followed by changes in the same direction so that the Fed does not "whipsaw" financial markets. While the Fed is generally viewed as adjusting the funds rate to achieve various economic goals, for our purposes it is sufficient to let the Fed's best guess of an unconstrained optimal interest rate target follow an exogenous process. For simplicity, let

$$\Delta r_t^* = \rho \Delta r_{t-1}^* + u_t, \tag{5}$$

where r_t^* is the unconstrained optimal interest rate. That is, it is the interest rate the Fed would choose before the arrival of new information if it were not constrained to move the funds rate discretely. One could think of r_t^* as arising from a reaction function, but equation (5), along with additional behavioral constraints, is sufficient for the purpose of our investigation. To capture Fed behavior, we model changes in the funds rate according to the following criteria:

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\begin{aligned}
r_t^f &= r_{t-1}^f + \;^1\!/_2 \text{ if } r_t^* - r_{t-1}^f \geq \;^1\!/_2, \\
r_t^f &= r_{t-1}^f + \;^1\!/_4 \text{ if } ^1\!/_4 \leq r_t^* - r_{t-1}^f < \;^1\!/_2, \\
r_t^f &= r_{t-1}^f \text{ if } -\,^1\!/_4 < r_t^* - r_{t-1}^f < \;^1\!/_4, \\
r_t^f &= r_{t-1}^f - \;^1\!/_4 \text{ if } -\,^1\!/_2 < r_t^* - r_{t-1}^f \leq \; -\,^1\!/_4, \\
r_t^f &= r_{t-1}^f - \;^1\!/_2 \text{ if } -\,^1\!/_2 > r_t^* - r_{t-1}^f.
\end{aligned}
\tag{6}
$$

The behavior described by equation (6) implies that at each decision point the Fed is guided by its overall macroeconomic goals as depicted by the

behavior of r_t^*. It adjusts its instrument r_t^f incrementally and discretely. Thus, for a big positive shock to r_t^*, the Fed would be expected to raise the funds rate at a number of decision points until r_t^f approximated r_t^*. There would also be only a small probability that the Fed would ever reverse itself (i.e., raise the funds rate one period and lower it the next).

We parameterize the variance of u_t and the parameter ρ so that the behavior of the funds rate target, r_t^f, is consistent with actual behavior over the period 1985:1 to 1993:12. The parameter ρ is set at 0.15, and u_t has a variance of 0.09. In particular, u_t is distributed uniformly on the interval $[-0.525, 0.525]$. A uniform distribution is used to facilitate the pricing of multiperiod bonds in the next section.

The behavior generated by a typical draw from our stochastic process and by the actual funds rate target are reasonably similar. These are depicted in Figures 1 and 2. Table 2 provides some additional methods of comparison.

Figure 1   Federal Funds Rate Target
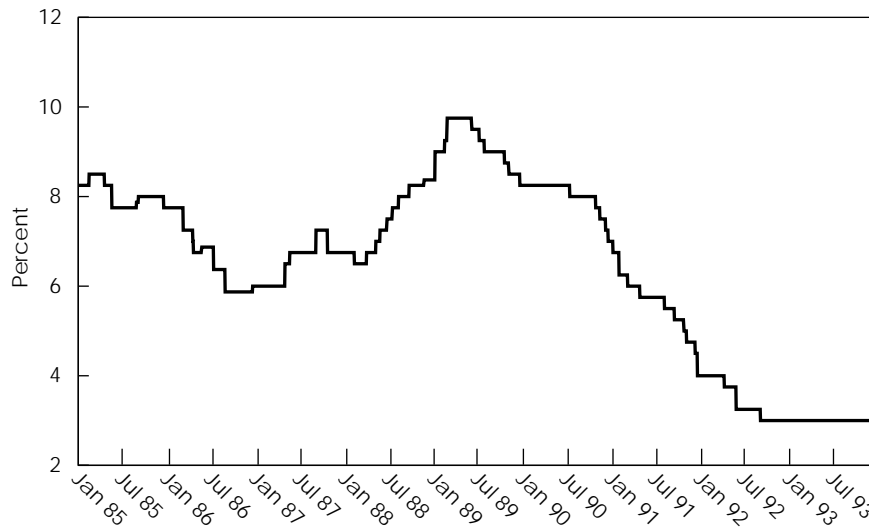
The low p-values of Fisher's exact test indicate that both actual and model data are consistent with targeted interest rate changes not being independent of the sign of previous changes.[4] However, a somewhat smaller percentage of interest rate changes are of the same sign in the model. The Fed, as modeled here, is

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[4] Model data are from 250 draws of a series with 300 observations.

Figure 2   Representative Draws of Funds Rate Target
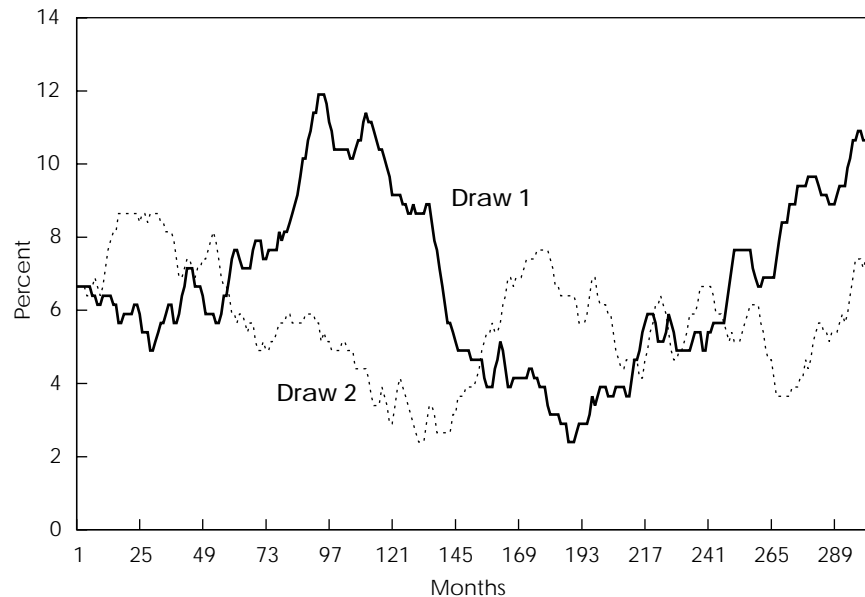
more likely to reverse itself than the Fed actually did over this period. Also, the Fed of our model is more likely to leave the funds rate unchanged. Finally, the correlation coefficient between funds rate changes in the model is not significantly different from the actual correlation coefficient displayed by the data.

We thus feel that equations (5) and (6) jointly represent a reasonable and tractable model of Federal Reserve behavior, especially if r_t^* is thought of as depending upon underlying economic behavior.

3.   MONETARY POLICY AND THE TERM STRUCTURE

As described by equations (5) and (6), the Federal Reserve determines the behavior of the one-period nominal interest rate. The FOMC meets formally eight times per year and informally via conference calls. Also, the chairman may act between FOMC meetings so that in actuality the term of the one-period rate is less than one month. Further, the timing between decision periods is stochastic and can be as little as one week or as long as an intermeeting period.[5] For simplicity, we model the decision period as monthly. Thus, the pricing of a

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[5] For a more detailed modeling of behavior along these lines, see Rudebusch (1994).

Table 2 Actual and Model Data Comparison

	Actual	**Model**
Percent of Changes of Same Sign	0.833	0.648
Fisher's Exact Text p-value (standard error)	2.6E-06	0.003600 (0.011)
$\text{Corr}(\Delta r^f_t, \Delta r^f_{t-1})$ (standard error)	0.2661	0.3242 0.04948
$\text{Std}(E_t r_{t+1} - r_t)$ (standard error)	0.2409	0.1234 (0.0034)
$\text{Std}[E_t r_{t+1}(2) - r_t]$ (standard error)	0.2562	0.1457 (0.0039)
$\text{Std}[E_t r_{t+1}(3) - r_t]$ (standard error)	0.2687	0.1565 (0.0040)
$\text{Std}[E_t r_{t+3}(3) - r_t(3)]^*$	0.1858	0.1306 (0.0052)

* Goldsmith-Nagan yields 0.2011.

Notes: Model data are from 250 draws of a series with 300 observations. The null of Fisher's exact test is that the sign of the change in the funds rate is independent of the sign of the previous change.

two-month bond or, more accurately, a two-month federal funds contract will obey

$$2r_t(2) = r^f_t + {}^1\!/_2(\text{Prob}[r^*_{t+1} - r^f_t \geq {}^1\!/_2])$$
$$+ {}^1\!/_4(\text{Prob}[{}^1\!/_4 \leq r^*_{t+1} - r^f_t < {}^1\!/_2])$$
$$- {}^1\!/_4(\text{Prob}[-{}^1\!/_2 < r^*_{t+1} - r^f_t \leq -{}^1\!/_4])$$
$$- {}^1\!/_2(\text{Prob}[r^*_{t+1} - r^f_t \leq -{}^1\!/_2]) + 2\phi_t(2,1). \tag{7}$$

In calculating the expectation of interest rates further than one period ahead, say, for example, two periods ahead, one needs to form time t expectations of terms such as $\text{Prob}[r^*_{t+2} - r^f_{t+1} > {}^1\!/_2]$. Assuming that u_t is uniformly distributed, the expressions we obtain for the various probabilities are linear in r^*_{t+j} and r^f_{t+j-1}. Thus, one can pass the expectations operator through the respective cumulative distribution functions.

For pricing three-, four-, five-, and six-month term federal funds, we use expressions analogous to equation (7). In order to examine the effect that our model of monetary policy has on tests of the rational expectations hypothesis of the term structure, we generate 250 simulations, each containing 300 values of each rate. The results are presented in Table 3, where standard errors have been corrected using the Newey-West (1987) procedure. We report results when

Table 3 Coefficient Estimates Using Model-Generated Data

Independent Variable	$r(2) - r$	$r(3) - r$	$r(4) - r$	$r(5) - r$	$r(6) - r$	$r(4) - r(2)$	$r(6) - r(3)$
(a) $\sigma(\phi) = 0$							
Coefficient	1.03	1.00	1.00	.99	.99	.97	.95
Standard Error	(.005)	(.118)	(.109)	(.158)	(.179)	(.22)	(.42)
(b) $\sigma(\phi) = .10$							
Coefficient	.28	.49	.57	.62	.64	.35	.31
Standard Error	(.055)	(.084)	(.105)	(.122)	(.138)	(.083)	(.114)

there is no term premium (row 1) and when there is a white-noise term premium with standard deviation 0.10 (row 2).

The results indicate that in the absence of time-varying term premia, there is no departure of estimates of β from one. This essentially serves as a check on our calculations, since all interest rates are calculated using REHTS. With a time-varying term premia, REHTS is rejected. However, the rejection of the model's data is not in keeping with the result on actual data. The estimates of β are increasing in $k = n/m$ rather than decreasing. Also, the results for $k = 2$ and $m =$ one month, two months, and three months, respectively, are almost identical for the model, while they are strikingly different for the data. Looking at Table 2 and equation (4) shows why. Table 2 indicates that $\sigma^2[E_t \Delta r_{t+1}(m)]$ is approximately the same for $m = 1$ and $m = 3$. (When $m = 2$, its value is 0.151.) With $\sigma^2[\phi(n,m)]$ equivalent by construction, the estimate of β will not vary much across experiments. For a model with white-noise term premia to replicate actual empirical results, it must generate $\sigma^2(E_t \Delta r_{t+1}) > \sigma^2[E_t \Delta r_{t+2}(2)] > \sigma^2[E_t \Delta r_{t+3}(3)]$, which does not happen in our particular model.

Interestingly enough, as shown in Table 2, the required behavior of $\sigma^2[E_t r_{t+1}(m) - r_t(1)]$ does not occur in the data either. We are therefore forced to conclude that our description of monetary policy, along with white-noise term premia, is insufficient to explain the empirical results in Roberds, Runkle, and Whiteman (1993) as well as in Campbell and Shiller (1991). Our failure could be due primarily to an insufficient model of policy or to an inadequate model of term premia. In the next section we modify our model of term premia and reexamine REHTS on data generated by our modified model.

4.   A DESCRIPTION OF TERM PREMIA

To generate term premia that potentially resemble the stochastic processes of actual term premia, we need some way of estimating term premia. For this we

turn to the multivariate ARCH-M methodology described in Bollerslev (1990). We use a multivariate model since the term premia generated from a univariate model are highly correlated. In essence, we estimate a multivariate ARCH-M model of excess holding period yields then use the estimated process to simulate time-varying term premia. The simulated processes, along with the model in Section 2, are used to generate data on interest rates. This simulated data is then used to estimate regressions like (3′).

In estimating term premia (for the case in which $k = 2$), first define the excess holding period yield, $y_t(n, m)$, as

$$2r_t(n) - r_{t+m}(m) - r_t(m).$$

From equation (1) we see that this is merely $E_t r_{t+m}(m) - r_{t+m}(m) + 2\phi_t(n, m)$, which is the sum of an expectational error and twice the actual term premium as defined in (1).

The multivariate ARCH-M specification that we estimate over the sample period 1983:1–1993:12 is given by

$$y_t = \beta + \delta \log h_t + \varepsilon_t, \tag{8}$$

where ε_t conditioned on past information is a normal random vector with variance-covariance matrix H_t. The elements of H_t are given by

$$h_{jj,t}^2 = \gamma_j + \alpha_j \sum_{i=1}^{12} w_i \varepsilon_{j,t-i}^2$$

$$h_{ij,t} = \rho_{ij} h_{ii,t} h_{jj,t}, \tag{9}$$

where y_t is a 3 by 1 vector of the ex-post excess holding period yields on Treasury bills that includes the two-month versus one-month bill, the three-month versus one-month bill, and the six-month versus three-month bill.[6] The w_i are fixed weights given by $(13 - i)/78$. In this specification of the model, the covariances h_{ij} are allowed to vary but the correlation coefficients, ρ_{ij}, between the errors are constant. The coefficient estimates are reported in Table 4. Almost all the coefficients are highly significant.

The term premia derived from this model are depicted in Figure 3 and are labeled T2, T3, and T6. Recall that T2 and T6 are twice $\phi(2, 1)$ and $\phi(6, 3)$, respectively, while T3 is three times $\phi(3, 1)$. One notices the term premia spike upward in 1984, in late 1987, and in early 1991. The term premium on two-month bonds also spikes in late 1988 and early 1989. The 1987 episode is associated with the October stock market crash. Interestingly, the 1984 and

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[6] We use T-bills rather than term federal funds because coefficient estimates using the federal funds rate are insignificant. One possible explanation for this result is that the excess holding period yield on federal funds involves both a term premia derived from a consumption-based asset pricing model as well as default risk that may be uncorrelated with the term premia. This default risk may add sufficient noise that it is difficult to estimate the term premia using ARCH-M type regressions.

**Table 4 Coefficient Estimates for ARCH-M Model**
        Log likelihood = 149.72

Coefficient	Estimate	Standard Error	Significance Level
β_2	.98	.13	.0000
δ_2	.46	.13	.0005
γ_2	.058	.018	.0009
α_2	.92	.20	.0000
β_3	1.46	.13	.0000
δ_3	1.25	.47	.0076
γ_3	.52	.089	.0000
α_3	.44	.13	.0010
β_6	.94	.45	.0389
δ_6	.79	.82	.331
γ_6	.25	.08	.0011
α_6	.23	.17	.0601
ρ_{23}	.92	.016	.0000
ρ_{26}	.48	.082	.0000
ρ_{36}	.67	.060	.0000

1988–89 episodes correspond to the inflation-scare episodes documented in Goodfriend (1993). The last spike in the term premia occurs around the time of the Gulf War and a recession.

Statistical data for the in-sample residuals, the estimated term premia, and the ex-post holding period yields are depicted in Table 5. In attempting to ascertain the joint importance of Fed behavior and time-varying term premia in explaining the regression results of Campbell and Shiller (1991) as well as Roberds, Runkle, and Whiteman (1993), we perform three experiments. First we generate a funds rate that is stationary and thus does not display the interest rate smoothing or discrete interest rate changes that are embodied in our model of Fed behavior. Longer-term interest rates are then derived using equation (1) and the rational expectations hypothesis. We do this to see if our model of term premia by itself can account for the actual regression results. Next we examine an interest rate process that includes a greater degree of smoothing but does not require discrete changes in the funds rate. Finally, we combine our model of term premia with our depiction of Fed behavior and investigate whether this model of interest rate determination can explain the regression results obtained using actual data. The results we analyze involve the cases in which $n = 2$, $m = 1$, and $n = 6$, $m = 3$ (i.e., the term spread between the two-month and one-month bills and the six-month and three-month bills).

To begin, we model the one-period interest rate as $r_t^* = 0.75r_{t-1}^* + u_t$. As in our actual model of Fed behavior, u_t is distributed uniformly on the interval $[-0.525, 0.525]$. Combining this behavior with term premia generated from our estimated ARCH-M model, we generate data on longer-term interest

Figure 3   Interest Rate Term Premia

T2 and T3

T2 and T6
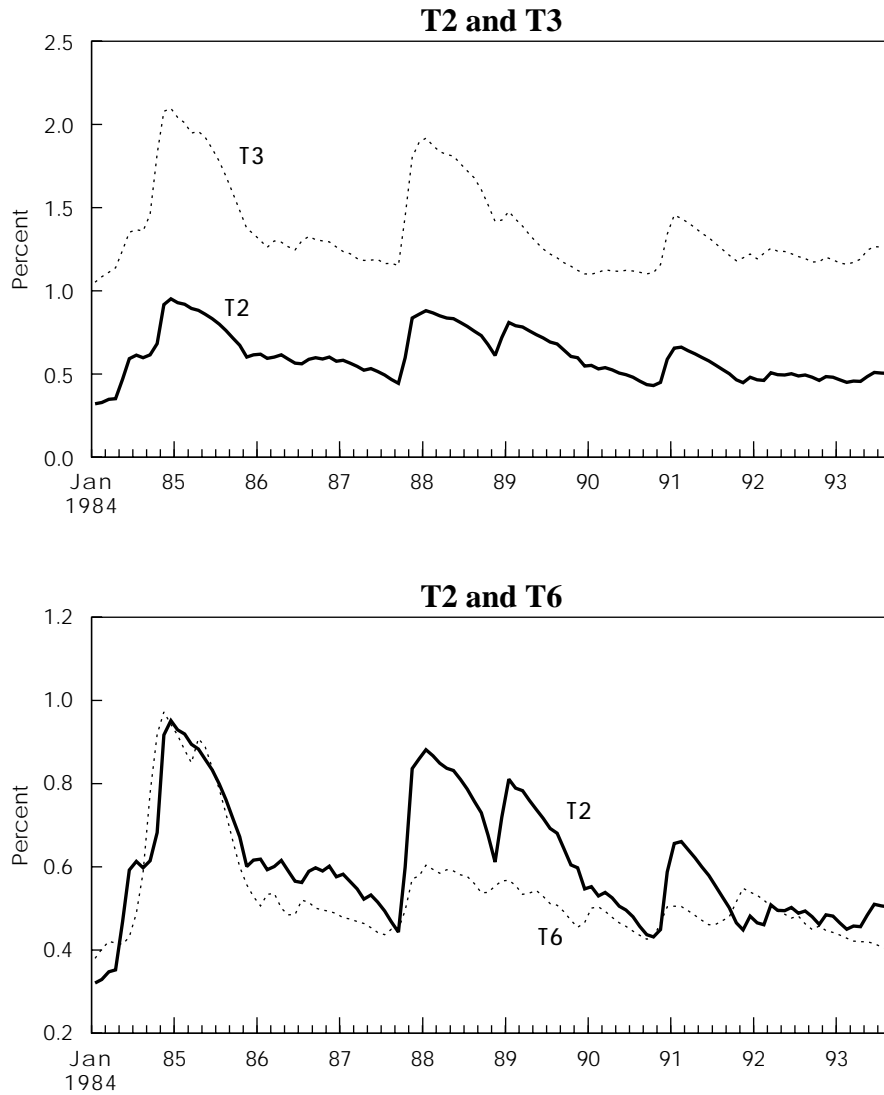
Notes: T2 is the term premium between the two-month and one-month bonds. T3 is the term premium between the three-month and one-month bonds. T6 is the term premium between the six-month and three-month bonds.

Table 5 Statistical Data from ARCH-M Model

Residuals		Standard Error	Correlation Matrix		
$\varepsilon 2$		.45	1.0		
$\varepsilon 3$		.99	.92	1.0	
$\varepsilon 6$		.64	.50	.70	1.0
Estimated Term Premia	**Mean**	**Standard Error**	**Correlation Matrix**		
T2	.60	.15	1.0		
T3	1.35	.25	.90	1.0	
T6	.53	.13	.77	.85	1.0
Actual Ex-post Yields	**Mean**	**Standard Error**	**Correlation Matrix**		
y2	.65	.47	1.0		
y3	1.51	1.04	.93	1.0	
y6	.62	.68	.55	.74	1.0

rates using equation (1). The regression results based on 500 simulations of 125 observations are

$$r_{t+1}(1) - r_t(1) = a_0 + 0.43 \, [r_t(2) - r_t(1)] + \varepsilon_{1t},$$
$$(0.16)$$

$$r_{t+3}(3) - r_t(3) = \alpha_0 + 1.00 \, [r_t(6) - r_t(3)] + \varepsilon_{3t},$$
$$(0.18)$$

where standard errors are in parentheses. REHTS is not rejected by the second regression, and the results of this regression are consistent with those documented in Mankiw and Miron (1986) and Choi and Wohar (1991) for the period prior to the founding of the Fed. One must therefore conclude that our model of term premia is not sufficient for generating data that are capable of replicating regression results using actual post-Fed data.

Next we model the short-term interest rates as $\Delta r_t^* = 0.15 \Delta r_{t-1}^* + u_t$, which is consistent with our modeling of r_t^* in equation (5). Thus, the only element lacking from our complete model of Fed behavior is the discrete nature of funds rate behavior given by equation (6). Generating data using this nonstationary model of r_t^*, along with our model of term premia, we obtain the following regression results:

$$r_{t+1}(1) - r_t(1) = b_0 + 0.10 \, [r_t(2) - r_t(1)] + e_{1t},$$
$$(0.19)$$

$$r_{t+3}(3) - r_t(3) = \beta_0 + 0.12 \, [r_t(6) - r_t(3)] + e_{3t}.$$
$$(1.24)$$

Here both coefficients are insignificantly different from zero. Thus, this experiment does not generate the statistically significant coefficient commonly found when using actual data on two- and one-month interest rates.

Finally, we combine the joint modeling of term premia using the ARCH-M process and Fed behavior given by equations (5) and (6). These regression results are the following:

$$r_{t+1}(1) - r_t(1) = c_0 + 0.46 \, [r_t(2) - r_t(1)] + w_{1t},$$
$$(0.10)$$

$$r_{t+1}(3) - r_t(3) = \gamma_0 + 0.64 \, [r_t(6) - r_t(3)] + w_{3t}.$$
$$(0.59)$$

Here the joint modeling of term premia and Fed behavior is capable of explaining a statistically significant coefficient that is less than one in the shorter-maturity regression, whereas the coefficient in the regression involving longer maturities is insignificantly different from zero. An explanation for the increased significance of the coefficient in the first regression from that estimated in the previous experiment goes as follows. Due to Fed behavior, the standard deviation of the expected change in the one-month rate has risen from a value of 0.095 to 0.123, while the standard deviation of the term premia has remained unchanged. However, there are only 35 episodes in which the coefficient in the first regression is greater than 0.5 while the coefficient in the second regression is also less than zero. Thus, the coefficient estimates that are consistent with the results presented in Roberds, Runkle, and Whiteman (1993) occur in approximately 7 percent of the trials.

The results presented above are not entirely satisfactory because the generated term premia do not exactly match the fitted term premia of the model (perhaps because the correlation coefficients are constrained to be time invariant). The standard deviations of the generated term premia are somewhat less than those depicted in Table 5, whereas the correlation coefficients are appreciably less. With generated data, $\sigma_{T2} = 0.17$, $\sigma_{T3} = 0.14$, and $\sigma_{T6} = 0.06$ while $\rho_{23} = 0.66$, $\rho_{26} = 0.20$, and $\rho_{36} = 0.41$.

To remedy this situation, we generate data by also allowing the correlation coefficients, ρ_{ij}, to vary intertemporally. We do this by allowing them to depend on the $h_{jj,t}$s in equation (9), producing standard deviations of $\sigma_{T2} = 0.14$, $\sigma_{T3} = 0.44$, and $\sigma_{T6} = 0.12$ and correlation coefficients of $\rho_{23} = 0.88$, $\rho_{26} = 0.73$, and $\rho_{36} = 0.83$. In a regression using data generated by this mechanism, the coefficient on the 2,1 term is 0.93(0.16) and on the 6,3 term is 0.79(0.63), where standard errors are in parentheses. Also, in 10 percent of the cases the 6,3 coefficient is less than zero, while the 2,1 coefficient is greater than 0.5. When there is no discretization of movements in the funds rate, these coefficients are 0.54(0.47) and 0.11(1.95). Both coefficients differ insignificantly from zero.

While the term premia in the last simulation do not come from any estimated model, the experiment at least shows that regression results that are in accord with those obtained in practice can be generated by the combination of (1) Fed behavior that both smooths the movements in interest rates and only moves interest rates discretely and (2) time-varying term premia that are calibrated to match data moments.

5. CONCLUSION

This article explores the linkage between Federal Reserve behavior and time-varying term premia and analyzes what effect these two economic phenomena have on tests of the rational expectations hypothesis of the term structure. Adding both these elements to a model of interest rate formation produces simulated regression results that are reasonably close to those reported using actual data. We thus feel that a deeper understanding of interest rate behavior will be produced by jointly taking into account the behavior of the monetary authority along with a more detailed understanding of what determines term premia. Reconciling theory with empirical results probably does not require abandonment of the rational expectations paradigm.

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