Testing Long-Run Neutrality

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Key classical macroeconomic hypotheses specify that permanent changes in nominal variables have no effect on real economic variables in the long run. The simplest "long-run neutrality" proposition specifies that a permanent change in the money stock has no long-run consequences for the level of real output. Other classical hypotheses specify that a permanent change in the rate of inflation has no long-run effect on unemployment (a vertical long-run Phillips curve) or real interest rates (the long-run Fisher relation). In this article we provide an econometric framework for studying these classical propositions and use the framework to investigate their relevance for the postwar U.S. experience.

Testing these propositions is a subtle matter. For example, Lucas (1972) and Sargent (1971) provide examples in which it is impossible to test long-run neutrality using reduced-form econometric methods. Their examples feature rational expectations together with short-run nonneutrality and exogenous variables that follow stationary processes so that the data generated by these models do not contain the sustained changes necessary to directly test long-run neutrality. In the context of these models, Lucas and Sargent argued that it was necessary to construct fully articulated behavioral models to test the neutrality propositions. McCallum (1984) extended these arguments and showed that low-frequency band spectral estimators calculated from reduced-form models were also subject to the Lucas-Sargent critique. While these arguments stand on firm logical ground, empirical analysis following the Lucas-Sargent prescriptions has not yet yielded convincing evidence on the neutrality propositions. This undoubtedly reflects a lack of consensus among macroeconomists on the appropriate behavioral model to use for the investigation.

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The specific critique offered by Lucas and Sargent depends critically on stationarity. In models in which nominal variables follow integrated variables processes, long-run neutrality can be defined and tested without complete knowledge of the behavioral model. Sargent (1971) makes this point clearly in his paper, and it is discussed in detail in Fisher and Seater (1993).[1] But, even when variables are integrated, long-run neutrality cannot be tested using a reduced-form model. Instead, what is required is the model's "final form," showing the dynamic response of the variables to underlying structural disturbances.[2]

Standard results from the econometric analysis of simultaneous equations show that the final form of a structural model is not econometrically identified, in general, because a set of a priori restrictions are necessary to identify the structural disturbances. Our objective in this article is to summarize the reduced-form information in the postwar U.S. data and relate it to the long-run neutrality propositions under alternative identifying restrictions. We do this by systematically investigating a wide range of a priori restrictions and asking which restrictions lead to rejections of long-run neutrality and which do not. For example, in our framework the estimated value of the long-run elasticity of output with respect to money depends critically on what is assumed about one of three other elasticities: (i) the impact elasticity of output with respect to money, (ii) the impact elasticity of money with respect to output, or (iii) the long-run elasticity of money with respect to output. We present neutrality test results for a wide range of values for these elasticities, using graphical methods.

Our procedure stands in stark contrast to the traditional method of exploring a small number of alternative identifying restrictions, and it has consequent costs and benefits. The key benefit is the extent of the information conveyed: researchers with strong views about plausible values of key parameters can learn about the result of a neutrality test appropriate for their beliefs; other researchers can learn about what range of parameter values result in particular conclusions about neutrality. The key cost is that the methods that we use are only practical in small models, and we demonstrate them here using

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[1] Also see Geweke (1986), Stock and Watson (1988), King, Plosser, Stock, and Watson (1991), and Gali (1992).

[2] Throughout this article we use the traditional jargon of dynamic linear simultaneous equations. By "structural model" we mean a simultaneous equations model in which each endogenous variable is expressed as a function of the other endogenous variables, exogenous variables, lags of the variables, and disturbances that have structural interpretation. By "reduced-form model" we mean a set of regression equations in which each endogenous variable is expressed as a function of lagged dependent variables and exogenous variables. By "final-form model" we mean a set of equations in which the endogenous variables are expressed as a function of current and lagged values of shocks and exogenous variables in the model. For the standard textbook discussion of these terms, see Goldberger (1964), chapter 7.

bivariate models. This raises important questions about effects of potential omitted variables, and we discuss this issue below in the context of specific empirical models.

We organize our discussion as follows. In Section 1 below, we begin with the theoretical problem of testing for neutrality in economies that are consistent with the Lucas-Sargent conclusions. Our goal is to show the restrictions that long-run neutrality impose on the final-form model, and how these restrictions are related to the degree of integration of the variables. In Section 2, we discuss issues of econometric identification. Section 3 contains an empirical investigation of (i) the long-run neutrality of money, (ii) the long-run superneutrality of money, and (iii) the long-run Fisher relation. Even with an unlimited amount of data, the identification problems discussed above make it impossible to carry out a definitive test of the long-run propositions. Instead, we investigate the plausibility of the propositions across a wide range of observationally equivalent models. In Section 4 we investigate the long-run relation between inflation and the unemployment rate, i.e., the slope of the long-run Phillips curve. Here, the identification problem is more subtle than in the other examples. As we show, the estimated long-run relationship depends in an important way on whether the Phillips curve slope is calculated from a "supply" equation, as in Sargent (1976) for example, or from a "price" equation, as in Solow (1969) or Gordon (1970).

Previewing our empirical results, we find unambiguous evidence supporting the neutrality of money but more qualified support for the other propositions. Over a wide range of identifying assumptions, we find there is little evidence in the data against the hypothesis that money is neutral in the long run. Thus the finding that money is neutral in the long run is robust to a wide range of identifying assumptions. Conclusions about the other long-run neutrality propositions are not as unambiguous: these propositions are rejected for a range of identifying restrictions that we find arguably reasonable, but they are not rejected for others. Yet many general conclusions are robust. For example, the rejections of the long-run Fisher effect suggest that a one percentage point permanent increase in inflation leads to a smaller than one percentage point increase in nominal interest rates. Moreover, a wide range of identifying restrictions leads to very small estimates of the long-run effect of inflation on unemployment. On the other hand, the sign and magnitude of the estimated long-run effect of money growth on the level of output depends critically on the specific identifying restriction employed.

1. THE ROLE OF UNIT ROOTS IN TESTS FOR LONG-RUN NEUTRALITY

Early empirical researchers investigated long-run neutrality by examining the coefficients in the distributed lag:

$$y_t = \Sigma \alpha_j m_{t-j} + \text{error} = \alpha(L) m_t + \text{error}, \tag{1}$$

where y is logarithm of output, m is logarithm of the money supply, $\alpha(L) = \Sigma\alpha_j L^j$, and L is the lag operator.[3] If m_t is increased by one unit permanently, then (1) implies that y_t will eventually increase by the sum of the α_j coefficients. Hence, investigating the long-run multiplier, $\alpha(1) = \Sigma\alpha_j$, appears to be a reasonable procedure for investigating long-run neutrality. However, Lucas (1972) and Sargent (1971) demonstrated that in models with short-run nonneutrality and rational expectations, this approach can be very misguided.

The Lucas-Sargent critique can be exposited as follows. Consider a model consisting of an aggregate supply schedule (2a); a monetary equilibrium condition (2b); and a money supply rule (2c):

$$y_t = \theta(p_t - E_{t-1}p_t), \tag{2a}$$

$$p_t = m_t - \delta y_t, \text{and} \tag{2b}$$

$$m_t = \rho m_{t-1} + \epsilon_t^m, \tag{2c}$$

where y_t is the logarithm of output; p_t is the logarithm of the price level; $E_{t-1}p_t$ is the expectation of p_t formed at $t-1$, m_t is the logarithm of the money stock, and ϵ_t^m is a mean-zero serially independent shock to money. The solution for output is

$$y_t = \pi(m_t - E_{t-1}m_t) = \pi(m_t - \rho m_{t-1}) = \pi(1 - \rho L)m_t = \alpha(L)m_t, \tag{3}$$

with $\pi = \theta/(1 + \delta\theta)$ and $\alpha(L) = \alpha_0 + \alpha_1 L = \pi(1 - \rho L)$.

As in Lucas (1973), the model is constructed so that only surprises in the money stock are nonneutral and these have temporary real effects. Permanent changes in money have no long-run effect on output. However, the reduced-form equation $y_t = \alpha(L)m_t$ suggests that a one-unit permanent increase in money will increase output by $\alpha_0 + \alpha_1 = \alpha(1) = \pi(1 - \rho)$. Moreover, as noted by McCallum (1984), the reduced form also implies that there is a long-run correlation between money and output, as measured by the spectral density matrix of the variables at frequency zero.

On this basis, Lucas (1972), Sargent (1971), and McCallum (1984) argue that a valid test of long-run neutrality can only be conducted by determining the structure of monetary policy (ρ) and its interaction with the short-run response to monetary shocks (π), which depends on the behavioral relations in the model (δ and θ). While this is easy enough to determine in this simple setting, it is much more difficult in richer dynamic models or in models with a more sophisticated specification of monetary policy.

[3] See Sargent (1971) for references to these early empirical analyses.

However, if $\rho = 1$, there is a straightforward test of the long-run neutrality proposition in this simple model. Adding and subtracting ρm_t from the right-hand side of (3) yields

$$y_t = \pi\rho\Delta m_t + \pi(1-\rho)m_t \tag{3'}$$

so that with $\rho = 1$ there is a zero effect of the level of money under the neutrality restriction. Hence, one can simply examine whether the coefficient on the level of money is zero when m_t is included in a bivariate regression that also involves Δm_t as a regressor.

With permanent variations in the money stock, the reduced form of this simple model has two key properties: (i) the coefficient on m_t corresponds to the experiment of permanently changing the level of the money stock; and (ii) the coefficient on Δm_t captures the short-run nonneutrality of monetary shocks. Equivalently, with $\rho = 1$, the neutrality hypothesis implies that in the specification $y_t = \Sigma\alpha_j m_{t-j}$, the neutrality restriction is $\alpha(1) = 0$, where $\alpha(1) = \Sigma\alpha_j$ is the sum of the distributed lag coefficients.

While the model in (2a) – (2c) is useful for expositing the Lucas-Sargent critique, it is far too simple to be used in empirical analysis. Standard macroeconomic models include several other important features: shocks other than ϵ_t^m are incorporated to capture other sources of fluctuations; the simple specification of an exogenous money supply in (2c) is discarded in favor of a specification that allows the money supply to respond to the endogenous variables in the model; and finally, the dynamics of the model are generalized through the incorporation of sticky prices, costs of adjusting output, information lags, etc. In these more general settings, it is still the case that long-run neutrality can sometimes be determined by examining the model's final form.

To see this, consider a macroeconomic model that is linear in both the observed variables and the structural shocks. Then, if the growth rates of both output and money are stationary, the model's final form can be written as

$$\Delta y_t = \mu_y + \theta_{y\eta}(L)\epsilon_t^\eta + \theta_{ym}(L)\epsilon_t^m \text{ and} \tag{4a}$$

$$\Delta m_t = \mu_m + \theta_{m\eta}(L)\epsilon_t^\eta + \theta_{mm}(L)\epsilon_t^m, \tag{4b}$$

where ϵ_t^η is vector of shocks, other than money, that affect output; $\theta_{mm}(L)\epsilon_t^m = \Sigma\theta_{mm,j}\epsilon_{t-j}^m$, and the other terms are similarly defined. Rich dynamics are incorporated in the model via the lag polynomials $\theta_{y\eta}(L)$, $\theta_{ym}(L)$, $\theta_{m\eta}(L)$, and $\theta_{mm}(L)$. These final-form lag polynomials will be functions of the model's behavioral parameters in a way that depends on the specifics of the model, but the particular functional relation need not concern us here.

The long-run neutrality tests that we conduct all involve the answer to the following question: does an unexpected and exogenous permanent change in the level of m lead to a permanent change in the level of y? If the answer is no, then we say that m is long-run neutral towards y. In equations (4a) and (4b), ϵ_t^m

are exogenous unexpected changes in money. The permanent effect of ϵ_t^m on future values of m is given by $\Sigma\theta_{mm,j}\epsilon_t^m = \theta_{mm}(1)\epsilon_t^m$. Similarly, the permanent effect of ϵ_t^m on future values of y is given by $\Sigma\theta_{ym,j}\epsilon_t^m = \theta_{ym}(1)\epsilon_t^m$. Thus, the long-run elasticity of output with respect to permanent exogenous changes in money is

$$\gamma_{ym} = \theta_{ym}(1)/\theta_{mm}(1). \tag{5}$$

Within this context, we say that the model exhibits long-run neutrality when $\gamma_{ym} = 0$. That is, the model exhibits long-run neutrality when the exogenous shocks that permanently alter money, ϵ_t^m, have no permanent effect on output.

In an earlier version of this article (King and Watson 1992) and in King and Watson (1994), we explored the relationship between the restriction $\gamma_{ym} = 0$ and the traditional notion of long-run neutrality using a dynamic linear rational expectations model with sluggish short-run price adjustment. We required that the model display theoretical neutrality, in that its real variables were invariant to proportionate changes in all nominal variables. We showed that this long-run neutrality requirement implied long-run neutrality in the sense investigated here. That is, unexpected permanent changes in m_t had no effect on y_t. Further, like the simple example presented in equations (2) and (3) above, the model also implied that long-run neutrality could be tested within a system like (4) if (and only if) the money stock is integrated of order one. Finally, in the theoretical model, long-run neutrality implied that $\gamma_{ym} = 0$.

In the context of equations (4a) – (4b), the long-run neutrality restriction $\gamma_{ym} = 0$ can only be investigated when money is integrated. If the money process does not contain a unit root, then there are no permanent changes in the level of m_t and $\theta_{mm}(1) = 0$. In this case, γ_{ym} in (5) is undefined, and the model's final form says nothing about long-run neutrality. This is the point of the Lucas-Sargent critique. The intuition underlying this result is simple: long-run neutrality asks whether a permanent change in money will lead to a permanent change in output. If permanent changes in money did not occur in the historical data (that is, money is stationary), then these data are uninformative about long-run neutrality. On the other hand, when the exogenous changes in money permanently alter the level of m, then $\theta_{mm}(1) \neq 0$, money has a unit root, γ_{ym} is well defined in (5), and the question of long-run neutrality can be answered from the final form of the model.

2. ECONOMETRIC ISSUES

In general, it is not possible to use data to determine the parameters of the final-form equations (4a) – (4b). Econometric identification problems must first be solved. We approach the identification problem in an unusual way. Rather than "solve" it by imposing a single set of a priori restrictions, our empirical strategy is to investigate long-run neutrality for a large set of observationally equivalent models. Our hope is that this will provide researchers with a clearer sense of the robustness of any conclusions about long-run neutrality. Before presenting the empirical results, we review the issues of econometric identification that arise in the estimation of sets of equations like (4a) and (4b). This discussion motivates the set of observationally equivalent models analyzed in our empirical work.

To begin, assume that $(\epsilon_t^{\eta\prime}\epsilon_t^m)'$ is a vector of unobserved mean-zero serially independent random variables, so that (4a) – (4b) can be interpreted as a vector moving average model. The standard estimation strategy begins by inverting the moving average model to form a vector autoregressive model (VAR). The VAR, which is assumed to be finite order, is then analyzed as a dynamic linear simultaneous equations model.[4] We will work within this framework.

Estimation and inference in this framework requires two distinct sets of assumptions. The first set of assumptions is required to transform the vector moving average model into a VAR. The second set of assumptions is required to econometrically identify the parameters of the VAR. These sets of assumptions are intimately related: the moving average model can only be inverted if the VAR includes enough variables to reconstruct the structural shocks. In the context of (4a) – (4b), if $\epsilon_t = (\epsilon_t^{\eta\prime}\epsilon_t^m)'$ is an $n \times 1$ vector, then there must be at least n variables in the VAR. But, identification of an n-variable VAR requires $n \times (n-1)$ a priori restrictions, so that the necessary number of identifying restrictions increases with the square of the number of structural shocks.

In our empirical analysis we will assume that $n = 2$, so that only bivariate VARs are required. To us, this seems the natural starting point, and it has been employed by many other researchers in the study of the neutrality propositions discussed below. We also do this for tractability: when $n = 2$, only 2 identifying restrictions are necessary. This allows us to investigate thoroughly the set of observationally equivalent models. The cost of this simplification is that some of our results may be contaminated by omitted variables bias. We discuss this possibility more in the context of the empirical results.

To derive the set of observationally equivalent models, let $X_t = (\Delta y_t, \Delta m_t)'$, and stack (4a) – (4b) as

$$X_t = \Theta(L)\epsilon_t, \tag{6}$$

where $\epsilon_t = (\epsilon_t^{\eta}\epsilon_t^m)'$ is the 2×1 vector of structural disturbances. Assume that

[4] Standard references are Blanchard and Watson (1986), Bernanke (1986), and Sims (1986). See Watson (1994) for a survey.

$|\Theta(z)|$ has all of its zeros outside the unit circle, so that $\Theta(L)$ can be inverted to yield the VAR: [5]

$$\alpha(L)X_t = \epsilon_t, \tag{7}$$

where $\alpha(L) = \Sigma_{j=0}^{\infty}\alpha_j L^j$, with α_j a 2×2 matrix. Unstacking the Δy_t and Δm_t equations yields

$$\Delta y_t = \lambda_{ym}\Delta m_t + \sum_{j=1}^{p}\alpha_{j,yy}\Delta y_{t-j} + \sum_{j=1}^{p}\alpha_{j,ym}\Delta m_{t-j} + \epsilon_t^{\eta} \text{ and} \tag{8a}$$

$$\Delta m_t = \lambda_{my}\Delta y_t + \sum_{j=1}^{p}\alpha_{j,my}\Delta y_{t-j} + \sum_{j=1}^{p}\alpha_{j,mm}\Delta m_{t-j} + \epsilon_t^{m}, \tag{8b}$$

which is written under the assumption that the VAR in (7) is of order p.

Equation (7) or equivalently equations (8a) and (8b) are a set of dynamic simultaneous equations, and econometric identification can be studied in the usual way. Writing $\Sigma_\epsilon = E(\epsilon_t\epsilon_t')$, the reduced form of (7) is

$$X_t = \sum_{i=1}^{p}\Phi_i X_{t-i} + e_t, \tag{9}$$

where $\Phi_i = -\alpha_0^{-1}\alpha_i$ and $e_t = \alpha_0^{-1}\epsilon_t$. The matrices α_i and Σ_ϵ are determined by the set of equations

$$\alpha_0^{-1}\alpha_i = -\Phi_i, i = 1, \ \ldots, \ p \text{ and} \tag{10}$$

$$\alpha_0^{-1}\Sigma_\epsilon\alpha_0^{-1\prime} = \Sigma_e = E(e_t e_t'). \tag{11}$$

When there are no restrictions on coefficients on lags entering (9), equation (10) imposes no restrictions on α_0; it serves to determine α_i as a function of α_0 and Φ_i. Equation (11) determines both α_0 and Σ_ϵ as a function of Σ_e. Since Σ_e (a 2×2 symmetric matrix) has only three unique elements, only three unknown parameters in α_0 and Σ_ϵ can be identified. Equations (8a) and (8b) place 1s on the diagonal of α_0, but evidently only three of the remaining parameters $\text{var}(\epsilon_t^m), \text{var}(\epsilon_t^\eta), \text{cov}(\epsilon_t^m, \epsilon_t^\eta), \lambda_{my}$ and λ_{ym} can be identified. We follow the standard practice in structural VAR analysis and assume that the structural shocks are uncorrelated. Since λ_{my} and λ_{ym} are allowed to be nonzero, the assumption places no restriction on the contemporaneous correlation between y and m. Moreover, nonzero values of λ_{my} and λ_{ym} allow both y and m to respond ϵ^m and ϵ^η shocks within the period. With the assumption that $\text{cov}(\epsilon_t^m, \ \epsilon_t^\eta) = 0$, only one additional identifying restriction is required.

Where might this additional restriction come from? One approach is to assume that the model is recursive, so that either $\lambda_{my} = 0$ or $\lambda_{ym} = 0$. Geweke (1986), Stock and Watson (1988), Rotemberg, Driscoll, and Poterba (1995), and Fisher and Seater (1993) present tests for neutrality under the assumption

[5] The unit roots discussion of Section 1 is important here, since the invertability of $\Theta(L)$ requires that $\Theta(1)$ has full rank. This implies that y_t and m_t are both integrated processes, and (y_t, m_t) are not cointegrated.

that $\lambda_{ym} = 0$; Geweke (1986) also presents results under the assumption that $\lambda_{my} = 0$. Alternatively, neutrality might be *assumed*, and the restriction $\gamma_{ym} = 0$ used to identify the model. This assumption has been used by Gali (1992), by King, Plosser, Stock, and Watson (1991), by Shapiro and Watson (1988), and by others to disentangle the structural shocks ϵ_t^m and ϵ_t^η. Finally, an assumption such as $\gamma_{my} = 1$ might be used to identify the model; this assumption is consistent with long-run price stability under the assumption of stable velocity.

The approach that we take in the empirical section is more eclectic and potentially more informative. Rather than report results associated with a single identifying restriction, we summarize results for a wide range of observationally equivalent estimated models. This allows the reader to gauge the robustness of conclusions about γ_{ym} and long-run neutrality to specific assumptions about $\lambda_{ym}, \lambda_{my}$, or γ_{my}. Our method is in the spirit of robustness calculations carried out by sophisticated users of structural VARs such as Sims (1989) and Blanchard (1989).

3. EVIDENCE ON THE NEUTRALITY PROPOSITIONS IN THE POSTWAR U.S. ECONOMY

While our discussion has focused on the long-run neutrality of money, we can test a range of related long-run neutrality propositions by varying the definition X_t in equation (7). As we have shown, using $X_t = (\Delta y_t, \Delta m_t)'$, with m_t assumed to follow an I(1) process, the model can be used to investigate the neutrality of money. If the process describing m_t is I(2) rather than I(1), then the framework can be used to investigate superneutrality by using $X_t = (\Delta y_t, \Delta^2 m_t)'$.[6] In economies in which rate of inflation, π_t, and the nominal interest rate, R_t, follow integrated processes, then we can study the long-run effect of inflation on real interest rates by setting $X_t = (\Delta \pi_t, \Delta R_t)'$. Finally, if both the inflation rate and the unemployment rate are I(1), then the slope of the long-run Phillips curve can be investigated using $X_t = (\Delta \pi_t, \Delta u_t)$.

We investigate these four long-run neutrality hypotheses using postwar quarterly data for the United States. We use gross national product for output;

[6] Long-run neutrality cannot be tested in a system in which output is I(1) and money is I(2). Intuitively this follows because neutrality concerns the relationship between shocks to the level of money and to the level of output. When money is I(2), shocks affect the rate of growth of money, and there are no shocks to the level of money. To see this formally, write equation (8a) as

$$\begin{aligned}\alpha_{yy}(L)\Delta y_t &= \alpha_{ym}(L)\Delta m_t + \epsilon_t^\eta \\ &= \alpha_{ym}(1)\Delta m_t + \alpha_{ym}^*(L)\Delta^2 m_t + \epsilon_t^\eta,\end{aligned}$$

where $\alpha_{ym}^*(L) = (1 - L)^{-1}[\alpha_{ym}(L) - \alpha_{ym}(1)]$. When money is I(1), the neutrality restriction is $\alpha_{ym}(1) = 0$. But when money is I(2) and output is I(1), $\alpha_{ym}(1) = 0$ by construction. (When $\alpha_{ym}(1) \neq 0$, output is I(2).) For a more detailed discussion of neutrality restrictions with possibly different orders of integration, see Fisher and Seater (1993).

money is M2; unemployment is the civilian unemployment rate; price inflation is calculated from the consumer price index; and the nominal interest rate is the yield on three-month Treasury bills.[7]

Since the unit root properties of the data play a key role in the analysis, Table 1 presents statistics describing these properties of the data. We use two sets of statistics: (i) augmented Dickey-Fuller (ADF) t-statistics and (ii) 95 percent confidence intervals for the largest autoregressive root. (These were constructed from the ADF statistics using Stock's [1991] procedure.)

The ADF statistics indicate that unit roots cannot be rejected at the 5 percent level for any of the series. From this perspective, output (y_t), money (m_t), money growth (Δm_t), inflation (π_t), unemployment (u_t), and nominal interest rates (R_t) all can be taken to possess the nonstationarity necessary for investigating long-run neutrality using the final form (7). Moreover, a unit root cannot be rejected for $r_t = R_t - \pi_t$, consistent with the hypothesis that R_t and π_t are not cointegrated.

However, the confidence intervals are very wide, suggesting a large amount of uncertainty about the unit root properties of the data. For example, the real GNP data are consistent with the hypothesis that the process is I(1), but also are consistent with the hypothesis that the data are trend stationary with an autoregressive root of 0.89. The money supply data are consistent with the trend stationary, I(1) and I(2) hypotheses. The results in Table 1 suggest that while it is reasonable to carry an empirical investigation of the neutrality propositions predicated on integrated processes, as is usual in models with unit root identifying restrictions, the results must be interpreted with some caution.

Our empirical investigation centers around the four economic interpretations of equation (7) discussed above. For each interpretation, we estimate the model using the following identifying assumptions:

(i) α_0 has 1s on the diagonal,

(ii) Σ_ϵ is diagonal,

and, defining $X_t = (x_t^1 \; x_t^2)$, one of the following:

(iii.a) the impact elasticity x^1 with respect to x^2 is known (e.g., λ_{ym} is known in the money-output system),

[7] Data sources: Output: Citibase series GNP82 (real GNP). Money: The monthly Citibase M2 series (FM2) was used for 1959–1989; the earlier M1 data were formed by splicing the M2 series reported in Banking and Monetary Statistics, 1941–1970, Board of Governors of the Federal Reserve System, to the Citibase data in January 1959. Inflation: Log first differences of Citibase series PUNEW (CPI-U: All Items). Unemployment Rate: Citibase Series LHUR (Unemployment rate: all workers, 16 years and over [percent, sa]). Interest Rate: Citibase series FYGM3 (yield on three-month U.S. Treasury bills). Monthly series were averaged to form the quarterly data.

Table 1 Unit Root Statistics

Variable	ADF $\hat{\tau}^{\tau}$	ADF $\hat{\tau}\mu$	95 Percent Confidence Intervals for ρ Detrended Data	Demeaned Data
y_t	-2.53	—	(0.89 1.02)	—
m_t	-2.40	—	(0.90 1.03)	—
Δm_t	-2.76	-2.90	(0.86 1.02)	(0.84 1.01)
π_t	-3.27	-2.86	(0.81 1.02)	(0.84 1.02)
u_t	-3.35	-2.34	(0.81 1.01)	(0.89 1.02)
R_t	-3.08	-1.87	(0.84 1.02)	(0.92 1.02)
r_t	-3.34	-2.94	(0.82 1.02)	(0.85 1.01)

Notes: The regressions used to calculate the ADF statistics included six lagged differences of the variable. All regressions were carried out over the period 1949:1 to 1990:4 using quarterly data except those involving u_t, which began in 1950:1. The variables y_t, m_t are the logarithms of output and money multiplied by 400, so that their first differences represent rates of growth at annual rates; similarly, π_t represents price inflation at an annual rate. The 95 percent confidence intervals were based on the ADF statistics using the procedure developed in Stock (1991).

(iii.b) the impact elasticity of x^2 with respect to x^1 is known (e.g., λ_{my} is known in the money-output system),

(iii.c) the long-run elasticity of x^1 with respect to x^2 is known (e.g., γ_{ym} is known in the money-output system),

(iii.d) the long-run elasticity of x^2 with respect to x^1 is known (e.g., γ_{my} is known in the money-output system).

The models are estimated using simultaneous equation methods. The details are provided in the appendix, but the basic strategy is quite simple and we describe it here using the money-output system. If λ_{ym} in (8a) were known, then the equation could be estimated by regressing $\Delta y_t - \lambda_{ym}\Delta m_t$ onto the lagged values of the variables in the equation. However, the money supply equation (8b) cannot be estimated by ordinary least squares regression since it contains Δy_t, which is potentially correlated with the error term. The maximum likelihood estimator of this equation is constructed by instrumental variables, using the residual from the estimated output supply equation together with lags of Δm_t and Δy_t as instruments. The residual is a valid instrument because of assumption (ii). In the appendix we show how a similar procedure can be used when assumptions (iii.b) – (iii.d) are maintained. Formulae for the standard errors of the estimators are also provided in the appendix.

We report results for a wide range of values of the parameters in assumptions (iii.a) – (iii.d). All of the models include six lags of the relevant variables. The sample period is 1949:1–1990:4 for the models that did not include the unemployment rate; when the unemployment rate was included in the model, the sample period is 1950:1–1990:4. Data prior to the initial periods were used as lags in the regressions. The robustness of the results to choice of lag length and sample period is discussed below. We now discuss the empirical evidence on the four long-run neutrality propositions.

Neutrality of Money

Figure 1 plots the estimates of the stochastic trends or permanent components in output and money. These were computed as the multivariate Beveridge-Nelson (1981) trends from the estimated bivariate VAR. Also shown in the graph are the NBER business cycle peak and trough dates. Changes in these series at a given date represent changes in the long-run forecasts of output and money associated with the VAR residuals at that date.[8] A scatterplot of these residuals, or innovations in the stochastic trends, is shown in Figure 2. The simple correlation between these innovations is -0.25. Thus, money and output appear to have a negative long-run correlation, at least over this sample period. The important question is the direction of causation explaining this correlation. Simply put, does money cause output or vice versa? This question cannot be answered without an identifying restriction, and we now present results for a range of different identifying assumptions.

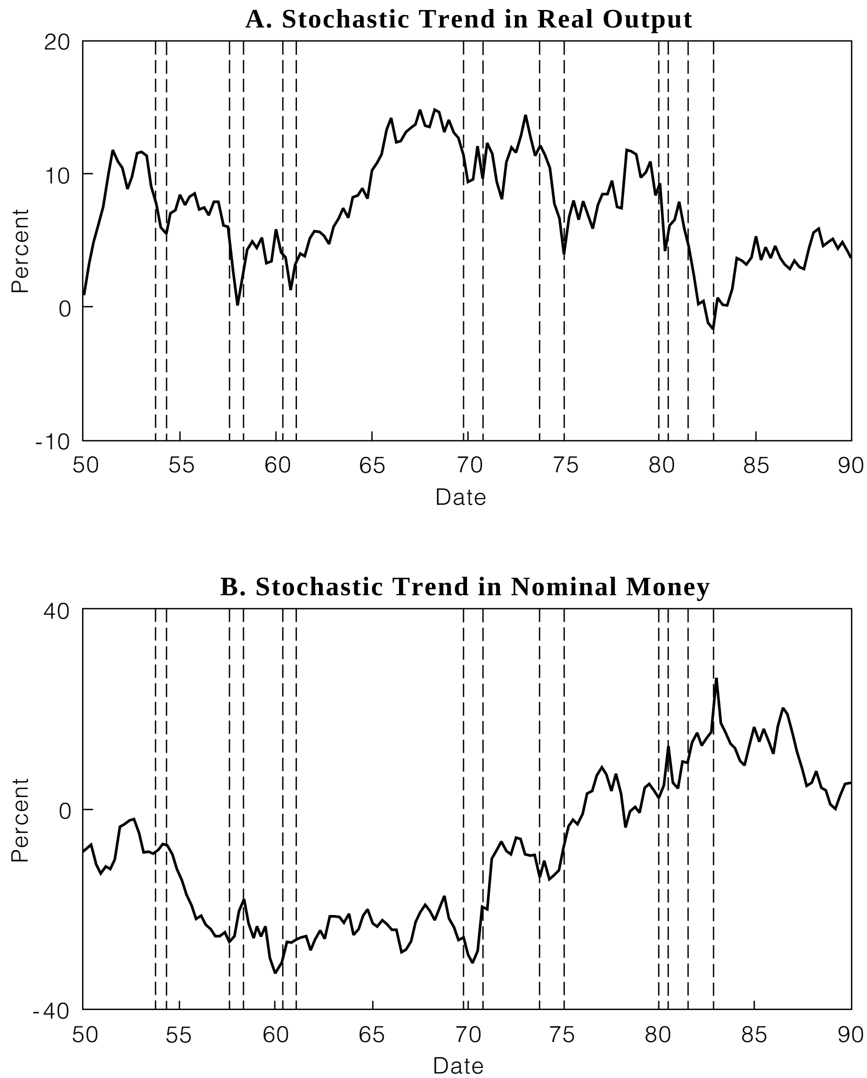
Since we estimate the final form (7) using literally hundreds of different identifying assumptions, there is a tremendous amount of information that can potentially be reported. In Figure 3 we summarize the information on long-run neutrality. Figure 3 presents the point estimates and 95 percent confidence intervals for γ_{ym} for a wide range of values of λ_{my} (panel A), λ_{ym} (panel B), and γ_{my} (panel C). Long-run neutrality is not rejected at the 5 percent level if $\gamma_{ym} = 0$ is contained in the 95 percent confidence interval. For example, from panel A, when $\lambda_{my} = 0$, the point estimate for γ_{ym} is 0.23 and the 95 percent confidence interval is $-0.18 \leq \gamma_{ym} \leq 0.64$. Thus, when $\lambda_{my} = 0$, the data do not reject the long-run neutrality hypothesis. Indeed, as is evident from the figure, long-run neutrality cannot be rejected at the 5 percent level for any value of $\lambda_{my} \leq 1.40$. Thus, the interpretation of the evidence on long-run neutrality depends critically on the assumed value of λ_{my}.

The precise value of λ_{my} depends on the money supply process. For example, if the central bank's reserve position is adjusted to smooth interest rates, then m_t will adjust to accommodate shifts in money demand arising from changes in y_t. In this case, λ_{my} corresponds to the short-run elasticity of money demand, and a reasonable range of values is $0.1 \leq \lambda_{my} \leq 0.6$. For all values of λ_{my} in this range, the null hypothesis of long-run neutrality cannot be rejected.

Panel B of Figure 3 shows that long-run neutrality is not rejected for values of $\lambda_{ym} > -4.61$. Since traditional monetary models of the business cycle imply

[8] Because the VAR residuals sum to zero over the entire sample, the trends are constrained to equal zero in the final period. In addition, they are normalized to equal zero in the initial period. This explains their "Brownian Bridge" behavior.

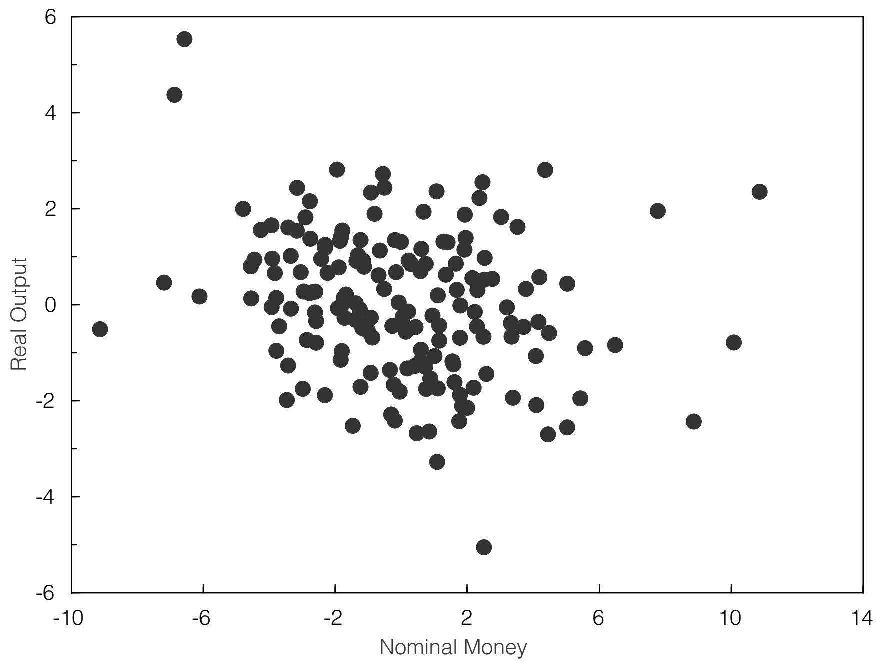
Figure 1 Stochastic Trends



that $\lambda_{ym} \geq 0$—output does not decline on impact in response to a monetary expansion—the results in panel B again suggest that the data are consistent with the long-run neutrality hypothesis.

Finally, the results in panel C suggest that the long-run neutrality hypothesis cannot be rejected for the entire range of values γ_{my} shown in Figure 3. To interpret the results in this figure, recall that γ_{my} represents the long-run response

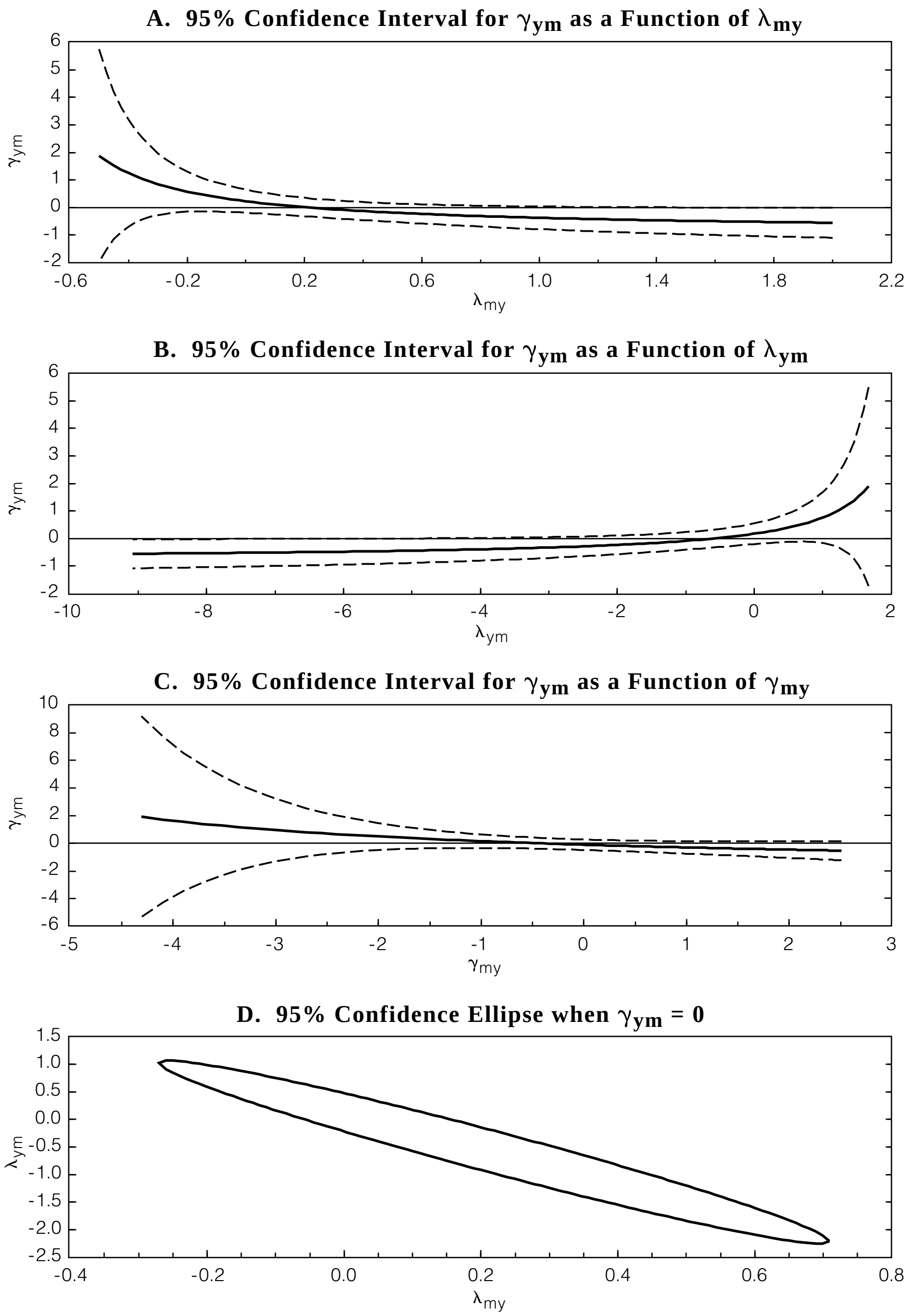
Figure 2 Innovations in Stochastic Trends



of m_t to exogenous permanent shifts in the level of y_t. If (M2) velocity is reasonably stable over long periods, then price stability would require $\gamma_{my} = 1$. Consequently, values of $\gamma_{my} < 1$ represent long-run deflationary policies and $\gamma_{my} > 1$ represent long-run inflationary policies. Thus, when $\gamma_{my} = 1 + \delta$, the long-run level of prices increase by δ percent when the long-run level of output increases by 1 percent. In the figure we show that long-run neutrality cannot be rejected for values of γ_{my} as large as 2.5; we have estimated the model using values of γ_{my} as large as 5.7 and found no rejections of the long-run neutrality hypothesis.

An alternative way to interpret the evidence from panels A – C of Figure 3 is to use long-run neutrality as an identifying restriction and to estimate the other parameters of the model. From the figure, when $\gamma_{ym} = 0$, the point estimates are $\hat{\lambda}_{my} = 0.22$, $\hat{\lambda}_{ym} = -0.59$, and $\hat{\gamma}_{my} = -0.51$, and the implied 95 percent confidence intervals are $-0.18 \leq \lambda_{my} \leq 0.62$, $-1.93 \leq \lambda_{ym} \leq 0.74$, and $-2.1 \leq \gamma_{my} \leq 1.06$. By definition, these intervals contain the true values of λ_{my}, λ_{ym}, and γ_{my} 95 percent of the time, if long-run neutrality is true. Thus, if the confidence intervals contain only nonsensical values of these parameters, then this provides evidence against long-run neutrality. We find that the

Figure 3 Money and Output



confidence intervals include many reasonable values of the parameters and conclude that they provide little evidence against the neutrality hypothesis.

Multivariate confidence intervals can also be constructed. Panel D of Figure 3 provides an example. It shows the 95 percent confidence ellipse for $(\lambda_{my},\ \lambda_{ym})$ constructed under the assumption of long-run neutrality.[9] If long-run neutrality holds, then 95 percent of the time this ellipse will cover the true values of the pair $(\lambda_{ym},\ \lambda_{my})$. Thus, if reasonable values for the pair of parameters are not included in this ellipse, then this provides evidence against long-run neutrality.

Table 2 summarizes selected results for variations in the specification. The VAR lag length (6 in the results discussed above) is varied between 4 and 8, and the model is estimated over various subsamples. Overall, the table suggests that the results are robust to these changes in the specification.[10]

These conclusions are predicated on the two-shock model that forms the basis of the bivariate specification. That is, the analysis is based on the assumption that money and output are driven by only two structural disturbances, here interpreted as a monetary shock and a real shock. This is clearly wrong, as there are many sources of real shocks (productivity, oil prices, tax rates, etc.) and nominal shocks (factors affecting both money supply and money demand). However, deducing the effects of these omitted variables on the analysis is difficult, since what matters is both the relative variability of these different shocks and their different dynamic effects on y and m. Indeed, as shown in Blanchard and Quah (1989), a two-shock model will provide approximately correct answers if the dynamic responses of y and m to shocks with large relative variances are sufficiently similar.

Superneutrality of Money

Evidence on the superneutrality of money is summarized in Figure 4 and in panel B of Table 2. Figure 4 is read the same way as Figure 3, except that now the experiment involves the effects of changes in the rate of growth of

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[9] This confidence ellipse is computed in the usual way. For example, see Johnston (1984), p. 190.

[10] These results are not robust to certain other changes in the specification. For example, Rotemberg, Driscoll, and Poterba (1995) report results using monthly data on M2 and U.S. Industrial Production (IP) for a specification that includes a linear time trend, 12 monthly lags, and is econometrically identified using the restriction that $\lambda_{my} = 0$. These authors report an estimate of $\gamma_{ym} = 1.57$ that is significantly different from zero and thus reject long-run neutrality. Stock and Watson (1988) report a similar finding using monthly data on IP and M1. The sample period and output measure seems to be responsible for the differences between these results and those reported here. For example, assuming $\lambda_{ym} = 0$ and using quarterly IP and M2 results in estimated values of γ_{ym} of 0.43 (0.31) using data from 1949:1 to 1990:4. (The standard error of the estimate is shown in parentheses.) As in Table 2, when the sample is split and the model estimated over the period 1949:1 to 1972:4 and 1973:1 to 1990:4, the resulting estimates are 0.56 (0.37) and 1.32 (0.70). Thus, point estimates of γ_{ym} are larger using IP in place of real GNP, and tend to increase in the second half of the second period.

Table 2 Robustness to Sample Period and Lag Length

A. Neutrality of Money
$X_t = (\Delta m_t,\ \Delta y_t)'$

Sample Period	Lag Length	Estimates of γ_{ym} when $\lambda_{my} = 0$		$\lambda_{ym} = 0$		$\gamma_{my} = 0$	
1949–1990	6	0.23	(0.21)	0.17	(0.19)	−0.12	(0.19)
1949–1972	6	0.15	(0.24)	0.13	(0.24)	0.04	(0.27)
1973–1990	6	0.77	(0.47)	0.65	(0.37)	0.02	(0.25)
1949–1990	4	0.24	(0.17)	0.20	(0.15)	−0.04	(0.17)
1949–1990	8	0.12	(0.19)	0.07	(0.17)	−0.18	(0.18)

B. Superneutrality of Money
$X_t = (\Delta^2 m_t,\ \Delta y_t)'$

Sample Period	Lag Length	Estimates of $\gamma_{y,\Delta m}$ when $\lambda_{\Delta m,y} = 0$		$\lambda_{y,\Delta m} = 0$		$\gamma_{\Delta m,y} = 0$	
1949–1990	6	3.80	(1.74)	3.12	(1.36)	−0.95	(1.57)
1949–1972	6	3.50	(1.66)	3.32	(1.49)	1.67	(1.99)
1973–1990	6	4.02	(4.57)	2.65	(2.62)	−4.11	(1.14)
1949–1990	4	1.81	(0.90)	1.31	(0.63)	−1.55	(0.97)
1949–1990	8	3.94	(1.81)	3.43	(1.53)	0.10	(1.66)

C. Long-Run Fisher Effect
$X_t = (\Delta \pi_t,\ \Delta R_t)'$

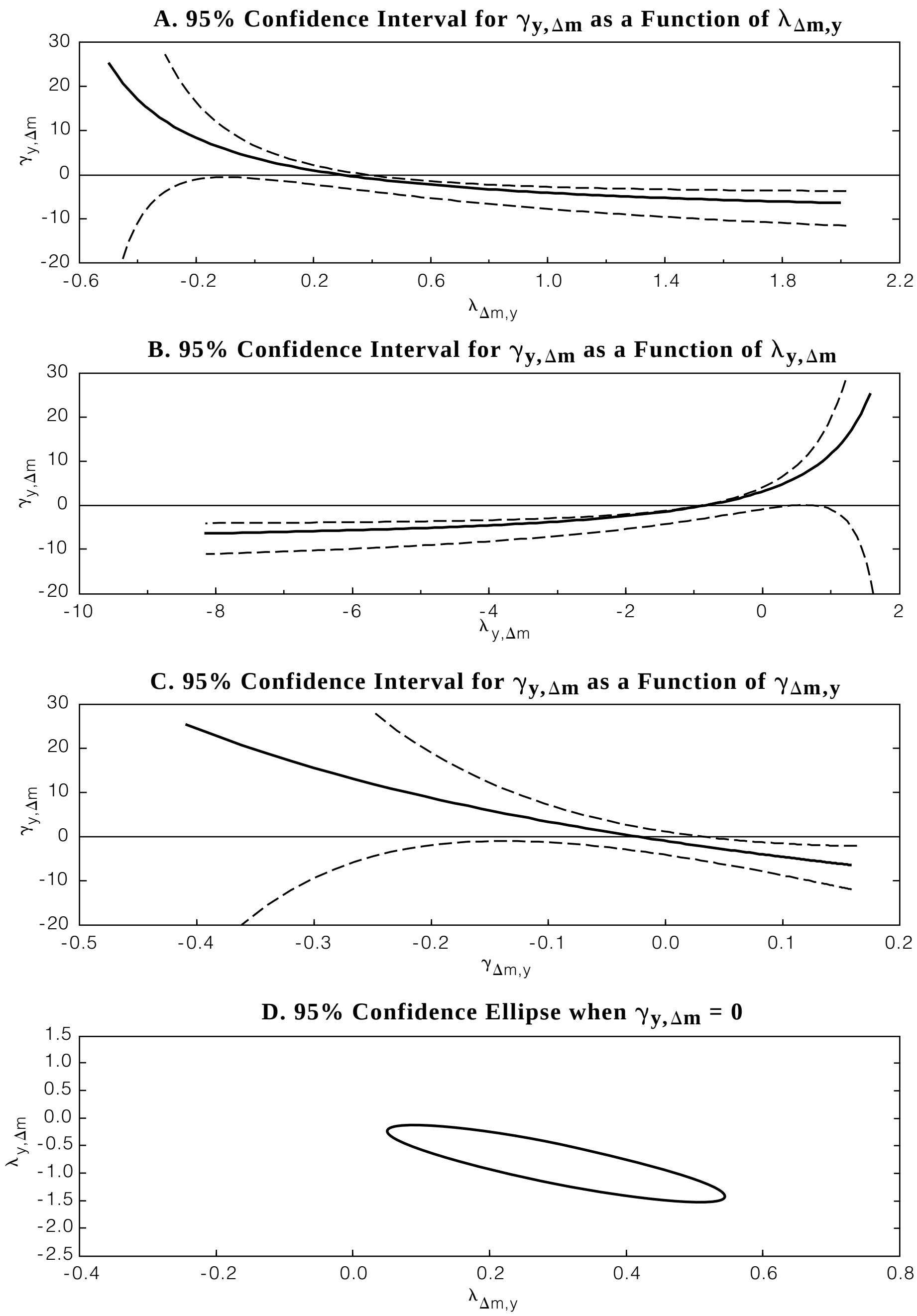
Sample Period	Lag Length	Estimates of $\gamma_{R\pi}$ when $\lambda_{\pi R} = 0$		$\lambda_{R\pi} = 0$		$\gamma_{\pi R} = 0$	
1949–1990	6	0.18	(0.09)	0.08	(0.08)	0.34	(0.12)
1949–1972	6	0.04	(0.06)	0.03	(0.05)	0.07	(0.09)
1973–1990	6	0.40	(0.16)	0.23	(0.18)	0.53	(0.20)
1949–1990	4	0.15	(0.07)	0.07	(0.06)	0.28	(0.09)
1949–1990	8	0.26	(0.09)	0.14	(0.08)	0.39	(0.13)

D. Long-Run Phillips Curve
$X_t = (\Delta \pi_t,\ \Delta u_t)'$

Sample Period	Lag Length	Estimates of $\gamma_{u\pi}$ when $\lambda_{\pi u} = 0$		$\lambda_{u\pi} = 0$		$\gamma_{\pi u} = 0$	
1950–1990	6	0.03	(0.09)	0.06	(0.09)	−0.17	(0.11)
1950–1972	6	−0.04	(0.10)	−0.03	(0.09)	−0.07	(0.14)
1973–1990	6	0.29	(0.35)	0.51	(0.56)	−0.21	(0.16)
1950–1990	4	−0.03	(0.06)	−0.00	(0.05)	−0.18	(0.07)
1950–1990	8	0.08	(0.09)	0.12	(0.09)	−0.11	(0.10)

Note: Standard errors are shown in parentheses.

Figure 4 Money Growth and Output



money, so that the parameters are $\lambda_{\Delta m,y}$, $\lambda_{y,\Delta m}$, $\gamma_{\Delta m,y}$, and $\gamma_{y,\Delta m}$. There are two substantive conclusions to be drawn from the table and figure.

The first conclusion is that it is possible to find evidence against superneutrality. For example, superneutrality is rejected at the 5 percent level for all values of $\lambda_{\Delta m,y}$ between -0.25 and 0.08, and for all values of $\lambda_{y,\Delta m}$ between -0.26 and 1.02. On the other hand, the figures suggest that these rejections are marginal, and the rejections are not robust to all of the lag-length and sample-period specification changes reported in Table 2. Moreover, a wide range of (arguably) reasonable identifying restrictions lead to the conclusion that superneutrality cannot be rejected. For example, superneutrality is not rejected for any value of $\lambda_{\Delta m,y}$ in the interval 0.08 to 0.53. Because of the lags in the model, the impact multiplier $\lambda_{\Delta m,y}$ has the same interpretation as λ_{my} in the discussion of long-run neutrality, and we argued above that the interval (0.08, 0.53) was a reasonable range of values for this parameter. In addition, from panel C, superneutality cannot be rejected for values of $\gamma_{\Delta m,y} < 0.07$. To put this into perspective, note that $\gamma_{\Delta m,y}$ measures the long-run elasticity of rate of growth of money with respect to permanent changes in the level of output. Thus a value of $\gamma_{\Delta m,y} = 0$ corresponds to a non-accelerationist policy.

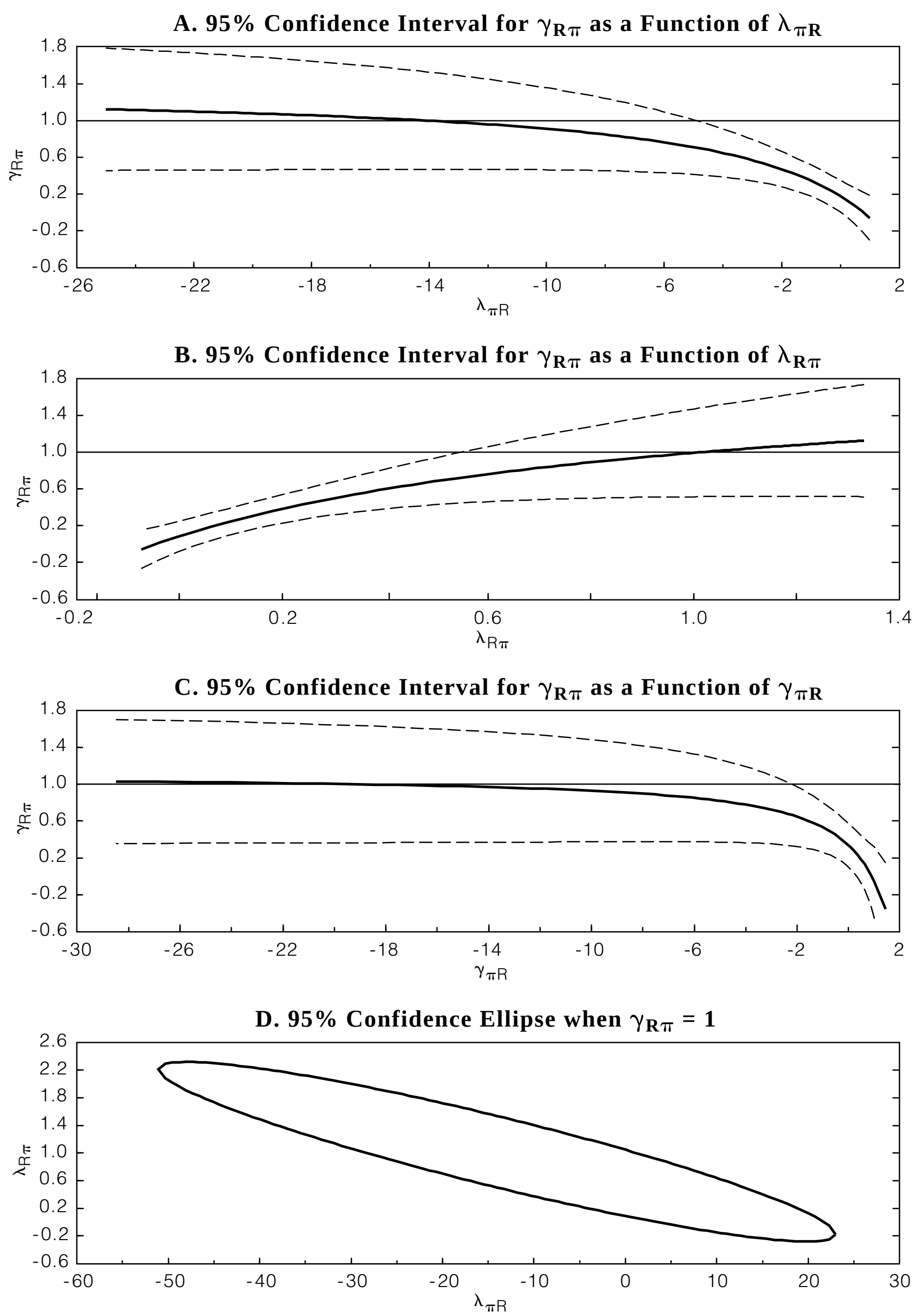
The second substantive conclusion is that the identifying assumption has a large effect on the sign and the magnitude of the estimated value of $\gamma_{y,\Delta m}$. For example, when $\lambda_{\Delta m,y} = 0$ the estimated value of $\gamma_{y,\Delta m}$ is 3.8. Thus, a 1 percent permanent increase in the money growth rate is estimated to increase the flow of output by 3.8 percent per year in perpetuity. Our sense is that even those who believe that the Tobin (1965) effect is empirically important do not believe that it is this large. The estimated value of $\gamma_{y,\Delta m}$ falls sharply as $\lambda_{\Delta m,y}$ is increased, and $\hat{\gamma}_{y,\Delta m} = 0$ when $\lambda_{\Delta m,y} = 0.30$. For values of $\lambda_{\Delta m,y} > 0.30$, the point estimate of $\gamma_{y,\Delta m}$ is negative, consistent with the predictions of cash-in-advance models in which sustained inflation is a tax on investment activity (Stockman 1981) or on labor supply (Aschauer and Greenwood 1983 or Cooley and Hansen 1989).

The Fisherian Theory of Inflation and Interest Rates

In the Fisherian theory of interest, the interest rate is determined as the sum of a real component, r_t, and an expected inflation component $E_t\pi_{t+1}$. A related long-run neutrality proposition—also suggested by Fisher—is that the level of the real interest rate is invariant to permanent changes in the rate of inflation. If inflation is integrated, then this proposition can be investigated using our framework: when $X_t = (\Delta\pi_t, \Delta R_t)$, then permament changes in π_t will have no effect on real interest rates when $\gamma_{R\pi} = 1$.

We find mixed evidence against the classical Fisherian link between long-run components of inflation and nominal interest rates, interpreted here as $\gamma_{R\pi} = 1$. For example, from Figure 5, maintaining a positive value of either

Figure 5 Inflation and Nominal Rates



$\lambda_{\pi R}$ or $\gamma_{\pi R}$ leads to an estimate of $\gamma_{R\pi}$ that is significantly less than 1. A mechanical explanation of this finding is that the VAR model implies substantial volatility in trend inflation: the estimated standard deviation of the inflation trend is much larger (1.25) than that of nominal rates (0.75). Thus, to reconcile the data with $\gamma_{R\pi} = 1$, a large negative effect of nominal interest rates on inflation is required.

However, from panel B of the figure, $\gamma_{R\pi} = 1$ cannot be rejected for a value of $\lambda_{R\pi} > 0.55$. One way to interpret the $\lambda_{R\pi}$ parameter is to decompose the impact effect of π on R into an expected inflation effect and an effect on real rates. If π has no impact effect on real rates, so that only the expected inflation effect was present, then $\lambda_{R\pi} = \partial\pi_{t+1}/\partial\epsilon_t^{\pi}$. For our data, $\partial\pi_{t+1}/\partial\epsilon_t^{\pi} = 0.6$ when the model is estimated using $\lambda_{R\pi} = 0.6$ as an identifying restriction, suggesting that this is a reasonable estimate of the expected inflation effect. The magnitude of the real interest effect is more difficult to determine since different macreconomic models lead to different conclusions about the effect of nominal shocks on real rates. For example, models with liquidity effects imply that real rates fall (e.g., Lucas [1990], Fuerst [1992], and Christiano and Eichenbaum [1994]), while the sticky nominal wage and price models in King (1994) imply that real rates rise. In this regard, the interpretation of the evidence on the long-run Fisher effect is seen to depend critically on one's belief about the impact effect of a nominal disturbance on the real interest rate. If this effect is negative, then there is significant evidence in the data against this neutrality hypothesis.

The confidence intervals suggest that the evidence against the long-run Fisher relation is not overwhelming. When $\gamma_{R\pi} = 1$ is maintained, the implied confidence intervals for the other parameters are wide ($-43.7 \leq \lambda_{\pi R} \leq 15.6, 0.0 \leq \lambda_{R\pi} \leq 2.1, -154.8 \leq \gamma_{\pi R} \leq 116.4$) and contain what are arguably reasonable values of these parameters. This is also evident from the confidence ellipse in panel D of Figure 5.

One interpretation is that these results reflect the conventional finding that nominal interest rates do not adjust fully to sustained inflation in the postwar U.S. data. This result obtains for a wide range of identifying assumptions. One possible explanation is that the failure depends on the particular specification of the bivariate model that we employ, suggesting the importance of extending this analysis to multivariate models. Another candidate source of potential misspecification is cointegration between nominal rates and inflation. This is discussed in some detail in papers by Evans and Lewis (1993), Mehra (1995), and Mishkin (1992).[11]

[11] These authors suggest that real rates $R_t - \pi_t$ are I(0). Evans and Lewis (1993) and Mishkin (1992) find estimates suggesting that nominal rates do not respond fully to permanent changes in inflation and attribute this to a small sample bias associated with shifts in the inflation process. Mehra (1995) finds that permanent changes in interest rates do respond one-for-one with

4. EVIDENCE ON THE LONG-RUN PHILLIPS CURVE

As discussed in King and Watson (1994), the interpretation of the evidence on the long-run Phillips curve is more subtle than the other neutrality propositions.[12] Throughout this article we have examined neutrality by examining the long-run multiplier in equations relating real variables to nominal variables. This suggests examining the neutrality proposition embodied in the long-run Phillips curve using the equation

$$\alpha_{uu}(L)u_t = \alpha_{u\pi}(L)\pi_t + \epsilon_t^u. \tag{12}$$

Of course, as in Sargent (1976), equation (12) is one standard way of writing the Phillips curve.

Figure 6 shows estimates $\gamma_{u\pi}$ for a wide range of identifying assumptions. When the model is estimated using $\lambda_{\pi u}$ as an identifying assumption, a vertical Phillips curve ($\gamma_{u\pi} = 0$) is rejected when $\lambda_{\pi u} > 2.3$.[13] Thus, neutrality is rejected only if one assumes that positive changes in the unemployment rate have a large positive impact effect on inflation. From panel B of the figure, $\gamma_{u\pi} = 0$ is rejected for maintained values of $\lambda_{u\pi} < -0.07$. Since $\lambda_{u\pi}$ can be interpreted as the slope of the short-run (impact) Phillips curve, this figure shows the relationship between maintained assumptions and conclusions about short-run and long-run neutrality. The data are consistent with the pair of parameters $\lambda_{u\pi}$ and $\gamma_{u\pi}$ being close to zero; the data also are consistent with the hypothesis that these parameters are both less than zero. If short-run neutrality is maintained ($\lambda_{u\pi} = 0$), the estimated long-run effect of inflation on unemployment is very small ($\hat{\gamma}_{u\pi} = 0.06$). If long-run neutrality is maintained ($\gamma_{u\pi} = 0$), the estimated short-run effect of inflation on unemployment is very small ($\hat{\lambda}_{u\pi} = -0.02$). This latter result is consistent with the small estimated real effects of nominal disturbances found by King, Plosser, Stock, and Watson (1991), Gali (1992), and Shapiro and Watson (1988), who all used long-run neutrality as an identifying restriction.

Several researchers, relying on a variety of specifications and identifying assumptions, have produced estimates of the short-run Phillips curve slope. For example, Sargent (1976) estimates $\lambda_{u\pi}$ using innovations in population, money, and various fiscal policy variables as instruments. He finds an estimate of $\lambda_{u\pi} = -0.07$. Estimates of $\lambda_{u\pi}$ ranging from -0.07 to -0.18 can

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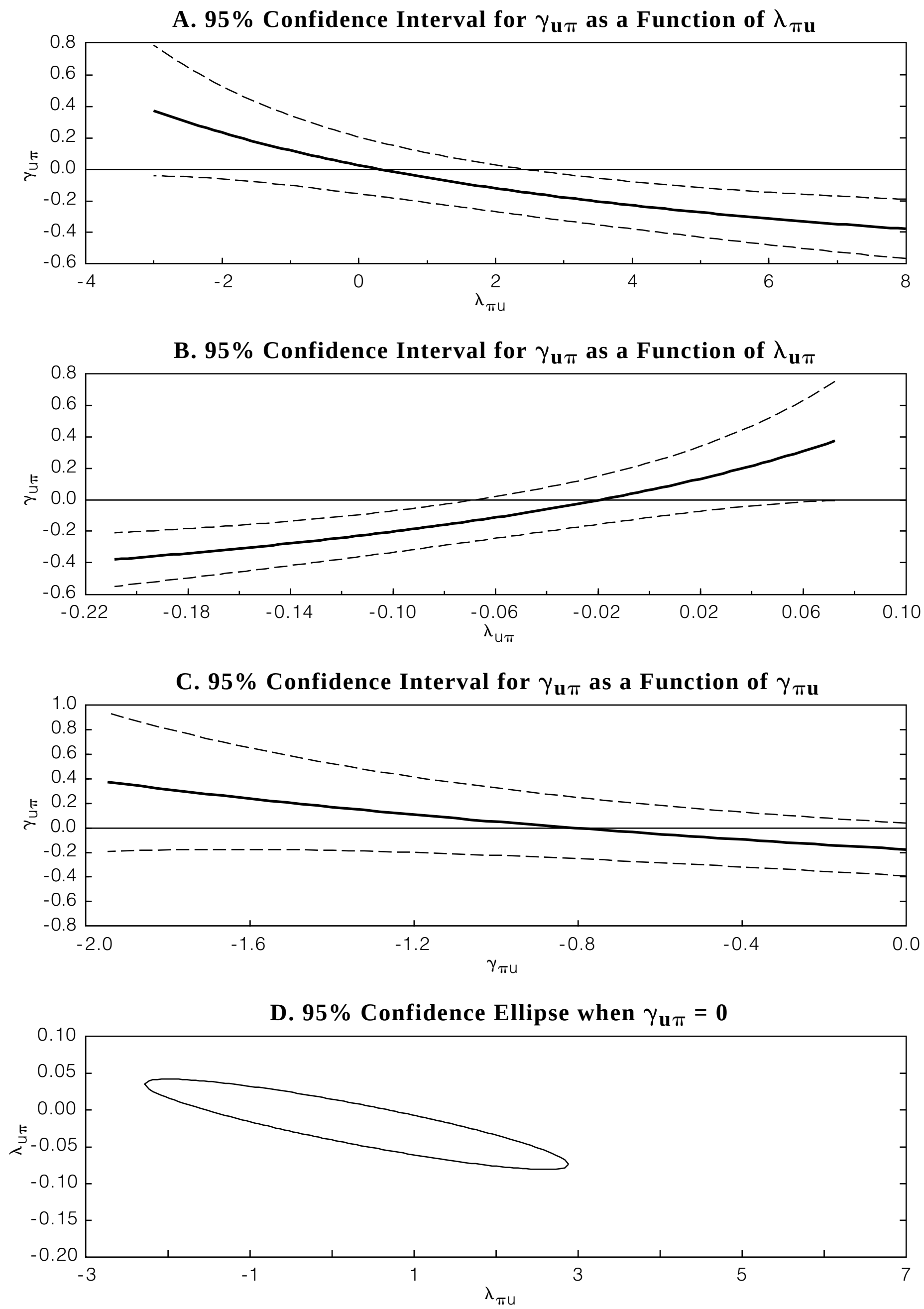
permanent changes in inflation. In contrast to these papers, our results are predicated on the assumption that π_t and R_t are I(1) and are not cointegrated over the entire sample. As the results in Table 1 make clear, both the I(0) and I(1) hypotheses are consistent with the data.

[12] A greatly expanded version of the analysis in this section is contained in King and Watson (1994).

[13] Recall that the Phillips curve is drawn with inflation on the vertical axis and unemployment on the horizontal axis. Thus, a vertical long-run Phillips curve corresponds to the restriction $\gamma_{u\pi} = 0$.

Figure 6 Inflation and Unemployment



be extracted from the results in Barro and Rush (1980), who estimated the unemployment and inflation effects of unanticipated money shocks. Values of $\lambda_{u\pi}$ in this range lead to a rejection of the null $\gamma_{u\pi} = 0$, but they suggest a very steep long-run tradeoff. For example, when $\lambda_{u\pi} = -0.10$, the corresponding point estimate of $\gamma_{u\pi} = -0.20$, so that the long-run Phillips curve has a slope of $-5.0 (= \gamma_{u\pi}^{-1})$.

By contrast, the conventional view in the late 1960s and early 1970s was that there was a much more favorable tradeoff between inflation and unemployment. For example, in discussing Gordon's famous (1970) test of an accelerationist Phillips curve model, Solow calculated that there was a one-for-one long-run tradeoff implied by Gordon's results. This calculation was sufficiently conventional that it led to no sharp discussion among the participants at the Brookings panel. Essentially the same tradeoff was suggested by the 1969 *Economic Report of the President*, which provided a graph of inflation and unemployment between 1954 and 1968.[14]

What is responsible for the difference between our estimates and the conventional estimates from the late '60s? Panel D in Table 2 suggests that sample period cannot be the answer: the full sample results are very similar to the results obtained using data from 1950 through 1972. Instead, the answer lies in differences between the identifying assumptions employed. The traditional Gordon-Solow estimate was obtained from a price equation of the form[15]

$$\alpha_{\pi\pi}(L)\pi_t = \alpha_{\pi u}(L)u_t + \epsilon_t^{\pi}. \tag{13}$$

The estimated slope of the long-run Phillips curve was calculated as $\gamma = \alpha_{\pi u}(1)/\alpha_{\pi\pi}(1)$. Thus, in the traditional Gordon-Solow framework, the long-run Phillips curve was calculated as the long-run multiplier from the inflation equation. In contrast, our estimate $(\gamma_{u\pi}^{-1})$ is calculated from the unemployment equation. The difference is critical, since it means that the two parameters represent responses to different shocks. Using our notation, the long-run multiplier from (13) is

$$\gamma_{\pi u} = \frac{\lim_{k\to\infty} \partial\pi_{t+k}/\partial\epsilon_t^u}{\lim_{k\to\infty} \partial u_{t+k}/\partial\epsilon_t^u},$$

while the inverse of the long-run multiplier from the unemployment equation (12) is

$$\gamma_{u\pi}^{-1} = \frac{\lim_{k\to\infty} \partial\pi_{t+k}/\partial\epsilon_t^{\pi}}{\lim_{k\to\infty} \partial u_{t+k}/\partial\epsilon_t^{\pi}}.$$

[14] See McCallum (1989, p. 180) for a replication and discussion of this graph.

[15] Equation (13) served as a baseline model for estimating the Phillips curve. Careful researchers employed various shift variables in the regression to capture the effects of demographic shifts on the unemployment rate and the effects of price controls on inflation. For our purposes, these complications can be ignored.

Thus, the traditional estimate measures the relative effect of shocks to unemployment, while our estimate corresponds to the relative effect of shocks to inflation. Figure 7 presents our estimates of $\gamma_{\pi u}$. Evidently, the Gordon-Solow value of $\gamma_{u\pi} = -1$ is consistent with a wide range of identifying restrictions shown in the figure.

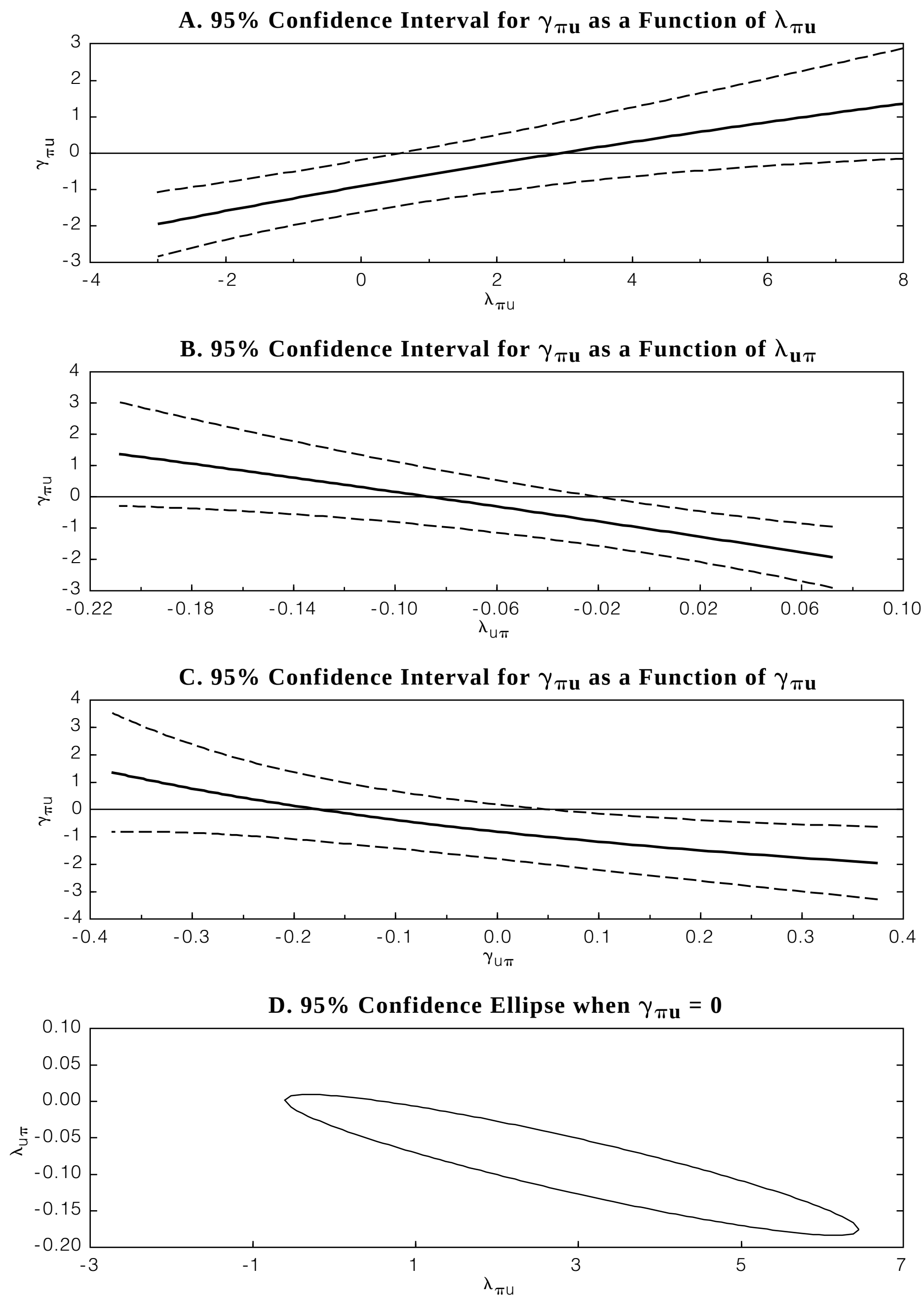
But the question is not whether the long-run multiplier is calculated from the unemployment equation, $\alpha_{uu}(L)u_t = \alpha_{u\pi}(L)\pi_t + \epsilon_t^u$, or from the inflation equation, $\alpha_{\pi\pi}(L)\pi_t = \alpha_{\pi u}(L)u_t + \epsilon_t^\pi$. By choosing between these two specifications under a specific identification scheme, one is also choosing a way of representing the experiment of a higher long-run rate of inflation, presumably originating from a higher long-run rate of monetary expansion. Under the Gordon-Solow procedure, the idea is that the shock to unemployment—the ϵ_t^u shock defined by a particular identifying restriction—is the indicator of a shift in aggregate demand. Its consequences are traced through the inflation equation since unemployment is the right-hand side variable in that equation. Under the Lucas-Sargent procedure, the idea is that the shock to inflation—the ϵ_t^π shock defined by a particular identifying restriction—is the indicator of a shift in aggregate demand.

To interpret the Gordon-Solow estimate of $\gamma_{\pi u}$ we must determine the particular identifying assumption that they used. Their assumption can be deduced from the way that they estimated $\gamma_{\pi u}$, namely from the ordinary least squares estimators of equation (13). Recall that OLS requires that the variables on the right-hand side of (13) are uncorrelated with the error term. Since u_t appears on the right-hand side of (13), this will be true only when $\lambda_{u\pi} = 0$. Thus, the particular identifying assumption employed in the Gordon-Solow specification in $\lambda_{u\pi} = 0$.

What does this identifying assumption mean? When $\lambda_{u\pi} = 0$, the Gordon-Solow interpretation implies that autonomous shocks to aggregate demand are one-step-ahead forecast errors in u_t. The other shocks in the system can affect prices on impact but cannot affect unemployment. Thus, in this sense, prices are flexible, since they can be affected on impact by all shocks, but unemployment is sticky, since it can be affected on impact only by aggregate demand shocks. For today's "new Keynesians" this may appear to be a very unreasonable identifying restriction (and so must any evidence about the Phillips curve that follows from it). However, the identifying restriction is consistent with the traditional Keynesian model of the late 1960s.[16]

[16] What we have in mind is a block recursive model in which the unemployment rate is determined in an IS-LM block, and wages and prices are determined in a wage-price block. This interpretation is further explored in King and Watson (1994).

Figure 7 Unemployment and Inflation



5. CONCLUDING REMARKS

We have investigated four long-run neutrality propositions using bivariate models and 40 years of quarterly observations. We conclude that the data contain little evidence against the long-run neutrality of money and suggest a very steep long-run Phillips curve. These conclusions are robust to a wide range of identifying assumptions. Conclusions about the long-run Fisher effect and the superneutrality of money are not robust to the particular identifying assumption. Over a fairly broad range of identifying restrictions, the data suggest that nominal interest rates do not move one-for-one with permanent shifts in inflation. The sign and magnitude of the estimated long-run effect of money growth on the level of output depends critically on the specific identifying restriction employed.

These conclusions are tempered by four important caveats. First, the results are predicated on specific assumptions concerning the degree of integration of the data, and with 40 years of data the degree of integration is necessarily uncertain. Second, even if the degree of integration were known, only limited "long-run" information is contained in data that span 40 years. This suggests that a useful extension of this work is to carry out similar analyses on long annual series. Third, the analysis has been carried out using bivariate models. If there are more than two important sources of macroeconomic shocks, then bivariate models may be subject to significant omitted variable bias. Thus another extension of this work is to expand the set of variables under study to allow a richer set of structural macroeconomic shocks. The challenge is to do this in a way that produces results that can be easily interpreted in spite of the large number of identifying restrictions required. Fourth, we have analyzed each of these propositions separately and yet there are obvious and important theoretical connections between them. Future work on multivariate extensions of this approach may allow for a unified econometric analysis of these long-run neutrality propositions.

APPENDIX

Estimation Methods

Under each alternative identifying restriction, the Gaussian maximum likelihood estimates can be constructed using standard regression and instrumental variable calculations. When λ_{ym} is assumed known, equation (8a) can be estimated by ordinary least squares by regressing $\Delta y_t - \lambda_{ym}\Delta m_t$ onto $\{\Delta y_{t-i}, \Delta m_{t-i}\}_{i=1}^{p}$. Equation (8b) cannot be estimated by OLS because Δy_t, one of the regressors, is potentially correlated with ϵ_t^m. Instrumental variables must be used. The appropriate instruments are $\{\Delta y_{t-i}, \Delta m_{t-i}\}_{i=1}^{p}$ together with the residual from the estimated (8a). This residual is a valid instrument because of the assumption that ϵ_t^η and ϵ_t^m are uncorrelated. When λ_{my} is assumed known, rather than λ_{ym}, this process was reversed.

When a value for γ_{my} is used to identify the model, a similar procedure can be used. First, rewrite (8b) as

$$\Delta m_t = \alpha_{my}(1)\Delta y_t + \beta_{mm}\Delta m_{t-1} + \sum_{j=0}^{p-1} \tilde{\alpha}_{my}^{j}\Delta^2 y_{t-j} + \sum_{j=0}^{p-1} \tilde{\alpha}_{mm}^{j}\Delta^2 m_{t-j} + \epsilon_t^m, \tag{A1}$$

where $\beta_{mm} = \sum_{j=1}^{p} \alpha_{mm}^{j}$. Equation (A1) replaces the regressors $(\Delta y_t, \Delta y_{t-1}, \ldots, \Delta y_{t-p}, \Delta m_{t-1}, \ldots, \Delta m_{t-p})$ in (8b) with the equivalent set of regressors $(\Delta y_t, \Delta m_{t-1}, \Delta^2 y_t, \Delta^2 y_{t-1}, \ldots, \Delta^2 y_{t-p+1}, \Delta^2 m_{t-1}, \ldots, \Delta^2 m_{t-p+1})$. In (A1), the long-run multiplier is $\gamma_{my} = \alpha_{my}(1)/(1 - \beta_{mm})$, so that $\alpha_{my}(1) = \gamma_{my} - \beta_{mm}\gamma_{my}$. Making this substitution, (A1) can be written as

$$\Delta m_t - \gamma_{my}\Delta y_t = \beta_{mm}(\Delta m_{t-1} - \gamma_{my}\Delta y_t) + \sum_{j=0}^{p-1} \tilde{\alpha}_{my}^{j}\Delta^2 y_{t-j} + \sum_{j=0}^{p-1} \tilde{\alpha}_{mm}^{j}\Delta^2 m_{t-j} + \epsilon_t^m. \tag{A2}$$

Equation (A2) can be estimated by instrumental variables by regressing $\Delta m_t - \gamma_{my}\Delta y_t$ onto $(\Delta m_{t-1} - \gamma_{my}\Delta y_t, \Delta^2 y_t, \Delta^2 y_{t-1}, \ldots, \Delta^2 y_{t-p+1}, \Delta^2 m_{t-1}, \ldots, \Delta^2 m_{t-p+1})$ using $\{\Delta y_{t-i}, \Delta m_{t-i}\}_{i=1}^{p}$ as instruments. (Instruments are required because of the potential correlation between Δy_t and the error term.) Equation (8a) can now be estimated by instrumental variables using the residual from the estimated (A2) together with $\{\Delta y_{t-i}, \Delta m_{t-i}\}_{i=1}^{p}$. When a value for γ_{ym} is used to identify the model, this process was reversed.

Two complications arise in the calculation of standard errors for the estimated models. The first is that the long-run multipliers, γ_{ym} and γ_{my}, are nonlinear functions of the regression coefficients. Their standard errors are calculated from standard formula derived from delta method arguments. The second complication arises because one of the equations is estimated using instruments that are residuals from another equation. This introduces the kind of "generated regressor" problems discussed in Pagan (1984). To see the problem in our context, notice that all of the models under consideration can be written as

$$y_t^1 = x_t^{1'}\delta_1 + \epsilon_t^1 \tag{A3}$$

$$y_t^2 = x_t^{2'}\delta_2 + \epsilon_t^2. \tag{A4}$$

Where, for example, when λ_{my} is assumed known, $y_t^1 = \Delta m_t - \lambda_{my}\Delta y_t, x_t^1$ represents the set of regressors $\{\Delta y_{t-i}, \Delta m_{t-i}\}_{i=1}^p, y_t^2 = \Delta y_t$, and x_t^2 represents the set of regressors $[\Delta m_t, \{\Delta y_{t-i}, \Delta m_{t-i}\}_{i=1}^p]$. Alternatively, when γ_{my} is assumed known, $y_t^1 = \Delta m_t - \gamma_{my}\Delta y_t, x_t^1$ represents the set of regressors $[\Delta m_{t-1} - \gamma_{my}\Delta y_t, \Delta^2 y_t, \{\Delta^2 y_{t-i}, \Delta^2 m_{t-i}\}_{i=1}^{p-1}]$, $y_t^2 = \Delta y_t$, and x_t^2 represents the set of regressors $[\Delta m_t, \{\Delta y_{t-i}, \Delta m_{t-i}\}_{i=1}^p]$.

Equations (A3) and (A4) allow us to discuss estimation of all the models in a unified way. First, (A3) is estimated using $z_t = \{\Delta y_{t-i}, \Delta m_{t-i}\}_{i=1}^p$ as instruments. Next, equation (A4) is estimated using $\hat{u}_t = (\hat{\epsilon}_t^1, z_t')$ as instruments, where $\hat{\epsilon}_t^1$ is the estimated residuals from (A3). If ϵ_t^1 rather than $\hat{\epsilon}_t^1$ is used as an instrument, standard errors could be calculated using standard formulae. However, when $\hat{\epsilon}_t^1$, an estimate of ϵ_t^1, is used, a potential problem arises. This problem will only effect the estimates in (A4) since $\hat{\epsilon}_t^1$ is not used as an instrument in (A3).

To explain the problem, some additional notation will prove helpful. Stack the observations for each equation so that the model can be written as

$$Y_1 = X_1\delta_1 + \epsilon_1 \tag{A5}$$

$$Y_2 = X_2\delta_2 + \epsilon_2, \tag{A6}$$

where Y_1 is $T \times 1$, etc. Denote the matrix of instruments for the first equation by Z, the matrix of instruments for the second equation by $\hat{U} = [\hat{\epsilon}_1 \; Z]$, and let $U = [\epsilon_1 \; Z]$. Since $\hat{\epsilon}_1 = \epsilon_1 - X_1(\hat{\delta}_1 - \delta_1), \hat{U} = U - [X_1(\hat{\delta}_1 - \delta_1) \; 0]$. Let $V_1 = \sigma_{\epsilon_1}^2$ plim $[T(Z'X_1)^{-1}(Z'Z)(X_1'Z)]$ denote the asymptotic covariance matrix of $T^{1/2}(\hat{\delta}_1 - \delta_1)$.

Now write,

$$\begin{aligned} T^{1/2}(\hat{\delta}_2 - \delta_2) &= (T^{-1}\hat{U}'X_2)^{-1}(T^{-1/2}\hat{U}'\epsilon_2) = (T^{-1}\hat{U}'X_2)^{-1}(T^{-1/2}U'\epsilon_2) \\ &\quad -(T^{-1}\hat{U}'X_2)^{-1}\begin{bmatrix} T^{1/2}(\hat{\delta}_1 - \delta_1)'(T^{-1}X_1'\epsilon_2) \\ 0 \end{bmatrix}. \end{aligned} \tag{A7}$$

It is straightforward to verify that plim $T^{-1}\hat{U}'\hat{U}$ = plim $T^{-1}U'U$ and that $T^{-1}\hat{U}'X_2$ = plim $T^{-1}U'X_2$. Thus, the first term on the right-hand side of (A7) is standard: it is asymptotically equivalent to the expression for $T^{1/2}(\hat{\delta}_2 - \delta_2)$ that would obtain if U rather than $\hat{U}$ were used as instruments. This expression converges in distribution to a random variable distributed as $N(0, \sigma^2_{\epsilon_2}$ plim $[T(\hat{U}'X_2)^{-1}(\hat{U}'\hat{U})(X_2'\hat{U})^{-1}])$, which is the usual expression for the asymptotic distribution of the IV estimator.

Potential problems arise because of the second term on the right-hand side of (A7). Since $T^{1/2}(\hat{\delta}_1 - \delta_1)$ converges in distribution, the second term can only be disregarded asymptotically when plim $T^{-1}X_1'\epsilon_2 = 0$, that is, when the regressors in (A3) are uncorrelated with the error terms in (A4). In our context, this will occur when λ_{my} and λ_{ym} are assumed known, since in this case x_t^1 contains only lagged variables. However, when γ_{my} or γ_{ym} are assumed known, x_t^1 will contain the contemporaneous value of Δy_t or Δm_t, and thus x_t^1 and ϵ_t^2 will be correlated. In this case the covariance matrix of $\hat{\delta}_2$ must be modified to account for the second term on the right-hand side of (A7).

The necessary modification is as follows. Standard calculations show that $T^{1/2}(\hat{\delta}_1 - \delta_1)$ and $T^{-1/2}U'\epsilon_2$ are asymptotically independent under the maintained assumption that $E(\epsilon_2|\epsilon_1) = 0$; thus, the two terms on the right-hand side of (A7) are asymptotically uncorrelated. A straightforward calculation demonstrates that $T^{1/2}(\hat{\delta}_2 - \delta_2)$ converges to a random variable with a $N(0, V_2)$ distribution where

$$V_2 = \sigma^2_{\epsilon_2} \text{ plim } [T(\hat{U}'X_2)^{-1}(\hat{U}'\hat{U})(X_2'\hat{U})^{-1}] \ + \ \text{plim } [T(\hat{U}'X_2)^{-1}D(X_2'\hat{U})^{-1}],$$

where D is a matrix with all elements equal to zero, except that $D_{11} = (\epsilon_2'X_1)TV_1(X_1'\epsilon_2)$, and where $TV_1 = \sigma^2_{\epsilon_1}(Z'X_1)^{-1}(Z'Z)(X_1'Z)^{-1}$. Similarly, it is straightforward to show that the asymptotic covariance between $T^{1/2}(\hat{\delta}_1 - \delta_1)$ and $T^{1/2}(\hat{\delta}_2 - \delta_2) = -\text{plim}[V_1(T^{-1}X_1'\epsilon_2)\ 0][T^{-1}X_2'\hat{U}]$.

An alternative to this approach is the GMM-estimator in Hausman, Newey, and Taylor (1987). This approach considers the estimation problem as a GMM problem with moment conditions $E(z_t\epsilon_t^1) = 0, E(z_t\epsilon_t^2) = 0$, and $E(\epsilon_t^1\epsilon_t^2) = 0$. The GMM approach is more general than the one we have employed, and when the errors terms are non-normal and the model is over-identified, it may produce more efficient estimates.

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