On the Aggregate Labor Supply

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Issues of labor supply are at the heart of macroeconomic studies of large cyclical fluctuations. The population puts forth more work effort in booms than in slumps. Economists' explanations of this phenomenon range from a pure market-clearing supply-and-demand view at one extreme to a dismissal of almost any role of supply and of market clearing at the other extreme. Disagreement is intense because labor markets' failure to clear may create a strong case favoring activist macroeconomic policy. According to the equilibrium business cycle models led by Lucas and Rapping (1969), people work more hours in some years than in others because the market rewards them for this pattern. Even in a non-equilibrium model in which the role of labor supply is dismissed in the short run, its slope is still important for the welfare cost of departing from the supply schedule. Labor supply elasticity is also crucial in evaluating the effect of taxes and government spending (e.g., Auerbach and Kotlikoff 1987; Judd 1987).

Figure 1 shows the cyclical components of total hours worked and wages for the U.S. economy for 1964:I–2003:II (detrended using the Hodrick-Prescott filter). Hours worked represent the total hours employed in the nonagricultural business sector. The wages are real hourly earnings of the production and nonsupervisory workers. Fluctuations of hours of work are much greater than those of wages.[1] If the intertemporal substitution hypothesis were to explain fluctuations in hours, it would require a labor supply elasticity beyond

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[1] Moreover, wages are not strongly correlated with hours, casting further doubt on the intertemporal substitution mechanism. While the contemporaneous and dynamic correlations between hours and wages are important for business cycle analysis, we focus on the slope of the labor supply schedule only in this article. See Chang and Kim (2004b) on this issue.

the admissible estimates from the empirical micro studies, which are typically less than 0.5.[2]

In this article, we demonstrate both qualitatively and quantitatively how the slope of the aggregate labor supply schedule is determined by the reservation wage distribution, rather than by the willingness to substitute leisure intertemporally.[3] Based on our recent studies (Chang and Kim 2004a, 2004b), we present a fully specified general equilibrium model economy where the reservation wage distribution is nondegenerate. While the model is parsimonious, it provides a laboratory in which we can investigate the mapping from individual to aggregate labor supply functions. The model economy is populated by many workers who face uninsurable idiosyncratic productivity shocks—as demonstrated in Aigagari's (1994) incomplete capital market—and make decisions on the labor market participation—as demonstrated by Rogerson's (1985) study of indivisible labor. The cross-sectional distributions of earnings and wealth are comparable to those in the U.S. data. We find that the aggregate labor supply elasticity of such an economy is around one, even though the intertemporal substitution elasticity of leisure at the individual level is assumed to be 0.4. This aggregate elasticity is greater than the typical micro estimates but smaller than those often assumed in the aggregate models.

The article is organized as follows: Section 1 provides various models of aggregate labor supply based on individuals' work decisions. Section 2 presents illustrative examples that demonstrate how the aggregate labor supply depends on the reservation wage distribution. Section 3 lays out the model economy where the reservation wage distribution is dispersed. In Section 4, we calibrate the model parameters using various microdata and investigate the properties of aggregate labor supply of the model. Section 5 summarizes our findings.

1. LABOR SUPPLY: INDIVIDUAL VERSUS AGGREGATE

In this section, we consider various models on individuals' labor supply decisions and derive the corresponding aggregate labor supply schedules. For

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[2] In his survey paper, Pencavel (1986) reports that most estimates are between 0.00 and 0.45 for men. In their parallel survey of research on the labor supply of women, Killingsworth and Heckman (1986) present a wide range of estimates, from -0.3 to 14.0; they do not venture a guess as to which is correct but conclude that the elasticity is probably somewhat higher for women than men. See Blundell and MaCurdy (1999) for a more recent review of the literature. An alternative (equilibrium) approach is to introduce shifts in labor supply through shifts in preference (Bencivenga 1992), home technology (Benhabib, Rogerson, and Wright 1991; Greenwood and Hercowitz 1991), or government spending (Christiano and Eichenbaum 1992).

[3] Hansen's (1985) indivisible labor economy, based on the theory of employment lotteries by Rogerson (1988), generates a very high aggregate labor supply elasticity—in fact, infinity—regardless of individual labor supply elasticity. However, the existence of employment lotteries is not strongly supported by the data, as the persons with greater hours or greater earnings per hour consume more. Our analysis illustrates that such an economy is a special case where the reservation wage distribution is degenerate.

the moment, we abstract from the intertemporal decisions. Hence, models in this section are static and of partial equilibrium. We will study a fully specified dynamic general equilibrium model in Section 3.

Homogeneous Agents with Divisible Labor

Suppose there is measure one of identical agents with the following preferences over consumption, c, and hours worked, h:

$$U = \max_{c,h} \; lnc - B \frac{h^{1+1/\gamma}}{1+1/\gamma} \tag{1}$$

subject to

$$c = wh + ra, \tag{2}$$

where w is the hourly wage; r, the interest rate; and a, asset holdings. The first order condition for hours of work is

$$Bh^{1/\gamma} = \frac{w}{c}. \tag{3}$$

The marginal disutility from additional hours of work equals the marginal utility of consumption from income earned. The labor supply function can be written as

$$h = \left(\frac{w}{Bc}\right)^{\gamma}. \tag{4}$$

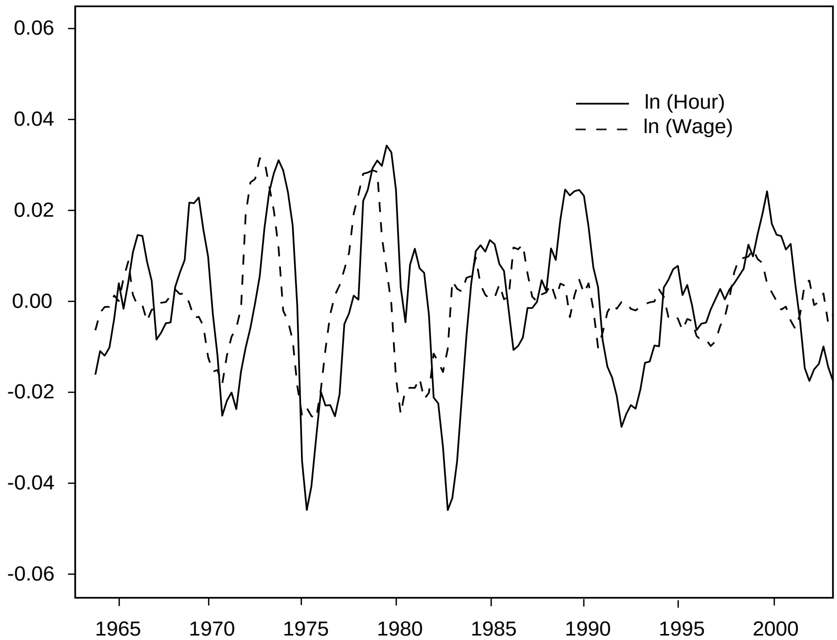
The Frisch elasticity—elasticity of hours with respect to wage holding wealth (consumption) constant—is γ. With homogeneous agents, the aggregate labor supply elasticity is also γ. According to the empirical micro studies, the labor supply is inelastic since a typical value of γ is less than 0.5. As Figure 1 illustrates, inelastic labor supply is hard to reconcile with the fact that hours fluctuate greatly without much variation in wages.

Homogeneous Agents with Indivisible Labor

A large fraction of cyclical fluctuations of total hours worked reflects the decisions to work or not (the so-called extensive margin), whereas the micro elasticities reflect the variation of hours for employed workers (intensive margin). The indivisible labor model has been developed to highlight the extensive margin of labor supply.[4] Suppose an agent supplies $\bar{h}$ hours if he works and zero hours otherwise. With homogeneous agents, the labor supply decision is

[4] In general, the labor supply decision operates on both the extensive and intensive margins. However, workers are rarely allowed to choose completely flexible work schedules or to supply a small number of hours.

Figure 1 Cyclical Components of Total Hours and Wages



Notes: Hours worked represent the nonagricultural private sector. Wage is real hourly earnings for nonsupervisory and production workers.

randomized (see Hansen 1985 and Rogerson 1988). An agent chooses probability of working, p, and the expected utility is $p(lnc - B\frac{\bar{h}^{1+1/\gamma}}{1+1/\gamma}) + (1 - p)(lnc - 0)$. Then the agent's maximization problem with the existence of a complete insurance market is

$$U = \max_{c,p}\ lnc - pB\frac{\bar{h}^{1+1/\gamma}}{1+1/\gamma},$$

subject to

$$c = wp\bar{h} + ra.$$

The equilibrium value of p is equal to the fraction of agents that work, and the aggregate labor supply is given by $H = p\bar{h}$. The aggregate labor supply elasticity is infinite, as the stand-in agent's utility is linear in p. While the aggregate labor supply is infinitely elastic in this environment, the underlying assumptions—homogeneity and complete market—are vulnerable even to casual empiricism.

Heterogeneous Agents with Indivisible Labor

Suppose workers differ in both preference (B) and asset holdings (a) and that the complete insurance of idiosyncratic risks is not available. With indivisible labor, the agent, i, works if

$$\log(w\bar{h} + ra_i) - B_i \frac{\bar{h}^{1+1/\gamma}}{1 + 1/\gamma} \geq \log(ra_i). \tag{5}$$

The reservation wage, $\widetilde{w}$, is

$$\widetilde{w} = \frac{ra_i}{\bar{h}}\Big(\exp(B_i \Delta) - 1\Big), \tag{6}$$

where $\Delta = \frac{\bar{h}^{1+1/\gamma}}{1+1/\gamma}$ is a constant, independent of individual characteristics. Workers with high B_i (those who value leisure more relative to commodity consumption) exhibit a higher reservation wage. The richer (a_i) a worker is, the higher his reservation wage. In general the reservation wage depends on various dimensions of cross-sectional heterogeneity. In Section 3, we investigate the fully specified dynamic general equilibrium model where the shape of $\Phi(\widetilde{w})$ is parsimoniously characterized by the microdata and depends on the agent's earnings ability as well as wealth.

2. ILLUSTRATIVE EXAMPLES

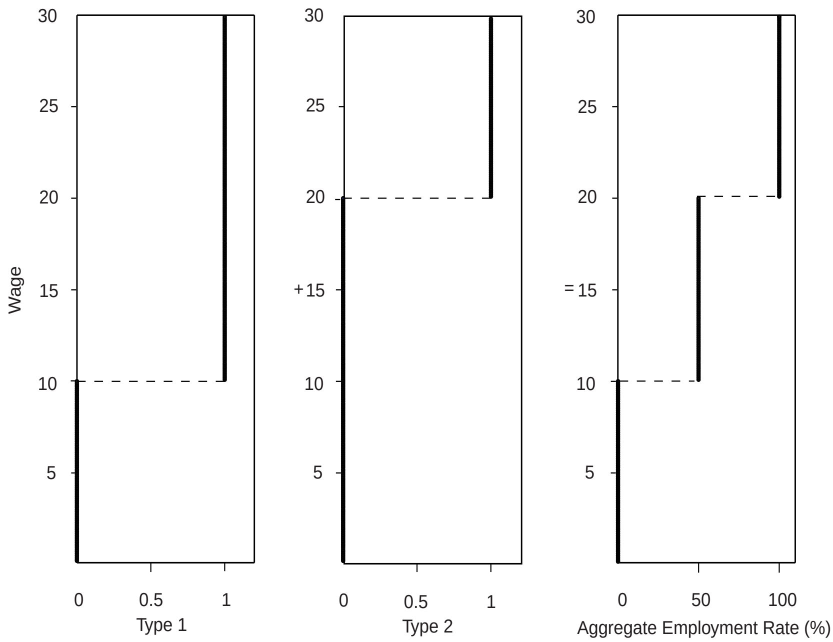
Before we present our formal analysis, we provide the examples that illustrate the dependence of aggregate labor supply on the shape of reservation wage distribution. Suppose that equal numbers of two types of workers exist in the economy, with reservation wages of \$10 and \$20, respectively. Suppose also that labor supply is indivisible in the sense that a worker supplies one unit of labor if he works. Figure 2 shows that the aggregate labor supply—the horizontal sum of individual labor supply—can have two elasticities. At a wage rate of \$10 and \$20, the elasticity is infinity. Otherwise, it is zero. Whenever a mass in the reservation wage distribution exists, the aggregate labor supply elasticity can take a large value. Suppose that many types of workers exist and that a worker works $\bar{h}$ hours if the market wage, w, exceeds the reservation wage, $\widetilde{w}$:

$$h(w) = \begin{cases} \bar{h} & \text{if } w \geq \widetilde{w}, \\ 0 & \text{otherwise.} \end{cases}$$

The aggregate labor supply function, $H(w)$, is

$$H(w) = \int_0^w \bar{h}\phi(\widetilde{w})d\widetilde{w} = \Phi(w)\bar{h}.$$

Figure 2 Individual and Aggregate Labor Supply



The aggregate labor supply elasticity, $\Gamma(w) = \frac{H'(w)w}{H(w)}$, is

$$\Gamma(w) = \frac{\Phi'(w)w}{\Phi(w)}.$$

The aggregate elasticity depends on the concentration of workers—the marginal density, $\Phi'(w)$, relative to the cumulative density, $\Phi(w)$. In the two-type workforce example, the aggregate elasticity is infinity where there is a mass of workers, ($\Phi'(10) = \Phi'(20) = \infty$), and zero elsewhere. In the lottery economy of Hansen (1985) and Rogerson (1988), the reservation wage distribution is degenerate (as the agents are identical) at the equilibrium wage rate ($\Phi'(w) = \infty$), and the aggregate elasticity becomes infinity.

The aggregate elasticity depends on the relative concentration of workers even when workers are allowed to work longer hours at higher wages. Suppose the labor supply of a worker is

$$h(w; \widetilde{w}) = \begin{cases} \bar{h} + \hat{h}(w; \widetilde{w}) & \text{if } w \geq \widetilde{w}, \\ 0 & \text{otherwise.} \end{cases}$$

Here $\hat{h}(w;\widetilde{w})$ is hours worked beyond the minimum hours, $\bar{h}$, and satisfies $\hat{h}(w;\widetilde{w}) \geq 0$ with equality when $w = \widetilde{w}$ and $\hat{h}'(w;\widetilde{w}) > 0$.[5] The aggregate labor supply function is

$$H(w) = \int_0^w h(w;\widetilde{w})\phi(\widetilde{w})d\widetilde{w} = \bar{h}\Phi(w) + \int_0^w \hat{h}(w;\widetilde{w})\phi(\widetilde{w})d\widetilde{w},$$

where the total hours worked consists of the sum of the extensive margins, $\bar{h}\Phi(w)$, and that of intensive margins, $\int_0^w \hat{h}(w;\widetilde{w})\phi(\widetilde{w})d\widetilde{w}$. Given that

$$H'(w) = h(w;w)\phi(w) + \int_0^w h'(w;\widetilde{w})\phi(\widetilde{w})d\widetilde{w},$$

the aggregate elasticity is

$$\Gamma(w) = \frac{[\bar{h}\phi(w) + \int_0^w h'(w;\widetilde{w})\phi(\widetilde{w})d\widetilde{w}]w}{\int_0^w h(w;\widetilde{w})\phi(\widetilde{w})d\widetilde{w}}.$$

For illustrative purposes, suppose the individual labor supply elasticity, γ, is constant across workers and wages: $\gamma = \dfrac{h'(w;\widetilde{w})w}{h(w;\widetilde{w})}$. Substituting $h'(w;\widetilde{w})$ with γ, the aggregate elasticity can be again expressed as the sum of the relative concentration of reservation wages and the individual elasticity:

$$\Gamma(w) = \frac{\bar{h}\Phi'(w)w}{\bar{h}\Phi(w) + \int_0^w \hat{h}(w;\widetilde{w})\phi(\widetilde{w})d\widetilde{w}} + \gamma.$$

These examples illustrate two important aspects of aggregate labor supply: the aggregate elasticity can be different from that of microelasticity and the aggregate labor supply elasticity is not time-invariant because the reservation wage distribution evolves over time as the wealth distribution and the level of employment change over time. However, these examples are silent about the magnitude of the aggregate labor supply elasticity for which the exact shape of the empirical reservation wage distribution must be uncovered. In the next section, we present a model economy—a simplified version of Chang and Kim (2004a)—where the reservation wage distribution, $\Phi(\widetilde{w})$, is determined by the asset accumulation of households that face different types of uninsurable income risks.[6] While the model is parsimonious, it allows for a complete characterization of the reservation wage distribution.

3. A FULLY SPECIFIED GENERAL EQUILIBRIUM MODEL

There is a continuum (measure one) of workers who have identical preferences but different productivity. Individual productivity varies exogenously

[5] The minimum-hours restriction can be easily justified, for example, by fixed costs, such as commuting time.

[6] In Chang and Kim (2004a), the economy consists of many households made up of a husband and wife. Here we present a model that is populated by many single-agent households.

according to a stochastic process with a transition probability distribution function, $\pi_x(x'|x) = Pr(x_{t+1} \leq x'|x_t = x)$. A worker maximizes his utility over consumption, c_t, and hours worked, h_t:

$$U = \max_{\{c_t,h_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \right\},$$

with

$$u(c_t, h_t) = lnc_t - B \frac{h_t^{1+1/\gamma}}{1 + 1/\gamma},$$

subject to

$$a_{t+1} = w_t x_t h_t + (1 + r_t)a_t - c_t. \quad (7)$$

Workers trade claims for physical capital, a_t, which yields the rate of return, r_t, and depreciates at the rate, δ. The capital market is incomplete. Physical capital is the only asset available to workers who face a borrowing constraint, $a_t \geq \bar{a}$ for all t. We abstract from the intensive margin and assume that the labor supply is indivisible. If employed, a worker supplies $\bar{h}$ units of labor and earns $w_t x_t \bar{h}$, where w_t is wage rate per effective unit of labor.

The representative firm produces output according to a Cobb-Douglas technology in capital, K_t, and efficiency units of labor, L_t:[7]

$$Y_t = F(L_t, K_t, \lambda_t) = \lambda_t L_t^{\alpha} K_t^{1-\alpha},$$

where λ_t is the aggregate productivity shock with a transition probability distribution function, $\pi_\lambda(\lambda'|\lambda) = Pr(\lambda_{t+1} \leq \lambda'|\lambda_t = \lambda)$.[8]

The value function for an employed worker, denoted by V^E, is

$$V^E(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}} \Bigg\{ lnc - B \frac{\bar{h}^{1+1/\gamma}}{1 + 1/\gamma} + \beta E\Big[\max \big\{ V^E(a', x'; \lambda', \mu'), V^N(a', x'; \lambda', \mu') \big\} \big| x, \lambda \Big] \Bigg\},$$

subject to

$$c = wx\bar{h} + (1 + r)a - a',$$

[7] This production function implicitly assumes that workers are perfect substitutes for each other. While this assumption abstracts from reality, it greatly simplifies the labor market equilibrium.

[8] In this model economy, the technology shock is the only aggregate shock. This restriction does not necessarily reflect our view on the source of the business cycles. As we would like to show that the preference residual contains a significant specification error rather than true shifts in preferences, we intentionally exclude shocks that may shift the labor supply schedule itself (e.g., shifts in government spending or changes in the income tax rate) from the present article.

$$a' \geq \bar{a}, \text{ and}$$

$$\mu' = \mathbf{T}(\lambda, \mu),$$

where $\mathbf{T}$ denotes a transition operator that defines the law of motion for the distribution of workers, $\mu(a, x)$.[9] The value function for a nonemployed worker, denoted by $V^N(a, x; \lambda, \mu)$, is defined similarly with $h = 0$. Then, the labor supply decision is characterized by

$$V(a, x; \lambda, \mu) = \max_{h \in \{0, \bar{h}\}} \left\{ V^E(a, x; \lambda, \mu), V^N(a, x; \lambda, \mu) \right\}.$$

Equilibrium consists of a set of value functions, $\{V^E(a, x; \lambda, \mu),\ V^N(a, x; \lambda, \mu),\ V(a, x; \lambda, \mu)\}$; a set of decision rules for consumption, asset holdings, and labor supply, $\{c(a, x; \lambda, \mu), a'(a, x; \lambda, \mu), h(a, x; \lambda, \mu)\}$; aggregate inputs, $\{K(\lambda, \mu), L(\lambda, \mu)\}$; factor prices, $\{w(\lambda, \mu), r(\lambda, \mu)\}$; and a law of motion for the distribution $\mu' = \mathbf{T}(\lambda, \mu)$ such that:

1. Individuals optimize:

   Given $w(\lambda, \mu)$ and $r(\lambda, \mu)$, the individual decision rules—$c(a, x; \lambda, \mu)$, $a'(a, x; \lambda, \mu)$, and $h(a, x; \lambda, \mu)$—solve $V^E(a, x; \lambda, \mu)$, $V^N(a, x; \lambda, \mu)$, and $V(a, x; \lambda, \mu)$.

2. The representative firm maximizes profits:

   $$w(\lambda, \mu) = F_1\big(L(\lambda, \mu), K(\lambda, \mu), \lambda\big), \text{ and}$$

   $$r(\lambda, \mu) = F_2\big(L(\lambda, \mu), K(\lambda, \mu), \lambda\big) - \delta$$

   for all (λ, μ).

3. The goods market clears:

   $$\int \left\{ a'(a, x; \lambda, \mu) + c(a, x; \lambda, \mu) \right\} d\mu = F\big(L(\lambda, \mu), K(\lambda, \mu), \lambda\big) + (1 - \delta)K$$

   for all (λ, μ).

4. Factor markets clear:

   $$L(\lambda, \mu) = \int x h(a, x; \lambda, \mu) d\mu, \text{ and}$$

   $$K(\lambda, \mu) = \int a d\mu$$

[9] Let $\mathcal{A}$ and $\mathcal{X}$ denote sets of all possible realizations of a and x, respectively. The measure $\mu(a, x)$ is defined over a σ-algebra of $\mathcal{A} \times \mathcal{X}$.

Table 1 Parameters of the Benchmark Model Economy

Parameter	Description
$\alpha = 0.64$	Labor share in production function
$\beta = 0.9785504$	Discount factor
$\gamma = 0.4$	Individual labor supply elasticity with divisible labor
$B = 151.28$	Utility parameter
$\overline{h} = 1/3$	Labor supply if working
$\bar{a} = -2.0$	Borrowing constraint
$\rho_x = 0.939$	Persistence of idiosyncratic productivity shock
$\sigma_x = 0.287$	Standard deviation of innovation to idiosyncratic productivity
$\rho_\lambda = 0.95$	Persistence of aggregate productivity shock
$\sigma_\lambda = 0.007$	Standard deviation of innovation to aggregate productivity

for all (λ, μ).

5. Individual and aggregate behaviors are consistent:

$$\mu'(A^0, X^0) = \int_{A^0, X^0} \left\{ \int_{\mathcal{A}, \mathcal{X}} 1_{a'=a'(a,x;\lambda,\mu)}\, d\pi_x(x'|x) d\mu \right\} da' dx'$$

for all $A^0 \subset \mathcal{A}$ and $X^0 \subset \mathcal{X}$.

4. QUANTITATIVE ANALYSIS

Calibration

We briefly explain the choice of the model parameters. The unit of time is a business quarter. We assume that x follows an AR(1) process: $lnx' = \rho_x lnx + \varepsilon_x$, where $\varepsilon_x \sim N(0, \sigma_x^2)$. As we view x as reflecting a broad measure of earnings ability in the market, we estimate the stochastic process of x based on the wages from the Panel Study of Income Dynamics (PSID) for 1979–1992. The values of $\rho_x = 0.939$ and $\sigma_x = 0.287$ reflect the persistence and standard deviation of innovations to individual wages.[10] The other parameters of the article are in accordance with the business cycle analysis and empirical labor supply literature. A working individual spends one-third of her discretionary time: $\bar{h} = 1/3$. The individual compensated labor supply elasticity of hours, γ, is 0.4. The labor share of output, α, is 0.64, and the depreciation rate, δ, is 2.5 percent. We search for the weight parameter on leisure, B, such

[10] These are maximum-likelihood estimates of Heckman (1979), correcting for a sample selection bias. Our estimate for income shocks does not purge the life-cycle effect. In our companion paper, Chang and Kim (2004a), we use both cases. When the life-cycle effect (accounted for by observed characteristics such as age, education, and sex) is purged, the aggregate labor supply elasticity becomes slightly bigger because the reservation wage distribution becomes less dispersed.

Table 2 Characteristics of Wealth Distribution

	Quintile					
	1st	2nd	3rd	4th	5th	Total
PSID						
Share of wealth	-0.52	0.50	5.06	18.74	76.22	100
Group average/population average	-0.02	0.03	0.25	0.93	3.81	1
Share of earnings	-7.51	11.31	18.72	24.21	38.23	100
Model						
Share of wealth	-2.05	2.46	10.22	23.88	65.49	100
Group average/population average	-0.10	0.12	0.51	1.19	3.27	1
Share of earnings	9.70	15.06	19.01	23.59	32.63	100

Notes: The PSID statistics reflect the family wealth and earnings levels published in their 1984 survey.

that the steady state employment rate is 60 percent, the current population survey average for 1967:II–2000:IV. The discount factor, β, is chosen so that the quarterly rate of return to capital is 1 percent. The aggregate productivity shock, λ_t, follows an AR(1) process: $ln\lambda' = \rho_\lambda ln\lambda + \varepsilon_\lambda$, where $\varepsilon_\lambda \sim N(0, \sigma_\lambda^2)$. We set ρ_λ equal to 0.95 and σ_λ equal to 0.007, following Kydland and Prescott (1982). Table 1 summarizes the parameter values of the benchmark economy.

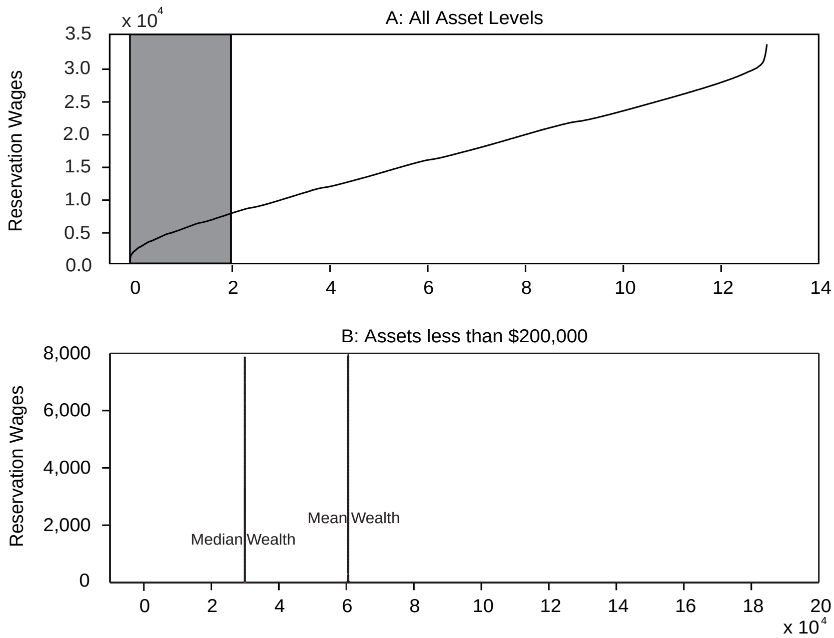
Cross-Sectional Earnings and Wealth Distribution

As we investigate the aggregation issue, it is desirable for the model economy to possess a reasonable amount of heterogeneity. We compare cross-sectional earnings and wealth—two important observable dimensions of heterogeneity in the labor market—found in the model and in the data.

Table 2 summarizes both the PSID and the model's detailed information on wealth and earnings. Family wealth in the PSID (1984 survey) reflects the net worth of houses, other real estate, vehicles, farms and businesses owned, stocks, bonds, cash accounts, and other assets. For each quintile group of wealth distribution, we calculate the wealth share, ratio of group average to economy-wide average, and the earnings share.

In both the data and the model, the poorest 20 percent of families in terms of wealth distribution were found to own virtually nothing. In fact, households in the first quintile of wealth distribution were found to be in debt in both the model and the data. The PSID found that households in the fourth and fifth quintile own 18.74 and 76.22 percent of total wealth, respectively, while, according to the model, they own 23.88 and 65.49 percent, respectively. The average wealth of those in the fourth and fifth quintile is, respectively, 0.93 and 3.81 times larger than that of a typical household, according to the

Figure 3 Reservation Wage Schedule



Notes: The graphs denote the reservation wage schedule of the benchmark model. Wages (quarterly earnings) and assets are in 1983 dollars.

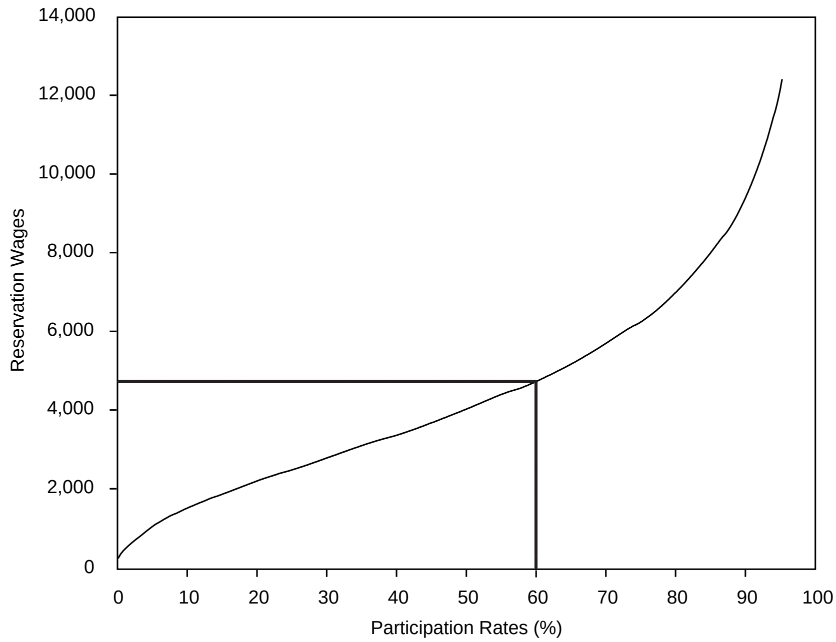
PSID. These ratios are 1.19 and 3.27 according to our model. The fourth and fifth quintile groups of the wealth distribution earn, respectively, 24.21 and 38.23 percent of total earnings, according to the PSID. The corresponding groups earn 23.59 and 32.63 percent, respectively, in the model.

Overall, the wealth distribution is found to be more skewed in the data. In particular, our model fails to match the highly concentrated wealth found in the right tail of the distribution. In the PSID, the top 5 percent of the population controls about half of total wealth (not shown in Table 2), whereas, in our model, they possess only 20 percent of total wealth. Since our primary objective is not to explain the top 1 to 5 percent of the population, we argue that the model economy presented in this article possesses a reasonable degree of heterogeneity, thus making it possible to study the effects of aggregation in the labor market.

Reservation Wage Distribution

The reservation wage distribution is crucial for the mapping from individual to aggregate labor supply. In Figure 3, we plot the reservation wage schedule

Figure 4 Reservation Wages and Participation Rates



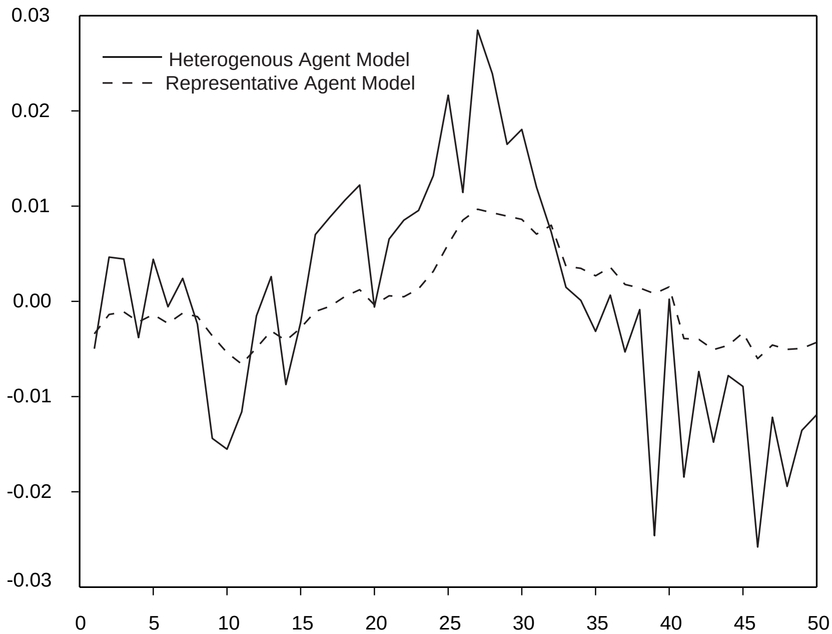
Notes: The graph denotes the inverse cumulative distribution functions of reservation wages. Wages are quarterly earnings in 1983 dollars.

of the benchmark model for all asset levels (panel A) and for assets less than \$200,000 (panel B). At a given asset level, workers with wages (productivity) above the line choose to work. The reservation wage increases as the asset level increases. To illustrate, we adjust the units so that the mean asset of the model matches the average asset in the 1984 PSID survey, \$60,524; thus, the values are in 1983 dollars.[11] Consider a worker whose assets are \$29,880, the median of the wealth distribution from the model. According to the model, he is indifferent between working and not working at quarterly earnings of \$3,287. Another worker whose assets are equivalent to the average asset holding of the economy, \$60,524 (which belongs to the 66th percentile of the wealth distribution in our model and to the 72nd percentile in the PSID), is indifferent about working at \$4,273 per quarter.

In Figure 4 we plot the inverse cumulative distribution of reservation wages of the model. In practice, the reservation wage distribution is neither observed nor constant over time. Based on the reservation wage schedule and

[11] The mean asset in our model is 14.48 units. The reservation wages in the vertical axis reflect quarterly earnings (the reservation wage rate multiplied by $\bar{h}$).

Figure 5 Total Hours Worked from the Models



invariant distribution, $\mu(x, a)$, we can infer the responsiveness of labor market participation. In Table 3 we compute the elasticities of participation with respect to the reservation wage around the steady state. These values may be viewed as the aggregate labor supply elasticity with zero wealth effect as they assume the *entire* wealth distribution held is constant. For the model economy, the elasticities are 1.12, 1.05, and 0.97, respectively, at the employment rates of 58, 60, and 62 percent. Overall, these values are bigger than typical micro estimates, but they remain in a moderate range. In particular, a very high elasticity—in fact, infinity—generated by a lottery economy with a homogeneous workforce (in which the reservation wage distribution is degenerate) does not survive serious heterogeneity.

Finally, we would like to emphasize that, when labor supply is indivisible, the slope of the aggregate labor supply schedule is mostly determined by the distribution of reservation wages rather than by the willingness to substitute leisure intertemporally. In fact, the aggregate labor supply is independent of γ in our economy. With a binary choice of hours, utility of market participants and non-participants differs by a constant term, $B\frac{\bar{h}^{1+1/\gamma}}{1+1/\gamma}$ [Recall (6)]. Given γ, we adjust B (the weight parameter on disutility from working) to match the 60 percent employment rate in the steady state, leaving the above constant

Table 3 Labor Supply Elasticity Implied by the Reservation Wage Distribution

Employment Rate		
$E = 58\%$	$E = 60\%$	$E = 62\%$
1.12	1.05	0.97

Notes: The numbers reflect the elasticity of the labor market participation rate with respect to reservation wage (evaluated at employment rates of 58, 60, and 62 percent) based on the reservation wage distribution in the steady state.

term unchanged. As a result, the steady state reservation-wage distribution remains the same regardless of γ.

Comparison with the Representative Agent Model

We compare the volatility of hours from our model economy to that of the representative agent economy. Both model economies will be subject to identical stochastic aggregate productivity shocks that resemble that of the post-war total factor productivity (Solow residual).

The value function of the representative agent, $V^R(K, \lambda)$, is

$$V^R(K, \lambda) = \max_{C,H} \left\{ lnC - B\frac{H^{1+1/\gamma}}{1+1/\gamma} + \beta E\Big[V^R(K', \lambda') \big| \lambda \Big] \right\},$$

subject to

$$K' = F(K, H, z) + (1-\delta)K - C.$$

Except for β, the same parameter values are used, $\beta = 0.99$.[12] Fluctuation of the heterogeneous agent model is solved by the method developed by Krusell and Smith (1998). Figure 5 shows the sample paths of total hours worked (percentage deviations from the steady states), respectively, from the heterogeneous agent economy and the representative agent economy, $\gamma = 0.4$. In the face of aggregate productivity shocks whose stochastic process resembles that of the post-war total factor productivity, hours of work from the heterogeneous agent economy exhibit a much greater volatility than those of the representative agent model.

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[12] B is a free parameter in a sense that it does not affect the dynamics around the steady state.

5. SUMMARY

We demonstrate that, at the aggregate level, the labor supply elasticity can significantly depart from the microelasticity. In an economy where households make decisions on labor market participation, the slope of the aggregate labor supply curve is determined by the distribution of reservation wages rather than by the willingness to substitute leisure intertemporally. We present a model economy where households face uninsurable idiosyncratic income shocks. While the model is parsimonious, the cross-sectional distributions of earnings and wealth are comparable to those in the U.S. data. We find that the aggregate labor supply elasticity of such an economy is around 1.0—despite the low intertemporal substitution elasticity of leisure, assumed to be 0.4. The equilibrium approach of business cycle analysis has been criticized on the grounds that it requires an elasticity higher than the intertemporal substitution elasticity estimated from the microdata. Our analysis shows that, while the aggregate labor elasticity can depart from a microelasticity, it remains in a moderate range as the reservation wage distribution is dispersed.

REFERENCES

Aiyagari, Rao S. 1994. “Uninsured Idiosyncratic Risk and Aggregate Savings.” *Quarterly Journal of Economics* CIX: 659–83.

Auerbach, A. J., and L. J. Kotlikoff. 1987. *Dynamic Fiscal Policy.* Cambridge, U.K.: Cambridge University Press.

Bencivenga, Valerie. 1992. “An Econometric Study of Hours and Output Variation with Preference Shocks.” *International Economic Review* (33): 448–71.

Benhabib, Jess, Richard Rogerson, and Randall Wright. 1991. “Homework in Macroeconomics: Household Production and Aggregate Fluctuations.” *Journal of Political Economy* 99: 1166–87.

Blundell, Richard, and Thomas MaCurdy. 1999. “Labor Supply: A Review of Alternative Approaches.” In *Handbook of Labor Economics*, Vol. 3A. Ed. O. Ashenfelter and D. Card. Amsterdam: North Holland: 1599–695.

Chang, Yongsung, and Sun-Bin Kim. 2004a. “From Individual to Aggregate Labor Supply: A Quantitative Analysis Based on a Heterogeneous Agent Macroeconomy.” Manuscript, Federal Reserve Bank of Richmond.

_____________. 2004b. “Heterogeneity and Aggregation in the Labor Market: Implications for Aggregate Preference Shocks.” Manuscript, Federal Reserve Bank of Richmond.

Christiano, Lawrence J., and Martin Eichenbaum. 1992. "Current Real-Business Cycle Theories and Aggregate Labor-Market Fluctuations." *American Economic Review* 82: 430–50.

Greenwood, Jeremy, and Zvi Hercowitz. 1991. "The Allocation of Capital and Time over the Business Cycle." *Journal of Political Economy* 99: 1188–215.

Hansen, Gary D. 1985. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics* 16: 309–27.

Heckman, James. 1979. "Sample Selection Bias as a Specification Error." *Econometrica* 47 (1): 153–62.

Judd, Kenneth L. 1987. "The Welfare Cost of Factor Taxation in a Perfect Foresight Model." *Journal of Political Economy* 95(4): 675–709.

Killingsworth, Mark R., and James Heckman. 1986. "Female Labor Supply." *Handbook of Labor Economics*, Vol. 1. Ed. O. Ashenfelter and R. Layards. Amsterdam: North Holland: 103–204.

Krusell, Per, and Anthony Smith. 1998. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy* 106: 867–96.

Kydland, Finn E., and Edward Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica* 50: 1345–70.

Lucas, Robert E. Jr., and Leonard Rapping. 1969. "Real Wages, Employment, and Inflation." *Journal of Political Economy* 77: 721–54.

Pencavel, John. 1986. "Labor Supply of Men." *Handbook of Labor Economics*, Vol. 1. Ed. O. Ashenfelter and R. Layards. Amsterdam: North Holland: 3–102.

Rogerson, Richard. 1988. "Indivisible Labor, Lotteries and Equilibrium." *Journal of Monetary Economics* 21: 3–16.