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Quantitative Models of Sovereign Default and the Threat of Financial Exclusion

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Business cycles in small emerging economies differ from those in developed economies. Emerging economies feature interest rates that are higher, more volatile, and countercyclical (interest rates are usually acyclical in developed economies). These economies also feature higher output volatility, higher volatility of consumption relative to income, and more countercyclical net exports.[1] Recent research is trying to develop a better understanding of these facts, as has been done for U.S. business cycles.

Because of the high volatility and countercyclicality of the interest rate, the (state-dependent) borrowing-interest rate menu is a key ingredient in any model designed to explain the cyclical behavior of quantities and prices in emerging economies. Some studies assume an exogenous interest rate.[2] Others provide microfoundations for the interest rate based on the risk of default.[3] This is the approach taken by recent quantitative models of sovereign default, which are based on the framework proposed by Eaton and Gersovitz (1981).[4] These articles build on the assumption that lenders can punish defaulting coun-

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[1] See Aguiar and Gopinath (2007), Neumeyer and Perri (2005), and Uribe and Yue (2006).

[2] See, for example, Aguiar and Gopinath (2007), Neumeyer and Perri (2005), Schmitt-Grohé and Uribe (2003), and Uribe and Yue (2006).

[3] Default episodes are not exceptional. Many nations have experienced episodes of sovereign default, some of the latest being Russia in 1998, Ecuador in 1999, and Argentina in 2001.

[4] See Aguiar and Gopinath (2006); Arellano (forthcoming); Arellano and Ramanarayanan (2006); Bai and Zhang (2006); Cuadra and Sapriza (2006a, b); Lizarazo (2005a,b); and Yue (2005).

tries by excluding them from international financial markets. The assumption is controversial on several grounds. First, it appears to be at odds with the existence of competitive international capital markets (which is assumed in these models). It is not obvious that competitive creditors would be able to coordinate cutting off credit to a country after a default episode.[5] Second, empirical studies suggest that once other variables are used as controls, market access is not significantly influenced by previous default decisions (see, for example, Gelos, Sahay, and Sandleris 2004, Eichengreen and Portes 2000, and Meyersson 2006).[6]

1. SUMMARY OF RESULTS

This article studies the role of the exclusion assumption for business cycle properties of emerging economies. It first describes the business cycle properties of a sovereign default model with exclusion and compares them with those of the same model without exclusion. The article finds that the presence of exclusion punishment is responsible for a high fraction of the sovereign debt that can be sustained in equilibrium. It also finds that the business cycle statistics of the model are not significantly affected by the exclusion punishment. The model without exclusion generates annual debt-output ratios of less than 2 percent. Whereas, the model with exclusion generates debt-output ratios between 4.8 and 6.3 percent. On the other hand, the cyclical behavior of consumption, output, interest rate, and net exports are not fundamentally different in the models with and without exclusion. An additional limitation shared by both model environments is that the volatility of the interest rate and (to a lesser extent) of the trade balance are too low compared to the data. This suggests that the exclusion assumption does not play an important role in these dimensions, and therefore future studies that do not rely on the threat of financial exclusion will not necessarily be handicapped in explaining the business cycle in emerging economies.

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5 This point is also raised by Cole, Dow, and English (1995) and Athreya and Janicki (2006).

6 Sturzenegger and Zettelmeyer (2005) discuss how holders of defaulted bonds succeeded in interfering with cross-border payments to other creditors who had previously agreed to a debt restructuring. From this, they infer that holders of defaulted bonds may have been able to exclude defaulting economies from international capital markets. On the other hand, they conclude that "legal tactics are updated all the time, and new ways are discovered both to extract payment from a defaulting sovereign as well as to avoid attachments." In particular, they expect that "the threat of exclusion may be less relevant for some countries or to all countries in the future." For example, they explain that after Argentina defaulted in 2001, "attempts to actually attach assets have so far turned out to be fruitless." In any case, other forms of financing are always available to defaulting economies (issuing bonds at home, aid, official credit, multilateral or bilateral financing, etc.). The discussion in Sturzenegger and Zettelmeyer (2005) suggests, therefore, that defaulting economies might face at most a higher borrowing cost, though it is not clear how important this cost differential may be.

The model studied in this article builds on the framework studied in Aguiar and Gopinath (2006), which in turn, quantifies the model presented by Eaton and Gersovitz (1981). The most appealing feature about this setup is that it reduces the default decision to a simple tradeoff between current and future consumption without a major departure from the workhorse model used for real business cycle analysis in the last decades. Recent quantitative studies on sovereign default have shown that this environment can potentially account for important business cycle features in emerging economies and that it can be extended to address other issues (such as the optimal maturity structure of sovereign debt).[7] The framework studied in Aguiar and Gopinath (2006) is the simplest among the ones presented in recent studies. This has the advantage of making the discussion of the role of the exclusion assumption more transparent.[8] On the other hand, this has the disadvantage of hurting the performance of the model along several dimensions. Where appropriate, the article explains how the simplifying assumptions hurt the performance of the model.

This article studies a small open economy endowed with a single tradable good. As in Aguiar and Gopinath (2006), two endowment processes are considered: a process with shocks to the endowment *level* and a process with shocks to the endowment *growth rate*. The objective of the government is to maximize the present value of future utility flows of the representative agent. The government has only one financial instrument available: it can save or borrow using one-period bonds. These assets are priced in a competitive market inhabited by a large number of identical, infinitely lived, risk neutral-lenders. Lenders have perfect information regarding the economy's endowment. The government makes two decisions in every period. First, it decides whether to refuse to pay previously issued debt. Second, it decides how much to borrow or save. The baseline model features two costs of defaulting. First, the country may be excluded from capital markets. Second, it faces an "output loss." The endowment is reduced in a fixed percentage in the period following a default. The assumption that countries experience an output loss after a default intends to capture the disruptions in economic activity entailed by a default decision. IMF (2002), Kumhof (2004), and Kumhof and Tanner (2005) discuss how financial crises that lead to severe recessions are triggered by sovereign default.

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[7] Arellano (forthcoming); Arellano and Ramanarayanan (2006); Bai and Zhang (2006); Cuadra and Sapriza (2006a,b); Lizarazo (2005a,b); and Yue (2005) extend the framework in Aguiar and Gopinath (2006) but maintain the basic assumptions (including the exclusion assumption).

[8] The only difference between the model presented in this article and the model in Aguiar and Gopinath (2006) is that here it is assumed that there is a unique period of output loss after default—in contrast with the stochastic number of periods of output loss assumed by Aguiar and Gopinath (2006). This allows us to eliminate the threat of exclusion without increasing the dimensionality of the state space. The Appendix shows that this departure does not have sizable effects on the results.

This article solves the model with and without the exclusion threat and compares their behavior. Mechanically, in the model with shocks to the endowment level, the default decision becomes relatively more sensitive to the endowment shock once the exclusion threat is eliminated and, therefore, less sensitive to the debt level. In turn, bond prices become less sensitive to the borrowing level. On the other hand, in the model with shocks to the growth rate the default decision becomes relatively less sensitive to the endowment shock, which increases the sensitivity of the bond price to the borrowing level. Given that in this class of models the high sensitivity of the default probability to the borrowing level limits their quantitative performance, the previous effects slightly improve the performance of the model with shocks to the endowment level and deteriorate the performance of the model with shocks to the growth rate. In spite of this, both models still do not replicate the default rates, the volatility of the trade balance, nor the volatility of the spread observed in the data.

The rest of the article proceeds as follows. Section 2 introduces the model. Section 3 presents the parameterization. Section 4 discusses the case in which the economy can be excluded from capital markets. Section 5 studies how the implications of the model change when the economy cannot be threatened with financial exclusion. Section 6 concludes the article.

2. THE MODEL

The environment studied in this article builds on the framework presented by Aguiar and Gopinath (2006), who study the quantitative performance of a model of sovereign default based on Eaton and Gersovitz (1981). Relative to Aguiar and Gopinath (2006), the only difference is that it is assumed here that there is a single period of output loss after default—in contrast with the stochastic number of periods of output loss assumed in their article. The Appendix shows that the results are not sensitive to this assumption.

The economy receives a stochastic endowment stream of a single tradable good. The endowment process has two components: a transitory shock and a trend shock, namely,

$$y_t = e^{z_t}\Gamma_t, \tag{1}$$

where y_t denotes the endowment realization in period t, z_t denotes the transitory shock, and Γ_t denotes the trend component.

The transitory shock z_t follows an AR(1) process with long-run mean μ_z, and autocorrelation coefficient $\left|\rho_z\right| < 1$, that is,

$$z_t = \left(1 - \rho_z\right)\mu_z + \rho_z z_{t-1} + \varepsilon_t^z, \tag{2}$$

where $\epsilon_t^z \sim N\left(0, \sigma_z^2\right)$.

The trend component evolves according to

$$\Gamma_t = g_t \Gamma_{t-1}, \tag{3}$$

where

$$ln\left(g_t\right) = \left(1-\rho_g\right)\left(ln\left(\mu_g\right)-m\right)+\rho_g ln\left(g_{t-1}\right)+\varepsilon_t, \tag{4}$$

$|\rho_g| < 1$, $\varepsilon_t \sim N\left(0, \sigma_g^2\right)$, and $\mu_g = \frac{1}{2}\frac{\sigma_g^2}{1-\rho_g^2}$.[9]

The objective of the government is to maximize the present value of future utility flows of the representative agent. The representative agent has preferences that display a constant coefficient of relative risk aversion:

$$u(c) = \frac{c^{(1-\sigma)}-1}{1-\sigma},$$

where σ denotes the coefficient of relative risk aversion. Let β denote the discount factor. To ensure a well-defined problem it is assumed that

$$E\left\{\lim_{t\to\infty}\beta^t\left(y_t\right)^{(1-\sigma)}\right\}=0.$$

The government makes two decisions in each period. First, it decides whether to refuse to pay previously issued debt. Second, it decides how much to borrow or save. As in previous quantitative studies, it is assumed that the government faces two penalties if it decides to default. One penalty is that it may be excluded from capital markets. The second penalty is that it faces an exogenous "output loss" of λ percent in the period following a default.

The exclusion state evolves as follows. In the default period, the economy is excluded from capital markets with probability $1-\phi_1$, with $\phi_1 \in [0, 1]$. In every period that follows a period of exclusion, the economy regains access to capital markets with probability $\phi \in [0, 1]$ or remains excluded for one more period with probability $1-\phi$.[10] This implies that the expected length of exclusion is given by $\frac{1-\phi_1}{\phi}$. If the economy was not excluded from financial markets at the end of the previous period, it is not excluded at the beginning of the current period.

The government can choose to save or borrow using one-period bonds. There is a continuum of risk-neutral lenders with "deep pockets." Each lender can borrow or lend at the risk-free rate r. Lenders have perfect information

[9] The endowment process is motivated by the work of Aguiar and Gopinath (2007). They find that shocks to trend growth (rather than transitory fluctuations around a stable trend) are the primary source of fluctuations in emerging markets.

[10] Previous quantitative studies of sovereign default assume that the government cannot borrow in the period it defaults ($\phi_1 = 0$), and it regains access to capital markets with a constant probability (ϕ) after that. In order to accommodate this possibility, it is assumed that ϕ_1 can be different from ϕ.

regarding the economy's endowment. The bond price is determined as follows. First, the government announces how many bonds it wants to issue. Then, lenders offer a price for these bonds. Finally, the government sells the bonds to one of the lenders who offered the highest price.

Let b denote the current position in bonds. A negative value of b denotes that the government was an issuer of bonds in the previous period. Each bond delivers one unit of good next period for a price of $q_d\left(b', z, \Gamma, g\right)$ this period. The price depends on the current default decision, d. This is due to the fact that a current default decreases future output and affects future default decisions.

The government compares two continuation values in order to decide whether to default or pay back the previously issued debt. The present discounted utility after a default is represented by $V_1(z, \Gamma, g, h)$. The variable h denotes the credit history of the government. It takes a value of 1 when the government defaulted in the previous period, and it takes a value of 0 when the government did not default in the previous period. The present discounted utility when all previously issued debt is paid back is represented by $V_0(b, z, \Gamma, g, h)$. The government defaults if the continuation value $V_1(z, \Gamma, g, h)$ is larger than $V_0(b, z, \Gamma, g, h)$ and does not default otherwise.

Let x denote the exclusion state. The variable x takes a value of 1 when the economy is excluded, and takes a value of 0 otherwise. Let $V(b, z, \Gamma, g, h, x)$ denote the government's value function at the beginning of a period.

In the period it defaults, the economy can or cannot be excluded from financial markets. Let $\tilde{V}_1(z, \Gamma, g, h, 0)$ denote the continuation value when the economy defaulted and is not excluded. Let $\tilde{V}_1(z, \Gamma, g, h, 1)$ denote the continuation value when it is excluded. Thus,

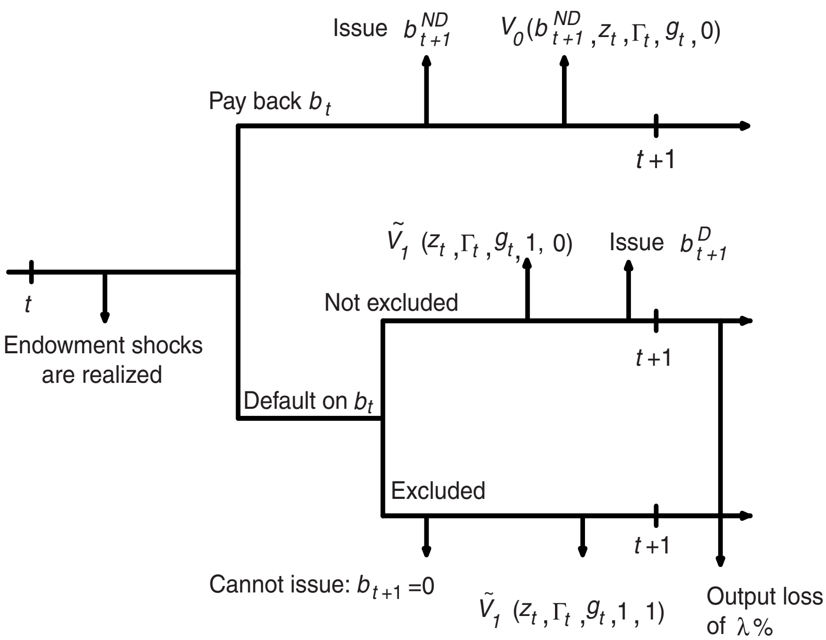
$$V_1(z, \Gamma, g, h) = \phi_1 \tilde{V}_1(z, \Gamma, g, h, 0) + (1 - \phi_1)\tilde{V}_1(z, \Gamma, g, h, 1). \quad (5)$$

The timing of the decisions within a period is summarized in Figure 1. At the beginning of the period the endowment shocks are realized. The realization in period t of a state variable x is denoted by x_t. After observing the endowment realization, the government decides whether to pay back previously issued debt. If it decides to pay the debt back, the government issues an amount b_{t+1}^{ND} of bonds and faces a continuation value of $V_0\left(b_{t+1}^{ND}, z_t, \Gamma_t, g_t, 0\right)$. If the government defaults, it may or may not be excluded from capital markets today. If it is not excluded, it faces a continuation value of $\tilde{V}_1(z_t, \Gamma_t, g_t, 1, 0,)$. If it is excluded, it faces a continuation value of $\tilde{V}_1(z_t, \Gamma_t, g_t, 1, 1,)$. If the government defaults and is not excluded today from capital markets, it issues an amount b_{t+1}^{D} of bonds. After a default, the government faces an output loss of λ percent in period $t+1$ regardless of whether it was excluded from capital markets in period t.

The value function of a defaulting economy that is excluded in the default period is computed as follows:

$$\tilde{V}_1(z, \Gamma, g, h, 1) = u\left(y\left(1 - h\lambda\right)\right) + \beta E\left[V(0, z', g'\Gamma, g', 1, e)\right], \quad (6)$$

Figure 1 Order of Events and Alternative Continuation Values in Period t



where

$$E\left[V(0,z',g'\Gamma,g',1,e)\right]=\int\int\left[\begin{array}{c}\phi V(0,z',g'\Gamma,g',1,0)+\\(1-\phi)\,V(0,z',g'\Gamma,g',1,1)\end{array}\right]F_z\left(dz'\mid z\right)F_g\left(dg'\mid g\right).$$

If the government has decided to default and is excluded in the period of default, it consumes the aggregate endowment (there are no financial transfers from or to the rest of the world) and carries zero debt to the next period. At the beginning of the following period, exclusion finishes with probability ϕ. The expected continuation value of this scenario is $V(0,z,g\Gamma,g,1,0)$. If the exclusion time is extended, the expected continuation value is $V(0,z,g\Gamma,g,1,1)$.

The dynamic programming problem for a defaulting economy that is not excluded in the default period is

$$\tilde{V}_1(z,\Gamma,g,h,0)=\max_{b'}\left\{\begin{array}{c}u\left(y(1-h\lambda)-q_1\left(b',z,\Gamma,g\right)b'\right)+\\\beta E\left[V(b',z',g'\Gamma,g',1,0)\right]\end{array}\right\}\qquad(7)$$

where

$$E\left[V(b',z',g'\Gamma,g',1,0)\right]=\int\int V(b',z',g'\Gamma,g',1,0)F_z\left(dz'\mid z\right)F_g\left(dg'\mid g\right).$$

In this case, the government must choose how much debt it will issue.

The value function of the government when it has decided to pay back its debt is obtained from the following Bellman equation:

$$V_0(b, z, \Gamma, g, h) = \max_{b'} \left\{ \begin{array}{c} u\left(y(1-h\lambda) + b - q_0\left(b', z, \Gamma, g\right) b'\right) + \\ \beta E\left[V(b', z', g'\Gamma, g', 0, 0)\right] \end{array} \right\}, \quad (8)$$

where

$$E\left[V(b', z', g'\Gamma, g', 0, 0)\right] = \int\int V(b', z', g'\Gamma, g', 0, 0) F_z\left(dz' \mid z\right) F_g\left(dg' \mid g\right).$$

The function $V(b, z, \Gamma, g, h, x)$ is computed as follows:

$$V(b, z, \Gamma, g, h, 0) = \max\{V_1(z, \Gamma, g, h), V_0(b, z, \Gamma, g, h)\}, \quad (9)$$

and

$$V(b, z, \Gamma, g, h, 1) = u(y(1-h\lambda)) + \beta E\left[\phi V(0, z', g'\Gamma, g', 0, e)\right],$$

where

$$E\left[V(0,z',g'\Gamma,g',0,e)\right] = \int\int \left[\begin{array}{c} \phi V(0, z', g'\Gamma, g', 0, 0) + \\ (1-\phi) V(0, z', g'\Gamma, g', 0, 1) \end{array} \right] F_z\left(dz' \mid z\right) F_g\left(dg' \mid g\right).$$

Let

$$d(b, z, \Gamma, g, h) = \left\{ \begin{array}{ll} 1 & \text{if } V_1(z, \Gamma, g, h) > V_0(b, z, \Gamma, g, h) \\ 0 & \text{if } V_1(z, \Gamma, g, h) \leq V_0(b, z, \Gamma, g, h) \end{array} \right. \quad (10)$$

denote the equilibrium default decision.

The price of a bond if a default decision d was made in the current period satisfies the lenders' zero profit condition. It is given by

$$q_d\left(b', z, \Gamma, g\right) = \frac{1}{1+r}\left[1 - E\left[d' \mid b', z, \Gamma, g, d\right]\right], \quad (11)$$

where

$$E\left[d' \mid b', z, \Gamma, g, d\right] = \int\int d\left(b', z', g'\Gamma, g', d\right) F_z\left(dz' \mid z\right) F_g\left(dg' \mid g\right)$$

denotes the probability that the government decides to default if it purchases b' bonds, and the current default decision is d.

Equilibrium Concept

Definition 1 *A recursive competitive equilibrium is characterized by*

1. a set of value functions $V(b, z, \Gamma, g, h, x)$, $V_1(z, \Gamma, g, h)$, and $V_0(b, z, \Gamma, g, h)$;

2. a set of policies for asset holdings $b_0'(b, z, \Gamma, g, h)$ and $b_1'(b, z, \Gamma, g, h)$, and a default decision $d(b, z, \Gamma, g, h)$; and

3. a bond price function $q_d\left(b', z, \Gamma, g\right)$,

such that

(a) $V(b, z, \Gamma, g, h, x)$, $V_1(z, \Gamma, g, h)$, and $V_0(b, z, \Gamma, g, h)$ satisfy the system of functional equations (5)–(9);

(b) the default policy $d(b, z, \Gamma, g, h)$ and the policies for asset holdings $b_0'(b, z, \Gamma, g, h)$ and $b_1'(b, z, \Gamma, g, h)$ solve the dynamic programming problem specified by equations (5)–(9); and

(c) the bond price function $q_d\left(b', z, \Gamma, g, h\right)$ is given by equation (11).

Discussion of the Environment

The model analyzed in this article relies on several simplifying assumptions. This has the advantage that the model remains tractable and that the main mechanisms can be presented in a more transparent way. The disadvantage of using such a stylized framework is that the model is ill-suited to account for the quantitative behavior of some key variables.[11] The rest of this section discusses several simplifications embedded in the environment presented in the previous section and extensions that have been studied in the literature.

Focusing on an endowment economy simplifies the analysis. A more comprehensive study of the business cycle would require incorporating capital and labor into the model. Aguiar and Gopinath (2006) also consider an extension of the basic model with labor as the only input in the production function. The results do not change significantly. More recently, Bai and Zhang (2006) study a production economy with capital.

The model assumes that the government issues one-period bonds. Allowing the government to issue long-term bonds would introduce nontrivial complications to the analysis. For instance, if the government can issue two-period bonds, it is necessary to keep track of how much debt was issued two periods ago (which is due today) and how much debt was issued one period ago (which will be due tomorrow). Alternatively, if the government only issues annuities there would only be one state: how many annuities have been issued since the last default. However, the pricing of the annuities issued today would be more complex than the price of a one-period bond. Lenders would not only need to compute the probability of a default in the following period, but also the probability of a default two-periods ahead, conditional on not observing a default tomorrow; the probability of observing a default three-periods ahead;

[11] Other authors have studied different extensions of this framework, which have improved its quantitative performance. See, for example, Arellano (forthcoming); Arellano and Ramanarayanan (2006); Bai and Zhang (2006); Cuadra and Sapriza (2006a, b); Lizarazo (2005a, b); and Yue (2005).

conditional on not observing a default in the next two periods; and so on. Arellano and Ramanarayanan (2006) allow the government to issue short and long bonds.

It was assumed that the government cannot issue bonds contingent on the future realization of its endowment. Even if creditors have perfect information regarding the economy's endowment, this would not imply that contracts contingent on the endowment realization could be written (the endowment may not be verifiable). In reality, one limitation for writing contracts contingent on real variables is that the government could manipulate the measurement of these variables (see Borensztein and Mauro 2004). Determining to what extent bonds can be state contingent in reality and studying some degree of state contingency in quantitative models of sovereign default are interesting avenues for future research.

The assumption that countries experience an exogenous output loss after defaulting intends to capture the disruptions in economic activity entailed by a default decision. In general, default episodes are not observed in economic booms but in recessions. This means that a fraction of the low economic activity that is observed after a default episode can be explained by weak fundamentals pre-existing the default decision. Thus, not all of the decrease in economic activity observed after a default is related to the default decision and cannot be considered as a cost of defaulting. On the other hand, default decisions are likely to introduce disruptions in economic activity of the defaulting economy. IMF (2002), Kumhof (2004), and Kumhof and Tanner (2005) discuss how financial crises that lead to severe recessions are triggered by sovereign default. This is due to the fact that government debt is not only held by foreigners but also by locals, and in particular by local banks—something that is not explicitly considered in the stylized model studied in this article. Thus, government default may hurt financial intermediation significantly (see IMF 2002 for a discussion of recent episodes).[12]

In the model, the output loss triggered by the default decision is independent of the size of the default. If the output loss represents the damage made by the default decision through the local financial system, it could be argued that the loss should depend (positively) on the amount that is not repaid by the government (in particular, it should depend on the amount held by the locals; see, for example, IMF 2002). Considering this would introduce additional complications to the analysis though it is an interesting avenue to be pursued

[12] In the stylized model discussed in this article, the output loss of λ percent in the period after a default intends to capture the cost of defaulting implied by the disruptions in economic activity triggered by the declaration of default. The calibration of the parameter λ should capture these disruptions and does not intend to match the overall decrease in output observed after a default. In contrast to the exclusion from capital markets, the output loss does not intend to capture a punishment imposed by creditors. Eaton and Gersovitz (1981) discuss output loss as the result of punishments.

in future work. If the output loss depends on the amount not paid by the government, it can be argued that the government should also be allowed to choose the size of the default.[13] Arellano (forthcoming) argues that the output loss depends on the state of the economy and, thus, introduces this into the model.

Previous quantitative studies assume that after default, the economy suffers the output loss for a stochastic number of periods (the periods in which the economy is excluded from capital markets). For simplicity, this assumption is modified in this article. Assuming that the output loss lasts for a stochastic number of periods in a context in which there is no financial exclusion raises the possibility of scenarios in which the government defaults before the duration of output losses triggered by the previous default has ended. This would require keeping track of the number of output losses the economy is suffering and would increase the dimensionality of the state space. The Appendix shows that the results are not sensitive to the modification of the output loss process utilized in this article.

The assumption that countries are excluded from capital markets after a default episode is motivated by evidence of a drainage in capital flows into countries that defaulted (see, for example, Gelos, Sahay, and Sandleris 2004). However, it very well may be that the difficulties in market access observed after a default episode respond to the same factors that triggered the default decision itself.[14] In support of this, Gelos, Sahay, and Sandleris (2004) document that once other variables are used as controls, market access is not significantly influenced by previous defaults (see also Eichengreen and Portes 2000 and Meyersson 2006). Moreover, it is not obvious that after a default episode competitive creditors would be able to coordinate cutting off credit to defaulting countries. Thus, the study of an environment in which a defaulting economy cannot be excluded from capital markets is the first building block of any work that attempts to explain the exclusion outcome as an endogenous outcome of the model.

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[13] In this article, the government must decide whether it honors all the debt issued in the previous period or whether it defaults on all of it. But this is not a restrictive assumption given that the costs of defaulting are orthogonal to the amount of debt that is repudiated. In this case, the government would never find it optimal to default on less than a 100 percent of the outstanding debt. This would not be the case if the cost of defaulting depends on the amount repudiated. Yue (2005) studies partial default in an environment in which the defaulted amount is decided in a bargaining process between the government and the lenders.

[14] For example, Hatchondo, Martinez, and Sapriza (2006b) analyze a model in which both default and the difficulties in market access after default may be triggered by a change in the policymaker in power.

Table 1 Parameter Values Specific to Models I and II

	Model I	Model II
ρ_g	–	0.17
σ_g	0%	3%
ρ_z	0.90	–
σ_z	3.4%	0%

Notes: A period in the model corresponds to a quarter.

3. PARAMETERIZATION

The model is solved numerically using value function iteration and interpolation as in Hatchondo, Martinez, and Sapriza (2006a).[15] Whenever possible, this article considers the same parameter values as in Aguiar and Gopinath (2006), which facilitates the comparison of the results. To solve the model numerically, Bellman equations are first recast in detrended form. All variables are normalized by $\mu_g \Gamma_{t-1}$ as in Aguiar and Gopinath (2006). This normalization implies that the mean of the detrended endowment is one.

Even though Aguiar and Gopinath (2007) argue that the best representation of the output process for emerging economies is characterized by equations (1) through (4), this specification requires keeping track of z and g as state variables. The computational method used in previous articles in the literature does not allow solving for this specification without incurring sizable approximation errors. Instead, they consider two alternative endowment processes. In Model I, the economy is hit only with transitory shocks (z shocks). In Model II, the economy is hit with shocks to the trend only (g shocks). Table 1 reports the parameter values specific to each of the two model alternatives.

Parameters values that are common across models are presented in Table 2. Aguiar and Gopinath (2006) assume an output loss of 2 percent during the exclusion period. This is based on empirical estimates of the output loss triggered by a default decision (see Chuhan and Sturzenegger 2005). As explained above, for simplicity this article assumes that all output loss occurs only in one period, the period that follows the decision. The value of λ is calibrated to make the output-loss cost of defaulting in this article equivalent to the one in Aguiar and Gopinath (2006). In particular, the value of λ is chosen to be such that for Model I, the mean debt level in the simulations is the same as the one in the original formulation of Aguiar and Gopinath (2006). We show that this value (of λ) enables Model II to generate a similar level of debt as in Aguiar and Gopinath (2006).

[15] The value functions V_0 and V_1 are approximated using Chebychev polynomials. Fifteen polynomials on the asset space and ten on the endowment shock are used. Results are robust to using more polynomials.

Table 2 Parameter Values Common to Models I and II

Risk Aversion	σ	2
International Interest Rate	r	1%
Probability of Redemption in the Same Period of Default	ϕ_1	0%
Probability of Redemption	ϕ	10%
Mean Growth Rate	μ_g	1.006
Mean (log) Transitory Productivity	μ_z	$-0.5\sigma_z^2$
Discount Factor	β	0.8
Loss of Output	λ	8.3%

Notes: A period in the model corresponds to a quarter.

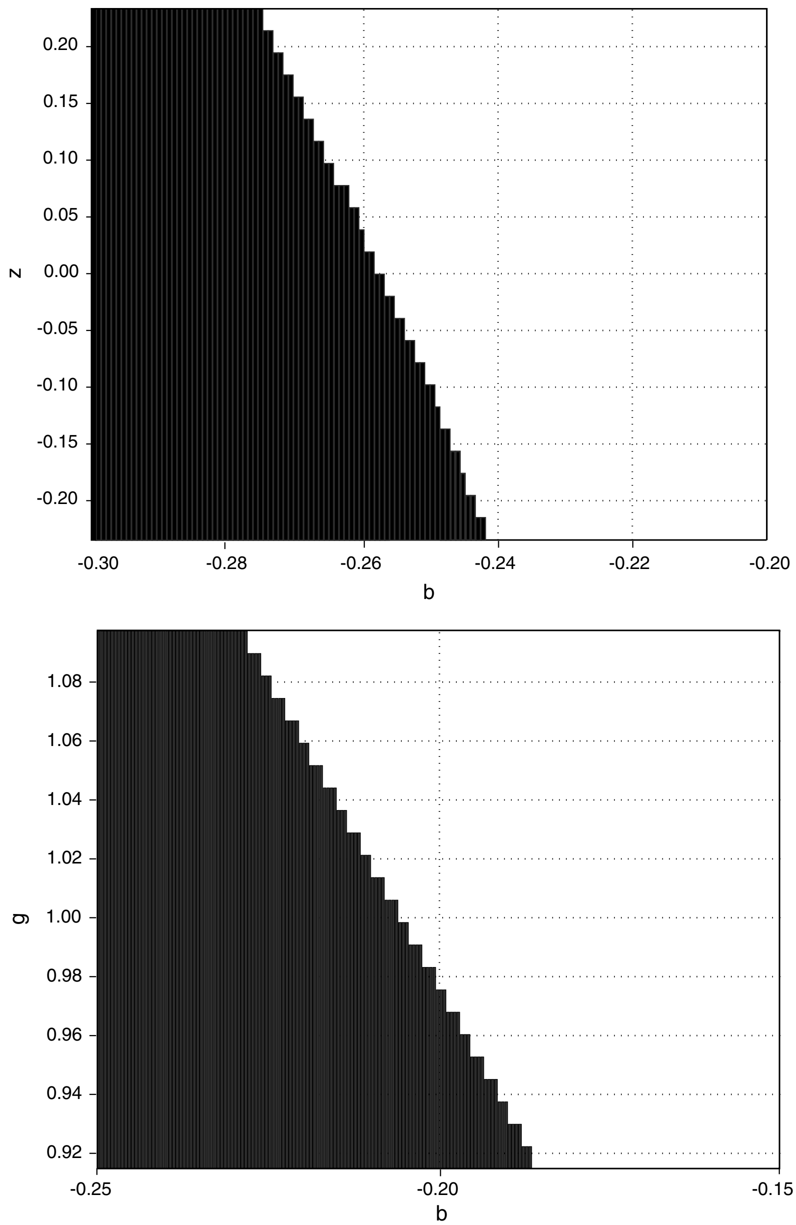
Except for the value of λ, the remaining parameters take the same values as in Aguiar and Gopinath (2006). The coefficient of relative risk aversion of 2 is within the range of accepted values. The probability of redemption implies an average autarky duration of 2.5 years (in the model, a period refers to a quarter), similar to the value estimated by Gelos, Sahay, and Sandleris (2004)—Section 5 presents the results when creditors cannot exclude a defaulting economy. The process of output is calibrated to match the process for Argentina from 1983 to 2000. The subjective discount factor is set to 0.8. This departs from standard macro models. As in Aguiar and Gopinath (2006), in the stylized framework discussed in this article, a low discount factor is needed to induce the economy to accumulate debt and be willing to accept a higher spread over the risk-free interest rate (the international interest rate). The limitations faced by the stylized framework to generate default with more reasonable discount factors may be a consequence of its simplifying assumptions.[16] As was mentioned previously, recent articles study different extensions of this framework that improve its quantitative performance (assuming that lenders can use financial exclusion as a punishment).

4. RESULTS WITH EXCLUSION

This section presents the results obtained when the parameters that determine the exclusion process are chosen as in Aguiar and Gopinath (2006), that is, $\phi_1 = 0$ and $\phi = 0.1$. This means that the government cannot issue bonds in the period it defaults, and after that it faces a constant probability ϕ of regaining access to capital markets.

[16] For example, the end of Section 4 describes how the assumption that the government can only issue one-period bonds increases the marginal issuance cost. If it were not for the low discount factor, the issuance volume and spreads observed in equilibrium would be even lower than what is observed in the data.

Figure 2 Default Regions



Notes: Default region when the endowment is hit with transitory shocks (top panel) and growth trend shocks (bottom panel).

Equilibrium Default Region and Bond Prices with Exclusion

The shaded areas in Figure 2 display the default regions (i.e., the combinations of endowment shocks and debt levels for which the economy would choose to default) of the model with transitory and trend shocks.[17] Both graphs show that the higher the endowment shock, the higher the minimum debt level at which it is optimal to declare a default. From another perspective, for a given initial debt level, the government finds it optimal to default only if the endowment shock is sufficiently low.

The benefit of defaulting is that resources that would have been allocated to pay back previously issued bonds are, instead, allocated to current consumption. There are two costs entailed by a default decision: a loss in output and the inability of the government to use international capital markets to smooth out domestic endowment shocks. It should be noticed that the "costs" of defaulting do not depend on the debt level at the time of default. Thus, a higher initial debt level increases the benefits of a default without increasing the costs. For sufficiently large debt levels, the benefits of defaulting offset the "fixed" costs. This explains why in Figure 2 it is optimal for the government to default on relatively large values of debt (low b).

A low endowment shock implies that there are less resources available in the current and subsequent periods.[18] Given that the output loss that follows a default decision is a constant fraction of the underlying potential output, it is more costly to default for high endowment realizations than for low endowment realizations. In the model with transitory shocks, this is the main force behind the negative relationship between the shock to the endowment and the debt threshold at which the government is indifferent between defaulting and not defaulting.[19] However, in the model with trend shocks, a high shock today signals higher growth rates in the future. This increases the desire to borrow as it allows bringing future resources to the current period. The fact that the ability to borrow is more valuable in good than in bad times helps explain why the government defaults only on larger debt volumes in good times.

Figure 3 shows the equilibrium bond prices faced by the government as a function of the current issuance level ($-b$) and the endowment shock. The

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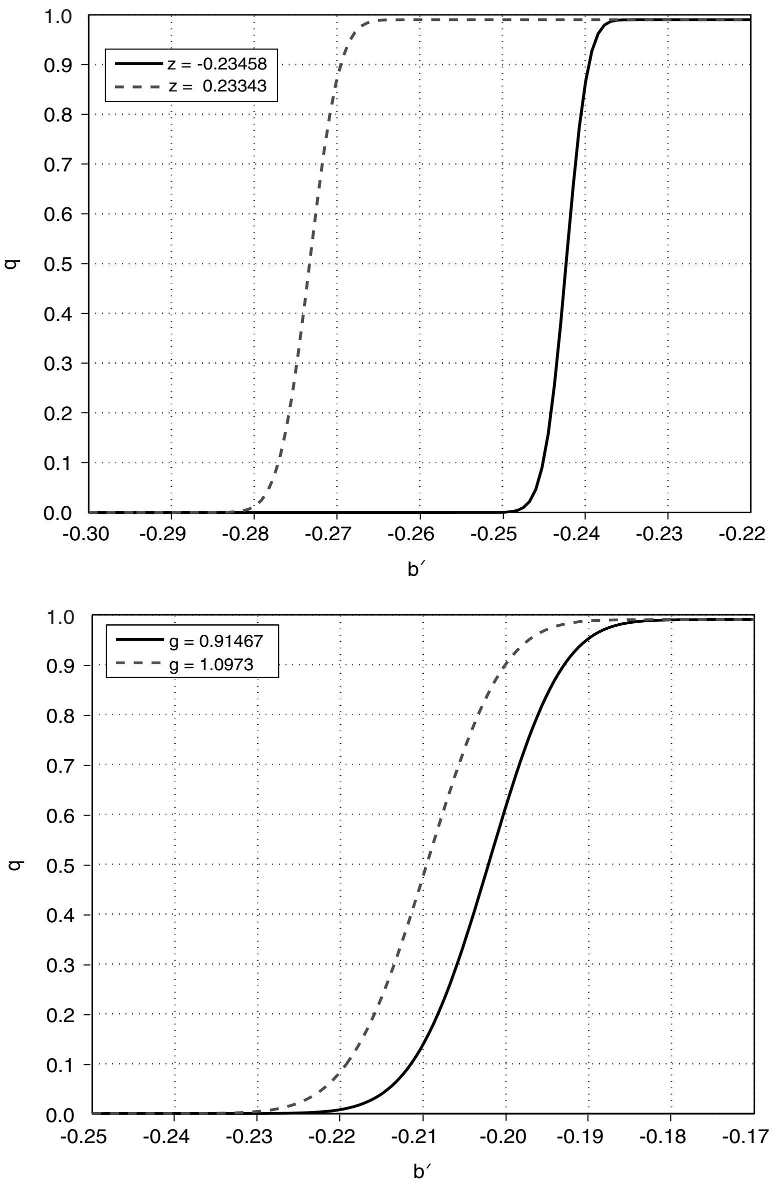
[17] The detrended output process has a mean of 1. This implies that the debt levels b in Figure 2 correspond to the ratio of debt-to-mean output.

[18] Both the trend shocks and the transitory shocks display positive autocorrelation.

[19] If the endowment is low, the marginal utility of consumption is high, and therefore the gain from defaulting is high. However, if the model with transitory shocks is solved assuming that the output loss is a fixed amount instead of a percentage of output, the default region becomes almost vertical but with a positive slope. This is due to the fact that with a high endowment shock the economy displays a less intense desire to issue debt, and therefore it assigns a lower value to retaining access to capital markets.

Figure 3 Bond Price Menus



Notes: Equilibrium bond price menu faced by the government at low and high endowment realizations. The low endowment realization is three standard deviations to the left of the unconditional mean. The high endowment realization is three standard deviations to the right of the unconditional mean. The top panel shows the equilibrium bond price in the model with shocks to the level. The bottom panel shows the equilibrium bond price in the model with shocks to the trend.

curves have a waterfall shape. For relatively low issuance levels there is no risk of default. In this case, competitive investors demand the risk-free rate in compensation for purchasing the government's bonds. For issuance volumes for which there is a positive probability of default tomorrow, the rate of return demanded for holding bonds is higher than the risk-free rate, i.e., the price offered is lower than $\frac{1}{1+r}$. Finally, for sufficiently high issuance volumes, it is common knowledge that the government would default in the following period for almost any endowment realization. In this case, investors offer a zero price for each bond issued today.

It should be noted that price q is nondecreasing in the current endowment realization. In other words, the higher the endowment, the higher the issuance level at which the price starts to fall. This is due to the persistence in the endowment process and the shape of the default regions. A higher endowment today implies that it is more likely to observe high endowments in the following period, and therefore it makes the default probability lower.

Business Cycle Properties With the Exclusion Punishment

The model is simulated for 750,000 periods (500 samples of 1,500 observations each). In order to compute business cycle statistics, 400 samples of the last 72 periods before a default episode are used. The samples selected are such that the last exclusion period was observed at least two periods before the first period in the sample. The number of periods in each sample is equal to the number of periods in the data compared with the simulations (Argentina 1983–2000). Restricting to samples at least two periods away from the last exclusion period helps avoid extreme observations that may distort the results.[20] The moments reported below correspond to the average across the 400 samples. The behavior of four series is analyzed: the logarithm of income (y), the logarithm of consumption (c), the ratio of the trade balance to output (tb), and the annualized spread (R_s). All series are filtered using the Hodrick-Prescott filter with a smoothing parameter of 1600. Standard deviations are denoted by σ and are reported in percentage terms; correlations are denoted by ρ.

Table 3 reports business cycle moments observed in the data (Argentina 1983–2000) and in Models I and II. With the exception of the debt-to-output

[20] In the periods that follow an exclusion period, the government inherits little or no debt from previous periods. The consequence is that in these periods the government borrows a relatively low amount, and thus it pays relatively low spreads compared to what is observed in the remaining observations. In the simulations, these outliers may appear up to two periods after the end of an exclusion period. It is judged that it is more appropriate to present results computed without considering these outliers. Simulation results are contrasted against data from Argentina during a period in which the economy was not excluded from capital markets. Moreover, these outliers can alter the calculations of business cycle statistics (see Hatchondo, Martinez, and Sapriza 2006a).

Table 3 Business Cycle Statistics

	Data	Model I Transitory Shocks	Model II Trend Shocks
$\sigma(y)$	4.08	4.14	4.15
$\sigma(c)$	4.85	4.23	4.38
$\sigma(tb)$	1.36	0.20	0.63
$\sigma(R_s)$	3.17	0.006	0.013
$\rho(c, y)$	0.96	0.99	0.99
$\rho(tb, y)$	-0.89	-0.43	-0.29
$\rho(R_s, y)$	-0.59	-0.80	-0.06
$\rho(R_s, tb)$	0.68	0.85	0.89
Mean Debt Output Ratio (%)	51	6.3	4.8
Rate of Default	75	6.6	24

Notes: The moments correspond to the average across 400 samples. The debt output ratio is measured as the stock of debt divided by the annual output level.

ratio, the business cycle moments for Argentina are taken from Aguiar and Gopinath (2006). The moments are chosen so as to evaluate the ability of the models to replicate the distinctive business cycle properties of emerging economies that were described in the beginning of the article. It must be said that the sample moments for Argentina display the same qualitative features observed in other emerging markets.[21]

The moments in the simulated samples generated by Models I and II are different from the moments reported in Aguiar and Gopinath (2006). The main reason is that they use a different computational method from the one used in this article. While, Aguiar and Gopinath (2006) use a discrete state space method, we use interpolation methods and a nonlinear optimization routine to find the optimal issuance levels. Hatchondo, Martinez, and Sapriza (2006a) demonstrate that the numerical errors incurred by the discrete state space technique may lead to misleading conclusions in some dimensions. The most important one is that the spread volatility becomes negligible once the model is solved using a more accurate method. Other statistics about the behavior of the spread over the business cycle are also susceptible to numerical errors when the model is solved using a discrete state space technique.[22]

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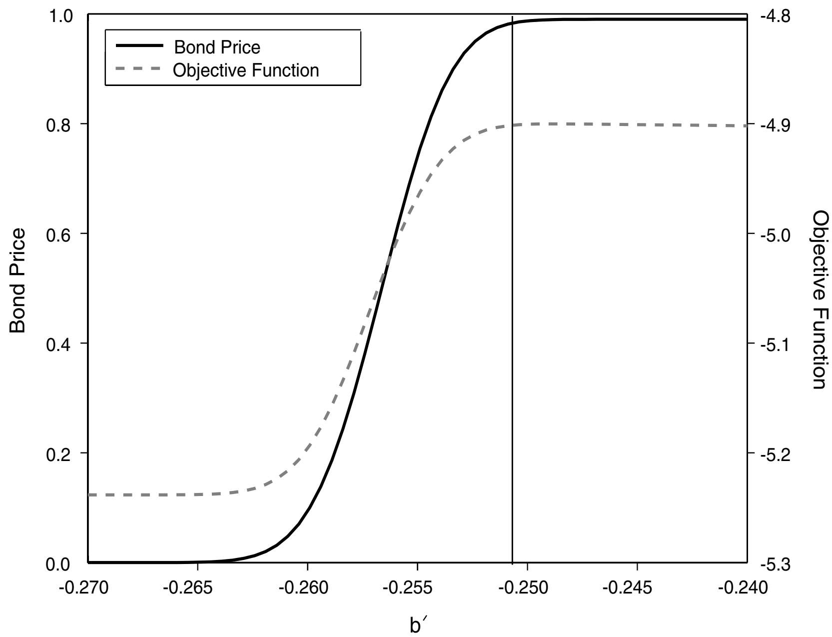
[21] As previously mentioned, emerging economies feature interest rates that are high, volatile, and countercyclical; high volatility of consumption relative to income (typically, higher than one); and countercyclical net exports (see, for example, Aguiar and Gopinath 2007; Neumeyer and Perri 2005; and Uribe and Yue 2006).

[22] A second reason behind the discrepancy between the moments in Table 3 and in Aguiar and Gopinath (2006) is that the setups are not exactly the same. In their model, the output loss lasts as long as the exclusion punishment. In the present setup, the output loss lasts for only one period. The Appendix shows that this difference accounts for only a small fraction of the discrepancy in the performance of the two models.

Figure 4 The Effect of the Bond Price Menu on the Objective Function



Notes: Objective function and price function faced by the government when $z = \mu_z$ and b $= -0.25$ (which is within the range observed in the simulations). The vertical line represents the optimal issuance level.

The table shows that both models fail to generate the volatilities of the trade balance and spread observed in the data. In particular, the standard deviations of the spread are two orders of magnitude lower than the value observed in the data. This is an important limitation of both models. Moreover, Models I and II generate 6.6 and 24 defaults in 10,000 periods, respectively. These values are below the ratio of 75 defaults in 10,000 periods computed by Reinhart, Rogoff, and Savastano (2003) using a sample of emerging markets from 1824 to 1999, though it is not clear that this is the frequency that the model should replicate.[23] An alternative procedure is to compare the default rate generated by the model with the default rate implicit in the average spread observed over the sample period (under the assumption of risk-neutral lenders). The value

[23] The model is calibrated to match the macroeconomic behavior of Argentina between 1983 and 2000, while the default rate computed in Reinhart, Rogoff, and Savastano (2003) is based on a different time period and a sample of various countries.

of the latter is 243 defaults for every 10,000 periods, which is even further away from the model predictions.

On the other hand, the models are able to generate a high volatility of consumption relative to income and the sign of the co-movements between the trade balance, spread, and output that are observed in the data.[24]

Discussion of the Results With the Exclusion Punishment

This section describes the tradeoffs the government faces when it decides how much debt to issue. This helps in understanding the logic behind the results presented in Table 3. The section focuses on the model with transitory shocks (z), though the same logic applies to the model with trend shocks.

Given the monotonicity of the default decisions (see Figure 2), the bond price function faced by the government when it decides how much to borrow can be written as

$$q\left(b', z\right) = \frac{1 - F\left(z^{*}\left(b'\right) \mid z\right)}{1 + r}, \tag{12}$$

where $z^{*}\left(b'\right)$ denotes the next period endowment shock that makes the government indifferent between defaulting and not defaulting on a debt level b', and F denotes the cumulative distribution function for the next period shock. If the endowment shock in the following period is lower than z^{*}, the government will default on b'. If it is higher than z^{*}, the government will pay back b'. In the top panel of Figure 2, $z^{*}\left(b'\right)$ represents the frontier of the shaded area. Equation (12) shows that the shape of price q mirrors the shape of the probability of observing a default the next period, namely, $F\left(z^{*}\left(b'\right) \mid z\right)$.

The solid line of Figure 4 displays the bond price menu faced by the government in a period in which the endowment realization is equal to the unconditional mean of the endowment process (μ_z) and the economy is not excluded from capital markets. The sensitivity of the bond price to the issuance volume ($b^{'}$) is given by

$$\frac{q\left(b', z\right)}{\partial b^{'}} = \frac{-f\left(z^{*}\left(b'\right) \mid z\right)}{1 + r} \frac{\partial z^{*}\left(b'\right)}{\partial b^{'}}, \tag{13}$$

where f denotes the density function of future shocks. This equation shows that the shape of the bond price depends on two factors: the probability distribution f and the sensitivity of z^{*} to changes in $b^{'}$ (the shape of the default region). The assumption that future endowment shocks are drawn from a

[24] The table does not report statistics about the current account. Given that both models generate a relatively stable debt level and a low volatility of the interest rate, interest payments also display low volatility. Therefore, the balance of the current account is almost perfectly correlated with the trade balance and inherits the statistical properties of the latter.

Gaussian distribution accounts for the flat portion of the price curve. The thin tails of f explain why the price is almost invariant to $b^{'}$ at issuance volumes such that the threshold z^* takes extreme values.

The bond price plays a central role in understanding the shape of the objective function of the government represented in Figure 4. Formally, the objective function is given by the right-hand side of the Bellman equation:

$$RHS\left(b'\right) = u\left(y\left(1-\lambda\right) + b - q_0\left(b', z\right)b'\right) + \beta \int V(b', z', 0, 0) F_z\left(dz' \mid z\right).$$

For the range of values of b' such that there is no default risk, the present discounted welfare increases with the issuance level, i.e., the burden of starting tomorrow with higher liabilities does not compensate for the extra resources collected for current consumption.[25] As the price per bond starts to fall, there is an extra factor that appears in the tradeoff between current and future consumption: an extra dollar of borrowing implies a lower bond price. In particular, an extra dollar of borrowing implies a decrease in price q received for all the bonds issued in the current period.

If the price function is steep, borrowing an extra dollar is quite costly due to the decrease in bond price received for all bonds issued in the current period. In the stylized model presented in this article, the price function becomes very steep at borrowing levels at which the government pays an interest rate close to the risk-free rate. Consequently, the borrowing levels observed in equilibrium are such that the economy pays low spreads. This explains the low default frequency reported in Table 3.

The top panel of Figure 3 shows that a higher endowment realization enables the government to borrow more without paying a higher spread, but the price function becomes steep at borrowing levels for which the spread is low independently of whether the endowment shock is "low" or "high." This feature contributes to the explanation of why in equilibrium the government chooses to pay low spreads at all endowment realizations, and thus the volatility of the spread is low.

The inability of the model to generate a higher default rate and spread volatility may be a consequence of its simplifying assumptions. Consider, for instance, the assumption that the government can only issue one-period bonds. Recall that as illustrated in Figure 4, the interest rate increases with the borrowing level (the bond price decreases). The assumption of one-period bonds implies that in every period the economy has to roll over its entire stock of debt. Thus, the increase in the interest rate that is due to an extra dollar of borrowing affects the entire stock of bonds and not only the last unit issued. More precisely, consider the decision of whether to borrow $x + 1$ dollars or x

[25] It should be stressed that this result is not general and critically depends on the parameter values chosen, especially the value of the subjective discount factor.

dollars. When the government is renewing its entire stock of debt, the higher interest implied by borrowing $x+1$ instead of x applies to the $x+1$ units issued. This is trivially larger than the cost induced by an increase in the interest rate paid for the last bond issued. This argument illustrates how restricting the government to issue one-period bonds increases the marginal issuance cost, and thus accounts for a fraction of the low spreads generated by the model.

Even though the government is risk averse and lenders are risk neutral, the volatility of consumption is higher than the volatility of output in both models. As discussed in the beginning of Section 4, price q is nondecreasing in the current endowment realization. That is, a higher endowment enables the government to issue more bonds without necessarily paying a higher spread. Given the low value of the discount factor, the economy will seize the opportunity to borrow more whenever it appears, explaining why the economy borrows more in good times. This explains why the model is able to generate a higher volatility of consumption relative to income. Formally, current consumption is determined by current income and net borrowing, namely,

$$c = y - \left(qb' - b\right).$$

Therefore,

$$\sigma^2(c) = \sigma^2(y) + 2\sigma[y - (qb' - b)] + \sigma^2\left(qb' - b\right).$$

The positive covariance between net borrowing $(b - qb')$ and income increases the volatility of consumption relative to income.

Table 3 shows that both models are able to replicate the sign of the comovements between trade balance, spread, and output observed in the data. The fact that the government borrows more in good times leads to a negative correlation between trade balance and output (as observed in the data).[26] The mechanics that determine the sign of the correlation between the spread and output are more complex. On the one hand, if the bond price function faced by the government is kept constant, a higher income realization today reduces the need to borrow, and therefore it reduces the spread that the government is willing to pay for its debt. This generates a negative correlation between income and spread. But the bond price function also changes with the income realization. If the price of the bond becomes less sensitive to the borrowing level at higher income levels, the government would be willing to pay a higher spread at higher income levels. The latter may change the sign of the correlation between the spread and income. For example, the next section shows that

[26] The trade balance is defined as

$$tb = y - c = qb' - b.$$

Thus, the positive correlation between net borrowing and income translates into a negative correlation between trade balance and income.

once the exclusion punishment is eliminated, the spread becomes pro-cyclical in Model II.

5. RESULTS WITHOUT EXCLUSION

This section studies the implications of removing the threat of financial exclusion. Formally, this implies setting the value of ϕ_1 equal to 1. The section describes how the default and saving decisions change when the government cannot be threatened with financial exclusion. This helps to understand how the business cycle statistics change when the exclusion assumption is abandoned, which is discussed later in the section entitled "Business Cycle Properties."

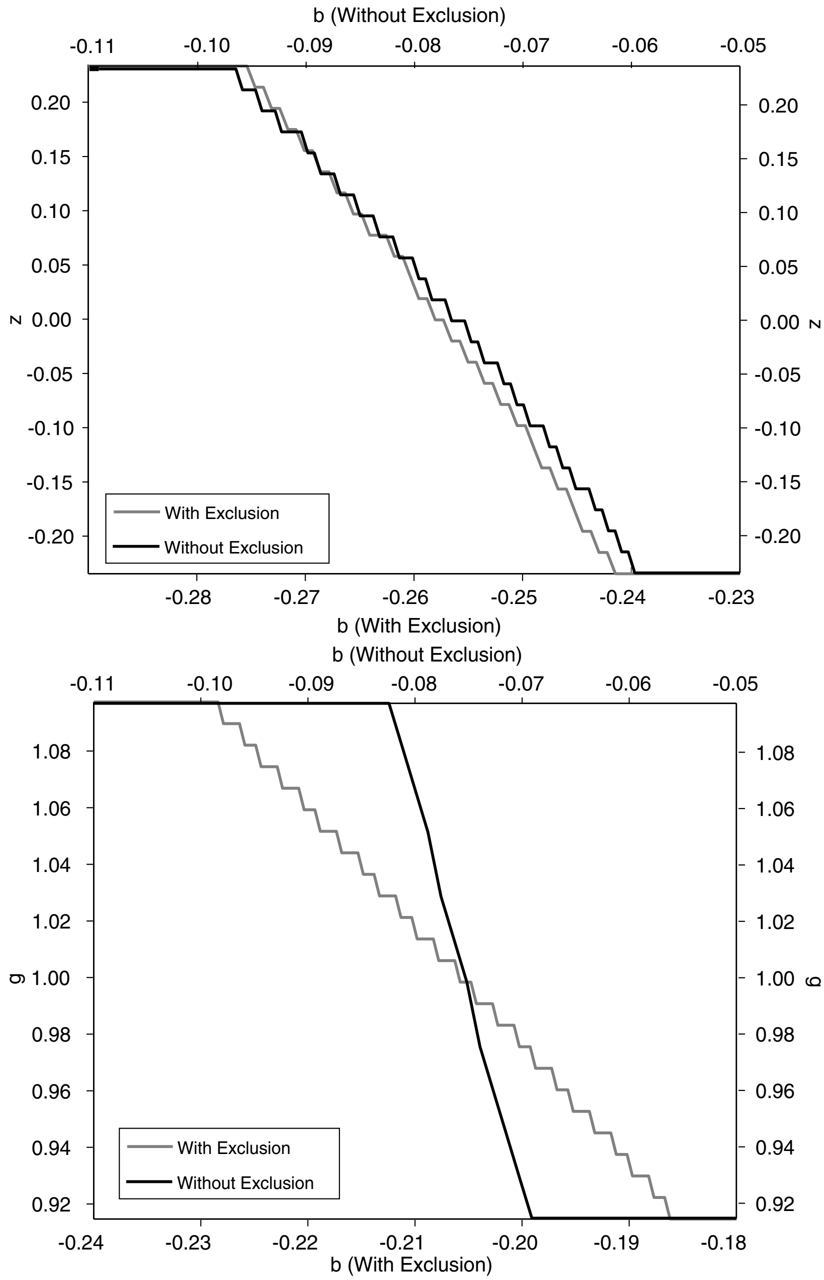
Equilibrium Choices Without the Exclusion Punishment

The common feature across Models I and II is that they are able to sustain a lower debt level when the government does not face the threat of financial exclusion. In other dimensions, the model with transitory shocks without exclusion features higher equilibrium spreads (lower equilibrium issuance prices) and spreads that are more responsive to the endowment shock compared to the model with transitory shocks and exclusion. On the other hand, the model with trend shocks features lower equilibrium spreads and spreads that are less responsive to trend shocks compared to the model with exclusion.

Figure 5 illustrates how the default decisions change when exclusion cannot be used as a punishment in Models I and II. The graphs illustrate that the government defaults at lower debt levels when the threat of financial exclusion is eliminated. The result is not surprising though. The possibility of being excluded from capital markets imposes a cost to every default decision (regardless of the size of the default). Once the threat of financial exclusion is eliminated, it becomes less costly to default, and thus the government finds it optimal to default at lower debt levels.

The top panel of Figure 5 shows that in Model I the default decision becomes relatively more sensitive to the endowment shock (and less sensitive to the debt level) when the threat of exclusion is eliminated. This is due to the fact that the possibility of going into financial autarky is more painful at lower endowment realizations than at higher endowment realizations. When the current shock to the endowment level is higher, the need to smooth out consumption by borrowing is weaker, and therefore the value assigned to retaining access to capital markets is lower. In other words, the government would suffer less from being excluded from capital markets if its endowment level is higher. When the threat of financial exclusion is eliminated, the overall cost of defaulting decreases more at lower endowment realizations than at

Figure 5 Default Regions With and Without Exclusion



Notes: Endowment shocks at which the government is indifferent between defaulting and not defaulting in the model with *transitory* shocks (top panel) and in the model with *trend* shocks (bottom panel). The scale on the bottom corresponds to the models *with exclusion*. The scale on the top corresponds to the models *without exclusion*.

higher endowment realizations. This accounts for the flatter default region in the top panel of Figure 5.

The picture looks different in the model with trend shocks. The bottom panel of Figure 5 shows that the default region becomes steeper without exclusion. In this case, the mechanism described in the previous paragraph is also present, but there is an additional effect. A higher growth rate in the current period not only means that there are more resources available for current consumption but also that future growth rates are likely to be high—recall that there is persistence in growth rates. Consequently, unlike a higher transitory shock, a higher growth shock introduces an incentive to borrow more on account of the future increases in the endowment. This means that the value assigned to retaining access to capital markets is larger when the current growth rate is higher. When the threat of financial exclusion is eliminated, the overall cost of defaulting decreases more at higher endowment realizations than at lower endowment realizations. This explains the change in the slope of the default regions in the model with trend shocks.

The change in the shape of the default region plays an important role in understanding the change in the shape of the price function. The formal link between the two is described in equation 13. For example, the steeper the default region (the higher the expression $\left|\frac{\partial z^*(b')}{\partial b'}\right|$), the steeper the price function.

Figure 6 shows the price functions faced by the economy in Model I with and without exclusion.[27] The charts in Figure 6 show that when the threat of exclusion is eliminated, the bond price starts to decrease at a lower issuance level. This mirrors the shift of the default region due to a lower cost of default. The graphs show that the moderate change in the slope of the default regions observed in the top panel of Figure 5 translate into a moderate change in the slope of the price functions.

Figure 7 shows the price function faced by the economy in Model II with and without exclusion.[28] The charts show that the steeper default regions that are observed when the threat of financial exclusion is eliminated translate into steeper price functions.

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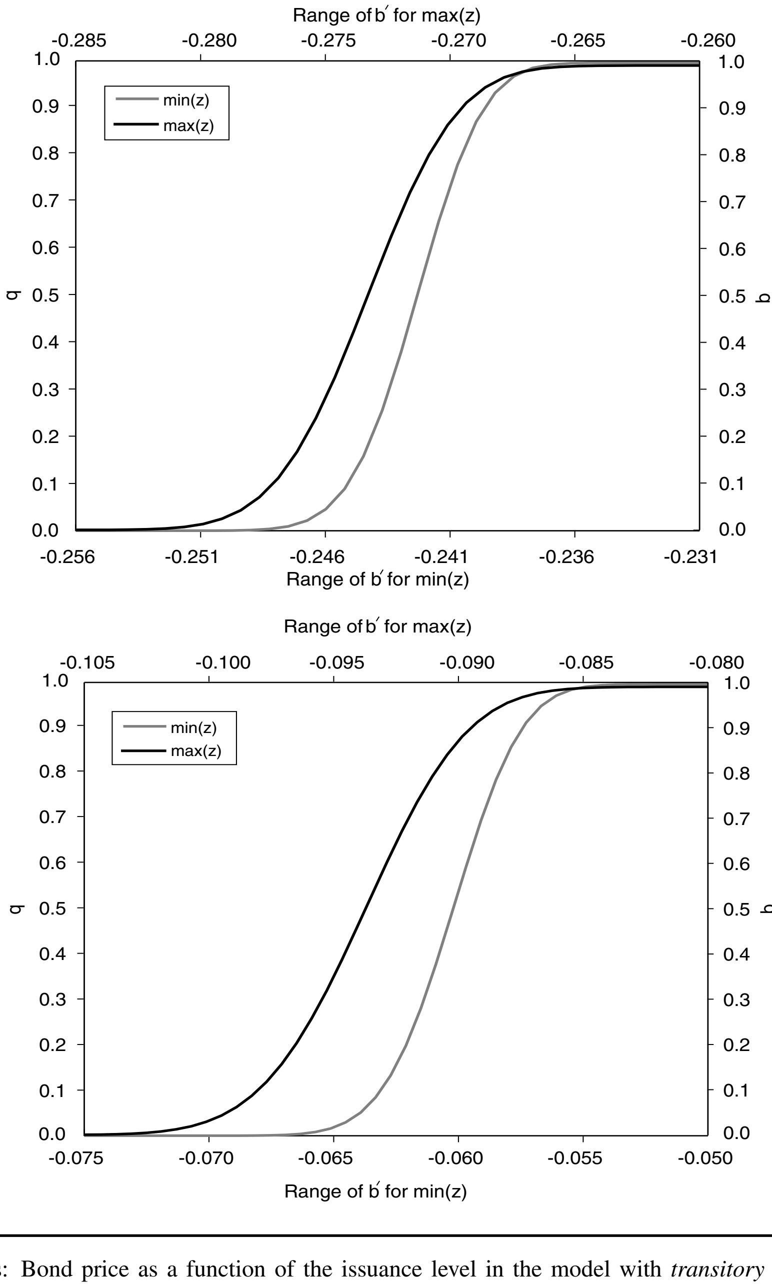
[27] Notice that the top panel of Figure 6 represents the same functions as the top panel of Figure 3 but with different scales. The graph is reproduced again in order to facilitate the comparison of the shape of the price functions in the models with and without exclusion.

[28] The top panel of Figure 6 represents the same functions as the bottom panel of Figure 3 but with different scales.

Figure 6 Bond Price Menus in Model I With and Without Exclusion



Notes: Bond price as a function of the issuance level in the model with *transitory* shocks and *exclusion* (top panel) and in the model with *transitory* shocks and *without exclusion* (bottom panel). The high (low) shock is three standard deviations higher (lower) than the unconditional mean of z. The scale of bond issuances when the endowment shock is low is described by the bottom horizontal axes. Likewise, the scale of bond issuances when the endowment shock is high is described by the top horizontal axes. The different scales used in the horizontal axes facilitate the comparison of the shape of the price functions.

Figure 8 displays the equilibrium issuance price in Model I as a function of the endowment shock realization in the specifications with and without exclusion. The graphs show that the price at which the government issues debt is lower and more sensitive to the endowment shock in the setup without exclusion.

Figure 9 shows the bond prices paid in equilibrium in the model with shocks to the trend with and without exclusion. The graphs show that the prices at which the government issues debt are higher in the setup without exclusion. The correlation with the endowment shock also changes. When the government can be threatened with financial exclusion the spread decreases with respect to the shock to the trend. But when the government cannot be threatened with financial exclusion, the spread increases with respect to the shock to the trend.[29]

The Mechanics of the Equilibrium Behavior of the Spread

This section discusses the differential behavior of the spread in Models I and II once the exclusion assumption is abandoned. Consider first the Euler equation that determines the optimal borrowing level

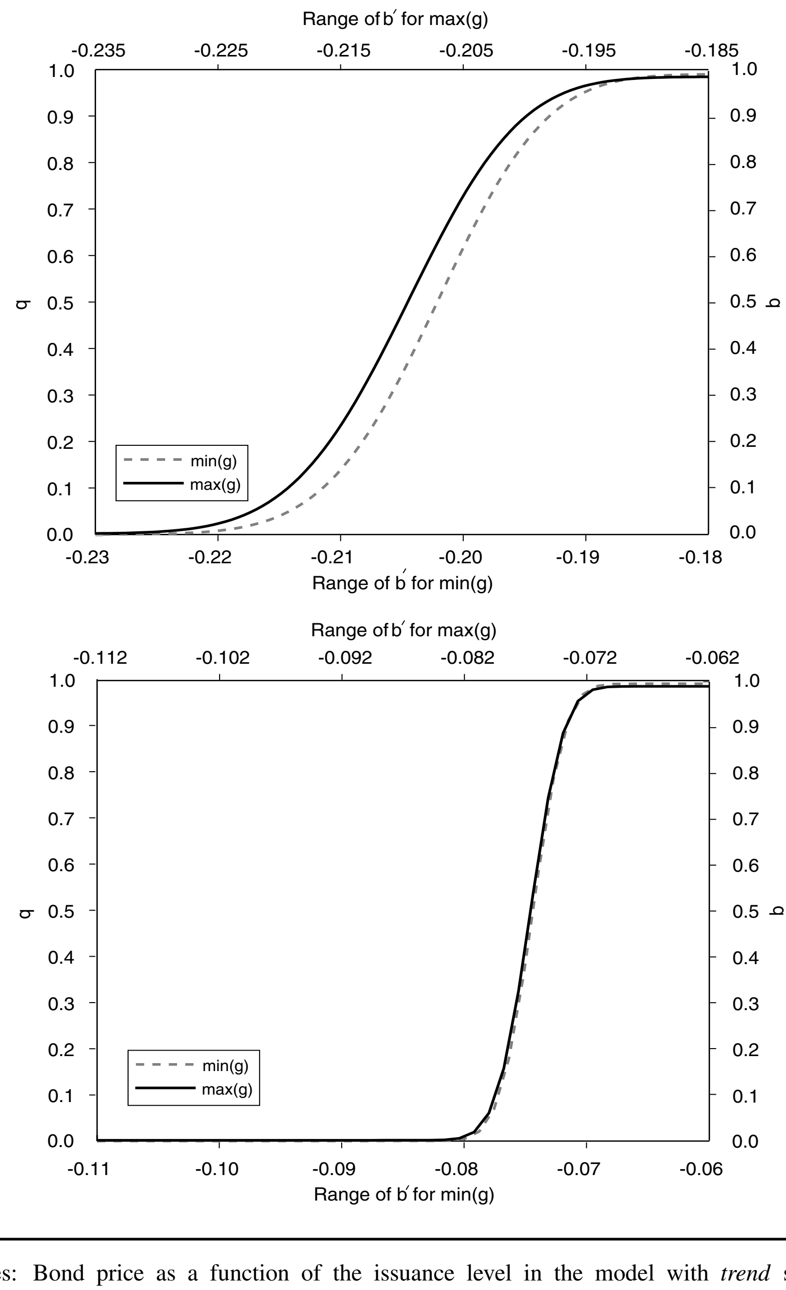
$$u'(c)\, q_0(b', z, \Gamma, g) = \left\{ \begin{array}{c} \beta \int \int \frac{\partial V(b',z',\Gamma',g',h)}{\partial b'} F(dz' \mid z) F(dg' \mid g) - \\ u'(c) b' \frac{\partial q_0(b',z,\Gamma,g)}{\partial b'}. \end{array} \right\} \quad (14)$$

The left-hand side of the equation captures the marginal benefit of issuing one more bond today, i.e., the increase in current consumption. The right-hand side captures the marginal costs. The first term represents the "future marginal cost." Issuing one more unit of debt today makes the economy poorer in the future—the government will either have to pay back its debt or face the cost of defaulting. The second term on the right-hand side represents the "present marginal cost." This is the cost derived from decreasing the price of all the bonds issued today. The role of the latter was discussed more extensively in the end of Section 4.

In both models the optimal issuance volume is lower when the government does not face the threat of default. An immediate consequence is that it depresses the present marginal cost (the second term in the right-hand side of equation (14) is the product of the borrowing level times the sensitivity of the price to the borrowing level). The decrease in the marginal cost induced by a lower borrowing level may be compensated, in part, by accepting a lower price for each bond issued, which reduces the marginal benefit of borrowing. This can explain the lower equilibrium bond prices (higher spread) that are observed in Model I when the threat of exclusion is eliminated (see Figure 8).

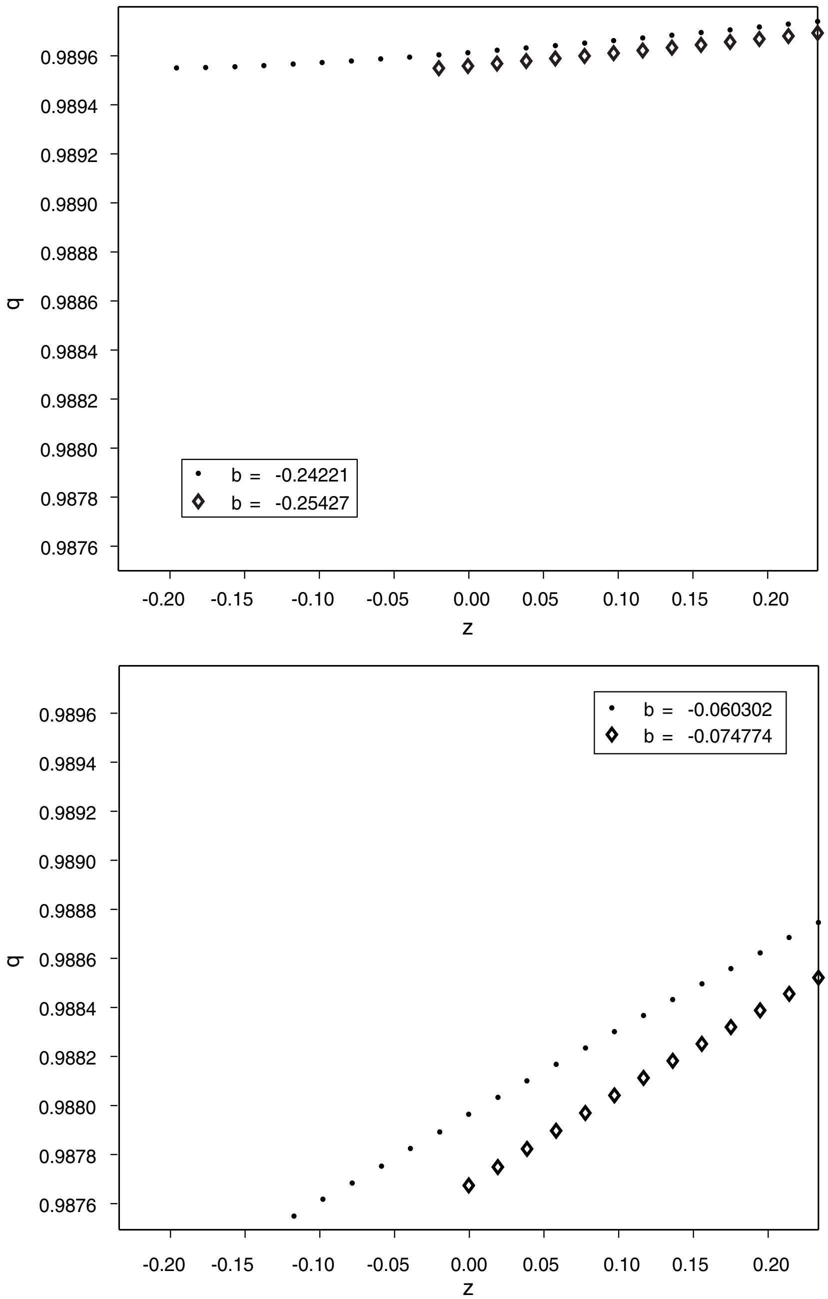
[29] The beginning of Section 5 provides some intuition for the differential behavior of the spread displayed by Models I and II once the exclusion assumption is abandoned.

Figure 7 Bond Price Menus in Model II With and Without Exclusion



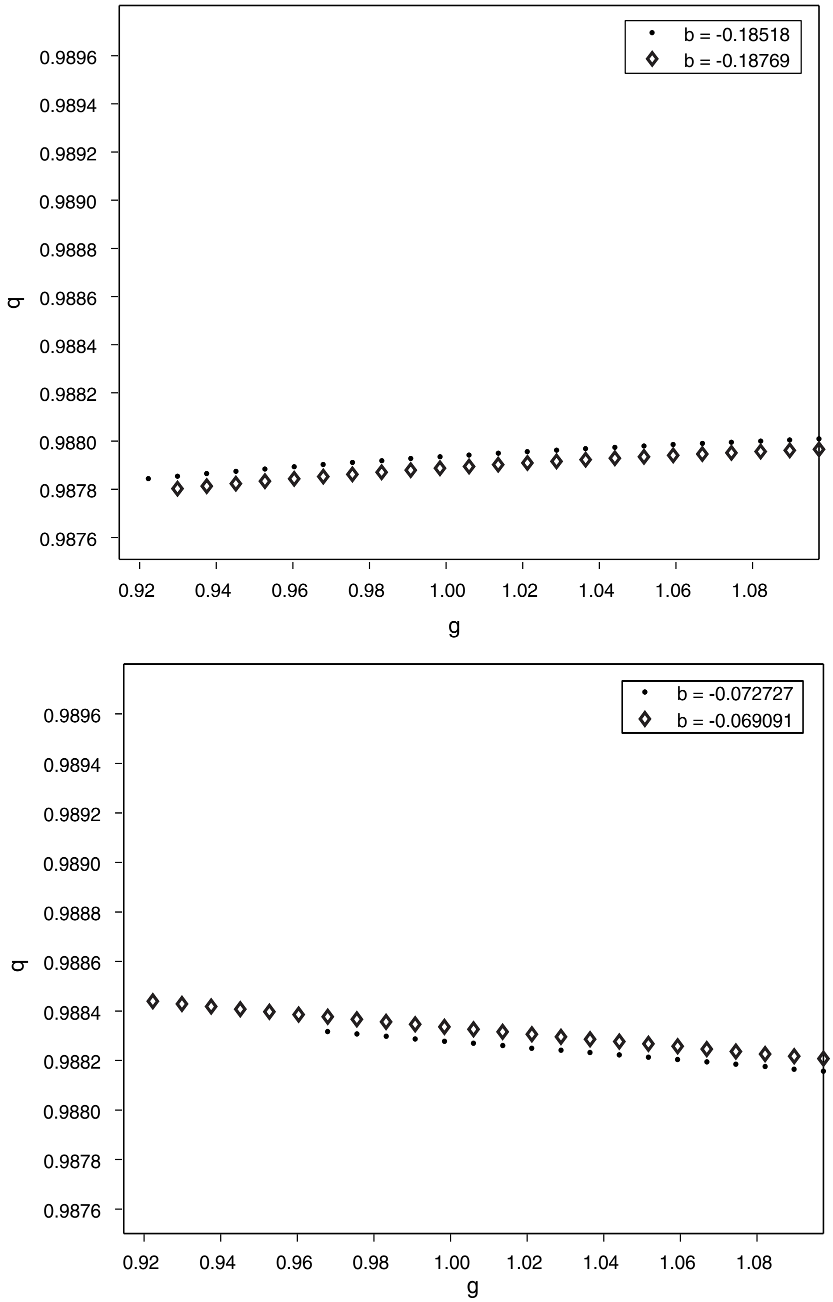
Notes: Bond price as a function of the issuance level in the model with *trend* shocks and *exclusion* (top panel) and in the model with *trend* shocks and *without exclusion* (bottom panel). The high (low) shock is three standard deviations higher (lower) than the unconditional mean of g. The scale of bond issuances when the trend shock is low is described by the bottom horizontal axes. Likewise, the scale of bond issuances when the trend shock is high is described by the top horizontal axes. The different scales used in the horizontal axes facilitate the comparison of the shape of the price functions.

Figure 8 Bond Prices Observed in Equilibrium in Model I



Notes: Bond price observed in equilibrium in the model with *transitory* shocks and *exclusion* (top panel) and in the model with *transitory* shocks and *without exclusion* (bottom panel).

Figure 9 Bond Prices Observed in Equilibrium in Model II



Notes: Bond price observed in equilibrium in the model with *trend* shocks and *exclusion* (top panel) and in the model with *trend* shocks and *without exclusion* (bottom panel).

Table 4 Business Cycle Statistics Computed With and Without Exclusion Punishment

		Transitory Shocks		Trend Shocks	
	Data	With Exclusion	Without Exclusion	With Exclusion	Without Exclusion
$\sigma(y)$	4.08	4.14	4.10	4.15	4.16
$\sigma(c)$	4.85	4.23	4.19	4.38	4.23
$\sigma(TB/Y)$	1.36	0.20	0.21	0.63	0.23
$\sigma(R_s)$	3.17	0.006	0.05	0.013	0.015
$\rho(c, y)$	0.96	0.99	0.99	0.99	0.99
$\rho(TB/Y, y)$	-0.89	-0.43	-0.43	-0.29	-0.29
$\rho(R_s, y)$	-0.59	-0.81	-0.95	-0.06	0.40
$\rho(R_s, TB/Y)$	0.68	0.85	0.69	0.89	-0.96
Mean Debt Output Ratio (%)	51	6.3	1.7	4.8	1.8
Rate of Default	75	6.6	25	24	20

Notes: The debt output ratio is measured as the stock of debt divided by the annual output level.

A similar effect is present in Model II. But in this case the bond price becomes steeper in the setup without exclusion. This effect alone tends to increase the present marginal cost of borrowing, and therefore unlike Model I, a lower bond price is not necessary to satisfy equation (14). This can explain why the levels of the equilibrium bond prices in Model II do not change significantly when the threat of exclusion is eliminated (see Figure 9).

The forces behind the changes in the slope of the spread with respect to the endowment shock are more difficult to tease out. As explained in the end of Section 4, the equilibrium relationship between the spread and the endowment shock depends on various effects, and the sign of the relationship does not necessarily need to be negative. In fact, Figure 9 shows that in the model with trend shocks and no exclusion, the spread increases with the growth shock.

Business Cycle Properties Under No Exclusion

The business cycle statistics reported in Table 3 are recalculated for an economy without the exclusion punishment and presented in Table 4 (the statistics in Table 3 are reproduced in order to facilitate comparison).

The implications of removing the exclusion punishment reported in Table 4 are consistent with the discussion of equilibrium choices in the beginning of Section 5. Table 4 shows that the assumption of exogenous exclusion is responsible for a high fraction of the debt level supported in the model with exclusion. Recall that in this model the government chooses borrowing levels that allow the government to pay very low spreads. These levels are lower when the default cost raised by the threat of financial exclusion is removed.

Table 4 shows that both models quantitatively fail along important dimensions with or without the assumption of financial exclusion: the default rate, the volatility of the spread, and the volatility of the trade balance are too low compared with the data. Even though the overall performance is poor, the behavior of the model with transitory shocks shows a moderate improvement, while the behavior of the model with trend shocks deteriorates when the exclusion assumption is eliminated. The model with endowment shocks and no exclusion displays a higher default rate and spread volatility compared to the model with exclusion (but still far below the data), while the remaining business cycle statistics are not substantially different. On the other hand, the sign of the correlation between output and the spread, and between the spread and the trade balance are reversed and become counterfactual in the model with trend shocks and no exclusion.

The higher default rate generated by Model I when the threat of financial exclusion is eliminated is consistent with the higher equilibrium spread described in Figure 8. The higher spread volatility observed in the setup without exclusion is also consistent with Figure 8, which shows a higher sensitivity of the spread with respect to output in the model without exclusion. The lower default rate and similar spread volatility generated by Model II when the threat of financial exclusion is eliminated are consistent with the adjustments illustrated in Figure 9.

6. CONCLUDING REMARKS

This article discusses the quantitative performance of sovereign default models and explains how the performance is affected by the assumption that countries can be exogenously excluded from capital markets after a default. The article compares the performance of a stylized model with and without the threat of exclusion. It is shown that the exclusion assumption explains a high fraction of the sovereign debt that can be sustained in equilibrium but does not significantly alter the remaining business cycle statistics of the model. In effect, the model without exclusion generates annual debt-output ratios of less than 2 percent. The model with exclusion generates annual debt-output ratios of 4.8 percent when the shocks hit the growth rate and of 6.3 percent when the shocks hit the endowment level. The article shows that in the model with shocks to the endowment level, the default decision becomes slightly more sensitive to the endowment shock and, therefore, less sensitive to the debt level. This helps reduce the sensitivity of the bond price to the borrowing level. On the other hand, in the model with shocks to the trend the default decision becomes relatively less sensitive to the endowment shock, which increases the sensitivity of the bond price to the borrowing level. Given that in this class of models the excessive sensitivity of the default probability to the borrowing level limits the models' quantitative performance, the previous effects may help explain why the performance of the model with shocks to the endowment level shows moderate improvement and why the performance of the model with shocks to the trend deteriorates. In spite of this, both models still fail along important dimensions. The default rate, the volatilities of the trade balance and of the spread, and the debt levels are too low compared to the data. These shortcomings suggest that the exclusion assumption does not play an important role, and therefore future studies that do not rely on the threat of financial exclusion will not necessarily be handicapped in explaining the business cycle in emerging economies. These shortcomings also suggest that other assumptions of the model must be modified in order to bring the model closer to the data.

APPENDIX: THE MODEL WITH STOCHASTIC DURATION OF OUTPUT LOSS

As mentioned before, the model introduced in Section 2 does not exactly coincide with the model presented in Aguiar and Gopinath (2006). They assume that following a default episode, the duration of lower output lasts as long as the time of exclusion. Table 5 shows that the business cycle statistics presented in Table 3 are not greatly affected by the choice of the process of output loss—statistics computed with a stochastic duration of output loss are taken from Hatchondo, Martinez, and Sapriza (2006a) who solve the model in Aguiar and Gopinath (2006) with the computational method used in this article.

Table 5 Business Cycles Under Different Specifications of the Output Loss

	Transitory Shocks		**Trend Shocks**	
	One-Period Output Loss	**Stochastic Duration of Output Loss**	**One-Period Output Loss**	**Stochastic Duration of Output Loss**
$\sigma(y)$	4.14	4.13	4.16	4.16
$\sigma(c)$	4.23	4.25	4.38	4.39
$\sigma\ (TB/Y)$	0.20	0.24	0.63	0.65
$\sigma\ (R_s)$	0.006	0.007	0.013	0.015
$\rho\ (c, y)$	0.99	0.99	0.99	0.99
$\rho\ (TB/Y, y)$	-0.43	-0.43	-0.29	-0.29
$\rho\ (R_s, y)$	-0.81	-0.74	-0.06	-0.09
$\rho\ (R_s, TB/Y)$	0.85	0.89	0.89	0.91
Mean Debt Output Ratio (%)	6.8	6.8	4.7	4.8
Rate of Default	6.6	7.8	24	22

Notes: Business cycle statistics from models with output loss in one period and models with a stochastic duration of the output loss. The debt output ratio is measured as the stock of debt divided by the annual output level.

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