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DSGE Model-Based Estimation of the New Keynesian Phillips Curve

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An important building block in modern dynamic stochastic general equilibrium (DSGE) models is the price-setting equation for firms. In models in which the adjustment of nominal prices is costly, this equation links inflation to current and future expected real marginal costs and is typically referred to as the New Keynesian Phillips curve (NKPC). Its most popular incarnation can be derived from the assumption that firms face quadratic nominal price adjustment costs (Rotemberg 1982) or that firms are unable to re-optimize their prices with a certain probability in each period (Calvo 1983). The Calvo model has a particular appeal because it generates predictions about the frequency of price changes, which can be measured with microeconomic data (Bils and Klenow 2004, Klenow and Kryvtsov 2008). The slope of the NKPC is important for the propagation of shocks and determines the output-inflation tradeoff faced by policymakers. The Phillips curve relationship can also be used to forecast inflation.

This article reviews estimates of NKPC parameters that have been obtained by fitting fully specified DSGE models to U.S. data. By now, numerous empirical papers estimate DSGE models with essentially the same NKPC specification. In this literature, the Phillips curve implies that inflation can be expressed as the discounted sum of expected future marginal costs, where marginal costs equal the labor share. We document that the identification of

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the Phillips curve coefficients is tenuous and no consensus about its slope and the importance of lagged inflation has emerged from the empirical studies.

We begin by examining how the NKPC parameters are identified in a DSGE model-based estimation. This is a difficult question. Many estimates are based on a likelihood function, which is the model-implied probability distribution of a set of observables indexed by a parameter vector. The likelihood function peaks at parameter values for which the model-implied autovariance function of a vector of macroeconomic time series matches the sample autocovariance function. Unfortunately, this description is not particularly illuminating. More intuitively, the NKPC parameters are estimated by a regression of inflation on the sum of discounted future expected marginal costs. The likelihood function corrects the bias that arises from the endogeneity of the marginal cost regressor. We show that if one simply uses ordinary least-squares (OLS) to regress inflation on measures of expected marginal costs, the slope coefficient is very close to zero. This finding is quite robust to the choice of detrending method and marginal cost measure. Hence, much of the variation in the estimates reported in the literature is due to the multitude of endogeneity corrections that arise by fitting different DSGE models that embody essentially the same Phillips curve specification.

The review of empirical studies distinguishes between papers in which marginal costs are included in the observations and, hence, are directly used in the estimation and studies that treat marginal costs as a latent variable. In the latter case, NKPC estimates are more sensitive to the specification of the households' behavior, the conduct of monetary policy, and the law of motion of the exogenous disturbances. Estimates of the slope of the Phillips curve lie between 0 and 4. If the list of observables spans the labor share, then the slope estimates fall into a much narrower range of 0.005 to 0.135. No consensus has emerged with respect to the importance of lagged inflation in the Phillips curve. We compare estimates of the relative movement of inflation and output in response to a monetary policy shock, which captures an important tradeoff for monetary policymakers. We find that the estimates in the studies that are surveyed in this article range from 0.07 to 1.4. A value of 0.07 (1.4) implies that a 1 percent increase in output due to a monetary policy shock is accompanied by a quarter-to-quarter inflation rate of 7 (140) basis points.

The remainder of this paper is organized as follows. We discuss the derivation of the NKPC as well as our concept of DSGE model-based estimation in Section 1. In Section 2, a simple DSGE model that can be solved analytically is used to characterize various sources of NKPC parameter identification. Any particular DSGE model-based estimation might exploit some or all of these sources of information. Section 3 provides empirical evidence from least-squares regressions of inflation on the discounted sum of future marginal costs as well as evidence from a vector autoregression (VAR) on the relative movement of output and inflation in response to a monetary policy shock. We thereby characterize some features of the data that are important for understanding the DSGE model-based parameter estimates reviewed in Section 4. Finally, Section 5 concludes.

1. PRELIMINARIES

This section begins with a brief description of the price-setting problem that gives rise to a Phillips curve in New Keynesian DSGE models. We then discuss some of the defining characteristics of DSGE model-based estimation of NKPC parameters.

Price Setting in DSGE Models

New Keynesian DSGE models typically assume that production is carried out by two types of firms: final good producers and intermediate goods producers. The latter hire labor and capital services from the households to produce a continuum of intermediate goods. The final good producers purchase the intermediate goods and bundle them into a single aggregate good that can be used for consumption or investment. The intermediate goods are imperfect substitutes and, hence, each producer faces a downward-sloping demand curve. Price stickiness is introduced by assuming that it is costly to change nominal prices. Rotemberg (1982) assumed that the price adjustment costs are quadratic, whereas Calvo (1983) set forth a model of staggered price setting in which the costs are either zero or infinite with fixed probabilities, i.e., only a fraction of firms is able to change or, more precisely, re-optimize prices.

Aggregating the optimal price-setting decisions of firms leads to the following expression for inflation in the price of the final good, referred to as the New Keynesian Phillips curve:

$$\tilde{\pi}_t = \gamma_b \tilde{\pi}_{t-1} + \gamma_f \mathrm{E}_t \left[\tilde{\pi}_{t+1} \right] + \lambda \widetilde{MC}_t + \tilde{\xi}_t. \tag{1}$$

Here $\widetilde{\pi}_t$ represents inflation, $\widetilde{MC}_t$ is real marginal costs, and $\widetilde{\xi}_t$ is an exogenous disturbance that is often called a mark-up shock. We use $\widetilde{z}_t$ to denote percentage deviations of a variable, z_t, from its steady state. The coefficients γ_b, γ_f, and λ are functions of model-specific taste and technology parameters. For instance, in Calvo's (1983) model of price stickiness

$$\gamma_b = \frac{\omega}{1 + \beta\omega}, \quad \gamma_f = \frac{\beta}{1 + \beta\omega}, \text{ and } \quad \lambda = \frac{(1 - \zeta)(1 - \zeta\beta)}{\zeta(1 + \beta\omega)},$$

where β is the households' discount factor and ζ is the probability that an intermediate goods producer is unable to re-optimize its price in the current period. In the derivation of (1), it was assumed that those firms that are unable to re-optimize their prices either adjust their past price by the steady-state inflation rate or by lagged inflation. The parameter ω represents the fraction of firms that indexes their prices to lagged inflation.

Assuming that $\beta = 0.99$, the sum of γ_b and γ_f is slightly less than 1 and the coefficient of lagged inflation lies between 0 (no dynamic indexation, $\omega = 0$) and 0.5 (full dynamic indexation, $\omega = 1$). If $\omega = 0$ and steady-state inflation is 0, then $1/(1 - \zeta)$ can be interpreted as the expected duration between price changes. For instance, $\zeta = \frac{2}{3}$ implies that the expected duration of a price set by an intermediate goods producer is three quarters, which leads to a slope coefficient of $\lambda = 0.167$. On the other hand, if $\zeta = \frac{7}{8}$, which means that the duration of a price is eight quarters, then the NKPC is much flatter: $\lambda = 0.018$.

Our survey of the empirical literature will focus on coefficient estimates for γ_b, γ_f, and λ rather than the model-specific preference-and-technology parameters. The slope, λ, determines the output-inflation tradeoff faced by central banks and affects, for instance, the relative response of output and inflation in response to an unanticipated monetary policy shock. A detailed exposition of the role that the NKPC plays in the analysis of monetary policy is provided in an article by Stephanie Schmitt-Grohé and Martín Uribe in this issue. The coefficient γ_b affects the persistence of inflation and, for instance, the rate at which inflation effects of shocks to marginal costs die out. This is an important parameter, particularly for central banks that pursue a policy of inflation targeting. If we rearrange the terms in (1), such that expected inflation appears on the left-hand side and all other terms on the right-hand side, then the Phillips curve delivers a forecasting equation for inflation.

DSGE Model-Based Estimation

This article focuses on estimates of γ_b, γ_f, and λ that are obtained by exploiting the full structure of a model economy. Thus, we consider approaches in which the researcher solves not only the decision problems of the firms but also those of the other agents in the economy and imposes an equilibrium concept. If the economy is subject to exogenous stochastic shocks, the DSGE model generates a joint probability distribution for time series such as aggregate output, inflation, and interest rates. Suppose we generically denote the vector of time, t, observables by x_t and assume that the DSGE model has been solved by log-linear approximation techniques. Then the equilibrium law of motion takes the form of a vector autoregressive moving average (VARMA) process of the form (omitting deterministic trend components)

$$x_t = \Phi_1 x_{t-1} + \dots \Phi_p x_{t-p} + R\epsilon_t + \Psi_1 R\epsilon_{t-1} + \dots \Psi_q R\epsilon_{t-q}. \qquad (2)$$

The matrices Φ_i, Ψ_j, and R are complicated functions of the Phillips curve parameters γ_b, γ_f, and λ, as well as the remaining DSGE model parameters, which we will summarize by the vector θ. The vector ϵ_t stacks the innovations to all exogenous stochastic disturbances and is often assumed to be normally and independently distributed.

A natural approach of exploiting (2) is likelihood-based estimation. Maximum likelihood (ML) estimation of optimization-based rational expectations models in macroeconomics dates back at least to Sargent (1989) and has been widely applied in the DSGE model literature (e.g., Altug [1989], Leeper and Sims [1994], and many of the papers reviewed in Section 4). The likelihood function is defined as the joint density of the observables conditional on the parameters, which can be derived from (2). Let $X^t = \{x_1, \ldots, x_t\}$, then

$$p(X^T|\gamma_b, \gamma_f, \lambda, \theta) = p(x_1|\gamma_b, \gamma_f, \lambda, \theta) \prod_{t=2}^{T} p(x_t|X^{t-1}, \gamma_b, \gamma_f, \lambda, \theta). \quad (3)$$

The evaluation of the likelihood function typically requires the use of numerical methods to solve for the equilibrium dynamics and to integrate out unobserved elements from the joint distribution of the model variables (see, for instance, An and Schorfheide [2007]). A numerical optimization routine can then be used to find the maximum of the (log-)likelihood function. The potential drawback of the ML approach is that identification problems can make it difficult to find the maximum of the likelihood function and render standard large sample approximations to the sampling distribution of the ML estimator and likelihood ratio statistics inaccurate.

A popular alternative to the frequentist ML approach is Bayesian inference. Bayesian analysis tends to interpret the likelihood function as a density function for the parameters given the data. Let $p(\gamma_b, \gamma_f, \lambda, \theta)$ denote a prior density for the DSGE model parameters. Bayesian inference is based on the posterior distribution characterized by the density

$$p(\gamma_b, \gamma_f, \lambda, \theta|X^T) = \frac{p(X^T|\gamma_b, \gamma_f, \lambda, \theta)p(\gamma_b, \gamma_f, \lambda, \theta)}{\int p(X^T|\gamma_b, \gamma_f, \lambda, \theta)p(\gamma_b, \gamma_f, \lambda, \theta)d(\gamma_b, \gamma_f, \lambda, \theta)}. \quad (4)$$

Notice that the denominator does not depend on the parameters and simply normalizes the posterior density so that it integrates to one. The controversial ingredient in Bayesian inference is the prior density as it alters the shape of the posterior, in particular if the likelihood function does not exhibit much curvature. On the upside, the prior allows the researcher to incorporate additional information in the time series analysis that can help sharpen inference. Many of the advantages of Bayesian inference in the context of DSGE model estimation are discussed in Lubik and Schorfheide (2006) and An and Schorfheide (2007). The implementation of Bayesian inference typically relies on Markov-chain Monte Carlo methods that allow the researcher to generate random draws of the model parameters from their posterior distribution. These draws can then be transformed—one by one—into statistics of interest. Sample moments computed from these draws provide good approximations to the corresponding population moments of the posterior distribution.

Notwithstanding all the desirable statistical properties of likelihood-based estimators, the mapping of particular features of the data into parameter estimates is not particularly transparent. Superficially, the likelihood function peaks at parameter values for which a weighted discrepancy between DSGE model-implied autocovariances of x_t and sample autocovariances is minimized. The goal of the next section is to explore the extent to which this matching of autocovariances can identify the parameters of the New Keynesian Phillips curve.

2. IDENTIFYING THE NKPC PARAMETERS

The identification of DSGE model parameters through likelihood-based methods tends to be a black box because the relationship between structural parameters and autocovariances or other reduced-form representations is highly nonlinear. This section takes a look inside this black box to develop some understanding about particular features of the DSGE model that contribute to the identifiability of NKPC parameters. Rather than asking whether there is enough variation in postwar data to estimate the NKPC parameters reliably, for now we focus on sources of identification in infinite samples. In practice, the estimation of a particular model might exploit several of these sources of information simultaneously.

Since the Phillips curve provides a relationship between marginal costs and inflation, the measurement of marginal costs is important for the identification of the NKPC parameters. A key feature of likelihood-based inference—as opposed to the single-equation methods reviewed by James Nason and Gregor Smith in this issue—is the exploitation of model-implied restrictions of contemporaneous correlations between variables, as well as the use of information from impulse responses. In many instances, higher-order autocovariances of inflation and marginal costs are an additional source of information.

While this section focuses on identifying the slope, λ, we also offer some insights into identifying γ_b and γ_f. For now we assume that $\gamma_b = 0$. In the context of the Calvo model this assumption implies that the fraction, ω, of firms that engage in dynamic indexation is zero. In this case, $\gamma_f = \beta$. Since β in a fully specified DSGE model is related to the steady-state real interest rate, the coefficient γ_f can be determined, for instance, by averaging interest rate data, and its identification is not a concern. Under our simplifications, the Phillips curve takes the form

$$\widetilde{\pi}_t = \beta \mathrm{E}_t[\widetilde{\pi}_{t+1}] + \lambda \widetilde{MC}_t + \widetilde{\xi}_t. \tag{5}$$

Solving this difference equation forward we find that today's inflation is a function of future expected marginal costs:

$$\widetilde{\pi}_t = \sum_{j=0}^{\infty} \beta^j \mathrm{E}_t[\lambda \widetilde{MC}_{t+j} + \widetilde{\xi}_{t+j}]. \tag{6}$$

Observed Versus Latent Marginal Costs

The identification of λ crucially depends on whether real marginal costs are treated as directly observable or as a latent variable. If $\widetilde{MC}_t$ is directly observed and, hence, is an element of the vector x_t in (2) and (4), then the main obstacle to the identification of λ is the endogeneity problem caused by the potential correlation between the mark-up shock, ξ_t, and marginal costs. The estimation of future expected marginal costs in (6) poses no real challenge because $\mathrm{E}_t[\widetilde{MC}_{t+j}]$ can be obtained from the reduced-form representation associated with the law of motion (2), which is always identifiable. The downside of including a direct measure of marginal costs in the set of observables is that measurement errors pertaining to the marginal cost series can potentially distort the inference about the NKPC parameters. Yet, identifying λ is more tenuous if marginal costs are not included in the vector x_t.

To make the discussion more concrete, imagine an economy in which labor is the only factor of production and, in log-linear terms,

$$\widetilde{Y}_t = \widetilde{Z}_t + \widetilde{H}_t.$$

Z_t is an unobserved total factor productivity process and H_t is hours worked. Marginal costs are given by

$$\widetilde{MC}_t = \widetilde{W}_t - \widetilde{Z}_t,$$

where W_t are wages. Moreover, suppose that the households' instantaneous utility function is of the form

$$U(C_t, H_t) = \frac{C_t^{1-1/\tau}}{1 - 1/\tau} - \phi H_t,$$

and μ_t denotes the marginal utility of consumption. Under these preferences labor supply is infinitely elastic, the wage has to satisfy $W_t = 1/\mu_t$, and the marginal utility of consumption is given by $\mu_t = C_t^{-1/\tau}$. Finally, assume that output is entirely used for household consumption such that $C_t = Y_t$. Then we obtain the following link between marginal costs and output:

$$\widetilde{MC}_t = \frac{1}{\tau}(\widetilde{Y}_t - \tau \widetilde{Z}_t).$$

If the vector of observables, x_t, contains output, wages, and hours worked, then the marginal costs are directly observed because

$$\widetilde{MC}_t = \widetilde{lsh}_t = \widetilde{W}_t + \widetilde{H}_t - \widetilde{Y}_t.$$

More generally, in models with Cobb-Douglas technology the vector x_t spans marginal costs as long as one can construct the labor share, $\widetilde{lsh}_t$, from the observables. If, however, the vector x_t only contains observations on output in addition to inflation and interest rates, then marginal costs are latent because they depend on the observed output as well as the unobserved technology process, $\widetilde{Z}_t$, and the unknown parameter, τ. Rewriting (5) in terms of inflation and output yields

$$\widetilde{\pi}_t = \beta \mathrm{E}_t[\widetilde{\pi}_{t+1}] + \frac{\lambda}{\tau}\widetilde{Y}_t - \widetilde{Z}_t + \widetilde{\xi}_t.$$

Two challenges arise. First, the presence of $\widetilde{Z}_t$ exacerbates the endogeneity problem that arises in the NKPC estimation. Moreover, the coefficient associated with $\widetilde{Y}_t$ in itself does not identify the original slope parameter, λ, since it also depends on the utility function parameter, τ, which needs to be identified from other equilibrium relationships.

In practice, likelihood-based estimation of DSGE models relies on the so-called state-space representation of the DSGE model, rather than the VARMA representation in (2). Omitting deterministic trend components, the state-space representation takes the form

$$x_t = As_t, \quad s_t = B_1 s_{t-1} + B_\epsilon \epsilon_t, \tag{7}$$

where x_t is the vector of observables, s_t is a vector of latent variables, and the matrices A, B_1, and B_ϵ are functions of the DSGE model parameters. The likelihood function associated with (7) can be computed with the Kalman filter. If the information in the vector x_t does not span marginal costs directly, then the Kalman filter constructs an estimate of the latent marginal costs (and technology, $\widetilde{Z}_t$, in our example) based on x_t and the parameters λ and θ. To the extent that the Kalman filter inference for the latent variables is sensitive to the assumed law of motion of the unobserved exogenous processes, inference about the slope of the Phillips curve is also sensitive to these auxiliary assumptions.

Identifying Information in Contemporaneous Correlations

Fully-specified DSGE models impose strong restrictions on the contemporaneous interactions of macroeconomic variables. We will show in the context of a simple example that these restrictions enter the likelihood function and potentially provide important identifying information that is not used in the single-equation approaches reviewed by James Nason and Gregor Smith in this issue. For the remainder of Section 2 we adopt the convention that all variables are measured in percentage deviations from a deterministic steady state and omit tildes to simplify the notation.

Consider the log-linear approximation of the Euler equation associated with the households' problem in the previous subsection:

$$Y_t = \mathrm{E}_t[Y_{t+1}] - \tau(R_t - \mathrm{E}_t[\pi_{t+1}]) + \epsilon_{\phi,t}. \tag{8}$$

$R_t - \mathrm{E}_t[\pi_{t+1}]$ is the expected real return from holding a one-period nominal bond. The parameter, τ, can be interpreted as the intertemporal substitution elasticity of the household and $\epsilon_{\phi,t}$ is an exogenous preference shifter. To complete the model, we characterize monetary policy by an interest rate feedback rule of the form

$$R_t = \psi\pi_t + \epsilon_{R,t}, \tag{9}$$

where $\epsilon_{R,t}$ is a monetary policy shock.

We now substitute the marginal cost expression derived in the previous subsection into the NKPC and obtain

$$\pi_t = \beta\mathrm{E}_t[\pi_{t+1}] + \frac{\lambda}{\tau}(Y_t - \tau Z_t) + \xi_t. \tag{10}$$

Since the unobserved technology shock, Z_t, and the mark-up shock, ξ_t, affect the equilibrium law of motion in a similar manner in this simple model, we set $Z_t = 0$ and let $\xi_t = \epsilon_{\xi,t}$. Moreover, we define $\kappa = \frac{\lambda}{\tau}$ and will direct our attention to the estimation of the output inflation tradeoff, κ, rather than λ. Thus, we are essentially abstracting from the two additional difficulties that arise if marginal costs are treated as a latent variable. Finally, it is assumed that the three exogenous shocks, $\epsilon_{R,t}$, $\epsilon_{\phi,t}$, and $\epsilon_{\xi,t}$ are independently and identically distributed zero mean normal random variables with standard deviations σ_R, σ_ϕ, and σ_ξ, respectively.

The linear rational expectations (LRE) model comprised of (8) to (10) can be solved with standard methods such as the one described in Sims (2002). To ensure that the LRE system has a unique stable solution, we impose $\psi > 1$, which implies that the central bank raises the real interest rate in response to an inflation rate that exceeds its steady-state level. Lubik and Schorfheide (2004) show that the equilibrium law of motion for the three observables is of the form

$$\begin{bmatrix} Y_t \\ \pi_t \\ R_t \end{bmatrix} = \frac{1}{1+\kappa\tau\psi}\begin{bmatrix} -\tau & 1 & -\tau\psi \\ -\kappa\tau & \kappa & 1 \\ 1 & \kappa\psi & \psi \end{bmatrix}\begin{bmatrix} \epsilon_{R,t} \\ \epsilon_{\phi,t} \\ \epsilon_{\xi,t} \end{bmatrix}. \tag{11}$$

Since our model lacks both endogenous and exogenous propagation mechanisms, output, inflation, and interest rates—the three variables observed by the econometrician—are serially uncorrelated in equilibrium. Thus, all the information about the slope of the Phillips curve must come from the contemporaneous correlations among the three observables.

The single-equation approach to the estimation of the NKPC reviewed by James Nason and Gregor Smith in this issue can be interpreted in two ways. First, one can write the NKPC as a regression of the form

$$\pi_{t+1} = \frac{1}{\beta}\pi_t - \frac{\kappa}{\beta}Y_t - \frac{1}{\beta}\eta_{t+1} - \frac{1}{\beta}\epsilon_{\xi,t} = \alpha_1\pi_t + \alpha_2 Y_t + resid_{t+1}. \quad (12)$$

Here we replaced the conditional expectation of inflation, $\mathrm{E}_t[\pi_{t+1}]$, by π_{t+1} and a forecast error $\eta_{t+1} = \pi_{t+1} - \mathrm{E}_t[\pi_{t+1}]$. The lack of serial correlation in the equilibrium dynamics implies that least-squares estimates of α_1 and α_2 converge in probability to zero. Hence, based on a large sample, an econometrician concludes that the slope of the Phillips curve is zero. The estimation of (12) with an instrumental variable estimator that tries to correct a potential bias due to the correlation between Y_t and $\epsilon_{\xi,t}$ is also bound to fail because in equilibrium, lagged values of output and inflation are uncorrelated with the regressors.

Alternatively, one can express the Phillips curve as a regression of the form

$$\pi_t = \alpha_1 \mathrm{E}_t[\pi_{t+1}] + \alpha_2 Y_t + resid_t. \quad (13)$$

However, even if the econometrician realizes that $\mathrm{E}_t[\pi_{t+1}] = 0$ and excludes the expected inflation regressor, it is not possible to estimate the slope of the Phillips curve consistently. The least-squares estimator of α_2 provides a biased estimate of κ because of the correlation between output and the mark-up shock, which is subsumed in the residual. Instrumental variable estimation is also uninformative because lagged endogenous variables are uncorrelated with current output. Notice that this failure of single-equation estimation is not directly apparent from (10). It is a consequence of the auxiliary assumptions about the other sectors in the economy and the law of motion of the exogenous disturbances. Nason and Smith (2008) show that the identification problems associated with single-equation methods prevail, even if the DSGE model is enriched with serially correlated exogenous disturbances.

DSGE model-based estimation of the Phillips curve parameters utilizes the information in the contemporaneous relationship between output, inflation, and interest rates.[1] Let $\theta = [\tau, \psi, \sigma_R, \sigma_\phi, \sigma_\xi]'$ and factorize the joint density of the observables as

[1] It is tempting to check identification by comparing the number of structural parameters to the number of free parameters in the covariance matrix of Y_t, π_t, and R_t. In the DSGE model described by the law of motion (11), the two parameter counts are equal to six. Unfortunately, having at least as many estimable reduced-form parameters as structural parameters is neither sufficient for identification, nor does it provide interesting insights about the sources of identification.

$$p(Y_t, \pi_t, R_t|\kappa, \theta) = p(Y_t|\kappa, \theta)p(\pi_t|Y_t, \kappa, \theta)p(R_t|Y_t, \pi_t, \theta). \tag{14}$$

The first term represents the marginal density of output and the third term is generated by the monetary policy rule. Key to understanding the DSGE model-based estimation of κ is the second term, that is, the conditional distribution of inflation given output. Since all the shocks are normally distributed,

$$\pi_t|Y_t \sim \mathcal{N}\left(\mathrm{E}\left[\pi_t \mid Y_t\right]\ ,\ var[\pi_t|Y_t]\right)$$

and we can focus our attention on the conditional mean and variance.

We begin with the derivation of $\mathrm{E}[\pi_t \mid Y_t]$. Solving the Phillips curve relationship forward as in (6) leads to

$$\pi_t = \kappa Y_t + \epsilon_{\xi,t}. \tag{15}$$

Taking expectations conditional on Y_t of the left-hand side and right-hand side of (15) yields

$$\mathrm{E}\left[\pi_t \mid Y_t\right] = \kappa Y_t + \mathrm{E}\left[\epsilon_{\xi,t} \mid Y_t\right].$$

Using (11) and the formula for the conditional moments of a joint normal distribution,[2] we obtain

$$\mathrm{E}\left[\epsilon_{\xi,t} \mid Y_t\right] = \mu_{\xi|y}(\theta)Y_t = -\frac{1}{\tau\psi}\underbrace{\frac{\tau^2\psi^2\sigma_\xi^2}{\tau^2\sigma_R^2 + \sigma_\phi^2 + \tau^2\psi^2\sigma_\xi^2}}_{sh(\sigma_y^2, \epsilon_\xi)} Y_t. \tag{16}$$

The conditional expectation depends on the intertemporal elasticity of substitution, the policy rule coefficient, and all the shock variances. Here $sh(\sigma_y^2, \epsilon_\xi)$ is the fraction of the variance of output that is due to the mark-up shock, $\epsilon_{\xi,t}$. We now turn to the calculation of the conditional variance of inflation. Notice that $var[\pi_t|Y_t] = var[\epsilon_{\xi,t}|Y_t]$. Thus,

$$var[\pi_t|Y_t] = \sigma^2_{\xi|y}(\kappa, \theta) = \sigma_\xi^2 - \frac{(\tau\psi\sigma_\xi^2)^2}{(1+\kappa\tau\psi)(\tau^2\sigma_R^2 + \sigma_\phi^2 + \tau^2\psi^2\sigma_\xi^2)}.$$

We deduce that

$$\begin{aligned} p(\pi_t|Y_t, \kappa, \theta) \quad \propto \quad & |\sigma^2_{\xi|y}(\kappa, \theta)|^{-1/2}\exp \\ & \left\{-\frac{1}{2\sigma^2_{\xi|y}(\kappa, \theta)}\left(\pi_t - [\kappa + \mu_{\xi|y}(\theta)]Y_t\right)^2\right\}, \end{aligned} \tag{17}$$

[2] Suppose that X and Y are jointly normally distributed with means μ_x and μ_y, variances v_{xx} and v_{yy}, and covariance v_{xy}; then the conditional mean $\mathrm{E}[X \mid Y = y] = \mu_x + v_{xy}v_{yy}^{-1}(y - \mu_y)$ and the conditional variance is $var[X|Y = y] = v_{xx} - v_{xy}^2/v_{yy}$.

where $\propto$ denotes proportionality.

We can draw several important conclusions from (17). First, the term $\mu_{\xi|y}$ given in (16) corrects for the endogeneity bias that arises in a regression of inflation and marginal costs. Suppose we set $\psi = 1.5$, which is Taylor's (1993) value, assume that $\tau = \frac{2}{3}$, which makes the agents slightly more risk-averse than agents with log preferences, and assume that 20 percent of the variation in output is due to mark-up or cost-push shocks. Then (16) implies that a simple least-squares regression of inflation on marginal costs, i.e. output, in our example model, would underestimate the slope, κ, by 0.2. Second, (17) implies that knowledge of the conditional distribution of inflation given output does not identify the slope of the Phillips curve. Moreover, the joint distribution of output and inflation is also not sufficient, because the marginal distribution of output only provides information about the variance of output, $\sigma_y^2(\kappa, \theta)$, which is insufficient to disentangle the values of all the θ elements. We will show below, however, that κ is identifiable with knowledge of the monetary policy reaction function.

To summarize, our simple example has a number of startling implications. First, a single-equation estimation based on (12) or (13) is unable to deliver a consistent estimate of κ. Second, an OLS regression of inflation on the sum of discounted future expected marginal costs generates a downward-biased estimate of κ. The magnitude of the bias is a function of central bank behavior, households' preferences, and, more generally, the importance of mark-up shocks for output fluctuations. Third, DSGE model-based estimation is promising but might require a prior that is informative about other model parameters, for instance those that control the law of motion of exogenous shocks or the conduct of monetary policy. We will subsequently elaborate on this last point.

Identifying Information in Impulse Response Functions

If the DSGE model embodies enough restrictions to identify a structural shock other than ξ_t from the observables, then one can potentially infer the Phillips curve slope from the impulse response function (IRF) associated with this shock. Consider the model analyzed in the previous subsection. Suppose that the policy rule coefficient, ψ, is known, which means that the sequence of monetary policy shocks can be directly obtained from interest rate and inflation data: $\epsilon_{R,t} = R_t - \psi\pi_t$. Recall from (15) that the forward solution of the Phillips curve takes the form

$$\pi_t = \kappa Y_t + \epsilon_{\xi,t}.$$

We previously showed that the correlation between the mark-up shock, $\epsilon_{\xi,t}$, and the regressor, Y_t, creates an endogeneity problem that complicates the identification of κ. The monetary policy shock can serve as an instrumental variable in the identification of κ. By assumption, the monetary policy shock is uncorrelated with $\epsilon_{\xi,t}$ but correlated with the regressor Y_t.

The argument can be formalized as follows. Suppose we factorize the likelihood function into[3]

$$
\begin{aligned}
p(Y_t, \pi_t, R_t|\kappa, \theta) &= p(R_t - \psi\pi_t|\kappa, \theta) p(Y_t|R_t - \psi\pi_t, \kappa, \theta) \\
&\quad p(\pi_t|Y_t, R_t - \psi\pi_t, \kappa, \theta). \qquad (18)
\end{aligned}
$$

$R_t - \psi\pi_t$ measures the monetary policy shock, $\epsilon_{R,t}$, and the first term corresponds to its density. The second factor captures the distribution of output given the monetary policy shock. The third conditional density represents the Phillips curve. From this factorization it is apparent that, in a linear Gaussian environment, the following conditional expectations (we replace $R_t - \psi\pi_t$ by $\epsilon_{R,t}$) are identifiable:

$$
\begin{aligned}
\mathrm{E}\left[Y_{t|}R_t - \psi\pi_t, \kappa, \theta\right] &= \alpha_{11}\epsilon_{R,t} \text{ and} \\
\mathrm{E}\left[\pi_t \mid R_t - \psi\pi_t, Y_t\right] &= \alpha_{21}\epsilon_{R,t} + \alpha_{22}Y_t,
\end{aligned}
$$

where α_{ij} is a function of κ and θ. Since

$$
\frac{\partial Y_t}{\partial \epsilon_{R,t}} = \alpha_{11}, \quad \frac{\partial \pi_t}{\partial \epsilon_{R,t}} = \alpha_{21} + \alpha_{22}\alpha_{11},
$$

it follows from (11) that κ is identified by the ratio of the output and inflation response $\alpha_{21}/\alpha_{11} + \alpha_{22}$.

In our simple example the identification of the monetary policy shock depends on the assumed knowledge of the parameter ψ, which the reader might find unconvincing. More interestingly, there are a number of papers that estimate DSGE models that are specified such that monetary policy shocks can be identified from exclusion restrictions. Most notably, Rotemberg and Woodford (1997), Christiano, Eichenbaum, and Evans (2005), and Boivin and Giannoni (2006) consider models in which the private sector is unable to respond to monetary policy shocks contemporaneously.[4] In a Gaussian vector autoregressive system, this exclusion restriction is sufficient to identify monetary policy shocks and the associated impulse response functions independently of the DSGE model parameters.

[3] The Jacobian associated with the transformation of $[R_t - \psi\pi_t, Y_t, \pi_t]'$ into $[R_t, Y_t, \pi_t]'$ is equal to one. We maintain that θ is defined as $\theta = [\tau, \psi, \sigma_R, \sigma_\phi, \sigma_\xi]'$ and, hence, includes ψ.

[4] Rather than conducting likelihood-based inference, all three papers use an estimation method that exclusively relies on the identification of model parameters from IRF dynamics. The structural parameters are directly estimated by minimizing the discrepancy between the model-implied impulse responses to a monetary policy shock and those obtained from estimating a structural VAR.

Identifying Information in the Reduced-Form Dynamics

The absence of equilibrium dynamics in (11) is clearly at odds with reality. Aggregate output, inflation, and interest rates tend to exhibit fairly strong serial correlation. This serial correlation opens up another avenue for identification as lagged endogenous variables can serve as instruments to correct endogeneity biases. In fact, it is this serial correlation that single-equation approaches rely on.

Suppose that the vector x_t contains inflation, a measure of marginal costs as well as other variables, denoted by z_t: $x_t = [\pi_t, MC_t, z_t']'$. Moreover, assume that the mark-up shock, ξ_t, is independently distributed and that the DSGE model-implied law of motion for x_t has a VAR(1) representation:

$$x_t = \Phi_1(\lambda, \theta)x_{t-1} + u_t, \quad \text{where} \quad u_t = R(\lambda, \theta)\epsilon_t. \tag{19}$$

The matrices Φ_1 and R are functions of the DSGE model parameters, the vector ϵ_t stacks the innovations to the exogenous driving processes of the model economy, and u_t can be interpreted as reduced-form, one-step-ahead forecast errors. While the assumption that ξ_t is serially uncorrelated is crucial for the subsequent argument, the VAR(1) representation is not.

Define the selection vectors M_1 and M_2 such that $M_1'x_t = \pi_t$ and $M_2x_t = MC_t$. Equation (15) implies that the slope of the Phillips curve has to solve the following restriction:

$$M_1{\prime}\Phi_1x_t - \lambda M_2{\prime}(I - \beta\Phi_1)^{-1}\Phi_1x_t = 0 \quad \text{for all } x_t. \tag{20}$$

Recall that under the assumption that ξ_t is independently distributed, the forward solution of the Phillips curve takes the form

$$\pi_t = \lambda \sum_{j=0}^{\infty} \beta^j \mathrm{E}_t[MC_{t+j}] + \xi_t.$$

Thus, the first term in (20) can be interpreted as the one-step-ahead VAR forecast of inflation. The second term in (20) corresponds to the one-step-ahead forecast of the sum of discounted expected future marginal costs, scaled by the Phillips curve slope. As long as ξ_t is serially uncorrelated, the two forecasts have to be identical. Notice that although it might be impossible to uniquely determine λ and θ conditional on the VAR coefficient matrix Φ_1, Φ_1 is always identifiable based on the autocovariances of x_t, provided that $\mathrm{E}\big[x_tx_t'\big]$ is invertible: $\Phi_1 = \mathrm{E}\big[x_{t-1}x_t'\big]\,(\mathrm{E}\big[x_tx_t'\big])^{-1}$. Hence, provided that inflation is serially correlated, the restriction (20) identifies λ.

Sbordone (2002, 2005) and Kurmann (2005, 2007) use (20) in conjunction with reduced-form VAR estimates of Φ to obtain estimates of the NKPC parameters. A system-based DSGE model estimation with serially uncorrelated mark-up shocks can be interpreted as simultaneously minimizing the discrepancy between an unrestricted, likelihood-based estimate of Φ_1 and the DSGE model-implied restriction function $\Phi_1(\lambda, \theta)$ and imposing the condition (20).

Identification of Backward-Looking Terms

Achieving identification becomes more difficult if we relax the restriction that $\gamma_b = 0$. Since insightful, analytical derivations are fairly complex, we offer a heuristic argument and point to some empirical evidence. Three factors contribute to the persistence of inflation: the backward-looking term $\gamma_b \widetilde{\pi}_{t-1}$, the persistence of marginal costs, and the persistence of the mark-up shock, ξ_t. Roughly speaking, we can measure inflation and marginal cost persistence from the data (provided observations on marginal costs are available). Hence, the challenge is to disentangle the relative contribution of γ_b and the mark-up shock to the persistence of inflation. Del Negro and Schorfheide (2006, Figure 8) display plots of the joint posterior distribution of γ_b and the autocorrelation of a latent mark-up shock obtained from the estimation of a DSGE model that is similar to the one developed by Smets and Wouters (2003). Not surprisingly, there is a strong negative correlation, suggesting that without strong *a priori* restrictions, it is difficult to measure the magnitude of γ_b. One widely used *a priori* restriction is the assumption that the mark-up shock is either absent or serially uncorrelated.

3. A (CRUDE) LOOK AT U. S. DATA

Before reviewing the DSGE model-based NKPC estimates reported in the literature, we will take a crude look at U.S. inflation, labor share, and output data. In view of the analysis presented in Section 2, two potentially important sources of variation in DSGE model-based estimates are (1) detrending methods for inflation data and marginal cost proxies and (2) endogeneity corrections. Thus, in the first subsection we construct different measures of steady-state deviations and compare the stochastic properties of the resulting $\widetilde{\pi}_t$, $\widetilde{MC}_t$, and $\widetilde{Y}_t$ series. We established that the estimation of the NKPC parameters amounts to a regression of inflation on future expected marginal costs. This regression, hidden within a complicated likelihood function, is plagued by an endogeneity problem, which, according to the simple model in Section 2, leads to a negative bias of least-squares estimates of the Phillips curve slope. It turns out that these least-squares estimates are relatively insensitive to data definitions (second subsection), which suggests that much of the variation across empirical studies is attributable to differences in the endogeneity correction.

We also showed that impulse response dynamics provide useful information about the NKPC coefficients. To the extent that a well-specified DSGE model is comparable in fit to a more densely parameterized VAR, evidence (reported in the third subsection) on the propagation of a monetary policy shock can be helpful to understand DSGE model-based estimates of NKPC parameters. Finally, the autocovariance restrictions exploited in the DSGE model-based estimation tend to nest those used by Sbordone (2002) to construct a VAR-based minimum distance estimator. Hence, we briefly review these minimum distance estimates in the fourth subsection.

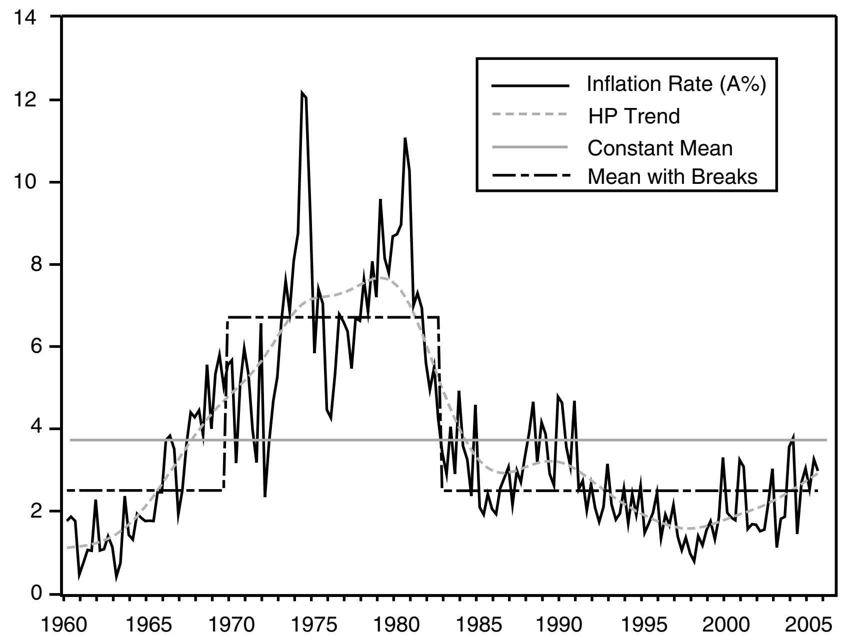
Measures of Inflation and Marginal Costs

Most authors use the gross domestic product (GDP) deflator as a measure of inflation when estimating New Keynesian DSGE models. Our subsequent review focuses on estimates obtained with DSGE models in which marginal costs equal the labor share. These estimates either include the labor share in the vector of observables or treat marginal costs as a latent variable. In the latter case, deviations of aggregate output from a trend or natural level are implicitly used as a marginal cost proxy. To study the stochastic properties of these series, we compile a small data set with quarterly U.S. observations. The raw data are taken from Haver Analytics. Real output is obtained by dividing the nominal series (GDP) by population 16 years and older and by deflating using the chained-price GDP deflator. Inflation rates are defined as log differences of the GDP deflator. The labor share is computed by dividing total compensation of employees (obtained from the National Income and Product Accounts) by nominal GDP. We take logs of real per capita output and the labor share. Our sample ranges from 1960:Q1 to 2005:Q4.

We will consider three measures of $\widetilde{\pi}_t$. First, $\widetilde{\pi}$ (mean) is obtained by subtracting the sample mean computed over the period 1960 to 2005 from the GDP deflator inflation. This calculation assumes that the target inflation rate has essentially stayed constant for the past 45 years. Second, we compute separate means for the subsamples 1960–69, 1970–1982, and 1983–2005. The break points are broadly consistent with the regime estimates obtained in Schorfheide (2005). The resulting measure of inflation deviations is denoted by $\widetilde{\pi}$ (break) and reflects the view that the target inflation rate rose in the 1970s because policymakers perceived an exploitable long-run output inflation tradeoff. Finally, we consider $\widetilde{\pi}$ (HP), which can be interpreted as deviations from a drifting target inflation rate.

We plot the inflation rate as well as the three versions of the target inflation in Figure 1. It is apparent from the figure that views about target inflation significantly affect the stochastic properties of $\widetilde{\pi}_t$. For instance, the first-order autocorrelations (see Table 1) are 0.88, 0.68, and 0.49 for $\widetilde{\pi}$ (mean), $\widetilde{\pi}$ (break), and $\widetilde{\pi}$ (HP), respectively. The two panels of Figure 2 depict $\widetilde{MC}_t$ as approximated by output movements or measured by labor share fluctuations. In models that treat marginal cost as a latent variable, the most common

Figure 1 Inflation and Measures of Trend Inflation

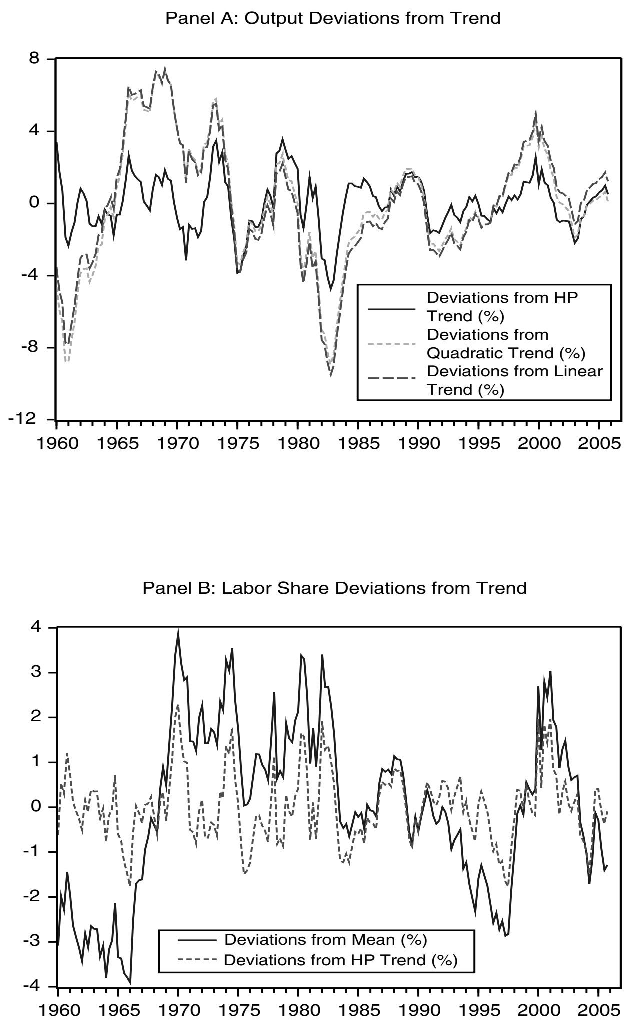


Notes: Inflation is measured as quarter-to-quarter changes in the log GDP deflator, scaled by 400 to convert it into annualized percentages. The sample ranges from 1960:Q1 to 2005:Q4.

marginal cost proxies are given by linearly detrended output, output deviations from a quadratic trend, and HP-filtered output. Since the potential output series produced by the Congressional Budget Office closely resembles the HP trend, we are not considering it separately.[5] Panel A clearly indicates that output deviations from a deterministic trend tend to be more volatile and persistent than deviations from the HP trend, since the HP filter removes more of the low frequency variation from the output series. Panel B shows time series for labor share deviations from a constant mean and an HP trend. As before, deviations from an HP trend tend to be smoother. First-order autocorrelations for the marginal cost measures are reported in Table 1. They range from 0.7 (HP-filtered labor share) to 0.97 (linearly detrended output).

[5] In some DSGE models, e.g., Schorfheide (2005), technology evolves according to a unit root process and the output term that appears in the Phillips curve refers, strictly speaking, to deviations from a latent stochastic trend. We do not consider this case in the regressions reported in this section.

Figure 2 Measures of Marginal Cost Deviations



Inflation and Marginal Cost Regressions

Under the assumptions that $\gamma_b = 0$, $\widetilde{\xi}_t$ is serially uncorrelated, $\beta = 0.993$, and marginal cost dynamics are well approximated by an AR(1) process with coefficient $\hat{\rho}$, one can express the forward solution of (6) as

Table 1 Persistence of Marginal Cost and Inflation Measures

Series	AR(1)
$\tilde{\pi}$ (mean)	0.88
$\tilde{\pi}$ (break)	0.68
$\tilde{\pi}$ (HP)	0.49
$\tilde{Y}$ (linear trend)	0.97
$\tilde{Y}$ (quadratic trend)	0.96
$\tilde{Y}$ (HP)	0.85
$l\tilde{s}h$ (mean)	0.94
$l\tilde{s}h$ (HP)	0.70

Notes: The table reports AR(1) coefficient estimates based on a sample from 1960:Q1 to 2005:Q4.

$$\widetilde{\pi}_t = \kappa \left(\frac{1}{1 - 0.993\hat{\rho}_Y} \right) \widetilde{Y}_t + \tilde{\xi}_t, \quad \text{or} \quad \widetilde{\pi}_t = \lambda \left(\frac{1}{1 - 0.993\hat{\rho}_{lsh}} \right) \widetilde{lsh}_t + \tilde{\xi}_t, \tag{21}$$

where lsh denotes the labor share. As in Section 2, the parameter, κ, confounds the slope, λ, and the elasticity of marginal costs with respect to output. Least squares regression results for (21) are summarized in Table 2. We report point estimates of κ and λ in (12) and R^2 statistics in parenthesis for the full sample as well as three subsamples: 1960–1969, 1970–1982, and 1983–2005.

Since there is no guarantee that the mean of inflation and marginal cost deviations is zero in the subsamples, we also include an intercept in the regression. As in other studies, e.g., Rudd and Whelan (2007), we find that the slope estimates and the R^2 statistics tend to be small. The largest estimate of κ is 0.03, obtained by regressing demeaned inflation on the HP-filtered output using the 1960–1969 subsample. If one regresses inflation on the labor share, the largest slope estimate is 0.05, which is obtained by using an HP-filter on both inflation and the labor share and restricting the sample to 1970–1982. The median slope estimate reported in the table is 0.002. The R^2 values range from nearly zero to 66 percent. If we assume that the target inflation rate has shifted in early 1970 and 1982 and use the demeaned labor share as a measure of marginal cost, then $\hat{\lambda} = .003$ and R^2 is 6 percent. The Durbin-Watson statistics (not reported in the table) for the OLS regressions indicate that the least-squares residuals have substantial positive serial correlation.

We draw two broad conclusions for the subsequent review of DSGE model-based estimates. First, since the least-squares estimates range from 0 to 0.03 for κ and 0 to 0.05 for λ, any variation beyond this range is most likely caused by the endogeneity correction. Second, for the Phillips curve to

Table 2 Inflation and Marginal Cost Regressions

Inflation	**Marginal Cost** Measure $\widetilde{MC}_t$	**Sample Period** 1960–2005	1960–1969	1970–1982	1983–2005
$\widetilde{\pi}$ (mean)	$\widetilde{Y}$ (lin trend)	1E-4 (4E-4)	.002 (0.64)	−8E-4 (0.02)	−.001 (0.08)
	$\widetilde{Y}$ (quad trend)	8E-4 (0.01)	.002 (0.65)	−8E-4 (0.01)	−.001 (0.05)
	$\widetilde{Y}$ (HP)	.006 (.008)	0.03 (0.35)	.009 (0.06)	.003 (.008)
	$\widetilde{lsh}$ (mean)	0.01 (0.40)	0.01 (0.66)	.007 (0.03)	.002 (0.03)
	$\widetilde{lsh}$ (HP)	0.04 (0.03)	0.03 (0.03)	0.03 (0.04)	.002 (.001)
$\widetilde{\pi}$ (break)	$\widetilde{Y}$ (lin trend)	5E-4 (0.02)			
	$\widetilde{Y}$ (quad trend)	8E-4 (0.03)			
	$\widetilde{Y}$ (HP)	0.01 (0.07)		same as $\widetilde{\pi}$ (mean)	
	$\widetilde{lsh}$ (mean)	.003 (0.06)			
	$\widetilde{lsh}$ (HP)	0.02 (0.02)			
$\widetilde{\pi}$ (HP)	$\widetilde{Y}$ (lin trend)	2E-4 (.005)	2E-4 (0.03)	−4E-4 (.007)	.001 (0.11)
	$\widetilde{Y}$ (quad trend)	2E-4 (.006)	2E-4 (0.03)	−4E-4 (.006)	.001 (0.11)
	$\widetilde{Y}$ (HP)	.004 (0.02)	.007 (0.12)	.002 (.005)	.007 (0.07)
	$\widetilde{lsh}$ (mean)	.002 (0.03)	.003 (0.16)	0.01 (0.17)	5E-4 (.002)
	$\widetilde{lsh}$ (HP)	0.03 (0.08)	0.02 (0.07)	0.05 (0.15)	0.01 (0.03)

Notes: We are reporting coefficient estimates, $\hat{\alpha}_1$ and R^2, in parenthesis for a regression of the form $\widetilde{\pi}_t = \alpha_0 + \alpha_1[\widetilde{MC}_t/(1 - \hat{\beta}\hat{\rho}_{MC})]$, where $\hat{\beta} = 0.993$, $\hat{\rho}_{MC}$ is the first-order autocorrelation of the marginal cost measure, and the marginal cost measure is either $\widetilde{Y}_t$ or $\widetilde{lsh}_t$.

capture the inflation persistence well, it has to be the case that lagged inflation enters the NKPC, that the mark-up shock is fairly persistent, or that inflation deviations are computed relative to a time-varying target inflation rate.

VAR-IRF Evidence

We explained in Section 2 that if the DSGE model imposes enough restrictions to unambiguously identify, say, a monetary policy shock, then the response of output and marginal costs to this shock provides useful information about the NKPC parameters. To the extent that we would expect a well-specified DSGE model to generate impulse responses that are similar to those obtained from a structural VAR analysis, it is informative to examine prototypical VAR responses to a monetary policy shock and to determine a range of NKPC parameterizations that are consistent with these responses.

Under the assumption that lagged inflation does not enter the NKPC and that marginal costs are proportional to output, the impulse responses to a monetary policy shock have to satisfy

$$\frac{\partial \tilde{\pi}_{t+h}}{\partial \epsilon_t^R} = \kappa \sum_{j=0}^{\infty} \beta^j \mathrm{E}_{t+h} \left[\frac{\partial \tilde{Y}_{t+h+j}}{\partial \epsilon_t^R} \right].$$

As in Section 2, we use κ to denote the slope of the Phillips curve with respect to output. The parameter, κ, absorbs the elasticity of marginal costs with respect to output into the definition of the slope. Suppose that the impulse responses are monotonic and the output response decays approximately exponentially at rate δ in response to a monetary policy shock. Then

$$\frac{\partial \tilde{\pi}_{t+h}}{\partial \epsilon_t^R} \approx \frac{\kappa}{1-\delta\beta} \frac{\partial \tilde{Y}_{t+h}}{\partial \epsilon_t^R}.$$

While a large literature exists (see Christiano, Eichenbaum, and Evans [1999] and Stock and Watson [2001] for surveys) that uses structural VARs to measure the effect of monetary policy shocks, we focus on a prominent recent study by Christiano, Eichenbaum, and Evans (2005).

The authors estimate a nine-variable VAR using data on real GDP, real consumption, the GDP deflator, real investment, the real wage, labor productivity, the federal funds rate, real profits, and the growth rate of M2. Christiano, Eichenbaum, and Evans (2005) find that a 15-basis point (bp) drop in the federal funds rate (quarterly percentage points) leads to a 5-bp increase in the quarterly inflation rate after 12 quarters and a 50-bp increase of output after nine quarters.[6] Hence, according to the mean impulse responses, κ should be about 0.1 if we set the decay factor, δ, to zero and 0.05 if we

[6] These numbers are approximate, based on Figure 1 in Christiano, Eichenbaum, and Evans (2005).

set $\delta = 0.5$. Suppose now that we ignore the dependence in the sampling distribution of the impulse response function estimators and let $\delta = 0$ again. Combining the lower bound of the reported 95 percent confidence band of the inflation response with the upper bound of the confidence band for the output response suggests that κ could be as low as 0.01. Combining the upper bound for the inflation response with the lower bound for the output response leads to a value of $\kappa = 0.5$. If we consider the labor share instead of the output response, we can obtain an estimate of λ instead of κ. Along the mean impulse response estimated by Christiano, Eichenbaum, and Evans (2005), the labor share appears to drop by about 25 bp, which for $\delta = 0$ and $\delta = 0.5$ leads to values of $\lambda = 0.2$ and $\lambda = 0.1$, respectively.

Evidence from Inflation and Marginal Cost Dynamics

Several papers, e.g., Sbordone (2002, 2005) and Kurmann (2005, 2007), exploit the restriction (20) to construct minimum-distance estimates of the NKPC parameters from the estimates of an unrestricted VAR that includes inflation and a measure of marginal costs. Using data from 1951 to 2002 on the labor share and inflation, Sbordone (2005) obtains an estimate of $\hat{\lambda} = 0.025$ in the purely forward-looking specification, and $\hat{\lambda} = 0.014$ and $\hat{\gamma}_b = 0.18$ if she allows lagged inflation to enter the NKPC. To the extent that the restriction (20) is also embodied in a DSGE model likelihood function, the DSGE model-based estimates of the NKPC parameters should be similar, provided that the same measure of marginal costs is used, the mark-up shock is assumed to be i.i.d., and the vector autoregressive approximation to the law of motion of the estimated DSGE model resembles the unrestricted VAR estimates.

4. REVIEW OF EMPIRICAL RESULTS

Broadly speaking, the empirical papers reviewed in this section fall into two categories: either marginal costs are treated as a latent variable or the set of observables spans the labor share and, hence, the model-implied measure of marginal costs. Consider once again the simple model of Section 2 and let us denote the labor share as lsh. Abstracting from inference about γ_b and γ_f, a study that estimates λ in

$$\tilde{\pi}_t = \beta \mathrm{E}_t[\tilde{\pi}_{t+1}] + \frac{\lambda}{\tau}\tilde{Y}_t - \tilde{Z}_t + \tilde{\xi}_t, \tag{22}$$

based on observations of $\tilde{\pi}_t$ and $\tilde{Y}_t$, falls in the first category. Identification of λ in (22) is tenuous because the presence of $\tilde{Z}_t$ exacerbates the endogeneity problem and the parameter, τ, has to be separately estimable from the observables for λ to be identifiable. On the upside, the use of (22) is more robust to the presence of measurement errors in the labor share (marginal cost) series. For some of the papers that fall into the first category, we will report estimates of the output coefficient, κ, which corresponds to $\frac{\lambda}{\tau}$ in the example, rather than λ. A paper that estimates λ in

$$\tilde{\pi}_t = \beta \mathrm{E}_t[\tilde{\pi}_{t+1}] + \lambda \widetilde{lsh}_t + \tilde{\xi}_t, \tag{23}$$

with observations on $\tilde{\pi}_t$ and $\widetilde{lsh}_t$, belongs to the second category.

Since the literature on estimated DSGE models is growing rapidly, we had to strike a balance between scope and depth. This survey is limited to models in which firms' price-setting equations are derived either under quadratic adjustment costs or under the Calvo mechanism. Ongoing research explores alternative sources of nominal rigidities that are not included in the subsequent review, for instance, menu costs and state-dependent pricing models (Dotsey, King, and Wolman 1999, Gertler and Leahy 2006), models with labor market search frictions (Gertler and Trigari 2006, Krause and Lubik 2007), and models with information processing frictions (Sims 2003, Mackowiak and Wiederholt 2007, Mankiw and Reis 2007, and Woodford 2008). Moreover, we focus on models in which the labor share is the model-implied measure of marginal costs.[7]

The numerical values reported in Tables 3–5 refer to point estimates that are obtained with one of four methods. In addition to papers that use Bayesian[8] and maximum likelihood methods as discussed in Section 1, we consider studies that estimate the DSGE model parameters by minimizing the discrepancy between impulse responses implied by the DSGE model and those obtained from the estimation of a structural VAR, or by minimizing the distance between sample moments obtained from U.S. data and DSGE model-implied population moments. The remainder of this section is organized as follows. We review estimates that are obtained by treating marginal costs as a latent variable. We examine studies in which the authors treat marginal costs as observable. For monetary policy analysis, the relationship between inflation and output is at least as important as the relationship between inflation and marginal costs. So we examine DSGE model-based estimates of the relative movements of inflation and output in response to a monetary policy shock. Finally, we discuss the role of wage stickiness.

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[7] Krause, López-Salido, and Lubik (2008) show that in a model with labor market search frictions, marginal costs are also affected by the labor market tightness. However, empirically they find that matching frictions in the labor market appear to affect the cyclical behavior of marginal costs only slightly in terms of co-movement, persistence, and volatility.

[8] Bayesian inference combines information contained in the likelihood function with prior information to form posterior estimates. Since it is difficult to disentangle the contribution of various sources of information ex post, we restrict our attention to the posterior estimates without examining the priors that were used to generate these posteriors.

Latent Marginal Costs

Table 3 summarizes parameter estimates of a Phillips curve specification in which marginal costs are replaced by output or a measure of the output gap:

$$\tilde{\pi}_t = \gamma_b \tilde{\pi}_{t-1} + \gamma_f \mathrm{E}_t[\tilde{\pi}_{t+1}] + \kappa \tilde{Y}_t + \tilde{\xi}_t, \tag{24}$$

where $\tilde{\xi}_t$ represents the latent variables that enter the NKPC in any particular model. These estimates are obtained by fitting New Keynesian DSGE models to observations of output, inflation, and interest rates. The models implicitly share the following features: household preferences are linear in labor and capital is not a factor of production. Estimates for κ range from values less than 0.001 (Cho and Moreno 2006) to 4.15 (Canova forthcoming). While the studies differ with respect to sample period as well as the detrending of inflation and output, our least-squares analysis in Section 3 suggests that most of the differences in $\hat{\kappa}$ are probably due to the treatment of latent variables. We showed that the likelihood function corrects for the endogeneity problem that arises in a regression of inflation on future expected output due to the correlation of the latent variables with expected output. This endogeneity correction appears to be very sensitive to the assumed correlation among the exogenous disturbances that enter the Phillips curve, the Euler equation, and the monetary policy rule. Models in which the shocks in the Euler equation and the Phillips curve are forced to be or allowed to be correlated tend to deliver larger estimates of κ than models in which these disturbances are assumed to be uncorrelated.[9]

We now turn to estimates of New Keynesian Phillips curves that are expressed in terms of marginal costs instead of output:

$$\tilde{\pi}_t = \gamma_b \tilde{\pi}_{t-1} + \gamma_f \mathrm{E}_t[\tilde{\pi}_{t+1}] + \lambda \widetilde{MC}_t + \tilde{\xi}_t. \tag{25}$$

These estimates are reported in Table 4. Rabanal and Rubio-Ramírez (2005) fit a canonical New Keynesian DSGE model without capital and habit formation using a data set that contains, in addition to inflation, interest rates, and detrended output, a measure of the real wage. For specifications in which γ_b is restricted to be zero, the authors obtain estimates of λ of about 0.015. If γ_b is estimated subject to the restriction that $\gamma_b + \gamma_f = 0.99$, the estimate of

[9] Correlation arises either because the structural model implies that, say, the government spending shock enters both the Phillips curve and the Euler equation (e.g., Schorfheide 2005), or because authors attach reduced-form disturbances to the Phillips curve and the Euler equation and assume that these disturbances are correlated (e.g., Lubik and Schorfheide 2004). In small-scale models, it is often "testable" whether the exogenous disturbances are correlated. In large DSGE models, parameters associated with the endogenous propagation mechanism and auxiliary parameters that generate correlation between exogenous disturbances are often not separately identifiable.

Table 3 Published NKPC Estimates: Latent Labor Share (Part 1)

Study	Sample Period	π_{t-1}	$E_t[\pi_{t+1}]$	Y_t	Method
No capital, no habit formation, output coefficient in Phillips curve					
Canova (forthcoming), Table 1	1955:Q1–2002:Q1		0.98	4.150	Bayes
Cho and Moreno (2006), Table 2	1980:Q4–2000:Q1	0.44	0.56	0.001	MLE
Cho and Moreno (2006), Table 2	1980:Q4–2000:Q1	0.44	0.56	0.001	MLE
Cho and Moreno (2006), Table 2	1980:Q4–2000:Q1	0.43	0.57	0.000	MLE
Del Negro and Schorfheide (2004), Table 2	1959:Q3–1979:Q2		1.00	0.314	Bayes
Del Negro and Schorfheide (2004), Table 2	1959:Q3–1979:Q2		1.00	0.249	Bayes
Lindé (2005), Table 5	1960:Q1–1997:Q4	0.72	0.28	0.048	MLE
Lindé (2005), Table 5	1960:Q1–1997:Q4	0.54	0.46	0.048	MLE
Lubik and Schorfheide (2004), Table 3	1960:Q1–1979:Q2		.997	0.770	Bayes
Lubik and Schorfheide (2004), Table 3	1960:Q1–1979:Q2		.997	0.750	Bayes
Lubik and Schorfheide (2004), Table 3	1982:Q4–1997:Q4		.993	0.580	Bayes
Rotemberg and Woodford (1997), Page 321	1980:Q1–1995:Q2		0.99	0.024	IRF-MD
Salemi (2006), Table 2	1965:Q1–2001:Q4	0.62	0.00	0.055	MLE
Salemi (2006), Table 2	1965:Q1–2001:Q4	0.43	0.57	0.003	MLE
Schorfheide (2005), Table 2	1960:Q1–1997:Q4		0.99	0.370	Bayes
Schorfheide (2005), Table 2	1960:Q1–1997:Q4		1.00	0.360	Bayes

Notes: We are providing point estimates of the New Keynesian Phillips curve, $\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t[\pi_{t+1}] + \kappa Y_t + \xi_t$, based on the information provided in the cited studies. Estimation methods: MLE = maximum likelihood estimation; Bayes = Bayesian analysis; and IRF-MD minimize discrepancy between impulse responses estimated with a structural VAR and those implied by a DSGE model.

λ drops to 0.004, whereas the weight on lagged inflation in the Phillips curve is 0.43.

The canonical three equation New Keynesian DSGE model that underlies, for instance, the analysis in Rabanal and Rubio-Ramirez (2005) lacks persistent dynamics, which makes it difficult to capture the serial correlation in U.S. data. The lack of persistence can be overcome in part by using household preferences that exhibit habit formation, that is, the instantaneous utility is a function of current consumption relative to some habit stock, which in turn depends on past consumption. Habit formation enriches the endogenous propagation mechanism of the model and enhances the model's ability to capture the persistence in output and consumption. More importantly for us, it changes the relationship between (observed) output and (latent) marginal costs. The marginal utility of consumption, and thereby marginal costs, depends not just on the current level of output, but also on past and expected future levels as well as the parameters that determine the degree of habit formation. The estimates of λ reported in the second section of Table 4 range from 0.004 to 0.437.

If capital is treated as a variable input, marginal costs remain equal to the labor share as long as the production function is of the Cobb-Douglas form. However, if labor share observations are not used directly in the estimation, the presence of capital affects the decomposition of marginal costs into an observed and an unobserved component. In other words, if the DSGE model is estimated based on observations of output, inflation, and interest rates, introducing variable capital changes the stochastic properties of $\tilde{\xi}_t$ in (24) and the relationship between κ in (24) and λ in (25). The third section of Table 4 reports NKPC estimates from six studies, ranging from 0.008 to 0.112. Among these papers, Fernandez-Villaverde and Rubio-Ramirez (2007) allow the parameters of the monetary policy rule and the parameters that determine the degree of price and wage stickiness to vary over time. This allows the authors to obtain a time series of the Phillips curve coefficient. If the slope estimates of the Phillips curve are converted into the probability that a firm is unable to change its price in a Calvo model (see Section 1), then the estimates can be summarized as follows. Prices stayed constant for an average of four quarters in the 1960s and 1970s, while inflation was relatively high and became a bit more rigid after the Volcker disinflation. Based on a casual inspection of the smoothed time series of the Phillips curve coefficients, λ appeared to be, on average, around 0.06 before 1979 and subsequently dropped to 0.03. The average estimate of γ_b pre-1979 is about 0.35 and decreased to 0.3 after 1979. This pattern is broadly consistent with the notion that the NKPC is not structural in the following sense: If a high target inflation rate makes it very costly for firms not to change their prices—and, hence, more attractive to incur the costs of adjusting the prices—we should observe a steeper Phillips curve relationship.

Table 4 Published NKPC Estimates: Latent Labor Share (Part 2)

Study	Sample Period	π_{t-1}	$E_t[\pi_{t+1}]$	MC_t	Method
No capital, no habit formation					
Rabanal and Rubio-Ramírez (2005), Table 2	1960:Q1–2001:Q4		0.99	0.015	Bayes
Rabanal and Rubio-Ramírez (2005), Table 2	1960:Q1–2001:Q4	0.43	0.56	0.004	Bayes
Rabanal and Rubio-Ramírez (2005), Table 2	1960:Q1–2001:Q4		0.99	0.016	Bayes
Rabanal and Rubio-Ramírez (2005), Table 2	1960:Q1–2001:Q4		0.99	0.017	Bayes
No capital, with habit formation					
Andres, López-Salido, and Nelson (2004), Table 1	1980:Q1–1999:Q2		0.99	0.014	MLE
Andres, López-Salido, and Nelson (2005), Table 2	1979:Q3–2003:Q3	0.50	0.50	0.437	MLE
Boivin and Giannoni (2006), Table 2	1959:Q2–1979:Q2	0.50	0.50	0.006	IRF-MD
Boivin and Giannoni (2006), Table 2	1979:Q3–2002:Q2	0.50	0.50	0.004	IRF-MD
Galí and Rabanal (2004), Table 4	1948:Q1–2002:Q4	0.02	0.97	0.413	Bayes
Lubik and Schorfheide (2006), Table 2	1983:Q1–2002:Q4		1.00	0.200	Bayes
Milani (2007), Table 2	1960:Q1–2004:Q2		0.99	0.024	Bayes
Models with capital					
Bouakez, Cardia, and Ruge-Murcia (2005), Table 1	1960:Q1–2001:Q2	0.50	0.50	0.015	MLE
Bouakez, Cardia, and Ruge-Murcia (forthcoming), Table 4	1964:Q1–2002:Q4		1.00	0.223	MD
Christensen and Dib (2008)	1979:Q3–2004:Q3		.993	0.092	MLE
Fernández-Villaverde and Rubio-Ramírez (2007), Table 6.1	1955:Q1–2000:Q4	0.13	0.87	0.008	Bayes
Fernández-Villaverde and Rubio-Ramírez (2007), Table 6.1	1955:Q1–1979:Q4	0.26	0.74	0.056	Bayes
Fernández-Villaverde and Rubio-Ramírez (2007), Table 6.1	1980:Q1–2000:Q4	0.13	0.87	0.030	Bayes
Laforte (2007), Table 3	1983:Q1–2003:Q1	0.40	0.59	0.112	Bayes
Rabanal (2007), Table 2	1959:Q1–2004:Q4	0.50	0.50	0.018	Bayes

Notes: We are providing point estimates of the New Keynesian Phillips curve, $\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t[\pi_{t+1}] + \lambda MC_t + \xi_t$, based on the information provided in the cited studies. Estimation methods: MLE = maximum likelihood estimation; Bayes = Bayesian analysis; IRF-MD minimize discrepancy between impulse responses estimated with a structural VAR and those implied by a DSGE model; and MD minimize discrepancy between sample moments and DSGE model-implied moments.

Observed Marginal Costs

We now turn to the Bayesian estimation of New Keynesian DSGE models based on a larger set of observables that spans the labor share and, hence, marginal costs as they appear in the Phillips curve. Intuitively, the use of labor share observations should lead to a sharper identification of λ. Table 5 summarizes empirical estimates from seven studies. Most estimates are based on a variant of the Smets and Wouters (2003) model, which augments a DSGE model by Christiano, Eichenbaum, and Evans (2005) with additional shocks to make it amenable to likelihood-based estimation. Smets and Wouters (2005), Levin et al. (2006), Del Negro et al. (2007), Smets and Wouters (2007), and Justiniano and Primiceri (2008) obtain estimates of λ of 0.01, 0.03, 0.10, 0.02, and 0.01, respectively. The estimates of the coefficient γ_b on lagged inflation are 0.25, 0.07, 0.43, 0.19, and 0.46, respectively. Compared to the numbers reported in Tables 3 and 4, the variation across studies is much smaller.

Impulse Response Dynamics

Much of our previous discussion focused on the marginal cost coefficient in the Phillips curve relationship. However, from a monetary policy perspective, equally important is the output-inflation tradeoff in the estimated DSGE model. This tradeoff not only depends on λ but also on the elasticity of marginal costs with respect to output. Thus, we will examine the relative movements of output and inflation in response to a monetary policy shock, that is, an unanticipated deviation from the systematic component of the monetary rule. Of course, these impulse responses do not merely depend on the slope of the NKPC, they also depend on other aspects of the model, such as labor market frictions and wage stickiness and the behavior of the central bank. Not all the papers for which we have reported estimates of the NKPC parameters in Tables 3 to 4 present impulse response functions. Those that do typically represent them in graphical form. The subsequent results are based on an inspection of impulse response plots and are summarized in Table 6.[10] We report the magnitude of the peak responses of the interest rate, inflation rate, and the output deviation from steady state. The interest rate response is measured in annualized percentages; that is, an entry of 0.25 implies that the monetary policy shock raises the interest rate 25 bp above its steady-state level. The inflation rate is not annualized and represents a quarter-to-quarter difference in the log price level, scaled by 100 to convert it into percentages. Output deviations are also reported in percentages. Since the length of a period in a DSGE model is typically assumed to be one quarter, in the context of the

[10] In a number of studies, it turned out to be difficult to determine whether interest rates and inflation rates are annualized. We tried to resolve this ambiguity.

Table 5 Published NKPC Estimates: Observed Labor Share

Study	**Sample Period**	π_{t-1}	$\mathrm{E}_t[\pi_{t+1}]$	MC_t	**Method**
Avouyi-Dovi and Matheron (2007), Tables 3–4	1955:Q1–1979:Q2	0.27	0.73	0.008	IRF-MD
Avouyi-Dovi and Matheron (2007), Tables 3–4	1982:Q3–2002:Q4	0.20	0.80	0.010	IRF-MD
Christiano, Eichenbaum, and Evans (2005), Table 2	1965:Q3–1995:Q3	0.50	0.50	0.135	IRF-MD
Del Negro et al. (2007), Table 1	1974:Q2–2004:Q1	0.43	0.57	0.100	Bayes
Justiniano and Primiceri (2008), Table 1	1954:Q3–2004:Q4	0.46	0.54	0.007	Bayes
Justiniano and Primiceri (2008), Table 1	1954:Q3–2004:Q4	0.46	0.54	0.005	Bayes
Levin et al. (2006), Table 1	1955:Q1–2001:Q4	0.07	0.92	0.033	Bayes
Smets and Wouters (2005), Table 1	1983:Q1–2002:Q2	0.25	0.74	0.007	Bayes
Smets and Wouters (2007), Table 1A/B	1966:Q1–2004:Q4	0.19	0.82	0.020	Bayes

Notes: We provide point estimates of the New Keynesian Phillips curve, $\pi_t = \gamma_b \pi_{t-1} + \gamma_f \mathrm{E}_t[\pi_{t+1}] + \lambda MC_t + \xi_t$, based on the information provided in the cited studies. Estimation methods: Bayes = Bayesian analysis; IRF-MD minimize discrepancy between impulse responses estimated with a structural VAR and those implied by a DSGE model.

"back-of-the-envelope" calculation in Section 3, the ratio of the inflation and output response, denoted by $\partial\pi/\partial y$, would correspond to $\kappa/(1-\delta\beta)$, where δ is the factor at which the output response decays to zero.

In Table 6 we report the number of periods it takes for the responses to reach their respective peaks, the ratio of the peak response of inflation and output, and the estimate of $\hat{\kappa}$ in the underlying model. Models without capital and with little endogenous propagation typically generate monotonic impulse response functions. For the models without capital, the relative responses of inflation and output range from 0.07 to 2.00. Once capital is included and the model is augmented by additional frictions, this range narrows to 0.08 to 0.17, which seems consistent with the VAR evidence provided by Christiano, Eichenbaum, and Evans (2005). Comparing the estimates reported in Del Negro et al. (2007) and Smets and Wouters (2007), it appears that these tradeoffs can be obtained with quite differently priced Phillips curve slopes, λ: 0.002 and 0.10.

Wage Versus Price Rigidity

This article has focused on estimates of the degree of price rigidity in New Keynesian DSGE models. Many authors believe that inflexible wages are another important source of nominal rigidities. In fact, the DSGE models that are based on the work of Smets and Wouters (2003), and Christiano, Eichenbaum, and Evans (2005) incorporate both price and wage stickiness. Following work by Erceg, Henderson, and Levin (2000), in order to generate wage stickiness in DSGE models, one typically assumes that households supply differentiated labor services that are aggregated by labor packers into homogenous labor services. These homogeneous labor services are in turn utilized by the intermediate goods-producing firms. Households act as monopolistically competitive suppliers and are subjected to a Calvo (1983) friction: only a fraction of households is allowed to re-optimize nominal wage. To clear the labor market ex post, one must assume that each household has to satisfy the demand for its labor service at the posted price.

For a joint estimation of price and wage rigidity to be meaningful, the set of observables needs to span inflation, labor share, and wages. The joint dynamics of inflation and the labor share provide information about the price Phillips curve, and the wage series, together with an implicit measure of the marginal disutility of work, contains information about the degree of wage stickiness. Del Negro and Schorfheide (2008) estimate a variant of the Smets and Wouters (2003) under three priors that differ with respect to a priori beliefs about nominal rigidities. The *low rigidities* prior assumes that the price and wage Calvo parameters have a beta-distribution centered at 0.45 with a standard deviation of 0.10. The *high rigidities* prior is centered at 0.75

Table 6 Impulse Responses to a Monetary Policy Shock

Study	**Interest Rate** [Annualized %]		**Inflation** [Quarterly %]		**Output** [% Dev. from Trend]		$\partial\pi/\partial y$
	Peak	After x Q's	Peak	After x Q's	Peak	After x Q's	
No capital, no habit formation, marginal costs function of current output deviations							
Cho and Moreno (2006), Figure 2	−0.80	0	0.01	9	0.12	9	0.08
Del Negro and Schorfheide (2004), Figure 2	−0.25	0	0.05	0	0.05	0	1.00
Ireland (2004a), Figure 1	−0.20	0	0.20	0	0.60	0	0.33
Ireland (2004b), Figure 1	−1.00	0	0.07	0	0.40	0	0.18
Ireland (2007), Figure 2	−0.40	0	0.10	0	0.30	0	0.33
Lubik and Schorfheide (2004), Figure 3	−0.70	0	0.12	0	0.16	0	0.75
Lubik and Schorfheide (2004), Figure 3	−0.60	0	0.17	0	0.16	0	0.76
Lubik and Schorfheide (2004), Figure 3	−0.60	0	0.20	0	0.16	0	1.25
Rotemberg and Woodford (1997), Figure 1	−0.80	0	0.03	2	0.38	2	0.07
Salemi (2006), Figure 3	−1.00	0	0.020	10	0.40	9	0.05
Salemi (2006), Figure 3	−1.00	0	0.002	8	0.30	8	0.01
No capital, with habit formation, marginal costs are function of current, past, and future output deviations							
Andres, López-Salido, and Nelson (2004), Figure 2	−0.30	0	0.17	0	0.40	3	0.43
Andres, López-Salido, and Nelson (2005), Figure 1	−0.50	0	0.15	0	0.08	1	2.00
Andres, López-Salido, and Nelson (2005), Figure 1	−0.70	0	0.06	0	0.40	1	0.15
Boivin and Giannoni (2006), Figure 1	−1.00	0	0.14	6	1.10	4	0.13
Boivin and Giannoni (2006), Figure 1	−1.00	0	0.02	4	0.30	4	0.07
With capital, no direct observations on labor share							
Christensen and Dib (2008), Figure 1	−0.48	2	0.14	2	0.50	2	0.28
Laforte (2007), Figure 2	−0.75	0	0.15	0	0.50	2	0.30
Rabanal (2007), Figure 4	−1.00	0	0.10	4	0.50	3	0.20
With capital, with direct observations on labor share							
Christiano, Eichenbaum, and Evans (2005), Figure 1	−0.60	4	0.05	11	0.60	6	0.08
Del Negro et al. (2007), Figure 3	−1.10	0	0.05	2	0.33	2	0.15
Smets and Wouters (2005), Figure 5	−0.70	4	0.05	3	0.45	5	0.11
Smets and Wouters, (2007) Figure 6	−0.72	0	0.05	2	0.30	3	0.17

Notes: Based on the graphical information provided in the cited studies, we determined the peak responses for interest rates (annualized percentage points), inflation (quarter-to-quarter percentage points), and output (percentage deviations from trend/steady state) to an unanticipated loosening of monetary policy.

with a standard deviation of 0.1. Finally, the *agnostic* prior is centered at 0.6 and is more diffuse—its standard deviation is 0.2.

Posterior inference based on these priors can be summarized as follows: both under the *agnostic* and the *low rigidities* prior, the posterior estimate of the wage stickiness is small. The Calvo parameter is around 0.25, which means that the households re-optimize their wages, on average, every four months. The estimated price stickiness translates into a value of λ of about 0.22. Under the *high rigidities* prior, the estimates of both the wage and the price Calvo parameter turn out to be substantially larger, namely about 0.8. Most interestingly, the time series fit of all three specifications is very similar, yet the policy implications are quite different. The results presented in Del Negro and Schorfheide (2008) suggest that the macro time series we typically consider is not informative enough to precisely measure the degree of nominal rigidity. This conclusion is consistent with the literature survey conducted in this section: the variation of parameter estimates reported in the literature is substantial. No clear consensus has emerged as of now.

5. CONCLUSION

While the literature on DSGE model-based estimation of the NKPC is still fairly young, a wide variety of results have been published in academic journals already. In most of these studies, the Phillips curve estimation is not a goal but rather a byproduct of the empirical analysis. DSGE model-based NKPC estimates tend to be fragile and sensitive to model specification and data definitions, in particular if marginal costs are treated as a latent variable. If the observations span the labor share, which is the model-implied measure of marginal costs in the studies that we reviewed, then the slope estimates are more stable. No consensus has emerged on the importance of lagged inflation in the Phillips curve. Estimates are sensitive to detrending methods for inflation and assumptions about the autocovariance structure of the exogenous disturbances in the DSGE model. Thus, from a policymaker's perspective, accounting for parameter and model uncertainty is important for prediction and decision making.

We attempted to understand the identification of Phillips curve parameters in estimated DSGE models. Unlike single-estimation approaches, DSGE model-based estimates are able to extract information about the structural parameters from the contemporaneous correlations of output, inflation, interest rates, and other variables, as well as from impulse responses to structural shocks that are identifiable based on exclusion restrictions hard-wired in model specifications. Unfortunately, the data do not speak loudly and clearly to us, and many DSGE models imply that if the model is "true," it is difficult to identify the NKPC parameters and the output-inflation tradeoff with only 20 to 40 years of observations.

Identification in the context of simultaneous equations models is well understood. To identify the slope of a supply curve we need variation in exogenous demand shifters. Identification in DSGE models is much more complicated. Variation in the data is created by unobserved shocks that in most cases shift both demand and supply. Our reading of the early literature on estimated DSGE models is that there was hope that the model-implied cross-coefficient restrictions were so tight that identification was not a concern. Over time the profession learned that, despite tight cross-equation restrictions, identification should not be taken for granted, in particular in New Keynesian DSGE models. While currently ongoing research is developing econometric techniques to try to diagnose identification problems, it might be time to go back to the drawing board and develop future DSGE models with parameter identifiability in mind.

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