

Limits to Redistribution and Intertemporal Wedges: Implications of Pareto Optimality with Private Information

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Traditionally an object of interest in microeconomics, models with privately informed agents have recently been used to study numerous topics in macroeconomics.¹ Characterization of Pareto-optimal allocations is an essential step in these studies, because the structure of optimal institutions of macroeconomic interest depends on the structure of optimal allocations. In models with privately informed agents, however, characterization of optimal allocations is a complicated problem, relative to models in which all relevant information is publicly available, especially in dynamic settings with heterogeneous agents, which are of particular interest in macroeconomics.

The objective of this article is to characterize Pareto-optimal allocations in a simple macroeconomic environment with private information and heterogeneous agents. We focus on the impact of private information on the implications of Pareto optimality. To this end, we consider two economies that are identical in all respects other than the presence of private information. In each

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¹ These topics include business cycle fluctuations (e.g., Bernanke and Gertler 1989); optimal monetary policy (Athey, Atkeson, and Kehoe 2005); unemployment insurance (Atkeson and Lucas 1995, Hopenhayn and Nicollini 1997, Stiglitz and Yun 2005); capital income and estate taxation (Kocherlakota 2005, Albanesi and Sleet 2006, Farhi and Werning 2006); disability insurance and social welfare (Golosov and Tsyvinski 2006, Pavoni and Violante 2007); social security design (Stiglitz and Yun 2005, Grochulski and Kocherlakota 2007); financial intermediation (Green and Lin 2003); and asset pricing (Kocherlakota and Pistaferri 2008).

economy, we fully characterize the set of all Pareto-optimal allocations. By comparing the structure of the sets of optimal allocations obtained in these two cases, we isolate the effect private information has on the implications of Pareto optimality.

The economic environment we consider is, on the one hand, rich enough to have features of interest in a macroeconomic analysis, and, on the other hand, simple enough to admit elementary, closed-form characterization of Pareto-optimal allocations, both with and without private information. The model we use is a stylized, two-period version of the Lucas (1978) pure capital income economy that is extended, however, to incorporate a simple form of agent heterogeneity. We assume that the population is heterogenous in its preference for early versus late consumption. In particular, we assume that a known fraction of agents are impatient, i.e., have a strong preference for consumption in the first time period, relative to the rest of the population. In the economy with private information, individual impatience is not observable to anyone but the agent. A detailed description of the environment is provided in Section 1.

In our analysis, we exploit the connection between Pareto-optimal allocations and solutions to so-called social planning problems, in which a (stand-in) social planner maximizes a weighted average of the individual utility levels of the two types of agents. These planning problems are defined and solved for both the public information economy and the private information economy in Section 2. The solutions obtained constitute all Pareto-optimal allocations in the two economies.

In the third section, we compare the Pareto optima of the two economies along two dimensions. First, we examine their welfare properties by comparing the utility levels provided to agents in the cross-section of Pareto-optimal allocations. The range of individual utility levels supported by Pareto optima in the private information economy turns out to be much smaller than that of the public information economy. In this sense, private information imposes limits to redistribution that can be attained in this economic environment. Then, we compare the structures of optimal intertemporal distortions, which are often called intertemporal wedges, across the Pareto optima of the two economies. With public information, all Pareto-optimal allocations are free of intertemporal wedges. In the economy with private information, we find Pareto-optimal allocations characterized by a positive intertemporal wedge, and others characterized by a negative intertemporal wedge. We close Section 3 with a short discussion of the implications of wedges for the consistency of Pareto-optimal allocations with market equilibrium outcomes, which are studied in many macroeconomic applications. Section 4 draws a brief conclusion.

1. TWO MODEL ECONOMIES

We consider two parameterized model economies. The two economies have the same preferences and technology. They differ, however, with respect to the amount of public information.

The following features are common to both economies. Each economy is populated by a unit mass of agents who live for two periods, $t = 1, 2$. There is a single consumption good in each period, c_t , and agents' preferences over consumption pairs (c_1, c_2) are represented by the utility function

$$\theta u(c_1) + \beta u(c_2),$$

where β is a common-to-all discount factor, and θ is an agent-specific preference parameter. Agents are heterogenous in their relative preference for consumption at date 1. We assume a two-point support for the population distribution of the impatience parameter θ . Agents, therefore, can be of two types. A fraction μ_H of the agents are impatient with a strong preference for consuming in period 1. Denote by H the value of the parameter θ representing preferences of the impatient agents. A fraction $\mu_L = 1 - \mu_H$ are agents of the patient type. Their value of the impatience parameter θ , denoted by L , satisfies $L < H$.²

In the economies we consider, the production side is represented by a so-called Lucas tree. We assume that the economy is endowed with a fixed amount of productive capital stock—the tree.³ Each period, the capital stock produces an amount Y of the consumption good—the fruit of the tree. The consumption good is perishable—it cannot be stored from period 1 to 2. The size of the capital stock, i.e., the tree, is fixed: the capital stock does not depreciate nor can it be accumulated.

In our discussion, we will focus attention on a particular set of values for the preference and technology parameters. This will allow for explicit analytical solutions to the optimal taxation problem studied in this article. In particular, we will take

$$u(\cdot) = \log(\cdot), \quad \beta = \frac{1}{2}, \quad H = \frac{5}{2}, \quad L = \frac{1}{2}, \quad \mu_H = \mu_L = \frac{1}{2}, \quad Y = 1. \quad (1)$$

Roughly, the model period is thought of as being 25 years. The value of the discount factor β of $\frac{1}{2}$ corresponds to the annualized discount factor of about 0.973. The fractions of the two patience types are equal, and preferences are logarithmic. With $\frac{H}{L} = 5$, we consider a significant dispersion of the

²A formulation of preferences with the two types having different discount factors would be equivalent.

³In our study of optimal allocations, we abstract from private ownership of capital. Given that (a) capital is publicly observable and seizable, and (b) the society does not value individual utilities differently on the basis of individual wealth, this abstraction has no bearing on the problem we study. That is, the set of Pareto optimal allocations does not depend on who holds wealth in the economy.

impatience parameter in the population. The per-period product of the capital stock is normalized to one.

The two economies we consider differ with respect to the scope of public knowledge of each agent's individual impatience parameter. In the first economy we consider, each agent's preference type is public information, i.e., it is known to the agent and everyone else. In the second economy, each agent's individual impatience is known only to himself.

2. PARETO-EFFICIENT ALLOCATIONS

An allocation in this environment is a description of how the total output (i.e., the economy's capital income Y) is distributed among the agents each period. We consider only type-identical allocations, in which all agents of the same type receive the same treatment. An allocation, therefore, consists of four positive numbers, $c = (c_{1H}, c_{1L}, c_{2H}, c_{2L})$, where $c_{t\theta}$ denotes the amount of the consumption good in period t assigned to each agent of type θ .

In this section, we describe the efficient allocations. We use the standard notion of Pareto efficiency applied to type-identical allocations. We say that an allocation c is Pareto-dominated by an allocation \hat{c} if all types of agents are at least as well off at \hat{c} as they are at c and some are strictly better off. In our model, allocation c is Pareto-dominated by an allocation \hat{c} if

$$\theta u(\hat{c}_{1\theta}) + \beta u(\hat{c}_{2\theta}) \geq \theta u(c_{1\theta}) + \beta u(c_{2\theta})$$

for both $\theta = H, L$, and if

$$\theta u(\hat{c}_{1\theta}) + \beta u(\hat{c}_{2\theta}) > \theta u(c_{1\theta}) + \beta u(c_{2\theta})$$

for at least one θ . An allocation c is Pareto-efficient in a given class of allocations if c belongs to this class and is not Pareto-dominated by any allocation \hat{c} in this class of allocations.

Pareto Optima in the Public Types Economy

In our economy with public preference types, resource feasibility is the sole constraint on the class of allocations that can be attained. An allocation is resource-feasible if the total amount consumed each period does not exceed the total available output. That is, in our model, allocation c is resource-feasible (RF) if for $t = 1, 2$,

$$\sum_{\theta=H,L} \mu_{\theta} c_{t\theta} \leq Y. \quad (2)$$

In the public types economy, therefore, we are interested in allocations that are Pareto-efficient in the class of all RF allocations. We will refer to such allocations as First Best Pareto optima.

Characterizing the Set of All First Best Pareto Optima

In order to find all First Best Pareto-optimal allocations, it will be useful to consider a social planning problem defined as follows:

First Best Planning Problem For each $\gamma \in [0, +\infty]$, find an allocation $c = (c_{1H}, c_{1L}, c_{2H}, c_{2L})$ that maximizes the value of the welfare objective

$$\gamma [Hu(c_{1H}) + \beta u(c_{2H})] + Lu(c_{1L}) + \beta u(c_{2L}), \quad (3)$$

subject to resource feasibility constraints (2).⁴

In this problem, which we will refer to as the First Best planning problem, γ represents the relative weight that the social welfare criterion (3) puts on the agents of type H . The constraint set of the First Best planning problem is defined by the RF constraints (2). It is easy to check that this constraint set is compact (i.e., closed and bounded). This, and the fact that the objective (3) is continuous, implies that a solution to the First Best planning problem exists for every $\gamma \in [0, +\infty]$. Also, since the RF constraints are linear in consumption, the constraint set is convex. The objective (3) is strictly concave for each $\gamma \in (0, +\infty)$. Thus, the First Best planning problem has a unique solution for every $\gamma \in [0, +\infty]$.⁵ Denote this unique solution by $c^*(\gamma)$.

The social planning problem is a useful tool for welfare analysis due to the following result: If the set of all feasible allocations is convex and the utility functions of all agent types are strictly increasing and strictly concave, then every solution $c^*(\gamma)$ to the social planning problem is a Pareto optimum, and every Pareto optimum is a solution to the social planning problem for some $\gamma \in [0, +\infty]$.⁶

Because of the concavity of u and the convexity of the set of RF allocations, this result applies in our economy with public types. Thus, we can exploit the connection between the set of Pareto optima and the set of solutions to the First Best social planning problem. We will solve the social planning problem for each $\gamma \in [0, +\infty]$. The solutions we obtain, $c^*(\gamma)$, will determine the set of First Best Pareto optima as we adjust the value of γ between zero and infinity.

Since the First Best planning problem is concave for each $\gamma \in [0, +\infty]$, the solution $c^*(\gamma)$ is given by the necessary and sufficient first-order conditions

⁴ Alternatively, we could write the social objective as $\alpha [Hu(c_{1H}) + \beta u(c_{2H})] + (1 - \alpha) [Lu(c_{1L}) + \beta u(c_{2L})]$, with $\alpha \in [0, 1]$. Our formulation (3) is equivalent when $\gamma = \alpha / (1 - \alpha)$. Thus, $\gamma = +\infty$ corresponds to $\alpha = 1$, i.e., the social objective under $\gamma = +\infty$ is given by $Hu(c_{1H}) + \beta u(c_{2H})$.

⁵ The optima for $\gamma = 0$ and $\gamma = \infty$, trivially, are unique as well, with optimal allocations assigning all consumption respectively to type L and type H .

⁶ The argument for this is entirely standard. See, e.g., section 16E of Mas-Colell, Whinston, and Green (1995).

of this problem. Thus, we can find the solution $c^*(\gamma)$ by taking the first-order conditions and solving for c . Denoting by ρ_t the Lagrange multiplier of the RF constraint at date $t = 1, 2$, the first-order conditions with respect to consumption are as follows:

$$\gamma H u'(c_{1H}) = \rho_1 \mu_H, \quad (4)$$

$$L u'(c_{1L}) = \rho_1 \mu_L, \quad (5)$$

$$\gamma \beta u'(c_{2H}) = \rho_2 \mu_H, \text{ and} \quad (6)$$

$$\beta u'(c_{2L}) = \rho_2 \mu_L. \quad (7)$$

The multipliers ρ_t must be strictly positive at the solution because the objective (3) is strictly increasing in consumption, i.e., both RF constraints bind. For each $\gamma \in [0, +\infty]$, the optimum $c^*(\gamma)$ is the solution to the system of equations consisting of the first-order conditions (4)–(7) and the RF constraints (2).

Using the parameterization (1), we can obtain a closed-form expression for the set of all First Best Pareto-optimal allocations, indexing the allocations in this set by γ . Solving for the optimal consumption values, as a function of γ , we get

$$c_{1H}^*(\gamma) = \frac{10\gamma}{1 + 5\gamma}, \quad (8)$$

$$c_{1L}^*(\gamma) = \frac{2}{1 + 5\gamma}, \quad (9)$$

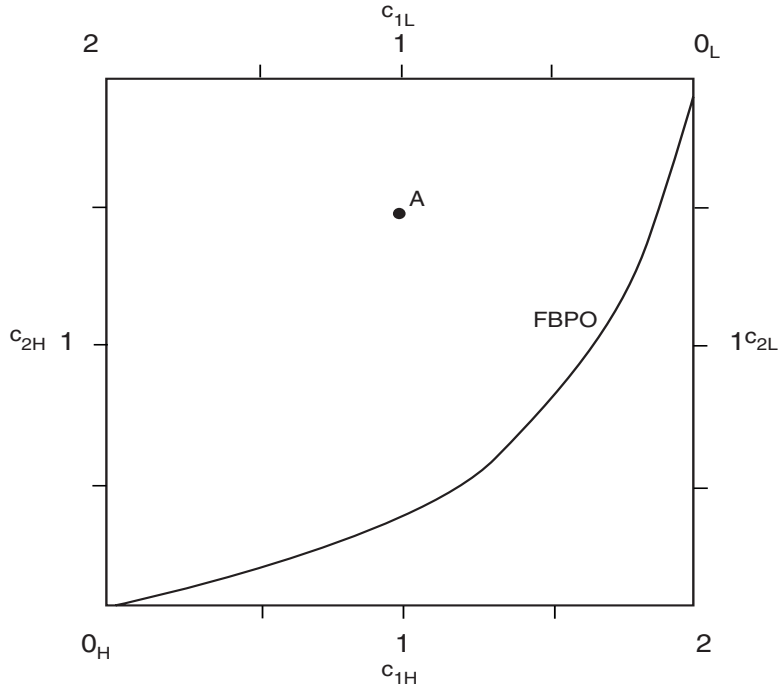
$$c_{2H}^*(\gamma) = \frac{2\gamma}{1 + \gamma}, \quad (10)$$

$$c_{2L}^*(\gamma) = \frac{2}{1 + \gamma}. \quad (11)$$

As we see, at any Pareto optimum, consumption allocated to the impatient type H is front-loaded, i.e., $c_{1H}^*(\gamma) > c_{2H}^*(\gamma)$, and consumption assigned to the less impatient type L is back-loaded, i.e., $c_{1L}^*(\gamma) < c_{2L}^*(\gamma)$. Looking across Pareto optima, consumption of the H -type is strictly increasing, at both dates, in the weight γ , while consumption of the L -type is strictly decreasing.

Figure 1 provides an Edgeworth-box representation of the set of all First Best Pareto optima. The Edgeworth box represents the set of all RF allocations at which the RF constraints (2) are satisfied as equalities (i.e., there is no waste of resources). In the Edgeworth box of Figure 1, the bottom-left corner represents the origin of measurement of consumption allocated to the agents of type H . The horizontal axis measures consumption in period 1. For example, point A in Figure 1, whose coordinates are (1, 1.5), represents an allocation at which the consumption of the H -type agents is $(c_{1H}, c_{2H}) = (1, 1.5)$.

Figure 1 The Set of First Best Pareto Optima



Note that since the fractions of the two types are equal and the resource constraints (2) are binding, we can write them as

$$c_{iL} = 2 - c_{iH} \tag{12}$$

for $t = 1, 2$. Thus, for a given consumption (c_{1H}, c_{2H}) allocated the H -type, the consumption allocated the L -type is given by

$$(c_{1L}, c_{2L}) = (2 - c_{1H}, 2 - c_{2H}).$$

Since the Edgeworth box represents only non-wasteful allocations, the top-right corner of the box of Figure 1, whose coordinates are $(2, 2)$, is the origin of measurement of consumption allocated to the agents of type L . Point A in Figure 1, for example, represents an allocation that assigns amounts $(2 - 1, 2 - 1.5) = (1, 0.5)$ to the agents of type L .

The solid curve in Figure 1 represents the set of all First Best Pareto-optimal allocations given in (8)–(11). The allocations in this set are indexed by γ with the Pareto optimum for $\gamma = 0$ being in the bottom-left corner of the box, and the one obtained for $\gamma = \infty$ in the top-right corner. The curve representing the Pareto set is strictly increasing, which reflects the fact that

consumption of the H -type is strictly increasing in γ . As we noted before, for any weight γ , it is efficient to front-load consumption of the H -type and back-load consumption of the L -type. In the Edgeworth box of Figure 1, this is reflected by the fact that the First Best Pareto set lies below the 45 degree line (not depicted).

Pareto Optima in the Private Types Economy

In the second economy we consider, agents have private knowledge of their own impatience type θ . This imposes additional constraints on the set of allocations that are feasible in this environment.

As an example, suppose that the social planner—or simply the government—wants to distribute the total output of the Lucas tree according to the Pareto-optimal allocation $c^*(0) = (c_{1H}^*(0), c_{1L}^*(0), c_{2H}^*(0), c_{2L}^*(0)) = (0, 2, 0, 2)$. At this particular Pareto optimum, type H agents are assigned zero consumption in both periods (as the social welfare criterion (3) with $\gamma = 0$ does not value their utility at all), and agents of type L consume the whole output of the Lucas tree $Y = 1$. (Each agent of the L -type consumes 2 units, and the mass of the L -type agents is $\frac{1}{2}$, so the total consumption of the L -type agents is 1.) It is clear that when agents' types are private information, it is impossible for the government to attain this distribution of consumption. How will the government know which agent should be assigned zero consumption, as agents themselves are the only possible source of information about their preference type? If revealing the preference type H to the government means consuming zero in both periods, no impatient agent will admit being impatient. Thus, the Pareto optimum $c^*(0)$ is not feasible for the social planner when the impatience type is private information.

As this example demonstrates, the set of allocations feasible in the economy with private information is smaller than the set of all allocations satisfying the resource feasibility constraints (2). In particular, in addition to being resource-feasible, a feasible allocation of consumption c must also be incentive compatible. This requirement states that when faced with an allocation c , agents of both types must be willing to reveal truthfully their type to the government.⁷

Formally, an allocation $c = (c_{1H}, c_{1L}, c_{2H}, c_{2L})$ is incentive compatible (IC) if it satisfies the following two constraints:

$$Hu(c_{1H}) + \beta u(c_{2H}) \geq Hu(c_{1L}) + \beta u(c_{2L}) \quad (13)$$

and

$$Lu(c_{1L}) + \beta u(c_{2L}) \geq Lu(c_{1H}) + \beta u(c_{2H}). \quad (14)$$

⁷A general result known as the Revelation Principle (see Harris and Townsend 1981) guarantees that imposing the incentive compatibility requirement is actually without loss of generality.

Using this definition, we can simply say that the Pareto optimum $c^*(0)$ is not feasible in the economy with private types because it is not IC, as

$$\begin{aligned} Hu(c_{1H}^*(0)) + \beta u(c_{2H}^*(0)) &= Hu(0) + \beta u(0) \\ &< Hu(2) + \beta u(2) \\ &= Hu(c_{1L}^*(0)) + \beta u(c_{2L}^*(0)), \end{aligned}$$

and thus the IC constraint for the H -type, (13), is violated. The example of allocation $c^*(0)$ demonstrates that the set of feasible allocations in the private information economy is a strict subset of the set of allocations feasible in the public information economy. Moreover, this restriction on the feasibility is not irrelevant from the welfare perspective, as $c^*(0)$ is a Pareto optimum.

Characterizing the Set of Feasible Allocations with Private Types

Using the parameter values in (1), we can further characterize the set of feasible allocations in the private information economy, i.e., the set of all allocations that are RF and IC. Substituting the values in (1), the IC constraints (13) and (14) are given by, respectively,

$$\frac{5}{2} \log(c_{1H}) + \frac{1}{2} \log(c_{2H}) \geq \frac{5}{2} \log(c_{1L}) + \frac{1}{2} \log(c_{2L})$$

and

$$\frac{1}{2} \log(c_{1L}) + \frac{1}{2} \log(c_{2L}) \geq \frac{1}{2} \log(c_{1H}) + \frac{1}{2} \log(c_{2H}).$$

Using the RF constraints (12), we can eliminate from these inequalities consumption of the L -type agents. Simplifying and solving for c_{2H} , we obtain the following expressions for the IC conditions for the type H and L , respectively,

$$c_{2H} \geq \frac{2(2 - c_{1H})^5}{c_{1H}^5 + (2 - c_{1H})^5} \tag{15}$$

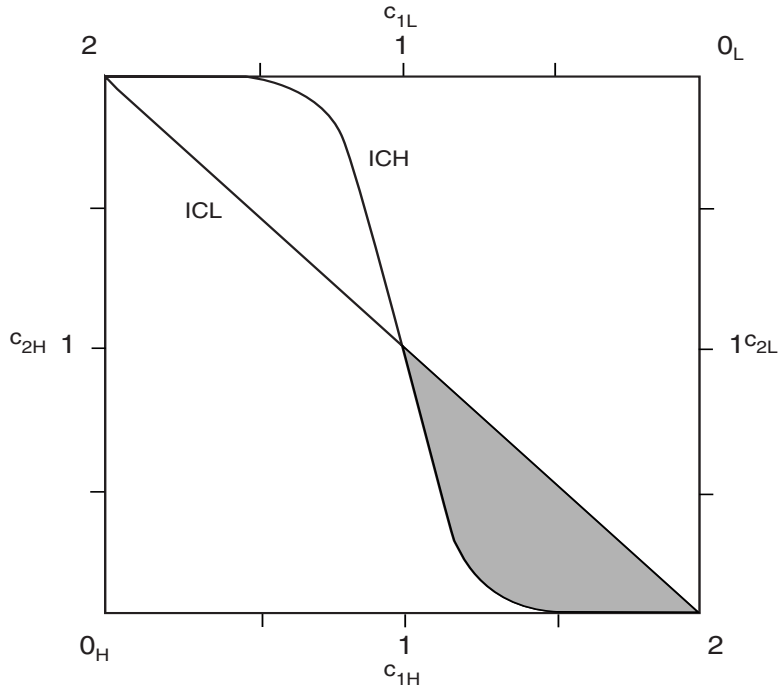
and

$$c_{2H} \leq 2 - c_{1H}. \tag{16}$$

Figure 2 depicts the set of all IC allocations in the Edgeworth box. The resource-feasible allocations that satisfy the IC constraint for type H , (15), lie on and above the curve ICH in Figure 2. Allocations that satisfy the IC constraint for type L , (16), lie on and below the line ICL. The shaded area, therefore, represents all IC allocations, i.e., those allocations that satisfy both IC conditions.

As we can see in Figure 2, the set of IC allocations (also satisfying the RF constraints as equalities) is convex. This property is not obvious a priori, as the IC constraints are given by nonlinear inequalities. Thus, the set of allocations

Figure 2 Incentive-Compatible Allocations in the Private Information Economy



feasible in the private information economy, i.e., those that satisfy the RF constraints as equalities and are incentive compatible, is convex.⁸ Similar to the case of public information, this property is valuable as we can characterize the set of all Pareto optima in the private information economy by solving a planning problem.

Characterizing the Set of All Second Best Pareto Optima

Consider a planning problem defined as follows:

⁸ Generally, the feasible set is not always convex in private information economies. Allocations involving lotteries over consumption bundles have been used in the literature to convexify the feasible set (see, e.g., Kehoe, Levine, and Prescott 2002). Also, when agents who misrepresent their type are more risk averse than those who report their type truthfully, lotteries may be welfare-improving even if the feasible set is convex (see Cole 1989). Neither of these reasons to consider lottery allocations, however, is present in the environment we consider in this article.

Second Best Planning Problem For each $\gamma \in [0, +\infty]$, find an allocation $c = (c_{1H}, c_{1L}, c_{2H}, c_{2L})$ that maximizes the value of the welfare objective (3) subject to resource feasibility constraints (2) and incentive compatibility constraints (13), (14).

Thanks to the convexity of the set of feasible allocations and the concavity of the objective, any solution to the Second Best planning problem is a Pareto optimum of the private information economy, and all such optima, referred to as the Second Best Pareto optima, can be obtained by solving this problem for all $\gamma \in [0, +\infty]$.⁹

Similar to the First Best planning problem, the Second Best planning problem is a concave maximization problem. Thus, for each γ , a unique solution exists. Let us denote this solution by $c^{**}(\gamma)$. As before, we can find $c^{**}(\gamma)$ using the first-order conditions.

There is, however, one difficulty in the private information economy that does not appear in the public information case: we do not know which, if any, IC constraints (13), (14) bind in the Second Best planning problem for a particular value of γ .

To determine which IC constraints bind for different values of γ , it will be helpful to return to the Edgeworth box. Figure 3 combines the curve representing the set of First Best Pareto optima from Figure 1, denoted by FBPO, with the set of IC allocations from Figure 2.

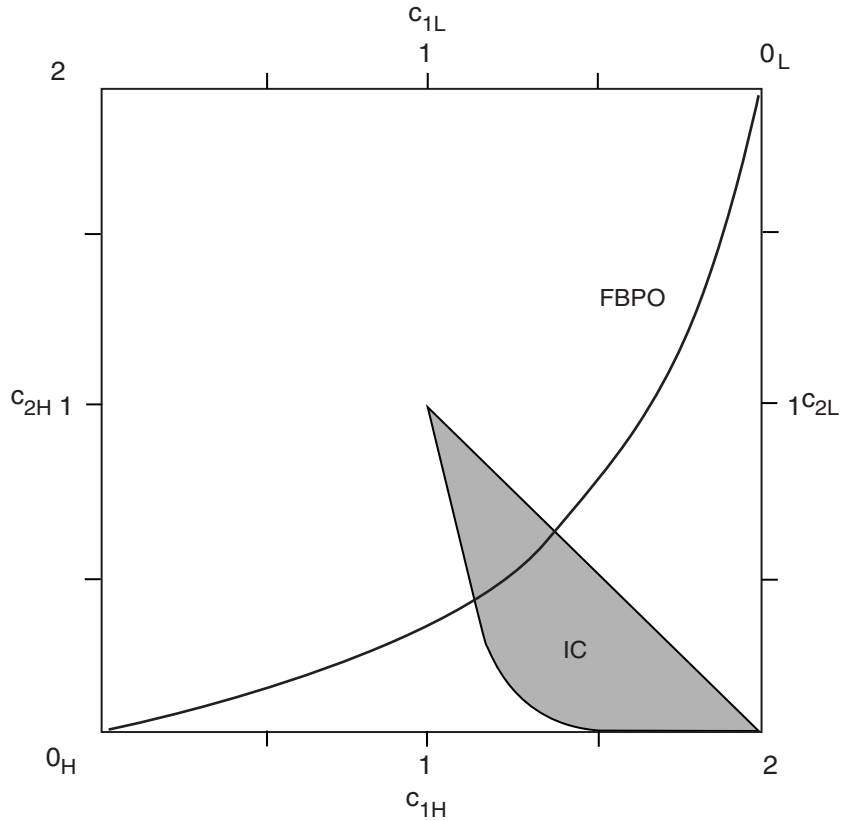
The first observation we make in Figure 3 is that a whole segment of the FBPO curve lies inside the IC set of the Second Best planning problem. Thus, for the values of the weight parameter γ for which the First Best Pareto optimum $c^*(\gamma)$ satisfies the IC constraints, the First Best Pareto optimum is also a solution to the Second Best planning problem, so $c^{**}(\gamma) = c^*(\gamma)$.

Second, we see that the First Best Pareto optima in the segment of the set FBPO that lies above the IC set are not incentive compatible because they violate the IC constraint of the L -type, (14). Similarly, the First Best Pareto optima in the segment of the set FBPO that lies below the IC set are not incentive compatible because they violate the IC constraint of the H -type, (13).

These observations suggest what the following lemma demonstrates formally. See the Appendix for a formal proof.

⁹ Second Best Pareto optima are often referred to in the literature as constrained Pareto optima.

Figure 3 Incentive-Compatible Allocations and the First Best Pareto Optima



Lemma 1 *In the Second Best planning problem, we have the following. For all $\gamma \in [\gamma_1, \gamma_2]$, where*

$$\begin{aligned} \gamma_1 &= 5^{-\frac{5}{8}} \approx 0.26, \\ \gamma_2 &= 5^{-\frac{1}{2}} \approx 0.45, \end{aligned}$$

no IC constraints bind.

For all $\gamma > \gamma_2$, the IC constraint of the L-type, (14), binds, and the IC constraint of the H-type, (13), does not.

For all $\gamma < \gamma_1$, the IC constraint of the H-type, (13), binds, and the IC constraint of the L-type, (14), does not.

By Lemma 1, the Second Best Pareto optimum $c^{**}(\gamma)$ coincides with the First Best Pareto optimum $c^*(\gamma)$ for all welfare weights $\gamma \in [\gamma_1, \gamma_2]$. Also,

for $\gamma < \gamma_1$, the Second Best Pareto optimum $c^{**}(\gamma)$ can be found by solving a relaxed Second Best planning problem in which the IC of the L -type, (14), is dropped and the IC constraint of the H -type, (13), holds as equality. Similarly, for $\gamma > \gamma_2$, the Second Best Pareto optimum $c^{**}(\gamma)$ can be found by solving a relaxed Second Best planning problem in which the IC constraint of the H -type is dropped and the IC constraint of the L -type holds as equality.

Taking the first-order conditions of the relaxed Second Best planning problem for $\gamma > \gamma_2$, we obtain

$$(\gamma - \lambda_L \frac{L}{H})Hu'(c_{1H}) = \rho_1\mu_H, \tag{17}$$

$$(1 + \lambda_L)Lu'(c_{1L}) = \rho_1\mu_L, \tag{18}$$

$$(\gamma - \lambda_L)\beta u'(c_{2H}) = \rho_2\mu_H, \tag{19}$$

$$(1 + \lambda_L)\beta u'(c_{2L}) = \rho_2\mu_L, \tag{20}$$

where $\lambda_L > 0$ is the multiplier on the IC constraint (14). For each $\gamma > \gamma_2$, the Second Best Pareto optimum $c^{**}(\gamma)$ is the solution to the system of equations consisting of the first-order conditions (17)–(20), the resource constraints (2), and the binding IC constraint (14). Using the parameter values in (1), we can solve explicitly for the optimum. After some algebra, we obtain

$$c_{1H}^{**}(\gamma) = c_{2L}^{**}(\gamma) = \frac{1 + 5\gamma}{1 + 3\gamma}, \tag{21}$$

$$c_{2H}^{**}(\gamma) = c_{1L}^{**}(\gamma) = \frac{1 + \gamma}{1 + 3\gamma}, \tag{22}$$

for all $\gamma > \gamma_2$. Similarly, taking the first-order conditions of the relaxed Second Best planning problem for $\gamma < \gamma_1$, we have

$$(\gamma + \lambda_H)Hu'(c_{1H}) = \rho_1\mu_H, \tag{23}$$

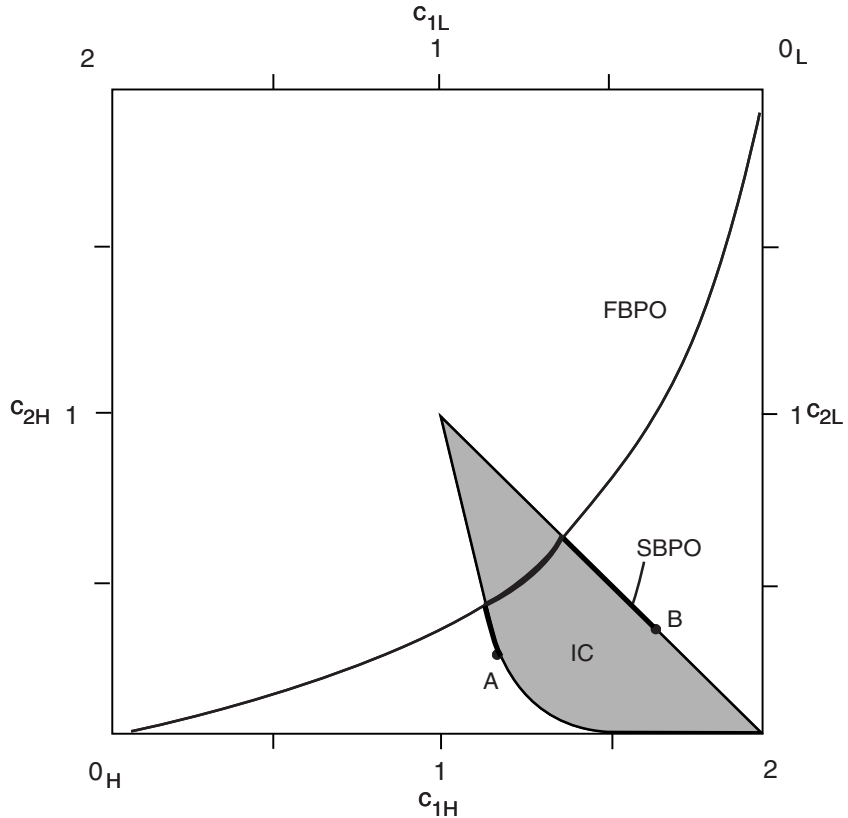
$$(1 - \lambda_H \frac{H}{L})Lu'(c_{1L}) = \rho_1\mu_L, \tag{24}$$

$$(\gamma + \lambda_H)\beta u'(c_{2H}) = \rho_2\mu_H, \text{ and} \tag{25}$$

$$(1 - \lambda_H)\beta u'(c_{2L}) = \rho_2\mu_L, \tag{26}$$

where $\lambda_H > 0$ is the multiplier on the IC constraint (13). Using the parameter values in (1), for each $\gamma < \gamma_1$, we can solve these first-order conditions, together with the resource constraints and the binding IC constraint, and obtain the Pareto optimum $c^{**}(\gamma)$.

Figure 4 represents the full set of Second Best Pareto-optimal allocations in the Edgeworth box. This figure also depicts the set of IC allocation and the set of First Best Pareto optima. For $\gamma \in [\gamma_1, \gamma_2]$, the Second and First Best Pareto optima coincide. The Second Best optima $c^{**}(\gamma)$ for $\gamma < \gamma_1$ lie on the lower edge of the IC set, where the IC constraint for the H -type binds. Point A represents the Second Best Pareto optimum $c^{**}(0)$. Similarly, the Second Best optima $c^{**}(\gamma)$ for $\gamma > \gamma_2$ lie on the upper edge of the IC set, where the

Figure 4 The Set of Second Best Pareto Optima

IC constraint for the L -type binds. Point B represents the Second Best Pareto optimum $c^{**}(\infty)$.

3. COMPARING PARETO OPTIMA IN THE TWO ECONOMIES

Having characterized the sets of optimal allocations in the public and private information economies, we can now compare their structures. In the first subsection, we compare the welfare properties of the two sets of Pareto optima. In the second subsection, we compare the structure of intertemporal wedges characterizing optimal allocations in the two economies.

Limits to Redistribution Under Private Information

Using the closed-form solutions we have obtained for the sets of First and Second Best Pareto optima, we can compute the value of utility optimally delivered to the two types of agents in the two economies. Denote by $V_\theta^*(\gamma)$ the lifetime utility delivered to each agent of type θ at the First Best Pareto optimum $c^*(\gamma)$ for $\gamma \in [0, \infty]$.¹⁰ By $V_\theta^{**}(\gamma)$ denote the lifetime utility delivered to each agent of type θ at the Second Best Pareto optimum $c^{**}(\gamma)$ for $\gamma \in [0, \infty]$.

Figure 5 depicts the so-called First Best Pareto frontier. The concave curve represents the pairs of values $(V_H^*(\gamma), V_L^*(\gamma))$ for γ between 0.025 and 40. Outside this range, the frontier extends toward negative infinity and converges to a horizontal and vertical line. Point A in Figure 5 represents the values $(V_H^*(\frac{1}{3}), V_L^*(\frac{1}{3}))$. Point B marks the values $(V_H^*(1), V_L^*(1))$.

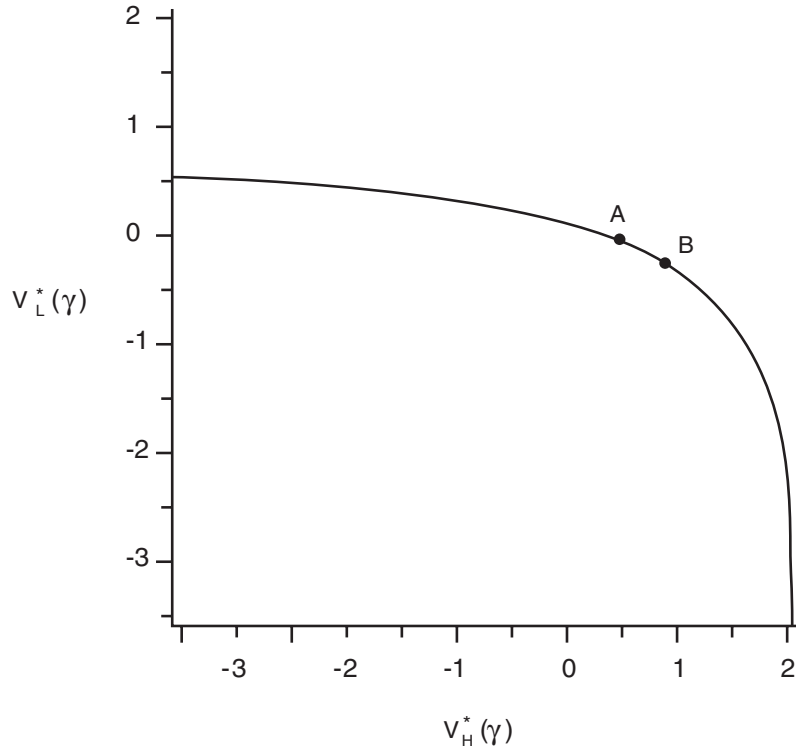
Figure 6 graphs the whole Second Best Pareto frontier, as well as a small section of the First Best frontier. The Second Best Pareto frontier consists of all points $(V_H^{**}(\gamma), V_L^{**}(\gamma))$ for $\gamma \in [0, \infty]$. As in Figure 5, points A and B represent the values $(V_H^*(\frac{1}{3}), V_L^*(\frac{1}{3}))$ and $(V_H^*(1), V_L^*(1))$. Because $1/3 \in [\gamma_1, \gamma_2]$, where First and Second Best Pareto optima coincide, point A belongs to the Second Best Pareto frontier. However, B lies outside of this set. The values $(V_H^{**}(1), V_L^{**}(1))$ are represented by point C in Figure 6.

Comparing Figures 5 and 6, we note that private information severely restricts the range of the utility levels that can be provided to the two agent types, relative to the public information economy. With public information, the impatient type H can be provided with welfare as high as $V_H^*(\infty) = 2.08$, while under private information, the maximum welfare for the impatient type is $V_H^{**}(\infty) = 0.78$. For the agents of the patient type L , these maximal values are $V_L^*(0) = 0.69$ and $V_L^{**}(0) = 0.17$, respectively. Private information, thus, puts limits on the amount of redistribution that can be attained by a social planner.¹¹

To gain some intuition on how these limits arise, we return to Figure 4 and consider the optimal allocation at the upper end of the range of γ for which private and public information optima coincide, i.e., $\gamma = \gamma_2$. The impact of private information on welfare attained in the two economies can be seen as we consider the values of $\gamma > \gamma_2$. In the public information economy, in order to increase welfare of the type H agents, the social planner simply increases consumption allocated to type H at both dates. That is, both $c_{1H}^*(\gamma)$ and $c_{2H}^*(\gamma)$ increase in γ , which of course means that both $c_{1L}^*(\gamma)$ and $c_{2L}^*(\gamma)$ decrease in γ . As γ grows, the consumption of the L -type becomes smaller

¹⁰ That is, $V_\theta^*(\gamma) = \theta u(c_{1\theta}^*) + \beta u(c_{2\theta}^*)$ for $\theta = H, L$ and $\gamma \in [0, \infty]$.

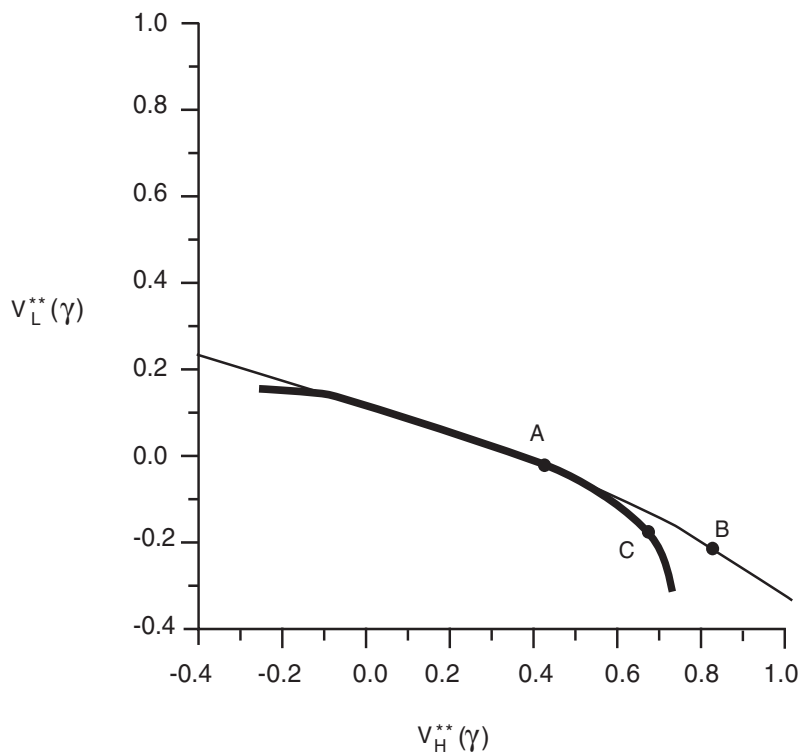
¹¹ Redistribution is measured here in terms of utility, relative to a benchmark level, which does not have to be explicitly specified as the statement is true for any benchmark.

Figure 5 The First Best Pareto Frontier

and smaller. Resource feasibility is the only limit to this process. Eventually, the H -type consumes the economy's whole output.

Private information, however, puts a much more stringent limit on how small consumption of the agents of type L can be. At the Second Best optimum with $\gamma = \gamma_2$, consumption of the L -type is already small enough that the agents of type L are indifferent between their allocation and that intended for the H -type. Maximizing the social welfare criterion with $\gamma > \gamma_2$, the planner cannot improve the H -types' welfare by increasing its consumption at both dates, as this would violate the incentive compatibility condition for the L -type, i.e., the agents of type L would misrepresent their type. As γ is raised above γ_2 , the planner increases H -types' welfare by increasing their consumption at date 1 and preserves incentive compatibility for the L -types by increasing their consumption at date 2. Because type H has a strong preference for consumption at date 1, relative to type L , it is possible to simultaneously compensate the L -type and increase the welfare of the H -type, to a point. In Figure 4, the Second Best Pareto optima $c^{**}(\gamma)$ for $\gamma > \gamma_2$

Figure 6 The First and Second Best Pareto Frontier



lie on the negative 45 degree line given by the upper edge of the set of IC allocations. At point B, which represents the Second Best optimum $c^{**}(\infty)$, the planner wants to further maximize H -types' welfare, regardless of type L 's welfare. However, no further increase in H -types' welfare is possible. At $c_H^{**}(\infty) = (\frac{5}{3}, \frac{1}{3})$, the marginal utility levels of H -types' consumption at dates 1 and 2 are equal.¹² Adding one unit of consumption at date 1 and subtracting one unit of consumption at date 2 is not going to improve H -types' welfare. But in order to preserve incentive compatibility, the planner has to compensate any increase in H -types' consumption at date 1 with a one-to-one increase of L -types' consumption at date 2. Preserving incentive compatibility for the L -type, therefore, becomes too expensive for the planner to be able to further increase H -types' welfare. Thus, even though the social welfare objective does

¹² It is easy to check that $Hu'(c_{1H}^{**}(\infty)) = \beta u'(c_{2H}^{**}(\infty)) = \frac{3}{2}$.

not value the utility of L -type at all, it is not feasible in the private information economy to further redistribute to the H -type agents.

The same intuition applies to the limit that private information puts on the value that can be delivered to the L -type. As γ decreases below γ_1 , the planner increases the utility of the L -types by increasing their consumption at date 2 and compensates the H -types with an increase in their consumption at date 1. At point A in Figure 4, the compensation for the H -type needed to preserve incentive compatibility becomes too large (and L -types' marginal utility of consumption at date 2 relative to marginal utility of consumption at date 1 too small) for a further increase in L -types' welfare to be feasible.

In Figure 6, we see that the presence of private information affects the value delivered to the disfavored type much more strongly than it affects the value delivered to the favored type, under any γ outside of $[\gamma_1, \gamma_2]$. When $\gamma = \infty$, the L -type consumes zero at the First Best Pareto optimum $c^*(\infty)$, i.e., $V_L^*(\infty) = -\infty$. In the private information economy, however, the L -type receives consumption $(\frac{1}{3}, \frac{5}{3})$ at the Second Best Pareto optimum $c^{**}(\infty)$, and $V_L^{**}(\infty) = -0.29 > -\infty$. Similarly, with $\gamma = 0$, welfare of the type H is $-\infty$ in the public information economy, but it is a finite number in the economy with private information.¹³

In addition, comparing points B and C in Figure 6, we see that when the social welfare objective is purely utilitarian, i.e., $\gamma = 1$, the L -type agents are better off in the private information economy. This observation generalizes. It is not hard to show that for all $\gamma > \gamma_2$, the disfavored L -types are better off when their type is private information, as in this case where the social planner's ability to redistribute to the H -type is hampered. Similarly, if $\gamma < \gamma_1$, i.e., when the H -types' utility receives a low weight in the social welfare criterion, we have that $V_H^{**}(\gamma) > V_H^*(\gamma)$, i.e., the disfavored H -type is better off in the private information economy.

Optimal Intertemporal Wedges

In order to gain further insight into the structure of the optimal allocations in the public and private information economies, we examine the intertemporal wedges in this subsection. Intertemporal wedge is defined as the difference between the social and the individual shadow interest rate. Wedges associated with a given Pareto optimum give us an understanding of the implicit distortions that are optimally imposed on the agents.

¹³ The value of negative infinity is specific to the logarithmic utility. Under a constant relative risk aversion utility function with relative risk aversion smaller than one, for example, this value would be zero, i.e., a finite number. That the value delivered to the disfavored type is strongly impacted by the presence of private information remains true, however, for any strictly concave utility function.

We clarify the definitions as follows: the social shadow interest rate R^* associated with a Pareto-optimal allocation c^* is the number R at which the planner would choose to not alter the allocation c^* if given a chance to re-solve the social planning problem with access to a borrowing and savings technology with gross interest rate R . Similarly, the private shadow interest rate R_θ^* for $\theta = H, L$ is the number R at which the agents of type θ would not find it beneficial to trade away from their individual consumption allocation c_θ^* if they could borrow and save at the gross interest rate R .

In the simple economic environment that we consider, characterization of social and private shadow interest rates is straightforward. The social shadow interest rate is given by the ratio $\frac{\rho_1}{\rho_2}$ of the Lagrange multipliers associated with the resource feasibility constraints (2) at dates 1 and 2.¹⁴ The private shadow interest rate of type θ at an optimum c^* is given by the ratio of type θ 's marginal utility at date 1 and 2, i.e., $\theta u'(c_{1\theta}^*)/\beta u'(c_{2\theta}^*)$.¹⁵

Public Information Economy

Directly from the first-order conditions (4)–(7), we obtain that the First Best optima $c^*(\gamma)$ satisfy

$$\frac{\theta u'(c_{1\theta}^*(\gamma))}{\beta u'(c_{2\theta}^*(\gamma))} = \frac{\rho_1}{\rho_2},$$

for both $\theta = H, L$ and any $\gamma \in [0, \infty]$. The intertemporal wedge, given by the difference between the social and private shadow interest rate, is zero. This means that it is never optimal to distort the private intertemporal margin in the public information economy.

Private Information Economy

In the private information economy, the intertemporal wedges are zero at the Second Best Pareto optimum $c^{**}(\gamma)$ for any $\gamma \in [\gamma_1, \gamma_2]$, because the Second Best Pareto optimum $c^{**}(\gamma)$ coincides with the First Best Pareto optimum $c^*(\gamma)$ for each γ in this range.

¹⁴ If the planner could borrow and lend at the gross interest rate R , the resource feasibility constraints of the social planning problem would be given by $\sum_\theta \mu_\theta c_{1\theta} + S \leq Y$ and $\sum_\theta \mu_\theta c_{2\theta} \leq Y + RS$, where S is the planner's saving at date 1. The first-order condition of this problem with respect to S is $-\rho_1 + R\rho_2 = 0$. This means that if $R = \rho_1/\rho_2$, the presence of the intertemporal saving technology does not alter the solution to the social planning problem, i.e., ρ_1/ρ_2 is the social shadow interest rate.

¹⁵ This follows from the first-order condition with respect to individual savings s , evaluated at $s = 0$, of the individual optimal re-trading problem $\max_s \theta u(c_{1\theta}^* - s) + \beta u(c_{2\theta}^* + Rs)$.

For $\gamma > \gamma_2$, the first-order conditions in the Second Best planning problem, (17)–(20), imply that

$$\frac{Lu'(c_{1L}^{**}(\gamma))}{\beta u'(c_{2L}^{**}(\gamma))} = \frac{\rho_1 \mu_L / (1 + \lambda_L)}{\rho_2 \mu_L / (1 + \lambda_L)} = \frac{\rho_1}{\rho_2},$$

which means that an intertemporal wedge of zero is optimal for the agents of type L . From the same first-order conditions we obtain that

$$\frac{Hu'(c_{1H}^{**}(\gamma))}{\beta u'(c_{2H}^{**}(\gamma))} = \frac{\rho_1 \mu_H / (\gamma - \lambda_L \frac{L}{H})}{\rho_2 \mu_H / (\gamma - \lambda_L)} < \frac{\rho_1 \mu_H / (\gamma - \lambda_L)}{\rho_2 \mu_H / (\gamma - \lambda_L)} = \frac{\rho_1}{\rho_2},$$

which means that a strictly positive intertemporal wedge is optimal for the agents of type H at each Second Best Pareto optimum $c^{**}(\gamma)$ with $\gamma > \gamma_2$. The positive wedge means that agents of type H are *savings-constrained* at the optimal allocation of the private information economy when $\gamma > \gamma_2$. If agents could trade away from the optimum by borrowing or saving at the social shadow interest rate, the agents of type H would like to save. Note that the L -type agents would choose to not trade away from their consumption allocation, as their intertemporal wedge is zero.

The literature studying Pareto-optimal allocations in multi-period economies with private information finds that the positive intertemporal wedge characterizes Pareto-optimal allocations in many such environments.¹⁶

For $\gamma < \gamma_1$, the first-order conditions (23)–(26) of the Second Best planning problem imply that

$$\frac{Hu'(c_{1H}^{**}(\gamma))}{\beta u'(c_{2H}^{**}(\gamma))} = \frac{\rho_1 \mu_H / (\gamma + \lambda_H)}{\rho_2 \mu_H / (\gamma + \lambda_H)} = \frac{\rho_1}{\rho_2},$$

and

$$\frac{Lu'(c_{1L}^{**}(\gamma))}{\beta u'(c_{2L}^{**}(\gamma))} = \frac{\rho_1 \mu_L / (1 - \lambda_H \frac{H}{L})}{\rho_2 \mu_L / (1 - \lambda_H)} > \frac{\rho_1 \mu_L / (1 - \lambda_H)}{\rho_2 \mu_L / (1 - \lambda_H)} = \frac{\rho_1}{\rho_2}.$$

This means that the optimal intertemporal wedge is zero for the H -type, and strictly negative for the L -type at any Second Best Pareto optimum $c^{**}(\gamma)$ with $\gamma < \gamma_1$. Therefore, we have that agents of type L are *borrowing-constrained* at the optimal allocation of the private information economy when $\gamma < \gamma_1$. If agents could borrow and lend at the social shadow interest rate, the L -type agents would like to borrow. This property is different from the intertemporal wedge typically found in the literature, in which, as we mentioned before, the positive intertemporal wedge is prevalent.

The intertemporal wedges associated with an optimal allocation give us an understanding of what distortions are optimal in agents' intertemporal consumption patterns. These distortions are relevant for the analysis of the welfare

¹⁶ Articles that find this property of the optimal allocations include Diamond and Mirrlees (1978); Rogerson (1985); and Golosov, Kocherlakota, and Tsyvinski (2003).

properties of equilibrium outcomes in market economies. In a market economy, by definition, agents can use markets to trade away from the socially optimal allocation. Therefore, the negative intertemporal wedge in the optimal allocation for the L -type, which we have at any Pareto optimum $c^{**}(\gamma)$ with $\gamma < \gamma_1$, can be consistent with market equilibrium only if agents of type L can be prevented from borrowing at the social shadow interest rate. At the same time, however, any such disincentive to borrow cannot affect the agents of type H , whose private shadow interest rate is aligned with the social shadow interest rate at any optimum $c^{**}(\gamma)$ with $\gamma < \gamma_1$.

Detailed analysis of the issue of consistency between Pareto optima and market equilibria is beyond the scope of this article. This issue, however, plays an important role in the macroeconomic applications of private information models. It is central, for example, in the study of information-constrained optimal taxation problems.¹⁷

4. CONCLUSION

Our analysis of a simple macroeconomic environment with heterogeneous agents provides an elementary exposition of the implications of Pareto optimality with private information. We obtain closed-form representation of all Pareto-optimal allocations with and without private information. We highlight the limits that private information puts on the utility distributions that can be attained in our environment. In addition, we provide a complete description of intertemporal distortions that are consistent with Pareto optimality in the private information case. Interestingly, we find that both negative and positive intertemporal distortions are consistent with Pareto optimality.

APPENDIX

Proof of Lemma 1

Note that removing the IC constraints (13) and (14) from the Second Best planning problem gives us exactly the First Best planning problem. Thus, neither of the two IC constraints binds at a solution to the Second Best planning problem with a given $\gamma \in [0, +\infty]$ if and only if the solution to the First Best planning problem, $c^*(\gamma)$, satisfies both IC constraints. We now show that this is the case if and only if $\gamma \in [\gamma_1, \gamma_2]$.

¹⁷ See Kocherlakota (2006) for a survey of recent articles studying these problems. In footnote 1, we mention other relevant applications and give further references.

Substituting the expression for the First Best optimum $c^*(\gamma)$ from (8)–(11) into the IC constraint for the H -type, (15), we get

$$\frac{2\gamma}{1+\gamma} \geq \frac{2(2 - \frac{10\gamma}{1+5\gamma})^5}{(\frac{10\gamma}{1+5\gamma})^5 + (2 - \frac{10\gamma}{1+5\gamma})^5}.$$

Solving for γ , we get

$$\gamma \geq 5^{-\frac{5}{6}}. \quad (27)$$

This means that the First Best optimal allocation $c^*(\gamma)$ satisfies the IC condition of the H -type if and only if $\gamma \geq 5^{-\frac{5}{6}} = \gamma_1$. Similarly, substituting $c^*(\gamma)$ into the IC constraint for the L -types, expressed as in (16), and solving for γ we get

$$\gamma \leq 5^{-\frac{1}{2}}.$$

Thus, the First Best optimum $c^*(\gamma)$ satisfies the IC condition of the L -type if and only if $\gamma \leq 5^{-\frac{1}{2}} = \gamma_2$. Furthermore, the First Best optimum $c^*(\gamma)$ satisfies both IC constraints if and only if $\gamma \in [\gamma_1, \gamma_2]$.

Therefore, no IC constraints bind in the Second Best planning problem if and only if $\gamma \in [\gamma_1, \gamma_2]$. Thus, at least one IC constraint binds in the Second Best planning problem for each $\gamma \notin [\gamma_1, \gamma_2]$. We now show that exactly one IC constraint binds in this problem for each $\gamma \notin [\gamma_1, \gamma_2]$.

Suppose to the contrary that both IC constraints bind at the solution to the Second Best planning problem for some γ . Then, (i) by complementary slackness conditions, both IC constraints must be satisfied as equalities, and (ii) the solution to the Second Best planning problem for this value of γ (as for all other values) must be a Second Best Pareto optimum. Using the fact that the RF constraints hold as equalities at any solution to the Second Best planning problem (which follows from the fact that the RF constraints always bind in this problem), it is easy to check (by simply solving the RF and IC constraints for c) that both IC constraints are satisfied as equalities at only one allocation: $c = (1, 1, 1, 1)$. But this allocation is not a Second Best Pareto optimum, because an allocation $c_\varepsilon = (1 + \varepsilon, 1 - \varepsilon, 1 - \varepsilon, 1 + \varepsilon)$ Pareto-dominates c for any $\varepsilon > 0$, as the H -type strictly prefers c_ε over c , and the L -type is indifferent. (It is straightforward to confirm that c_ε is incentive compatible for ε small enough.) Thus, (i) and (ii) are inconsistent—we have a contradiction—so both IC conditions cannot bind at a solution to the Second Best planning problem for any γ .

Thus, for each $\gamma \notin [\gamma_1, \gamma_2]$ exactly one IC constraint binds in the Second Best planning problem.

Suppose now that for some $\bar{\gamma} > \gamma_2$, the IC constraint for the L -type does not bind at a solution to the Second Best planning problem, and consider a relaxed planning problem obtained from the Second Best planning problem by dropping the IC constraint of the L -type. Since this IC constraint does not

bind in the Second Best planning problem, the solution to the relaxed problem coincides with the solution to the Second Best planning problem. We know from (27) that for all $\gamma \geq \gamma_1$ the First Best optimal allocation $c^*(\gamma)$ satisfies the IC condition of the H -type. Thus, since $\bar{\gamma} > \gamma_2 > \gamma_1$, the First Best optimal allocation $c^*(\bar{\gamma})$ solves the relaxed planning problem. But then $c^*(\bar{\gamma})$ must also be the solution to the Second Best planning problem, which we know it is not, because $\bar{\gamma} \notin [\gamma_1, \gamma_2]$, a contradiction. Thus, the IC constraint of the L -type must bind in the Second Best planning problem for all $\gamma > \gamma_1$.

Similarly, supposing that the IC constraint for the H -type does not bind at a solution to the Second Best planning problem for some $\bar{\gamma} < \gamma_1$, we construct a relaxed planning problem by dropping this constraint from the Second Best planning problem, which leads to a false conclusion that $c^*(\bar{\gamma})$ solves the Second Best planning problem for a $\bar{\gamma} < \gamma_1$, a contraction. Thus, the IC constraint for the H -type must bind at a solution to the Second Best planning problem for all $\gamma < \gamma_1$.

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