

# Bailouts

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In the United States during 2008–2009, as in previous episodes here and other countries, supplying funding to financial intermediaries and other firms was a component of the government’s response to a financial crisis. Some of these funding initiatives have been characterized—and, in some quarters, heavily criticized—as being *bailouts*: transfers from the government, made to firms (and sometimes other entities such as city governments) or to their creditors in order to avert insolvency or mitigate its effects, that the recipients are not anticipated to repay. Note that this definition distinguishes bailouts from bona fide government loans.<sup>1</sup> Henry Thornton (1802) and Walter Bagehot (1877) explained why it is good public policy for government to lend to firms (particularly to banks) in a financial crisis, and today that justification is widely accepted. Bailouts remain highly controversial, however.

Many economists perceive bailouts to be a costly manifestation of time inconsistency on the part of policymakers. That is, the government threatens that an entity that becomes insolvent must fail rather than being rescued, but subsequently, perhaps out of fear that insolvency would harm many people who bear no responsibility for it, the entity will be rescued when push comes to shove. Anticipating this denouement, the owners and managers of the entity

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<sup>1</sup> Even if a transfer takes the explicit form of a loan, it has an implicit bailout component if the interest rate is too low to compensate the lender at market terms for the risk of default. In particular, if the market price of risk is set by risk-neutral traders, then a loan has a bailout component unless it is actuarially sound—that is, unless the lender is making a bet at fair odds that it will be repaid. Specifically, suppose that a loan of size  $\$L$  is made at interest rate  $r$ , and is anticipated to be repaid with probability  $p$  when it is made. Assume for simplicity that the loan will be repaid either in full, or else not at all. Then the loan is actuarially sound if  $p \cdot r \cdot L \geq (1 - p) \cdot L$ , or equivalently if  $p \cdot r \geq 1 - p$ .

make inadvisable investments, taking risks that they would have avoided if the threat not to assist had been taken seriously. This view, formalized by Kareken (1983) and more recently elaborated by Stern and Feldman (2004), is a cogent, *prima facie* reason to judge that bailouts are a socially inefficient form of government intervention in the economy.

Nevertheless, despite this logic, numerous academic economists, policy-makers, and market participants argued publicly that the 2008–2009 bailout was an indispensable policy action. In their view, there was considerable risk that the economy would have suffered serious, long-term harm if the finance, automobile, and housing industries had not been subsidized. Presumably they were concerned that millions of people would face the sort of immediate harm that expositions of the time-inconsistency argument typically cite, but they spoke of a greater, more persistent harm. In their view, if government did not provide a bailout in circumstances where to do so was vital, then incentives for socially beneficial investment would be impaired in a way that might take decades to repair. This vision is the polar opposite of the time-inconsistency vision, which sees investment incentives being harmed by the occurrence of bailouts rather than by their nonoccurrence.

The goal of this article is to formulate an economic model, in terms of which the concern just described can be understood. This is a very limited goal. It is not even to provide a *prima facie* argument that conducting a bailout is likely to be good policy. To meet the goal, the model need only establish that a bailout would be economically efficient under some conceivable conditions in some economy that shares salient features of the actual one.

In an economy in which a bailout of firms might be efficient, there must be some reason for production to be undertaken by firms that issue financial claims against which they might default. This feature is necessary because, if there were no good reason for firms ever to become insolvent, then an optimal policy would be to prevent them from ever taking that risk, rather than to allow them to take it and to help them when insolvency occurs.

In particular, besides firms being able to do something for their investors that the investors cannot do for themselves, there must be some constraint on a firm's ability to issue financial claims that would only have to be paid in those states of nature where the firm had the capacity to pay them. The Modigliani-Miller theorem (cf. Stiglitz 1969) states that, if a firm could contract *ex ante* for the payments that it could make and receive at every date, in any state of the world, then any production plan could be financed in such a way that the firm could not possibly become insolvent. Thus, a threshold condition for an economic model to be suitable for studying insolvency is that it must rule out some contracts that a firm might make in principle so that the no-insolvency implication of the Modigliani-Miller theorem will be avoided.

A well-known model with these features is the model of bank runs formulated by John Bryant (1980) and Douglas Diamond and Philip Dybvig (1983).

A firm (which those authors interpret to be a bank) can improve on autarkic production by pooling its investors' risks of idiosyncratic shocks to their respective preferences. It is assumed that the firm can only fund its production by issuing standard debt contracts rather than by issuing financial claims in a completely flexible manner.

The model to be formulated here closely resembles the Diamond-Dybvig model. Although those authors (and also Bryant) were particularly concerned with the possibility that a solvent firm might become illiquid—indeed, to formalize that distinction was an important aspect of their contribution—the model can be parameterized in such a way that optimal financial and production decisions must lead to insolvency in some states of the world.

Rather than assuming that a standard debt contract is the only available financial claim, financial flexibility will be constrained in the present model by assuming that the firm is a limited-liability corporation. In fact, Modigliani and Miller cited limited liability as a consideration that arguably prevents their theorem from holding precisely in an actual economy. Like the Diamond-Dybvig model, the present model is a partial-equilibrium model in the sense that it assumes a constraint on financing opportunities, rather than deriving that constraint as an implication of, or as an optimal policy response to, economic primitives such as tastes, technology, and privacy of information. For an informal discussion of the history and economic rationale of limited liability, see Easterbrook and Fischel (1985).

The firm is modeled here as making payouts of the good it produces, and those payouts cannot exceed in the aggregate the firm's output in any state of nature. However, there is an equivalent way of describing the way in which the allocation is implemented. That is, the firm promises state-contingent payouts to investors that exceed, in the aggregate, its output in some states of nature. Then, in those states of nature, the firm receives a tax-funded subsidy to bridge the gap between its output and its aggregate liabilities. According to this description, the tax/transfer scheme is a bailout of the insolvent firm. The tax is collected on investors' endowments at the date when the subsidy is paid. The limited-liability constraint specifies that the firm cannot claim those endowments directly, so the government's authority to tax them must be invoked in order to substitute for the promised payouts that the firm is unable to make.

It might be asked, what sense does it make to tax investors' endowments and then return them? The answer is that the tax is a lump-sum tax but the indemnification is dependent on the investors' continued participation in the firm. Thus, the tax-subsidy scheme can affect incentives. From an ex-ante perspective, it may be essential for providing sufficient incentive to invest along with others, some of whom (that is, those who suffer an adverse preference shock) foreseeably will liquidate their investments prematurely.

Since there is only one firm in the entire economy in this model, it should be interpreted to represent the entire firm sector of the economy, including banks, other financial firms, and nonfinancial firms. A main insight of Diamond and Dybvig is that banks contribute to economic welfare by engaging in *maturity transformation*, that is, by “borrowing short and lending long.” Banks are not the only firms that do this, however. Recent research (cf. Acharya, Gale, and Yorulmazer 2009) emphasizes that a nonfinancial firm can engage in maturity transformation on its own behalf through the market for short-term corporate debt, with essentially the same implication as if it had borrowed from a maturity-transforming bank. The terminology adopted in this paper—“firm,” rather than “bank”—reflects a view that the welfare analysis of bailouts as public policy is largely the same, whether the recipient is a financial or a nonfinancial firm.<sup>2</sup> Like the Diamond-Dybvig model, the present model concerns a policy response to a problem in a broad sector of an economy. Regardless of whether there is one direct recipient of government funds or there are many, and regardless of whether those direct recipients are financial firms or nonfinancial ones, a bailout affects the position of the firm sector (including its investors) in the aggregate.

## 1. THE ENVIRONMENT

There are three dates, denoted by 0, 1, 2. There is a large population of investors, each of whom randomly has one of two utility functions. Each investor behaves atomistically, and in particular, ascribes zero probability to the event that he could be a pivotal liability holder whose decision to demand payment might force the firm to default.

There is one good at each date, which can be either consumed or, except at the terminal date 2, transformed into the good at the next date by the technologies described below.<sup>3</sup> Each investor is endowed with  $\bar{x}_0 > 0$  units of good at date 0 and  $\bar{x}_2 > 0$  units of good at date 2 but is not endowed with any of the date-1 good.

If he is *impatient* (type 1), then an investor wants to maximize his consumption at date 1 until it has reached a high threshold. If he is *patient* (type 2), then he wants to maximize the sum of his consumption at date 1 and date 2. Date 0 is a date at which each investor can invest his endowment or exchange for a liability of the firm, which invests it, but at which no consumption takes

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<sup>2</sup> Diamond and Dybvig introduced the sequential-service constraint, a feature of their model environment that prevented market transactions from decentralizing the same allocation as banking contracts implement. Analogously, the limited-liability constraint prevents market transactions from substituting for a combination of contracts and bailouts in the model to be analyzed here.

<sup>3</sup> There will be no explicit technology for transforming date-1 consumption to date-2 consumption, but refraining from early liquidation of illiquid investment is tantamount to such a technology.

place. Each agent privately learns his own type at date 1 but remains ignorant of others' types.

An investor's utility function has three arguments: consumption at date 1, consumption at date 2, and the investor's type. An impatient investor has utility

$$\begin{aligned} u(c_1, c_2, 1) &= v(c_1) + c_2 \\ v(x) &= \begin{cases} \eta x & \text{if } x \leq \theta; \\ \eta\theta + (x - \theta) & \text{if } x > \theta; \end{cases} \\ \eta &> 1. \end{aligned} \quad (1)$$

A patient investor has utility

$$u(c_1, c_2, 2) = c_1 + c_2. \quad (2)$$

At date 0, before he knows his type, and at date 1, if the consequences of a decision depend on other investors' types of which he is ignorant, an investor maximizes expected utility as explained below.

There is a risk that concerns the fraction of the population that is patient. In every state of the world, either a fraction  $\mu_\Gamma$  or  $\mu_B$  of the investors are impatient, where  $0 < \mu_\Gamma < \mu_B < 1$ . In subsequent analysis, it will be assumed that  $\bar{x}_0/\mu_B < \theta < \bar{x}_0/\mu_\Gamma$ .

An *aggregate state* of the world in which fraction  $\mu_B$  of investors are impatient occurs with probability  $\beta$ , and  $0 < \beta < 1$ . Denote this aggregate state by  $B$ , and denote its complement by  $\Gamma$ . (Strictly speaking,  $B$  is the event that comprises all of the *bad* states in which some group of  $\mu_B$  investors is impatient, and the *good* aggregate state  $\Gamma$  is the complementary event.)

All investors are equally likely to be patient, and no investor's type is more highly correlated with the aggregate state than any other's type is. Thus, if  $\mu_1^*$  is the probability that a particular agent is impatient, then the following equation is satisfied:<sup>4</sup>

$$\mu_1^* = \beta\mu_B + (1 - \beta)\mu_\Gamma. \quad (3)$$

It follows that, if an investor knows that he is impatient but knows nothing of other investors' types, then his probability belief that event  $B$  has occurred is<sup>5</sup>

$$\beta_1^* = \beta\mu_B/\mu_1^*. \quad (4)$$

<sup>4</sup>An asterisk superscript indicates in this article that a probability will appear explicitly in an investor's expected-utility calculation.

<sup>5</sup>Probability  $\beta_1^*$  is calculated according to Bayes' Theorem. Game theorists call this the investor's *interim probability*, to distinguish it from *posterior probability*, which reflects knowledge of both the investor's own type and also the other investors' types. *Interim expected utility* is the mean of the investor's utility function with respect to the interim probability measure.

Similarly, if the probability that a particular investor is patient is denoted by  $\mu_2^*$ , then

$$\mu_2^* = \beta(1 - \mu_B) + (1 - \beta)(1 - \mu_\Gamma), \quad (5)$$

and a patient investor believes with probability  $\beta_2^*$  that event  $B$  has occurred, where

$$\beta_2^* = \beta(1 - \mu_B)/\mu_2^*. \quad (6)$$

Intuitively, an investor assigns higher probability to  $B$  if he discovers that he is impatient than if he finds that he is patient. A routine computation shows that, correspondingly,  $\beta_1^* > \beta_2^*$ .

Investment must be undertaken at date 0. There are two technologies, each of which has return that is linear in investment. *Liquid technology* is just storage: it returns one unit of output at date 1 for each unit of investment. *Illiquid technology* returns  $R > 1$  units of consumable output at date 2 for each unit of investment. However, if a unit is withdrawn at date 1, then it only yields  $r < 1$  units of consumable output. Assume that

$$R/r < \eta. \quad (7)$$

The economic implication of this inequality is that, for an impatient investor whose date-1 consumption is below  $\theta$  (so that the marginal utility of consumption at date 1 is  $\eta$ ), the marginal rate of substitution of  $c_1$  for  $c_2$  (that is,  $R/r$ ) is higher than the marginal rate of transformation of  $c_1$  into  $c_2$  by choosing between alternate uses of the illiquid technology.

## 2. EXPECTED UTILITY

The probabilities defined and calculated above provide the basis for calculating investors' prior and conditional expected utilities of state-contingent allocations. In the calculations below, and throughout the rest of this article,  $s$  will denote an individual investor's state,  $\sigma$  will denote an aggregate state, and  $t$  will denote a date in the model economy. For  $s \in \{1, 2\}$ ,  $\sigma \in \{B, \Gamma\}$ , and  $t \in \{1, 2\}$ , let  $c_t^{s\sigma}$  denote the consumption level at date  $t$  in the event that the investor's state is  $s$  and the aggregate state is  $\sigma$ . An *allocation* specifies the consumption level in each of the eight possible combinations of date, investor's state, and aggregate state.<sup>6</sup>

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<sup>6</sup>As is typical in models in which there are many agents whose individual states are i.i.d. conditional on the aggregate state, so that a law of large numbers can be presumed to hold, it is unnecessary to distinguish formally between an economy-wide allocation in a generic state of the world and a bundle of state-contingent commodities for an individual agent in the economy.

Denote the prior expected utility of allocation  $c$  by  $U_0(c)$ , which is

$$\begin{aligned} & \beta [\mu_B u(c_1^{1B}, c_2^{1B}, 1) + (1 - \mu_B) u(c_1^{2B}, c_2^{2B}, 2)] \\ & + (1 - \beta) [\mu_\Gamma u(c_1^{1\Gamma}, c_2^{1\Gamma}, 1) + (1 - \mu_\Gamma) u(c_1^{2\Gamma}, c_2^{2\Gamma}, 2)]. \end{aligned} \quad (8)$$

If an agent learns that he is impatient (type 1), then his interim expected utility is  $U_1(c)$ , which is

$$\beta_1^* u(c_1^{1B}, c_2^{1B}, 1) + (1 - \beta_1^*) u(c_1^{1\Gamma}, c_2^{1\Gamma}, 1). \quad (9)$$

If he learns that he is patient (type 2), then his interim expected utility is  $U_2(c)$ , which is

$$\beta_2^* u(c_1^{2B}, c_2^{2B}, 2) + (1 - \beta_2^*) u(c_1^{2\Gamma}, c_2^{2\Gamma}, 2). \quad (10)$$

### 3. COOPERATIVE AND AUTARKIC PRODUCTION FEASIBILITY

The endowment and technologies described above imply a set of technically feasible cooperative production outcomes if all investors' endowments are invested jointly, and a set of technically feasible individual production outcomes if a single investor invests autarkically. Assume free disposal: If a production outcome is feasible, then any outcome that provides less consumption at each date (and, in the case of individual feasibility, for each type) is also feasible.

Since technology is linear, cooperative feasibility can be considered in per capita terms. A *cooperative production plan* specifies an amount,  $\iota$ , of the endowment to be invested in the illiquid technology and amounts,  $\epsilon_B$  and  $\epsilon_\Gamma$ , of that illiquid investment to be liquidated at date 1 in the two possible aggregate states. *Technical feasibility* requires that, for each  $\sigma \in \{B, \Gamma\}$ ,

$$0 \leq \epsilon_\sigma \leq \iota \leq \bar{x}_0. \quad (11)$$

Given production plan  $\pi$ , let  $y_{\sigma t}$  denote the output at date  $t$  in aggregate state  $\sigma$ . Then, for  $\pi = (\iota, \epsilon_B, \epsilon_\Gamma)$ ,  $y$  is the vector satisfying

$$\begin{aligned} y_{\sigma 1} &= (\bar{x}_0 - \iota) + \epsilon_\sigma r; \\ y_{\sigma 2} &= (\iota - \epsilon_\sigma) R. \end{aligned} \quad (12)$$

**Proposition 1** *If  $\pi = (\iota, \epsilon_B, \epsilon_\Gamma)$  is technically feasible and  $\min(\epsilon_B, \epsilon_\Gamma) > 0$ , then another technically feasible production plan provides strictly more output at both dates and in both aggregate states than  $\pi$  does.*

**Proof.** Let  $\pi = (\iota, \epsilon_B, \epsilon_\Gamma)$  be a technically feasible production plan, and define  $\pi' = (\iota - \min(\epsilon_B, \epsilon_\Gamma), \epsilon_B - \min(\epsilon_B, \epsilon_\Gamma), \epsilon_\Gamma - \min(\epsilon_B, \epsilon_\Gamma))$ . Then

$\pi'$  is also technically feasible, and it has weakly higher output than  $\pi$  at both dates and in both aggregate states, because resources that  $\pi$  allocates to early-liquidated illiquid-technology production are reallocated in  $\pi'$  to liquid-technology production of the same goods (that is, goods at date 1 in the two aggregate states). If both  $\epsilon_B$  and  $\epsilon_\Gamma$  are positive, then  $\pi'$  produces strictly more output at date 1 in each aggregate state than  $\pi$  does, and it produces identical output to  $\pi$  at date 2 in each aggregate state. Then, by devoting a slightly higher investment than  $\iota - \min(\epsilon_B, \epsilon_\Gamma)$  to illiquid production, a new, technically feasible production plan can be constructed that provides strictly more output at both dates and in both aggregate states than  $\pi$  does. ■

An allocation is *technically feasible for cooperative production* if there is a technically feasible cooperative production plan such that, at each date and in each aggregate state, the impatient and patient agents together consume no more than the sum of the output of that plan and the endowment at that date (that is, 0 at date 1 or  $\bar{x}_2$  at date 2). Specifically,  $c$  is technically feasible for cooperative production if, for some feasible production plan  $\pi$ ,

$$\begin{aligned}\mu_\sigma c_1^{1\sigma} + (1 - \mu_\sigma) c_1^{2\sigma} &\leq y_{\sigma 1}; \\ \mu_\sigma c_2^{1\sigma} + (1 - \mu_\sigma) c_2^{2\sigma} &\leq y_{\sigma 2} + \bar{x}_2.\end{aligned}\quad (13)$$

An *autarkic production plan* specifies a fraction of  $\bar{x}_0$  to be invested in the illiquid technology and fractions  $\epsilon_1$  and  $\epsilon_2$  of that investment to be liquidated at date 1 if the investor is impatient or patient, respectively. Note that, since an individual investor does not observe the aggregate state, an autarkic production plan cannot depend on it. The output of an autarkic production plan is defined analogously to (12):

$$\begin{aligned}y_{s1} &= (x_0 - \iota) + \epsilon_s r; \\ y_{s2} &= (\iota - \epsilon_s) R.\end{aligned}\quad (14)$$

Moreover, since the aggregate state is irrelevant to either the production possibility set or the preferences of an investor of either type, there is no reason for an autarkic investor's allocation to depend on it. Because an autarkic investor does not need to acquire private information from anyone else in order to implement his plan, technical constraints are the only feasibility constraints. Thus, define allocation  $c$  to be *feasible for autarkic production* if, for some autarkic production plan  $\pi$ ,

$$\begin{aligned}c_1^{sB} &\leq y_{s1}; \\ c_2^{sB} &\leq y_{s2} + \bar{x}_2; \\ c_t^{s\Gamma} &= c_t^{sB}.\end{aligned}\quad (15)$$



**Proposition 2** *If allocation  $c$  is feasible for autarkic production, then there is a technically feasible cooperative production plan with sufficient output per capita to provide every investor with the same level of consumption at both dates, in every state, as  $c$  provides. If at least one type of investor would liquidate a positive amount of illiquid investment at date 1 in an autarkic production plan for  $c$ , then there is a cooperative plan with sufficiently high output to provide every investor with higher consumption at both dates, in every state, than  $c$  provides.*

**Proof.** If  $(\iota, \epsilon_1, \epsilon_2)$  is an autarkic production plan that produces sufficient output for  $c$  according to (15), then  $(\iota, \mu_B \epsilon_1 + (1 - \mu_B) \epsilon_2, \mu_\Gamma \epsilon_1 + (1 - \mu_\Gamma) \epsilon_2)$  is a cooperative production plan that produces sufficient output for  $c$  according to (12). The second assertion in this proposition follows from Proposition 1 since, if at least one of  $\epsilon_1$  and  $\epsilon_2$  is positive, then  $\epsilon_B = \mu_B \epsilon_1 + (1 - \mu_B) \epsilon_2$  and  $\epsilon_\Gamma = \mu_\Gamma \epsilon_1 + (1 - \mu_\Gamma) \epsilon_2$  imply that both  $\epsilon_B$  and  $\epsilon_\Gamma$  are positive. ■

#### 4. OPTIMAL AUTARKIC PRODUCTION

Consider the problem of optimizing expected utility,  $U_0(c)$ , among allocations that are feasible for autarkic production. Since the feasibility condition (15) requires that  $c_t^{s\Gamma} = c_t^{sB}$ , the definition (8) of  $U_0(c)$  reduces to

$$U_0(c) = \mu_1^* u(c_1^{1B}, c_2^{1B}, 1) + \mu_2^* u(c_1^{2B}, c_2^{2B}, 2). \quad (16)$$

Since  $U_0$  is strictly increasing in all of its consumption arguments, the feasibility constraints will all hold with equality in (15). That is,  $c$  is the entire output of some autarkic production plan  $(\iota, \epsilon_1, \epsilon_2)$ , together with the endowment  $\bar{x}_2$  at date 2. Making this substitution into (16), and expanding  $u$  according to its defining equations (1) and (2) yields

$$\begin{aligned} U_0(c) = & \mu_1^* [v((\bar{x}_0 - \iota) + \epsilon_1 r) + (\iota - \epsilon_1) R + \bar{x}_2] \\ & + \mu_2^* [v(\bar{x}_0 - \iota) + \epsilon_2 r + (\iota - \epsilon_2) R + \bar{x}_2]. \end{aligned} \quad (17)$$

Recall that, by (1),  $1 \leq v' \leq \eta$ , so, if  $\Delta_\iota$  denotes the derivative of the right side of (17) with respect to  $\iota$ , then<sup>7</sup>

$$R - (\mu_1^* \eta + \mu_2^*) \leq \Delta_\iota \leq R - 1. \quad (18)$$

<sup>7</sup> Function  $v$  is differentiable except at  $\theta$ , where the rightmost and leftmost terms in (18) are the left and right directional derivatives of  $v$ , respectively.

If

$$R - (\mu_1^* \eta + \mu_2^*) > 0, \quad (19)$$

then the optimal level of  $\iota$  is the maximal investment level  $\bar{x}_0$ . Condition (19) will be assumed henceforth, in order to focus on this case.

The derivative of the right side of (17) with respect to  $\epsilon_2$  is  $\mu_2^*(r - R) < 0$ , so the optimal level of  $\epsilon_2$  is the minimum level 0. Let  $\Delta_\epsilon$  denote the derivative of the right side of (17) with respect to  $\epsilon_1$ :

$$\Delta_\epsilon \begin{cases} \mu_1^*(\eta r - R) & \text{if } \epsilon_1 r < \theta; \\ \mu_1^*(r - R) & \text{if } \epsilon_1 r > \theta. \end{cases} \quad (20)$$

By assumption (7),  $\Delta_\epsilon > 0$  if  $\epsilon_1 r$  is to the left of  $\theta$ , so the optimal autarkic production plan must set  $\epsilon_1 = \bar{x}_0$  if  $\bar{x}_0 r < \theta$ . Subsequent analysis will focus on this case.

The following proposition recapitulates what has been established in this section.

**Proposition 3** *If  $R - (\mu_1^* \eta + \mu_2^*) > 0$  and  $\bar{x}_0 r < \theta$ , then  $(\bar{x}_0, \bar{x}_0, 0)$  is the optimal autarkic production plan. If allocation  $c$  is the output of this plan, then  $U_0(c) = \bar{x}_0 (\mu_1^* \eta r + \mu_2^* R) + \bar{x}_2$ .*

## 5. OPTIMAL COOPERATIVE PRODUCTION

Consider the optimal allocation that is technically feasible for cooperative production. Recall that impatient investors who receive date-1 consumption less than  $\theta$  have marginal utility  $\eta > 1$  for consumption at that date, all other investors have marginal utility 1 for date-1 consumption, and all investors have marginal utility 1 for date-2 consumption. Recall that output is described in per capita terms, so the greatest amount of output  $y_{\sigma 1}$  that can be given to each investor of type 1 at date 1 in aggregate state  $\sigma$  is  $y_{\sigma 1}/\mu_\sigma$ . It follows that, to maximize expected utility among allocations that distribute  $y$ , it is necessary and sufficient that, for  $\sigma \in \{B, \Gamma\}$ ,  $\min(y_{\sigma 1}/\mu_\sigma, \theta) \leq c_1^{1\sigma}$ . In particular, it is optimal to allocate all production output to the impatient investors, at both dates and in both aggregate states, and to allow every investor to consume his own endowment,  $\bar{x}_2$ , of the date-2 good. The level of ex-ante expected utility that this allocation provides is

$$\begin{aligned} & \beta \mu_B (\eta \min(y_{B1}/\mu_B, \theta) + \max(y_{B1}/\mu_B - \theta, 0) + y_{B2}/\mu_B) \\ & + (1 - \beta) \mu_\Gamma (\eta \min(y_{\Gamma 1}/\mu_\Gamma, \theta) + \max(y_{\Gamma 1}/\mu_\Gamma - \theta, 0) + y_{\Gamma 2}/\mu_\Gamma) \\ & + \bar{x}_2. \end{aligned} \quad (21)$$

If  $y$  is the output of cooperative production plan  $(\iota, \epsilon_B, \epsilon_\Gamma)$ , then (21) is equivalent to

$$\begin{aligned} & \beta \mu_B \eta \min ((\bar{x}_0 - \iota) + \epsilon_B r) / \mu_B, \theta) \\ & + \max ((\bar{x}_0 - \iota) + \epsilon_B r) / \mu_B - \theta, 0) + (\iota - \epsilon_B) R / \mu_B) \\ & + (1 - \beta) \mu_\Gamma \eta \min ((\bar{x}_0 - \iota) + \epsilon_\Gamma r) / \mu_\Gamma, \theta) \\ & + \max ((\bar{x}_0 - \iota) + \epsilon_\Gamma r) / \mu_\Gamma - \theta, 0) + (\iota - \epsilon_\Gamma) R / \mu_\Gamma) + \bar{x}_2. \end{aligned} \quad (22)$$

Assume that it is technically feasible to provide consumption at least as high as the low-marginal-utility threshold to impatient investors at date 1 in aggregate state  $\Gamma$ , but not in  $B$ . That is,

$$\bar{x}_0 / \mu_B < \theta < \bar{x}_0 / \mu_\Gamma. \quad (23)$$

It is optimal to liquidate all investment in state  $B$ . The reason is that, regardless of the value of  $\iota$ , date-1 output with complete liquidation will be  $(\bar{x}_0 - \iota) + \iota r$ , which is not greater than  $\bar{x}_0$ . If this output is all given to impatient investors to consume, then each of them receives  $[(\bar{x}_0 - \iota) + \iota r] / \mu_B \leq \bar{x}_0 / \mu_B < \theta$ , at which level the marginal utility of date-1 consumption in state  $B$  is  $\eta$ , versus 1 for date-2 consumption. The marginal rate of transformation of date-0 endowment to date-1 consumption by means of making illiquid investment but liquidating it early is  $r$ , while the marginal rate of transformation to date-2 consumption by not liquidating is  $R$ , so (7) entails that early liquidation is optimal.

Under some circumstances, it is optimal to make illiquid investment up to the point where just enough is left over to provide every impatient investor with  $\theta$  units of consumption at date 1 in state  $\Gamma$  and not to liquidate any of the investment in that state. That is,  $(\bar{x}_0 - \mu_\Gamma \theta, \bar{x}_0 - \mu_\Gamma \theta, 0)$  is the optimal cooperative investment plan. These circumstances are now characterized.

Let  $\Delta^-$  and  $\Delta^+$  denote the left- and right-hand derivatives of (22) with respect to  $\iota$ , evaluated at  $(\iota, \epsilon_B, \epsilon_\Gamma) = (\bar{x}_0 - \mu_\Gamma \theta, \bar{x}_0 - \mu_\Gamma \theta, 0)$ . Then

$$\begin{aligned} \Delta^- &= \beta (R - \eta) + (1 - \beta) (R - 1); \\ \Delta^+ &= R - \eta. \end{aligned} \quad (24)$$

Now assume that

$$R - \eta < 0 < R - (\beta \eta + 1 - \beta). \quad (25)$$

This entails that  $\Delta^-$  and  $\Delta^+$  are positive and negative, respectively, so the maximum is achieved at  $\bar{x}_0 - \mu_\Gamma \theta$ , where these directional derivatives were evaluated. Also, at that level of  $\iota$ , the right-hand derivative of (22) with respect to  $\epsilon_\Gamma$  is  $r - R < 0$ , which is sufficient, given the concavity of the objective

function, for 0 to be the optimum. The following proposition restates the conclusions of this argument.

**Proposition 4** *If  $\bar{x}_0/\mu_B < \theta < \bar{x}_0/\mu_\Gamma$  and  $R - \eta < 0 < R - (\beta\eta + 1 - \beta)$ , then  $(\bar{x}_0 - \mu_\Gamma\theta, \bar{x}_0 - \mu_\Gamma\theta, 0)$  is the unique cooperative production plan, the output of which can be allocated to maximize expected utility  $U_0$  among the allocations that are technically feasible for aggregate production. Allocation  $c$  is optimal among allocations that are feasible from this plan if and only if  $c_1^{1B} = (\mu_\Gamma\theta + (\bar{x}_0 - \mu_\Gamma\theta)r)/\mu_B$ ,  $c_1^{1\Gamma} = \theta$ ,  $c_1^{2B} = c_1^{2\Gamma} = 0$ ,  $\mu_B c_2^{1B} + (1 - \mu_B)c_2^{2B} = \bar{x}_2$ , and  $\mu_\Gamma c_2^{1\Gamma} + (1 - \mu_\Gamma)c_2^{2\Gamma} = \bar{x}_2 + R(\bar{x}_0 - \mu_\Gamma\theta)$ .*

Note that it is possible for both premise (19) of Proposition 3 and also (25) to be satisfied, that is,

$$R - \eta < 0 < R - \max(\mu_1^*\eta + \mu_2^*, \beta\eta + 1 - \beta). \quad (26)$$

If (23) and (26) both hold, then, by Proposition 2 and Proposition 3, the optimal level of expected utility that it is technically feasible to obtain from the cooperative production characterized in Proposition 4 is strictly higher than the optimal autarkic level.

## 6. FIRM SECTOR, GOVERNMENT, AND FORMALIZATION OF A BAILOUT

The model being formulated and analyzed here is a rather abstract one. It makes no explicit mention of institutions, particularly of firms or of a government. Yet, the model is being proposed as a tool for gaining insight about government bailouts of firms. It is now time to discuss the intended interpretation of the model, in order to justify how a bailout is formalized within it.

The intended interpretation of cooperative production is that it is the activity of a limited-liability firm. Investors voluntarily give their initial endowments,  $\bar{x}_0$ , to the firm in return for state-contingent claims against it—the firm’s liabilities. However, the firm is not empowered to come back to the investors at date 2 and demand part or all of their endowments,  $\bar{x}_2$ . Neither are the firm’s creditors so empowered, if the firm defaults on its liabilities.

A firm does not have to be incorporated so that its investors have limited liability, but this is the typical legal arrangement, especially for large firms, in the United States and other industrialized countries. Historically, the widespread existence of limited-liability firms only goes back for about a century and a half. Until well into the twentieth century, U.S. banks were required by law to be chartered with shareholders having “double liability,” whereby they could be required to contribute up to the par value of their

equity, if necessary, toward meeting the bank's corporate liabilities.<sup>8</sup> Although some firms today continue to be chartered as general partnerships or other forms of company with at least some investors having unlimited liability, it is widely accepted that the corporate form of organization confers benefits that society would forgo in an unlimited-liability regime (cf. Easterbrook and Fischel 1985).

Alongside the limited-liability corporations in a modern economy is the government, which can tax some investors and redistribute the proceeds to others. In particular, taxation can force an investor's consumption at date 2 below  $\bar{x}_2$ . Given an allocation  $c$ , for each individual state  $s$  and aggregate state  $\sigma$  there are unique  $\rho^{s\sigma} \geq 0$  and  $\tau^{s\sigma} \geq 0$  such that

$$\begin{aligned} c_2^{s\sigma} &= \bar{x}_2 + \rho^{s\sigma} - \tau^{s\sigma}; \\ \rho^{s\sigma} + \tau^{s\sigma} &= \min\{\rho + \tau \mid \rho \geq 0 \text{ and } \tau \geq 0 \text{ and } c_2^{s\sigma} = \bar{x}_2 + \rho - \tau\}. \end{aligned} \quad (27)$$

(The second equation means that, at most, one of  $\rho^{s\sigma}$  and  $\tau^{s\sigma}$  can be positive, and that both must be zero if  $c_2^{s\sigma} = \bar{x}_2$ .) The quantity  $\rho^{s\sigma}$  represents the investor's receipts from both corporate payouts and government subsidies, and  $\tau^{s\sigma}$  represents the amount of tax that the investor has paid. Feasibility of an allocation implies a government budget constraint that subsidies cannot exceed taxes. In particular, if  $\tau^{1\sigma} = \tau^{2\sigma} = 0$ , then no tax is collected in aggregate state  $\sigma$  and therefore no subsidy can be paid out in that state. Allocation  $c$  exhibits subsidy if, for some  $s$  and  $\sigma$ ,  $\tau^{s\sigma} > 0$ .

Intuitively, not every subsidy is a bailout. A bailout occurs when an extraordinarily high level of liquidation occurs and also (perhaps subsequently to the liquidation) an extraordinarily high level of subsidy is provided. Formally, a *bailout* is an aggregate state  $\sigma \in \{B, \Gamma\}$  such that

$$\begin{aligned} \text{Either } 0 &< \min(\epsilon_B, \epsilon_\Gamma) \text{ and } \mu_\sigma \tau^{1\sigma} + (1 - \mu_\sigma) \tau^{2\sigma} > 0, \\ \text{or, for } \sigma' &\neq \sigma, \epsilon_\sigma > \epsilon_{\sigma'} \text{ and } \mu_\sigma \tau^{1\sigma} + (1 - \mu_\sigma) \tau^{2\sigma} > \mu_{\sigma'} \tau^{1\sigma'} \\ &+ (1 - \mu_{\sigma'}) \tau^{2\sigma'}. \end{aligned} \quad (28)$$

The two clauses of this definition represent situations with different welfare characteristics. In the first clause, early liquidation occurs in both aggregate states, so the allocation is technically inefficient. The clause states that, in that context, every aggregate state in which there is positive taxation (and associated subsidy) is a bailout state. Such a bailout resembles a bank run in Diamond and Dybvig's model.<sup>9</sup> In contrast, the second clause stipulates that

<sup>8</sup> Macey and Miller (1992) provide a history of this requirement, and they argue that it worked reasonably well as a prudential regulatory regime for banks.

<sup>9</sup> In Diamond and Dybvig (1983), there is a "run equilibrium" in which early liquidation takes place in both aggregate states. The allocation resulting from this equilibrium is inefficient, by the same logic as applies here.

early liquidation occurs only in one of the two aggregate states, specifically in the one in which tax revenue is highest. That is, liquidation is accompanied by a higher level of taxation, measured as tax revenue *per capita*, than is imposed in the non-bailout state. An allocation in which such a bailout occurs might, or might not, be optimal. Proposition 5, to be proved in Section 8, states that there is an economy in which a bailout occurs in one of the optimal allocations. Proposition 6, to be proved in Section 9, states that, when the model is modified by positing a convex deadweight cost of taxation, there is an economy in which a bailout must occur in the unique optimal allocation.

The relationship between the formal definition of a bailout provided here and the informal definition stated in the introduction deserves comment. The informal definition refers to subsidy for the purpose of preventing or mitigating insolvency. The formal definition refers to a correlation between subsidy and early liquidation of investments. The idea that links the two definitions is that early liquidation is a drastic measure that must be taken to avert or minimize insolvency under *laissez faire*, and that it has adverse effects *prima facie*. Especially in the case that a subsidy would be completely successful in averting insolvency without having recourse to early liquidation, there could not be any correlation of the subsidy with early liquidation. Thus, a subsidy that fits the intuitive definition of subsidy would not fit the formal definition. Conversely, if a firm can meet its obligations by means of early liquidation, and if a subsidy is provided when early liquidation is used for that purpose, then—providing that the firm could be forced to liquidate rather than having to be bribed with the subsidy to do so—the subsidy is not necessary to avert insolvency. That is, the subsidy would fit the formal definition of a bailout but not the informal definition. Nevertheless, although some adjustment of both the informal and formal definitions of a bailout to reconcile their meanings would be desirable in principle, the formal definition succeeds well in capturing the intent of the informal definition in the examples to be studied below. The issue of definitional fit here is typical, not exceptional. Formal concepts introduced in scientific theories seldom match exactly the informal concepts that they supplant.

## 7. INCENTIVE COMPATIBILITY

Since each investor's information about his own state is private, investors must be willing to report it truthfully in order for an aggregate production plan and the allocation that distributes its output to distinguish between  $B$  and  $\Gamma$  (cf. Myerson 1979). That is, evaluated according to the conditional expected utility of the investor's true type, what the allocation gives to that type is better than what it gives to the opposite type (that is, than what he would get if he were to report his type falsely). To formalize this idea, let  $\tilde{c}$  be the allocation that, at each date and in each aggregate state, gives an impatient investor what

$c$  gives a patient investor and vice versa. That is, for all  $t$  and  $\sigma$ , and for each  $s \in \{1, 2\}$ ,  $\tilde{c}_t^{s\sigma} = c_t^{(3-s)\sigma}$ .<sup>10</sup> Allocation  $c$  is incentive compatible if, for each  $s \in \{1, 2\}$ ,<sup>11</sup>

$$U_s(c) \geq U_s(\tilde{c}). \quad (29)$$

Note that, according to the terminology adopted in this article, *feasible* means technically feasible, and does not imply incentive compatibility. *Optimal* means maximal with respect to expected utility among feasible production plans, rather than among plans that are both technically feasible and incentive compatible. However, an allocation cannot actually be implemented in a private-information environment unless it is both technically feasible and incentive compatible. The reason is that, unless the allocation is incentive compatible, investors will not voluntarily make the state-contingent choices that are required to implement it, and those choices must be made voluntarily because they depend on contingencies that would have to be—but cannot be—observed by some third party in order to be enforced coercively. The remainder of the article will be a study of the questions: Can an optimal allocation be incentive compatible and, if so, must a subsidy or even a bailout be provided in some state of the world in order to satisfy the incentive-compatibility constraint (29)?

## 8. OPTIMALITY, INCENTIVE COMPATIBILITY, AND SUBSIDY

Here is an example of an economy in which there is an allocation that is both optimal for cooperative production (as in Proposition 4) and incentive compatible, and in which every allocation that satisfies both of these conditions exhibits subsidy. Consider the following parameter values:

$$\bar{x}_0 = 3; \bar{x}_2 = 1; \beta = 0.06; \mu_B = \frac{5}{6}; \mu_\Gamma = \frac{1}{2}; \theta = 4; R = 2; r = \frac{1}{2}; \eta = 5. \quad (30)$$

<sup>10</sup> It would not necessarily be technically feasible to give the consumption specified by  $\tilde{c}$  to every investor. The point of defining  $\tilde{c}$  is to specify what an investor would get by deviating unilaterally from truthful revelation. Such a unilateral deviation, by an investor whose individual consumption is infinitesimal compared to aggregate consumption, would not cause incentive compatibility to be violated.

<sup>11</sup> To spell out condition (29),

$$\begin{aligned} \beta_1^* u(c_1^{1B}, c_2^{1B}, 1) + (1 - \beta_1^*) u(c_1^{1\Gamma}, c_2^{1\Gamma}, 1) &\geq \beta_1^* u(c_1^{2B}, c_2^{2B}, 1) + (1 - \beta_1^*) u(c_1^{2\Gamma}, c_2^{2\Gamma}, 1); \\ \beta_2^* u(c_1^{2B}, c_2^{2B}, 2) + (1 - \beta_2^*) u(c_1^{2\Gamma}, c_2^{2\Gamma}, 2) &\geq \beta_2^* u(c_1^{1B}, c_2^{1B}, 2) + (1 - \beta_2^*) u(c_1^{1\Gamma}, c_2^{1\Gamma}, 2). \end{aligned}$$

These values satisfy (23) and (26). By Proposition 4, the optimal cooperative production plan is  $(\iota, \epsilon_B, \epsilon_\Gamma) = (1, 1, 0)$ , and an optimal allocation,  $c$ , of the product,  $y$ , of this plan, if it does not exhibit subsidy, must satisfy

$$\begin{aligned}
c_1^{1B} &= y_{B1}/\mu_B = 3; \\
c_1^{1\Gamma} &= \theta = 4; \\
c_1^{2B} &= c_1^{2\Gamma} = 0; \\
\mu_\Gamma c_2^{1\Gamma} + (1 - \mu_\Gamma) c_2^{2\Gamma} &= \bar{x}_2 + y_{\Gamma 2} = 3; \\
c_2^{sB} &= 1 \text{ for } s \in \{1, 2\}; \\
c_2^{s\Gamma} &\geq 1 \text{ for } s \in \{1, 2\}.
\end{aligned} \tag{31}$$

For the allocation to be incentive compatible for a patient investor, according to (29),  $U_2(c) - U_2(\tilde{c}) \geq 0$  must be satisfied.  $U_2(c) - U_2(\tilde{c})$  is increasing in  $c_2^{2\Gamma}$  and decreasing in  $c_2^{1\Gamma}$ . In an optimal allocation, by (31), a patient investor can only consume at date 2 and must consume exactly one unit at that date in aggregate state  $B$ . Therefore, if any optimal allocation can be incentive compatible for the patient investor but not exhibit subsidy, then one such allocation will give all date-2 output to patient investors. That is, the following allocation should be checked for incentive compatibility:

$$\begin{aligned}
c_1^{1B} &= y_{B1}/\mu_B = 3; \\
c_1^{1\Gamma} &= 4; \\
c_1^{2B} &= c_1^{2\Gamma} = 0; \\
c_2^{1B} &= c_2^{1\Gamma} = 1; \\
c_2^{2B} &= 1; \\
c_2^{2\Gamma} &= R\iota/\mu_\Gamma + 1 = 5.
\end{aligned} \tag{32}$$

But this allocation is obviously not incentive compatible. A patient investor's utility function is  $u(c_1, c_2, 2) = c_1 + c_2$ , and this quantity is identical for a patient and an impatient investor in aggregate state  $\Gamma$  and strictly higher for an impatient investor in state  $B$ . The consequence for condition (32) is that

$$U_2(c) - U_2(\tilde{c}) = -3\beta_2^* \approx -.063. \tag{33}$$

Taxing impatient investors' endowments at date 2, and transferring the tax revenue to patient investors, converts  $c$  to a new allocation that is equal to  $c$  with regard to ex ante expected utility, and that is incentive compatible. Specifically, define allocation  $d$  by taxing one unit of impatient investors' endowment in state  $\Gamma$  and transferring it to patient investor. That is,



$$\begin{aligned}
d_t^{s\sigma} &= c_t^{s\sigma} \text{ if } t = 1 \text{ or } \sigma = B; \\
d_2^{1\Gamma} &= 0; \\
d_2^{2\Gamma} &= 6.
\end{aligned} \tag{34}$$

It is obvious from (32) that, when the date-2 consumption of patient investors in  $c$  is increased by two relative to impatient investors with probability close to one, the resulting allocation,  $d$ , is incentive compatible for patient investors. To be precise, the incentive-compatibility constraint (29) for patient investors evaluates (after rounding) to  $5.9 > 4.0$ , so the constraint is satisfied. For impatient investors, (29) evaluates to  $6.9 > 5.5$ , so their incentive-compatibility constraint is also satisfied.

There is no bailout in allocation  $d$ , however, because liquidation occurs in one state but a tax is levied (and subsidy is distributed) only in the other. Consider optimal allocation  $e$ , in which taxation occurs in the bad state along with liquidation. That is, a bailout occurs in this allocation:

$$\begin{aligned}
e_t^{s\sigma} &= c_t^{s\sigma} \text{ if } t = 1 \text{ or } \sigma = \Gamma; \\
e_2^{1B} &= 0; \\
e_2^{2B} &= 6.
\end{aligned} \tag{35}$$

The incentive-compatibility condition (29) evaluates to  $5.02 > 4.96$  for patient investors and to  $7.8 > 5.1$  for impatient investors. The following proposition summarizes these findings.

**Proposition 5** *Every technically feasible allocation of the economy described by (30) either is suboptimal, violates incentive compatibility, or exhibits subsidy. The economy has some allocations that are technically feasible, optimal, and incentive compatible. All such allocations exhibit subsidy. In some of them, a bailout occurs.*

## 9. ESSENTIAL BAILOUTS

Bailouts are defined as essential in an economy if one occurs in every allocation of that economy that is optimal subject to incentive compatibility constraints. In this section, the model of an economy is modified in such a way that there is an example in which bailouts are essential.

One way to make such a change would be, in effect, to gerrymander the model. We specify that the marginal utility of consumption for impatient investors at date 2 is lower in aggregate state  $B$ , but higher in state  $\Gamma$ , than that for patient investors. We also specify that each investor's date-2 endowment is state contingent, and is perfectly correlated with the investor's preference type, specifically with an investor having a larger endowment when impatient

than when patient. Optimality subject to incentive compatibility would then require the transfer from impatient to patient investors to be maximized at date 2 in  $B$ , and it would require the transfer from patient to impatient agents at date 2 in  $\Gamma$  to be maximized, subject to incentive compatibility. If early liquidation is required in  $B$  to achieve the optimal level of ex-ante utility, subject only to technical feasibility, and if incentive-compatibility constraints do not bind in that allocation, then bailouts are essential in the economy.

This sketch of an example shows that, in principle, either the definition of a bailout or the definition of essentiality needs to be tightened. That is, the two definitions together should express the idea that subsidy is being used in the bailout state to solve an incentive problem created intrinsically by early liquidation, rather than playing a distinct role having to do with insurance. Since all investors' utility for consumption at date 2 is assumed to be linear and identical across individual states (implying that there is no possibility of increasing ex-ante welfare by equalizing different investors' marginal utilities at date 2) in the example studied in Section 8, the current definitions seem satisfactory, as long as that assumption is maintained.

Consider an alternative modification of the model: the introduction of a convex deadweight cost of taxation. Let  $\delta$  be a convex function satisfying  $\delta(\tau) = 0$  for all  $\tau \leq 0$ , and  $\delta$  is strictly convex at positive tax levels. This function specifies, for each investor, how much consumption is lost to the economy when tax is collected from him.<sup>12</sup> To formalize this idea, replace the definition (13) of technical feasibility for cooperative production with

$$\begin{aligned} \mu_\sigma c_1^{1\sigma} + (1 - \mu_\sigma) c_1^{2\sigma} &\leq y_{\sigma 1}; \\ \mu_\sigma c_2^{1\sigma} + (1 - \mu_\sigma) c_2^{2\sigma} &\leq y_{\sigma 2} + \bar{x}_2 - (\mu_\sigma \delta(\tau^{1\sigma}) + (1 - \mu_\sigma) \delta(\tau^{2\sigma})). \end{aligned} \quad (36)$$

A calculus result, Jensen's inequality, implies the following lemma.

**Lemma 1** *If  $\delta$  is strictly convex for  $\tau > 0$ , and if technical feasibility of an allocation for aggregate production is defined by (36), then  $\tau^{1B} = \tau^{1\Gamma}$  and  $\tau^{2B} = \tau^{2\Gamma}$  in an allocation that is optimal subject to technical feasibility and incentive compatibility. By strict convexity of  $\delta$ , this constrained-efficient allocation is generically unique.<sup>13</sup>*

Using this lemma, it is routine to calculate an allocation  $f$ , analogous to  $e$  in Section 8, that is optimal among technically feasible, incentive-compatible

<sup>12</sup> The cost includes the direct cost of collecting and enforcing taxes and the indirect cost (in an actual economy, as opposed to the highly simplified model economy) of agents shifting resources to low-productivity, but tax-favored, investments. Embedding a costly state verification model of tax collection (along the lines of Townsend [1979]) in the model economy would provide a foundation for this reduced-form specification.

<sup>13</sup> *Generically* means that, for any parameter vector having more than one such allocation, the economy corresponding to an arbitrarily small perturbation of that vector in a random direction will have a unique optimum.

allocations of the economy that is identical to the one studied in Section 8 (with parameters specified in [30]), except that technical feasibility is defined according to (36). By the lemma, this can be taken to be the unique constrained-efficient allocation of the economy. In the allocation,  $\tau^{1\sigma} > 0 = \tau^{2\sigma}$ . By the lemma, the tax does not depend on  $\sigma$ . Thus, let  $\hat{\tau}$  denote the tax levied on impatient investors in both states. The amount of tax levied is  $\hat{\tau}/2$  in state  $\Gamma$  and  $5\hat{\tau}/6$  in state  $B$ . This means that  $B$  is the high-subsidy state, as well as being the early-liquidation state, so there is a bailout. Since the allocation in question is the unique constrained-efficient allocation, bailouts are essential in this economy.

To carry out the details of this construction, let  $\zeta$  be a small, positive number, and define

$$\delta(\tau) = \begin{cases} 0 & \text{if } \tau < 0; \\ \zeta \tau^2 & \text{if } \tau \geq 0. \end{cases} \quad (37)$$

Consider the economy with parameters specified in (30) and with the set of feasible allocations specified to incorporate a deadweight cost of taxation according to (36) and (37). Modify allocation  $c$ , defined in (32), to specify a feasible allocation  $f$  of this economy, defined in terms of a positive parameter  $\hat{\tau}$ , as follows:

$$\begin{aligned} f_1^{s\sigma} &= c_1^{s\sigma}; \\ f_2^{1\sigma} &= \bar{x}_2 - (\hat{\tau} + \delta(\hat{\tau})); \\ f_2^{2B} &= \bar{x}_2 + 5\hat{\tau}; \\ f_2^{2\Gamma} &= \bar{x}_2 + 2Rt + \hat{\tau}. \end{aligned} \quad (38)$$

That is, set  $f$  equal to  $c$  at date 1, set the consumption level of an impatient investor at date 2 to be the investor's date-2 endowment minus the sum of a tax  $\hat{\tau}$  and the deadweight cost of its imposition, and set the consumption level of a patient investor at date 2 to be the sum of the investor's endowment and the investor's share of both the date-2 investment proceeds from plan  $(1, 1, 0)$  and the receipt from the taxation of impatient investors.

If  $\zeta = 0$  and  $\hat{\tau} < \bar{x}_2$ , then allocation  $f$  is optimal. If  $\zeta > 0$  and  $\hat{\tau} > 0$ , then  $f$  is not optimal because  $\delta(\hat{\tau}) > 0$ , and this deadweight cost must be deducted from consumption. However, for the parameter values specified in (30), a subsidy is necessary to achieve incentive compatibility, and logically this is true under an assumption that  $\zeta > 0$ , since the set of feasible allocations for positive  $\zeta$  is a subset of those for  $\zeta = 0$ . Optimality subject to incentive compatibility is achieved when the tax,  $\hat{\tau}$ , is minimized, subject to the constraint that the resulting allocation should be incentive compatible. That value of  $\hat{\tau}$  is the one that makes the incentive-compatibility constraint for patient investors hold with equality, that is,

$$\begin{aligned}
0 &= U_2(f) - U_2(\tilde{f}) \\
&= \beta_2^* [(f_1^{2B} - f_1^{1B}) + (6\hat{\tau} + \delta(\hat{\tau}))] \\
&\quad + (1 - \beta_2^*) [(f_1^{2\Gamma} - f_1^{1\Gamma}) + (2Rl + 2\hat{\tau} + \delta(\hat{\tau}))] \\
&= \zeta \hat{\tau}^2 + (2 + 4\beta_2^*) \hat{\tau} - 4\beta_2^*.
\end{aligned} \tag{39}$$

By the quadratic formula and the positivity of  $\hat{\tau}$ ,

$$\hat{\tau} = \frac{-(2 + 4\beta_2^*) + \sqrt{(2 + 4\beta_2^*)^2 + 12\zeta\beta_2^*}}{2\zeta}. \tag{40}$$

Using Taylor's formula to approximate the square-root term in the numerator of (40),

$$\hat{\tau} = \frac{4\beta_2^*}{2 + 4\beta_2^*} = 0.03. \tag{41}$$

It can easily be computed that, when the tax is set at this level, the incentive-compatibility constraint for impatient investors does not bind. Thus, allocation  $f$  is the unique allocation that is technically feasible and is also optimal subject to the incentive-compatibility constraints for both patient and impatient investors. In  $f$ , since  $\hat{\tau}$  is collected from  $\frac{5}{6}$  of the investors in state  $B$  but only from  $\frac{1}{2}$  of them in state  $\Gamma$ , aggregate tax revenue in  $B$  is  $\frac{5}{3}$  times aggregate tax revenue in  $\Gamma$ . That is, given that early liquidation occurs in  $B$  but not in  $\Gamma$ , allocation  $f$  exhibits a bailout in  $B$ , and this bailout is essential. The following proposition summarizes this result.

**Proposition 6** *Under the assumption that taxing an investor has convex dead-weight cost, there is an economy in which bailouts are essential.*

## 10. CONCLUSION

Occasional bailouts of insolvent firms that are ultimately financed by taxation—notably including bailouts of financial intermediaries—are a fact of life in virtually every country. On one side of a debate about the welfare assessment of such bailouts are economists, such as Kareken (1983) and Stern and Feldman (2004), who emphasize that inefficient risk-taking results from a combination of time inconsistency on the part of the government and moral hazard on the part of firms' owners, liability holders, and managers. On the other side, there has been only an amorphous plea, albeit a sincere one from some distinguished economists and sophisticated policymakers and financial-market participants, that unspecified but very serious and long-term harms would result if government were to refrain from a bailout. At first sight, such a plea seems to be

a reflection of precisely the time inconsistency that is pivotal to the critics' arguments. However, there is another possible interpretation of the point that apologists for bailouts are trying to make. Namely, once a regime has been established that favors the incorporation of limited-liability firms, bailing out those firms in some states of the world may be the only way to make ex-ante efficient investments incentive compatible. While critics believe that it would be time inconsistent to conduct a bailout, apologists believe that it would be time inconsistent to refrain from a bailout in some circumstances. The long-term harm that they fear is impairment, after an ex-ante commitment to incentive-enhancing bailouts had been shown not to be credible, of investors' willingness to fund socially beneficial projects. This paper, particularly in Proposition 6, develops the logic of that position.

It should be kept in mind that this article has explored the logic of an economic argument, rather than having advocated a policy. Issues of first-rank importance in an actual economy, such as the effect that anticipating a bailout to be available will have on firm owners' and managers' incentive to take risk, do not arise in the model economy studied here. Nevertheless, this analysis shows that public discussion regarding the bailout of firms by the U.S. government during the financial crisis in 2008–2009 has had shortcomings. It has generally been asserted by critics of the bailout, and conceded by its proponents, that a tax-financed subsidy to firms is ex ante a bad policy. This assertion is not sound with respect to the model economy analyzed here. It may well be sound with respect to the U.S. economy, but that judgment should be given a supporting argument rather than taken as a starting point. A tradeoff has to be made between the potential benefits of a bailout emphasized in the present model and the costs that are emphasized in other models. It is an oversimplification to presume that a bailout is necessarily all bad.

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