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Instability and Indeterminacy in a Simple Search and Matching Model

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The search and matching model of Mortensen and Pissarides (1994) has become a popular and successful framework for analyzing labor market dynamics in dynamic stochastic general equilibrium (DSGE) models.[1] In this article, we point out a potentially problematic feature of this framework. We show that the solution to the dynamic model can be nonexistent or indeterminate. In particular, uniqueness problems arise when endogenous matching in response to labor market pressures is not elastic enough. In such a scenario, extraneous uncertainty, "sunspots," can lead to business cycle fluctuations even in the absence of any other disturbances. However, a solution does not exist when matching is too elastic. While these determinacy problems are plausible outcomes, we argue that they are not likely, as they are associated with regions of the parameter space that are at the extremes of typical calibrations.

Indeterminacy in search and matching models has previously been addressed by Giammarioli (2003). Her article differs from ours in that it introduces increasing returns in the matching function, which is a well-known mechanism to generate multiplicity in DSGE models (see Farmer and Guo [1994]). We show that indeterminacy in the search and matching model can arise even under constant returns. Our paper is similar to Burda and Weder

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[1] A nonexhaustive list of references includes Merz (1995); Andolfatto (1996); Cooley and Quadrini (1999); den Haan, Ramey, and Watson (2000); Krause and Lubik (2007); and Trigari (2009).

(2002) in this respect. Their indeterminacy results are driven, however, by the existence of labor market distortions, such as taxes, and the associated fiscal policy functions, and not by the features of the matching process per se. More recently, Hashimzade and Ortigueira (2005) analyzed the determinacy properties of a real search and matching model with capital. They show numerically how, for a given parameterization, the model admits sunspot equilibria. Zanetti (2006) incorporates the standard search and matching model into a New Keynesian DSGE model, where monetary policy is governed by an interest rate feedback rule. He shows that this expands the region of the parameter space where the Taylor principle, and thus equilibrium uniqueness, is violated. However, his paper focuses on the monetary policy rule as a source of indeterminacy. Labor market search and matching only provides a transmission mechanism, but is not analyzed as an independent factor of determinacy problems.

This article proceeds as follows. We present a canonical DSGE model with search and matching frictions in the next section. This is a bare-bones version of the model that does not rely on any increasing returns to scale in the functional forms. Our model specification has the advantage that the determinacy regions can be characterized largely analytically. Section 2 discusses issues related to the calibration of this model, while Section 3 derives its determinacy properties, both analytically and numerically. The final section briefly summarizes and concludes.

1. A CANONICAL DSGE MODEL OF LABOR MARKET SEARCH AND MATCHING

We develop a simple version of a discrete-time DSGE model with search and matching frictions in the labor market.[2] Key to the search and matching model is that new employment relationships are the result of time-consuming searches, both by firms and potential workers. In order to hire workers, firms first have to advertise open positions; they have to post vacancies, which is assumed to be costly. Existing matches between workers and firms are subject to job destruction, which leads to a flow of workers into the unemployment pool. The behavior of the aggregate economy is governed by the choices of a representative household, which engages in consumption smoothing. The household engages in perfect risk-sharing between its employed and unemployed members. The latter enjoy unemployment benefits while searching for a job. We employ some simplifying assumptions later on that lead to steady-state and dynamic equations that can be solved analytically. The properties of the full model are then analyzed numerically.

[2] The model is similar to Lubik (2009), to which we refer the reader for additional discussion and references.

Time is discrete. One period in the model is assumed to be a quarter. There is a continuum of identical firms that employ workers, who each inelastically supply one unit of labor.[3] Output, y, of a typical firm is linear in employment, n:

$$y_t = n_t. \tag{1}$$

The matching process is represented by a constant-returns matching function, $m(u_t, v_t) = mu_t^{\xi} v_t^{1-\xi}$, of unemployment, u, and vacancies, v, with parameters $m > 0$ and $0 < \xi < 1$. It captures the number of newly formed employment relationships that arise out of the contacts of unemployed workers and firms seeking to fill open positions. Unemployment is defined as

$$u_t = 1 - n_t, \tag{2}$$

which is the measure of all potential workers in the economy who are not employed at the beginning of the period and are thus available for job search activities.

Inflows to unemployment arise from exogenous job destruction at rate $0 < \rho < 1$. Employment therefore evolves according to

$$n_t = (1 - \rho)[n_{t-1} + m(u_{t-1}, v_{t-1})]. \tag{3}$$

Note that newly matching workers who are separated from their job within the period reenter the matching pool immediately. We can define $q(\theta_t)$ as the probability of filling a vacancy, or the firm-matching rate, where $\theta_t = v_t/u_t$. We refer to θ as the degree of labor market tightness. In terms of the matching function, we can write this as $q(\theta_t) = m(u_t, v_t)/v_t = m\theta_t^{-\xi}$. Similarly, the probability of finding a job, the worker-matching rate, is $p(\theta_t) = m(u_t, v_t)/u_t = m\theta_t^{1-\xi}$. An individual firm is atomistic in the sense that it takes the aggregate matching rate, $q(\theta_t)$, as given. The employment constraint on the firm's decision problem is therefore

$$n_t = (1 - \rho)[n_{t-1} + v_{t-1}q(\theta_{t-1})], \tag{4}$$

that is, it is linear in vacancy postings.

Firms maximize profits using the discount factor $\beta^t \frac{\lambda_t}{\lambda_0}$ (to be determined below):

$$\max_{\{v_t, n_t\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} [n_t - w_t n_t - \kappa v_t] + \\ + \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \mu_t \left[(1 - \rho)[n_{t-1} + v_{t-1}q(\theta_{t-1})] - n_t\right]. \tag{5}$$

[3] For expositional convenience, we present the problem of a representative firm only, and abstract from indexing the individual form and aggregation issues.

Wages paid to the workers are w, while $\kappa > 0$ is a firm's cost of opening a vacancy. μ is the Lagrange multiplier on the firm's employment constraint. It can be interpreted as the marginal value of a filled position. Firms decide how many vacancies to post (which can be turned into employment relationships) and how many workers to hire. The first-order conditions are

$$n_t \quad : \qquad \mu_t = 1 - w_t + \beta(1-\rho)\frac{\lambda_{t+1}}{\lambda_t}\mu_{t+1}, \tag{6}$$

$$v_t \quad : \qquad \kappa = \beta(1-\rho)\frac{\lambda_{t+1}}{\lambda_t}\mu_{t+1}q(\theta_t), \tag{7}$$

which imply a job-creation condition

$$\frac{\kappa}{q(\theta_t)} = (1-\rho)\beta\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left[1 - w_{t+1} + \frac{\kappa}{q(\theta_{t+1})}\right]. \tag{8}$$

This optimality condition trades off the expected hiring cost (which depends on the success probability $q(\theta_t)$) against the benefits of a productive match (which consists of the output accruing to the firms net of wage payments and the future savings on hiring costs when the current match is successful).

We assume that the economy is populated by a representative household. The household is composed of workers who are either unemployed or employed. If they are unemployed they are compelled to search for a job, but they can draw unemployment benefits, b. Employed members of the household receive pay, w, but share this with the unemployed. They do not suffer disutility from working and supply a fixed number of hours.[4] The household's only choice variable is consumption, so that its optimization problem is trivial:

$$\max_{\{C_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma}\right], \tag{9}$$

subject to

$$C_t = Y_t, \tag{10}$$

where C is consumption and Y is income earned from labor and residual profits from the firms; $0 < \beta < 1$ is the discount factor, and σ^{-1} is the intertemporal elasticity of substitution. From the household's (trivial) first-order condition we find that $\lambda_t = C_t^{-\sigma}$, where λ is the multiplier on the household's budget constraint. In equilibrium, total income accruing to the household equals net output in the economy, which is composed of production less real resources lost in the search process:

$$Y_t = y_t - \kappa v_t. \tag{11}$$

[4] We thus assume income pooling between employed and unemployed households and abstract from potential incentive problems concerning labor market search. This allows us to treat the labor market separate from the consumption choice. See Merz (1995) and Andolfatto (1996) for discussion of these issues.

Finally, we need to derive how wages are determined. We assume that wages are set according to the Nash bargaining solution.[5] Firms and workers maximize the bargaining function

$$\max_{w_t} \left(\mathcal{W}_t\right)^{\eta} \left(\mathcal{J}_t\right)^{1-\eta}, \tag{12}$$

with respect to the variable over which the two parties bargain, namely the wage, w_t. This results in the sharing rule:

$$\eta \mathcal{J}_t = (1-\eta)\, \mathcal{W}_t. \tag{13}$$

$\mathcal{W}_t$ denotes the match surplus accruing to the worker, while $\mathcal{J}_t$ is the firm's surplus, that is, the value of a filled job. The latter can be found from the firm's optimization problem. It is equal to the Lagrange multiplier on the employment constraint, μ_t, and is the shadow value of a filled position; to wit, $\mathcal{J}_t = \mu_t$. From the first-order condition with respect to employment we find that

$$\mathcal{J}_t = 1 - w_t + \beta(1-\rho)\frac{\lambda_{t+1}}{\lambda_t}\mathcal{J}_{t+1}. \tag{14}$$

The expression states that the value of a filled job is its marginal product, 1, net of wage payments, w_t; but it also has a continuation value $\mathcal{J}_{t+1}$, which is discounted at the time preference rate, β, and assuming that the filled job is still there next period. The latter is captured by the survival rate $(1-\rho)$.

We can derive the worker's surplus as follows. The worker receives payment in the form of the wage, w_t. But while he is working, he loses the value of being unemployed, b. The latter can be interpreted as the money value of enjoying leisure, engaging in household production, or simply unemployment benefits. Therefore, the current period net return is $w_t - b$. In the next period, the worker receives the continuation value $\mathcal{W}_{t+1}$, which is discounted at rate β. The worker has to take into account that he might not be employed next period, which is captured by the survival rate $(1-\rho)$, adjusted for the fact that a separated worker might not find a job again with probability $[1 - p(\theta_t)]$. Putting it all together, we have

$$\mathcal{W}_t = w_t - b + \beta(1-\rho)\left[1 - p(\theta_t)\right]\frac{\lambda_{t+1}}{\lambda_t}\mathcal{W}_{t+1}. \tag{15}$$

The two marginal values can now be substituted into the sharing rule and, after some algebra using the firm's first-order conditions, we can find the Nash-bargained wage:

$$w_t = \eta\left(1 + \kappa\theta_t\right) + (1-\eta)b. \tag{16}$$

[5] This is a standard assumption in the literature. Shimer (2005) provides further discussion.

We can now use this wage equation to derive the job-creation condition:

$$\frac{\kappa}{q(\theta_t)} = (1-\rho)\beta\frac{Y_t^{\sigma}}{Y_{t+1}^{\sigma}}\left[(1-\eta)(1-b) - \eta\kappa\theta_{t+1} + \frac{\kappa}{q(\theta_{t+1})}\right], \quad (17)$$

where we have used the first-order conditions of the household to eliminate the Lagrange multiplier, λ, from the discount factor. The dynamics of the model are given by the five equations in five unknowns: (2), (3), (11), (17), and the definition of labor market tightness, θ_t.

2. STEADY STATE AND CALIBRATION

We first compute the deterministic steady state of the model. We then linearize the dynamic system around the steady state and analyze the local determinacy properties of the economy. The equations describing the steady state are

$$u = 1-n, \quad (18)$$

$$\theta = \frac{v}{u}, \quad (19)$$

$$n = \frac{1-\rho}{\rho}mv^{1-\xi}u^{\xi}, \quad (20)$$

$$Y = n - \kappa v, \quad (21)$$

$$(1-\eta)(1-b) = \frac{1-\beta(1-\rho)}{\beta(1-\rho)}\frac{\kappa}{m}\theta^{\xi} + c\eta\theta. \quad (22)$$

(20) is the employment accumulation equation. It stipulates that inflows and outflows of the unemployment pool have to be equal. In a steady-state equilibrium, the number of separated workers, ρn, has to equal newly hired workers. Equation (22) is the job-creation condition, while the other equations are definitions.

There are five endogenous variables (u, n, v, θ, y) and seven structural parameters (ρ, m, ξ, κ, β, η, b). Because of the nonlinearity in the last equation, there is no analytical solution to this system. Given values for the parameters, however, we can compute a numerical solution. Using a nonlinear equation solver we determine θ from equation (22).[6] From equation (20) we can find $u = \left(1 + \frac{1-\rho}{\rho}m\theta^{1-\xi}\right)^{-1}$, and the solution for the other variables follows immediately.

We find it more convenient, however, to calibrate the model by fixing the steady-state unemployment rate, $u = \overline{u}$. This implies that one parameter has to be determined endogenously. Additionally, we can fix the endogenous

[6] Since the function in θ is monotonically increasing for nonnegative θ, there is a unique solution to this equation as long as $0 \leq b < 1$. This reflects the fact that the outside option of the worker, namely staying unemployed, cannot be larger than the worker's marginal product, i.e., the maximum rent that the worker can extract from the firm.

matching rate, $\overline{q} = m\theta^{-\xi}$, by using evidence on the rates at which firms fill vacancies. Hence, another parameter has to be determined endogenously. Using $n = 1 - \overline{u}$ in (20), we find that the match efficiency parameter is $m = \left(\frac{\rho}{1-\rho}\frac{1-\overline{u}}{\overline{u}}\right)^{\xi}\overline{q}^{1-\xi}$ and labor market tightness is $\theta = \left(\frac{m}{\overline{q}}\right)^{1/\xi}$. From (22) we can then also compute $\frac{1-b}{\kappa} = \frac{\eta}{1-\eta}\theta + \frac{1}{1-\eta}\frac{1-\beta(1-\rho)}{\beta(1-\rho)}\frac{\theta^{\xi}}{m}$. Note, however, that this condition does not pin down b and κ independently, nor does any other restriction in the model. Equation (21) helps only insofar as it restricts κ such that y remains positive. We chose to fix the vacancy cost parameter, κ, and let the benefit parameter, b, be determined endogenously.

For our calibration exercise we set the discount factor as $\beta = 0.99$. We chose a separation rate of $\rho = 0.1$. This is consistent with the evidence reported in Shimer (2005) and Lubik (2010), who use various econometric methods to estimate this parameter from U.S. labor market data. We agnostically set the bargaining parameter as $\eta = 0.5$ and follow most of the literature in this respect. Similarly, the match elasticity is $\xi = 0.5$, which is on the low end of estimates in the literature. Note that this benchmark calibration implements the Hosios condition, under which the market allocation in the model is socially efficient. The value for the match elasticity is at the low end of the plausible range as reported in the empirical study by Petrongolo and Pissarides (2001). We set the intertemporal substitution elasticity as $\sigma = 1$.

Finally, the two steady-state values are chosen as follows. We fix the unemployment rate, $\overline{u}$, at 12 percent. Our idea is to capture both measured unemployment in terms of recipients of unemployment benefits and potential job searchers that are only marginally attached to the labor force, but are open to job search. Since we do not model labor force participation decisions, this is a shortcut to capturing effective labor market search. This approach has been taken by Cooley and Quadrini (1999) and Trigari (2009). In choosing the steady-state job-matching rate, we follow den Haan, Ramey, and Watson (2000) who set $\overline{q} = 0.7$. In the numerical determinacy analysis below we conduct robustness checks for selected parameters and the calibrated steady-state values by varying them over their admissible range.

3. INDETERMINACY AND NONEXISTENCE

We now proceed by linearizing the dynamic equilibrium conditions around the steady state. It is a well-known feature of linear rational expectations models that they can have multiple equilibria, or that the solution may not even exist. We show that both scenarios are possible outcomes in the standard search and matching model, but they are associated with regions at the fringes of the parameter space. The linearized system is as follows (where $\widehat{x_t} = \log x_t - \log x$

denotes the percentage deviation of the variable x_t from its steady state x):

$$u\,\widehat{u}_t = -n\,\widehat{n}_t, \tag{23}$$
$$\widehat{\theta}_t = \widehat{v}_t - \widehat{u}_t, \tag{24}$$
$$\widehat{n}_{t+1} = (1-\rho)\widehat{n}_t + \rho(1-\xi)\widehat{v}_t + \rho\xi\widehat{u}_t, \tag{25}$$
$$\widehat{Y}_t = \frac{n}{y}\widehat{n}_t - \frac{\kappa v}{y}\widehat{v}_t, \tag{26}$$
$$\xi\widehat{\theta}_t - \sigma\widehat{Y}_t = \left(\frac{\kappa\xi}{m}\theta^{\xi} - \eta\kappa\theta\right)X^{-1}\widehat{\theta}_{t+1} - \sigma\widehat{Y}_{t+1}, \tag{27}$$

where $X = \frac{1}{\beta(1-\rho)}\frac{\kappa}{m}\theta^{\xi}$.

It is straightforward to substitute out $\widehat{u}_t$, $\widehat{v}_t$, and $\widehat{Y}_t$, so that we are left with

$$\begin{bmatrix} \widehat{\theta}_{t+1} \\ \widehat{n}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{\xi+\sigma\frac{\kappa v}{y}}{\alpha_1} + \rho(1-\xi)\frac{\alpha_2}{\alpha_1} & -\frac{\alpha_2}{\alpha_1}\frac{\rho}{u} \\ \rho(1-\xi) & \frac{u-\rho}{u} \end{bmatrix} \begin{bmatrix} \widehat{\theta}_t \\ \widehat{n}_t \end{bmatrix}, \tag{28}$$

where $\alpha_1 = \beta(1-\rho)(\xi - \eta m\theta^{1-\xi}) + \sigma\frac{\kappa v}{y}$ and $\alpha_2 = \sigma\frac{n}{y}(1+\kappa\theta)$. This reduced form is expressed in terms of the state (or predetermined) variable, $\widehat{n}_t$, and the jump variable, $\widehat{\theta}_t$, which is a function of vacancy postings, $\widehat{v}_t$. The stability properties of the solution depend on the eigenvalues of the coefficient matrix. A unique solution requires that one root be inside the unit circle and the other root outside. Indeterminacy arises when both roots are inside the unit circle, while nonexistence occurs with both roots being explosive. In the former case, both equations are dynamically stable and an infinite number of paths (starting from arbitrary initial conditions) toward the unique steady state exist. In the latter case, both equations are explosive, which implies that, from any arbitrary initial condition, employment and vacancies would grow without bounds. This violates transversality or boundary conditions and can therefore not be an equilibrium.

The coefficient matrix is sufficiently complicated to prevent simple analytical derivations of the equilibrium regions. For illustrative purposes and for gaining intuition, we therefore make the simplifying assumption that the representative household is risk neutral, $\sigma = 0$. Later on, we discuss the general case using simulation results. Under risk neutrality, the coefficient matrix reduces to

$$\begin{bmatrix} \frac{\xi}{\beta(1-\rho)(\xi-\eta p)} & 0 \\ \rho(1-\xi) & \frac{u-\rho}{u} \end{bmatrix}. \tag{29}$$

Since the coefficient matrix is triangular, the eigenvalues can be read off the principal diagonal. Recall that the worker matching rate is $p = m\theta^{1-\xi}$, which is equal to $\frac{\rho}{1-\rho}\frac{1-u}{u}$. Since we are treating the unemployment rate as a parameter to be calibrated, the determinacy conditions therefore only depend on structural parameters.

We establish the determinacy properties in the following proposition.

Proposition 1

1. *The model solution is indeterminate if and only if*
   (a) $0 < \rho < 2u$,
   (b) $0 < \xi < \frac{\beta(1-\rho)}{1+\beta(1-\rho)}\eta p$.
2. *The model solution is nonexistent if and only if*
   (a) $\rho > 2u > 0$,
   (b) $\frac{\beta(1-\rho)}{1+\beta(1-\rho)}\eta p < \xi < 1$.
3. *The model solution is unique if and only if either*
   (a) $0 < \rho < 2u$,
   (b) $\frac{\beta(1-\rho)}{1+\beta(1-\rho)}\eta p < \xi < 1$,
   or
   (c) $\rho > 2u > 0$,
   (d) $0 < \xi < \frac{\beta(1-\rho)}{1+\beta(1-\rho)}\eta p$.

Proof. Indeterminacy requires both roots inside the unit circle. Call $\lambda_2 = \frac{u-\rho}{u}$. It is straightforward to verify that $|\lambda_2| < 1$ over the permissible range iff $0 < \rho < 2u$. Call the other root $\lambda_1 = \frac{\xi}{\beta(1-\rho)(\xi-\eta p)}$. We have to distinguish two cases: if $\xi > \eta p$, no parameter combination can be found such that $|\lambda_1| < 1$. If $\xi < \eta p$, we can write $-\beta(1-\rho)(\xi-\eta p) > \xi > \beta(1-\rho)(\xi-\eta p)$. Simple algebra in combination with $\xi > 0$ then yields 1(b). Nonexistence requires that both roots be outside the unit circle. This is just the opposite scenario discussed before. Part 2 of the proposition follows immediately. Uniqueness requires one stable and one unstable eigenvalue. The parameter regions are consequently implied by those not considered in part 1 and 2. ■

The proposition shows that indeterminacy is a potential outcome in this model. It arises when the job destruction rate is less than twice the (calibrated) unemployment rate. For instance, at a separation rate of 10 percent, the unemployment rate would have to be less than 5 percent to definitely rule out indeterminacy on account of condition 1(a). This value is not implausible, given historical data for the United States where the average post-war unemployment rate is 4.8 percent. However, it has been argued (e.g., Trigari [2009]) that the proper corresponding concept for model unemployment includes not only the registered unemployed but also all workers potentially available for employment, such as discouraged workers or workers loosely attached to the labor force. Consequently, $\overline{u}$ should be assigned a much higher value (for instance, 26 percent as in Trigari [2009]), which raises the possibility of equilibrium indeterminacy.[7]

[7] Calibrating $\overline{u}$ to a different value implies that benefits, b, and match efficiency, m, would have to change, too, since they are computed endogenously from the steady-state conditions. Higher

Condition 1(b) imposes an upper bound on the match elasticity, ξ. In the benchmark calibration, this upper bound is 0.147. Since ξ is typically calibrated to be above 0.5, this would rule out indeterminacy. However, this observation comes with the caveat that values for the match elasticity below 0.5 have some support in the literature. For instance, Cooley and Quadrini (1999) argue that a low elasticity in the range of $\xi = 0.1$ is necessary to match labor market cyclicality. Using likelihood-based econometric methods, Lubik (2010) finds that there is, in fact, substantial probability mass on low values of ξ. We also note that the upper bound is increasing in the Nash bargaining parameter. But even if $\eta \rightarrow 1$, indeterminacy would not occur for the typical parameter choices in the literature. Suppose, however, that the unemployment rate were set to $\overline{u} = 0.06$. In this case, the upper bound increases to 0.816, which would imply indeterminacy for typical search elasticity choices. Clearly, the interpretation of the pool of searchers in the matching model matters for determinacy questions.

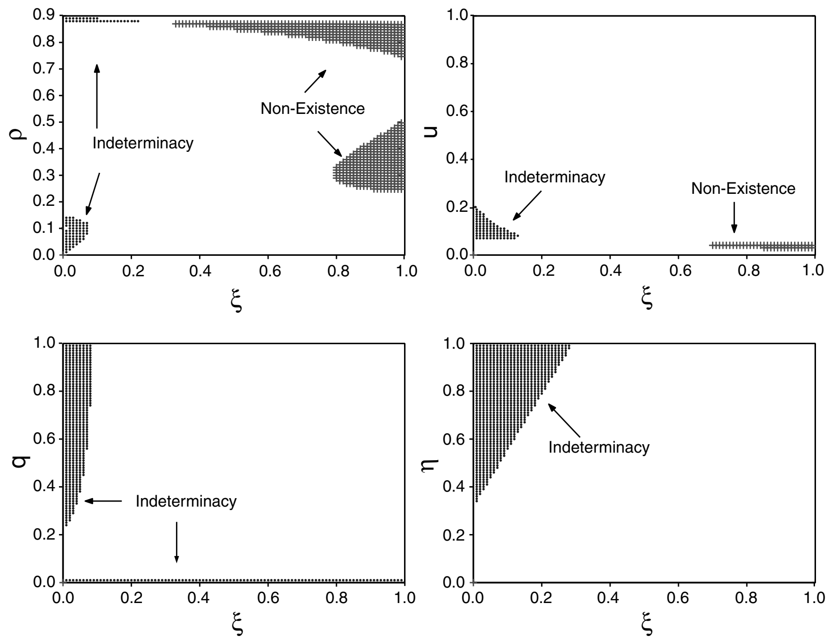
Intuitively, we can think about a sunspot equilibrium in the following way. Firms are willing to incur vacancy posting costs if they expect to recoup them through the proceeds from production net of wages and the savings on future hiring, as captured by the job-creation condition (17). The equilibrating mechanism is the behavior of the matching rate, $q(\theta)$. An increase in vacancy posting raises labor market tightness and lowers the probability that an individual firm is successful in finding an employee. This, in turn, raises effective hiring costs, $\kappa/q(\theta)$, which would have to be offset by higher expected returns. It is this externality, namely the fact that firms do not internalize the effect of their posting decisions on aggregate match probabilities, that is at the heart of the determinacy issue.[8]

Now suppose that a firm believes that future profits will be higher than is warranted by the fundamentals, such as the level of productivity. Beliefs of this kind can be triggered by sunspot shocks, as in the interpretation by Lubik and Schorfheide (2003). This belief would compel the firm to post more vacancies. If other firms were to do the same, aggregate tightness would increase and match probability would fall, raising effective hiring cost. What tends to rule out a sunspot equilibrium is that expected future benefits are not consistent with the higher posting costs. Consequently, rational firms do not act on sunspot beliefs. This argument breaks down in an environment where future benefits rise to accommodate higher current costs. The proposition

steady-state unemployment corresponds to a higher value of b and lower m. This can be interpreted as an implication of different labor market institutions.

[8] This has similar characteristics to the notion of an upward-sloping labor demand schedule in Farmer and Guo (1994). In their model, production exhibits constant returns to scale at the individual firm level, but increasing returns in the aggregate. An individual firm hiring more workers raises the marginal product of workers in the aggregate, thereby stimulating more labor demand. The job-creation condition can be thought of as a vacancy-demand curve.

Figure 1 Determinacy Regions



stipulates that indeterminacy arises when both the separation, ρ, and the match elasticity, ξ, are too small. When the former applies, the unemployment pool is small, while the latter makes new matches, and thereby future employment, highly elastic to vacancy postings. Consequently, the savings on future hiring costs react more than current effective costs, which helps validate sunspot beliefs.

A similar argument applies for the case of nonexistence of equilibrium. In general, nonexistence problems would arise for unemployment rates that are too low for given separation rates, in combination with excessively high match elasticities. In more technical terms, this combination makes the employment equation explosive. Any disturbance to a steady-state equilibrium would result in excessive job destruction (due to high separation rates) and matching that is inconsistent with the job-creation condition.

We now turn to the full model solution with risk-averse households ($\sigma > 0$). We compute the determinacy regions numerically for combinations of the match elasticity, ξ, and various other structural parameters. The results are presented in Figure 1, where we have plotted determinacy regions for different subsets of the parameter space. The parameters are calibrated at the benchmark values discussed above. In each panel we vary two parameters over their admissible range while keeping the other parameters at their benchmark values.

As a general conclusion, determinacy problems tend to arise when the match elasticity, ξ, is either too small or too big. For small ξ, the equilibrium is indeterminate when the job destruction rate, ρ, or the unemployment rate, u, is too small. This is related to the analytical condition found in Proposition 1. Furthermore, firm-matching rates, q, above 0.2 and a Nash parameter that puts more weight on workers also lead to multiplicity. No equilibrium exists for large ξ and either a small unemployment rate or ρ above 0.2. We also analyze the sensitivity of the regions with respect to σ (not reported). As $\sigma \to 0$, the indeterminacy regions expand. In particular, any q implies multiple equilibria when $\xi < 0.2$. In the limit the boundaries between regions are given in the proposition. As the household becomes more risk averse, however, regions of indeterminacy disappear entirely.

An interesting special case to consider is a calibration with the Hosios condition, where $\eta = \xi$. This can be represented by a 45-degree line in the lower-right panel of Figure 1. In the absence of outside information on the value of the bargaining parameter, η, the Hosios calibration is often chosen in the literature. In this case, indeterminacy and nonexistence are ruled out and become highly unlikely for other parameter combinations. For instance, equilibrium nonexistence requires a separation rate of $\rho = 0.81$. Moreover, if $\eta = \xi$, we can rule out indeterminacy in the case of $\sigma = 0$ because condition 1(b) of the proposition never holds. The equilibrium could still be nonexistent, but this would require very high separation rates. In principle, these could obtain when the model period is much longer than a quarter since eventually all workers turn over within a long enough time horizon.[9]

Interpreting these results in light of standard calibrations used in the literature, we would argue that indeterminacy and nonexistence do not present serious problems for the search and matching framework. Hence, it is unlikely that sunspot equilibria would be helpful in explaining labor market dynamics (as claimed in Hashimzade and Ortigueira [2005]). This is not to say that labor search and matching frameworks cannot support indeterminate equilibria. Mildly increasing returns to scale in the matching function (Giammarioli 2003) lead to widely expanded indeterminacy regions, while a New Keynesian model with search and matching frictions in the labor market has broader indeterminacy properties than the standard New Keynesian model (Zanetti 2006).

[9] Incidentally, the continuous-time version of this simple search and matching model always has a unique solution (see Shimer [2005]), as the separation-relevant time horizon is infinitesimally small. We are grateful to Andreas Hornstein for pointing this out.

4. CONCLUSION

We show in this article that for most plausible parameterizations the simple search and matching model does not suffer from determinacy problems. Specifically, we argue that it is unlikely that the model has multiple equilibria so that extraneous uncertainty, i.e., animal spirits, can cause business cycles. Parameterizations that lead to indeterminacy can be found, but they lie at the boundaries of the region that the empirical literature would consider plausible. We identify the match elasticity and the separate rate as crucial parameters in that respect.

These properties are obviously model specific, but our conclusions are likely robust to modifications such as endogenous job destruction. While the boundaries of the determinacy regions are likely to shift, the dynamic mechanism stays unaffected. The main caveat to our study is that our analysis applies to a local equilibrium in the neighborhood of the steady state. However, the underlying model is nonlinear and local results may therefore not adequately describe the global equilibrium properties. Naturally, this is a topic for further investigation. Moreover, researchers may actually be interested in the business cycle implications of indeterminacy that do not depend on policy rules or externalities. It appears plausible that actual labor market decisions are characterized to some extent by animal spirits. Further research should shed some light on this issue.

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