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Risk Sharing, Investment, and Incentives in the Neoclassical Growth Model

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The amount of risk sharing among households, regions, or countries is crucial in determining aggregate welfare. For example, pooling resources at the national level can help regions better deal with natural disasters like floods. Similarly, pooling resources with an insurance company can help individuals deal with shocks like a house fire or a car accident.

Capital accumulation and economic growth also are crucial in determining aggregate welfare. In particular, they determine the stock of wealth available for consumption and investment. Importantly, wealthier households, regions, or countries possess a buffer stock of precautionary assets, a form of self-insurance.

These two important factors in determining welfare have interesting interactions with one another. An important one is how insurance and savings substitute for each other. For example, individuals may want to save more when they do not have access to insurance than when they do because the extra savings can protect against the consequences of an uninsured shock. Therefore, capital accumulation and growth would be faster in an economy without perfect insurance than in one with perfect insurance.

This article explores the tradeoffs between insurance and growth in the neoclassical growth model with two agents and preference shocks. Most of the analysis reviews the full information version of the model, where there are no limits on insurance between the two agents, though there is still aggregate uncertainty that affects aggregate savings behavior. Private information is

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then added to the model to limit the ability to insure the two agents. This is a much harder problem, as has been observed in the literature, and only a partial characterization is provided.

Literature Review

Our article relates to the voluminous consumption/savings/capital accumulation literature on two levels. On one hand, there has been a growing literature focusing on the accumulation effects of demand side shocks in dynamic stochastic general equilibrium models, following the pioneering work of Baxter and King (1991) and Hall (1997). In general equilibrium models, demand side shocks (such as preference shocks to consumption demand) have a strong tendency to crowd out investment.[1] On the other hand, there is literature on the impact of inequality on capital accumulation. If preferences aggregate in the Gorman sense, the distribution of wealth does not affect the evolution of aggregate variables—see Chatterjee (1994) and Caselli and Ventura (2000). In our setting, preferences do not aggregate in that strong sense. Thus, distribution matters for aggregate savings and the corresponding dynamics of the aggregate stock of capital.[2]

The literature analyzing economic growth and private information is not as large, and the valuable contributions have relied on different simplifying assumptions to make the analysis tractable. This article is related to those articles because we are interested in understanding when information is (more) important to implement the full information allocation. However, we solve the full information model to obtain the full information allocation and characterize only the incentives to misreport the shocks under that allocation.

Pioneering contributions in the literature on constrained efficient allocations with private information abstracted from capital accumulation, as the main goal was to study wealth distribution. In a pure exchange economy setting, Green (1987) and Atkeson and Lucas (1992) show that (constrained) efficient allocations, independent of the feasibility technologies, display extreme levels of "immiserization": The expected utility level of (almost) every agent in the economy converges to the lower bound with probability one. This result is also present in Thomas and Worrall (1990). Then, in an early contribution that includes capital accumulation, Marcet and Marimon (1992) examine a two-agent model where a *risk-neutral* investor with *unlimited resources* invests in the technology of a risk-averse producer whose output is subject to privately observed productivity shocks. They show that the full information investment policy can be implemented in the private information

[1] See Wen (2006) for an overview and references therein.

[2] See Lucas and Stokey (1984) for a general early discussion and, more recently, Sorger (2002).

environment. That is, in their setting, a risk-neutral investor can make the risk-averse entrepreneur follow the full information investment policy and allocate his consumption conditional on output realizations. Thus, they find that growth levels are as high as with perfect information. The key simplification in this article is that the second agent in the economy is risk-neutral with unlimited resources.

Khan and Ravikumar (2001) extend Marcet and Marimon (1992) to impose a period-by-period feasibility constraint and endogenous growth. In particular, they examine the impact of incomplete risk sharing on growth and welfare in the context of the AK model. The source of market incompleteness is private information since household technologies are subject to idiosyncratic productivity shocks not observable by others. Risk sharing between households occurs through contracts with intermediaries. This sort of incomplete risk sharing tends to reduce the rate of growth relative to the complete risk-sharing benchmark. However, "numerical examples indicate that, on average, the growth and welfare effects on incomplete risk sharing are likely to be small." One key simplification in this case is that the allocation solved is not necessarily the best incentive-compatible allocation.

Recently, Greenwood, Sanchez, and Wang (2010a) embedded the costly state verification framework into the standard growth model.[3] The relationship between the firm and lender is modeled as a static contract. In the economy in which information is too costly, undeserving firms are overfinanced and deserving ones are underfunded. A reduction in the cost of information leads to more capital accumulation and a redirection of funds away from unproductive firms toward productive ones. Greenwood, Sanchez, and Wang (2010b) show that this mechanism has quantitative significance to explain cross-country differences in capital-to-income ratios and total factor productivity.

Other studies use similar models for other purposes. Espino (2005) studies a neoclassical growth model that includes a discrete number of agents, like the one presented in this article. However, he uses the economy with private information about the preference shock to analyze the validity of Ramsey's conjecture about the long-run allocation of an economy in which agents are heterogeneous in their discount factor. Clementi, Cooley, and Giannatale (2010) study a repeated bilateral exchange model with hidden action, along the lines of Spear and Srivastava (1987) and Wang (1997), that includes capital accumulation. The two agents in the economy are a risk-neutral investor and a risk-averse entrepreneur. They show that the incentive scheme chosen by the investor provides a rationale for firm decline.

This article is organized as follows: Section 1 presents the physical environment and the planner's problem, and derives the optimal allocation. Section

[3] See also Khan (2001) and Chakraborty and Lahiri (2007).

2 describes the calibration and the numerical solution of the full information allocation. Section 3 studies in which cases the full information allocation would be incentive compatible in an economy with private information. Section 4 offers concluding remarks.

1. MODEL

Environment

There is a constant returns to scale aggregate technology to produce the unique consumption good that is represented by a standard neoclassical production function, $F(K, L)$, where K is the current stock of capital and L denotes units of labor. There are two agents in the economy, $h = 1, 2$. Agent h is endowed with one unit of time each period and does not value leisure, i.e., the time endowment is supplied inelastically in the labor market. The initial stock of capital at date 0 is denoted by $K_0 > 0$. Capital depreciates at the rate $\delta \in (0, 1)$.

At the beginning of date t, agent 1 faces an idiosyncratic preference shock $s_t \in S_t = \{s_L, s_H\}$, where $s_H > s_L$. This shock is assumed to be i.i.d. across time, where $\pi_i > 0$ is the probability of s_i, $i = L, H$. Notice that s_t is also the aggregate preference shock at date t. The aggregate history of shocks from date 0 to date t, denoted $s^t = (s_0, ..., s_t)$, has probability at date 0 given by $\pi(s^t) = \pi(s_0)...\pi(s_t)$.

Given a consumption plan $\{c_{1,t}\}_{t=0}^{\infty}$ such that $c_{1,t} : S^t \to \mathbb{R}_+$, agent 1's state-dependent preferences are represented by

$$\mathcal{U}_1(c_1) = E\left\{\sum_{t=0}^{\infty} \beta^t \; u_1(s_{1,t}, c_{1,t})\right\} = \sum_{t=0}^{\infty}\sum_{s^t} \beta^t \pi(s^t) \; u_1(s_t, c_1(s^t)),$$

where $u_1 : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, strictly concave, and twice differentiable, $\lim_{c_t \to 0} u'(c_t) = +\infty$, and $\beta \in (0, 1)$. Similarly, given $\{c_{2,t}\}_{t=0}^{\infty}$, agent 2's preferences are represented by

$$\mathcal{U}_2(c_2) = E\left\{\sum_{t=0}^{\infty} \beta^t u_2(c_{2,t})\right\} = \sum_{t=0}^{\infty}\sum_{s^t} \beta^t \pi(s^t) u_2(c_2(s^t)).$$

Planner's Problem

Consider the problem of a fictitious planner choosing the best feasible allocation. Let $K' = \{K_{t+1}\}_{t=0}^{\infty}$ be an *investment plan* that every period allocates next period's capital for all t. Similarly, let $C = \{C_t\}_{t=0}^{\infty}$ be a *consumption plan* where $C_t = (c_{1t}, c_{2t})$. Given K_0, a sequential allocation (C, K') is feasible if, for all s^t,

$$K_t(s^t) + c_{1t}(s^t) + c_{2t}(s^t) \leq F(K_t(s^{t-1}), 1) + (1 - \delta)K_t(s^{t-1}).$$

We will assume throughout the article that the production function F is Cobb-Douglas with exponent γ.

The Pareto-optimal allocation in this economy is a feasible allocation such that there is no other feasible allocation that provides all the agents the same or more lifetime utility. One reason to be interested in these allocations is that, under certain conditions, they are equivalent to competitive equilibrium allocations. Under our assumptions, Pareto-optimal allocations can be obtained by solving the following problem:

$$\max_{(C,K')} \alpha \mathcal{U}_1(c_1) + (1-\alpha)\mathcal{U}_2(c_2)$$

subject to

$$K(s^t) + c_1(s^t) + c_2(s^t) \leq F(K(s^{t-1}), 1) + (1-\delta)K(s^{t-1}), \forall s^t,$$

where K_0 is given and $\alpha \in [0, 1]$ is the weight that the planner assigned to agent 1—referred to hereafter as Pareto weight. Notice that different values of α characterize different points in the Pareto frontier. Later, we will consider a different allocation varying the value of α.

To characterize the problem further, it is simpler to consider the methods developed by Lucas and Stokey (1984) to solve for Pareto-optimal allocations in growing economies populated with many consumers. It is actually simple to adapt their method to analyze this economy. The idea is to make next period welfare weights conditional on the current shock.[4]

The planner's recursive problem is a fixed point, V, of the function equation

$$\begin{aligned} V(k,\overline{\alpha}) \;=\; & \max_{c,k',w} \alpha \left\{\pi_L \left[u_1(s_L, c_{1L}) + \beta w_{1L}\right] + \pi_H \left[u_1(s_H, c_{1H}) + \beta w_{1H}\right]\right\} + \\ & (1-\alpha)\left\{\pi_L \left[u_2(c_{2L}) + \beta w_{2L}\right] + \pi_H \left[u_2(c_{2H}) + \beta w_{2H}\right]\right\} \qquad (1) \end{aligned}$$

subject to

$$f(k) + (1-\delta)k \;\geq\; k'_L + c_{1L} + c_{2L}, \qquad (2)$$

$$f(k) + (1-\delta)k \;\geq\; k'_H + c_{1H} + c_{2H}, \qquad (3)$$

$$\min_{\overline{\alpha}'_L} V(k'_L, \overline{\alpha}'_L) - \alpha'_L w_{1L} - (1-\alpha'_L) w_{2L} \;\geq\; 0, \qquad (4)$$

$$\min_{\overline{\alpha}'_H} V(k'_H, \overline{\alpha}'_H) - \alpha'_H w_{1H} - (1-\alpha'_H) w_{2H} \;\geq\; 0, \qquad (5)$$

where $\overline{\alpha} = \{\alpha, 1-\alpha\}$ and w are the from-tomorrow-on utilities. The idea in (1)–(5) is to represent the problem of choosing an optimal allocation for a given stock of capital k and a vector of Pareto weights $(\alpha, 1-\alpha)$ as one of choosing a feasible current period allocation of consumption $c = \{c_{1L}, c_{1H}, c_{2L}, c_{2H}\}$ and capital goods $k' = \{k'_L, k'_H\}$, and a vector of from-tomorrow-on utilities $w =$

[4] See Beker and Espino (2011) for a discussion about the implementation and the corresponding technical details.

$\{w_{1L}, w_{1H}, w_{2L}, w_{2H}\}$, subject to the constraint that these utilities be attainable given the capital accumulation decision, as guaranteed by constraints (4)–(5). As in Lucas and Stokey (1984), the weights $\{\overline{\alpha}'_L, \overline{\alpha}'_H\}$ that attain the minimum in (4) and (5) will be the new weights used in selecting tomorrow's allocation, and so on, ad infinitum.

Characterization

Assume preferences are represented by

$$u_1(s, c) = s\frac{c^{1-\sigma}}{1-\sigma} \text{ and } u_2(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

The first-order conditions (FOC) for consumption are

$$\begin{aligned} \alpha\pi_L s_L (c_{1L})^{-\sigma} &= \lambda_L, \\ \alpha\pi_H s_H (c_{1H})^{-\sigma} &= \lambda_H, \\ (1-\alpha)\pi_L (c_{2L})^{-\sigma} &= \lambda_L, \\ (1-\alpha)\pi_H (c_{2H})^{-\sigma} &= \lambda_H, \end{aligned}$$

where λ_i is the Lagrange multiplier in the resource constraints in state $i = L, H$. From these equations it is simple to obtain that the consumption of each agent will be a share of the aggregate consumption, $\mathbf{C}_i$,

$$\begin{aligned} c_{1L} &= \frac{(\alpha s_L)^{1/\sigma}}{(\alpha s_L)^{1/\sigma} + (1-\alpha)^{1/\sigma}}\mathbf{C}_L, \\ c_{1H} &= \frac{(\alpha s_H)^{1/\sigma}}{(\alpha s_H)^{1/\sigma} + (1-\alpha)^{1/\sigma}}\mathbf{C}_H, \\ c_{2L} &= \frac{(1-\alpha)^{1/\sigma}}{(\alpha s_L)^{1/\sigma} + (1-\alpha)^{1/\sigma}}\mathbf{C}_L, \\ c_{2H} &= \frac{(1-\alpha)^{1/\sigma}}{(\alpha s_H)^{1/\sigma} + (1-\alpha)^{1/\sigma}}\mathbf{C}_H. \end{aligned} \tag{6}$$

The FOC with respect to w are

$$\begin{aligned} \alpha\pi_L\beta &= \mu_L\alpha'_L, \\ \alpha\pi_H\beta &= \mu_H\alpha'_H, \\ (1-\alpha)\pi_L\beta &= \mu_L(1-\alpha'_L), \\ (1-\alpha)\pi_H\beta &= \mu_H(1-\alpha'_H). \end{aligned}$$

These imply that

$$\alpha\pi_L\beta + (1-\alpha)\pi_L\beta = \mu_L\alpha'_L + \mu_L(1-\alpha'_L),$$

and therefore $\pi_L\beta = \mu_L$ and $\pi_H\beta = \mu_H$. Using the FOC with respect to w again, these two conditions imply $\alpha = \alpha'_L = \alpha'_H$. Thus, the Pareto weights will be constant in this problem.

Using the fact that individual consumption is a share of aggregate consumption and that Pareto weights are constant, this problem can be rewritten as one solving for the consumption (or capital accumulation) of a representative consumer with aggregate preference shocks. In that case, the state-dependent utility of the representative consumer, u_R, would be

$$u_R(s, \mathbf{C}) = \left(\left(s\alpha\right)^{1/\sigma} + (1-\alpha)^{1/\sigma}\right)^{\sigma} \frac{\mathbf{C}^{1-\sigma}}{1-\sigma}.$$

Notice here that the level of the shock depends not just on the size of s, but also on α. This representation is useful to understand that the optimal investment decision is affected by the realization of the preference shock and the distributional parameter α. When s is larger, the representative agent prefers to increase consumption today and decrease investment. Given the same shock, the size of the drop in investment depends on the Pareto weight of the agent that received the shock.

The FOC with respect to capital accumulation are

$$\begin{aligned}\lambda_L &= \mu_L \frac{\partial V(k'_L, \overline{\alpha}'_L)}{\partial k'_L},\\ \lambda_H &= \mu_H \frac{\partial V(k'_H, \overline{\alpha}'_H)}{\partial k'_H}.\end{aligned}$$

An application of the envelope conditions makes these conditions imply the standard Euler equations determining capital accumulation,

$$\begin{aligned}1 &= (F'(k'_L) + (1-\delta))\beta \frac{\left(\pi_L s_L (c'_{1L})^{-\sigma} + \pi_H s_H (c'_{1H})^{-\sigma}\right)}{s_L\,(c_{1L})^{-\sigma}},\\ 1 &= (F'(k'_H) + (1-\delta))\beta \frac{\left(\pi_L s_L (c'_{1L})^{-\sigma} + \pi_H s_H (c'_{1H})^{-\sigma}\right)}{s_H\,(c_{1H})^{-\sigma}}.\end{aligned}$$

2. NUMERICAL SOLUTION

This model can be solved in the computer once the values of the parameters are determined. Most of the parameters are standard in the neoclassical growth model and take standard values. Others, such as the size of the preference shock and the probability of occurrence, were chosen only to illustrate the behavior of the model. In particular, a high preference shock happens on average every 6.7 years, but it is large enough to demand a significant amount of resources. Think, for example, that a country in an economic union requires help or assistance on average every 6.7 years. Table 1 presents the values for all the parameters of the model.

The right-hand side of (1)–(5) defines a contraction. The computation is based on value function iteration as follows. Guess a function V. Then solve for $\max_{c,w',k'}$ using V, the FOC described above, and numerical maximization.

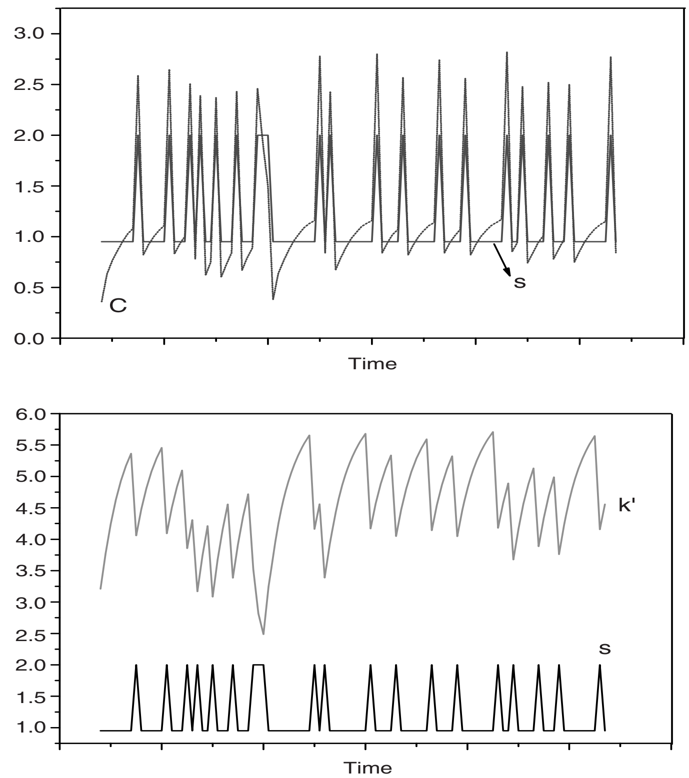
Table 1 Parameter Values

	Parameter	Value
γ	Exponent of capital in production function	0.30
δ	Depreciation rate of capital	0.07
β	Discount factor	0.97
σ	Relative risk aversion	0.50
s_L	Low value of the preference shock	0.95
s_H	High value of the preference shock	1.05 and 2.00
π_L	Probability of low value of the preference shock	0.85
π_H	Probability of high value of the preference shock	0.15

With this solution, construct a new function V' and restart the maximization unless V' is sufficiently close to V. Now we discuss the results using the parameters in Table 1 with $s_H = 2$ and Pareto weights $\{0.75, 0.25\}$. Figure 1 presents time series for aggregate consumption and capital accumulation in the steady state of this economy. On the top panel that aggregate consumption jumps after a preference shock and then returns slowly to a relatively constant value until a new shock hits. As a consequence, capital accumulation drops after a high preference shock to accommodate larger aggregate consumption, as shown on the top panel. The effect of this change on the incentives to misreport a shock—if it would be unobservable—is discussed in the next section. The distribution of consumption among agents is determined by equations (6), i.e., agent 1's share of aggregate consumption increases with the value of the shock. More on this later.

Figure 2 depicts the stationary distribution of the main variables for the same example analyzed in Figure 1. The top left panel shows that 15 percent of the time there is a large preference shock equal to 2 and most of the time (85 percent) a low shock equal to 0.95. The top right panel presents the stationary distribution of capital. It is somehow surprising that very different values (e.g., 3 and 6) are reached with positive probability. Most of its mass is accumulated on the higher values, however. Those correspond to periods with low preference shocks. The lowest values of capital correspond to periods of several consecutive high preference shocks. Something similar happens with c_2, on the bottom right panel. A priori, these properties could have been expected since k' and c_2 are the two sources to finance transfers to agent 1 after a high preference shock. The distribution of c_1, presented on the bottom left panel, has most of the mass around lower values and some mass at higher values. The highest values correspond to a high preference shock hitting the economy after a long period of low shocks.

Figure 1 Consumption and Capital Paths in the Stationary Distribution



Notes: These histograms were computed from time series data of these variables for 5,000 periods after deleting the first 500 realizations.

3. THE ROLE OF INFORMATION

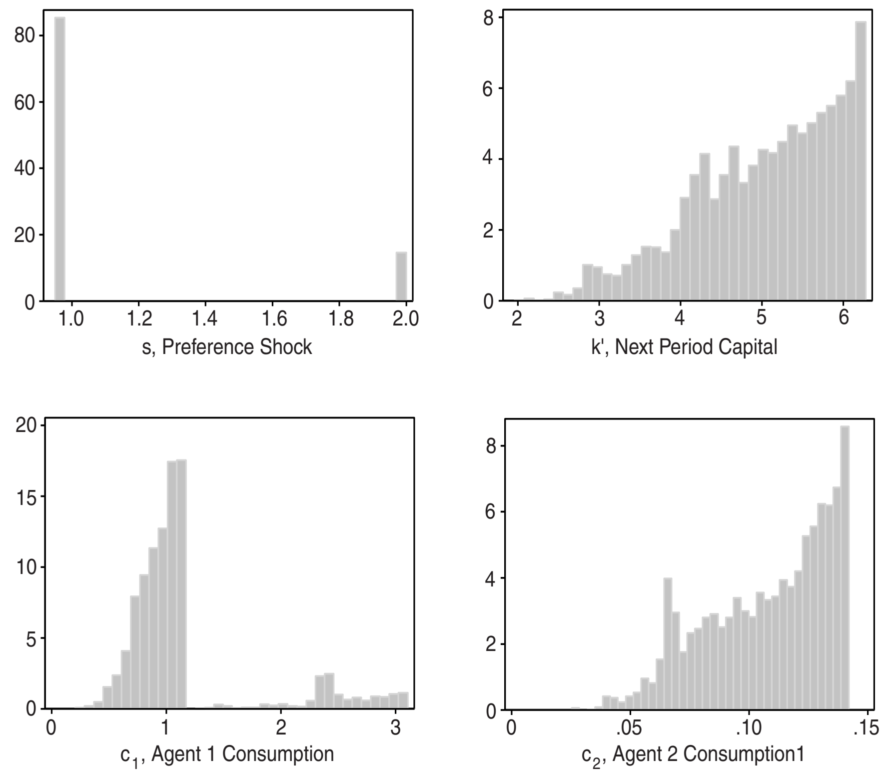
This section investigates the incentives to misreport preference shocks by agent 1 whenever the full information allocation described above is the target to be implemented. To do so, consider the value of the following (implicit) incentive compatibility constraints:

$$icc_{HL} = s_H u(c_{1H}) + \beta w_{1H} - [s_H u(c_{1L}) + \beta w_{1L}], \quad (7)$$

$$icc_{LH} = s_L u(c_{1L}) + \beta w_{1L} - [s_L u(c_{1H}) + \beta w_{1H}]. \quad (8)$$

The interpretation of these variables is very important for the analysis hereafter. If the variable icc_{HL} is positive, it means that when the state H realizes, agent 1 would prefer truthfully reporting a high preference shock and obtaining $\{c_{1H}, w_{1H}\}$ instead of misreporting it and receiving $\{c_{1L}, w_{1L}\}$. Similarly, a

Figure 2 Stationary Distribution, Main Variables

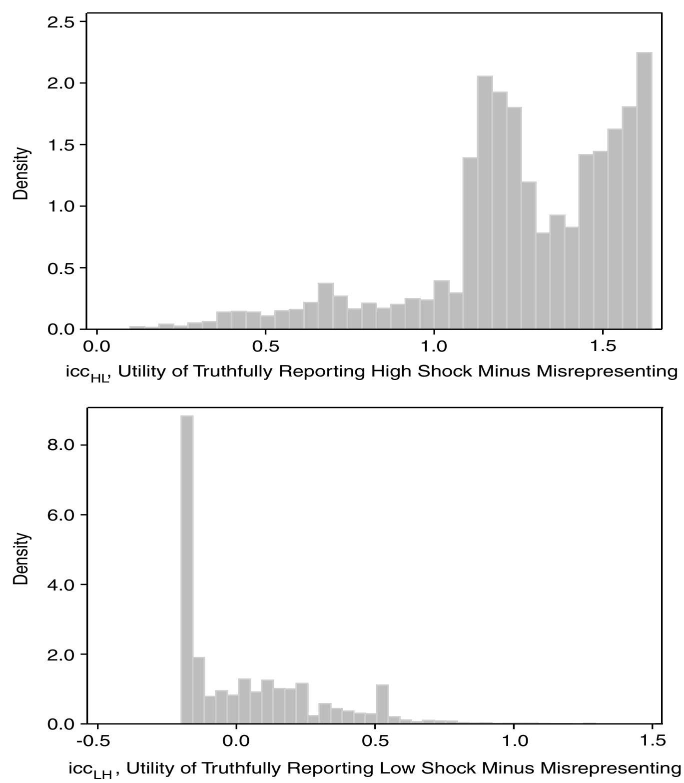


Notes: These histograms were computed from time series data of these variables for 5,000 periods after deleting the first 500 realizations.

negative value of icc_{LH} means that agent 1 would prefer misreporting a high preference shock and obtaining $\{c_{1H}, w_{1H}\}$ to truthfully reporting a low shock and receiving $\{c_{1L}, w_{1L}\}$.

Since $c_{1H} > c_{1L}$, one may expect that there is no incentive to report the low shock when the high shock was actually realized, i.e., a positive value of icc_{HL}. This is actually what happens in the stationary distribution, as shown on the top panel of Figure 3. In contrast, agent 1 may be tempted to misreport a high preference shock to obtain higher consumption. Remember that this would imply that $icc_{LH} < 0$. This does not need to always be the case, however. Since k' is lower after a high preference shock, agent 1's prospects worsen after a high preference shock. Thus, it will be a race between more consumption today, $c_{1H} > c_{1L}$, and less future consumption, $w_{1L} > w_{1H}$. The results for

Figure 3 Incentive Compatibility in the Stationary Distribution

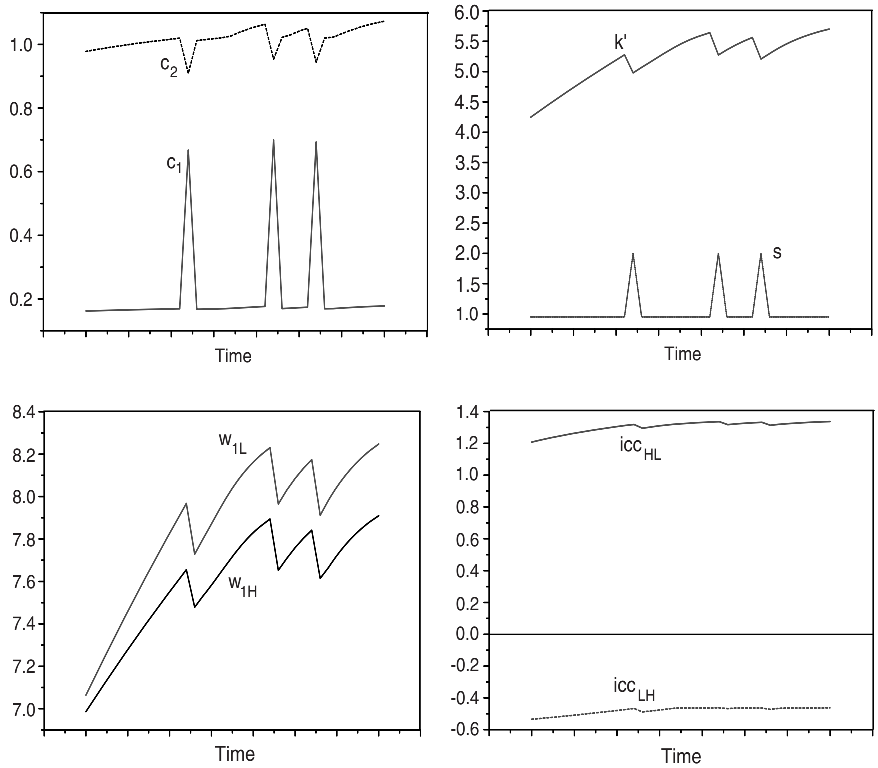


Notes: These histograms were computed from time series data of these variables for 5,000 periods after deleting the first 500 realizations.

the example described above are presented in the bottom panel of Figure 3. There, icc_{LH} is negative more than 80 percent of the time but positive in some instances. This means that in all such instances, the drop in from-tomorrow-on utilities caused by reporting a high preference shock is enough to compensate for the difference in current consumption. What determines whether icc_{LH} is negative or positive will be studied next by analyzing different examples.

The next two examples capture the role of the size of redistribution versus disinvestment. The first example is presented in Figure 4. This is the same example in all the previous figures, but the difference is that the Pareto weight of agent 1 is only 0.33 (instead of 0.75) and the weight of agent 2 is 0.67. This implies that agent 2's consumption is larger, as shown in the top left panel. The top right panel presents the behavior of capital accumulation. Notice that

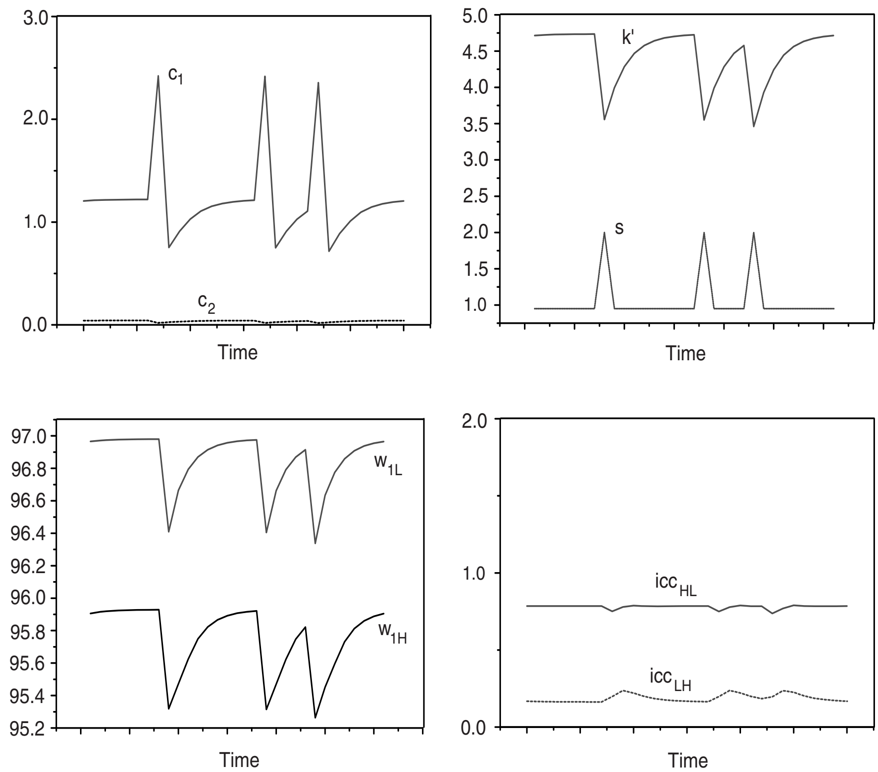
Figure 4 Paths with Large Redistribution of Aggregate Consumption



Notes: In this economy, the weights on agents 1 and 2 are 0.33 and 0.67, respectively. The time series data in all four graphs correspond to the initial 35 periods after the economy is started with a stock of capital smaller than the steady-state level.

the time series in the graphs correspond to the transition toward a higher level of capital. From these two figures it is clear that a nontrivial part of the rise in agent 1's consumption after a high preference shock comes from redistribution of consumption across agents. As a consequence, the promised utilities from next period on are not that different after a report of high or low preference shock, as shown in the bottom left panel. In turn, this implies that icc_{LH} is always negative, as presented in the right bottom panel. Thus, this is an example in which the full information allocation would not be implementable under private information: After a low preference shock, agent 1 would prefer to falsely report a high preference shock.

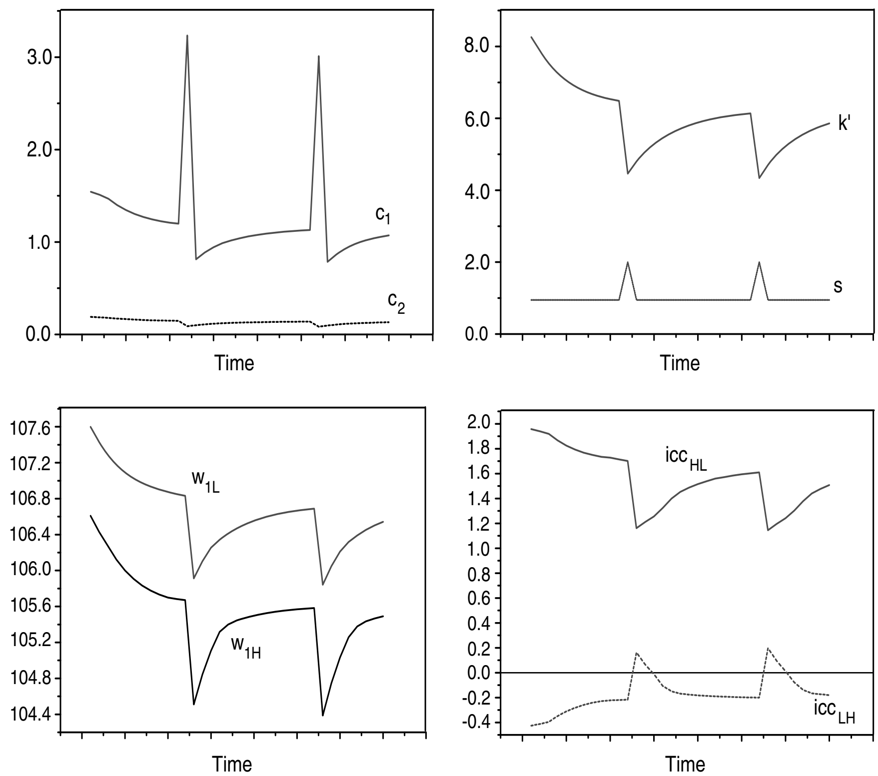
Figure 5 Paths with Large Variation in Investment



Notes: In this economy, the weights on agents 1 and 2 are 0.85 and 0.15, respectively. The time series artificial data in all four graphs correspond to 30 periods created after the steady-state level of capital is reached.

Now consider the example presented in Figure 5. Here, the behavior of the same series is presented for an economy in which the Pareto weight of agent 1 is 0.85 and the steady-state distribution of capital is reached. This implies that agent 1's consumption is much larger than that of agent 2, as shown in the top left panel. As a consequence, capital accumulation must vary significantly to provide more consumption to agent 1 after the realization of a high preference shock. This is shown in the top right panel. Therefore, as presented in the bottom left panel, the difference in from-tomorrow-on utilities associated with low and high preference shocks is large. Thus, both incentive compatibility constraints are positive in the stationary distribution

Figure 6 Incentive Compatibility and Capital Accumulation



Notes: In this economy, the weights on agents 1 and 2 are 0.75 and 0.25, respectively. The time series data in all four graphs correspond to the initial 35 periods after the economy is started with a stock of capital larger than the steady-state level.

of this economy (see bottom right panel), and the full information allocation would be implementable under private information.

The previous two examples are useful to understand that the relative importance of the agent who privately observes the shock matters for the role of private information. When this agent is more important, her share of aggregate consumption is larger, and the rise of that agent's consumption after a shock comes mainly from disinvestment. This makes misreporting a high preference shock too costly in terms of her own future consumption, and hence the full information allocation is implementable under private information. Thus, the *size* of disinvestment, determined by the importance of the agent with the preference shock, matters for the provision of incentives under private information. This *suggests* that in a fully specified model with private information, the planner would like to increase the Pareto weight of the agent with private information to reduce the incidence of this friction.

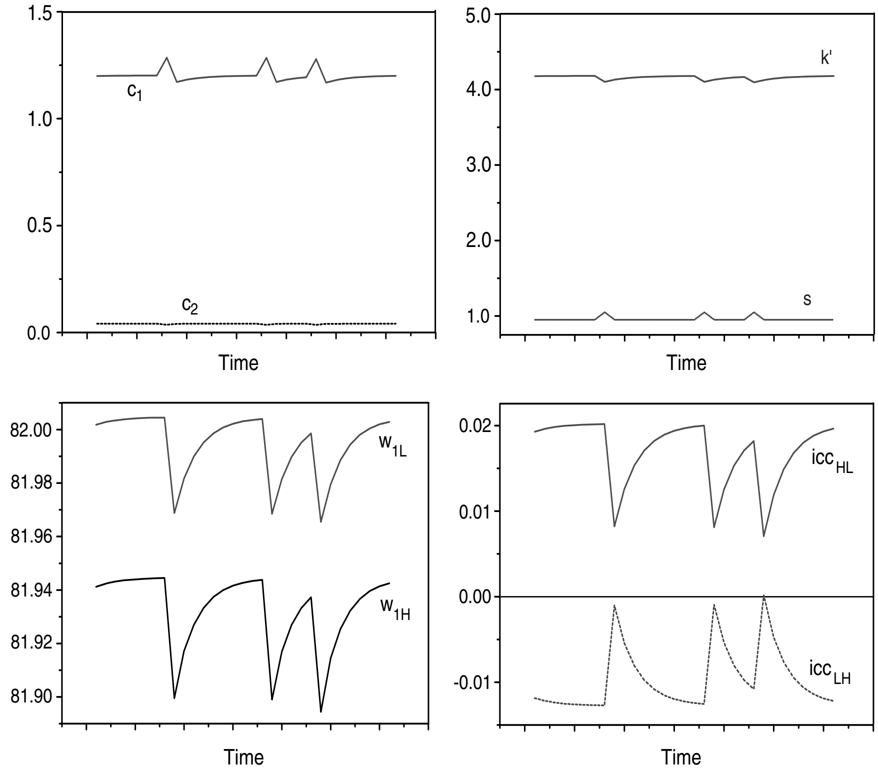
The next example illustrates the role of the outlook for economic growth at the time of disinvestment in preventing misrepresentation of preference shocks. First, consider the example in Figure 6. It displays the transition to the steady state from a larger stock of capital. The weights of agents 1 and 2 are 0.75 and 0.25, respectively. Initially, consumption, capital, and from-tomorrow-on utilities decrease. During this initial phase, while capital is large and decreasing, icc_{LH} is negative and increasing. This means that when there is extra capital in the economy, as compared to the stationary distribution, the optimal drop in capital that a high preference shock would require (and its corresponding drop in promised utility) is not large enough to provide incentives to make the report of that shock incentive compatible. Eventually, a high preference shock hits the economy, the consumption of agent 1 jumps, and capital drops significantly. Now, the economy is expected to grow in the coming years, which implies that another high preference shock would hurt both agents more. Therefore, reporting a high preference shock becomes incentive compatible for a few years, until the stock of capital reaches a higher level. The same story occurs again in a few years, when a high preference shock hits the economy again. Thus, this example illustrates the interaction of growth and information. Misrepresentation of preference shocks is more costly if the economy is expected to grow. This finding suggests that a planner solving for the best incentive-compatible allocation would delay growth to facilitate the provision of incentives.

The last example confirms the importance of the size of disinvestment and the outlook for economic growth. Consider the time series artificial data presented in Figure 7. The Pareto weight for agent 1 is larger than in previous examples, 0.85, but the value of the high preference shock is smaller, $s_H = 1.05$. First, notice that this example confirms the result in the previous figure: It is easier to provide incentives (icc_{LH} is larger) when the economy is expected to grow. However, in this case, icc_{LH} is never greater than zero. Notice that this happens despite agent 1's weight being larger than in all other examples. The key difference is that the shock is not that large. Thus, the size of the drop in capital accumulation is not very relevant, and therefore the difference between w_{1L} and w_{2L} is small.

4. CONCLUSIONS

This article studies the interaction between growth and risk sharing. First, it answers how investment is affected by insurance needs. A stochastic growth model with two agents and preference shocks is used to answer this question. Only one of the agents (or groups, regions, countries) is affected by this shock,

Figure 7 Paths for the Model with Small Shocks



Notes: In this economy, the weights on agents 1 and 2 are 0.85 and 0.15, respectively. The time series artificial data in all four graphs correspond to 30 periods created after the steady-state level of capital is reached.

which basically increases the need of consumption for this agent. When both agents are risk-averse, the socially optimal response to this shock requires both decreasing the consumption of other agents and decreasing capital accumulation. Thus, the occurrence of this shock slows down the convergence toward the stationary distribution of capital.

Then, we analyze if the best path of capital accumulation and consumption allocation is implementable if needs are privately observed by the agents. That is, if the shocks are privately observed by individuals, do they have incentive to misrepresent? The value of the incentive compatibility constraints implied by the full information allocation is used to answer this question. Because investment drops when an agent reports a high preference shock, the prospects of all agents deteriorate after such a report. This may be enough to prevent misreporting. The size of disinvestment after the report of a high preference shock and the outlook for economic growth at the time of disinvestment are important to induce individuals to report a low realization of the preference shock truthfully. This analysis *suggests* that in a fully specified model with private information, the best incentive compatible allocation would tend to hurt growth, by decreasing investment, and increase inequality, by augmenting the share of consumption of the agent with private information. Of course, this is only a conjecture. Solving for the constrained-efficient allocation in this environment is necessary to verify the validity of this conjecture. This is left for future research.

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