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On the Benefits of GDP-Indexed Government Debt: Lessons from a Model of Sovereign Defaults

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Whether governments should issue GDP-indexed sovereign debt—that promise payments that are a function of the gross domestic product (GDP)—continues to be the subject of policy debates. On the one hand, several studies highlight possible benefits from tying sovereign debt obligations to domestic GDP.[1] One benefit from GDP-indexation is that issuing debt that promises lower payments when GDP takes low values may facilitate the financing of automatic stabilizers (such as an increase in unemployment benefits during economic downturns) and countercyclical fiscal policy. Another benefit is that GDP indexation could diminish the likelihood of fiscal crises for governments that face a countercyclical borrowing cost (in part because of a countercyclical default risk). Kamstra and Shiller (2010) argue that GDP indexation would help investors who want exposure to income growth (for instance, to protect relative standards of living in retirement) and protection against inflation.

On the other hand, there are several difficulties in the implementation of the basic idea described in the previous paragraph. First, GDP-indexed bonds may introduce moral hazard problems by weakening the government's incentives to implement growth-promoting policies (see, for instance, Krugman [1988]). Second, GDP may not be easily verifiable. This is in part because the

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[1] See, for instance, Shiller (1993), Borensztein and Mauro (2004), Borensztein et al. (2004), Griffith-Jones and Sharma (2005), and the references therein.

government could manipulate the GDP calculation (however, reporting lower GDP figures may imply a political cost). Moreover, even without manipulation, final GDP data are available with a significant lag.[2] This could force a government to make a high payment during a low GDP period because the previous year GDP was high (problems created by lags in GDP statistics could be mitigated by provisions on the government's accounts; see United Nations [2006]).[3] Third, gains from indexing sovereign debt to GDP may be limited because domestic GDP is not the only determinant of default risk and the government's borrowing cost (think, for instance, about contagion, shocks to the investors' risk aversion, political shocks, etc.; see Tomz and Wright [2007]). Perhaps because of the implementation difficulties described above, the majority of sovereign debt is not GDP indexed. However, past experiences show that issuing GDP-indexed debt is feasible. For instance, Argentina issued GDP warrants in 2005, during a period of renewed interest in these contracts (see United Nations [2006]). The 2012 debt restructuring in Greece also included the issuance of bonds carrying detachable GDP warrants.[4]

This article contributes to the debate on GDP-indexed sovereign debt by discussing the effects of using this debt contract. We study a model in which the government faces a countercyclical borrowing cost because of a countercyclical default risk. We use this model to discuss the effects of introducing GDP-indexed bonds.

We introduce income-indexed sovereign bonds into the equilibrium default model studied by Aguiar and Gopinath (2006) and Arellano (2008), who extend the framework proposed by Eaton and Gersovitz (1981) to analyze its quantitative performance. We study a small open economy that receives a stochastic endowment stream of a single tradable good. The government's objective is to maximize the expected utility of a representative private agent. Each period, the government makes two decisions. First, it decides whether to default on previously issued debt. Second, it decides how much to borrow or save. The cost of defaulting is given by an endowment loss and temporary exclusion from capital markets. We study two versions of this model. First, we assume that the government issues one-period bonds that promise a non-contingent payment. Second, we assume the government can issue a one-period income-indexed bond that promises a payment function of next-period

[2] For instance, payments for the GDP warrants issued by Argentina during its 2005 debt restructuring are made effective with a one-year lag.

[3] These problems could be addressed by indexing debt contracts to variables that are correlated to GDP and that the government cannot control (such as commodity prices or trading partners' growth rates; see Caballero [2002]).

[4] Other experiences with GDP indexation include various "Value Recovery Rights" indexed to GDP issued by Bosnia and Herzegovina, Bulgaria, Costa Rica, Nigeria, and Venezuela in the early 1990s as part of the Brady bonds restructuring (Sandleris, Sapriza, and Taddei 2011). For instance, Bulgaria issued, in 1994, bonds with a potential premium if Bulgaria's GDP exceeded 125 percent of its 1993 level.

income. In both cases, bonds are priced in a competitive market inhabited by risk-neutral investors.

We solve the model using the calibration in Arellano (2008), which is based on an economy facing significant default risk: Argentina before its 2001 default. The ex-ante welfare gain from the introduction of income-indexed bonds when there is no initial debt is equivalent to an increase of 0.5 percent of consumption. Introducing income-indexed bonds results in welfare gains because it allows the government to:

1. *Eliminate defaults.* In the model, debt and income are the only determinants of default. With income-indexed bonds, the government makes a different payment promise for each level of next-period income, which means that there is no uncertainty about whether a government promise will be paid. Then, lenders would never pay for a payment promise on which they know the government would default and a bond making such a promise is not traded. In contrast, with non-contingent bonds, when the government borrows it promises the same payment for all next-period income levels. The government defaults in the next period at income levels that are sufficiently low.

2. *Increase its indebtedness from 4 percent to 18 percent of mean income.* The government is assumed to be eager to borrow (it discounts future consumption at a rate higher than the risk-free interest rate). With indexed bonds, the government can bring forward resources from future high-income states without increasing the default probability in low-income states (the cost of defaulting is assumed to be lower in low-income states). In contrast, with non-contingent bonds, the future resources the government can bring forward are limited by default risk. If the government issued a non-contingent bond equivalent to 18 percent of mean income, for most current income levels the revenue it would collect from that debt issuance would be even smaller than the revenue it would collect from issuing debt equivalent to 4 percent of mean income. The reason is that lenders would internalize that, at a debt of 18 percent of mean income, there is a significant mass of income realization states at which the government would default, and lenders would thus offer to buy those bonds at a significant discount.

3. *Reduce the ratio of standard deviations of consumption relative to income from 1.07 to 0.79.* With income-indexed bonds, the government chooses to smooth consumption by buying claims that pay in states with lower income and borrowing against states with higher income. Furthermore, the borrowing cost is constant because the government does not pay a default premium. Thus, the government chooses to borrow more when income is lower. In contrast, with non-contingent bonds, the borrowing cost is countercyclical. In bad times, the cost

> of defaulting is assumed to be lower and, therefore, the probability of default and the cost of borrowing are higher. Consequently, optimal borrowing becomes procyclical: In bad times, since the cost of borrowing is higher, the government chooses to finance more of its debt service obligations by lowering consumption instead of borrowing.[5]

It should be noted that our analysis does not consider the implementation difficulties of GDP-indexed bonds that we mentioned above: We assume that the government cannot affect GDP growth, that bond payments can be determined using current income, and that income is the only determinant of sovereign defaults. Thus, the gains from introducing GDP-indexed bonds measured in this article should be seen as an upper bound. Relaxing the simplifying assumptions that limit our analysis increases the dimensionality of the model's state space and thus augments the computation time required to solve the model. Relaxing these simplifying assumptions is the subject of our ongoing research but is beyond the scope of this article.

In spite of the interest in GDP-indexed bonds among policymakers, there are few formal studies of the effects of introducing these bonds. Athanasoulis and Shiller (2001) and Durdu (2009) also study the effects of GDP-indexed debt but in frameworks without endogenous borrowing constraints determined by default risk.

Chamon and Mauro (2006) study the effects of introducing GDP-indexed bonds using a debt sustainability framework, commonly used in policy institutions. Because of the low computation cost of solving this framework, Chamon and Mauro (2006) can study a set of debt instruments richer than the one we study in this article. However, a disadvantage of the sustainability framework is that the government's borrowing (the primary balance) is estimated using past data and is not the result of an optimization problem. Thus, the analysis assumes that the government's borrowing does not change when indexed bonds are introduced (in contrast with our findings). Furthermore, their debt sustainability framework does not allow default risk to affect the borrowing cost. The framework is also not suitable for the derivation of the optimal indexation. As we do, Chamon and Mauro (2006) find that indexation could reduce default risk.

Faria (2011); Sandleris, Sapriza, and Taddei (2011); and Hatchondo, Martinez, and Sosa Padilla (2012) study the effects of introducing GDP-indexed sovereign debt in an environment with equilibrium default risk. Comparing quantitative predictions of these studies is difficult because of differences in the parameterizations and the reported statistics. Faria (2011) and Sandleris, Sapriza, and Taddei (2011) present the effects of introducing

[5] This is consistent with evidence of procyclical fiscal policy in emerging economies (that pay a high and volatile interest rate), as documented by Gavin and Perotti (1997); Kaminsky, Reinhart, and Vegh (2004); Talvi and Vegh (2005); Ilzetzki and Vegh (2008); and Vegh and Vuletin (2011).

an income-indexation that is not chosen by the government and is constant over time. As in this article, Hatchondo, Martinez, and Sosa Padilla (2012) allow the government to choose how to index its debt to future income in each period. Hatchondo, Martinez, and Sosa Padilla (2012) compare the effects of introducing income indexation with the ones of introducing interest-rate indexation. The latter form of indexation is the main focus of that article.

The rest of the article proceeds as follows. Section 1 introduces the model. Section 2 discusses the parameterization. Section 3 presents the results. Section 4 concludes.

1. THE MODEL

There is a single tradable good. The economy receives a stochastic endowment stream of this good y_t, with

$$\log(y_t) = \log(A) + \rho \log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$, and $\varepsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$.

The government's objective is to maximize the present expected discounted value of future utility flows of the representative agent in the economy, namely

$$\mathbb{E}_t \left[\sum_{j=t}^{\infty} \beta^{j-t} u\left(c_j\right) \right], \tag{1}$$

where $\mathbb{E}$ denotes the expectation operator, β denotes the subjective discount factor, and the utility function is assumed to display a constant coefficient of relative risk aversion denoted by γ. That is,

$$u(c) = \begin{cases} \frac{c^{(1-\gamma)}-1}{1-\gamma} & \text{if } \gamma \neq 1, \\ log(c) & \text{if } \gamma = 1. \end{cases} \tag{2}$$

Each period, the government makes two decisions. First, it decides whether to default. Second, it chooses the number of bonds that it purchases or issues in the current period.[6]

There are two costs of defaulting (Hatchondo, Martinez, and Sapriza [2007a] discuss the costs of sovereign defaults). First, a defaulting sovereign is excluded from capital markets. In each period after the default period, the country regains access to capital markets with probability $\psi \in [0, 1]$.[7] Second, if a country has defaulted on its debt, it faces an income loss of $\phi(y)$

[6] Bianchi, Hatchondo, and Martinez (2012) study a sovereign default framework where the government can issue debt and accumulate assets simultaneously.

[7] Hatchondo, Martinez, and Sapriza (2007b) solve a baseline model of sovereign default with and without the exclusion cost and show that eliminating this cost affects significantly only the

units in every period in which it is excluded from capital markets. Following Arellano (2008), we assume that

$$\phi(y) = \begin{cases} y - \lambda & \text{if } y > \lambda \\ 0 & \text{if } y \leq \lambda. \end{cases} \tag{3}$$

With this income loss function, the default cost rises more than proportionately with income. This property of the income loss triggered by defaults helps the equilibrium default model to match the high sovereign spreads—defined as the difference between the sovereign bond yield and a risk-free interest rate—observed in the data (see, for instance, the discussion of the effects of the income loss function in Chatterjee and Eyigungor [forthcoming]). This is also a property of the income loss triggered by default in Mendoza and Yue (2012).[8]

We focus on Markov perfect equilibrium. That is, we assume that in each period, the government's equilibrium default and borrowing strategies depend only on payoff-relevant state variables. As discussed by Krusell and Smith (2003), there may be multiple Markov perfect equilibria in infinite-horizon economies. In order to avoid this problem, we solve for the equilibrium of the finite-horizon version of our economy, and we increase the number of periods of the finite-horizon economy until value functions for the first and second periods of this economy are sufficiently close. We then use the first-period equilibrium functions as the infinite-horizon-economy equilibrium functions.

Government bonds are priced in a competitive market. Lenders can borrow or lend at the risk-free rate r, are risk neutral, and have perfect information regarding the economy's income.

We study two versions of this model. First, we assume the government can issue non-contingent bonds. Each bond is a promise to deliver one unit of the good in the next period. Second, we assume the government can issue an indexed bond that promises a next-period payment that is a function of next-period income.

Recursive Formulation with Non-Contingent Bonds

Let b denote the government's current bond position, and b' denote its bond position at the beginning of the next period. A negative value of b implies that the government was a net issuer of bonds in the previous period. Let d denote

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debt level generated by the model. Hatchondo, Martinez, and Sapriza (2009) argue that lower borrowing levels after a default could be explained by political turnover that triggered a default (see, also, Hatchondo and Martinez [2010] for a discussion of the interaction between political factors and default decisions).

[8] Mendoza and Yue (2012) introduce an endogenous channel through which defaults decrease output in the defaulting economy: They assume that when the government defaults, local firms lose access to foreign credit, which is necessary to finance the purchases of foreign inputs.

the current-period default decision. We assume that $d = 1$ if the government defaulted in the current period and $d = 0$ if it did not. Let V denote the government's value function at the beginning of a period, that is, before the default decision is made. Let V_0 denote the value function of a sovereign not in default. Let V_1 denote the value function of a sovereign in default. Let F denote the conditional cumulative distribution function of the next-period endowment y'. Let h and g denote the optimal default and borrowing rules followed by the government. The default rule h takes one of two values: 0 if the rule prescribes to pay back, and 1 if the rule prescribes to default.

The price of a bond equals the payment a lender expects to receive discounted at the risk-free rate. The bond price is given by the following functional equation:

$$q(b', y) = \frac{1}{1+r} \int \left[1 - h\left(b', y'\right)\right] F\left(dy' \mid y\right). \tag{4}$$

This bond price satisfies a lender's expected-zero-profit condition and is equal to the payment probability discounted by the risk-free interest rate. Recall a bond promises to pay one unit of the consumption good next period. Thus, the payment the holder of a bond will receive next period with the state $\left(b', y'\right)$ is given by $\left[1 - h\left(b', y'\right)\right]$.

For a given price function q, the government's value function V satisfies the following functional equation:

$$V(b, y) = \max_{d \in \{0,1\}} \{dV_1(y) + (1-d)V_0(b, y)\}, \tag{5}$$

where

$$V_1(y) = u\left(y - \phi(y)\right) + \beta \int \left[\psi V(0, y') + (1-\psi) V_1(y')\right] F\left(dy' \mid y\right), \tag{6}$$

$$V_0(b, y) = \max_{b'} \left\{u\left(y + b - q(b', y)b'\right) + \beta \int V(b', y') F\left(dy' \mid y\right)\right\}. \tag{7}$$

Definition 1 *A Markov perfect equilibrium is characterized by*

1. a set of value functions V, V_1, and V_0,
2. a default rule h and a borrowing rule g,
3. a bond price function q,

such that:

(a) given h and g, V, V_1, and V_0 satisfy functional equations (5), (6), and (7), when the government can trade bonds at the bond price function q;

(b) the bond price function q is given by equation (4); and

(c) the default rule h and borrowing rule g solve the dynamic programming problem defined by equations (5) and (7) when the price at which the government can trade bonds is given by the bond price function q.

Recursive Formulation with the Indexed Bond

With the income-indexed bond, the government can choose what to promise to pay next period for each realization of next-period income y' (payments can be negative). Let $\hat{b}'$ denote the payment function promised by the government. Let $\hat{g}$ and $\hat{h}$ denote the government's borrowing and default rules, respectively.

As in the previous subsection, a bond price is equal to the expected payment a lender will receive, discounted at the risk-free rate. For the indexed bond, this price is given by

$$\hat{q}(\hat{b}', y) = \frac{1}{1+r} \int \hat{b}'(y') \left[1 - \hat{h}(\hat{b}'(y'), y')\right] F\left(dy' \mid y\right). \tag{8}$$

Note that, with N possible income levels $\{y_1, y_2, ..., y_N\}$, we could think about the government choosing a portfolio of N defaultable Arrow-Debreu securities instead of the payments of an income-indexed bond. For all $i \in \{1, 2, ..., N\}$, security i promises to deliver one unit of the good in the next period if and only if $y' = y_i$. The price of each of these securities is equal to the expected payment the lender will receive. Let b_i denote the number of securities issued by the government promising to pay if and only if $y' = y_i$. Let $P_i(y)$ denote the probability of $y' = y_i$ given current income y. The price of a security promising to pay if and only if $y' = y_i$ is equal to the likelihood of $y' = y_i$ multiplied by the payment the lender would receive $\left(1 - \hat{h}(b_i, y_i)\right)$, and discounted at the risk-free rate:

$$\tilde{q}(b_i, y) = \frac{1}{1+r} \left[1 - \hat{h}(b_i, y_i)\right] P_i(y). \tag{9}$$

Without loss of generality, we assume that the government only promises payments $\hat{b}'(y')$ for which it would not choose to default. Since the government makes a different promise for each level of next-period income, and debt and income are the only determinants of default, there is no uncertainty about whether a government promise will be paid. Note that, for any payment $\hat{b}'(y')$ on which the government would choose to default ($\hat{h}(\hat{b}'(y'), y') = 1$), the contribution of $\hat{b}'(y')$ to the bond price in equation (8) is equal to zero. Then, the government cannot gain from promising a payment $\hat{b}'(y')$ on which it would choose to default.[9] In contrast, without income indexation, the government may issue a bond promising a payment on which it will default next period in some states (y') but not in other states. Since the government may pay next period, lenders are willing to pay for a defaultable bond.

Let W_1 denote the value function of a government in default. Since a defaulting government does not pay its debt, W_1 is not a function of the debt level.

Let W_0 denote the value function of a government not in default. When the government pays it debt, its expected utility is a decreasing function of its debt level.[10]

Since W_0 is decreasing with respect to the government's debt level and W_1 is not a function of the government debt level, for any income level y, there exists a debt level $B(y)$ such that the government defaults if and only if its debt level is higher than $-B(y)$. This debt threshold satisfies $W_0(B(y), y) = W_1(y)$, where

$$W_0(b, y) = \max_{\hat{b}'} \left\{ u(c) + \beta \int W_0(\hat{b}'(y'), y') F\left(dy' \mid y\right) \right\} \tag{10}$$

$$s.t. \quad c = y + b - \frac{1}{1+r} \int \hat{b}'(y') F\left(dy' \mid y\right), \tag{11}$$

$$\hat{b}'(y') \geq B(y') \text{ for all } y', \tag{12}$$

and

$$W_1(y) = u(y - \phi(y)) + \beta \int \left[\psi W_0(0, y') + (1-\psi) W_1(y')\right] F\left(dy' \mid y\right). \tag{13}$$

One way of thinking about the government's lack of commitment to its future default decisions is to suppose that, each period, decisions are made by a different government, and that the current government has no control over future governments' decisions. For instance, the borrowing constraint in equation (12) is exogenous to the current government because $B(y')$ is determined by the next-period government's default decision and the current government cannot control that decision.

The borrowing constraint in equation (12) is the only difference between the economy with indexed bonds and an Arrow-Debreu economy. A binding borrowing constraint would be the source of inefficiency in the indexed-debt economy.

Definition 2 *A Markov perfect equilibrium is characterized by*

[9] Equivalently, with Arrow-Debreu securities, if the government chooses a b_i for which it would choose to default next period ($\hat{h}(b_i, y_i) = 1$), lenders would not pay for b_i ($\tilde{q}(b_i, y) = 0$).

[10] This is also a property of V_0. Chatterjee et al. (2007) provide a formal characterization of equilibrium functions in a default model.

1. a set of value functions W_0 and W_1,

2. a borrowing rule $\hat{g}$,

3. debt thresholds $B(y')$,

such that:

(a) given the borrowing rule $\hat{g}$ and debt thresholds $B(y')$, W_0 and W_1 satisfy functional equations (10) and (13);

(b) given debt thresholds $B(y')$, the borrowing rule $\hat{g}$ solves the dynamic programming problem defined by equation (10); and

(c) $B(y)$ satisfies $W_0(B(y), y) = W_1(y)$.

2. PARAMETERIZATION

We solve the model for the parameterization presented by Arellano (2008). This parameterization was chosen to mimic some moments of the Argentinean economy: properties of the GDP time series and the standard deviation of the trade balance from 1993–2001, an average debt service-to-GDP of 5.53 percent between 1980 and 2001, and a default frequency of 3 defaults per 100 years chosen after counting 3 defaults in the last 100 years for Argentina. Each period corresponds to a quarter. Table 1 presents the parameter values.

The parameterization studied by Arellano (2008) is a common reference for quantitative studies of sovereign defaults. However, some important limitations of this parameterization have been documented in the literature. A model with one-period bonds targeting the average debt service-to-GDP ratio results in debt levels that are too low compare to the data (Hatchondo and Martinez [2009]; Arellano and Ramanarayanan [2012]; Hatchondo, Martinez, and Roch [2012]; and Chatterjee and Eyigungor [forthcoming] study frameworks with long-term debt). Targeting a default frequency of 3 defaults per 100 years implies that the model generates sovereign spreads that are lower than the ones observed in Argentina before its 2001 default. This occurs in part because the model assumes risk-neutral lenders (Lizarazo [2006], Arellano [2008], and Borri and Verdelhan [2009] present models with risk-averse lenders).

We solve the models numerically using value function iteration. We find two value functions: one for a government not in default, and one for a government in default (i.e., V_0 and V_1, or W_0 and W_1). We discretize endowment levels and we use spline interpolation for asset positions. The stochastic process for the endowment is discretized using Tauchen (1986) on a uniformly distributed grid of endowment realizations. We center points around the mean

Table 1 Parameter Values

Sovereign's Risk Aversion	γ	2
Interest Rate	r	0.017
Income Autocorrelation Coefficient	ρ	0.945
Standard Deviation of Innovations	σ_ϵ	0.025
Income Scale	A	10
Exclusion Length	ψ	0.282
Discount Factor	β	0.953
Default Cost	λ	0.969 $E(y)$

and we use a width of three standard deviations. We use 200 endowment grid points.[11]

3. RESULTS

Table 2 reports moments in the simulations of the models with non-contingent and indexed bonds. Statistics correspond to the mean of the value of each moment in 500 simulation samples. Each sample consists of 32 periods before a default episode. The simulations in the economy with state-contingent claims are computed using the same 500 samples of 32 periods that were used to compute the simulations in the benchmark economy. The interest rate spread (r_s) is expressed in annual terms. The trade balance (income minus consumption) is expressed as a fraction of income ($tb = \frac{y-c}{y}$). The logarithm of income and consumption are denoted by $\tilde{y}$ and $\tilde{c}$, respectively. The standard deviation of x is denoted by $\sigma(x)$ and is reported in percentage terms. The coefficient of correlation between x and z is denoted by $\rho(x, z)$. Moments are computed using detrended series. Trends are computed using the Hodrick-Prescott filter with a smoothing parameter of 1,600.

Table 2 shows that the income-indexed bond allows the government to avoid defaults. With non-contingent bonds, the government, when it borrows, promises payments for which it would choose to default if next-period income is low. In contrast, with income-indexed bonds the government cannot gain from promising a payment for which it would choose to default.

Table 2 also shows that income-indexed bonds allow the government to increase its mean level of indebtedness from 4 percent to 18 percent of mean income. With non-contingent bonds, if the government were to promise to pay 18 percent of mean income, the probability of default would be very high and the government would have to pay a very high interest rate to compensate

[11] We do not find significant differences in the welfare gains from introducing indexed debt when we use 100 grid points instead (Hatchondo, Martinez, and Sapriza [2010] discuss the sensitivity of a default model's predictions to changes in the grid specification).

Table 2 Simulation Statistics

	Non-Contingent Bonds	Indexed Bonds
$\sigma(\tilde{y})$	5.58	5.58
Defaults per 100 Years	2.82	0.00
$E(r_s)$	3.24	0.00
$\sigma(r_s)$	2.92	0.00
Mean Debt (% of Mean Income)	3.94	17.89
$\sigma(\tilde{c})/\sigma(\tilde{y})$	1.07	0.79
$\sigma(tb)$	1.13	1.81
$\rho(tb, \tilde{y})$	−0.24	0.69
$\rho(r_s, \tilde{y})$	−0.36	0.00
$\rho(\tilde{c}, \tilde{y})$	0.98	0.96

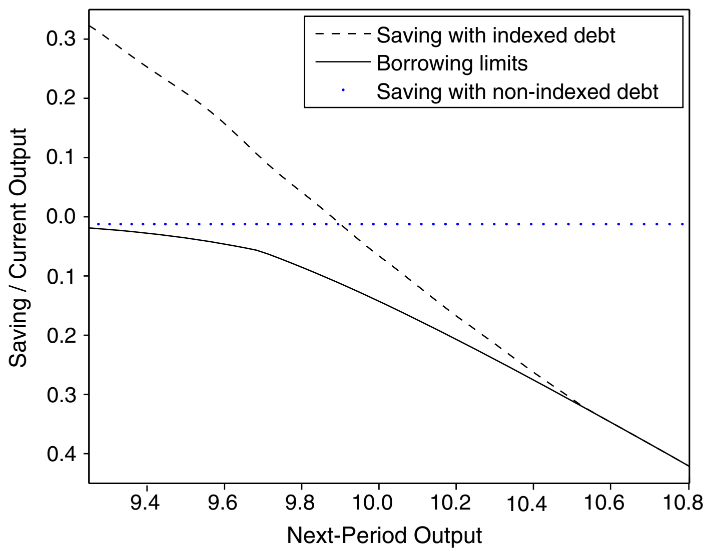
lenders for default risk. That interest rate would be high enough to deter the government from choosing such high debt levels. In contrast, with indexed bonds, the government can promise to pay more when next-period income is higher, which implies a higher cost of defaulting (see equation (3)). That is, with indexed bonds, the government can bring to the present resources from future high-income states without increasing the probability of default in low-income states. Figure 1 illustrates how this is in fact what the government chooses to do.[12] Recall that in the model the government is eager to borrow because it discounts future consumption at a rate higher than the risk-free interest rate.

In addition, Table 2 shows that income-indexed bonds allow the government to reduce the ratio of standard deviations of consumption relative to income from 1.07 to 0.79. A mirror result is that the trade balance is procyclical with income-indexed bonds and countercyclical with non-contingent bonds. To account for this result, note first that income-indexed bonds allow the government to smooth consumption by buying claims that pay in states with lower next-period income and borrowing against states with higher next-period income (see Figure 1).

Furthermore, as shown in Table 2, the spread is countercyclical in the economy with non-contingent bonds. In bad times, the cost of defaulting is lower (see equation (3)) and, therefore, the probability of default and the cost of borrowing are higher. Consequently, optimal borrowing becomes procyclical: In bad times, since the cost of borrowing is higher, the government chooses to finance more of its debt service obligations by lowering consumption instead of borrowing. In contrast, with indexed bonds, the cost of borrowing is constant and thus the government chooses to borrow more when income is lower.

[12] The figure also shows that the indexed-debt borrowing limit binds for sufficiently high next-period income. Furthermore, the figure shows that with non-contingent debt, the government only issues debt with a face value of 1.2 percent of current income.

Figure 1 Borrowing Decisions in a State with Zero Debt and Income Equal to its Unconditional Mean



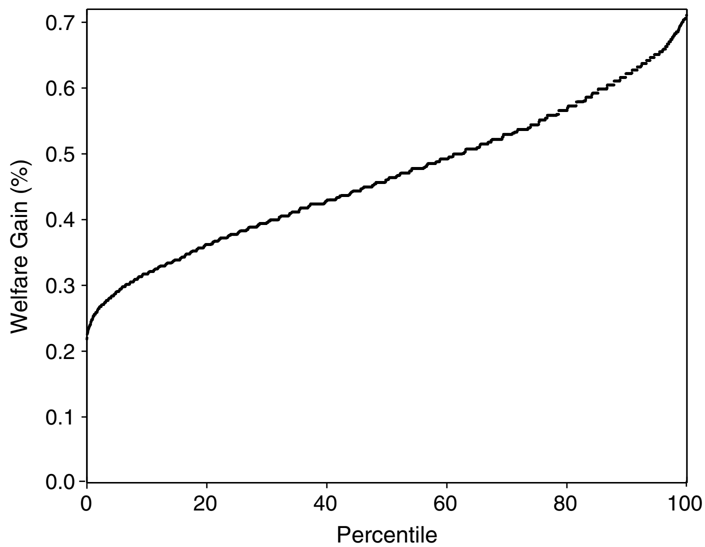
Notes: The dashed line represents the demand for claims contingent on next-period income chosen by the government. The solid line represents the thresholds at which the government will be indifferent between defaulting and not defaulting in the next period. The dotted line represents the saving decision in the economy with non-indexed debt.

Figure 2 presents the distribution of welfare gains from implementing indexed bonds. We compute this distribution using all combinations of income and debt levels in the simulations with non-contingent bonds, for periods with access to capital markets. For each combination of debt and income, we measure welfare gains as the constant proportional change in consumption that would leave a consumer indifferent between living in the economy with non-contingent debt and in the economy with income-indexed bonds. This consumption change is given by

$$\left(\frac{W_0(b, y)}{V_0(b, y)}\right)^{\left(\frac{1}{1-\gamma}\right)} - 1,$$

and can be easily derived from equations (1) and (2). A positive value means that agents prefer the economy with income-indexed bonds. For instance, the figure shows that for 50 percent of the combinations of income and debt levels we consider, welfare gains are higher than 0.45 percent.

Figure 2 Distribution of Consumption Compensation that Makes Domestic Agents Indifferent between Living in the Economy with Non-Contingent Debt and the Economy with Income-Indexed Bonds

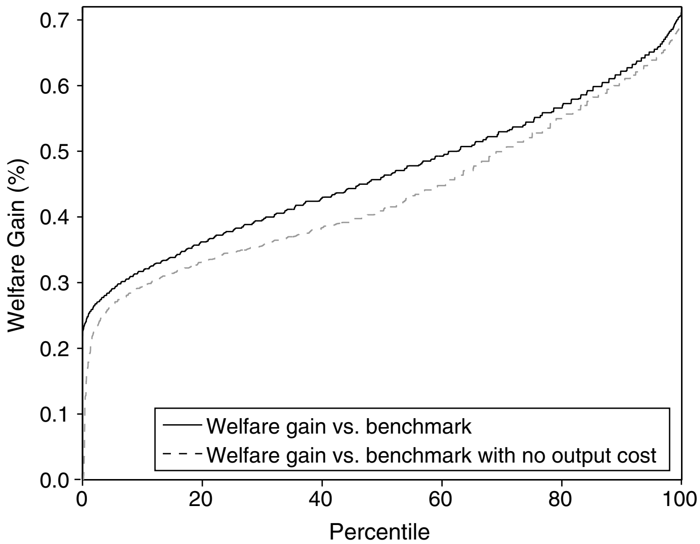


Notes: The distribution (in percentage terms) is computed using the distribution of income and debt levels observed in the economy with non-contingent bonds in periods with market access. For instance, the graph shows that, for half of the combinations of income and debt levels observed in the simulations, the welfare gain is no larger than 0.45 percent.

Figure 2 shows that, for all combination of income and debt levels we consider, the welfare gain from introducing indexed bonds is positive. On average, this gain is equivalent to an increase of 0.46 percent of consumption.

Figure 3 depicts the distribution of welfare gains computed comparing the economy with indexed debt with a hypothetical economy in which there are no income losses triggered by defaults but in which the government follows the saving and default rules of the benchmark economy with non-contingent debt. The figure indicates that income losses triggered by defaults play a relatively small role in accounting for the welfare gains from introducing indexed debt. Most welfare gains from intruding indexed debt come from the relaxation of the government's borrowing constraint: Indexed debt allows the government to borrow more and smooth consumption. The small role of income losses triggered by defaults is not surprising since defaults are infrequent and occur in periods where income losses are small (see equation (3)).

Figure 3 Distribution of Consumption Compensation that Makes Domestic Agents Indifferent between Living in Each of Two Economies with Non-Contingent Debt and the Economy with Income-Indexed Bonds



Notes: The first economy with non-contingent debt is our benchmark economy (welfare gains are represented with a dark line). The second economy with non-contingent debt is a hypothetical economy in which there are no income losses triggered by defaults, but the government follows the saving and default rules of the benchmark economy with non-contingent debt (welfare gains are represented with a gray line).

4. CONCLUSIONS

We introduced income-indexed bonds into a standard sovereign default model and illustrated how a government may benefit from using these bonds instead of non-contingent bonds. Income-indexed bonds allow the government to avoid costly default episodes, increase its level of indebtedness, and improve consumption smoothing.

There are difficulties from issuing income-indexed bonds that are not present in our setup. First, we do not consider difficulties that may arise in the verifiability of the state on which the debt contracts are written. Second, there may be other shocks that could affect the willingness to repay. Third, we circumvent moral hazard problems that could be created by the introduction of GDP-indexed bonds. Expanding our analysis would enhance the understanding of the effects of introducing indexed sovereign bonds and is the subject of our ongoing research.

REFERENCES

Aguiar, Mark, and Gita Gopinath. 2006. "Defaultable Debt, Interest Rates and the Current Account." *Journal of International Economics* 69 (June): 64–83.

Arellano, Cristina. 2008. "Default Risk and Income Fluctuations in Emerging Economies." *American Economic Review* 98 (June): 690–712.

Arellano, Cristina, and Ananth Ramanarayanan. 2012. "Default and the Maturity Structure in Sovereign Bonds." *Journal of Political Economy* 120: 187–232.

Athanasoulis, Stefano G., and Robert J. Shiller. 2001. "World Income Components: Measuring and Exploiting International Risk Sharing Opportunities." *American Economic Review* 91 (September): 1,031–54.

Bianchi, Javier, Juan Carlos Hatchondo, and Leonardo Martinez. 2012. "International Reserves and Rollover Risk." Mimeo.

Borensztein, Eduardo, Marcos Chamon, Oliver Jeanne, Paolo Mauro, and Jeromin Zettelmeyer. 2004. "Sovereign Debt Structure for Crisis Prevention." IMF Occasional Paper 237.

Borensztein, Eduardo, and Paolo Mauro. 2004. "The Case for GDP-Indexed Bonds." *Economic Policy* 19: 165–216.

Borensztein, Eduardo, and Ugo Panizza. 2008. "The Costs of Sovereign Default." IMF Working Paper 08/238 (October).

Borri, Nicola, and Adrien Verdelhan. 2009. "Sovereign Risk Premia." Manuscript, MIT.

Caballero, Ricardo. 2002. "Coping with Chile's External Vulnerability: A Financial Problem." Central Bank of Chile Working Paper 154.

Chamon, Marcos, and Paolo Mauro. 2006. "Pricing Growth-Indexed Bonds." *Journal of Banking & Finance* 30 (December): 3,349–66.

Chatterjee, Satyajit, and Burcu Eyigungor. Forthcoming. "Maturity, Indebtedness and Default Risk." *American Economic Review*.

Chatterjee, Satyajit, Dean Corbae, Makoto Nakajima, and Jose-Victor Rios-Rull. 2007. "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default." *Econometrica* 75 (November): 1,525–89.

Durdu, Ceyhun Bora. 2009. "Quantitative Implications of Indexed Bonds in Small Open Economies." *Journal of Economic Dynamics and Control* 33 (April): 883–902.

Eaton, Jonathan, and Mark Gersovitz. 1981. "Debt with Potential Repudiation: Theoretical and Empirical Analysis." *Review of Economic Studies* 48 (April): 289–309.

Faria, André L. 2007. "Growth-Indexed Bonds in Emerging Markets: A Quantitative Approach." Mimeo.

Gavin, Michael, and Roberto Perotti. 1997. "Fiscal Policy in Latin America." In *NBER Macroeconomics Annual 1997 Volume 12*, edited by Ben S. Bernanke and Julio J. Rotemberg. Cambridge, Mass.: National Bureau of Economic Research, 11–71.

Griffith-Jones, Stephany, and Krishnan Sharma. 2005. "GDP Indexed Bonds: Making it Happen." United Nations Department of Economic and Social Affairs Working Paper 21 (April).

Hatchondo, Juan Carlos, and Leonardo Martinez. 2009. "Long-Duration Bonds and Sovereign Defaults." *Journal of International Economics* 79 (September): 117–25.

Hatchondo, Juan Carlos, and Leonardo Martinez. 2010. "The Politics of Sovereign Defaults." Federal Reserve Bank of Richmond *Economic Quarterly* 96 (3): 291–317.

Hatchondo, Juan Carlos, Leonardo Martinez, and César Sosa Padilla. 2010. "Debt Dilution and Sovereign Default Risk." Federal Reserve Bank of Richmond Working Paper 10-08R.

Hatchondo, Juan Carlos, Leonardo Martinez, and Francisco Roch. 2012. "Fiscal Rules and the Sovereign Default Premium." Federal Reserve Bank of Richmond Working Paper 12-01 (March).

Hatchondo, Juan Carlos, Leonardo Martinez, and Horacio Sapriza. 2007a. "The Economics of Sovereign Default." Federal Reserve Bank of Richmond *Economic Quarterly* 93 (Spring): 163–97.

Hatchondo, Juan Carlos, Leonardo Martinez, and Horacio Sapriza. 2007b. "Quantitative Models of Sovereign Default and the Threat of Financial Exclusion." Federal Reserve Bank of Richmond *Economic Quarterly* 93 (Summer): 251–86.

Hatchondo, Juan Carlos, Leonardo Martinez, and Horacio Sapriza. 2009. "Heterogeneous Borrowers in Quantitative Models of Sovereign Default." *International Economic Review* 50 (November): 1,129–51.

Hatchondo, Juan Carlos, Leonardo Martinez, and Horacio Sapriza. 2010. "Quantitative Properties of Sovereign Default Models: Solution Methods Matter." *Review of Economic Dynamics* 13 (4): 919–33.

Ilzetzki, Ethan, and Carlos A. Vegh. 2008. "Procyclical Fiscal Policy in Developing Countries: Truth or Fiction?" Working Paper 14191. Cambridge, Mass.: National Bureau of Economic Research (July).

Kaminsky, Graciela L., Carmen M. Reinhart, and Carlos A. Vegh. 2004. "When It Rains, It Pours: Procyclical Capital Flows and Macroeconomic Policies" In *NBER Macroeconomics Annual 2004, Volume 19*, edited by Mark Gertler and Kenneth Rogoff. Cambridge, Mass.: National Bureau of Economic Research, 11–53.

Kamstra, Mark J., and Robert J. Shiller. 2010. "Trills Instead of T-Bills: It's Time to Replace Part of Government Debt with Shares in GDP." *The Economists' Voice* Volume 7, Issue 3, Article 5 (September).

Krugman, Paul R. 1988. "Financing vs. Forgiving a Debt Overhang." *Journal of Development Economics* 29 (November): 253–68.

Krusell, Per, and Anthony A. Smith, Jr. 2003. "Consumption-Savings Decisions with Quasi-Geometric Discounting." *Econometrica* 71 (January): 365–75.

Lizarazo, Sandra. 2006. "Contagion of Financial Crises in Sovereign Debt Markets." ITAM Working Paper.

Mendoza, Enrique G., and Vivian Z. Yue. 2012. "A General Equilibrium Model of Sovereign Default and Business." *Quarterly Journal of Economics* 127 (2): 889–946.

Sandleris, Guido, Horacio Sapriza, and Filippo Taddei. 2011. "Indexed Sovereign Debt: An Applied Framework." Collegio Carlo Alberto Working Paper 104 (November).

Shiller, Robert. 1993. *Macro Markets: Creating Institutions for Managing Society's Largest Economic Risks*. New York, N.Y.: Oxford University Press.

Talvi, Ernesto, and Carlos A. Vegh. 2005. "Tax Base Variability and Procyclical Fiscal Policy in Developing Countries." *Journal of Development Economics* 78 (October): 156–90.

Tauchen, George. 1986. "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions." *Economics Letters* 20 (2): 177–81.

Tomz, Michael, and Mark L. J. Wright. 2007. "Do Countries Default in 'Bad Times?"' *Journal of the European Economic Association* 5: 352–60.

United Nations. 2006. "GDP-Indexed Bonds: An Idea Whose Time has Come." Seminar Report held at the International Monetary Fund, Washington, D.C., April 21.

Vegh, Carlos A., and Guillermo Vuletin. 2011. "How is Tax Policy Conducted Over the Business Cycle?" University of Maryland Working Paper.