CDS Auctions: An Overview

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credit default swap (CDS) is a credit derivative that can be used as insurance against a reference entity's credit risk, where a reference entity is either a government or corporation that has issued debt. If a party owns equal amounts of bonds and CDSs for a particular reference entity, then the party is completely insured against a negative credit event. However, unlike insurance, it is possible to own more of the CDS protection than of the underlying bond. In this way, CDS contracts make it possible to trade on an entity's credit risk without having exposure to the entity's actual bonds.

Figure 1 summarizes how CDS contracts work. A CDS contract is a bilateral agreement between a protection seller and a protection buyer. The former is taking a short position in the CDS, while the latter is taking a long position. The protection seller compensates the protection buyer if there is a credit event with respect to any of the bonds issued by the contract's reference entity. Credit events include bankruptcy, failure to pay, and restructuring, among others. In exchange, the protection buyer makes periodic interest payments to the protection seller until the contract expires.

CDS auctions are the main settlement mechanism for CDS contracts. The auction provides a unique price for the defaulted bond, which directly impacts the amount that the protection seller needs to pay the protection buyer if a credit event occurs. In this way, CDS auctions have direct influence on payouts in the CDS market, a market that had approximately \$10 trillion in contracts outstanding by the end of 2007.¹ Considering the size of the CDS market, understanding how CDS auctions function is extremely important for CDS users and regulators.

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¹ See Aldasoro and Ehlers (2018).



Figure 1 A CDS Contract

In this paper, we discuss the historical background of the CDS market, why CDS auctions were developed, and the most recent CDS literature. We then describe the auction rules and use the recent Toys R Us auction as an example. In order to illustrate frontier research on the topic, we discuss the theoretical results presented in Chernov et al. (2013) and extend their empirical findings.

Chernov et al. (2013) highlight important incentives that participants have during CDS auctions.² In particular, they show that dealers have an incentive to manipulate the auction price to get better terms when they settle their CDS contracts. These theoretical predictions can be empirically tested, and Chernov et al. (2013) successfully tested one of them. After extending their data to include more recent auctions, we show that their empirical results are also consistent when more recent data are included.

One difficulty that Chernov et al. (2013) face in testing some of their empirical predictions is that they do not observe dealers' CDS positions. If dealers do not actually own CDSs, they have little incentive to manipulate the auction. We further extend their work using regula-

 $^{^2}$ A CDS auction "participant" is anyone who wants to make a bid or offer during the auction, including dealers. "Dealers" are typically big banks that participate directly in the auction. Dealers make bids for themselves and the other auction participants.

tory data from the Depository Trust and Clearing House Corporation (DTCC) on dealers' CDS positions. Using these data, we show that some dealers have large CDS positions. This supports the theoretical findings in Chernov et al. (2013) that some dealers have an incentive to manipulate the auction price.

1. HISTORICAL BACKGROUND AND RECENT LITERATURE

JPMorgan created the first CDS contract to help manage its credit risk. After the 1989 Exxon Valdez oil spill, Exxon needed a large loan to pay for the spill's damages. Since Exxon was an important client, JPMorgan wanted to serve them but did not want the risk associated with the loan. In order to both serve Exxon and not take risk, JPMorgan made the loan to Exxon and entered into a CDS contract with the European Bank of Reconstruction and Development (EBRD). The EBRD was now responsible for covering losses resulting from Exxon defaulting on its obligation to JPMorgan. In exchange, JPMorgan agreed to pay the EBRD for the protection. In this way, CDS contracts allowed JPMorgan to transfer the loan's risk off of its books, while the EBRD was able to get exposure to the loan without having Exxon as a client.

We define a CDS's *underlying assets* as the assets (usually bonds or loans) that trigger a CDS payout if a credit event occurs with respect to them. In the example above, JPMorgan's loan to Exxon is the underlying asset.

CDS contracts allow banks to manage credit risk without trading, or even owning, the CDS's underlying bonds; however, banks are not the only possible users of CDSs. A wide variety of users, such as hedge funds, that wish to exchange credit risk trade CDSs. The initial lack of common standards for CDS contracts made it hard to trade CDSs; as a result, the market lacked liquidity. In the late 1990s, the International Swaps and Derivatives Association (ISDA) issued a set of standard credit derivatives definitions for use in connection with the ISDA Master Agreement. Combined with guidance from financial regulators, these standards helped the market grow from \$632 billion in the early 2000s to \$20 trillion in 2006. During the same time period, other financial and nonfinancial investors joined the CDS market, decreasing the market share of the banks. In 2000, banks accounted for 81 percent of all protection sold and 63 percent of all protection bought through CDS contracts. In 2006, banks' respective market shares fell to 59 percent and 44 percent, respectively.³

 $^{^{3}}$ See Mengle (2007).

In the early 1990s, most protection buyers (that is, those making a payment to transfer the credit risk) were banks carrying the underlying asset. In case of a credit event, the protection buyer could transfer the asset to the protection seller and get paid the protection amount, which was the par value of the bond. This agreement is called a *physical* settlement. Of course, if both sides agree to a payment, they could also settle the contract with a cash transfer and no asset transfer.⁴ This agreement is called a *cash* settlement.

With the market growing rapidly in the late 1990s and early 2000s, issues regarding the settlement of CDS contracts emerged. By the mid-2000s, the CDS market was very different than it was in the 1990s. Investors were not necessarily carrying both the underlying asset (which at this point was mostly bonds) and the CDS. Many investors only held the CDS (naked CDS holders). For naked CDS holders, a physical settlement was not attractive. Protection buyers would have to buy bonds in the market in order to settle the CDS, and protection sellers would have to sell the bonds in order to cash its value. Buying and selling the bonds exposed both the long and short positions to price fluctuations. Moreover, with the volume of CDSs outstanding higher than the volume of bonds issued, the same bond had to be traded many times in the market to settle all CDS contracts. Given the over-the-counter nature of the bond market, the rush to buy the deliverable bonds artificially raised the price well above the expected recovery value. A particularly striking example of this followed the bankruptcy of Delphi Corporation in 2005. Delphi only had \$2 billion in deliverable bonds for \$28 billion in CDS contracts outstanding.⁵

Starting in 2005, CDS auctions were designed to solve these problems by providing a unified settlement mechanism. Investors can use CDS auctions for both physical and cash settlements. The auction identifies a price for the underlying bond, which then can be used for a cash settlement and the exchange of bonds. In a cash settlement, protection sellers pay protection buyers the par value of the underlying asset minus the auction price. Auction participants who prefer physical settlement sell their bonds in the auction and then settle in cash. The auction's cash settlement mechanism, combined with selling the bond at the auction price, replicates the payout of a physical settlement.

To see how this works, consider a participant with \$100 in a particular bond and equivalent CDS protection in the same amount. Say the result of the auction is p dollars per 100 notional. If the participant submitted an order to sell his bonds in the auction, then his payout

⁴ Typically, the cash transfer will be 100 minus bond price per 100 notional.

⁵ See Augustin et al. (2014).

would be 100-*p* from the cash settlement, plus *p* from selling the bond, resulting in a 100-*p*+*p*=100 payout. This is the same outcome of a physical settlement, regardless of the final auction price, $p^{.6}$

With CDS auctions determining the payouts of trillions of dollars in contracts, market participants, policymakers, and researchers began analyzing the auction. In particular, dealers and other large CDS auction participants can manipulate the price of the bond to favor their own CDS position. For example, a participant with a long (short) position in CDSs receives a higher return as the bond's auction price decreases (increases). As a result, a net buyer (seller) of protection has incentive to manipulate the auction price downward (upward). The ISDA recognized the possibility of price manipulation and designed the auction to prevent it from happening,⁷ but whether the current design prevents price manipulation is a matter of theoretical and empirical research.

On the theoretical side, three recent papers have made significant progress toward understanding CDS auctions. Du and Zhu (2017) study a model of CDS auctions with restrictions to participation. They show that these restrictions bias the auction price, where bias is the difference between the asset price and its fundamental value. On the other hand, investors do not have price impact and do not bid strategically in order to manipulate the auction price because the authors consider an economy with a continuum of investors.⁸ Peivandi (2017) allows for endogenous participation in the auction and solves for the optimal auction design. Peivandi shows that a CDS trader has incentives to prevent his counterparties from participating in the auction. By settling contracts in advance of the auction at better terms for his counterpart, the CDS trader can manipulate the auction price to his advantage. The better price resulting from the auction more than compensates the trader for the pre-auction settlement losses. As a result, neither full participation nor an unbiased price can be achieved. In Chernov et al. (2013), the authors consider an environment where the auction participants not only have price impact, but also have restrictions when buying/selling assets. They show that participants have

 $^{^{6}}$ Buying or selling bonds in the auction and then performing a cash settlement mimics a physical settlement regardless of the bond and CDS position of the participant.

⁷ We interpret price manipulation as any participation in the auction with the intention to move the auction price of the bond away from its market price. We discuss the auction design later in the paper.

 $^{^{8}}$ Another interesting feature of Du and Zhu's (2017) model is that investors are heterogeneous in valuations, which has implications for the efficiency of the asset allocation. We find this interesting; however, we focus on the determinants of the auction price.

incentive to manipulate the auction price to profit from their existing CDS positions, resulting in a biased final price.

On the empirical side, four papers study bias in the auction price. That is, whether the auction price differs from the true fundamental price. Helwege et al. (2009) and Coudert and Gex (2010) investigate an early sample of CDS auctions, while Gupta and Sundaram (2012) and Chernov et al. (2013) investigate a more current sample. To approximate the fundamental price, these papers compare the market bond price near the time of the auction (the true price) to the final auction price. In their early sample, Helwege et al. (2009) do not find evidence of bias in the auction price. Coudert and Gex (2010), Gupta and Sundaram (2012), and Chernov et al. (2013) all find evidence of some bias in the auction price.

2. THE AUCTION

Participants in CDS auctions include nondealer participants (investors) and dealers. Since dealers are the only entities allowed to participate directly in the auction, nondealer participants must submit their requests and orders through the dealers.

CDS auctions have two stages, and each stage focuses on pricing the bonds deliverable in the auction. The auction's result is a uniform price for the auction's underlying bonds, which is the bond price used to cash settle all CDS contracts. That is, all CDS protection holders are paid 100 minus auction price per 100 notional by participants who are protection sellers. The final auction price is also used to settle all bids (offers) to buy (sell) the underlying bonds in the auction. This process is designed to mimic physical settlement, even though all CDS contracts are settled via cash.

We use the Toys R Us CDS auction as an example to clarify how the auction proceeds. We review the events leading up to their auction here. After Toys R Us filed for Chapter 11 bankruptcy on September 18, 2017, a set of fifteen dealers voted that a credit event did occur with respect to Toys R Us. This vote triggered CDS payouts and therefore a CDS auction. The Toys R Us CDS auction took place on October 11, 2017, twenty-three days after the company filed for bankruptcy.

The First Stage of the Auction

Participants may submit physical delivery requests and dealers must submit initial market quotations during the auction's first stage. Physical delivery requests are bids (offers) to buy (sell) the auction's underlying bonds at the auction's final price. The requests are used to find the net open interest (NOI), which determines whether participants will be submitting bids or offers in the second stage. For the initial market quotations, all of the auction's dealers submit both bids and offers on a predetermined amount of the underlying bonds. These quotes help find the initial market midpoint (IMM), a price for the underlying bond that restricts second-stage orders. In this section, we detail how to calculate the NOI and IMM from the physical settlement requests and initial market quotations. Subsequently, we demonstrate how the price cap and adjustment amounts are calculated.

Physical Settlement Requests

Participants make physical settlement requests restricted by their CDS position. These physical settlement requests are used to calculate the NOI, which is carried over to the auction's second stage. Submitting Physical Settlement Requests

All participants may submit a physical settlement request—an order to buy or sell the underlying bond at the price determined through the two-stage auction. Participants can only submit the quantity of bonds they are willing to buy or sell in these requests. They do not submit a price.

Dealers submit their physical settlement requests directly; meanwhile, nondealer participants may submit physical settlement requests through a dealer. Additionally, all participants are not allowed to submit requests above, or in the opposite direction of, their CDS position. That is, the participant can only offer (bid) to sell (buy) the underlying bonds using physical settlement if they are net buyers (sellers) of CDSs.⁹ For example, a net buyer of \$100 in protection can only offer to sell bonds via a physical settlement request, and the request cannot exceed \$100. Note that a participant is not obligated to submit a physical settlement request of bond position.

Figure 2 displays the physical settlement requests submitted by the ten participating dealers in the Toys R Us auction. In total, four dealers submitted physical settlement requests. Bank of America, BNP Paribas, and Goldman Sachs submitted physical settlement offers. These offers could not exceed the amount of CDSs the dealers owned, indicating that these three dealers either had a long position in Toys R Us CDSs (they owned protection) or another participant who submitted a physical settlement offer through one of these dealers did. In particular, several news stories around that time mentioned that

 $^{^9}$ Recall from Section 1 that CDS protection holders must give the protection seller the underlying bond in exchange for par value during physical settlement. As a result, protection sellers bid to buy bonds and protection holders offer to sell bonds.



Figure 2 Toys R Us Auction Physical Settlement Requests by Dealer

Goldman Sachs had a large long position in Toys R Us CDSs, which is consistent with their large offer to sell the underlying bond via physical delivery. Barclays was the only dealer who submitted a physical settlement bid.

Net Open Interest (NOI)

Once the physical settlement requests have been received, we calculate the NOI. Any offer during physical settlement is considered positive, while any bid is negative. We sum the physical settlement requests to find the NOI. If the sum of physical settlement offers exceeds bids (NOI is positive), then the NOI is to sell by the difference between the offers and bids. Likewise, the NOI is to buy by the difference between the bids and offers whenever physical settlement bids exceed offers (NOI is negative). If the NOI is zero, the auction ends in the first stage and the final price is set equal to the IMM. Otherwise, the NOI is taken to the second stage of the auction, where participants can bid (offer) to buy (sell) the remaining open interest depending on whether the NOI is to sell (buy).

In the Toys R Us auction, there were \$5.12 million in bids and \$86.292 million in offers via physical settlement requests. When offers exceed bids, the NOI is to sell and equals the amount of offers less the

amount of bids. As a result, the NOI was \$81.172 million to sell,¹⁰ so dealers submitted limit order bids in the second stage of the auction to fill the positive NOI.

Initial Market Quotations

In addition to submitting physical settlement requests during the first stage, each dealer must submit an initial market quotation, which is composed of a bid-offer pair for the underlying bonds. The first-stage quotations are used to calculate the IMM and a price cap, both of which affect the auction's second round. Additionally, either the dealer's bid or offer is carried over to the second stage, depending on the NOI. Using the Toys R Us example, we first describe the process through which dealers submit their initial market quotations; subsequently, we show how both the IMM and price cap are calculated directly from the initial quotations.

Submitting Initial Market Quotations

Dealers must submit both bids to buy and offers to sell a predetermined amount of the auction's deliverable bonds. Nondealer participants cannot submit initial quotations, even via the dealers. ISDA sets the maximum bid-ask spread prior to the auction based on asset liquidity. Any rational dealer would maximize their bid-ask spread in their quotation; therefore, each dealer's bid-ask spread should equal the maximum allowable bid-ask spread. Moreover, ISDA also sets the predetermined quotation size, which is the amount of underlying bonds for which dealers submit initial quotations.

In the Toys R Us auction, the quotation size was \$2 million and the maximum bid-offer spread was 2 percent. Figure 3 displays the initial market quotations for the ten dealers who participated in the Toys R Us auction, with their bids in blue and their offers in green. As expected, each dealer's bid exceeded their offer by exactly 2 percent. Bank of America had the highest bid-offer pair (highest bid and highest offer), while Goldman Sachs and Barclays tied for the lowest (lowest bid and lowest offer).

The Initial Market Midpoint (IMM)

The dealers' initial quotations are used to calculate the IMM. Consider ordering the initial bids in descending order and the initial offers in ascending order. Then, pair the highest bid with the lowest offer, the second highest bid with the second lowest offer, and so on, until the lowest bid is paired with the highest offer. We define the bid-ask

 $^{^{10}}$ Offers are positive, and bids are negative. NOI = Offers-Bids = 86.292 million - 5.12 million = 81.172 million.



Figure 3 Toys R Us Auction Initial Quotations by Dealer

pairs created in this process as ordered bid-offer pairs. Any ordered bid-offer pair whose bid is greater than or equal to its offer is defined as a crossing bid-offer pair. In contrast, any bid-offer pair whose bid is less than its offer is a noncrossing bid-offer pair. The best half is the set of ordered bid-offer pairs whose bids are the highest among the noncrossing bid-offer pairs. In the case of an odd number of non-crossing bid-offer pairs, we round up. For instance, if there are seven noncrossing bid-offer pairs, the best half would be the four noncrossing bid-offer pairs with the highest bids (or lowest offers).

The IMM is the average of the bids and offers that make up the ordered bid-offer pairs in the best half. In other words, we first discard ordered bid-offer pairs until the ordered offer is strictly higher than the ordered bid. Then, the IMM is the average of the highest half of the remaining bids and the lowest half of the remaining offers.

Figure 4 illustrates how the IMM was calculated in the Toys R Us auction. Bids were put in descending order and offers were put in ascending order to create ordered bid-offer pairs (a bid and offer that were part of the same bid-offer pair have the same x-axis value). From the graph, a crossing bid-offer pair was identified when the bid was as high, or higher, than the offer in the ordered pair. In ordered pair 1, the highest bid (Bank of America) was equal to the lowest offer (either Barclays or Goldman Sachs) at a price of 30.5 per 100 notional. Because the bid was greater than or equal to the offer, we removed



Figure 4 Toys R Us Initial Quotation Cross to Find the IMM

this ordered bid-offer pair (as represented by the triangle on Figure 4). There were no other crossing bid-offer pairs; therefore, only the first ordered bid-offer pair was removed. The other nine ordered bid-offer pairs made up the noncrossing pairs.

Once the crossing bids were removed, we found the best half of the remaining ordered bid-offer (noncrossing) pairs by dividing the noncrossing bid-offer pairs by two and excluding the half of pairs farthest from the cross. Since there were nine noncrossing bid-offers pairs in the Toys R Us auction, we divided by two and rounded up to get five ordered bid-offer pairs in the best half. Thus, the four ordered bid-offer pairs farthest from the cross (represented by squares in Figure 4) were also excluded from the IMM calculation. In the Toys R Us auction, there were five ordered bid-offer pairs (ten total bids and offers, represented by circles in Figure 4) comprising the best half. The IMM was the average of these ten quotations, which is 30.25 per 100 notional.

First-Stage Calculations Utilizing both the NOI and IMM

The NOI and IMM are calculated exclusively from an auction's physical settlement requests and initial quotes, respectively. In what follows, the calculation of an auction's price cap and adjustment amounts use both the NOI and IMM.

Price Cap

The price cap (floor) sets the maximum (minimum) possible auction price. When calculating a price cap, the direction of the NOI determines whether a price cap or price floor is imposed; meanwhile, the IMM determines the amount. Specifically, a price cap (floor) equal to the IMM plus (minus) half the size of the predetermined bid-offer spread is imposed if the NOI is to sell (buy). Therefore, the auction price cannot exceed (be lower than) the price cap (floor).

In the Toys R Us auction, the NOI was to sell; therefore, the auction had a price cap. The IMM was 30.25, and the bid-offer spread was 2 percent. Therefore, the second-stage price cap was 31.25 per 100 notional.¹¹ The brown line on Figure 4 represents the price cap in the Toys R Us auction.

Adjustment Amounts

The auction also penalizes dealers that submit initial quotes in the wrong direction of the market, via adjustment amounts. In contrast to price caps, adjustment amounts are onetime fees paid by certain dealers and therefore do not influence the auction's second stage.

The NOI determines the direction of the market, while the IMM impacts the cutoff at which dealers must pay an adjustment amount. Explicitly, if the NOI is to sell (buy), then we analyze all initial bids (offers). If a dealer's bid (offer) is higher (lower) than the IMM, then the dealer's quote is in the wrong direction of the market and they must pay the adjustment amount. It is only possible for a dealer to pay an adjustment amount if there are crossing ordered bid-offer pairs. Therefore, not every auction has a dealer that needs to pay an adjustment amount. If a dealer pays an adjustment amount, it equals the quotation amount multiplied by the amount the dealer's quote differed from the IMM.

In the Toys R Us auction, the NOI was to sell; therefore, we searched Figure 3 for any initial bids that exceeded the IMM. Bank of America's bid of 30.5 was the only initial bid (blue dots) that exceeded the IMM of 30.25 (the black horizontal line). Only Bank of America was on the wrong side of the market and paid an adjustment amount equal to the quotation amount (\$2 million) multiplied by the amount their bid differed from the IMM (0.25 percent). As a result, Bank of America paid an adjustment amount of \$5,000 to penalize them for being off-market.¹²

¹¹ Price Cap = IMM + (Bid-Offer Spread)/2 = 30.25 + 2/2 = 31.25.

 $^{^{12}}$ ISDA says that they round the IMM to the nearest 0.125. When calculated with greater precision, the IMM was actually 30.3125, which is equidistant to 30.25 and 30.375. We found no official rules that indicate why the IMM was rounded down instead



Figure 5 Toys R Us Initial Quotation Cross to Find the IMM

First-Stage Logistics

Participants have fifteen minutes to submit their physical settlement requests and initial quotations online via ISDA's electronic platform. Within thirty minutes of the end of this period, ISDA publishes the IMM, NOI, and any adjustment amounts on creditfixings.com. Then, participants have two to three hours to evaluate the results before the second stage begins.

The Second Stage of the Auction

In the second stage, dealers submit limit orders to fill the NOI established in the first stage to find the final auction price. This price is used to settle all outstanding CDS contracts and the auction's bond trades. In what follows, we discuss how the direction of the second stage is determined, the two ways that limit orders are submitted, and the method for determining the auction's final price. Throughout, we reference the Toys R Us auction and Figure 5.

of up. However, rounding down, as opposed to up, cost Bank of America \$2,500, or 50 percent of their adjustment amount. What a rounding tragedy!

Determining the Direction of the Second Stage

The direction of the second stage depends on the NOI. If the NOI is to sell (buy), participants can only submit bids (offers) to buy (sell) the bonds. In this way, all first-round physical settlement requests that were not matched by other physical settlement requests in the first stage (NOI) are filled by limit orders in the second stage. In the Toys R Us auction, the NOI was to sell; therefore, participants submitted bids in the second stage.

Limit Orders Carried Over From First Round

The relevant side of dealers' initial quotes carry over to the second stage at the predetermined size of the initial quotation. ISDA calls these carried over quotes: *limit orders that were derived from inside markets*. If the NOI is to sell (buy), then dealers' initial bids (offers) are automatically submitted in the second stage. More specifically, as long as the relevant quote was noncrossing,¹³ the relevant side of the dealer's initial quote is carried directly to the second stage. If the relevant quotation is part of a crossing bid-offer pair, then that quotation is carried forward in a different way. If this is the case and the NOI is to sell (buy), then the limit order is the minimum (maximum) of the initial bid (offer) and the IMM.

In the Toys R Us auction, dealers' initial bids for \$2 million of the underlying bond were automatically carried over to the second round, since the NOI was to sell. All dealers, except Bank of America, had their initial bids carried directly to the second stage. Bank of America had the only crossing bid. For Bank of America, their mandatory second-stage bid was the minimum of the IMM (30.25) and their initial bid (30.5). As a result, Bank of America's bid derived from inside markets was 30.25 per 100 notional.

Submitting Limit Orders in the Second Stage

Any participant can submit additional limit orders by submitting a price-quantity pair in the relevant direction of the market. Unlike firststage initial quotations, these limit orders have no predetermined size. Dealers may submit both their own limit orders and those of other participants. Each dealer can submit as many unique price-quantity pairs to buy or sell the underlying asset as they (or the participants they are

 $^{^{13}}$ Recall that a quote is noncrossing if the ordered offer is strictly greater than the ordered bid.

submitting for) want. In the Toys R Us auction, there were forty-five total limit order bids. Ten of them were derived from inside markets; meanwhile, the other thirty-five bids were submitted by participants specifically for the auction's second stage. None of the forty-five bids exceeded the price cap of 31.25 per 100 notional (brown horizontal line on Figure 5).

Some of a dealer's orders are their own, while others are those of other participants. On creditfixings.com, it is impossible to tell the orders of nondealer participants from a dealer's own orders.¹⁴ For example, Goldman Sachs bid \$81.172 million at a price of 25 in the Toys R Us auction and had smaller bids at other prices. Although it is likely that most of the smaller bids were from nondealer participants submitting bids through Goldman Sachs, it is impossible to be certain.

Finding the Auction Price

Second-stage limit orders determine the auction's bond price, which is used to cash settle the CDS contracts. The auction price is also used to settle any request to trade the underlying bond during the auction, whether through first-stage physical settlement requests or second-stage limit orders.

When the NOI is to sell (buy), we match the highest bid (lowest offer) to the amount of open interest that is equivalent to the size associated with the limit order. We continue matching the limit orders in this fashion until we match the entire NOI or run out of limit orders. In the likely case that there are sufficient limit orders to fill the NOI, the last limit order that fills the NOI is the auction price. In the unlikely scenario that there are not enough limit orders to fill the NOI, the bond price is zero (par value) when the NOI is to sell (buy). The final price is compared with the price cap. If the NOI is to sell (buy) and the final auction price is higher (lower) than the price cap (floor), then the final price is the price cap (floor)—reducing second-round price manipulation.

Dealers who submit bids (offers) that are higher (lower) than the auction price are obligated to buy (sell) the underlying bond at the final auction price along with those who submitted first-stage physical settlement requests. In this way, those who submit either a physical settlement request or a limit order in the auction exchange the relevant amount of bonds at the auction price. If there are multiple participants who made a limit order at the auction price, the amount of bonds each exchanges is proportional to the size of their limit order at the auction

¹⁴ With the exception of the orders derived from insider markets.

price (pro rata at the margin rule). These are *partially filled orders*. All CDS contracts are then settled via cash, with the underlying bond price being the final auction price.

In the Toys R Us auction, second-stage bids were ordered from largest to smallest, creating a downward-sloping demand curve of bids (blue line in Figure 5). The NOI was a vertical supply line at \$81.172 million. The point at which the downward-sloping demand curve of bids and the NOI meet is the price and quantity pair that filled the NOI (black dot in Figure 5). The equilibrium price was 26 per 100 notional, which was below the price cap of 31.25; therefore, the intersection point was the final auction price. Moreover, Barclays bid \$25 million and Goldman Sachs bid \$2 million at the final price of 26. When settling bond trades, Barclays and Goldman Sachs split the remaining NOI on a pro rata basis, meaning Barclays bought 25/27 of the remaining NOI from another participant who submitted a physical settlement offer in the first round.

How the Auction Prevents Manipulation

Five auction rules reduce the extent to which participants can manipulate the auction. The first three rules reduce the amount that participants can manipulate the Initial Market Midpoint (IMM) and the NOI. Rules four and five directly constrain participants' second-stage behavior.

- 1. Dealers' initial quotes are limited to a predetermined size and maximum spread.
- 2. Participants are not allowed to submit settlement requests exceeding, or in the opposite direction of, their CDS position.
- 3. Dealers are penalized for submitting quotes in the wrong direction of the market.
- 4. Based on the direction of the NOI determined in the first stage, there is a price cap (floor) on the final auction price.
- 5. The relevant side of all dealers' initial quotes is carried over to the second stage of the auction.

3. THE MODEL

This section and the subsequent section that together describe Chernov et al.'s (2013) model and theoretical results may be skipped. The empirical results and main findings of this paper in Section 5 can be understood without Section 3 and 4.

In this section, we describe the basic environment (players, payoffs, etc.), the auction game, and trading frictions of CDS auctions, as discussed in Chernov et al. (2013). This section formalizes the auction rules discussed in Section 2.

Environment

Chernov et al. (2013) build their auction analysis on the work of Wilson (1979) and Back and Zender (1993). CDS auctions are two-sided auctions. There are two periods, t = 1, 2. The first stage of the auction happens in period t = 1, and the second in period t = 2. There is a set N of participants, from which a subset $N_d \subset N$ are dealers. Each participant $i \in N$ starts period 1 holding an endowment n_i of CDS contracts and b_i of bonds. One unit of the bond pays a value ν between zero and 100 in period t = 1. If ν is 100, the bond is paying the par value; if ν is zero, the bond has no residual value after default.

Holdings of CDS contracts can be zero, positive, or negative; meanwhile, bond holdings cannot be negative. Participants with positive holdings of CDS contracts are protection buyers, and participants with negative holdings are protection sellers. The net supply of CDS contracts is zero, while the supply of bonds is strictly positive. Participants have common knowledge of CDS and bond holdings.

The Auction Game

We start by describing the actions that an auction's participants can take in the first part of the auction (period t = 1), the second part of the auction (period t = 2), and the payoffs associated with such actions.

The First Stage of the Auction

In the first stage of the auction, each participant $i \in N$ submits a settlement request y_i . When y_i is positive (negative), the settlement request is an order to sell (buy) y_i units of the bond at the auction price, p^A .

The auction has restrictions on the settlement requests participants are allowed to make. A participant *i* with a long (short) CDS position, $n_i > 0$ ($n_i < 0$), can only submit selling (buying) orders. These orders cannot exceed their CDS position. That is, a participant with a long position must submit a request $y_i \in [0, n_i]$, and a participant with a short position must submit a request $y_i \in [-n_i, 0]$.

In addition to settlement requests, each dealer $i \in N_d$ submits a quote π_i . Dealers must be ready to buy L units of the bond at their quoted price π_i minus a spread s or to sell L units of the bond at their quoted price π_i plus a spread s. The quotation size L and the spread s are parameters determined in advance of the auction; however, dealers do select their quote price, π_i .

After all participants submit their settlement requests and dealers submit their initial quotes, we compute the NOI and the IMM. The NOI is the sum of all settlement requests; that is, $NOI = \sum_{i \in N} y_i$. The IMM is the average of dealers' quotes, π_i , after excluding any crossing bids and offers.

The Second Stage of the Auction

There are three possibilities for the auction's second stage depending on the NOI. If the NOI = 0, the auction ends and the price p^A is set to the IMM. If NOI > 0, participants bid to buy NOI units of the bond. If NOI < 0, participants offer to sell |NOI| units of the bond.

When NOI > 0, each participant $i \in N$ submits a left-continuous weakly decreasing demand schedule $x_i(p) : [0, IMM + s] \to \mathbb{R}_+$. Note that the price cap is IMM + s. Let $X(p) = \sum_i x_i(p)$ denote the aggregate demand for the asset at the price p. The equilibrium price is the highest price $p \in [0, IMM + s]$ such that the aggregate demand X(p) matches the supply, which is the NOI. That is,

$$p^{A} = \max\{p | p \in [0, IMM + s] \text{ and } X(p) \ge NOI\}.$$
(1)

If X(p) < NOI for all price $p \in [0, IMM + s]$, then p^A is set to zero. Let $q_i(p^A)$ be the asset allocation to participant $i \in N$ at the auction price p^A . The allocation is determined using a pro rata at the margin rule. Formally,

$$q_i(p^A) = x_i^+(p^A) + \frac{x_i(p^A) - x_i^+(p^A)}{X(p^A) - X^+(p^A)} (NOI - X^+(p^A)), \qquad (2)$$

where $x_i^+(p^A) = \lim_{p \downarrow p^A} x_i(p)$ and $X^+(p^A) = \lim_{p \downarrow p^A} X(p)$.

When NOI < 0, each participant $i \in N$ submits a left-continuous weakly increasing supply function $x_i(p) : [IMM - s, 100] \to \mathbb{R}_-$; IMM - s is the price floor. In this case, let $X(p) = \sum_i x_i(p)$ denote the aggregate supply for the asset at the price p. The equilibrium price is the lowest price $p \in [IMM - s, 100]$ such that the aggregate supply X(p) matches the demand NOI. That is,

$$p^{A} = \min\{p | p \in [IMM - s, 100] \text{ and } X(p) \le NOI\}.$$
 (3)

If X(p) > NOI for all price $p \in [IMM - s, 100]$, then p^A is set to 100. Let $q_i(p^A)$ be the bond allocation to participant $i \in N$ at the auction price p^A . As before, the allocation is determined using a prorate at the margin rule. Formally,

$$q_i(p^A) = x_i^-(p^A) + \frac{x_i^-(p^A) - x_i(p^A)}{X^-(p^A) - X(p^A)} (NOI - X^+(p^A)), \qquad (4)$$

where $x_i^-(p^A) = \lim_{p \uparrow p^A} x_i(p)$ and $X^-(p^A) = \lim_{p \uparrow p^A} X(p)$.

Preferences

Agents are risk neutral and maximize their total payoff from the auction. The payoff of a player *i* given his CDS position n_i , bond holding b_i , settlement request y_i , bond allocation in the auction q_i , and final auction price p^A is

$$\Pi_i = q_i(\nu - p^A) + (n_i - y_i)(100 - p^A) + y_i(100 - \nu) + b_i\nu, \quad (5)$$

where $\nu \in [0, 100]$ is the player's valuation for the bond. The term $q_i(\nu - p^A)$ is the player's gain from buying q_i units of bonds at the auction price p^A , $(n_i - y_i)(100 - p^A)$ is the player's gains from the cash settlement of $n_i - y_i$ units of CDS contracts. $y_i(100 - \nu)$ is the player's gains from the physical settlement of y_i units of CDS contracts, and $b_i\nu$ is the player's gains from his bond holdings.

Trading Frictions

To add realism to the environment, Chernov et al. (2013) consider two trading frictions. First, because short-selling bonds is extremely hard in practice, the authors impose the following assumption.

Assumption 1 Each player $i \in N$ can sell at most his endowment b_i of bonds.

Second, because some investors, such as pension funds, are not allowed to hold defaulted bonds, the authors impose the following assumption.

Assumption 2 Only a subset of players $N_+ \subset N$, satisfying $N_+ \neq \emptyset$, can hold a positive amount of bonds after the auction.

Solution

We analyze the model by backward induction; consequently, we start from the second stage of the auction. In the second stage, participants take all CDS positions, $\{n_i\}_i$, physical settlement requests, $\{y_i\}_i$, and the *NOI* as given. Each participant *i* chooses his demand/supply schedule $x_i(p)$ in the auction to maximize his utility in equation (5), given the demand schedules of all players in the auction other than player *i*. In the first stage of the auction, participants again take all CDS positions, $\{n_i\}_i$, as given and submit settlement requests, $\{y_i\}_i$. Dealers also optimally submit their quotes $\{\pi_i\}_i$ in the first part of the auction.

All players understand that the settlement requests and dealers' quotes will determine price and quantities in the auction's second stage. This happens directly since the price in equations 1 and 3, as well as the quantities in equations 2 and 4, depend on the *NOI*. It also occurs indirectly since the demand/supply schedules that players submit in the auction's second stage are a function of the outcomes in the first stage. Therefore, an equilibrium is then composed of settlement requests $(\{y_i\}_i)$, dealers quotes $(\{\pi_i\}_{i\in N_d})$, and demand/supply schedules $(\{x_i\}_i)$ that maximize players profits given in equation (5).

4. THEORETICAL RESULTS

In this section, we describe the main theoretical results of Chernov et al. (2013). Additionally, we provide intuition for why participants have incentive to manipulate the auction price and why under/over pricing in the auction can be an equilibrium outcome. We refer the reader to Chernov et al. (2013) for the formal arguments and proofs.

Price Manipulation and the Frictionless Economy

The trading frictions implied by Assumptions 1 and 2 create limits on arbitrage. This is necessary for the result that some players manipulate the auction price to their advantage. Without market frictions, the price cannot differ from fundamentals in a meaningful way. We can conclude this result from Proposition 2 in Chernov et al. (2013), which we restate below.

Proposition 1 (2 in Chernov et al. [2013]) Suppose there are no trading frictions, that is, Assumptions 1 and 2 are not imposed. Then, in any equilibrium, one of the following three outcomes can be realized: (1) $p \in (\nu, 100]$ and $NOI \ge 0$; (2) $p \in [0, \nu)$ and $NOI \le 0$; and (3) $p = \nu$ and any NOI. Moreover, in all equilibria, players achieve the same expected utility as in the equilibrium with $p = \nu$.

Proposition 1 states that, without imposing Assumptions 1 and 2, the equilibrium price for the auction, p^A , can differ from the fundamental value of ν but not in a meaningful way. In this case, every equilibrium is payoff equivalent to an equilibrium where $p^A = \nu$. Intuitively, if some equilibrium has $p^A \neq \nu$ and is not payoff equivalent to an equilibrium where $p^A = \nu$, players can gain from buying/selling bonds at the auction price. As a result, the initial allocation cannot be an equilibrium.

Price Manipulation and Trading Frictions

Once we impose trading frictions—that is, Assumptions 1 and 2—then participants can manipulate the auction price to their advantage. In this subsection, we show that manipulation is possible for the more empirically relevant case with a positive NOI (NOI > 0). An analogous result can be obtained when NOI < 0.

Proposition 2 (4 in Chernov et al. [2013]) Suppose that there are trading frictions, that is, Assumptions 1 and 2 are imposed. Moreover, assume that

$$\sum_{i:n_i > 0} n_i + \sum_{i \in N_+: n_i < 0} n_i > 0$$

and that for any player i who is a protection buyer, n_i satisfies

$$n_i > \frac{\sum_{j:n_j > 0} n_j + \sum_{j \in N_+: n_j < 0} n_j}{K+1}$$

where K is the total number of players with initial long positions $(n_i > 0)$. Then, there exist a multitude of equilibria for the two-stage auction, in which NOI > 0 and p^A is decreasing in the NOI. In particular, there exists a subset of equilibria in which the second stage of the auction leads to a final price that is a linear function of the NOI:

$$p^A = \nu - \delta \times NOI, \tag{6}$$

where δ is defined in Chernov et al. (2013).

In general, if participants anticipate underpricing $(p^A < \nu)$, protection buyers will prefer to settle in cash because they gain $100 - p^A$ from the settlement instead of $100 - \nu$, while protection sellers will prefer physical settlement because it costs them only $100 - \nu$ instead of $100 - p^A$. This would lead to a negative NOI (*NOI* < 0). However, if some of the protection sellers cannot hold bonds at the end of the auction, they will not be able to do a physical settlement. Moreover, if the protection buyers anticipate that the auction price is a negative function of the NOI, then they will have incentive to do some physical settlement to lower the price. As a result, under some parametric restrictions, there exist equilibria where the NOI is positive and the auction price is a decreasing function of the NOI. This results in underpricing when the NOI is positive.

5. EMPIRICAL RESULTS

We replicate the test in Table 4 of Chernov et al. (2013), but we extend their data. We have their original twenty-six auctions that occurred prior to December 2011 and an additional thirteen auctions that occurred between January 2012 and December 2017.

We also look at the CDS position of dealers to verify that some dealers have long CDS positions and therefore have incentive to lower the auction price. For the dealers' CDS positions, we only have data for 2013 forward since this is when reporting CDS positions became mandatory.¹⁵

Data

We collect data for our regressions to replicate Chernov et al.'s (2013) results with our extended dataset. We also find dealer-level CDS positions to add to their findings.

Replication Data

To replicate Chernov et al.'s (2013) regressions, we need data for the auction's NOI, number of participants (N), aggregate CDS position (NETCDS), notional amount of bonds outstanding (NAB), bond price the day before the auction (p^{-1}) , and the auction price (p^A) .

We use creditfixings.com to find NOI, p^A , and N. They directly report each auction's NOI and p^A . Finding the number of participants is more tricky. Creditfixings.com lists all second-stage orders by dealer. Nondealers submit their bids through dealers, so we use the amount of second-stage orders and the number of dealers to estimate a lower bound for the total number of participants, following an approach proposed by Chernov et al. (2013).¹⁶

 $^{^{15}}$ In fact, our data have positions prior to 2013; however, since the report was not mandatory (the regulation was not in place), we are not confident the data are accurate for this period.

 $^{^{16}}$ Specifically, we get the second-stage bid/offer list, remove all orders from inside markets (dealer bids derived from the first round), and add an additional participant to the auction if a dealer submits two bids/offers at the same price (unless the dealer

The NAB represents the sum of the notional amount of bonds outstanding among each auction's deliverable obligations. Deliverable obligations are the sets of bonds that are exchanged and priced via the CDS auction at the uniform auction price, p^A . To find the NAB, we first find the Committee on Uniform Security Identification Procedures (CUSIP) number for the deliverable obligations, as listed in each auction's protocol. Then, on Fidelity's "CUSIP Look Up," we determine the initial amount of bonds offered for each deliverable obligation. Subsequently, we sum the respective initial amounts of bonds for all deliverable obligations in each auction to obtain the auction's NAB. The NAB could differ from the initial amount of bonds issued; however, the two values are unlikely to be significantly different for distressed entities. While Chernov et al. (2013) use a slightly different method for calculating the NAB, using the initial amount of bonds outstanding is a good estimate, since there is little difference between our NABestimates and theirs for the sixteen auctions common to both samples.

We obtain an auction's NETCDS from the DTCC. NETCDS is at the market level. We download the weekly CDS trade data and sum the trades by week and by entity to find the NETCDS position the week prior to the auction. Because Chernov et al. (2013) include auctions in which there were no NETCDS data, there are fewer than thirty-seven observations of NETCDS with nonmissing values.

We get the bond prices the day before the auction for the deliverable obligations (p^{-1}) using the Trade Reporting and Compliance Engine, which we access using Wharton Research Data Services. To find p^{-1} , we exclude trades below \$100,000 because these trades are likely to be noninstitutional. We also remove deliverable obligations that were not the cheapest to deliver (which we define as the obligations whose prices are two standard deviations above the average price the day before the auction). Finally, p^{-1} is the average of the remaining trades.

To make sure that our data are consistent with Chernov et al. (2013), we compare the variables of interest for the twenty-six auctions that are in both datasets. Even though we do not always use the same data source as Chernov et al. (2013),¹⁷ all the variables match well.

was submitting for another party, there is no reason they would submit two bids/offers at the same price). In this sense, this estimate of auction participants is a lower bound of the total possible participants.

 $^{^{17}}$ For example, Chernov et al. (2013) use Mergent data to compute the NAB and we use Fidelity's CUSIP Look Up.

Dealers' NETCDS Position

When extending Chernov et al.'s (2013) results, we find the position of individual dealers in each auction. The Dodd-Frank Wall Street Reform and Consumer Protection Act requires real-time reporting of all swap contracts to a registered swap data repository (SDR). The DTCC operates a registered SDR on CDS. The Dodd-Frank Act also requires SDRs to make all reported data available to the appropriate prudential regulators.¹⁸ As a prudential regulator, the Federal Reserve has access to the transactions and positions involving individual parties, counterparties, or reference entities that are regulated by the Federal Reserve. Using the DTCC data, we recover the CDS position for dealers (Dealers' *NETCDS*) in fifteen auctions since 2013—a total of seventythree observations of auction/dealer pairs or about five dealers per auction.

Regressions

In their empirical analysis, Chernov et al. (2013) investigate whether equation (6) holds in CDS auctions (Table 4). To be specific, they estimate the linear regression:

$$\frac{p^A}{p^{-1}} = \alpha + \beta \times \frac{NOI}{S} + \epsilon, \tag{7}$$

where p^A is the auction price, p^{-1} is the bond price the day before the auction, NOI is the net open interest, and S is a variable to normalize the net open interest. Since it is not clear what the normalization should be, the authors try four different specifications: no normalization (S = 1), number of auction participants (S = N), net notional amount of bond outstanding (S = NAB), and net CDS (S = NETCDS).

Table 1 depicts the empirical results in Chernov et al. (2013) and our extended dataset. The main prediction of the theory is that the β coefficient is negative. That is, p^A relative to p^{-1} is decreasing in the *NOI*.

In general, our results are consistent with the findings from Chernov et al. (2013). The exception is the regression in which we do not normalize the *NOI*. In this case, we obtain a coefficient β that is not significant, while Chernov et al. (2013) get significance at the 10 percent level. Note, however, that our p-value is 15 percent, while their

 $^{^{18}}$ See Sections 727 and 728 of the Dodd-Frank Wall Street Reform and Consumer Protection Act.

	NOI	NOI/N	NOI/NETCDS	NOI/NAB
		Chernov et	al. (2013)	
α	0.90^{***}	0.93^{***}	0.98^{***}	0.99^{***}
β	(20.38) -0.07^{*} (-1.65)	(21.33) -3.32*** (-2.77)	(-3.07)	(23.29) -0.91*** (-4.85)
		Using data up to	December 2017	
α	0.87***	0.90***	0.92***	0.93***
β	(23.89) -0.06 (-1.51)	(26.48) -7.36*** (-3.43)	(17.99) - 0.37^{**} (-2.65)	(23.96) -0.71*** (-3.26)

Table 1 Regression (7) for Chernov et al. (2013) andExtended Dataset

Notes: Sample in Chernov et al. (2013) includes twenty-six CDS auctions up to December 2011. Extended dataset includes thirty-seven CDS auctions up to December 2017. *** p<0.01, ** p<0.05, * p<0.1.

p-value is 10 percent, so our results are relatively close. Overall, we both find evidence of downward price manipulation when the NOI is positive.

Dealers' CDS Positions

The above results are consistent with the theory of price manipulation we discussed in Section 4. It also has implications for the CDS position of participants. Proposition 2 tells us that there is an equilibrium in the auction where the NOI is positive and participants with positive CDS holdings bid to lower the auction price. In our fifteen auctions, the average auction price relative the bond price the day before is 0.87, suggesting underpricing of 13 percent. Since we do observe underpricing, according to the theory, we should also have participants who are protection buyers.

Figure 6 depicts the distribution of dealers' CDS positions for the fifteen auctions for which we have dealer-level CDS positions.¹⁹ On average, we have data for five dealers per auction; in comparison, each auction usually has ten dealers. The NOI is positive in thirteen of the fifteen auctions. After netting long and short positions for each

 $^{^{19}}$ We only have data for the auction's dealers (those directly participating in the auction). This is okay as they are likely the only ones who could have large enough positions to manipulate the auction.



Figure 6 CDS Position of Dealers in the Auction

dealer, dealers own \$8.23 million in protection per auction on average. Not only do dealers own protection on average, dealers also own protection in 68.5 percent of the observations. Sometimes, dealers have very large positive positions. Moreover, the CDS positions of dealers are not small in comparison to the *NAB* or *NETCDS*. The total notional amount of CDS holdings of dealers is 34 percent of the deliverable bonds' *NAB*, or 11.5 percent of the *NETCDS*—just among the dealers we observe. It seems some dealers have significant positive CDS positions and therefore have incentive to manipulate the auction price downward when the *NOI* is positive—supporting Chernov et al.'s (2013) empirical findings.

6. CONCLUSION

We first introduced the historical background of CDSs and CDS auctions. We then explained the auction's rules in great detail, including an example of the Toys R Us auction. These auctions are under the radar, difficult to understand, and rarely explained fully; as a result, we believe the auction details provided here will be a helpful starting point for those looking to understand CDS auctions.

After discussing three relevant CDS auction papers, we focus on Chernov et al. (2013). We provide a summary of their theoretical model and test their empirical predictions using data through 2017. Our findings concur with theirs and indicate that CDS auction prices are being manipulated in the downward direction when the NOI is to sell.

Finally, we use regulatory data on CDS positions from the DTCC to demonstrate that dealers sometimes hold CDS positions significant enough to provide incentive to manipulate the auction. This finding provides further support for the conclusions in Chernov et al. (2013). In future work, we aim to leverage our DTCC dataset to analyze CDS auctions at the dealer holdings and bid level in a much more quantitative manner. This analysis will provide better insight into how CDS auctions are manipulated.

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