ALGEBRAIC QUANTITY EQUATIONS BEFORE FISHER AND PIGOU

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Perhaps the most basic tool of monetary analysis is the quantity equation of exchange MV = PQ, where M is the money stock, V its average turnover velocity, P the price level, and Q the quantity of goods exchanged against money. This equation has at least three alternative interpretations. Stated as the identity MV = PQ (where velocity is defined as V = PQ/M so as to render the equation a tautology), it reminds us that expenditures must equal receipts, that the sum total of monetary payments (MV) must just add up to the aggregate value of goods sold (PQ). Written as M/P = Q/V or M = (Q/V) P, where velocity is now defined independently of the other variables such that the equation is non-tautological, it states that the price level P must adjust to equate the real or price-deflated value of the given nominal money stock M with the given real demand for it, this real demand being the fraction l/V of real transactions Q that the public wishes to hold in the form of real cash balances. In other words, it states that the price level P is determined by the nominal money supply M and real money demand (Q/V), varying directly with the former and inversely with the latter. Alternatively formulated as P = MV/Q, it says that prices are determined by total expenditure (MV) relative to output Q, that is by aggregate demand and supply. Most often the equation is used to expound the celebrated quantity theory of money, which says that, given real money demand, changes in the money supply cause equiproportional changes in prices.

The equation's applications are of course well-known. Not so well-known, however, is its origin and early history. For the most part, textbooks typically treat it as a product of 20th century monetary thought, usually identifying it with Irving Fisher and A. C. Pigou, its most influential 20th century formulators. Fisher, in his *Purchasing Power of Money* (1911), wrote the equation in its transaction velocity form :

(1) $MV + M'V' = \Sigma pQ = PT$

where M is the stock of currency, V its velocity, M' is the volume of checking deposits, V' their velocity, EpQ is the sum of the quantities Q of goods and services sold valued at their market prices p, P is the weighted average of these prices or the general price level, and T is the aggregate of real transactions or the sum of all the Qs. Similarly, Pigou, in his 1917 article "The Value of Money," wrote the equation in its alternative Cambridge cash balance form :

(2) $\dfrac{1}{P} = \dfrac{kR}{M}$

where l/P, the inverse of the price level, is the value (purchasing power) of the monetary unit ; R denotes real resources; k, the reciprocal of velocity, is the the proportion of those resources that people wish to hold in the form of money; and M is the money stock.[1] Neither Fisher nor Pigou, however, were the first to write such equations. On the contrary, the cash balance equation preceded Pigou by more than thirty years, having been presented by Léon Walras in 1886. Likewise, the transactions velocity equation predated Fisher by more than 100 years, having been fully enunciated in 1804.

In fact, quantity equations are even older than the preceding discussion implies. For rudimentary proto-typal versions began to appear as early as the late 17th and 18th centuries, followed by increasingly sophisticated versions in the 19th and early 20th centuries-versions that were often more elaborate and complete than those associated with Fisher and Pigou. These earlier contributions have been largely overlooked. In an effort to correct this oversight and to set the record straight, this article traces the pre-Fisherian, pre-Pigovian development of the quantity

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[1] Pigou's equation is virtually the same as the celebrated cash balance expression presented by John Maynard Keynes in his A Tract on Monetary Reform (1923). Keynes's equation is

$$n = pk$$

where n is the nominal money stock (Pigou's M), p is the price level (Pigou's P), and k is that part of real output the command over which people wish to hold in the form of cash balances (Pigou's kR).

equation in the British, German, Italian, French, and American monetary literature. It covers only writers who presented the equation in explicit algebraic form. Unmentioned are the host of analysts (including, among others, Locke, Hume, Smith, Thornton, Ricardo, Mill, and Marshall) who employed the equation in merely arithmetic or verbal form. It shows that earlier economists not only formulated the equation and specified its components; they also interpreted it as an equilibrium condition between the price level (or value of money) and money supply and demand. That is, they viewed it as an algebraic model of equilibrium price level determination.

British Writers

The first rudiments of the quantity equation have their origin in the 17th and 18th century British monetary literature. To John Briscoe [3] in 1694 and Henry Lloyd [14] in 1771 go the credit for presenting the first such equation and also for being the first to interpret it as a model of equilibrium price level determination.[2]   Their equation, however, lacked a velocity term, being written in the form

$$(3) \quad M = PQ \text{ or } P = \frac{M}{Q}$$

where P denotes the price level, M the money stock, and Q the quantity of goods exchanged for money. They omitted the velocity term (or implicitly assigned it a magnitude of unity) because they viewed prices as being determined in a single transaction involving the one-time exchange of the entire stock of money for the entire stock of goods. They did not understand that price level determination is a continuous process and that the stock of money turns over several times per period in purchasing goods. Nor did they realize that the volume of goods exchanged against money is a flow and not a stock, and that the stock of money must therefore be multiplied by its average velocity of circulation to make it dimensionally comparable with the flow of goods. Despite this shortcoming, they were able to draw correct conclusions from their equation, namely that prices vary in direct proportion with money and in inverse proportion with output. Lloyd even gave an algebraic proof of this latter conclusion, pointing out that if the quantity of goods increases by a scale factor y while the money stock is held constant, then prices will fall

by the inversely proportional scale factor 1/y according to the equation

$$(4) \quad P(\frac{1}{y}) = \frac{M}{yQ}.$$

Lloyd also viewed his equation as embodying an aggregate demand/aggregate supply theory of price determination, a theory in which M serves as the demand variable and Q as the supply variable. That is, he saw M as affecting P through demand just as Q influences P through supply. Although his work had no apparent impact on his fellow countrymen, it did influence his Italian contemporary, the mathematician P. Frisi. The latter, in his review of Lloyd's equation, proposed multiplying the money/goods (M/Q) ratio by the ratio of the number of buyers to the number of sellers (a proxy for real demand and supply) in a crude effort to account for all nonmonetary market forces affecting general prices.[3] He failed to see that Lloyd's commodity (Q) variable already comprehends these forces so that additional variables are superfluous.

After Lloyd, the next British writer to present a quantity equation was Samuel Turner, who, in his *A Letter Addressed to the Right Hon. Robert Peel with Reference to the Expediency of the Resumption of Cash Payments Fixed by Law* (1819), wrote the expression

$$(5) \quad a = bc$$

in which a is the value of commodities exchanged (that is, PQ) over a period of time such as a year, b is the quantity of metallic money in circulation (or M), and c is the circulating power of money or the number of times it changes hands during the year (or V). Turner's formula does not divide the, nominal transactions variable into its price and quantity components.[4] But it does incorporate a velocity term and therefore constitutes an improvement over the primitive equations of Briscoe and Lloyd. Turner also expanded his equation to include a term for paper money, resulting in the augmented expression

$$(6) \quad a = (b+p)c$$

where p is paper money and b is metallic coin. Here is the first quantity equation to contain separate variables denoting different components of the money stock, each multiplied by the same velocity coefficient c.

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[2] On Briscoe, see Schumpeter [25, pp. 314-5]. On Lloyd, see Schumpeter [25, p. 315] and Theocharis [26, pp. 30-31].

[3] On Frisi's modification of Lloyd's equation, see Theocharis [26, p. 31] and Marget [18, pp. 154, 270-1, 277].

[4] On Turner's formula, see Theocharis [26, pp. 120-1].

Twenty-one years after Turner, Sir John Lubbock presented in his On Currency (1840) the first quantity equation to incorporate separate velocity coefficients for the different items comprising the media of exchange.[5] Lubbock's equation, which also includes a term for transactions (such as gifts) that do not involve market prices, is

$$(7) \quad \Sigma a\alpha + E = 1D + mB + nC$$

where $\Sigma a\alpha$ is the sum of market transactions a at prices α during a given period (Fisher's ΣpQ), E is the sum of transactions or transfers not involving prices (such as gifts, tax payments, the repayment of principal on debts, etc.), D is the amount of checking deposits (Fisher's M'), B is the total amount of bills of exchange (Fisher's M''), C the total amount of cash, or money narrowly defined (Fisher's M), and 1, m, and n are velocity coefficients corresponding to the V', V'' and V terms of Fisher's equation. Note that Lubbock distinguishes between the money and near-money (or money-substitute) components of the media of exchange-the near-money component being defined as deposits and bills of exchange. He further decomposes the money or cash component C into its bank note, coin, and cash-reserve constituents according to the equation

$$(8) \quad C = f + g + \frac{D}{k}$$

where f denotes bank notes in circulation, g denotes coin in circulation, and D/k denotes the coin and bullion reserves backing banks' note and deposit liabilities, these reserves being expressed as the ratio of deposits D to the deposit expansion multiplier k.[6] Substituting this last formula into the one immediately preceding it yields the augmented quantity equation

$$(9) \quad \Sigma a\alpha + E = 1D + mB + n(f+g+\frac{D}{k}).$$

Here is the first appearance of the deposit expansion multiplier and separate velocity coefficients in a quantity equation.

From his equation, Lubbock concluded as follows: given output and velocities, prices move equiproportionally with changes in the means of payment-but only if the volume of unilateral transfers E is zero or also moves equiproportionally with the means of payment. In these cases the quantity theory holds. It may not hold, however, if E responds disproportionately to monetary changes since such responses require offsetting disproportionate movements in prices. In particular, with E invariant to monetary shocks (a situation Lubbock thought most likely) prices would tend to move in greater proportion than money.

German Writers

In the 48-year interval separating the contributions of Lloyd and Turner, German economists made significant advances in the formulation and analysis of algebraic quantity equations. Claus Kröncke, in his *Das Steuerwesen* nach *seiner Natur und* seinen *Wirkungen untersucht* (1804), was the first writer to introduce a velocity term into the equation, doing so fifteen years before Turner.[7] Kröncke's equation is

$$(10) \quad r = \frac{\phi}{m}$$

where r is the money stock needed in a country, ø is the nominal value of all goods sold during a certain period of time, and m is the number of times on the average that money turns over in purchasing goods during the period. This is the same as the conventional quantity equation M = PQ/V, where r = M, ø = PQ, and m = V. Although Kröncke did not divide his nominal transactions variable into its price and quantity components, he did state that if output and velocity are given, prices must vary directly with the money stock. That is, he used his equation to help illustrate the quantity theory of money. He also recognized that monetary contraction could occur without depressing nominal activity only if there were offsetting rises in velocity. Because he wished to maintain the level of activity while simultaneously minimizing the quantity of gold in circulation (so that the excess could be exported for consumption goods), he advocated policies to increase velocity.

In 1811, Kröncke's compatriot Joseph Lang made two key contributions to quantity-equation analysis.[8] He was the first to include separate terms for the four crucial variables M, V, P, and Q, thereby improving upon Kröncke's three-variable formulation. He was also the first mathematical economist to employ finite difference notation in deriving the quantity theory prediction that prices vary equiproportionally with money. He writes the equation in his *Grundlinien der politischen Arithmetik* (1811) as

[5] On Lubbock, see Marget [17, pp. 11-12].

[6] See Marget [17, pp. 152-3].

[7] On Kröncke, see Theocharis [26, pp. 102-3].

[8] On Lang, see Theocharis [26, pp. 109-10].

(11) $yZ = Px$

where according to his symbols, y is velocity, Z is money, P is real output, and x is the price level. His equation can be translated into the conventional formula MV = PQ. Having written the equation, he then solves for the price level or

(12) $P = \dfrac{MV}{Q}$

from which he concludes that prices P vary in direct proportion to M and V and in inverse proportion to Q.

Then, for the first time in the history of mathematical economics, he employs finite difference, or delta (A), notation to demonstrate rigorously that, with V and Q given, prices vary in exact proportion to money.[9] Starting with his equation

(13) $MV = PQ$

he supposes money to increase by a small amount AM, where the delta symbol denotes an incremental change in the attached variable. Assuming V and Q fixed, he notes that only prices can respond. Denoting this price response by ΔP and inserting it and the corresponding monetary increment ΔM into his quantity equation yields

(14) $(M + \Delta M)V = (P + \Delta P)Q.$

Expanding this equation, subtracting the preceding equation MV = PQ from the result, and then solving for the increment in prices gives him the expression

(15) $\Delta P = (\dfrac{V}{Q}) \Delta M$

which states that the incremental variation in prices is exactly proportional to that of money, with the ratio V/Q (the inverse of the demand for real balances) being the factor of proportion. Here is the first rigorous algebraic statement of the quantity theory of money.

Other 19th century German writers who employed quantity equations include K. Rau and W. Roscher.[10] Little need be said about them, however, as they added virtually nothing to the earlier formulations of Kröncke and Lang. Rau, in his *Grundsätze der Volkswirtschaftslehre* (1841), stated the formula MV = PQ, prompting Friedrich Lutz in 1936 to suggest that it thereafter be called the "Rau-Fisher

equation."[11] Similarly, Roscher, in his *Grundlagen der Nationalökonomie* (1854), presented an equation similar to Kröncke's, namely

(16) $u = ms$

where u is the monetary sum of transactions (or PQ), m is the quantity of money (or M), and s is the velocity of circulation (or V). Roscher's three-variable formula, of course, was already obsolete at the time he published it, having been superseded by Lang's four-variable formulation forty-three years before. Nevertheless, the quantity equation's appearance in the popular textbooks of Rau and Roscher indicates that it had gained thorough acceptance in Germany by the middle of the 19th century.

Italian Writers

At least three pre-twentieth century Italian writers presented versions of the quantity equation. They include P. Frisi in 1772, L. Cagnazzi in 1813, and M. Pantaleoni in 1889. Of these, Frisi has already been discussed above and for that reason will be treated only briefly here. As previously mentioned, his equation, as presented in his review of Henry Lloyd's *An Essay on the Theory of Money* (1771), is

(17) $P = \dfrac{MC}{QV}$

where P is price, M is money, Q is quantity of goods, C is number of buyers (a crude proxy for real demand), and V is number of sellers (a proxy for real supply).   In essence, Frisi's equation constitutes a naive attempt to decompose the price level into its nominal (monetary) and real determinants. In this connection, he argues that the ratio of money to output M/Q captures the monetary factors affecting prices while the ratio of buyers to sellers C/V captures the real factors.[12] What he overlooks is that the real factors underlying prices are already accounted for by the output variable Q so that the other variables C and V are unnecessary. This, plus the omission of a velocity term, renders his equation defective.

Also defective is Cagnazzi's equation, but for a different reason : it omits the price variable. No price term appears in his formula, which he presents in his *Elementi di Economia Politica* (1813), namely

[9] See Theocharis [26, p. 109].

[10] On Rau and Roscher, see Marget [17, pp. 10-11].

[11] Marget [17, p. 10].

[12] On Frisi's equation, see the references cited in footnote 3.

$$(18) \quad Mc = DC$$

where M is the money stock, c its velocity of circulation, D the quantity of goods, and C their velocity of circulation.[13] Cagnazzi claims that his equation describes market equilibrium between the flow of money and the flow of goods. Without a price term, however, his equation makes little sense since it equates dimensionally dissimilar magnitudes. It equates one flow having the dimensions dollars per unit of time with another flow having the dimensions real quantity per unit time. To render the latter flow dimensionally comparable to the former, he should multiply goods by their dollar prices.

Cagnazzi's equation was the first to include a velocity coefficient on the goods variable. Conventional quantity equations of course dispense with that coefficient (or implicitly assign it a magnitude of unity). They do so on the grounds that since the PQ side of the equation summarizes a continuing process, i.e., an ongoing flow of physical goods and services sold, each item transferred should be treated as if it were sold but once before disappearing from economic circulation. That is, each good should be treated as if it had a turnover velocity of one. On this logic, items transferred more than once are to be counted as additional goods each time they are sold. For example, if a single item such as a house were sold four times during the period for which PQ is measured, it would be counted in the Q variable as four houses. In this way, the goods variable itself registers commodity turnover; no velocity coefficient is needed. Cagnazzi, however, proposed that such transfers be registered by a velocity coefficient. Here is the first appearance in the equation of a term denoting the velocity of circulation of goods, a concept later embodied in the quantity equations of the Frenchmen Levasseur and Walras and of the Americans Bowen and Kemmerer.

Maffeo Pantaleoni, in his *Pure Economics* (1898), also endorsed the goods-velocity concept. The volume of business transactions, he said, resolves itself into two elements: the quantity of goods offered for sale and the number of times each good is bought and sold for money. Having acknowledged the goods turnover concept, however, he failed to assign it a specific symbol in his quantity equation

$$(19) \quad v = \frac{m}{qr}$$

where v is the value of the monetary unit (or inverse of the price level l/P), m is the volume of business

transactions (or Q), q is the quantity of money (or M), and r is its rapidity of circulation (or V). He did, however, present his equation as a money demand/money supply theory of price level determination. He defined the numerator m of the right hand side of his equation as real money demand and the denominator qr as nominal money supply. Today we would define m/r as real money demand and q as nominal money supply. Their quotient-the ratio of money demand to money supply-determines the value of money and hence the price level. He also stated the quantity theory of money according to which, for given values of the transactions and velocity variables, the value of money varies equiproportionally with its quantity.

French Writers

Quantity equations made their debut in the English, German, and Italian literature no later than the early 1800s. Not until the middle of the century, however, were they first seen in French monetary texts. E. Levasseur in his *La Question de l'Or* (1858) was the first French writer to present a quantity equation.[14] Like Pantaleoni, he argued that the value of money is determined by the ratio of real money demand to nominal money supply, the former defined by him as the quantity of goods for sale times their rate of turnover and the latter defined as the money stock times its circulation velocity. To illustrate this proposition he writes the equation

$$(20) \quad \text{value of money} =$$
$$\frac{1}{P} = \frac{TC}{(M-R)C' + C_r}$$

where P is the price level, T is the total sum of goods and services for sale, C their circulation velocity, (M-R) is the portion of the total quantity of precious metals M that circulates as money-the remainder R being reserved for nonmonetary uses-, C' is the circulation velocity of metallic money, and C_r denotes credit instruments serving as nonmetallic means of payment multiplied by their velocities. Except for the inclusion of the velocity of circulation of goods C, Levasseur's equation is virtually the same as Irving Fisher's equation. This can be seen by omitting the C variable and replacing Levasseur's terms T, (M-R), C', and C_r by their Fisherian counterparts T, M, V, and M'V' to obtain Fisher's formula

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[13] On Cagnazzi, see Marget [17, p. 11] and Theocharis [26, pp. 39-40].

[14] On Levasseur, see Wu [31, pp. 191-3].

$$(21) \quad \frac{1}{P} = \frac{T}{MV + M'V'}$$

which implies that the price level adjusts to equilibrate money demand and supply.

Sixteen years after Levasseur, Leon Walras, in the first edition of his *Eléments d' économie politique pure* (1874), also presented a Fisherian equation.[15] In addition, he formulated the quantity equation in its alternative cash balance form, becoming the first person to do so. Also, he augmented the latter equation with a base/multiplier component to account for the relationship between high-powered (metallic) money and the rest of the money stock. His contributions are outlined below.

Regarding the Fisherian equation, he derives it in two steps. First, he assumes that the means of payment consists solely of metallic money so that the equation is :

$$(22) \quad \alpha'' Q_a'' = \alpha' Q_a' + \beta Q_b p_b + \gamma Q_c p_c + \delta Q_d p_d \ldots$$

where Q_a'' is the stock of (metallic) money; Q_a' the quantity of the monetary metal in its nonmonetary commodity uses; Q_b, Q_c, Q_d .... the quantities of the other goods B, C, D .... exchanged against money; α'' is the velocity of money; α' is the velocity of monetary metal in its commodity uses; β, γ, δ .... are the velocities of circulation of the goods B, C, D ....; and p_b, p_c, p_d .... are the prices of goods B, C, D .... stated in terms of money (p_a, the price of the monetary metal in terms of itself, by definition being unity).[16] In Fisher's notation :

$\alpha'' = V$,

$Q_a'' = M$,

$Q_a' + Q_b p_b + Q_c p_c + Q_d p_d + \ldots = \Sigma p Q$ and.

α', β, γ, δ .... $= v$ (the velocity of circulation of goods).

Expressed this way, Walras' equation is :

$$(23) \quad MV = \Sigma p Q v.$$

As a second step, Walras adds to the left-hand side of his equation the term F (or M'V' in Fisher's notation) to represent the value of exchanges effected by means of fiduciary (nonmetallic) money. The result is the augmented expression

[15] What follows draws heavily from Marget's [16] classic study of Walras' work on quantity equations.

[16] Marget [16, p. 577].

$$(24) \quad MV + M'V' = \Sigma p Q v$$

which, except for the v or goods-velocity term, is the same as Fisher's formula.

Walras' next contribution is his cash balance equation. This states that the nominal stock of money M must just equal the demand for it, this demand being the aggregate nominal value of goods kPQ the command over which people desire to hold in the form of cash. In his *Théorie de la Monnaie* (1886) he writes the cash balance equation as

$$(25) \quad Q_a'' = \alpha + \beta p_b + \gamma p_c + \delta p_d .$$

where α, β, γ, δ .... denote the respective quantities of the goods A, B, C, D .... the money value of which people require to hold in the form of cash ; Q_a'' is the quantity of money needed to satisfy these requirements ; and p_b, p_c, p_d .... are the money prices of the goods B, C, D.[17] This expression is essentially the same as Keynes' famous cash balance equation n = kp presented almost 37 years later in his *Tract on Monetary Reform* (1923), where n is money, k is the collection of goods the command over which people desire to hold in money form, and p is the price of those goods. Indeed, Walras elsewhere presents his equation in Keynesian form, writing it as

$$(26) \quad Q_a'' P_a = H$$

where H is the demand for real balances (Keynes' k), Q_a'' is the quantity of metallic money (Keynes' n), and P_a is the value of money (the inverse of Keynes' p).   From this equation Walras reached the strict quantity theory conclusion: given the demand for real balances H, the value of money P_a varies in inverse proportion to its quantity Q_a''.

Finally, in the 2nd ed. of his *Eléments* and in his 1898 *Etudes* [30], Walras adds to his cash balance equation the term F resulting in the augmented expression

$$(27) \quad (Q_a'' + F) P_a = H$$

where F is defined as the stock of fiduciary (nonmetallic) money in circulation. Denoting such fiduciary money F as a fixed multiple f of the stock of metallic money Q (i.e., F = fQ) and substituting this expression into the one preceding it yields

$$(28) \quad Q_a'' (1+f) P_a = H.$$

Here are four key ingredients of modern monetarist analysis, namely the stock of high-powered or base money Q_a'', a money multiplier (1 + f), the demand

[17] Marget [16, p. 580].

for real balances H, and the value of money P_a or its inverse, the general price level.[18] All this in an equation presented in 1898, fully 13 and 19 years, respectively, before the appearance of Fisher's and Pigou's equations.

Additional evidence that French monetary theorists had fully developed algebraic quantity equations before Fisher and Pigou comes from A. de Foville. His book *La Monnaie* (1907) contains the expression

$$(29) \quad \frac{P}{p} = \frac{M}{m} \frac{c}{C} \frac{V}{v}$$

where P denotes prices, M the money stock, V its velocity, C the quantity of commodities exchanged against money, and the upper-case and lower-case letters refer to the magnitudes of these variables on any two different dates. Written in ratio form, Foville's expression explains the relative change in the price level between any two dates as the product of the underlying relative changes in its money, velocity, and output determinants.

American Writers

After a late start, the quantity equation developed rapidly in the United States, progressing from an initial incomplete version in the mid-1850s to an elaborate disaggregated version in the early 1900s. The major steps in this progression can be outlined briefly. In 1856 Francis Bowen presented in his *The Principles of Political Economy* the equation

$$(30) \quad gs = mr$$

where g is the quantity of goods sold, s is the number of times the goods are sold, m is the quantity of money in circulation, and r is its rapidity of circulation. Bowen's equation, expressing as it does an equivalence between a flow of goods and a flow of money, is the same as that presented earlier by Cagnazzi and suffers from the same defect, namely the omission of a price-level variable necessary to render the two sides dimensionally comparable.

Simon Newcomb corrected this defect in his "equation of societary circulation" which he presented in his *Principles of Political Economy* (1885). Newcomb's equation is

$$(31) \quad VR = KP$$

where V is the volume of the currency, R its average rapidity of circulation, K the number of real trans-

actions, and P the price level. From his equation he concluded that prices vary equiproportionally with changes in the money stock since the latter can have no lasting effect on the steady-state levels of the real variables R and K. Equilibrium values of these real variables, he said, are immune to monetary change such that the latter registers its full impact on prices only. To explain how money affects prices, he constructs an aggregate demand function from the components of his quantity equation. Like modern monetarists who define real aggregate demand as money stock times velocity divided by prices (MV/P) he writes the demand function as:

$$(32) \quad D = \frac{NVR}{P} \text{ [19]}$$

where D is the quantity of goods demanded, N is a fixed constant, and V, R, and P are the volume-of-currency, rapidity-of-circulation, and price-level variables as defined above. This equation says that, whereas demand varies directly with money and inversely with prices, it is unaffected by equiproportional changes in both variables. Thus, according to Newcomb, a monetary expansion initially puts upward pressure on real demand. But the resulting rise in demand subsequently bids up prices, which eventually rise equiproportionally with money, thus restoring real demand to its original level. In steady-state equilibrium, prices vary proportionally with money, and the latter is neutral in its effect on real variables-just as the quantity theory predicts.

While endorsing the quantity theory, however, he was quick to point out that it holds only if prices are flexible. He put his quantity equation VR = KP to work in demonstrating that price inflexibility would render monetary changes nonneutral in their effect on real activity. For, with prices P slow to adjust to monetary shocks, the real transactions K term of the equation VR = KP would have to bear some of the burden of adjustment to currency contraction. Also, he noted that, with velocity given, autonomous rises in prices P engineered by monopolistic sellers would result in compensating falls in real activity K if the money stock V were held constant. Despite this, he warned that a policy of validating or

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[18] Marget [16, p. 585].

[19] More precisely, he writes

$$D = \frac{NF}{P}$$

where F ("the flow of the currency") is defined as
$$F = VR.$$

Substituting this latter expression into the former yields equation 32 of the text.

underwriting such price increases with money growth in an effort to maintain full employment would only serve to perpetuate inflation. The full employment guarantee, he claimed, would encourage sellers to raise prices repeatedly. Each time accommodating money growth would follow. In this way prices and money would chase each other upward ad infinitum in a cumulative inflationary spiral. Newcomb's work strongly influenced Irving Fisher, who derived his famous equation of exchange from Newcomb's formulation and who dedicated his *The Purchasing Power, of Money* (1911) to Newcomb.

Following Newcomb, the quantity equation appeared with increasing frequency in the U. S. monetary literature. Arthur Hadley helped to popularize it by incorporating it into his well-known textbook *Economics* (1896) in the form

$$(33) \quad RM = PT$$

with M being money, R its rapidity of circulation, P prices, and T the volume of real transactions. Similarly, Edwin W. Kemmerer employed it in his *Money and Credit Instruments in Their Relation to General Prices* (1907), stating it alternatively as a price equation and a money supply/demand equation. The price equation version he writes as

$$(34) \quad P = \frac{MR + CR_c}{NE + N_cE_c}$$

where P is the price level; M the money stock (coin, currency, bank notes), R its average rate of turnover, C the dollar amount of checks in circulation, R_c the average rate of check turnover, N and N_c are the number of commodities exchanged by means of money and checks, respectively, and E and E_c are the average number of exchanges of these goods (that is, their velocities of circulation). This equation expresses the equilibrium price level as the quotient of its monetary and real determinants, which he identifies with money supply and demand. He obtains his alternative money supply/demand expression by rewriting the equation as

$$(35) \quad MR + CR_c = P(NE + N_cE_c).$$

According to him, the left-hand side measures money supply, the right-hand side measures money demand, and the price level P adjusts to equilibrate the two. Except for the inclusion of velocity in his concept of the money supply, his analysis is the same as modern monetarists'.

Five years before Kemmerer, John P. Norton, in his *Statistical Studies in the New York Money Market* (1902), presented perhaps the most elaborate version of the quantity equation to be found in the literature. He includes separate terms for each type of coin and currency in circulation. He distinguishes between the velocity of demand deposits and the velocity of coin and currency. He expresses the total of demand deposits, in terms of its three underlying components, namely bank reserves, the deposit-expansion multiplier, and the proportion of maximum allowable deposits that banks actually create. Lastly, he shows the effect of loan extension and repayment on the equation. He does all this in the following way.

First, he starts with the equation's money or MV side, writing it as

$$(36) \quad E = (MV+DU)T$$

where E is total monetary expenditure, M is money narrowly defined (coin and notes), V is its velocity, D is the volume of demand deposits, U is the turnover velocity of deposits, and T is the number of units of time for which these variables are measured. He then disaggregates the money variable M into its constituent components, namely gold coin G, silver coin S, silver certificates C, United States notes N, National bank notes B, and all other forms of currency L. That is, he defines money M as

$$(37) \quad M = G+S+C+N+B+L.$$

Third, having expressed money in terms of its coin and currency components, he next expresses demand deposits in terms of the reserves backing them. More precisely, he defines such deposits D as the product of the reserve base R, the deposit-expansion multiplier Z (which determines the maximum of deposits per dollar of reserves), and the proportion K of maximum allowable deposits that banks actually create. In short, D = ZKR. He multiplies these deposits by their turnover velocity U and aggregates over the four classes of banks in existence in 1902 to obtain the expression

$$(38) \quad DU = \sum_{i=1}^{4} Z_iK_iR_iU_i$$

where the subscript i indexes the type of bank (country, reserve city, central reserve, and state). He then substitutes this equation and the one immediately preceding it into equation (36) to get his final expression for the MV side:

(39) $\quad E = [(G+S+C+N+B+L)V +$

$$\sum_{i=1}^{4} Z_i K_i R_i U_i]T$$

where E is total expenditure.[20]

He next attempts to show how bank loan extension and repayment affects the equation. He argues that loan repayment temporarily absorbs expenditures that otherwise would be directed toward goods just as loan extension expands them. To show the effect of loan retirement, he adds to the goods or PQ side of the equation the term ΣM_s denoting banks' receipt of "spot" or current dollars M_s as borrowers repay loans. Similarly, to show how loan creation increases expenditure he adds to the opposite side of the equation the term $\Sigma(1-d)M_f$, denoting banks' acquisition of loan assets or "claims to future dollars" M_f, each such dollar valued at its discounted price (1-d), where d is the discount rate on loans. For convenience, he then transposes this term to the goods side of the equation such that the latter reads

(40) $\quad \Sigma pQ + \Sigma M_s - \Sigma(1-d)M_f$

where the first term denotes expenditure on goods and the last two terms denote net debt repayment. Finally, he equates both sides of the equation to obtain the entire expression

(41) $\quad \Sigma pQ + \Sigma M_s - \Sigma(1-d)M_f =$

$[(G+S+C+N+B+L)V +$

$$\sum_{i=1}^{4} Z_i K_i R_i U_i]T.$$

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[20] Note that Norton defines the velocity V term of equation (39) as the average of the individual velocities of each currency component weighted by each component's share in the entire stock. He writes

$$V = \frac{GV_G + SV_S + CV_C + NV_N + BV_B + LV_L}{G+S+C+N+B+L}.$$

Here is another example of his elaborate derivation of the equation's components.

This equation, together with those of Newcomb, Hadley, and Kemmerer, prepared the way for the appearance of Fisher's equation in 1911.

Conclusion

Irving Fisher and A. C. Pigou presented their famous quantity equations in the second decade of the 20th century. By that time, however, their contributions had already been largely or fully anticipated by at least 19 writers located in five countries over a time span of at least 140 years. Except for some primitive initial versions, these writers formulated equations that in all essential respects were virtually the same as their Fisherian and Pigovian counterparts, and in at least two cases were even more detailed and sophisticated than the latter.

Not only did these earlier equations include the same variables and possess the same properties as their celebrated modern counterparts, they also embodied the same analysis. Their authors presented them either as price equations expressing P as a mathematical function of the variables M, V, and Q, or as money-supply-and-demand equations expressing an equilibrium condition between the money stock and the underlying determinants of the demand to hold it. In any case, earlier writers perceived their quantity equations as functional relationships and not as mere identities, just as Fisher and Pigou likewise were to do. Recognition of this fact renders invalid the typical textbook identification of Fisher and Pigou as "the original sources of the equation of exchange and the cash balance equation" [1, p. 98]. Far from being the source of such equations, those writers were the recipients or inheritors of them. In short, whereas quantity equations may have *culminated* in the writings of Fisher and Pigou, they did not begin there. As documented in this article, their source is to be found elsewhere.

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