

# THE MONETARY RESPONSIBILITIES OF A CENTRAL BANK

*Robert L. Hetzel\**

## I. INTRODUCTION

In today's world of paper money, money consists of currency created by the printing press and bank deposits created by the bookkeeping operations of bankers. What limits the ability of the printing presses and the pens of bankers to create money? The currency component of money is central bank money (dollar bills). The bank deposits component of money is backed by central bank money (bookkeeping entries at the central bank). It is the central bank's monopoly of its own money that allows it to limit creation of the public's money. In turn, limitation of central bank money and the public's money limits the price level. The essence of central banking lies in the responsibility to limit the money stock in order to tie down the price level.

In practice, central banks typically do not decide explicitly how much of their money to create. Instead, they create and extinguish their money in response to the current behavior of financial markets. The particular nature of this process of central bank money creation determines how the money stock and the price level are actually limited. The nature of this process depends in turn upon the macroeconomic goals of the central bank. How then does the way the central bank selects macroeconomic goals and weights their relative importance determine the behavior of the price level?

In order to answer these questions, it is necessary to have a model that captures the connection between the goals of the central bank and nominal (dollar) variables: the monetary base (central bank money), the money stock and the price level. The purpose of this article is to lay out such a model. The model is general in that it applies to any central bank that operates in a regime of paper money, although occasional specific references are made to the Federal Reserve System.

---

\* The author is indebted to Marvin Goodfriend for patient encouragement and to Alfred Broadus, Timothy Cook, William Cullison, Michael Dotsey, and Carl Walsh for critical comments.

Section II presents an intuitive discussion of the model. The actual model is presented in the Appendix.<sup>1</sup> Section III elucidates the working of the model by showing how the central bank smooths interest rate fluctuations. Section IV uses the model to discuss money stock and price level determination. This section states the two major responsibilities of the modern central bank. First, the central bank gives the price level an equilibrium value. Specifically, the central bank gives the price level a well-defined or particular value such that market forces operate to eliminate departures of the actual price level from this value. Second, the central bank determines how the equilibrium value of the price level changes over time. The article concludes with suggestions for clarifying the responsibility of the central bank for the behavior of the price level.

## II. THE MODEL

### The Structure of the Economy and the Interest Rate

The model gives substance to the natural rate hypothesis. This hypothesis summarizes the inherent limitations on the central bank's ability to influence real variables. These limitations derive from the fact that paper money creation, or monetary base creation, does not alter the real resources available to the economy. The public cares only about real variables, while the central bank only determines the behavior of a nominal variable, the monetary base.

In the literature that follows the work of Lucas (1972), the natural rate hypothesis is given content by allowing only changes in money and the price level not expected by the public to affect real variables. Furthermore, the public is assumed to form its expectations "rationally," that is, in a way that is consistent with assumptions made about the structure

---

<sup>1</sup> This model has been worked on especially by economists associated with the Federal Reserve Bank of Richmond. [See Dotsey and King (1983), McCallum (1981 and 1986), and Goodfriend (1987)].

of the economy and the behavior of the central bank. The natural rate hypothesis then implies that the central bank cannot systematically affect the level of real variables. For example, the central bank cannot systematically lower the level of the real (inflation-adjusted) rate of interest [Sargent (1973)]. Through the policy process it chooses for determining the monetary base, however, it can influence the way random macroeconomic disturbances affect fluctuations in real variables.

Equation (1) summarizes the determinants of the market rate ( $r_t$ ).<sup>2</sup>

$$(1) \quad r_t = \left( \frac{E_t P_{t+1}}{P_t} - 1 \right) + F_{\pi}[c, (P_t - E_{t-1}P_t)] \cdot Q_t$$

This equation is derived from two more fundamental relationships. One, the IS function, summarizes the conditions under which the goods market clears. For different values of real output, it shows the values of the expected real rate of interest that cause investment and saving to be equal. The other function makes the supply of real output depend upon the contemporaneous price level error ( $P_t - E_{t-1}P_t$ ). The IS function and the aggregate supply function are equated, and output is eliminated from the resulting expression. (The goods market must clear at a level compatible with the aggregate supply of output.) The resulting partially reduced form, when solved for the market rate, is equation (1).

The first right-hand term of (1) equals the rate of inflation the public expects. In the second right-hand term, the function  $F_{\pi}$  is the expected real rate of interest. The real rate depends upon a constant ( $c$ ) and the contemporaneous price level error ( $P_t - E_{t-1}P_t$ ). This functional form derives from the particular form of the natural rate hypothesis, which makes fluctuations in output respond to discrepancies between the contemporaneous price level and the public's prior expectation of the contemporaneous price level. When the price level is higher than the public had expected in the prior period, that is, when  $P_t$  exceeds  $E_{t-1}P_t$ , real output and saving rise, and the real rate of interest falls, and conversely. (These real effects of inaccurate forecasts of the price level can be thought of as deriving from the existence of one-period contracts fixed in dollar terms.) Finally, the expected real rate is affected by real sector disturbances ( $Q_t$ ).

<sup>2</sup>  $E$  indicates an expectation formed by the public and the subscript  $t$  indicates the time period when the public formed that expectation. The subscript  $t$  is the contemporaneous time period, while  $t-1$  and  $t+1$  are the prior period and the following period, respectively.

## The Demand and Supply of Money

Equation (2) is a money demand function.

$$(2) \quad M_t^d = P_t \cdot F_{md}[r_t, (P_t - E_{t-1}P_t)] \cdot V_t$$

Nominal money demand ( $M_t^d$ ) equals the product of the price level ( $P_t$ ), real money demand (given by the function  $F_{md}$ ), and a random disturbance term ( $V_t$ ). Real money demand varies inversely with the market rate of interest ( $r_t$ ) and positively with real output. The function  $F_{md}$ , instead of showing real output as a variable, shows the variable ( $P_t - E_{t-1}P_t$ ) because real output varies positively with this variable, the contemporaneous price level error.

The money supply function has the form of a money-multiplier formula.

$$(3) \quad M_t^s = B_t \cdot F_{mm}[r_t] \cdot X_t$$

The money supply ( $M_t^s$ ) equals the product of the monetary base ( $B_t$ ) and the multiplier, which is given by the function  $F_{mm}$ . This function depends upon the market rate ( $r_t$ ). There is a positive relationship between the market rate and the multiplier because of the effect of the market rate on the reserves-deposit ratio desired by banks and the currency-deposit ratio desired by the nonbank public. The multiplier is also affected by a random term ( $X_t$ ).

## The Monetary Policy Process

The monetary policy process is summarized by the procedure the central bank puts into place for creating and extinguishing the monetary base ( $B_t$ ). This procedure is shown in equation (4).

$$(4) \quad B_t = B_{t-1}(1 + \theta_{trend}) + \theta_{smooth}(r_t - E_{t-1}r_t) - \theta_{drift}(B_{t-1} - E_{t-2}B_{t-1})$$

The three  $\theta$  parameters of (4) determine the time-series behavior of the monetary base.<sup>3</sup> They summarize the information the public needs about monetary policy to form an expectation of the future price level.

<sup>3</sup> Equation (4) summarizes the policy process through the time-series behavior of the monetary base, as determined by the  $\theta$  parameters. Equation (4) could, alternatively, be solved in a way that makes the market rate ( $r_t$ ) the left-hand variable. With this formulation, the policy process would be summarized by the time-series behavior of the market rate. The model is unaffected by the choice of whether to summarize the policy process in terms of the behavior of the base or the market rate. Although monetary policy can be summarized by the behavior of the interest rate, the effect of monetary policy on the economy is transmitted solely through the process that generates the monetary base [Goodfriend and King (1988)].

The first parameter,  $\theta_{\text{trend}}$ , specifies the trend rate of growth of the monetary base. With the simplifying assumption that real output does not grow over time, this rate of growth is also the trend rate of inflation. The second parameter,  $\theta_{\text{smooth}}$ , is the rate smoothing parameter.  $\theta_{\text{smooth}}$  specifies the change in the monetary base the central bank makes in response to deviations in the market rate from a reference rate  $E_{t-1}r_t$ . It follows from the natural rate hypothesis that the central bank must set this reference rate equal to the model's expected real rate plus the trend rate of inflation ( $c + \theta_{\text{trend}}$ ).

Each period, random disturbances impact the economy and move the market rate away from the reference rate. When the market rate exceeds the reference rate, the central bank increases the monetary base by an amount that depends upon the value of  $\theta_{\text{smooth}}$ , and conversely. The result is that each period there is a change in the monetary base that could not have been predicted in the previous period. The base drift parameter,  $\theta_{\text{drift}}$ , specifies how much of the prior period's unpredictable change in the monetary base the central bank offsets in the subsequent period. There are two general cases. In the case of either no offset or only a partial offset ( $\theta_{\text{drift}}$  not equal to one), the level of the monetary base will be affected permanently each period by some random amount. Because there is then no path to which the base tends to return, the monetary base follows a random walk (superimposed on the persistent movement given by the value of the growth rate parameter  $\theta_{\text{trend}}$ ). In the case of a complete offset, the base fluctuates over time around a well-defined path. These two cases are also said to produce, respectively, nonstationary and stationary behavior of the monetary base.

### Central Bank Objectives

Assume now that the central bank possesses two macroeconomic objectives: an economic stabilization objective and a monetary stabilization objective. These objectives can be expressed by the loss function (5).

$$(5) \quad C = \beta \text{var}[P_t - E_{t-1}P_t] + \gamma \text{var}\left[\frac{E_t P_{t+1}}{P_t} - 1\right]$$

The first right-hand term in (5) measures the variability of contemporaneous price level errors. The central bank considers the fluctuations in output caused by these errors to be undesirable. It therefore attempts to limit the variability of these errors. The

second right-hand term measures the variability in the rate of inflation the public expects. The central bank also attempts to minimize this variability. The coefficients on the two right-hand terms reflect the relative importance of the economic stabilization and inflation stability objectives. The central bank chooses the values of the  $\theta$  parameters in (4) in order to minimize the value of  $C$  in (5).<sup>4</sup>

### The Complete Model

The equations of the model are listed below.

- (1)  $r_t = \left(\frac{E_t P_{t+1}}{P_t} - 1\right) + F_{\pi}[c, (P_t - E_{t-1}P_t)] \cdot Q_t$
- (2)  $M_t^d = P_t \cdot F_{md}[r_t, (P_t - E_{t-1}P_t)] \cdot V_t$
- (3)  $M_t^s = B_t \cdot F_{mm}[r_t] \cdot X_t$
- (4)  $B_t = B_{t-1}(1 + \theta_{\text{trend}}) + \theta_{\text{smooth}}(r_t - E_{t-1}r_t) - \theta_{\text{drift}}(B_{t-1} - E_{t-2}B_{t-1})$
- (5)  $C = \beta \text{var}[P_t - E_{t-1}P_t] + \gamma \text{var}\left[\frac{E_t P_{t+1}}{P_t} - 1\right]$

With the constraints imposed by rational expectations and the assumption that money demand equals money supply, equations (1)-(4) can be solved for  $r_t$ ,  $P_t$ ,  $M_t$ ,  $B_t$ ,  $E_{t-1}P_t$  and  $E_t P_{t+1}$ . The resulting values for  $[P_t - E_{t-1}P_t]$  and  $[E_t P_{t+1}/P_t - 1]$  are substituted into the central bank's loss function (5). The loss function is now expressed in terms of the structural parameters of the model, the disturbances, and the  $\theta$  parameters. Finally, the central bank sets the  $\theta$  values in order to minimize this expression for the loss function.

<sup>4</sup> The model is intended for policy analysis. Policy analysis involves the conceptual exercise of assuming different objective functions for the central bank as a way of discussing how the central bank affects the behavior of the economy. In contrast to this kind of analysis, the model could be used to forecast, say, inflation. In this case, it would be necessary to use the actual objective function of the central bank and to be explicit about the way this function changes over time. This kind of exercise is more difficult because of the need to understand how in the real world the policy process affects the way the public forms its expectations. In a world in which monetary policy evolves in unexpected ways, it will be inherently difficult to model realistically the way in which the expectations formation of the public is shaped by the policy process. In order to form expectations, the public must evaluate how the central bank's objective function will change and how such changes will alter the time series behavior of the monetary base.

## Policy Analysis

The nature of the model imposes a discipline on policy analysis. The model is dynamic, that is, it is concerned with how the monetary aggregates and the price level change over time. Furthermore, the public's expectations of the future values of these variables are shaped by the *process* the central bank uses to generate changes in the monetary base. (That is, the public's expectations depend upon the  $\theta$  parameters the central bank chooses to govern monetary base creation.) It follows that one can use the model to ask what happens when the central bank takes a particular *policy action* only if the *policy process* that generated the particular action is also specified. That is, one must know the  $\theta$  values of (4). For example, the model cannot be used to predict the effect on the money stock and the price level of a change in the monetary base of a given amount, if that change is all that is specified. The reason is that the effect of a particular policy action depends upon the public's expectation of subsequent policy actions, and this expectation depends upon the nature of the policy process.<sup>5</sup>

### III. RATE SMOOTHING

Insight into the way the model works can be gained by using it to understand how the central bank smooths interest rates. The central bank can smooth fluctuations in the market rate in two ways. Assume that the policy process that governs the behavior of the monetary base makes  $\theta_{\text{smooth}}$  positive. Assume also that  $\theta_{\text{drift}}$  is greater than zero. (There is at least some subsequent offset of random variations in the monetary base.) Consider an unanticipated, positive real sector disturbance ( $Q_t$ ), for example, a technological innovation that increases investment. This disturbance increases the market rate and the central bank responds by increasing the monetary base. The money stock and the price level rise. The price level will now exceed the value the public had predicted last period, so real output rises. Because

<sup>5</sup> In the model, for example, the effect of a change in the monetary base can only be predicted with an understanding of how the market rate is affected. The public, however, in order to set the market rate, must form an expectation of the future price level. (It needs this expectation to estimate the inflation premium to put into the market rate.) In order to form an expectation of the future price level, it must know the value of the base drift parameter. The reason is that the base drift parameter determines the extent to which the change in the monetary base will be incorporated permanently into the future level of prices.

of the rational expectations assumption, both the error in predicting the price level and the associated rise in output will be transitory. Because the rise in output is transitory, the public saves a relatively high proportion of it. This increased saving offsets to some extent the initial rise in the real rate of interest and in the market rate.

The market rate is also smoothed as a consequence of the interaction between the rate smoothing and base drift parameters. As noted above, with a positive  $\theta_{\text{smooth}}$  parameter, the positive real sector disturbance increases the money stock and raises the price level. Because the central bank is assumed not to allow complete base drift, the public will expect that the central bank will offset next period at least some of the current period's increase in money. The public will then expect that, after adjusting for trend growth, the money stock and the price level will be higher in the present period than in the next period. The expected future one-period inflation rate will fall below trend. A fall in the premium in the market rate for expected inflation will mitigate the rise in the market rate caused by the real disturbance.

### IV. MONEY STOCK AND PRICE LEVEL DETERMINATION

#### Graphical Analysis and Determinacy of the Price Level

The determination of the money stock and the price level is shown graphically in Figure 1. The inverse of the price level (the goods price of money) is shown on the vertical axis. The nominal amounts of money demanded and supplied are shown on the horizontal axis. The nominal money demand and supply schedules are derived by substituting (1) into (2) and (3), respectively. The money demand (supply) schedule then expresses the relationship between the price level and nominal money demand (supply) given a partially-reduced form that assumes fixed values for price level expectations ( $E_{t-1}P_t$  and  $E_tP_{t+1}$ ) and the monetary base, but allows the interest rate and output to vary.

Before discussing these schedules, it is useful to note that, given the public's prior expectation of the contemporaneous price level ( $E_{t-1}P_t$ ), a rise in the contemporaneous price level ( $P_t$ ) produces a positive price level forecast error, that is,  $[P_t - E_{t-1}P_t]$  becomes positive. As a result, there is a transitory increase in output. Also, under the assumption that both  $E_{t-1}P_t$  and  $E_tP_{t+1}$  are fixed, a rise in the price level lowers the market rate of interest in two ways.

First, the transitory increase in output just described increases saving, which lowers the real rate of interest. Second, an increase in the price level reduces the expected one-period rate of inflation, that is,  $(E_t P_{t+1} - P_t)$  declines. The market rate then declines from a reduction in the inflation premium.

With this discussion in mind, now consider the ways in which money demand is increased by a rise in the price level (a fall in the inverse of the price level). First, a rise in the price level produces a direct proportional increase in the demand for money. Second, money demand is increased by the increase in output produced by a positive price level prediction error. Third, the fall in the market rate of interest produced by the price rise increases money demand.

Consider next the effect of a price rise on nominal money supply. A rise in the price level causes the market rate to decline for the reasons mentioned above. This decline in the market rate decreases the money supply by lowering the value of the money multiplier function,  $F_{mm}$ , for a given value of the monetary base.

The money stock and the price level are endogenously determined through the intersection of the money demand and supply schedules. These variables possess well-defined equilibrium values because

of the existence of these schedules. If the price level falls below its equilibrium level, the nominal amount of money supplied exceeds the nominal amount of money demanded, and the price level returns to its equilibrium value, and conversely. The nominal money demand and supply schedules exist because the central bank's policy process (4) permits the public to form an expectation of the future price level ( $E_t P_{t+1}$ ).

This policy process specifies the  $\theta$  parameters upon which  $E_t P_{t+1}$  depends.<sup>6</sup> These parameters derive from the objectives of the central bank as summarized in (5). One can, therefore, ask the question, "How are nominal variables made well defined?" by asking "What characteristics must the central bank's objective (loss) function possess in order to permit the public to form an expectation of the future price level?" With the loss function (5), the central bank cares about the contemporaneous price level (through the first right-hand term) and the change in the price level (through the second right-hand term). This loss function, therefore, constrains the behavior of nominal variables sufficiently for the public to be able to form an expectation of the future price level. In short, it is the central bank that gives nominal variables (the price level and money stock) equilibrium values.

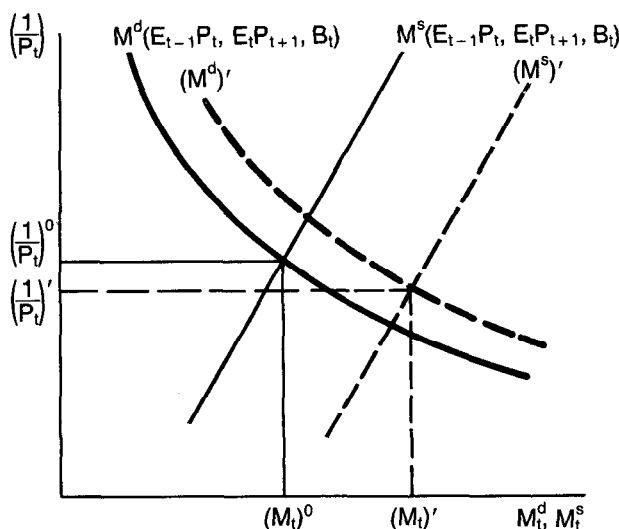
### The Effect of Macroeconomic Disturbances on the Money Stock

Consider first the way in which an unexpected, positive real sector disturbance ( $Q_t$ ) influences the money stock and the price level with rate smoothing ( $\theta_{smooth}$  greater than zero) and base drift ( $\theta_{drift}$  less than one). As the market rate begins to rise, the central bank supplies reserves and the money supply schedule shifts rightward. In Figure 1,  $M^s$  shifts to  $(M^s)'$ . Two opposing forces shift the position of the nominal money demand schedule. On the one hand, an increase in the market rate shifts it leftward. On the other, the unexpected increase in the monetary base and the money stock requires a higher price level than the public had expected, so real output rises. The increase in output shifts the money demand schedule rightward. In Figure 1, the net result is assumed to yield a rightward shift from  $M^d$  to  $(M^d)'$ .

<sup>6</sup> The solution for  $E_t P_{t+1}$  yielded by (1)–(4) includes values of the  $\theta$  parameters in all its terms. These terms are a) a constant; b) the value of the monetary base in the prior period multiplied by the two-period growth rate  $(1 + \theta_{trend})^2$ ; c) a negative term,  $\theta_{drift}$ , multiplied by the prior period's unexpected change in the monetary base; d) a term,  $\theta_{smooth}(1 - \theta_{drift})$ , multiplied by a linear combination of the monetary and real disturbances.

Figure 1

### NOMINAL MONEY DEMAND AND SUPPLY SCHEDULES



Note: The variables in parentheses ( $E_{t-1}P_t$ ,  $E_tP_{t+1}$ ,  $B_t$ ) are held constant while  $P_t$  and  $M_t$  change. The dashed lines show the effect of a positive real sector disturbance.

The rightward shift in the money supply schedule dominates the rightward shift in the money demand schedule, and the price level rises.<sup>7,8</sup>

Consider next the effect of a positive money demand disturbance ( $V_t$ ) with significant rate smoothing ( $\theta_{smooth}$  large) and significant base drift ( $\theta_{drift}$  near zero). Because the model is dynamic and accounts for the way the policy process affects the expectations of the public, it yields strikingly different results than the standard static models of money stock determination. The following example illustrates that, when there is base drift, rate smoothing does not insulate the price level and the real sector from money demand shocks. The positive disturbance to money demand causes an incipient increase in the market rate. For a large value of  $\theta_{smooth}$ , the central bank increases the monetary base by enough to make the money supply schedule shift rightward in line with the money demand schedule. In Figure 2,  $(M^d)$  and  $(M^s)$  shift rightward by the same amount to  $(M^d)'$  and  $(M^s)'$ , with no effect on the price level. In the absence of base drift, there are no further effects. The price level and real variables are unaffected.

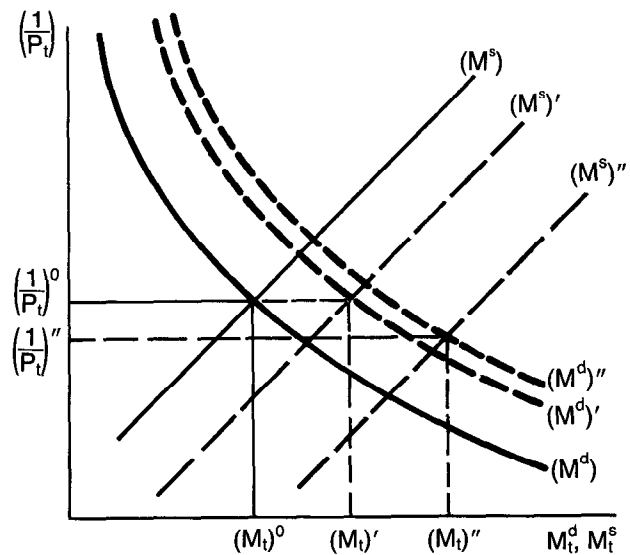
If the central bank allows base drift, however, the money demand disturbance will increase permanently the level of the monetary base and the money stock. Because the model assumes that the increase in money demand due to the monetary disturbance is transitory, the public will expect a higher price level next period. The expected one-period inflation rate will rise, and the market rate will start to rise further

<sup>7</sup> These shifts in the money demand and supply schedules are the primary shifts due to a  $\theta_{smooth}$  greater than zero. There are secondary shifts (not shown in Figure 1) due to the interaction between  $\theta_{smooth}$  and  $\theta_{drift}$ . The real sector disturbance produces a higher money stock in the contemporaneous period. The increase in the price level required by the higher money stock, however, is mitigated by the rise in the demand for real money produced by the higher level of output. The rise in real output is transitory. The existence of base drift in the monetary aggregates implies that, in contrast, at least some of the increase in the money stock is permanent. Consequently, the public will expect (adjusting for trend growth) a price level in the future that is higher than the contemporaneous price level. The inflation premium in the market rate will rise. The consequent rise in the market rate will cause the central bank to increase further the monetary base. The shifts in the money demand and supply schedules shown in Figure 1 are then amplified. The initial change in the money stock is proportional to  $\theta_{smooth}$ . The additional change is proportional to the product  $\theta_{smooth}(1 - \theta_{drift})$ . With no base drift ( $\theta_{drift}$  equal to one), there are no secondary effects.

<sup>8</sup> As noted above, the rise in the price level, relative to both the prior period's expectation of the price level and the contemporaneous expectation of next period's price level, affects the public's savings behavior and inflationary expectations in a way that mitigates the rise in the market rate.

Figure 2

## NOMINAL MONEY DEMAND AND SUPPLY SCHEDULES



Note: The dashed lines show the effect of a positive disturbance to money demand. The dashed lines marked by a double prime show that part of the effect due to base drift.

due to an increase in the inflation premium. In response, the central bank will then increase the monetary base again, and the money supply schedule will shift rightward again. In Figure 2,  $(M^s)'$  shifts rightward to  $(M^s)''$ . As in the case of the real sector disturbance, the price level rises and real output is stimulated. It then follows that  $(M^d)'$  shifts to  $(M^d)''$ .<sup>9</sup> Rate smoothing does not insulate the real sector from monetary disturbances.<sup>10</sup>

<sup>9</sup> The increase in output increases saving. Increased saving lowers the real rate and offsets the increase in the market rate caused by the increase in the inflation premium. The rise in the market rate caused by the money demand disturbance is, therefore, mitigated. These secondary effects from the money demand disturbance are analogous to those described in footnote 7.

<sup>10</sup> The model is constructed with nominal money demand and supply schedules that derive from different behavioral relations. The money demand schedule comes from (2), the real money demand function. The money supply schedule comes from (3), the money-multiplier function. The determinants of real money demand and nominal money supply are different. The model, therefore, makes the quantity-theory assumption that macroeconomic disturbances will produce divergent shifts in the nominal money demand and supply schedules. In the jargon of econometrics, the model assumes that the money demand and supply schedules are identified. Independent shifts in these schedules occur that permit the econometrician to use actual observations on the money stock and the price level to identify separate demand and supply schedules.

## The Central Bank and the Behavior of the Price Level

Although with rate smoothing the monetary base is determined endogenously, the procedure the central bank puts into place for altering the monetary base determines how the monetary base, the money stock, and the price level are affected by macroeconomic disturbances. Furthermore, while particular random realizations of the monetary base are produced by macroeconomic disturbances, the timeseries behavior of the monetary base is largely determined by the central bank. The  $\theta$  parameters, which are set by the central bank, determine the general behavior over time of the monetary base and also the time series behavior of the money stock and the price level.

The rate smoothing parameter ( $\theta_{\text{smooth}}$ ) determines the variability of the monetary aggregates and price level. A higher value of  $\theta_{\text{smooth}}$  requires increased variability in the monetary aggregates and, after some point, increased variability in the price level. The trend growth rate parameter ( $\theta_{\text{trend}}$ ) determines the trend growth rate of the monetary aggregates and the price level. With a positive value of  $\theta_{\text{trend}}$ , the money supply schedule ( $M^s$ ) shifts rightward over time down the money demand schedule ( $M^d$ ) at the rate given by  $\theta_{\text{trend}}$ . The price level rises at the rate given by  $\theta_{\text{trend}}$ . Sustained inflation is always and everywhere a monetary phenomenon [Friedman (1968)].

The base drift parameter  $\theta_{\text{drift}}$  determines how a change in the money stock shifts the initial position of the money supply schedule in the subsequent period. With a value of  $\theta_{\text{drift}}$  different from one, transitory macroeconomic disturbances shift permanently the position of the money supply schedule. In this way, transitory disturbances are incorporated permanently into the price level. An implication of the model is that a random walk in prices is always and everywhere a monetary phenomenon. The model is special in that it does not allow for a permanent component to real sector and money demand disturbances. If these disturbances possessed a significant permanent component, base drift in the price level could still occur even in the absence of base drift in the monetary aggregates. There would, however, still be truth to the statement that a random walk in prices is a monetary phenomenon. The central bank can have any time series behavior of the price level it desires. For example, nonstationary behavior in the price level could never arise if the central bank had price stability as one of its objectives. Such an objective would introduce into the central bank's objective function a term like  $k(P_t - \bar{P})$ , where  $k$  is a constant and  $\bar{P}$  is the central bank's stable price level objective.

## V. POLICY CHOICES FACED BY THE CENTRAL BANK

The model makes possible a comparison of alternative policies by elucidating the trade-offs made in selecting one policy rather than another. First, the model identifies those policies that do not require the policymaker to make trade-offs among objectives. When it is necessary to make trade-offs, the model clarifies their nature. The policymaker can ask, "In order to gain the benefits from adoption of a particular policy, what benefits must be foregone by rejection of alternative policies?"

### When Must the Policymaker Trade Off?

The standard discussion of trade-offs in policymaking is by Tinbergen (1967). Tinbergen points out that the policymaker with multiple objectives need not make compromises when seeking to attain these objectives if he possesses as many policy instruments as he has objectives. Attainment of the objectives of policy is then constrained only by the structure of the economy. If the number of objectives exceeds the number of policy instruments, the policymaker must make a choice about the relative importance of each objective. An increase in the significance attached to one objective necessarily decreases the significance that can be attached to the other objectives. This section reformulates Tinbergen's discussion in terms of the dynamic model used here.

In order to discuss policy choices, it is necessary to posit an objective function. An objective function makes explicit the central bank's objectives and the relative importance it assigns to achievement of its different objectives. In (5), the objective function is expressed as a loss function that the central bank attempts to minimize through the choice of the  $\theta$  parameters in (4). The parameters  $\beta$  and  $\gamma$  express the relative importance the central bank assigns to achievement of the two objectives of economic stabilization and inflation stabilization.

The central bank has available two degrees of freedom ( $\theta_{\text{smooth}}$  and  $\theta_{\text{drift}}$ ) to use in pursuit of its objectives. It can vary these parameters in order to influence the way macroeconomic disturbances affect the relationship between the contemporaneous price level and the prior period's expectation of this variable. Also, it can vary these parameters in order to influence the way macroeconomic disturbances affect the relationship between the contemporaneous price level and the contemporaneous expectations of next period's price level. These variations in the

price level and the contemporaneous expectation of next period's price level. These variations in the policy process can only be effected through changes in  $\theta_{\text{smooth}}$  and  $\theta_{\text{drift}}$ . Under the assumption that the public's expectations are formed rationally, the central bank's choice of the trend growth-rate parameter ( $\theta_{\text{trend}}$ ) cannot affect the first relationship and affects the second relationship only by the addition of a constant. The choice of a value for  $\theta_{\text{trend}}$  greater than zero does not help the central bank attain any of its macroeconomic objectives.<sup>11</sup>

With the loss function (5), the central bank possesses two objectives and possesses two degrees of freedom for manipulating the behavior of the monetary base. The central bank is not forced to trade off between achievement of its objectives. It can minimize the variability of inaccurate forecasts of the price level without reducing its ability to minimize the variability of expected inflation, and vice versa. Its pursuit of each objective is constrained only by the inherent uncertainty caused by random macroeconomic disturbances. Formally, this result shows up in the central bank's choices of  $\theta_{\text{smooth}}$  and  $\theta_{\text{drift}}$  that minimize (5). The optimal values of the  $\theta$ s do not depend upon the relative magnitudes of  $\beta$  and  $\gamma$ . Even if the central bank were to weight heavily one objective, it would not have to sacrifice achievement of its other objective.

### The Optimal Choice of $\theta_{\text{smooth}}$

The optimal value of the rate smoothing parameter increases as the variability of money demand disturbances (the variability of the  $V_t$ ) rises relative to the variability of the real sector disturbances (the variability of the  $Q_t$ ). Increases in the value of the rate-smoothing parameter up to its optimal value reduce variability in the price level and reduce undesirable fluctuations in output. Further increases raise the variability of the price level and increase fluctuations in output. This result can be understood by considering the allocative role played by the interest rate in the price system.

The real rate of interest is a price (the price of current output in terms of future output) whose varia-

tions distribute aggregate demand across time. The interest rate varies in order to cause the goods market to clear at a level of output compatible with aggregate supply. A change in the interest rate due to a disturbance in money demand, however, offers a misleading signal for intertemporal resource allocation. The greater the importance of disturbances from the monetary sector relative to disturbances from the real sector, the more frequently changes in interest rates will be misleading guides to resource allocation and the higher the optimal value of the rate-smoothing parameter. If monetary disturbances are large relative to real disturbances, it is desirable for the central bank to supply the monetary base in a way that smooths fluctuations in the market rate.

### The Optimal Choice of $\theta_{\text{drift}}$

One striking result derived from minimizing (5) is that it is optimal for the central bank to eliminate completely base drift. This result can be understood intuitively. The optimal value of the rate smoothing parameter puts an amount of interest rate sensitivity into the monetary base that reflects the likelihood that an interest rate fluctuation is due to a money demand disturbance. Because such disturbances are assumed to be transitory, there is no reason to allow fluctuations in the monetary aggregates due to fluctuations in the market rate to affect permanently the money stock.<sup>12</sup> Base drift would increase the variability of expected inflation, the second right-hand term in (5), without reducing the variability of inaccurate forecasts of the price level, the first right-hand term in (5).

### Trade-offs in the Choice of Policies

The loss function (5) cannot explain the actual time series behavior of the monetary aggregates and the price level. An obvious problem with (5), given the result noted in the preceding paragraph, is that it cannot explain the significant amount of base drift

<sup>11</sup> A loss function like (5) that contains only macroeconomic objectives cannot rationalize a positive rate of inflation. Barro and Gordon (1983) attempt to explain the existence of positive inflation in a model like the one here in that the central bank understands the structure of the economy. Their explanation turns on the discretionary character of policy (the inability of the central bank to precommit itself to a particular objective function) and a desire by the central bank to lower persistently the value of a real variable like unemployment. Hetzel (1988) explains inflation as a way of generating revenue through an inflation tax.

<sup>12</sup> If there is a permanent component to either money demand disturbances or real output disturbances and if the central bank desires to render the price level stationary, it needs to allow some amount of base drift in the monetary base and the money stock [Walsh (1986)]. Whether shocks to the money demand function exercise a transitory or a permanent effect upon the demand for money is an empirical issue. (In fact, it appears to depend upon the monetary aggregate considered. M1 velocity appears to be a random walk, but M2 velocity is stationary. There may be a permanent element to disturbances in real output, although the time series behavior of output is disputed by economists.) In any event, the nonstationarity in the price level that appeared after countries abandoned the gold standard for a paper money standard can only be explained by the nonstationarity introduced into the monetary base at this time.



in these variables [Broaddus and Goodfriend (1984)]. A loss function [from Goodfriend (1987)] that can explain base drift is shown in (6).

$$(6) \quad C = \alpha \text{ var}[r_t - E_{t-1}r_t] + \beta \text{ var}[P_t - E_{t-1}P_t] + \gamma \text{ var}\left[\frac{E_t P_{t+1}}{P_t} - 1\right]$$

With (6), the central bank attempts to minimize the variability of three variables: the market rate of interest, inaccurate price level forecasts, and expected inflation. Minimization of this loss function can generate the kind of base drift that has characterized nominal variables in the post-World War II era.

Now, while the central bank possesses three objectives, it still has only two degrees of freedom for varying its policy process; consequently, it must trade off among achievement of its objectives. Minimization of (6) implies that in general the central bank will allow base drift. It will also set a higher value for the rate smoothing parameter than is optimal for minimizing fluctuations in real output. The central bank trades off achievement of reduced variability in price level forecast errors and expected inflation in order to obtain a reduction in variability in the market rate. With (6), in contrast to (5), the optimal values of the  $\theta$  parameters depend upon the ratios of the trade-off parameters:  $\alpha$ ,  $\beta$  and  $\gamma$ .

## VI.

### CLARIFYING THE MONETARY RESPONSIBILITIES OF THE CENTRAL BANK

#### The Role of Money in the Formulation of Monetary Policy

There is an ongoing debate over the importance to assign to the behavior of money in the formulation of monetary policy. With the financial deregulation of the early 1980s and the resulting uncertainty over the behavior of the public's M1 demand function, this debate has centered on the contention that the role of money should be reduced when money demand is highly variable.<sup>13</sup> For example, Stephen

<sup>13</sup> The nationwide introduction of NOW accounts in 1981 and their incorporation into M1 altered the character of the public's demand for M1. Because NOWs pay explicit interest, they have caused M1 to become more highly substitutable with deposits used for saving, rather than for transactions. Because that part of M2 that is not included in M1 contains primarily savings-related deposits, M1 including NOWs has become more highly substitutable with the non-M1 component of M2. The result has been to alter the character of the public's M1 demand function. [The character of the M2 demand function has remained un-

Axilrod (Staff Director of the Office for Monetary and Financial Policy at the Board of Governors until July 1986) uses the increased uncertainty over the behavior of money demand to explain the de-emphasis of money in the policy process after 1982:

... money became less useful as a policy instrument because of a combination of market developments and attitudinal shifts that made it more unstable in relation to the economy and its own history. So money was de-emphasized after 1982 for pragmatic economic reasons. [Axilrod (1988), p. 59]

[See also Axilrod (1985), p. 17.]

The most important aspect of monetary policy is the central bank's objective function. The objective function determines the policy process (4) through the values set for the  $\theta$  parameters. This policy process can be formulated with the monetary base as the left-hand variable or the interest rate as the left-hand variable. In the former formulation (the one employed here), it is natural to discuss monetary policy in terms of the behavior of the monetary aggregates. In the latter formulation, it is natural to discuss monetary policy in terms of the behavior of the interest rate. In actual practice, central banks have not usually formulated policy discussions in terms of the behavior of the monetary aggregates. Instead, they have discussed monetary policy in terms of the behavior of the discount rate and its effect on money market rates.<sup>14</sup>

There is, however, a reason to discuss monetary policy in terms of the behavior of the monetary aggregates. The time series behavior of the aggregates translates into the time series behavior of the price level more directly than is the case with the interest rate. A given increase in the trend rate of growth of the monetary aggregates ( $\theta_{\text{trend}}$ ) implies the same increase for the trend rate of inflation. An increase in rate smoothing ( $\theta_{\text{smooth}}$ ) beyond an optimal value implies an increase in the variability of the price level. Base drift in the monetary aggregates ( $\theta_{\text{drift}}$  different from one) implies base drift in the price level (apart

changed [Hetzl and Mehra, 1988].] As a result, a debate has occurred over the usefulness of M1 in the policy process. Also, M1 has often been accepted as *the* definition of money. Consequently, the debate has often taken the form of whether money can play a role in the formulation of monetary policy when money demand is highly variable. [Angell (1987) and Johnson (1988), for example, have sought replacements, at least temporarily, for money in policy discussions. Their work concentrates on the role of money as an indicator variable.]

<sup>14</sup> In the United States, the Federal Reserve System has modified the traditional discount rate procedure by choosing a combination of the discount rate and borrowed reserves. The market rate is then determined as the sum of the discount rate and some positive amount that is proportional to the level of borrowed reserves.

from whatever drift is allowed to compensate for any permanent component in disturbances to money demand and output). This connection between the time series behavior of the monetary aggregates and the price level holds regardless of the degree of variability in money demand.

The importance of clarifying the implications of monetary policy for the behavior of the price level is increased due to the indirect way the behavior of the price level is produced in actual practice. Typically, the central bank possesses multiple macroeconomic objectives.<sup>15</sup> It has, however, only the two degrees of freedom for pursuing these objectives given by the two parameters ( $\theta_{smooth}$  and  $\theta_{drift}$ ) that alter the time series behavior of the monetary base. The behavior of the price level (and the monetary aggregates) emerges as a by-product of the trade-offs the central bank must make in order to pursue multiple objectives with a more limited number of degrees of freedom to manipulate in setting the policy process. Furthermore, the central bank's objective function is not made explicit in policy discussions. The trade-offs that must be made in pursuit of multiple objectives are not discussed explicitly. The link between these trade-offs and the price level is obscured when the central bank's objective function is not made explicit in policy discussions and when policy is discussed in terms of interest rates (or a money market proxy). In contrast, the implications for the price level of these trade-offs are clearer when policy is discussed in terms of the monetary aggregates.

### Explicit Targets for the Money Stock

Determination of the monetary base in part on the basis of current conditions in the money market obscures the responsibility of the central bank for the behavior of the monetary base, the money stock, and the price level. The consequent endogeneity of

---

<sup>15</sup> The *Federal Reserve Act* stipulates that the Federal Open Market Committee should set its ranges for the monetary aggregates "taking account of past and prospective developments in employment, unemployment, production, investment, real income, productivity, international trade and payments, and prices" [Board of Governors(1984)].

the monetary base facilitates special factors explanations of inflation, that is, explanations that confine the causes of inflation to the macroeconomic disturbances that impinge upon the economy.<sup>16</sup> Endogenous determination of the monetary base also obscures the way the central bank gives the money stock and the price level well-defined equilibrium values. In the absence of explicit limitation of the monetary base, the money stock and the price level are made well-defined economic variables by the way the central bank determines the public's expectation of the future price level. This indirectness obscures central bank responsibility.

In order to clarify the way it determines the behavior of prices, the central bank could formulate the policy process in terms of the monetary aggregates. The central bank could select a single definition of the money stock as a substantive intermediate target and explain to the public the relationship that it believes will hold over time between the money stock and an explicitly formulated path for the price level.<sup>17</sup> The central bank could also use the monetary base as the policy variable it sets in order to achieve its intermediate money target. [See Black (1986).]

### Conclusion

Clarification of the monetary responsibilities of the central bank requires an explicit statement of the objectives of the central bank and the relative importance attached to these objectives. It also requires an explicit statement of how the central bank believes its monetary policy will achieve its objectives. It is necessary to make explicit the consequences of monetary policy for the behavior of the price level. Hopefully, the model presented in this article will aid in discussion of the monetary responsibilities of the central bank.

---

<sup>16</sup> See Cullison (1988) for a discussion of the possible influence on monetary policy of special factors explanations of inflation in the 1970s.

<sup>17</sup> In order to make the money stock a substantive intermediate target, it would be necessary to make the decision whether to allow base drift in its targeted value an explicit part of the decision-making process.

## References

- Angell, Wayne D. "A Commodity Price Guide to Monetary Aggregate Targeting." Speech given at the Lehman Institute, New York, December 10, 1987. Board of Governors of the Federal Reserve System. Processed.
- Axilrod, Stephen H. "U.S. Monetary Policy in Recent Years: An Overview." *Federal Reserve Bulletin* 71 (January 1985): 14-24.
- . "What Really Went on in the Temple." *Across the Board* (March 1988), pp. 58-61.
- Barro, Robert J., and David B. Gordon. "A Positive Theory of Monetary Policy in a Natural Rate Model." *Journal of Political Economy* 91 (August 1983): 589-610.
- Black, Robert P. "A Proposal to Clarify the Fed's Policy Mandate." *Cato Journal* 5 (Winter 1986): 787-95.
- Board of Governors of the Federal Reserve System. *Federal Reserve Act and Other Statutory Provisions Affecting the Federal Reserve System, As Amended Through April 20, 1983*. Washington, D.C.: Board of Governors, 1984.
- Broaddus, Alfred, and Marvin Goodfriend. "Base Drift and the Longer Run Growth of M1: Experience from a Decade of Monetary Targeting." Federal Reserve Bank of Richmond *Economic Review* 70 (November/December 1984): 3-14.
- Cullison, William. "On Recognizing Inflation." Federal Reserve Bank of Richmond *Economic Review* 74 (July/August 1988): 4-12.
- Dotsey, Michael, and Robert G. King. "Monetary Instruments and Policy Rules in a Rational Expectations Environment." *Journal of Monetary Economics* 12 (September 1983): 357-82.
- Friedman, Milton. "Inflation: Causes and Consequences." In *Dollars and Deficits*. Englewood Cliffs, New Jersey: Prentice Hall, 1968.
- Goodfriend, Marvin. "Discount Window Borrowing, Monetary Policy, and the Post-October 6, 1979, Federal Reserve Operating Procedure." *Journal of Monetary Economics* 12 (September 1983): 343-56.
- . "Interest Rate Smoothing and Price Level Trend-Stationarity." *Journal of Monetary Economics* 19 (May 1987): 335-48.
- Goodfriend, Marvin, and Robert G. King. "Financial Deregulation, Monetary Policy, and Central Banking." Federal Reserve Bank of Richmond *Economic Review* 74 (May/June 1988): 3-22.
- Hetzel, Robert L. "The Political Economy of Inflation." Federal Reserve Bank of Richmond, April 1988. Processed.
- Hetzel, Robert L., and Yash Mehra. "The Behavior of Money Demand in the 1980s." Federal Reserve Bank of Richmond, June 1988. Processed.
- Johnson, Manuel H. "Recent Economic Developments and Indicators of Monetary Policy." Speech given to the Money Marketeers of New York University, New York, March 15, 1988. Board of Governors of the Federal Reserve System. Processed.
- Keynes, John Maynard. *A Tract on Monetary Reform* (1923). In *The Collected Writings of John Maynard Keynes*, vol. 4. London: The Macmillan Press, 1971.
- Lucas, Robert E., Jr. "Econometric Policy Evaluation, A Critique" (1976). Reprinted in *Studies in Business-Cycle Theory*. Cambridge, Mass.: MIT Press, 1983a, pp. 104-30.
- . "Econometric Testing of the Natural Rate Hypothesis" (1972). Reprinted in *Studies in Business-Cycle Theory*. Cambridge, Mass.: MIT Press, 1983b, pp. 90-103.
- McCallum, Bennett T. "Price Level Determinacy with an Interest Rate Policy Rule and Rational Expectations." *Journal of Monetary Economics* 8 (November 1981): 319-29.
- . "Some Issues Concerning Interest Rate Pegging, Price Level Determinacy, and the Real Bills Doctrine." *Journal of Monetary Economics* 17 (January 1986): 135-60.
- Poole, William. "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model." *Quarterly Journal of Economics* 84 (May 1970): 197-216.
- Sargent, Thomas J. "The Demand for Money during Hyperinflations under Rational Expectations" (1977). In *Rational Expectations and Econometric Practice*, edited by Robert E. Lucas, Jr. and Thomas J. Sargent, vol. 2. Minneapolis, Minn.: The University of Minnesota Press, 1981a, pp. 429-52.
- . "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment" (1973). In *Rational Expectations and Econometric Practice*, edited by Robert E. Lucas, Jr. and Thomas J. Sargent, vol. 1. Minneapolis, Minn.: The University of Minnesota Press, 1981b, pp. 159-98.
- Tinbergen, Jan. *Economic Policy: Principles and Design*. Chicago: Rand McNally, 1967.
- Walsh, Carl E. "In Defense of Base Drift." *American Economic Review* 76 (September 1986): 692-700.

## APPENDIX

Equation (1) is an IS function. It shows the combinations of real output and the real rate of interest that equate the public's desired saving and investment.

$$(1) \quad y_t = d_0 + d_1[r_t - (E_t p_{t+1} - p_t)] + w_t \\ d_1 < 0 < d_0$$

Real output is  $y_t$ ; the interest rate  $r_t$ ; and the price level  $p_t$ . (All the variables are logarithms, except for  $r_t$ .) The subscript  $t$  indicates the time period.  $E$  is the expectations operator with the subscript indicating the time period at which the expectation was formed. The variable  $w_t$  is a serially-uncorrelated, zero-mean random shock. Equation (2) is an aggregate supply function.

$$(2) \quad y_t^s = d_2 + d_3[p_t - E_{t-1}p_t] \quad 0 < d_2, d_3$$

The public's supply of goods varies positively with its error in predicting the contemporaneous price level. It is assumed that this monetary nonneutrality arises from one-period contracts specified in nominal terms.

Next, equate the IS function (1) and the aggregate supply function (2) in order to eliminate output  $y_t$ ; solve the resulting expression for  $r_t$ ; and simplify the notation for the coefficients and error term ( $q_t$  is a transformation of  $w_t$  and remains a serially-uncorrelated, zero-mean random error).

$$(3) \quad r_t = (E_t p_{t+1} - p_t) + c_1 \\ + c_2[p_t - E_{t-1}p_t] + q_t \quad c_2 < 0 < c_1$$

The market for the quantity of money is described by a money demand function and a money supply function. The money demand function is

$$(4) \quad m_t^d = p_t + a_0 + a_2 r_t + a_4 y_t + v_t. \\ a_2 < 0 < a_0, a_4$$

Nominal money demand ( $m_t^d$ ) depends positively upon the price level ( $p_t$ ) and real output ( $y_t$ ) and negatively upon the market rate of interest ( $r_t$ ). The variable  $v_t$  is a serially-uncorrelated, zero-mean random shock.

The money supply function is

$$(5) \quad m_t^s = b_1 + B_t + b_2 r_t + x_t. \quad 0 < b_1, b_2$$

$B_t$  is the (log of the) monetary base. The term  $b_1$  is a constant that captures the effect on the money-multiplier of the legal required reserve ratio. The term

$b_2 r_t$  captures the effect on the multiplier of the interest sensitivity of excess-reserves and currency-deposit ratios. The variable  $x_t$  is a serially-uncorrelated, zero-mean shock to the value of these ratios.

Equation (6) describes the behavior of the central bank.

$$(6) \quad B_t = B_{t-1} + \theta_1[r_t - E_{t-1}r_t] \\ - \theta_2[B_{t-1} - E_{t-2}B_{t-1}] + \theta_3$$

The central bank specifies three parameters (the three  $\theta$ s) that determine the time series behavior of the monetary base. The parameter  $\theta_3$  is the trend rate of growth of the base. The parameter  $\theta_1$  determines the interest elasticity of the monetary base. It specifies the extent to which the central bank smooths movements of the market interest rate around a reference level  $E_{t-1}r_t$ . The central bank cannot smooth the market rate around an arbitrary level. It is constrained to smooth around the prior period's expectation of the market rate. This expectation is the sum of the expected real rate and of the trend rate of inflation. (The solution of the model is determinate only for this value.) Specifically, the central bank must smooth the market rate around the value  $(c_1 + \theta_3)$ , which is  $E_{t-1}r_t$  from (3). The variable  $[r_t - (c_1 + \theta_3)]$  measures innovations (unpredictable changes) in the market rate. Interest-rate innovations cause the central bank to produce innovations in the monetary base, which, from (6), equal  $\theta_1$  times the interest rate innovations.

The third term on the right side of (6) measures the extent to which the central bank offsets, in the contemporaneous period, last period's innovation in the monetary base. There are two general cases. In one case,  $\theta_2$  differs from one, so that there is not an exact offset to last period's innovation. The monetary base then behaves like a random walk with a persistence over time given by  $\theta_3$ . In the second case,  $\theta_2$  is one so that the central bank offsets exactly last period's innovation. In this case,  $E_{t-2}B_{t-1}$  can be defined as  $\theta_0 + \theta_3(t-1)$ , where  $t$  is the number of time periods that have elapsed since a base period 0. The constant  $\theta_0$  is the (log of the) monetary base at time 0. This expression defines a path for the monetary base that grows over time at the rate  $\theta_3$  and around which the monetary base fluctuates.

The model's equations are listed below. [Equation (7) comes from substituting  $y_t^s$  from the aggregate supply function (2) into (4), the money demand function, and simplifying the notation for the coefficients.]

$$(7) \quad m_t^d = p_t + a_1 + a_2 r_t + a_3 [p_t - E_{t-1} p_t] + v_t$$

$$a_2 < 0 < a_1, a_3$$

$$(8) \quad m_t^s = b_1 + B_t + b_2 r_t + x_t \quad 0 < b_1, b_2$$

$$(9) \quad B_t = B_{t-1} + \theta_1 [r_t - (c_1 + \theta_3)]$$

$$- \theta_2 [B_{t-1} - E_{t-2} B_{t-1}] + \theta_3$$

$$(10) \quad r_t = (E_t p_{t+1} - p_t) + c_1$$

$$+ c_2 [p_t - E_{t-1} p_t] + q_t \quad c_2 < 0 < c_1$$

The model is completed with a cost function [from Goodfriend (1987)] for the central bank. Var is variance.

$$(11) \quad C = \beta \text{var}[p_t - E_{t-1} p_t]$$

$$+ \gamma \text{var}[E_t p_{t+1} - p_t] \quad 0 < \beta, \gamma$$

With the assumption that money demand equals money supply and the assumption of rational expectations, equations (7)-(10) can be solved for  $m_t$ ,  $p_t$ ,  $r_t$ , and  $B_t$ . The solutions for the contemporaneous price level prediction error  $[p_t - E_{t-1} p_t]$  and for expected inflation  $[E_t p_{t+1} - p_t]$  are substituted into (11). With these substitutions, the central bank's cost function is expressed in terms of the  $\theta$  parameters that characterize the behavior of the monetary base. The central bank chooses these parameters in order to minimize (11).

The  $\theta$  parameters that minimize (11) are

$$(12) \quad \theta_1 = a_2 - b_2 + [(1 + a_3)\sigma_q^2]^{-1}[(1 - c_2)\sigma_{v-x}^2]$$

$$(13) \quad \theta_2 = 1.$$

$\sigma_q^2$  and  $\sigma_{v-x}^2$  are the variance of  $q_t$  and  $(v_t - x_t)$ . [See Poole (1970, p. 208) for a solution for  $\theta_1$  in a static model.] The model does not determine a value for  $\theta_3$ . Note that the values of  $\theta_1$  and  $\theta_2$  that minimize

(11) also minimize each term of (11) separately. Consequently, the  $\beta$  and  $\gamma$  parameters do not enter into the expression for the  $\theta$ s. The central bank is not forced to trade off among conflicting objectives.

Consider now the cost function (14).

$$(14) \quad C = \alpha \text{var}[r_t - (c_1 + \theta_3)]$$

$$+ \beta \text{var}[p_t - E_{t-1} p_t]$$

$$+ \gamma \text{var}[E_t p_{t+1} - p_t] \quad 0 < \alpha, \beta, \gamma$$

The  $\theta$  values that minimize this cost function involve the  $\alpha$ ,  $\beta$  and  $\gamma$  parameters. The central bank chooses the process for generating the monetary base in a way that reflects the relative importance it assigns to achieving conflicting objectives. With (14), the optimal rate smoothing parameter  $\theta_1$  is larger than the  $\theta_1$  in (12). The base drift parameter  $\theta_2$  is in general different from one. Its value is greater or less than one depending upon the parameters of the cost function (14) and the structural parameters of the model. For example,  $\theta_2$  is less than one if  $\alpha$  and  $\gamma$  are large and the magnitude of  $c_2$  is large.

(The structural parameters of the model,  $a_2$ ,  $a_3$ ,  $b_2$ , and  $c_2$ , are functions of the  $\theta$  parameters. Different objective functions imply different structural coefficients [Lucas (1976)].) For some issues, it is important to model how the policy process affects the structural relationships that summarize the economy. For example, the model incorporates a money demand function and a money supply function. Sargent (1981a) and Goodfriend (1983) discuss, respectively, the way in which a change in the monetary policy process affects the structural form of the money demand function and the money supply function. The major policy issue of interest here is the way different objective functions affect the general time series behavior of the price level. For this issue, the main assumption that must be made is that the signs of the structural parameters remain unchanged when the policy process that generates the monetary base changes.