Takeovers and Stock Price Volatility

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I. INTRODUCTION

There is now a large literature documenting the statistical relation between stock prices and dividends at the aggregate level. A robust finding is that stock prices are too volatile to be explained by subsequent changes in dividends. Observations of large market swings, like the crash of October 1987 and the mini-crash of October 1989, encourage the popular perception that stock prices are excessively volatile. While these observations have provoked a great deal of analysis, there has been little discussion of the possible link between excess stock price volatility and the fact that changes in the control of large corporations often take place via market acquisition of the outstanding shares. These transactions—takeovers—are often associated with dramatic increases in the price of the shares of the firm being acquired; these are called "takeover premia." In fact, some commentators argue that movements in the stock market in the 1980s, including the large market declines of October 1987 and October 1989, were linked to changes in takeover activity. In this article we explore the possible link between takeover activity and stock price volatility.

The explanation we propose relies on recent advances in our understanding of imperfections in the monitoring of firm managers. These imperfections imply that there is a "value of control" (we make this term more precise below) that is appropriable by the managers of a large corporation. This private value of control arises out of the delegation of decision-making authority that is intrinsic to the separation of ownership and control in the modern corporation. The value of control explains, in part, the premium often paid to shareholders to acquire control of a firm.

We will argue that the value of control, along with the probability that someone will be willing to pay it, can vary independently of the expected present value of dividends. This adds an independent source of variation to the price of the traded shares of publicly held corporations.

The plan of the paper is to first describe the Martingale Model of stock prices, often referred to as the "efficient markets theory." This serves as a benchmark, both for the excess volatility findings and for the alternative model we propose. We then survey some of the key empirical regularities concerning stock prices; these include the excess volatility finding in time series of aggregate stock prices, as well as the behavior of individual stock prices before and after control change transactions.

We then proceed to outline the essential elements of our model of the link between takeovers and stock prices. First, we describe imperfections in the relationship between a large firm's managers and the people who hold claims issued by the firm. Next, we describe the implications of these imperfections for some of the characteristics of the claims issued by the firm—specifically, the legal control mechanism associated with them. We argue that traded shares are bundled claims giving the holder the right to help determine the control of the firm as well as a claim on a stream of dividends. We then show how such shares can display excess volatility because of variations in the expected future value of the control right embedded in the claim. The final section describes some of the implications of these insights for policy and for economic theory. The appendix provides a more rigorous derivation of our model of excess volatility.

II. THE MARTINGALE MODEL OF STOCK PRICES

As a benchmark, consider a simple but general model of the determination of stock prices, the Martingale Model. The empirical findings of excess volatility that we describe below are essentially
contradictions between the properties of the Martingale Model and those of actual stock market data; stock prices are more volatile relative to dividends than is predicted by the Martingale Model. When we describe an alternative explanation for stock price volatility, a comparison of the predictions of the alternative model with those of the Martingale Model will be useful.

According to the Martingale Model—often referred to as the “efficient markets theory” or the “expected present value relation”—the price of a given stock at any given time is equal to the expected present value of the stream of future dividends that will accrue on that stock. To be more explicit, let \( p_t \) be the price of a share of stock at time \( t \) (after payment of dividends due at time \( t \)); let \( d_{t+s} \) be the amount paid in dividends paid at time \( t+s \), where the index \( s \) takes on the values \( 1, 2, 3, \ldots \). We abstract from inflation, and so we assume that \( d_{t+s} \) is the real value of dividends at time \( t+s \). We also abstract from stock splits or repurchases, and so the sequence of dividends, \( d_{t+s} \) for \( s = 1, 2, 3, \ldots \), captures the total return to an investor who purchases the share at time \( t \) and holds it to eternity. Note that from the point of view of an investor at the current date, the future stream of dividends is a sequence of random variables.

The Martingale Model asserts that there is a constant rate \( r \), where \( r > 0 \), at which future expected returns are discounted back to the present, and that the current price is related to next period’s price and next period’s dividend by the equation

\[
(1) \quad p_t = (1 + r)^{-1} E_t[pt+1 + d_{t+1}],
\]

where \( E_t[w_{t+s}] \) is notation for the expected value of a random variable \( w_{t+s} \), with the expectation taken using only information available at period \( t \). Equation (1) states that the current price of a stock equals the expected value of the sum of next period’s price and dividends, discounted back to the present at rate \( r \).\(^1\)

Equation (1) can be used to derive an equation relating the current stock price to the entire stream of future dividends. First, update equation (1) one period, replacing \( t+1 \) by \( t+2 \) and \( t \) by \( t+1 \), and substitute the resulting expression in (1) for \( p_{t+1} \) to get

\[
(2) \quad p_t = (1 + r)^{-1} E_t[ (1 + r)^{-1} E_{t+1}[p_{t+2} + d_{t+2} + d_{t+1}],
\]

where \( E_{t+1}[w_{t+2}] \) is the expected value of the random variable \( w_{t+2} \) given information available at time \( t+1 \). The law of iterated expectations implies that \( E_t[E_{t+1}[p_{t+2}]] = E_t[p_{t+2}] \). Equation (2) can then be rewritten as

\[
(3) \quad p_t = (1 + r)^{-1} E_t[d_{t+1}] + (1 + r)^{-2} E_t[p_{t+2} + d_{t+2}].
\]

If one repeats this substitution \( n \) times, the result is an equation relating \( p_t \) to the stream of dividends from period \( t+1 \) to period \( t+n \), plus the term \( (1 + r)^{-n} E_t[p_{t+n}] \). One can assume that this term converges to zero as \( n \) approaches infinity. This assumption rules out speculative bubbles. (We'll discuss this assumption below.) Under this assumption, the equation obtained as the limit of this repeated substitution is

\[
(4) \quad p_t = v^d_t,
\]

where \( v^d_t = \sum_{s=1}^{\infty} (1 + r)^{-s} E_t[d_{t+s}] \).

Equation (4) states that the current price of a stock equals \( v^d_t \), the present value of expected future dividends.

This model was first advanced by Paul Samuelson (1965), and is often called the “efficient capital markets model,” a term associated with Eugene Fama’s (1970) exposition. The model can be viewed as arising in particular classes of artificial economies. An artificial economy is just a particular mathematical specification of the preferences, technological opportunities, and informational abilities of economic agents, together with some notion of the mutual consistency of plans, or equilibrium. In one class of artificial economies that gives rise to the Martingale Model, agents are risk-neutral, discount the future at the same rate, and share common information and beliefs about future returns (see Lucas (1978)). In another such class there is a perfectly risk-free asset, and all randomness in stock returns is idiosyncratic to individual stocks (see Connor (1984)).

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\(^1\) Stephen F. Leroy (1989) calls this theory the Martingale Model. His article also contains an excellent description, history, and survey of empirical tests of the theory. This section follows his exposition.

\(^2\) A martingale is any random series \( \{w_t\} \) that always satisfies \( w_t = E_t[w_{t+s}] \). The model is called the Martingale Model because there is a simple variable that is a martingale—the present value of the value of a mutual fund that reinvests all dividend earnings. See Leroy (1989, pp. 1589-90).
In many other artificial economies, equation (1) does not always hold. However, there is usually a more general version of equation (1) that does hold. In general, the current stock price is related to the entire probability distribution governing the sum of next period's price and dividends, rather than just the mean, as in (1). This implies that risk premia can affect the current price of a stock, as in the Capital Asset Pricing Model or the Arbitrage Pricing Theory (see Connor (1984)). More general economies also imply that discount rates can vary over time, rather than remain constant as in (1). It turns out, however, that empirical contradictions of (1) or (4) do not seem to be attributable to risk premia or time-varying risk premia (see West (1988b)).

Even in economies in which equation (1) holds, the stock price may not satisfy equation (4) because of the presence of a speculative bubble. A stock price is said to contain a bubble if it can be written as

\[ p_t^b = p_t + b_t, \]

where \( p_t \) is given by equation (4), and \( b_t \) is the bubble term. In order for \( p_t^b \) to satisfy (1), it must be true that \( b_t = (1 + r)^{-n} \text{E}(b_t + 1) \). In fact, any random \( b_t \) series that satisfies this condition implies that \( p_t^b \) satisfies (1). There are an infinite number of \( b_t \) random variable series that satisfy this condition, so there are an infinite number of solutions, \( p_t^b \), that satisfy equation (1). Only one solution is consistent with (4), however, and that is the solution in which \( b_t = 0 \). Recall that in deriving equation (4) we assumed that the expression \( (1 + r)^{-n} \text{E}(p_t + 1) \) converges to zero as \( n \) grows very large. This effectively rules out any solution other than \( p_t^b = p_t \).

A negative value for \( b_t \) implies that there is a positive probability that the stock price is eventually negative, which is inconsistent with the free disposal of stocks. This case is conventionally ruled out. A positive value for \( b_t \) implies that the stock price is always above the fundamental value, given by equation (4). It is useful to keep in mind the properties of bubble solutions to equation (1) because our model of takeovers and stock prices predicts that an econometrician would be unable to reject the hypothesis that stock prices contain a bubble term.

III. SOME EMPIRICAL REGULARITIES IN STOCK PRICES

Aggregate Stock Prices

The Martingale Model has some strong implications for the joint behavior of stock prices and dividends. One of the most striking of these is an upper bound on the variability of stock prices relative to dividends. There is now a large literature, beginning with the seminal papers by Leroy and Portier (1981) and by Shiller (1981), that documents the failure of empirical data on stock prices and dividends to satisfy this inequality; see West (1988b) and Leroy (1989) for recent surveys.

To understand this variance bound, first define a variable \( c_{t+s} \) as the difference between the actual and expected dividends in period \( t+s \). In other words,

\[ c_{t+s} = d_{t+s} - \text{E}(d_{t+s}), \quad \text{for } s=1,2,3, \ldots \]

Then define a variable \( d_t^* \) as the present discounted value of actual dividends. Shiller called this the "ex post rational" stock price. This is what the price of the stock would be if the entire stream of future dividends were known with perfect certainty, and the Martingale Model, equation (4), determined the price. More explicitly,

\[ d_t^* = \sum_{s=1}^{\infty} (1 + r)^{-s} d_{t+s}. \]

Using these two definitions, we can obtain the following relation between \( p_t \) and \( d_t^* \)

\[ d_t^* = p_t + x_t^d, \]

where \( x_t^d = \sum_{s=1}^{\infty} (1 + r)^{-s} c_{t+s}. \)

Equation (8) states that the ex post rational price is equal to the actual current price plus the present value of the unexpected component of future dividends.

One immediate implication is that the current price is an unbiased forecast of the ex post rational price; in other words, \( p_t = \text{E}(d_t^*). \) This follows from the fact that \( \text{E}(c_{t+s}) = 0 \) because of the optimality of forecasts of future dividends. The optimality of forecasts also implies that the forecast errors, \( c_{t+s}, \quad s=1,2, \ldots, \) are uncorrelated with \( p_t \), and this implies that \( p_t \) and \( x_t^d \) are uncorrelated. Therefore,
we can derive the following relation between the variances of \( p_t \) and \( d_t^* \):

\[
(9) \quad \text{var}(d_t^*) = \text{var}(p_t) + \text{var}(x_t^d).
\]

Since variances are positive, equation (9) implies that the variance of stock prices has an upper bound:

\[
(10) \quad \text{var}(d_t^*) \geq \text{var}(p_t).
\]

The variance of the actual stock price can be no greater than the variance of the present value of actual future dividends.

The original tests of the inequality (10) were first published in 1981 by Leroy and Porter, and by Shiller; both papers reported violations that were large in magnitude and statistically significant. A large number of papers have appeared since developing or applying this inequality test (see the recent survey by West (1988b)). Some initial work questioned the finding of excess volatility on econometric grounds, arguing that small sample bias and/or the presence of unit roots in dividends may explain the results (see Flavin (1983), Marsh and Merton (1986), and Kleidon (1986)). Subsequent studies taking account of the possibility of unit roots and small sample bias "still tend to find substantial excess volatility" (West (1988b), p. 639).

Recent work has examined the possibility that risk aversion causes stock prices to deviate from the Martingale Model, as might be expected from more general theories of asset pricing (see Singleton (1987)). Allowing risk averse investors, however, fails to explain excess volatility. Other recent work has examined the possibility that the expected rate of return, \( r \), varies over time (see Campbell and Shiller (1988a and 1988b), and West (1988a)). Although this line of work is at a very preliminary stage, initial results suggest that the variance of the expected rate of return would have to be implausibly large to explain the excess volatility results. Consequently, many of the simplifications inherent in the Martingale Model do not seem to be responsible for the inconsistency between the model and the data.

Some researchers have examined whether the finding of excess volatility could be caused by speculative bubbles. It appears that empirical evidence on stock prices is consistent with the presence of bubbles, which is not surprising, because bubbles can take many forms (see West (1987 and 1988a), and Shiller (1984)). Bubbles are often associated (in many people's minds) with large sustained increases in asset prices followed by a sharp collapse, as in Tulipmania, the South Sea Bubble, and other famous cases (see Mackay (1852), but see also Garber (1989)).

Bubbles need not take such a spectacular form, however. In the model of takeovers and stock prices that we consider below, an econometrician examining data generated by the model would be unable to reject the conclusion that the stock price includes a bubble. But in our model, what appears to the econometrician to be a bubble term is uniquely determined and has an economic rationale—it is actually part of the fundamental of the stock, properly defined. Therefore, one way of interpreting our explanation of stock price volatility is that the characteristics of the financial claims of the modern corporation could give rise to what appears to be a bubble in stock prices. This exemplifies the point made by Hamilton and Whiteman (1985) and Hamilton (1986) that movements in the true fundamental that are unobserved by the econometrician are indistinguishable from bubbles.

**Takeovers and Individual Stock Prices**

The research discussed above focuses on the behavior of the aggregate stock price and dividend series. At the level of the individual firm, the relationship between the market for corporate control and stock prices has been extensively investigated using the "event study" methodology. This approach examines the behavior of share prices of participating firms around the date of the announcement of a takeover or other change in control. To the extent that stock price changes cannot be explained by a market model (the Capital Asset Pricing Model, for example), these abnormal changes are attributed to the takeover event. Much of the event study literature on takeovers was surveyed by Jensen and Ruback (1983). Averaging over the results of a large number of studies, Jensen and Ruback find that there is a 30 percent abnormal increase in the stock price of a target firm in the event of a tender offer takeover (a takeover executed by a direct purchase of shares). In the case of mergers, when there is agreement on the acquisition between the management of the acquiring and target firms, the gains in the target's stock price are substantially lower (20 percent). One might conclude, in these cases, that part of the premium that the acquirer is willing to
pay is going in some form to the incumbent management. When a change in control is executed through a proxy contest, with little or no direct purchase of shares by those acquiring control, the abnormal stock price change is much smaller (8 percent). In the cases of tender offers and mergers, Jensen and Ruback report that the abnormal changes in the stock prices of bidding firms are much smaller than those for the target firms; there is a 4 percent change for bidding firms in tender offers and no significant change in mergers.

Jensen and Ruback interpret the results from the event study literature as providing evidence that the market for corporate control reallocates productive resources from less to more efficient users (managements). That is, takeovers create value for shareholders because they result in an improved use of resources. One might call this the "inefficient management hypothesis." This hypothesis suggests a world in which some managements are better matched than others to the assets and activities of any given firm. Hence, in this view, the market for corporate control is a market in which managers search for and acquire firms to which they are well matched.

Like the inefficient management hypothesis, the process described in this paper is also one of searching and matching. In our view, however, a manager can earn private benefits from an improved match between management and assets. If managers are motivated by this private value, then one would expect to see acquiring managements pay a premium for control. At the same time, one would not necessarily expect acquisitions to generate value for shareholders of the acquiring firm. These expectations are supported by the distribution of stock price gains observed in the event study literature; large gains accrue to target firm shareholders in tender offer takeovers and little or no gains accrue to acquiring firm shareholders. Similarly, one would not necessarily expect acquisitions to drive the value of control to result in improved profitability after the acquisition. An extensive literature, surveyed by Mueller (1987), has examined post-merger performance using accounting data. The most notable result is the failure to find evidence of improved performance after mergers. While this evidence has been used to discredit the inefficient management hypothesis, it is consistent with the approach described in this paper based on the private value of control.

**Takeovers and Aggregate Stock Price Movements**

If one accepts the existence of a control premium in a takeover transaction, there are sharp implications for the time series behavior of an individual firm's stock price; the price would fluctuate not only with information about future dividends, but also with information about the probability of a change in control of the firm. The existence of a control premium does not, by itself, have any implications for aggregate stock price behavior. If the probability of a takeover were independent across firms and over time, then the effect on stock prices would average out across firms. Stock price indices would, then, be expected to vary only with information about expected future aggregate dividends. If, however, there are systematic movements in aggregate takeover activity over time, then takeover activity (or expected future levels of takeover activity) will affect aggregate stock prices.

There is evidence suggesting that aggregate takeover activity is subject to systematic movements over time. Shughart and Tollison (1984) examine annual data on the number of takeovers in the U.S. from 1895 to 1979. They find that they cannot reject the hypothesis that merger activity follows a random walk. If this is so, then an unexpected rise in takeover activity has persistent effects. Hence, future expected rates of takeover activity will depend on the current rate. If a higher aggregate rate of takeovers implies a higher probability that a randomly selected firm will face a challenge for control, then the random walk behavior of takeover activity has implications for the behavior of aggregate stock prices. A rise in takeover activity implies a rise in the rate at which control premia are realized in changes of control. This, in turn, implies higher stock prices in the aggregate.

The notion that there is a link between takeover activity and aggregate stock prices is certainly consistent with casual observation of the behavior of stock prices in the 1980s. The decade witnessed an unprecedented wave of activity in the market for corporate control, coinciding with a sustained and substantial rise in stock prices. The two large declines in the market in the late 1980s, in October 1987 and October 1989, both came at times when many were beginning to suspect that the takeover and buyout boom might be coming to an end. In fact, much of the discussion surrounding the mini-crash of October 1989 centered on the collapse of a single deal, the UAL buyout. It was feared that the failure of the pilots' union to raise the financing for their offer was a signal of similar problems arising for future deals. Many commentators attributed the preceding increase in overall stock prices from January to August of 1989 in part to expectations of increased takeover activity. Most notably, some recent research seems to indicate that the over 10 percent decline in the
stock market on October 14-16, 1987, which arguably triggered the crash of October 19, 1987, was caused by U.S. House Ways and Means Committee consideration and approval of a tax bill containing restrictive antitakeover provisions (Mitchell and Netter (1989)).

IV.
AN ALTERNATIVE EXPLANATION OF STOCK PRICE VOLATILITY

The previous section summarized the empirical literature on the volatility of aggregate stock prices and argued that volatility is too large to be consistent with the Martingale Model described in Section II. In this section we present a theory of stock price volatility that is based on takeovers. The theory is also consistent with the empirical regularities displayed by individual stock prices around control change events. In addition, the theory offers an explanation for the broad comovements in stock prices and control change activity described above.

The key to the relationship between takeovers and the volatility of stock prices is the value of control of a firm. In this section we discuss the concept of "the value of control," and describe how the value of control can affect stock prices.

The Nature of the Firm and the Value of Control

To make precise just what we mean by the term "value of control," we briefly describe some important features of the way the modern, publicly held corporation is organized.

The diverse activities associated with the modern large corporation involve a large number of people: employees, directors, and the individuals and institutions holding the contractual liabilities of the firm, to name just a few. We focus on two main groups. We refer to the individual or group of individuals exercising effective control over the firm's operations as the management or managers: the chief executive officer, for example. We will refer to the people or institutions that own the explicit financial claims associated with ownership of the firm as claimholders: for example, shareholders, bondholders, or banks that have made loans to the firm.

The relationship between managers and claimholders is a complex one, governed by a variety of legal (and other) arrangements. For example, loan and bond contracts often contain explicit covenants that restrict future actions of the firm, including investment decisions, financial restructuring, or excessive dividend payouts (see Smith and Warner (1979)). Publicly held firms generally have a rather elaborate and explicit governance structure. Holders of shares of stock have the right to vote periodically on various matters affecting the firm. A board of directors, formally elected by the shareholders, is charged with the responsibility of overseeing the operation of the firm, and has the vested authority to hire and dismiss the managers of the firm. Managers submit important policy decisions to the board at regular meetings for formal approval. While holders of various forms of claims do have some ability to monitor and, perhaps, affect the actions of managers via these mechanisms, managers in the typical large corporation have wide discretion over how they use the firm's productive resources.

A more detailed description of these complex arrangements is beyond the scope of this paper. There is an extensive literature on the design of the arrangements between managers and claimholders, much of which draws its inspiration from Berle and Means (1932) (see, for instance Jensen and Meckling (1976) and Fama and Jensen (1983). From this literature, one can identify an important tradeoff between two opposing forces: sharing risk widely versus minimizing conflicts of interest.

The desire to allocate risk efficiently leads to widely dispersed ownership of the (risky) residual claim usually associated with ownership of the firm. The dispersion of ownership leads immediately to the need for delegated decision making authority. The communication and coordination costs which would be associated with direct decision making by a large number of claimholders makes the appointment of professional managers (with relatively small ownership stakes) a virtual necessity. This is a key characteristic distinguishing the modern corporation from the sole proprietorship in which the owner and manager are one individual.

The delegation of decision-making authority is not without its costs. The fact that management's ownership stake is relatively small suggests that the goals and incentives of managers may not always coincide perfectly with those of the claimholders. In addition, managers, who are directly involved in the operation of the firm, are likely to have a significant informational advantage over claimholders regarding alternative uses of the firm's resources. The delegation of decision-making allows managers to pursue private objectives that might harm the long-term interests of the firm.
Many of the legal arrangements between claimholders and the firm's managers alluded to earlier are designed to mitigate the misalignments of incentives. Managerial compensation schemes are often explicitly tied to the performance of the firm. This strategy imposes part of the residual risk associated with managerial decisions on the managers themselves. This type of compensation, however, works against the goal of efficient risk sharing which originally led to the dispersion of ownership and the delegation of decision-making authority, since managers are made to bear the risk rather than claimholders. Some managerial decisions can be directly mandated by claimholders through, for instance, covenants in bond and loan contracts. More specifically, covenants give the claimholder certain rights—to declare bankruptcy for example—in certain predetermined circumstances. This presumably discourages the firm's managers from taking the undesirable actions.

The manager's informational advantage, however, makes the monitoring of such agreements imperfect at best. And finally, the board of directors, ostensibly representing shareholders' interests, supervises managers and attempts to ensure that managerial decisions are in the interest of shareholders. The limitations of the supervisory role of boards of directors are apparent: because they devote very little time to a given firm, they are unable to duplicate the managers' knowledge, and so must rely on limited and self-serving reports by managers in evaluating managers' performance. In short, the nature of the relationship between corporate management and corporate claimholders leaves management with wide discretion in allocating the firm's productive resources.

The problems associated with the separation of ownership and control suggest that managers may be able to extract private benefits, or "rents," from their insider positions. There may be actions that managers can take that benefit themselves without adding to the value of the firm and, therefore, to the wealth of the claimholders. The value of control, then, is the value of the stream of benefits which necessarily accrue to those in control of the firm. This is a private value in the sense that those in control cannot credibly commit to transfer these benefits to claimholders. These benefits may take the form of private consumption of "perks" or of the pursuit of private goals distinct from value maximization. It has also been suggested by Jensen (1986) that managers can derive private benefits from the discretionary control over the firm's free cash flow. For instance, in order to pursue firm growth as an end in itself, a manager may use retained earnings to fund investments with negative net present value. More generally, access to internal funds for investment shelters managers' decisions from the scrutiny they would receive in obtaining external sources of finance.

Allowing management to extract private value may, in fact, be part of the (imperfect) scheme for providing managers with correct incentives. If managers are able to extract more rents during good (profitable) times than bad—because, for example, managers' actions come under more direct scrutiny during bad times—then managers have an incentive to take actions that make good times more likely. In addition, control of a large organization may be valuable in and of itself, quite apart from any resources directly obtained thereby. It could provide utility directly for managers in the form of enhanced prestige or ego gratification.

Corporate Financial Claims

We can now describe how the value of control of a firm affects the nature of the financial claims issued by the firm. It is essential to our argument that a financial claim is a contract between the issuer (the corporation) and the holder of the claim. This contract specifies payments to be made by the corporation under a variety of contingencies. Sometimes these specifications are explicit, as in the case of bank loans or corporate bonds. In other cases, promised payments are implicit, as in the expectation of dividend payments to equity holders based on an announced dividend policy. In addition to stipulating payments, the financial claim gives the holder certain rights. A debt holder may have the right to directly monitor some of the actions taken by corporate management, as specified in a bond covenant. Debt claims also carry important rights in the case of bankruptcy. The main right attached to a standard common stock equity claim is the right to vote on some corporate governance matters on a one-share-one-vote basis. Most important, shareholders have the collective ability to choose corporate management through the election of the board of directors.

Debt and common stock equity are the predominant forms of financial claims issued by the modern corporation. Other forms of claims can be viewed as hybrid varieties, such as preferred stock or convertible debt. Uncovering the determinants of the mix of claims issued by corporations remains one of the major challenges of financial economics. A recent paper by Harris and Raviv (1988) is particularly relevant to the concerns of this paper. They
assume that managers derive private value from the control of a firm and examine the implications of this assumption for the design of securities. They find that if claims are to be issued with an interest in promoting efficiency-enhancing changes in control but deterring efficiency-reducing changes, then rights to vote on changes in control should be attached to equity claims and not debt claims. This is, of course, exactly the allocation of rights observed.

Given our arguments above that managers derive private rents or value from the control of firms, it is useful to view equity claims as bundled claims. The voting feature of tradable equity shares implies that control can be acquired through the purchase of shares; buy enough shares, and you can install yourself or anyone of your choosing in top management positions. Hence, the claim to a stream of dividends is bundled with a claim to the premium that a potential manager might pay to acquire enough votes to take control of the firm. Note that this feature is unique to equity; one cannot acquire control of a firm by buying all of its debt. Hence, the equity claim is necessarily linked to the process of change in control, regardless of how those changes come about.

It is interesting to note that firms often issue voting and nonvoting classes of equity. While nonvoting equity is relatively unimportant in publicly held firms in the U.S., in some other countries it is more important. The relative prices of voting and nonvoting shares often reflect the value of control. For instance, Hermann and Santoni (1989) show that when Swiss firms began allowing foreign investors to hold voting shares, the value of the voting shares increased relative to the value of outstanding nonvoting shares by as much as 20 percent. While there may be other explanations of this increase, allowing foreign purchases of voting shares may have increased the likelihood of an acquirer buying shares in order to obtain control.

Takeovers and the Value of Control

When the control of a corporation changes hands, the value of control is often transferred as well. The way in which a change in control takes place determines how the value of control is transferred and how the financial claims on the corporation are affected. One form of change in control is, of course, internal succession to the top management positions. When a vacancy at the top is filled by promotion from within, the value of control need not be "purchased" from shareholders. The internal transfer of control might represent an implicit contract between generations of managers; new managers may have "paid for" control through a period of apprenticeship. Alternatively, one might view the value of control as accruing to a coalition or team of managers (such as the CEO, the board of directors, and other top executives). Internal succession then amounts to keeping control in the hands of the same coalition. Similarly, the board of directors hiring a CEO from outside the firm, for instance, is a transaction between the controlling coalition and an individual who is joining the coalition.

In the cases of internal succession and external hiring discussed above, there is no change in the designation of the delegated decision-making authority. There is, therefore, no need for those engaged in the change of control to purchase control through the acquisition of shares. However, sometimes a change in the delegation of control becomes desirable to at least some shareholders. They may feel that incumbent management has not responded well to a change in the economic environment or that an alternative management would perform better. In such cases, the shareholders' voting rights become important.

The various ways in which a change in the delegation of control might be brought about were discussed by Manne (1965) in an effort to outline the economic role of corporate takeovers. Manne views all changes in control as attempts to replace less efficient with more efficient management. The nature of the equity claim gives an unsatisfied shareholder a number of options. First, one could try to unseat the incumbent board of directors through a proxy contest. Proxy contests, however, are relatively infrequent. This may be because of the costs involved in soliciting votes; incumbent management can use corporate resources to fight its battle, but dissidents must use their own resources. Having incurred the expense, the outcome of the contest remains uncertain until the actual vote is held. One way in which a challenger for control can reduce the uncertainty is through his or her own ownership of shares. This, of course, suggests an alternative route to obtaining control. By acquiring enough shares, one can dispense with the need for a prolonged and potentially unsuccessful proxy contest.

Faced with a challenge to its (valuable) control, incumbent management can be expected to spend resources resisting the change. This is true in the case of a proxy contest or an acquisition of shares. When a challenger attempts to gain control through
the acquisition of shares, or when an incumbent seeks to protect control through the acquisition of shares, the share price is bid up to reflect all or part of the private value of control. In a friendly merger as opposed to a hostile takeover, shareholders may realize a smaller part of the value of control; this would be so if the acquiring management obtained the incumbent management’s consent through some form of implicit or explicit payment. In short, the extent to which a change in control results in value accruing to shareholders depends on the extent to which there is competition for control.

In the absence of frictions or barriers to open competition in the market for control, the market price of equity would always fully reflect the value of control. There are, however, some important frictions built into the market for control. Many of these derive from the very nature of the relationship between corporate ownership and management. An unrestricted market for control could expose managers to too much employment risk; managers might then have an insufficient incentive to accumulate firm-specific human capital. Shareholders have an interest in giving their delegated decision makers an incentive to make themselves well matched to the particular firm they are managing. On the other hand, complete protection from the market for control is not good from the shareholders’ point of view. Entrenched management can receive the private benefits of control with no concern for the firm’s performance on shareholders’ behalf. These opposing forces suggest an optimal intermediate degree of protection for incumbent managers. Such protection may take the form of golden parachutes, or provisions in the corporate charter giving the manager the right to take certain defensive actions in the event of a takeover attempt.

In addition to the frictions built into the form of corporate governance, government regulations can create barriers to takeover activity. A variety of federal and state regulations restrict the actions of a raider in a contest for control. A prime example at the Federal level is the 1968 Williams Amendment, which restricts the actions of bidding firms by, for instance, requiring that tender offers be outstanding for a minimum number of days. Such restrictions can add to the cost of attempts to acquire control, thereby making such attempts less frequent.

One might label barriers to takeovers that arise from legal restrictions or the contractual relationship between ownership and management “artificial” barriers. There may also be important “natural” barriers in the market for control. Both the private value and the public profitability that a manager can achieve with a firm may depend on how well-matched that manager is to the firm’s organization, array of activities, and “corporate culture.” Time and resources may be required to investigate the quality of such a match. Hence, the potential acquirer’s behavior may best be viewed as a process of costly search. Both the costs of search and the likely costs of making an acquisition (once a match is found) affect the raider’s willingness to search for targets.

Viewing the market for control as a market in which buyers or raiders search for targets has implications for the effects of the value of control on stock prices. The extent to which a share price reflects the value of control depends on the probability that a potential raider finds the firm to be worth challenging for control. This probability, in turn, depends on the overall level of ongoing search activity. In addition to the artificial and natural frictions suggested above, the level of search activity is likely to depend on what might be called the “infrastructure” of the market for corporate control. By this we mean, for instance, the conditions under which a raider could obtain financing for a deal. Casual observation suggests that the takeover boom of the 1980s was fed, in part, by innovations in the market for below-investment-grade corporate debt (junk bonds). In short, the availability of a full array of financial and legal services facilitates the search process. Variation over time in these infrastructure services might contribute to variation in the level of search and takeover activity, and thus to variations in stock price volatility over time.

We are not aware of a theoretical explanation of the variations in aggregate takeover activity, although it has been suggested (e.g., by Gort (1969)) that waves of mergers are driven by large disturbances to the economic environment. For our purposes, it is enough to take as given that takeover activity varies over time according to a random process which can reasonably be described by a random walk. With this assumption, in periods of high takeover activity, such activity is expected to remain high. Hence, the perceived probability that a randomly selected firm faces a challenge for control in the near future is high, and the value of a stock price index significantly exceeds the expected value of the underlying stream of dividends. By similar reasoning, in periods of low takeover activity, stock prices are closer to the value of future dividends. These arguments lead directly to our excess volatility results.
Takeovers and Stock Prices

The descriptive analysis above can be made quite rigorous. To be specific, one can formally specify an artificial economy that displays the forces described above, albeit in relatively stark and simple form. We have done this in a forthcoming paper (Lacker, Levy and Weinberg 1990), where we specify agents' preferences, their production and investment technologies, and, most crucially, the informational opportunities available to them. The critical feature of the economy is that the agent that manages an asset also has the ability to manipulate the observed return on the asset. The appendix of this paper describes a similar economy in more detail. Here we present the main implications.

For our economy, we can derive the equilibrium price of shares of stock in any given asset. Let \( p_t \) now be the price of a share if no takeover occurs during period \( t \), and let \( q_t \) be the price paid if there is a takeover in period \( t \). Both \( p_t \) and \( q_t \) are determined by general equilibrium conditions in our economy: \( p_t \) by the value investors place on a share knowing no takeover will occur until next period at the earliest, and \( q_t \) by the value to a new manager of acquiring and subsequently controlling the firm. In equilibrium \( q_t > p_t \), meaning that a new manager is willing to pay a premium to acquire control of the firm. The critical feature of the economy is that the agent that manages an asset also has the ability to manipulate the observed return on the asset. The appendix of this paper describes a similar economy in more detail. Here we present the main implications.

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The variable \( \pi_{t+1} \) is the probability that a takeover occurs during period \( t+1 \), given information available during period \( t \). Equation (11) states that the current stock price depends on the expected value of control in a takeover as well as on expected dividends and expected price as before:

\[
(11) \quad p_t = (1 + r)^{-1}E_t[p_{t+1} + d_t + \pi_{t+1} + \eta_{t+1}].
\]

The variable \( \pi_{t+1} \) is the probability that a takeover occurs during period \( t+1 \), given information available during period \( t \). Equation (11) states that the current price of a share equals the expected discounted value of the sum of the price, dividends and the value of control, with the latter weighted by the probability of a takeover.

As in the Martingale Model, we can use this equation to derive an expression for the current stock price in terms of the entire stream of future dividends. As before, the derivation requires repeated substitution for \( p_{t+1}, p_{t+2}, \) and so on. The result is

\[
(12) \quad p_t = v_t^d + v_t^f
\]

where \( v_t^f = \sum_{s=1}^{\infty} (1 + r)^{-s}E_t[\pi_{t+1} + \eta_{t+s}] \)

and \( v_t^d \) is defined as before. Comparing equation (12) to equation (4) reveals that the present value relation is now augmented by a term related to the value of control. The current stock price is equal to the expected present value of dividends plus the expected present value of the premium associated with control, adjusted by the probability that shareholders realize that premium. One immediate implication of (12) is that the variance of \( p_t \) can be written in terms of the variances and covariance of \( v_t^d \) and \( v_t^f \):

\[
(13) \quad \text{var}(p_t) = \text{var}(v_t^d) + \text{var}(v_t^f) + 2\text{cov}(v_t^d, v_t^f).
\]

The possibility of excess volatility in stock prices is now easily demonstrated:

**Proposition 1:** If \( \text{var}(v_t^d) + 2\text{cov}(v_t^d, v_t^f) > 0 \), then \( \text{var}(p_t) > \text{var}(v_t^f) \), and the variance of the stock price is greater than the variance of the present value of expected dividends. For example, if \( v_t^d \) is not negatively correlated with \( v_t^f \), then \( \text{var}(p_t) > \text{var}(v_t^f) \).

Therefore, the price of a stock can vary by more than is justified by variations in expected future dividends. The condition that \( v_t^d \) and \( v_t^f \) are positively correlated is stronger than required; all that is needed is for the correlation between \( v_t^d \) and \( v_t^f \) to be not too large a negative number. This condition seems reasonable. If the actual realized value of control is positively correlated with realized dividends, this assumption is satisfied. One would think that the expected value of controlling a firm would be larger if the firm is expected to do better.

We have not yet shown how the variance in stock prices compares with the variance of \( d_t^* \), the ex post rational price. The Martingale Model predicts that \( \text{var}(p_t) \leq \text{var}(d_t^*) \), but this inequality is violated empirically. Can our economy display violations of this inequality? To find out, first recall that because \( d_t^* = v_t^d + x_t^d \), and \( \text{cov}(v_t^d, x_t^d) = 0 \) because of the optimality of forecasts of \( d_t^* \), we know that

\[
(14) \quad \text{var}(d_t^*) = \text{var}(v_t^d) + \text{var}(x_t^d).
\]

The variance of \( p_t \) can be written as follows:

\[
(15) \quad \text{var}(p_t) = \text{var}(v_t^d) + \text{var}(v_t^f) + 2\text{cov}(v_t^d, v_t^f) - \text{var}(d_t^*) - \text{var}(x_t^d) + \text{var}(v_t^f) + 2\text{cov}(v_t^f, v_t^f).
\]
This expression gives us a condition under which the variance bounds condition in the Martingale Model is violated.

**Proposition 2:** If \( \text{var}(v^t) + 2 \text{cov}(v^d, v^t) > \text{var}(\Delta x^d) \), then \( \text{var}(p_t) > \text{var}(\Delta d^t) \), and the variance of the stock price is greater than the variance of the present value of actual dividends.

The condition in Proposition 2 states that the variance of the expected value of control plus the covariance of the expected value of control and the expected value of dividends must exceed the variance of the error in forecasting the present value of dividends. This condition can be understood by comparing \( \text{var}(p_t) \) in our model, equation (15), with \( \text{var}(\Delta d^t) - \text{var}(\Delta x^d) \) from equation (9) in the Martingale Model. In the latter, \( \text{var}(p_t) \) is equal to \( \text{var}(v^d) \), which is less than \( \text{var}(\Delta d^t) \) by the amount \( \text{var}(\Delta x^d) \). In our model, \( \text{var}(p_t) \) is greater than \( \text{var}(v^d) \) by the amount \( \text{var}(v^d) + 2 \text{cov}(v^d, v^t) \). For this effect to dominate, making \( \text{var}(p_t) > \text{var}(\Delta d^t) \), \( \text{var}(v^d) + 2 \text{cov}(v^d, v^t) \) must be larger than \( \text{var}(\Delta x^d) \).

More intuitively, a share of stock in our model is a bundled claim, consisting of the right to a stream of dividends plus a share of the right to control the firm. The latter is a value that can be realized by the shareholder in the event of a takeover, and it adds a variance to the stock price above and beyond the variance in expected dividends. It contributes a variance of its own to the price of the stock, and in addition could well be correlated with the expected present value of dividends. These two effects could add enough to the variance of the stock price to make it larger than the variance in the present value of actual dividends, consistent with the empirical violations of the Martingale Model's variance bounds condition.

This explanation of excess stock price volatility does not rely on some other explanations that have recently been advanced. Some economists have suggested that fads or irrational "noise traders" are responsible for observed anomalies in stock prices (see Shiller (1984), Black (1986), DeLong et al., (1987), and Campbell and Kyle (1988)). In our economy, all agents are fully forward-looking and expectations are rational. There are no unexploited arbitrage opportunities because the future control premia are rationally anticipated and incorporated into the current price of the stock. There are no externalities, and no restrictions on the contracts agents can write except those that follow from the technological and informational constraints agents face. In fact, our equilibrium is Pareto optimal, meaning that no agents can be made better off without making some other agents worse off. The key feature of the economy that gives rise to excess volatility is the friction affecting the contractual arrangements between managers and claimholders; managers' privileged position in control of the asset implies a positive value of control.

The evidence from event studies of tender offers and mergers, described in Section III, subheading "Takeovers and Individual Stock Prices" above, is consistent with the model presented here. The large abnormal increase in the stock price of the target firm represents the control premium \( \eta_t \). The fact that the stock price of the bidding firm changes very little suggests that a substantial part of the increased productivity or private value of control associated with the acquisition is captured internally by the acquiring firm and is not passed on to the acquiring firm's shareholders.

Our model is also consistent with one of the most striking features of the empirical variance bounds literature. Shiller's first paper contained graphs plotting \( \Delta x^d \), the present value of actual dividends (he called it \( p_t^* \)), against \( p_t \), actual stock prices, for the Standard and Poor's Composite Price Index and for the Dow Jones Industrial Average. The path of \( p_t^* \) is fairly smooth, while the path of \( p_t \) takes large persistent swings away from \( p_t^* \). An analogous graph, using more sophisticated techniques for removing trends, appears in a recent paper by Campbell and Shiller (1987, Figure 2, p. 1083). Both Shiller's and Campbell and Shiller's plots show that the difference \( p_t - p_t^* \) was largest during four time periods: the first decade of this century, the late 1920s, the mid-1960s, and the early 1980s. The peak in the mid-1960s is particularly large. All four of these periods correspond to merger waves, periods in which changes in corporate control were particularly frequent. This suggests that the economy might experience periods in which the probabilities of takeover for a broad range of stocks move together and exhibit long persistent swings. These swings might be caused by accelerations of technological shifts as some have argued (Gort 1969), periodic shifts in the regulatory environment affecting changes in corporate control, or innovations in the infrastructure of financial markets.

**V. SOME IMPLICATIONS OF OUR THEORY**

In this section we briefly discuss some of the implications of our theory, first for recent events and
trends in financial markets, and then for proposals to alter the regulations governing takeovers and markets for traded financial claims.

Recent Developments in Financial Markets

Dramatic changes have occurred in the markets for corporate financial claims in the last decade. Stock prices displayed a broad upward trend through the 1980s, albeit with setbacks in the fall of 1987 and the fall of 1989. It is a widely held perception that volatility has increased. An entirely new market has emerged for below-investment-grade, tradeable corporate debt, or "junk bonds." And the pace of changes in corporate control via acquisition of outstanding shares has increased dramatically.

Simultaneously explaining all of these trends is far beyond the scope of the present paper. However, our theory is able to cast a new light on many of these developments and their interrelations. One plausible interpretation is that, for some reason, perhaps linked to technological improvements in the ability of investors to monitor firm performance, investors are now much more willing to hold risky, high-yield corporate debt such as junk bonds. While not all of these securities have been associated with corporate takeovers, it seems clear that they were essential to many of the control transactions of the 1980s. The shift in investor demand for these securities facilitated takeovers that would not have been possible without the market for these securities. This improvement in the ability of acquirers to finance takeovers led in turn to a secular rise in the probability of a takeover for a broad range of stocks, \( \pi_{t+1} \) in our setup, and so led to a broad upswing in stock prices.

The theory might also illuminate some recent short-run swings in stock prices. In recently published research, Mitchell and Netter implicate Congressional consideration of antitakeover legislation in the October 19, 1987 crash in stock prices. They argue that "a tax bill containing antitakeover provisions proposed by the U.S. House Ways and Means Committee on October 13, 1987, and approved by the Committee on October 15 was the fundamental economic event causing the greater than 10% decline in the stock market on October 14-16, which arguably triggered the October 19 crash." By making takeovers more costly, such a bill would reduce the probability of future takeovers and thus depress current stock prices, consistent with our theory.

Analogously, the role of the junk bond market in facilitating changes in corporate control might explain why information about the willingness of investors to hold below-investment-grade securities would affect stock prices so strongly, as they seemed to in 1989. At many times during that year, particularly during the late summer and early fall, reports of broad stock price declines cited sharp declines in junk bond prices as the proximate cause. Similarly, the collapse of one well-publicized deal, the bid for UAL, was cited often for the broad decline in stock prices in the fall. Finally, we note that the fall of broad measures of stock prices since Summer 1989 has coincided with a rise in the use of proxy fights in corporate control contests, a method of control change that does not provide shareholders with an immediate monetary payment.

Regulations to Curb Takeovers and Reduce Stock Price Volatility

The finding of excess volatility of stock prices is often taken as evidence of capital market imperfections or the presence of irrationality in the determination of asset prices (see Shiller (1984), Black (1986), DeLong et al., (1987), and Campbell and Kyle (1988)). Such imperfections, in turn, are often adduced in support of various policy proposals that would legislatively alter the way financial markets currently operate. For example, some advocate that "circuit breakers" or "collars" be imposed on the stock market to halt or restrict trading in the event that prices change by more than some prespecified amount (see, for example, Greenwald and Stein (1988)). The argument is that such restrictions would reduce price volatility and improve the efficiency of financial markets. Similarly, some have suggested policy changes to discourage takeovers, either by making the financing of takeovers more difficult or costly, or by erecting barriers to changes in control via acquisition of shares (Scherer (1988), for example).

A complete evaluation of these many proposals is beyond the scope of this paper. We can point out, however, that in our model takeovers regularly occur, and are responsible for excess stock price volatility. Excess volatility arises because of the mechanisms by which the complex agency problems inherent in the management and financing of the modern corporation are resolved. These mechanisms thus have a positive allocation role. In fact, excess volatility is consistent with full market efficiency in our model, and there is no constructive role for government intervention. The lesson, then, is that

---

5 There may be plausible alternative explanations, of course.
the empirical finding that stock price volatility is larger than can be explained by the Martingale Model does not by itself justify regulatory intervention in financial markets.

Of course, a wide range of government policies already in place have important effects on the phenomena our model attempts to describe. The requirements imposed on corporate charters constrain the legal forms that corporate governance can take. The Securities and Exchange Commission significantly constrains the financial structure and conduct of publicly held firms, requiring, for example, that votes be strictly proportional to shareholdings. SEC regulations also impose severe restrictions on tender offers. Underlying all financial claims, of course, the structure of bankruptcy law has an important and sometimes neglected influence on financial arrangements.

Our model is not rich enough, as yet, to be able to fully assess the role of these and other regulations affecting the market for control. We suspect that they have important effects on the way various legal rights are allocated among the claimants of a firm, and thereby have important effects on the market for corporate control. Altering these regulations may well reduce stock price volatility, but would most likely alter the efficiency with which the control of assets is allocated. Any assessment of the impact of altering such regulations must look far beyond the effect on stock price volatility.

VI. CONCLUDING REMARKS

Our analysis contains a broader message for the understanding of financial markets. Traditional approaches to asset pricing treat an asset as nothing more than a claim to a stream of payments. The starting point of our analysis is the view that a financial asset is a contractual relation between various parties. A direct implication of this view, as our model illustrates, is that financial assets in general, and traded stock shares in particular, are bundled claims tying together fragments of governance rights with titles to streams of payments. Building upon this view may provide us with new insights into the diverse financial arrangements characteristic of developed economies.

APPENDIX

In this appendix, we develop a simple model that delivers an equilibrium pricing equation of the form of equation (11). The model is similar in spirit to the one in Lackey, Levy and Weinberg (1990). That paper was concerned with demonstrating that excess volatility was possible in principle. The model described here is somewhat more general in that it allows for periodic swings in takeover activity and shows how these might lead to coincident swings in stock prices.

In this economy there is a large number of durable productive assets (projects) and an even larger number of people (agents). Some people are claimholders, and others are managers. Together with the services of a manager, a project can produce a stream of output \( \{z_t\}, t=1,2, \ldots \), where \( z_t = d_t + y_t \). The portion \( d_t \) of the project's output is publicly observed. A manager can commit to paying out (all or a part of) \( d_t \) to claimholders. The remainder of the project's output, \( y_t \), is privately observed by the manager and is not verifiable by any outsider. Hence, \( y_t \) is simply consumed by the person who controls the firm, and cannot be contractually transferred to claimholders. These are the rents that accrue to managers and correspond to the private value of control posited by Harris and Raviv (1989).

The per period value of control, \( y_t \), and dividends, \( d_t \), are assumed to follow stochastic processes given by

\[
(A.1) \quad y_t = a_0 y_{t-1} + e_t, \quad \text{and} \quad d_t = a_1 d_{t-1} + u_t,
\]

where \( a_0, a_1 \leq 1 \), and \( e_t \) and \( u_t \) are independent, mean-zero random variables, independently and identically distributed over time.\(^6\)

\(^6\) One could assume a more general joint process for \( \{y_t, d_t\} \) without altering the results. Under more general assumptions, claimholders would need to be able to form expectations about future values of unobservables, \( y_t \), based only on publicly observed variables. Our assumptions allow us to avoid the filtering problem which arise with a more general specification.
Claimholders hold claims to the dividend stream \( \{d_t\} \), and these claims are attached to voting rights allowing claimholders, collectively, to delegate control of the productive project. For simplicity, we assume that a change in control requires a unanimous vote. Hence, a raider can acquire control by purchasing all claims to a particular project. We assume, however, that there are agents engaged in search activity to obtain information about the value of controlling projects. We do not model this search behavior explicitly. Rather, we simply assume that at any point in time there is a probability \( \phi_t \), that a raider arrives on the scene and obtains information about the value of control. We assume that \( \phi_t \) follows a first-order stationary Markov process, that is, that the probability distribution of \( \phi_{t+1} \), given the entire history of realizations up to and including period \( t \) for \( j = 0, 1, 2, \ldots \), depends only on \( \phi_t \).

The raider observes the incumbent's current value, \( y_t \), and also learns what his own value would be if he took control; call this \( y_f \). We assume that if a raider arrives in period \( t \), then \( y_f = y_{t-1} + e_t \), where \( e_t \) satisfies the same assumptions as does \( e_t \), but is drawn independently of \( e_t \). Thus the raider's current-period value of control could be different from the incumbent manager's current-period value of control.

The value of control is the present discounted value of the stream of per period values of control, weighted, for each future period, by the probability that the manager will still be incumbent in that period. The value of control is calculated by the incumbent manager, yielding the amount the incumbent manager would accept to forego continued control of the asset. The value of control is also calculated by the raider; it is the amount the raider would pay to acquire control of the asset. Both quantities are influenced by the past experience of the project through the influence of \( y_{t-1} \), and for both the incumbent manager and the raider the future of the value of control evolves according to (A.1). But because \( y_f \) can differ from \( y_t \), and because these influence the expected values of \( y_{t+1} \) and \( y_{t+1} \), there can be a discrepancy between the value of control to the incumbent and the value of control to the raider.

Once \( y_t \) and \( y_f \) are observed, the raider can choose to initiate a bid for control through the acquisition of shares. We assume that there is an arbitrarily small but nonzero cost of initiating a challenge for control. Hence, the raider only does so if his own value of control is greater than the incumbent's.\(^7\) Define \( \Phi_t \) as the probability that a raider appears in period \( t \) and has a greater value of control than the incumbent. The value of \( \Phi_t \) depends on the raider's expectations of future per period values of the control, \( y_{t+1} \), and the probability that some other raider will come along and acquire the asset in the future. We take the series \( \Phi_t \) as given for now. Let \( \eta^0 \) be the value of losing control: the expected present value of the manager's earnings from the next-best alternative occupation in the event of losing control of the asset. Then the value of control of an asset can be written as

\[
(A.2) \quad \eta_t = (1 + r)^{-1} E_t \{ \Phi_t + \eta^0 \} + (1 - \Phi_t)(y_{t+1} + \eta^0) \]

\( y_{t+1} \) is the value of being in control at the end of period \( t+1 \). Equation (A.2) states that the value of control is the present value of the value of losing control, multiplied by the probability of losing control next period, plus the value of remaining in control at the end of the next period, multiplied by the probability of remaining in control. An identical expression determines the value of control for a raider, \( \eta_f \). Note that if a raider assumes control this period, at the end of the next period he is an incumbent, so \( \eta_{t+1} \) appears on the right side of the expression for \( \eta_f \):

\[
(A.3) \quad \eta_f = (1 + r)^{-1} E_t \{ \Phi_t + \eta^0 \} + (1 - \Phi_t)(y_{t+1} + \eta^0) \]

If a raider arrives in period \( t \), a change of control takes place only if \( \eta_f > \eta_t \), the value of control to the raider exceeds the value of control to the incumbent. Because \( y_{t+1} \) evolves according to a stationary process, one can show that \( \eta_f > \eta_t \) if and only if \( e_t > e_t \), the current-period value of control is larger for the raider than for the incumbent. Therefore, the probability that a change in control occurs if a raider arrives in period \( t \) is \( \Phi_t = \Phi_t E_t [e_t > e_t] \). The probability that a change in control actually occurs in period \( t \) is then \( \Phi_t = \Phi_t E_t [e_t > e_t] \), the probability that a raider arrives times the probability that a change occurs given that a raider has arrived. Given our assumptions about \( \phi_t \), \( e_t \), and \( e_t \), the expected future rate of change in control depends only on the current value of \( \Phi_t \).

\(^7\) Relaxing this assumption, so that there is a contest for control whenever a raider "arrives," would not change the nature of the results but would complicate the computation of the present discounted value of control.
We define $\pi_{t+s}$ to be the probability, given that the firm does not face a challenge to control before or during period $t$, of the first such challenge occurring in period $t+s$. For $s = 1$, $\pi_{t+1} = \Phi_{t+1}$. For $s \geq 2$, $\pi_{t+s}$ is given by

$$
\pi_{t+s} = \Phi_{t+s} \prod_{j=1}^{s-1} (1 - \Phi_{t+j}).
$$

Equation (A.2) can now be solved forward to yield

$$
\eta_i = E_t \left\{ \sum_{s=1}^{\infty} (1+r)^{-s} \pi_{t+s} \eta_i^0 + \left( \pi_{t+s}(1 - \Phi_{t+s})/\Phi_{t+s} \right) \eta_i^{t+s} \right\}
$$

Notice that $\eta_i$ depends on the expected future rate of takeover activity, as well as on the expected future values of $\gamma_i$.

If a raider arrives in period $t$ and draws a current value of control, $e_i$, that is larger than the incumbent’s, then the raider outbids the incumbent by paying a premium of $\gamma_i$ for the equity shares of the firm. In the event of a takeover, the purchase price (ex dividend) of the shares is

$$
q_t = v_i^d + \eta_i.
$$

In the event that there is no takeover attempt in period $t$, the (ex dividend) stock price is

$$
p_t = (1+r)^{t-1}E_t [d_{t+1} + \pi_{t+1} q_{t+1} + (1 - \pi_{t+1}) p_{t+1}].
$$

Using equation (A.4) and solving forward, we have

$$
p_t = v_i^d + \eta_i^t,
$$

where

$$
\theta_i^t = E_t \left\{ \sum_{s=1}^{\infty} (1+r)^{-s} \pi_{t+s} \eta_i^{t+s} \right\}.
$$

For convenience, $\eta_i^{t+s}$ is written as $\eta_i^{t+s}$ in Section IV, subheading “Takeovers and Stock Prices.”

Suppose that there are a large number of identical versions of the asset that we have just described. The stochastic processes governing $d_i$, $e_i$, $e_i^t$, and $\phi_i$ are the same, although the realizations of these random processes are independent across assets. If the number of these assets is quite large, then the fraction that experience a change in control is very close to the population probability that a change in control occurs (by the Law of Large Numbers). Now define $\pi_{t+s}$ as the probability that a takeover occurs in period $t+s$ to any given firm selected at random, given the information known in period $t$. Imagine calculating a stock price index as a weighted average of individual stock prices; the weights are not important—any arbitrary weights will do. Then the formula derived above will also apply to the stock index, where $p_i$ is the value of the stock price index, and $\eta_i$, $v_i^d$, and $v_i^s$ are interpreted as weighted averages across stocks.

REFERENCES


Harris, Milton, and Raviv, Artur, 1988, The Design of Securities, Department of Finance, Kellogg Graduate School of Management, Northwestern Univ., Working Paper No. 64.


