

Credit Rationing by Loan Size in Commercial Loan Markets

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I. INTRODUCTION

Ample evidence exists suggesting that banks ration credit with respect to loan size.¹ For example, Evans and Jovanovic (1989) find evidence of loan size rationing in data from the National Longitudinal Survey of Young Men.² Further, the Federal Reserve Board's quarterly Survey of Terms of Bank Lending consistently indicates that the average interest rate charged on commercial loans (i.e., the rate per dollar lent) is inversely related to loan size. This evidence suggests two questions. First, why might loan size rationing occur? Second, why might loan size rationing have the particular interest rate and loan size pattern reported in the Survey of Terms of Bank Lending? Economists generally believe that higher average interest rates are charged on smaller loans because small borrowers are greater credit risks or because loan administration costs are being spread over a smaller base. This paper presents a counterexample to that belief. It shows that, even if credit risk and loan administration costs are the same for all borrowers, a lender with market power and

imperfect information about borrowers' characteristics still will offer quantity-dependent loan interest rates of exactly the type reported in the Survey of Terms of Bank Lending.³

The quantity-dependent loan interest rates that we derive are a form of second-degree price discrimination. Price discrimination is said to occur in a market when a seller offers different units of a good to buyers at different prices. This type of pricing is commonly used by private firms, governments and public utilities. For example, many firms have "bulk rate" pricing schemes, whereby they offer lower marginal rates for large quantity purchases. The income tax rates in the U.S. federal income tax schedule depend on the level of reported income; higher marginal tax rates are levied on higher-income taxpayers. In addition, the price per unit of electricity often depends on how much is used.

Both market power—a firm's ability to affect its product's price—and imperfect information regarding borrowers' characteristics are essential for producing the loan size-interest rate patterns observed in commercial loan markets.⁴ To see why, suppose that a lender has market power and perfect information about borrowers' loan demand. In this case, we would observe first-degree (or "perfect") price discrimination: the lender would charge each borrower the most he/she is willing to pay and would lend to all that are willing to pay at least the marginal cost of the loan. Suppose instead that a lender has imperfect information and operates in a competitive market. Milde and Riley (1988, p. 120) have shown that such a lender may not ration credit, even if borrowers can send the lender a signal about their characteristics.

In this paper, we provide an explicit analysis of the information aspects of price discrimination in

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¹ Jaffee and Stiglitz (1990) present alternative definitions of "credit rationing." Broadly defined, credit rationing occurs when there exists an excess demand for loans because quoted interest rates differ from those that would equate the demand and supply of loans.

² This evidence is contrary to most recent theoretical models of credit rationing. That literature derives loan *quantity* rationing, whereby some borrowers obtain loans while other observationally identical borrowers do not, in the spirit of Stiglitz and Weiss (1981). While some quantity rationing does occur, the evidence suggests that size rationing is more common.

³ Of course, the theory we will present is not inconsistent with differential credit risk and loan administration costs, although these factors are not necessary to obtain the observed interest rate-loan size pattern.

⁴ See Jaffee and Modigliani (1960) for an early distinction between types of price discrimination and credit rationing.

commercial loan markets. We interpret a lender's price discrimination with respect to loan size as a form of credit rationing that limits borrowing by all but the largest borrowers. Further, because we show that such credit rationing arises from rational, profit-maximizing lender behavior, our analysis has normative implications. We find that small borrowers are more credit constrained than large borrowers and thus bear a larger share of the distortion induced by the market imperfections.⁵ In the next section, we describe a simple prototype economy with a single lender and many different types of borrowers about whom the lender has limited information. We then present the lender's profit-maximization problem. Section III follows, describing the loan size-total repayment schedule that solves the lender's problem and explaining why the solution involves credit rationing with respect to loan size. Section IV concludes.

II. A SIMPLE MODEL ECONOMY

Consider an endowment economy with a single lender that may be thought of either as a local monopolist or as a price leader in the industry.⁶ Suppose also that there are n types of borrowers, where n is a positive and finite number. There are N_i borrowers of each type i ($i = 1, \dots, n$) who live for only two periods. The borrowers may be thought of as privately owned firms that differ only with respect to their fixed endowments of physical good.⁷ All firms have the same first-period endowment: $w_1^i = 0$ for all i ;⁸ however, higher-index firms have larger second-period endowment: $w_2^{i+1} > w_2^i$. In addition, each firm's second-period endowment is positive and known with certainty at the beginning of the first period.

⁵ Price discrimination in loan markets is facilitated by banks' use of "base rate pricing" practices: banks quote a prime rate (the base) and price other loans off that rate. With a base rate pricing scheme, banks price loans competitively for large borrowers with direct access to credit markets, while they act as price-setters on loans to smaller borrowers. Goldberg (1982, 1984) finds substantial evidence for such pricing practices.

⁶ The changing of the prime rate has been interpreted by banking industry insiders as an example of price leadership and called "the biggest game of follow-the-leader in American business" [Leander (1990)].

⁷ This interpretation is consistent with Prescott and Boyd (1987), which models the firm as a coalition of two-period lived agents with identical preferences and endowments; the coalition in our model consists of only one agent.

⁸ We assume $w_1^i = 0$ for simplicity to guarantee that firms borrow in the first period.

We assume that the welfare of each type i firm (i.e., borrower) is represented by a utility function, $u(x_1^i, x_2^i)$, where x_t^i is the amount of period t good consumed by the owner of the firm, for $t = 1, 2$. The utility function $u(\bullet)$ indicates the satisfaction that the owner gets from various combinations of consumption in the two time periods. We assume that the owner's utility function is twice differentiable, strictly increasing and strictly concave. These mathematical properties imply that the owner prefers more consumption to less and prefers relatively equal levels of consumption in the two time periods. We also assume that x_1^i is a normal good, which means that owner i 's demand for good x increases with his/her income. Given these assumptions and the endowment pattern specified, all firms will borrow in the first period and higher-index firms will be larger borrowers.⁹

The economy's single lender wishes to maximize profit, which is the difference between revenues (i.e., funds received from loan repayments) and costs (funds lent). Assume that the lender's capital at time 1, measured in units of physical good, is sufficient to support its lending policy, and suppose that the following information restriction exists: the lender and all borrowers know the utility function, the endowment pattern, and the number of borrowers of each type, but cannot identify the type of any individual borrower. Thus, a borrower's type is private information. This information restriction prevents perfect price discrimination by the lender but allows for the possibility of imperfect discrimination via policies that result in borrowers correctly sorting themselves into groups by choosing the loan package designed for their type.¹⁰ Finally, we assume that borrowers are unable to share loans.

The lender's problem is to choose a total repayment (i.e., principal plus interest) schedule for period 2, denoted by $P(q)$, such that any firm that borrows amount q in period 1 must repay amount P in period 2. Let $R_i(q)$ denote the reservation outlay for loans of size q by a type i borrower; that is, $R_i(q)$ indicates

⁹ Because endowment patterns are deterministic, there is no default risk in this model if the lender induces each type of borrower to self-select the "correct" loan size-interest rate package. We will specify self-selection constraints to ensure that all agents prefer the "correct" package. Consequently, we obtain price discrimination in the form of quantity discounts despite the absence of differences in default risk across borrowers.

¹⁰ With complete information about borrowers' endowments, the lender would use perfect price discrimination, offering each borrower a loan at the highest interest rate the borrower would willingly pay.

the maximum amount a type i borrower is willing to pay at time 2 for a time 1 loan of size q . Let $R'_i(q)$ denote the derivative of $R_i(q)$, which is the inverse demand for loans of size q . The inverse demand curve gives, for each loan size q , the total repayment amount that the lender must request for the borrower to choose that particular loan size. Further assume that the lowest-index group borrows nothing ($q_0 = 0$) and that the reservation value from borrowing zero is zero for all groups [$R_i(0) = 0$]. The lender's two-period profit-maximization problem can now be stated as follows:

$$\max_{(q_1, P(q_1)), \dots, (q_n, P(q_n))} \sum_{i=1}^n N_i [P(q_i) - q_i] \quad (1)$$

subject to

$$\begin{aligned} R_i(q_i) - P(q_i) &\geq R_i(q_j) - P(q_j) \\ \text{for all } i \text{ and all } j \neq i. \end{aligned} \quad (2)$$

Equation (1) is the lender's profit function, which is the aggregate amount repaid at time 2 by all borrowers (i.e., the lender's total revenue) minus the aggregate amount lent at time 1 (i.e., the lender's total cost). Equation (2) summarizes constraints for all types of borrowers that would induce a borrower of type i to willingly select a loan of size q . These constraints indicate that borrower i 's gain from choosing a loan of size q_i [the left-hand side of (2)] must be at least as great as the gain received from choosing a loan of some other size q_j [the right-hand side of (2)]. If (2) is satisfied, then only a type i borrower would prefer a loan of size q_i with total repayment $P(q_i)$. By choosing loan size q_i , a type i borrower reveals his/her type to the lender. Thus, the lender's two-period problem is to choose an amount to lend at time 1, q_i , and a total repayment schedule for time 2, $P(q_i)$, for every type of borrower.

III. PROPERTIES OF THE OPTIMAL SOLUTION

We can solve the lender's profit-maximization problem as follows. (A formal derivation of the solution appears in the appendix.) When the lender is maximizing profit, equation (2) is satisfied with equality because the lender need only ensure that borrower i is no worse off by selecting loan size q_i instead of any other loan size q_j , $j \neq i$. Using this fact and the assumptions that $q_0 = 0$ and $R_i(0) = 0$, and making successive substitutions into (2), one can show that

$$P(q_i) = \sum_{j=1}^i [R_j(q_j) - R_j(q_{j-1})]. \quad (3)$$

Equation (3) gives the lender's profit-maximizing repayment schedule, $P(q)$, for the loan sizes q_1, \dots, q_n .

The profit-maximizing loan sizes now can be determined as follows. Define

$$M_i = \sum_{j=i}^n N_j, \quad i = 1, \dots, n,$$

where M_i measures the total number of borrowers of types i through n ; thus $M_{n+1} = 0$ because n is the highest endowment group. Substituting (3) into (1), differentiating with respect to q_i and using the definition of M_i yields

$$\begin{aligned} R'_i(q_i) &= \left[\frac{M_{i+1}}{N_i + M_{i+1}} \right] R'_{i+1}(q_i) \\ &+ \left[\frac{N_i}{N_i + M_{i+1}} \right] \text{ for } i = 1, \dots, n, \end{aligned} \quad (4)$$

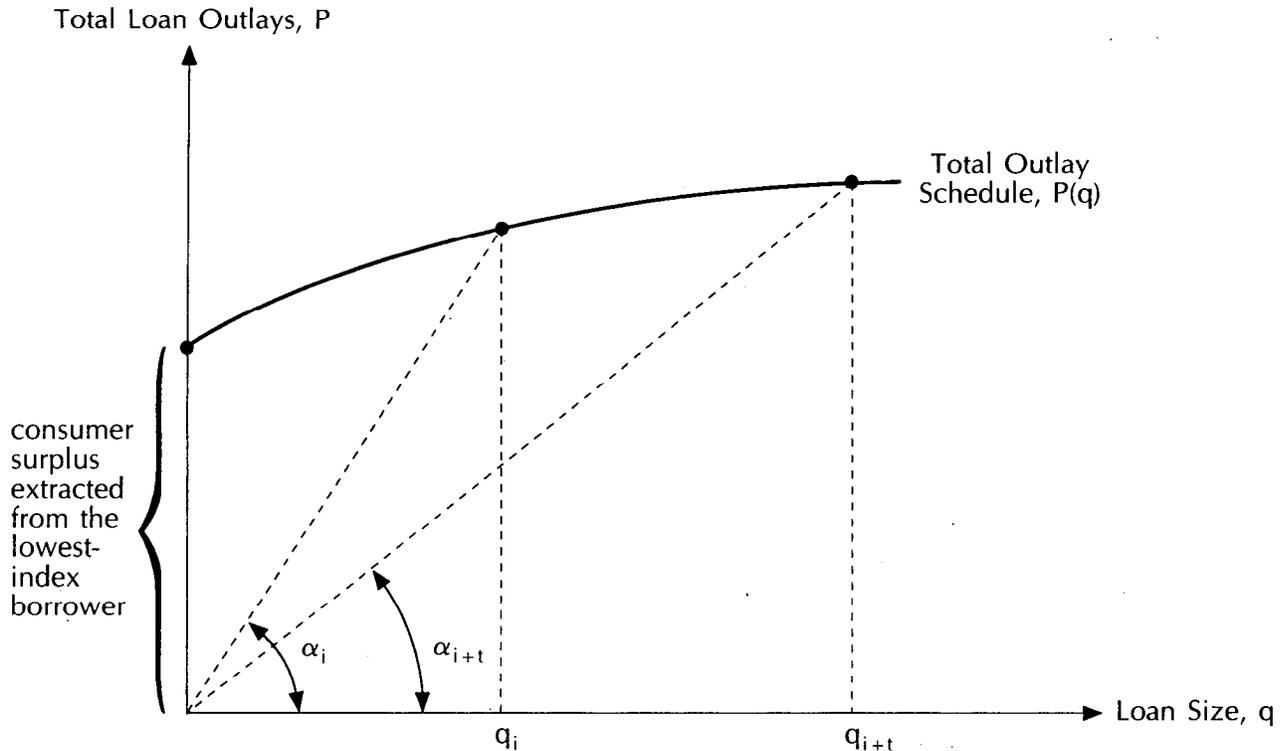
which can be solved for the lender's choice of loan sizes. Thus, equations (3) and (4) together give the solution to the lender's profit-maximization problem. This solution, which takes the form of a quantity-dependent interest rate schedule, is illustrated in Figure 1.

Equation (4), the formula for the optimal loan sizes, has the following properties. It indicates that the loan size, q_i , offered to borrowers of type $i = 1, \dots, n-1$ is strictly less than the size available in a perfectly competitive market for all groups except the largest. To see why, observe that equation (4) indicates that the profit-maximizing loan size for each group should be chosen so that the implicit marginal value of a loan of size q_i to type i borrowers, $R'_i(q_i)$, equals a weighted average of the implicit marginal value of the loan to the next highest group, $R'_{i+1}(q_i)$, and the marginal cost of lending, which is one. The weights are $M_{i+1}/(N_i + M_{i+1})$ and $N_i/(N_i + M_{i+1})$, respectively. In the perfectly competitive market, the lender instead would equate the loan's marginal value to its marginal cost.

Observe that a profit-maximizing lender will provide the perfectly competitive loan size to the largest borrowers, those in group $i = n$, because $M_{n+1} = 0$, which implies that $R'_n(q_n) = 1$ for group n . However, for all other borrower types the weight on the first term on the right-hand side of equation (4) is positive. This indicates that the marginal value of a loan to the next highest borrower (i.e., the next highest endowment firm) must be considered if the lender is to maximize profit. Thus, the implicit marginal price of a loan to group

Figure 1

OPTIMAL QUANTITY-DEPENDENT INTEREST RATE SCHEDULE



Note: Unlike a typical demand function, the total outlay schedule in Figure 1 slopes upward. This occurs because the loan outlay schedule, $P(q_i) = p_i q_i$, is the total amount that a borrower pays for a loan of size q_i . In contrast, an ordinary demand function represents the size of a loan requested as a function of price only (p_i). The total outlay schedule in Figure 1 is "quantity-dependent" in the sense that any quantity increase implies a decrease in the average interest rate charged by the lender, $\alpha_i = P(q_i)/q_i$. Thus, in Figure 1 the average interest rate charged on a loan of size q_{i+t} is lower than the average interest rate charged on a (smaller) loan of size q_i . Of course total outlays are higher for the larger loan (q_{i+t}) than the smaller loan (q_i). The average price will be the perfectly competitive price (i.e., a constant, or uniform, per unit price) only when the outlay schedule is a straight line through the origin.

$i = 1, \dots, n-1$ borrowers exceeds the marginal cost of the loan.

Equation (4) and $M_{n+1} = 0$ indicate that borrowers of type n (those with the largest endowment) clearly obtain the same loan size that they would receive in a perfectly competitive market. However, the degree of credit rationing experienced by borrowers from all other groups, $i = 1, \dots, n-1$, is regressive (i.e., inversely related to their index). To establish that the pattern of distortion is regressive, we prove in the appendix that our assumptions on preferences and net worth imply that $R'_{i+1}(q_i) > R'_i(q_i)$, which means that higher-index borrowers have a higher implicit value for a loan of size q_i than lower-index borrowers. This result and the restrictions on the distribution of borrower types (i.e., on the N_i) mean

that equation (4) implies that low-index (small) borrowers are relatively more constrained than high-index (large) borrowers.¹¹ This is confirmed by the first term on the right-hand side of equation (4), which is relatively higher for low-index groups.¹²

The final result pertains to the welfare properties of the discriminatory price and quantity scheme given

¹¹ See Spence (1980, p. 824) for a discussion of constraints on the distribution of consumer types.

¹² For example, suppose $N_i = 10$ for all borrower groups. Further, consider an economy with only two different borrower groups, $i = 1, 2$. Let $M_2 = 0.1$ and $M_3 = 0.9$. Then clearly $M_2/(N_1 + M_2) = 0.1/10.1$, which exceeds $M_3/(N_2 + M_3) = 0.9/10.9$, showing that the implicit marginal price of the loan to group 1 is higher than the implicit marginal price to group 2; the marginal cost is one in both cases. This pricing pattern is a general feature of the policy.

by equations (3) and (4). For any single price different from marginal cost, there is a discriminatory outlay schedule that benefits, or at least does not harm, all borrowers and the lender without side payments.¹³ In other words, if the borrowers and lender were given a choice between (i) any single interest rate policy that differs from the competitive interest rate and (ii) a quantity-dependent array of interest rates, with one rate appropriate for each group, then they would all prefer or at least be indifferent to the latter policy without coercion. This result indicates that there exists some quantity-dependent interest rate policy that makes all individuals at least as well off as any uniform interest rate policy, except for the single rate that prevails in a competitive market.

Two other features of the solution warrant discussion. Because imperfect information prevents perfect price discrimination, the lender must ensure that the loan size-interest rate package designed for each group satisfies equation (2). The ordering of loan sizes so that $q_i \geq q_{i-1}$ for all i , which is illustrated in Figure 1, is necessary for this constraint to be satisfied. This condition states that the lender must offer loans to high-index (i.e., large-endowment) borrowers that are at least as large as those offered to low-index borrowers. Further, $P(q)/q$ is weakly decreasing in q , which indicates that large borrowers pay lower average interest rates than small borrowers; the declining sequence of α_i in Figure 1 illustrates this. These features of the solution stem from the lender's need to ensure that each group selects the "correct" loan size-interest rate package. The lender must make the selection of a small loan undesirable for high-index borrowers. It does this by allowing the average interest rate to fall with loan size, thus letting larger borrowers keep some of their gains from trade. The lender must also ensure that small borrowers do not select loans designed for large borrowers. Such loan sharing is ruled out by assumption here.

We interpret the preceding results on loan size and interest rate distortions as credit rationing. All but the largest borrowers are prohibited from obtaining loans as large as they would choose if the lender had no market power and all agents had perfect information. Further, the lower a borrower's net worth, the more troublesome (i.e., distorting) the loan size

constraints imposed. These theoretical predictions appear to be consistent with the empirical results noted in the introduction. The intuition behind them is as follows. The model consists of numerous borrowers who differ along a single dimension, namely, second-period endowment. The lender has market power and wishes to maximize profit. It knows the distribution of borrower types in the economy, but does not know the identity of any particular borrower. This information restriction prohibits policies such as perfect price discrimination. However, the lender can exploit the correlation of borrowers' market choices with their endowment and does so by offering a discriminatory interest rate schedule that rations loan sizes to all but the largest group. The information implicitly revealed by borrowers' choices allows the lender to partially offset its inability, because of imperfect information about borrower characteristics, to design borrower-specific interest-rate schedules. Thus, the quantity constraints, which we interpret as credit rationing, arise endogenously as an optimal response to the information restriction in an imperfectly competitive market.

IV. CONCLUSION

This paper has presented a theoretical model of a commercial loan market characterized by imperfect information and imperfect competition. The model shows that a profit-maximizing lender operating in such a market will choose to price discriminate (or credit ration) by setting an inverse relationship between the loan sizes offered and the interest rates charged. This loan size-interest rate pattern is consistent with empirical evidence regarding commercial lending. In addition, it is a good example of how, as Friedrich von Hayek argued, the price system can economize on information in a way that brings about desirable results. Hayek (1945, pp. 526-27) noted that "the most significant fact about [the price] system is the economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action." The analysis here shows that a lender with imperfect information about borrower types can set an interest rate schedule that reveals borrowers' characteristics through their borrowing decisions. Interestingly, all loan market participants—the lender and all borrowers—are at least as well off with this discriminatory interest rate schedule as they would be if faced with any uniform interest rate other than the competitive rate.

¹³ See Spence (1980, pp. 823-24) for a formal proof.

APPENDIX

We adapt an argument in Villamil (1988) to show that our model is a special case of the widely used Spence nonuniform pricing model. Recall that $R_i(q) = pq$ is the borrowers' reservation outlay function, where p denotes the "reservation interest rate" that a borrower would be willing to pay for a loan of size q . We prove that the assumptions of our model imply reservation outlay functions that satisfy Spence's (1980, pp. 821-22) assumptions. We suppress the q_i and p_i notation because it is unnecessary; indeed, we prove our result for every nonnegative loan amount q . In equilibrium each q is associated with a particular p . Thus, the index i is implicit.

Spence's assumptions are

- S.1: Borrower types can be ordered so that for all q , $R_{i+1}(q) > R_i(q)$ and $R'_{i+1}(q) > R'_i(q)$.
- S.2: Firms need not borrow, and if they do not, $P(0) = 0$ and $R_i(0) = 0$.

Property S.1 implies that borrowers' reservation outlay schedules can be ordered so that a schedule representing $R_{i+1}(q)$ as a function of q lies above a schedule representing $R_i(q)$ and has a steeper slope. From S.2, it follows that the consumer surplus of a borrower of type i from a loan of size $q \geq 0$,

$R_i(q) - P(q)$, is at least as great as the reservation price for purchasing nothing, which is zero. The following proposition shows that our model satisfies these assumptions.

Proposition: The assumptions on preferences and endowments made in Section II imply reservation outlay functions for consumption in excess of endowment in the first period that satisfy S.1 and S.2.

Proof: Let p denote the per unit price of date $t+1$ good in terms of date t good. Let q denote the amount borrowed, i.e., the amount of first-period consumption in excess of w_1 , and let $h_i(p)$ denote the excess demand for first-period consumption by a type i borrower. From the assumptions that $u(\bullet)$ is concave and that consumption is a normal good, $h_i(p)$ is single-valued and decreasing in p where $h_i(p) > 0$. Thus, for all $q \geq 0$, $h_i(p)$ has an inverse that we shall denote by $R'_i(q)$. From the assumptions on preferences and net worth, $h_{i+1}(p) > h_i(p)$, and consequently, $R'_{i+1}(q) > R'_i(q)$ for all $q \geq 0$. Further, letting $R_i(q) = \int_0^q R'_i(z) dz$, we have that $R_{i+1}(q) > R_i(q)$ for all $q \geq 0$. Clearly, S.1 is satisfied. Property S.2 is also satisfied because any borrower can refuse to apply for a loan, in which case his/her repayment obligation and reservation outlay are zero [i.e., $P(0) = R_i(0) = 0$].

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