A Simple Model of Irving Fisher's Price-Level Stabilization Rule

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INTRODUCTION

It is now well understood that a price-level stabilization policy is more ambitious than a zero-inflation policy. Targeting zero inflation means that the central bank brings inflation to a halt but leaves the price level where it is at the end of the inflation. By contrast, targeting stable prices means that the central bank ends inflation and also rolls back prices to some fixed target level. By reversing inflated prices and restoring them to their pre-existing level, a price-stabilization policy eradicates the upward drift of prices that can occur under a zero-inflation policy. It follows that a stable-price policy is more stringent than a zero-inflation policy.

The history of monetary thought abounds with price-level (as opposed to inflation-rate) stabilization rules. Not all of these policy rules were sound; some would have destabilized prices rather than stabilizing them. Notoriously flawed was John Law's 1705 proposal to back the quantity of money dollar for dollar with the nominal value of land. His rule guaranteed that changes in the price of land would induce equiproportional changes in the money stock. Equally flawed was the celebrated *real bills doctrine* advanced by the antibullionist writers during the Bank Restriction period of the Napoleonic Wars. It tied money's issue to the "needs of trade" as represented by the nominal quantity of commercial bills presented to banks as loan collateral. It failed to note that since the nominal volume of bills supplied (or loans demanded) varies directly with general prices, accommodating the former with money creation meant accommodating prices as well. Seen by their proponents as price-stabilizing, both rules in fact would have expanded or contracted the money supply in response to shock-induced price-level changes, thus underwriting or validating those changes (see Mints, 1945, pp. 30, 47-48).

Ruling out such inherently fallacious schemes leaves the remaining valid ones. These fall into two categories. The first consists of non-activist policy rules that fix the money stock or its rate of change at a constant level. Milton Friedman's *k*-percent rule, which would establish the money stock at a fixed level when output's growth rate is zero, is perhaps the best known example of this type of rule. The second includes activist feedback rules which dictate predetermined corrective responses of the money supply and/or central-bank interest rates to price deviations from target. The proposals of David Ricardo, Knut Wicksell, and Irving Fisher exemplify this type of rule. Given England's 1810 regime of inconvertible paper currency and floating exchange rates, Ricardo (1810) advocated lock-step money-stock contraction in proportion to price-level increases as proxied by exchange-rate depreciation and the premium (excess of market price over mint price) on gold. Wicksell (1898, p. 198) proposed an interest-rate feedback rule: raise the bank interest rate when prices are rising, lower it when prices are falling, and keep it steady when prices are neither rising nor falling. Fisher suggested not one rule but two. His 1920 *compensated dollar plan* called for the policymakers to adjust the gold weight of the dollar equiproportionally to changes in the preceding month's general price index. In essence, he posited the relationship: dollar price of goods = dollar price of gold x gold price of goods. Official adjustments in the dollar price of gold, he thought, would offset fluctuations in the world gold price of goods (as proxied by the preceding month's general price index), thus stabilizing the dollar price of goods. His second rule (1935) was more conventional. Much like stabilization rules proposed today, it dictated automatic variations in the money stock to correct price-level deviations from target.

This article examines Fisher's second policy rule, particularly its dynamic properties. Fisher himself failed to investigate these properties. He provided no analytical model in support of his rule. Such a model is needed to show (1) that the rule would indeed force prices to converge to target, (2) how fast they would converge, and (3) whether the resulting path is oscillatory or monotonic. Lastly, only the model can demonstrate rigorously whether Fisher's rule is capable of outperforming rival rules such as the constant money-stock rule.

The following paragraphs attempt to provide the missing model underlying Fisher's scheme. In so doing, they contribute three innovations to the stabilization literature. First, they express Fisher's scheme in equations, something not done before. They represent his proposal in the form of price-change equations and policy-reaction functions suggested by A. W. Phillips' (1954, 1957) classic work on closed-loop feedback control mechanisms.

Second, they thus extend Phillips' analysis to encompass monetary models of price-level stabilization. Heretofore, Phillips' work has been applied exclusively to the design of output-stabilizing fiscal rules in Keynesian multiplier-accelerator models. (See the texts of Allen, 1959, 1967; Meade, 1972; Nagatani, 1981; and Turnovsky, 1977, for examples.) In finding a new use for Phillips' work, the article incorporates his notions of proportional, derivative, and integral control into Fisher's policy-response functions. The result is to show how the money stock in Fisher's scheme can be programmed to respond automatically (1) to the discrepancy between actual and target prices, (2) to the speed with which that discrepancy is rising or falling, and (3) to the cumulative value of the discrepancy over time beginning with the inauguration of his scheme.

Last but not least, the article compares within a single model the relative performance of Fisher's activist feedback rule with that of a non-activist constant money-stock rule. Policymakers of course must be convinced that Fisher's rule dominates rival candidate rules before they would consider adopting it. A related issue concerns the doctrinal accuracy of the model. To ensure that the model faithfully captures Fisher's thinking, one must outline his scheme to determine the appropriate variables and equations to use.

FISHER'S SCHEME

Fisher presented his proposal in his 1935 book *100% Money*. He argued that the monetary authorities could stabilize prices at a fixed target level via open market operations.

> If money became scarce, as shown by a tendency of the price level to fall, more could be supplied instantly; and if superabundant, some could be withdrawn with equal promptness. . . . The money management would thus consist . . . of buying [government securities] whenever the price level threatened to fall below the stipulated par and selling whenever it threatened to rise above that par. (P. 97)

He reasoned that price movements stem from excess money supplies or demands. Since money is employed for spending, these excess supplies spill over into the commodity market in the form of excess aggregate demand for goods, thus putting upward pressure on prices. Prices continue to rise until the surplus money is absorbed by higher cash balances needed to purchase the same real output at elevated prices. Likewise, excess money demands, manifested in increased hoarding and decreased spending, cause aggregate demand contractions and downward pressure on prices in the goods market. Prices and the associated need for transaction balances continue to fall until money demand equals money supply. In either case, appropriate variations of the money stock could, Fisher thought, correct the resulting price-level deviations from target. In other words, the Federal Reserve expands the money stock when prices fall below target and contracts the money stock when prices rise above target. Clearly, money constitutes the policy instrument and the price level the goal variable in Fisher's scheme.

A policy instrument of course is only as good as the Fed's ability to control it. In Fisher's view this ability was absolute—or at least it could be made so by the 100 percent reserve regime advocated in his book. As he saw it, the Fed exercises direct control over the high-powered monetary base. And since a 100 percent reserve regime renders the base and the money stock one and the same aggregate, it follows that tight command of the one constitutes perfect regulation of the other. In sum, when deposit money is backed dollar for dollar with bank reserves as prescribed in his book, there can be no slippage in money-stock control to disqualify money as the policy instrument. For that matter, little slippage would occur in a fractional reserve system or even a system of no reserve requirements as long as deposit money bore a stable relationship to high-powered money.

Nor did Fisher see policy lags as a problem. He knew that his rule to be effective required two things: prompt direct response of prices to money and equally prompt feedback response of money to prices. He was sanguine about both, although less so about the former. In *Stabilizing the Dollar* (1920), he stated that prices seem to follow money with "a lag of one to three months" (p. 29). As for money's corrective response to price misbehavior, he found virtually no lag at all. Money, he insisted, can be "supplied instantly" or "withdrawn with equal promptness" in reaction to price deviations from target. His scheme admits of no significant delays to retard the Fed from achieving its desired target setting of the money stock. Such setting occurs immediately. Accordingly, the model below omits all policy lags. On this point he was quite clear.

He was not so clear on other matters, however. For example, he did not specify the exact price indicator to which the Fed should react. Should it adjust the money stock in response to the differential between actual and target prices? To changes in that differential? To both? Suppose it has missed the target in every period since it initiated its policy. Should it forgive these past misses? Or should it allow them to influence current policy by linking the money stock to the cumulative sum of the past price errors? Which price indicator and associated policy response yields the smoothest and quickest path to price stability? Fisher did not say. Moreover, as noted above, he offered no proof that his feedback rule could in fact deliver price stability or that it would outperform other candidate rules.

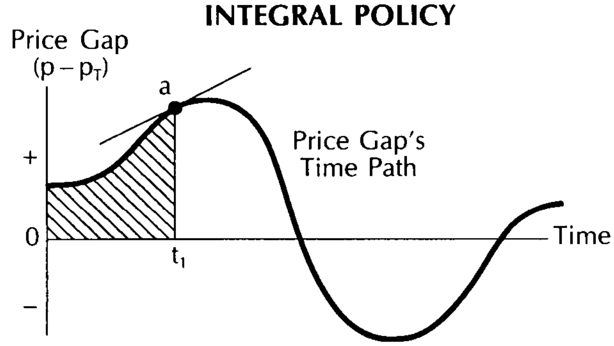
Modeling Fisher's Scheme

Addressing these issues requires an explicit analytical framework. The one supplied here employs the four-step technique pioneered by A. W. Phillips (1954, 1957) in his celebrated analysis of stabilization policy.

Step one models how the price level would behave if uncorrected by policy. Consistent with Fisher's exposition, the model treats prices as moving in response to excess money supplies and demands.

Step two incorporates policy-response functions embodying elements of *proportional*, *derivative*, and *integral control*. A proportional feedback control rule adjusts the money stock in response to current price deviations from target. Derivative control adjusts money in response to the deviation's rate of change. Integral control adjusts money in response to the cumulative sum of deviations over time. It seeks to correct the time integral of all past misses from target. For example, suppose prices since the inauguration of corrective policy have fluctuated about target. Or, what is the same thing, the discrepancy between actual and target prices $p - p_T$ has fluctuated about zero, as shown in the figure. The application of proportional policy at time t_1 requires money-stock contraction in proportion to the price gap t_1a. Derivative policy contracts the money stock by a fixed proportion of the rate of price rise indicated by the slope of the tangent to the curve at time t_1. Finally, integral policy aimed at correcting cumulative deviations from target contracts the money stock in

PROPORTIONAL, DERIVATIVE, AND INTEGRAL POLICY



Proportional policy contracts the money stock in proportion to the price gap t_1a. Derivative policy contracts the money stock in proportion to the rate of price rise as indicated by the slope of the tangent to the curve at point a. Integral policy contracts the money stock in proportion to the cumulative value of the price gap over time as indicated by the shaded area under the curve.

proportion to the shaded area under the curve up to time t_1. In short, proportional policy focuses on current price gaps, derivative policy on the direction of movement of such gaps, and integral policy on the sum of all gaps, past and present. Once applied, each policy produces a different policy-corrected path for prices.

Step three solves the model for these alternative policy-corrected paths. Step four uses a loss function to measure the cost (in terms of reputational damage suffered by the Fed when prices deviate from target) of adhering to each path. This procedure allows one to rank the alternative policy rules according to how smoothly and quickly they stabilize prices. As shown below, at least one of the feedback rules dominates the fixed money-stock rule.

Price-Change Equation

The first step is to model the non-policy determinants of price-level behavior. To Fisher, price changes emanated from excess money supplies and demands caused, say, by policy mistakes and/or shifts in the amount of money people want to hold at existing prices and real incomes. In his words, prices fall when money is "scarce" relative to the demand for it and rise when money is "superabundant" relative to demand. Accordingly, one seeks the simplest equation that captures his hypothesis.

That equation is $\dot{p} = \alpha(m - kpq)$, where the time derivative $\dot{p}$ denotes a change in prices, m denotes the money supply, the product kpq denotes money demand consisting of velocity's inverse or the Cambridge k times the price level p times real output q, and α is a positive goods-market reaction coefficient expressing the speed of response of prices to excess money supply.[1] Let the model's time unit be a calendar quarter. Define $T=1/\alpha$ as the number of quarters required for prices to adjust to clear the market for money balances. Then Fisher's 1920 finding that prices follow money with a lag of close to three months implies that both α and T are approximately equal to unity in magnitude. As for the other items in the equation, output q and the Cambridge k are taken as given constants at their long-run equilibrium values. In his 1935 book, Fisher mentioned no other money-demand determinants such as interest rates or price-change expectations. For that reason they are omitted here.[2]

Policy Rules

The next step is to specify the alternative policy-response functions the Fed might use to bring prices to target in the model. These functions are absolutely essential. Without them, prices p would adjust in response to an excess money supply until they reached an equilibrium level $p=m/kq$ different from the target level p_T. Permanent price gaps would result.

Under a *constant money-stock rule*, the Fed merely sets the money stock at the level $m_T=kqp_T$ that equilibrates money supply and demand at the target price level p_T and then leaves it there. Other than setting m_T consistent with p_T, the Fed does nothing else. Thus the money-stock equation is simply $m_c=m_T$, where m_c denotes constant money-stock policy.

Activist feedback rules attempt to improve upon the constant money-stock rule. Thus under a *proportional feedback rule* the Fed adjusts the money stock above or below the desired long-run equilibrium level m_T as prices are below or above their target level. In other words, the money stock is set to counter price gaps or deviations from target. The resulting policy-response equation is $m_p=m_T-\beta(p-p_T)$, where m_p denotes proportional policy and β is the proportional correction coefficient.

With a *derivative feedback rule* the Fed adjusts the money stock in response to the speed with which the price gap $p-p_T$ is increasing or decreasing in size. Or, what is the same thing (since the target price level p_T is fixed), it adjusts the money stock above or below money's long-run equilibrium level to counter falls or rises in the price level $\dot{p}$. The resulting policy-response equation is $m_d=m_T-\gamma\dot{p}$, where m_d denotes derivative policy and γ is the derivative correction coefficient.

With an *integral feedback rule* the Fed adjusts the money stock to correct cumulative price gaps or the sum of all past policy misses over time. The Fed learns from these price errors. It uses the information given by their integral to determine the current setting of the money stock. Accordingly, the policy equation is $m_i=m_T-\delta\int_0^t(p-p_T)dt$, where m_i denotes integral policy, $\int_0^t(\ \ )dt$ denotes the integral operator, and δ is the integral correction coefficient. This rule can be given an alternative expression. When differentiated to get rid of the integral, it becomes $\dot{m}=-\delta(p-p_T)$, stating that the Fed sets the money stock's *rate of change* opposite to the direction that prices are currently deviating from target.

Finally, the Fed may employ a *mixed feedback rule* involving various combinations of the foregoing equations. For example, a mixed proportional-derivative rule would yield the policy-response function $m_m=m_T-\beta(p-p_T)-\gamma\dot{p}$ embodying gap and gap-change terms together with their policy correction coefficients.

Price Time Paths and Loss Functions

The third step is to substitute each of the foregoing policy rules into the Fisherian price-change

[1] This equation admits of a straightforward derivation. Define an excess supply of money x as the surplus of money m over the demand for it kpq, or $x=m-kpq$. Assume this excess money supply spills over into the commodity market to underwrite an excess demand for goods g. In short, $x=g$. The excess demand for goods in turn puts upward pressure on prices, which rise in proportion to the excess demand, or $\dot{p}=\alpha g$, where α is the factor of proportionality. Substitution of x for g and $m-kpq$ for x yields the equation of the text.

[2] It is tempting to write Fisher's equation in expectations-augmented form as $\dot{p}=\alpha[m-(kpq-\tau\dot{p}^e)]$ where the term $-\tau\dot{p}^e$ captures the effect of price-change expectations $\dot{p}^e$ on money demand. This term says that people expecting money to depreciate in value will hold smaller cash balances than they would if they expected no such depreciation. One could further assume that people form their expectations rationally such that expected price changes equal actual ones $(\dot{p}^e=\dot{p})$ in this non-stochastic model and the equation reduces to $\dot{p}=[\alpha/(1-\alpha\tau)](m-kpq)$. But such an equation would be inconsistent with Fisher's own views on the subject. True, in his *The Purchasing Power of Money* (1911), he recognized that inflation perceptions affect money demand. When money "is depreciating," he wrote, "holders will get rid of it as fast as possible" (p. 63). But he did not spell out how such perceptions are formed or how they get incorporated into money demand functions. Nor did he believe in rational expectations. He held that people are subject to money illusion and so revise their expectations slowly in the face of actual price changes such that when those changes occur expectations lag behind. Indeed, he coined the phrase "money illusion" and used it as the title of his 1928 book.

equation $\dot{p}=\alpha(m-kpq)$. Doing so produces expressions whose solutions are the policy-corrected time paths of prices.

Thus substitution of the constant money-stock rule into the price-change equation yields $\dot{p}=\alpha kq(p_T-p)$. Solving this first-order expression for the time path of prices gives $p=p_T+(p_0-p_T)e^{-at}$, where p_0 denotes the (perturbed) price level at time $t=0$, t denotes time, e is the base of the natural logarithm system, and the parameter $a=\alpha kq$ denotes speed of convergence to target. This expression says that prices converge to target at a rate of $a=\alpha kq$ per unit of time. In short, as time passes and t gets large, the last term on the right-hand side of the equation goes to zero so that only the first term p_T remains. In this way the path to price stability terminates when $p=p_T$.

Associated with the path are certain costs to the Fed. The Fed's objective is to keep prices as close to target as possible over time. Society penalizes it for failing to do so. It suffers losses in reputation, credibility, and prestige that vary directly and disproportionally with the duration and size of its policy errors. These losses can be measured by the quadratic cost function expressing the Fed's reputational loss L as the cumulative squared deviation of prices from their desired target level, or $L=\int_0^\infty (p-p_T)^2dt$. Substituting the price path into this loss function and integrating yields the cost of adhering to the non-activist constant money-stock rule, or $L_c=(p_0-p_T)^2/2a$. As a hypothetical numerical example, let $p_T=1$, $p_0=2$, $\alpha=1$, $k=1/2$, $q=100$, and $a=50$. Then the quantitative measure of the loss is $L_c=1/100$.

EFFECTIVENESS OF THE ACTIVIST PROPORTIONAL RULE

To compare the foregoing loss with those of the activist feedback policy rules, one must derive the price paths and loss measures associated with the latter rules. Thus the proportional feedback rule yields the price path $p=p_T+(p_0-p_T)e^{-bt}$ with associated loss measure $L_p=(p_0-p_T)^2/2b$. Here $b=\alpha(kq+\beta)$ is the speed-of-convergence parameter. Since parameter b is larger than parameter a computed above, prices under the proportional rule converge to target faster than they do under the constant money-stock rule. It follows that prices deviate from target for a shorter time under the proportional rule. Consequently that rule yields the smallest loss of the two and thus dominates the constant money-stock rule. Numerically, $L_p=1/102$, assuming the proportional correction coefficient $\beta=1$ and the constants α, k, q, p_0, and p_T possess their values as assigned above. This loss compares favorably with the corresponding loss of 1/100 associated with the constant money-stock rule. Given a choice between the two rules, the Fed will select the proportional rule.

A word of warning is in order here. The proportional rule's superiority rests heavily on Fisher's assumption of no policy lags. By slowing convergence to target, policy lags could reverse the ranking of the two rules. Such lags would delay the Fed's adjustment $\dot{m}$ of the money stock m to its desired proportional setting m_p. A new equation $\dot{m}=\lambda(m_p-m)$, where the positive coefficient λ represents the policy lag, would have to be added to the model. The resulting reduced-form expression for $\ddot{p}$ would be a second-order differential equation whose solution—the time path of prices—would be more complicated than before. Overshooting and oscillations would be a distinct possibility; slower convergence a certainty. These considerations highlight the importance of Fisher's assumption of zero policy lags. Embodied in the model, his assumption ensures the superiority of the proportional rule such that the Fed will select it.

OPTIMAL VALUE OF THE β COEFFICIENT

Given the proportional rule's capability of outperforming the constant money-stock rule, a natural question to ask next is whether the proportional correction coefficient β has been assigned its optimal value. The preceding numerical example assumed $\beta=1$. But a glance at the proportional rule's loss measure $L_p=(p_0-p_T)^2/2\alpha(kq+\beta)$ suggests that the Fed should make β as large as possible $(\beta\rightarrow\infty)$ and L_p negligibly small. That is, optimality considerations would seem to compel instantaneous monetary contraction in amounts sufficient to force prices to target immediately.

Fisher, however, would have rejected this implication of the mathematical formulation of his scheme. He would have condemned the violent monetary contraction implied by high values of β. To him such contraction spelled devastating losses to output and employment. In his *The Purchasing Power of Money* (1911) and his 1926 paper on price changes and unemployment, he ascribed these losses to the failure of sticky nominal wage and interest rates to respond as fast as product prices to monetary shocks. He attributed nominal wage rigidities to fixed contracts and the inertia of custom; nominal interest rate rigidities to price misperceptions and sluggishly adjusting inflation expectations. Because of these inhibiting forces, a sharp fall in money and prices would transform sticky nominal wage and interest rates into rising real rates, thus depressing economic activity. In his 1933 debt-deflation theory of great depressions, he cited still another reason to fear violent monetary contraction. He argued that price deflation could, by raising the real burden of nominal debt, precipitate a wave of business bankruptcies with all its adverse repercussions for the real economy. To avoid these consequences, Fisher would have recommended relatively gradual monetary contraction implied by moderate values of β (such as $\beta = 1$). Consistent with his views, β's value here is restricted to unity.

Relative Effectiveness of the Derivative Rule

As for the other candidate rules, the Fed will reject as inferior to proportional policy the derivative rule. That rule calls for money-stock adjustments opposite to the direction prices are moving. It exerts stabilizing pressure when prices are moving away from target; less so when they are moving toward target. When prices fall toward target, the derivative rule interferes perversely by expanding the money stock. In so doing, it retards convergence and becomes a relatively unattractive option. In symbols, derivative policy results in the price path $p = p_T + (p_0 - p_T)e^{-ct}$ and loss function $L_d = (p_0 - p_T)^2/2c$, with speed of convergence denoted by the parameter $c = \alpha kq/(1 + \alpha\gamma)$. This parameter is smaller than its counterparts a and b, signifying slower convergence and larger reputation loss with a derivative rule than with a constant or proportional rule. Numerically, the derivative rule's loss is $L_d = 1/50$, assuming the derivative correction coefficient γ is assigned a value of one and the other constants possess the same magnitudes as defined above. This loss is twice that of the constant money-stock rule and more than twice that of the proportional rule. The Fed, therefore, would hardly opt for a derivative rule.

Relative Effectiveness of the Integral Rule

Nor would the Fed opt for an integral rule that seeks to correct the sum of all past and present misses of target. The past misses would continue to influence policy even when the current price level was close to target. Too strong a response to them would cause overshooting. Conversely, too weak a response would cause prices to move too slowly to target. Suppose past misses totaled 5 when the current miss was 0. Integral policy in this case would push the price level below target. And if the same past misses were exactly counterbalanced by a current miss of -5, integral policy would fail at that moment to put corrective pressure on prices. Of course continuation of the current miss would eventually activate corrective pressure. But this pressure would be slow in coming. Thus while stabilizing prices in the long run, integral policy might do so sluggishly and via damped oscillatory paths.

Integral policy yields the time path $p = p_T + A_1 e^{r_1 t} + A_2 e^{r_2 t}$. Here A_1, A_2 are constants of integration determined by initial conditions. And r_1, r_2 = $(-\alpha kq/2) \pm \sqrt{(\alpha kq)^2 - 4\alpha\delta}/2$ are the characteristic roots of the left-hand or homogeneous part of the second-order expression $\ddot{p} + (\alpha kq)\dot{p} + \alpha\delta p = \alpha\delta p_T$, obtained by differentiating the Fed's policy-reaction function to eliminate the integral and substituting the result into the Fisherian price-change equation. The roots r_1, r_2 possess negative real parts $(-\alpha kq/2)$ thus ensuring convergence. Damped oscillations about target occur if $(\alpha kq)^2 < 4\alpha\delta$ or, in other words, if the integral correction coefficient δ is larger than $\alpha k^2 q^2/4$. Monotonic convergence results from smaller values of that coefficient. But convergence, whether cyclical or monotonic, is slower under the integral rule than under the proportional and constant money-stock rules. The above-mentioned characteristic roots reveal as much. Dwarfed by the speed-of-convergence parameters of the other rules, their relatively small size renders the integral rule inferior to its rivals. Confirmation comes from the loss function, which shows a large reputational loss associated with the integral rule. One computes the integral policy's loss function as $L_i = -[A_1^2/2r_1] - [2A_1 A_2/(r_1 + r_2)] - [A_2^2/2r_2]$, where $A_1 = [\dot{p}_0 - r_2(p_0 - p_T)]/(r_1 - r_2)$ and $A_2 = [r_1(p_0 - p_T) - \dot{p}_0]/(r_1 - r_2)$. Although hard to evaluate analytically, this function yields to numerical computation. Let $\alpha = \delta = p_T = 1$, $\dot{p}_0 = -1$, $p_0 = 2$, $k = 1/2$, and $q = 100$ as before. Then the function in this hypothetical illustrative example produces a numerical value of 26, several hundred times the losses under the other rules.

Relative Effectiveness of a Mixed Rule

Finally, the Fed might try a mixed rule embodying proportional and derivative elements. The mixed rule yields the price path $p = p_T + (p_0 - p_T)e^{-dt}$ and associated loss function $L_m = (p_0 - p_T)^2/2d$, where the speed-of-convergence parameter $d = \alpha(\beta + kq)/(1 + \alpha\gamma)$. With the coefficient values as assigned above, the loss L_m is 2/102, ranking the mixed rule inferior to the proportional and constant money-stock rules, but superior to the derivative and integral rules. This ranking, however, depends on the values assigned to the coefficients. Giving γ a value of 1/52 instead of 1 would reverse the order of the mixed and constant money-stock rules. In general, the mixed rule ranks below the constant money-stock rule if $\gamma > \beta/\alpha kq$ and above it if that inequality is reversed.

Conclusion

Fisher's feedback policy rule delivers price stability in the simple money-demand-and-supply model presented here. And it does so whether the rule is expressed in terms of proportional, derivative, integral, or mixed control. All the rules yield paths that converge to target, albeit at different speeds. Proportional policy yields the quickest and smoothest path, followed by mixed, derivative, and integral policies in that order. Of these four activist feedback rules, only the proportional always outperforms the nonactivist constant money-stock rule. For this reason, proportional policy's loss measure is the smallest of the lot. Indeed, one can rank the loss measures to show that the proportional feedback rule dominates the constant money-stock rule, which in turn dominates the derivative and integral rules. While the mixed rule may, at certain values of the γ coefficient, outrank the constant money-stock rule, it can never dominate the proportional rule. Provided policy lags are short or nonexistent, these results create a presumption in favor of a proportional feedback rule.

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