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AN ALTERNATE METHOD OF ESTIMATING THE CAGAN MONEY DEMAND  
FUNCTION IN HYPERINFLATION UNDER RATIONAL EXPECTATIONS

Marvin Goodfriend

Federal Reserve Bank of Richmond

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1. Introduction

This paper contains a description and implementation of a new strategy for estimating the Cagan money demand function under rational expectations. The procedure has three main virtues. First, it is implemented without imposing restrictions on the money supply process. Second, the procedure is extremely simple and economical. Third, it admits a simple test of a restriction implied by the Cagan money demand function.

The technique presented in this paper utilizes only (1) the assumed Cagan structure of money demand, (2) the assumption that anticipations are formed rationally in the sense of Muth [1961], (3) the assumption that accurate contemporaneous information on the price level and money stock is available to individuals, and (4) the assumption that unobservable noise in the portfolio balance schedule is small. Under these assumptions, the proposed estimation strategy delivers consistent estimates of  $\alpha$ , the slope of the log of the demand for real balances with respect to anticipated inflation, in the Cagan money demand function.

The remainder of this paper is organized in four sections. Section 2 contains a description of the estimation strategy. The estimation is carried out on the Cagan hyperinflation data in Section 3. Section 4 relates issues that have been raised in recent hyperinflation studies to the estimation results of this paper. A summary follows.

## 2. Description of the Estimation Strategy

The Cagan money demand function is specified as follows:

$$(1) \quad \ln M_t - \ln P_t = \lambda - \alpha (\Delta \ln P_{t+1}^e) + v_t$$

where  $\ln M_t \equiv$  the natural log of the money supply in period  $t$

$\ln P_t \equiv$  the natural log of the price level in period  $t$

$\Delta \ln P_{t+1}^e \equiv$  the subjective anticipation formed in period  $t$  of the period  $t+1$  rate of inflation

$v_t \equiv$  an unobservable disturbance (velocity shock) in period  $t$

$\alpha \equiv$  the slope of the log of the demand for real balances with respect to anticipated inflation,  $\alpha > 0$

$\lambda$  is a constant

It is useful for the following discussion to write the subjective anticipated rate of inflation as the sum of the actual ex post rate of inflation and an ex post anticipation error as follows:

$$(2) \quad \Delta \ln P_{t+1}^e \equiv \Delta \ln P_{t+1} - u_{t+1}$$

where  $\Delta \ln P_{t+1} \equiv$  the realized rate of inflation in period  $t+1$

$u_{t+1} \equiv$  a downward ex post anticipation error in period  $t+1$

Substituting for  $\Delta \ln P_{t+1}^e$  in (1) with (2) and rearranging the result yields:

$$(3) \quad \ln P_{t+1} = -\frac{\lambda}{\alpha} + \frac{1+\alpha}{\alpha} \ln P_t - \frac{1}{\alpha} \ln M_t + \frac{1}{\alpha} v_t + u_{t+1}$$

Note that the coefficients on the lagged price level and money stock are greater than one and negative, respectively. It is useful to explain these restrictions. Consider the lagged price level coefficient first. Other right side variables held constant, a rise in the contemporaneous (period  $t$ ) price level is associated with a reduction in contemporaneous

real balances. The Cagan structure of money demand together with the implicit assumption of stock monetary equilibrium implies that the market is satisfied with reduced real balances only if anticipated inflation has risen. From (2), for  $u_{t+1}$  held constant, this means  $\Delta \ln P_{t+1}$  must be higher. This, in turn, means that a given rise of  $\ln P_t$  must be associated with a more than proportionate rise of  $\ln P_{t+1}$ . Hence, the coefficient  $\ln P_t$  is greater than one.

Now consider the coefficient on the lagged money stock. Again other right side variables held constant, a rise in the contemporaneous (period  $t$ ) money stock is associated with a rise in contemporaneous real balances. In this case anticipated inflation must be lower if the market is to be satisfied holding greater real balances. For given  $u_{t+1}$  and  $\ln P_t$ , this means  $\ln P_{t+1}$  must be lower. Hence the negative coefficient on  $\ln M_t$ .

Note that the sum of coefficients on  $\ln P_t$  and  $\ln M_t$  is one. This is because an equiproportionate rise of the period  $t$  price level and money stock leaves real balances unaffected. Therefore, this disturbance must be associated with an unchanged anticipated rate of inflation. From (2), for  $u_{t+1}$  held constant, this implies  $\Delta \ln P_{t+1}$  must be unchanged, so that  $\ln P_{t+1}$  must rise proportionally with  $\ln P_t$  and  $\ln M_t$ .

Can equation (3) be consistently estimated? The answer to this question depends on the relative importance of velocity shocks in the equation. Two cases, one where velocity shocks are zero and another where they are nonnegligible, are discussed below.

Case #1: No Velocity Shocks ( $\sigma_v^2 = 0$ )

Suppose that the velocity shocks ( $v$ 's) are small, i.e., noise in the portfolio balance schedule is insignificant, then (3) can be rewritten as:

$$(4) \quad \ln P_{t+1} = -\frac{\lambda}{\alpha} - \frac{1}{\alpha} \ln M_t + \frac{1+\alpha}{\alpha} \ln P_t + u_{t+1}.$$

Equation (4) can be consistently estimated only if the ex post anticipation error  $u_{t+1}$  is uncorrelated with  $\ln M_t$  and  $\ln P_t$ . Specifically, for (4) to be consistently estimated it is necessary that

$$E_t \left[ u_{t+1} \mid \ln M_t, \ln P_t \right] = 0.$$

The rational expectations assumption can help guarantee that this condition holds. Muth's rational expectations assumption says that the market's subjective anticipation of inflation should equal the mathematical expectation conditional on information available to the market in the period when the anticipation is formed. If it is also assumed that the market has accurate contemporaneous information on the price level and the money stock, then the ex post anticipation error must be uncorrelated with the lagged price level and money stock. In other words,

$E_t \left[ u_{t+1} \mid \ln M_t, \ln P_t \right] = 0$  as required. Therefore, the disturbance term in equation (4) is distributed independently of the two explanatory variables.

Since the contemporaneous price level is in the market's information set, the contemporaneous anticipation error is in its information set as well. The assumption that anticipations are formed rationally therefore implies that  $E_t \left[ u_{t+1} \mid \ln M_t, \ln P_t, u_t \right] = 0$ . The  $u_{t+1}$  expectation error must be orthogonal to the  $u_t$  expectation error. Furthermore, this orthogonality extends to all lagged anticipation errors, prices, and money stocks, since these lagged values are also in the information set. In other words, the disturbances in (4) must be serially uncorrelated at all lags. It follows that the coefficient in (4) can be estimated consistently with OLS. Moreover, the predicted absence of serial correlation of the residuals

in (4) is an important testable implication of the joint hypothesis underlying the equation.

The price level and money stock are highly correlated, especially in hyperinflation. This multicollinearity could lead to low precision on the coefficients of  $\ln M_t$  and  $\ln P_t$  if estimated separately. However, Cagan's money demand specification places a restriction across the two coefficients, requiring them to sum to one. More than that, both coefficients are specific functions of  $\alpha$ . A restricted version of equation (4) can be estimated so that only one parameter,  $\alpha$ , need be recovered from the coefficients of the two explanatory variables. This avoids the difficulties that multicollinearity poses for the estimation process, since the estimation procedure is not required to extract the separate effects of money and prices, but rather only the joint effect operating through the single parameter  $\alpha$ .

The restriction that Cagan's money demand specification imposes can serve as the basis for a test of the joint hypothesis underlying equation (4). A likelihood ratio can be calculated to check whether the sum of squared residuals is significantly larger for the restricted compared to the unrestricted fit of equation (4). If it is, this constitutes evidence against the restriction and the entire set of hypotheses underlying equation (4).

What makes this estimation technique work? If an econometrician believes that the market forms anticipations rationally, then in order to consistently estimate a model involving anticipations formation, it seems that the econometrician would first have to decide himself what the rationally anticipated future money supply movements were during his

sample period. Even if the econometrician were willing to maintain strong assumptions on the information set (such as the assumed availability of perfect contemporaneous information on the price level and money stock), it seems he would still have to place restrictions on, and estimate, an actual money supply rule as a basis for rational predictions of future money growth and inflation to use in his estimation procedure.

My technique demonstrates that by maintaining the assumption that  $\sigma_v^2 = 0$ , so that the noise in the portfolio balance schedule is zero, a great simplification can be achieved.<sup>2</sup> In this case, the econometrician is willing to believe that changes in real money balances are due entirely to changes in anticipated rates of inflation.<sup>3</sup> The econometrician can then "turn the money demand function around" and use observations on contemporaneous money stock and price level to infer the rate of inflation that the market anticipates in each period. If he further is willing to believe that the market forms anticipations rationally as if it had accurate contemporaneous observations on money and prices, then, if he knew the coefficient values in (4) he could use that equation to make unbiased predictions of the rate of inflation. On the other hand, if he does not know the coefficient values but instead wants to estimate them, he could use (4) as a "regression equation" to estimate the parameters of the money demand function itself, without having to develop particular restrictions on the money supply process.

#### Case #2: Velocity Shocks ( $\sigma_v^2 \neq 0$ )

Suppose that velocity shocks are not negligible,  $\sigma_v^2 \neq 0$ . In this case, the error term in equation (4) will, in general, be correlated with

right side variables. To illustrate, suppose  $u_{t+1}$  is zero so there is no ex post anticipation error. Equation (4) could still show an error in period  $t+1$  if  $v_t$  is nonzero. The velocity shock would cause simultaneous adjustment in the current price level as well as the anticipated price level. If (4) were estimated with OLS, then correlation between the unobservable disturbance and  $\ln P_t$  could introduce a bias into the estimated coefficient  $\alpha$ . Velocity shocks will in general be correlated with the contemporaneous price level and possibly the contemporaneous money stock as well, depending on the money supply rule. When velocity shocks are significant, the proposed estimation technique will, in general, not deliver consistent estimates of the parameters in equation (4).

### 3. Empirical Results

Table 1 summarizes the results of estimating equation (4) on Cagan's [1956] hyperinflation data. The variable of primary interest is  $\alpha$ , the elasticity of the demand for real balances with respect to anticipated inflation. An estimate of  $\alpha$  is obtained from equation (4) by a nonlinear least squares procedure under the restrictions that the coefficients of  $\ln P_t$  and  $\ln M_t$  equal  $\frac{1+\alpha}{\alpha}$  and  $-\frac{1}{\alpha}$ , respectively.<sup>4</sup> The estimated values of  $\alpha$  are shown in the column  $\hat{\alpha}$ . For comparison, Cagan's [1956] and Sargent's [1977] estimates of  $\alpha$  for the same data are reported in Tables 2 and 3, respectively.<sup>5</sup>

The estimates for  $\alpha$  reported in Table 1 are all economically reasonable and in fact, as a group, lie in roughly the same range as Cagan's and Sargent's estimates.<sup>6</sup> If anything, my estimates are grouped more tightly together than either Cagan's or Sargent's. Except for



TABLE 1

## SUMMARY OF ESTIMATION OF EQUATION (4) FOR CAGAN'S DATA

Country	NOB	restricted equation					unrestricted equation				
		$\hat{\alpha}$ (s.e.)	D-W	$R^2$	SEE	$SSR_r$	$\frac{\hat{1+\alpha}}{\alpha}$ (s.e.)	$-\frac{\hat{1}}{\alpha}$ (s.e.)	$\hat{\alpha}$	$SSR_u$	$\chi^2(1)$
Austria Feb. '21 to Aug. '22	19	-3.09 (1.22)	.92 (1.18, 1.40)	.982	.094	.149	.907 (.409)	.265 (.556)		.139	1.319
Germany Oct. '20 to June '23	33	-5.27 (1.37)	1.75 (1.38, 1.51)	.990	.101	.315	1.18 (.091)	-.18+ (.140)	-5.56	.315	= 0
Greece Feb. '43 to Aug. '44	19	-2.34 (.47)	2.19	.991	.126	.268	.612 (.171)	.771 (.245)		.104	17.99
Hungary Aug. '22 to Feb. '24	19	-4.08 (2.0)	1.38	.986	.078	.103	1.35 (.171)	-.40+ (.214)	-2.50	.098	.9455
Poland Aug. '22 to Jan. '24	18	-2.78 (1.18)	1.75	.986	.123	.240	.891 (.328)	.245 (.406)		.205	2.837
Russia Feb. '22 to Jan. '24	24	-4.75 (2.5)	.54 (1.27, 1.45)	.991	.102	.227	1.23 (.111)	-.21+ (.111)	-4.76	.212	1.641

COMMENTS: NOB  $\equiv$  number of observations;  $\chi^2(q) \equiv \text{NOB} \cdot \left\{ \ln \frac{SSR_r}{SSR_u} \right\}$ , where  $q = 1$  degree of freedom since only one restriction is imposed; the numbers below each D-W statistic are appropriate ( $d_1, d_u$ ) for 5% level of significance; since the dependent variable is in logs, SEE x 100 is the percent estimation error; Hungary (2) from Cagan's data is not used because the sample size is too small.

TABLE 2  
CAGAN'S ESTIMATES OF  $\alpha$

	$\hat{\alpha}$	$(\alpha_e, \alpha_u)^a$
Austria Jan. '21 to Aug. '22	-8.55	-(4.43, 31.0)
Germany Sept. '20 to July '23	-5.46	-(5.05, 6.13)
Greece Jan. '43 to Aug. '44	-4.09	-(2.83, 32.5+) <sup>b</sup>
Hungary July '22 to Feb. '24	-8.70	-(6.36, 42.2+) <sup>b</sup>
Poland Apr. '22 to Nov. '23	-2.30	-(1.74, 3.94)
Russia Dec. '21 to Jan. '24	-3.06	-(2.66, 3.76)

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SOURCE: Cagan [1956], Table 3, page 43.

<sup>a</sup> $(\alpha_e, \alpha_u) \equiv$  90 percent confidence band calculated by Cagan.

<sup>b</sup> $\alpha_u$  exceeds right-hand figure in parentheses.

TABLE 3  
SARGENT'S ESTIMATES OF  $\alpha$  USING CAGAN'S DATA

	<u><math>\hat{\alpha}</math></u>	<u>Standard Error</u>
Austria Feb. '21 to Aug. '22	-0.311	1.570
Germany Oct. '20 to July '23	-5.97	4.615
Greece Feb. '43 to Aug. '44	-4.09	2.970
Hungary Aug. '22 to Feb. '24	-1.84	.3978
Poland May '22 to Nov. '23	-2.53	.8562
Russia Feb. '22 to Jan. '24	-9.75	10.742

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SOURCE: Sargent's [1977], Table 2, p. 76.

Hungary, the standard errors of my estimates are either roughly equivalent to or much below those reported by Sargent. On the other hand, judging by reported confidence intervals, Cagan appears to have estimated  $\alpha$  more precisely than I in some cases and less precisely in others.

In the important German case, Cagan's, Sargent's, and my estimates of  $\alpha$  are -5.46, -5.97, and -5.27, respectively.<sup>7</sup> The estimate of  $\alpha$  from my procedure is certainly reasonable when compared with theirs. As far as precision of the estimates, judging by his confidence interval, Cagan's appears to be greater than mine, while Sargent's appears much worse.<sup>8</sup> Again my procedure appears to give reasonable results by comparison.

One important implication of the joint hypothesis underlying my estimation strategy is that the residuals from the restricted fit of equation (4) should display no evidence of autocorrelation. As a check on the presence of residual autocorrelation, the Durbin-Watson statistic is reported under D-W in Table 1. The Durbin-Watson statistic has been shown to be biased toward two, i.e., toward accepting the hypothesis of no serial correlation, when lagged dependent variables appear on the right side of a regression.<sup>9</sup> This means that the Durbin-Watson statistic should not be taken as evidence against the presence of autocorrelated residuals in this case. Nevertheless, a value widely different from two can be interpreted as evidence of residual autocorrelation.

The Durbin-Watson statistic shows no evidence of residual autocorrelation in the German, Greek, and Polish cases. For Hungary the statistic is inconclusive. But in the Austrian and Russian cases the statistic does indicate residual autocorrelation.

Since the Durbin-Watson statistic is biased in this context, and since it is only useful as a check on first order autocorrelation, an additional test for residual autocorrelation is presented. This involves estimating first, second, and third order autoregressive coefficients for the residuals. These estimates, together with their standard errors, are reported in Table 4. Residual autocorrelation appears to be significant only for Austria and Russia. In both cases autocorrelation is significant only at lag one. Therefore, taking into account both the Durbin-Watson statistic and estimated autocorrelation coefficients, the hypothesis that residuals are not serially correlated cannot be rejected for four of the six hyperinflations. In particular, the important German case is one for which no evidence of residual autocorrelation is detected. On the whole, autocorrelation checks constitute reasonably favorable evidence for the joint hypothesis underlying the estimation of equation (4).

Turn to the columns in Table 1 that report results of estimating an unrestricted version of equation (4). Here,  $\frac{\widehat{1+\alpha}}{\alpha}$  and  $-\frac{\widehat{1}}{\alpha}$  are estimates of the unrestricted coefficients of  $\ln P_t$  and  $\ln M_t$ , respectively. As is apparent from these restricted representations, the joint hypothesis upon which the estimation of equation (4) is based implies that the coefficient of  $\ln P_t$  should exceed one, the coefficient of  $\ln M_t$  should be negative, and these unconstrained coefficients should sum to one. These hypotheses are in fact closely borne out in the German, Hungarian, and Russian cases. It is remarkable that these hypotheses are verifiable in three of the six cases in spite of the extremely short samples and high multicollinearity of the price level and money growth. It is even more remarkable that in the German and Russian cases, the implied estimate of  $\alpha$ , obtained by

TABLE 4

OLS ESTIMATES OF AUTOREGRESSIVE COEFFICIENTS FOR THE RESIDUALS  
FROM THE RESTRICTED REGRESSIONS

<u>Austria</u>	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$
AR(1)	.55 (.24) <sup>a</sup>		
AR(2)	.55 (.28)	.00 (.29)	
AR(3)	.58 (.27)	.23 (.32)	-.48 (.31)
<u>Germany</u>			
	.06 (.20)		
	.07 (.20)	-.31 (.20)	
	.08 (.21)	-.32 (.20)	.07 (.21)
<u>Greece</u>			
	-.21 (.24)		
	-.27 (.29)	.02 (.31)	
	-.26 (.34)	.06 (.37)	.04 (.37)
<u>Hungary</u>			
	.28 (.26)		
	.40 (.27)	-.44 (.26)	
	.30 (.31)	-.36 (.31)	-.20 (.31)
<u>Poland</u>			
	.11 (.26)		
	.13 (.28)	-.07 (.28)	
	.13 (.30)	-.06 (.30)	-.15 (.30)
<u>Russia</u>			
	.74 (.16)		
	.94 (.22)	-.34 (.22)	
	.84 (.24)	-.12 (.32)	-.24 (.25)

<sup>a</sup>Standard errors are in parentheses.

inverting the estimated coefficient of  $\ln M_t$ , is very close to the estimate of  $\alpha$  obtained in the restricted estimation of equation (4).<sup>10</sup>

One important test of the joint hypothesis underlying the specification of equation (4) involves checking whether relaxing the restriction across the  $\ln P_t$  and  $\ln M_t$  coefficients leads to a significant improvement in the "fit" of that equation. A likelihood ratio statistic, presented in Zellner and Palm [1974], is employed here to test the null hypothesis that the restricted equation is correct.<sup>11</sup> The statistic is distributed as chi-square with one degree of freedom:

$$T \cdot \left\{ \ln \frac{SSR_U}{SSR_R} \right\} \sim \chi^2(1)$$

where  $SSR_R \equiv$  the sum of squared residuals  
from the restricted regression

$SSR_U \equiv$  the sum of squared residuals  
from the unrestricted regression

$T \equiv$  the length of the sample  
of observations on the residuals

High values of the test statistic indicate that the data reject the restriction. In particular, the restriction is rejected at a 5% level of significance if  $\chi^2(1)$  exceeds 3.84.

The chi-square values for this test are reported under  $\chi^2(1)$  in Table 1. Except for Greece, values are small, indicating that the restriction can't be rejected at the 5% level.<sup>12</sup> In fact, in the German case, the restricted and unrestricted SSR values were identical out to the number of decimal places reported by TROLL. The Greek chi-square value is at least six times larger than any of the others and indicates a clear rejection of the restriction at very low significance levels for

Greece. Except for Greece then, the chi-square tests provide extremely impressive evidence supporting the joint hypothesis underlying the specification of equation (4).

The tendency for the restriction to be consistent with data from all the hyperinflations except Greece is interesting in light of a potential inadequacy of the Greek data relative to other hyperinflation data pointed out by Cagan. Cagan's money series for Greece consists of an index of the quantity of bank notes issued by the Bank of Greece. It does not include data on bank deposits which presumably were not available. In describing this money series Cagan writes:

Bank deposits should not be dismissed as entirely insignificant, though their effects in other hyperinflations were minor, because deposits in Greece were as large in value as the quantity of bank notes in circulation during the hyperinflation.<sup>13</sup>

This suggests a reasonable explanation for the otherwise puzzling rejection of the restriction in the Greek case. It may be that the relatively inadequate coverage of the Greek money data compared to that collected for other hyperinflations is responsible for rejection of the restriction for Greek data.

#### 4. Related Issues in Recent Hyperinflation Studies

A convenient starting point for this discussion is Sargent [1977]. Sargent analyzes Cagan's model of hyperinflation under circumstances in which Cagan's adaptive scheme for forming anticipations of inflation is "rational" in the sense of Muth [1961]. Under these conditions, Sargent is able to show that Cagan's estimator of  $\alpha$  is generally inconsistent unless there is no noise in the portfolio balance schedule.<sup>14</sup> Admitting



noise in the portfolio balance schedule, Sargent is able to derive consistent estimates of  $\alpha$  under the assumption that disturbances to the demand and supply for money are uncorrelated.

Sargent's estimates of  $\alpha$  are interesting in the present context in two ways. First, Sargent's calculations show Cagan's estimates of  $\alpha$  should be downward biased if there were significant noise in the portfolio balance schedule. Since Sargent's estimates of  $\alpha$  are consistent, a comparison of the two sets of estimates reported in Tables 2 and 3 should show a tendency for Cagan's estimates to fall below Sargent's. But no such tendency is apparent.<sup>15</sup> This suggests that, at least if Sargent's framework is correct, noise in the portfolio balance schedule may in fact be relatively low. This evidence may be taken to support the assumption underlying my estimation strategy, that noise in the portfolio balance schedule is small.

One notable characteristic of Sargent's estimates is that, except for Hungary and Poland, they are accompanied by large estimated standard errors when compared to the standard errors of my estimates. This suggests that Sargent's estimator of  $\alpha$  is less efficient than mine. Given little evidence that Sargent's procedure obtains any reduction in bias, his procedure, as a technique for estimating  $\alpha$ , may not be worth the cost in efficiency when compared to mine.

Sargent has applied his theoretical framework to evaluating Jacobs' [1975] estimates of the Cagan model in Sargent [1976]. For present purposes it is sufficient to say that Sargent shows Jacobs' estimates to be consistent only if there is no feedback from inflation to money growth. Since both Sargent and Wallace [1973] and Evans [1978]

have found significant evidence that inflation does feed back to money growth, Sargent's critique seems to imply that Jacobs' estimation strategy is inappropriate.

In his reply, Jacobs [1976] emphasizes that Sargent's critique is developed under special restrictions for which Cagan's adaptive expectations assumption is "rational." Jacobs argues that these ad hoc restrictions may not be correct and so the implications of Sargent's analytical framework can not be trusted. Rather than assuming a money supply process sufficient to make the ad hoc adaptive expectations "rational," Jacobs argues that the money process should be modeled directly.<sup>16</sup> Then, if desired, the model could be solved and estimated under rational expectations consistent with the estimated money supply rule.

While the above issues are interesting and important, they are also difficult. A major attribute of my estimation strategy is that it provides a means of estimating  $\alpha$  without having to pay attention to the money supply rule. My estimation strategy for  $\alpha$  need not be embedded in a model of the entire inflationary process. My technique therefore obtains a potential separation of the problem of estimating  $\alpha$  in the money demand function from the far more difficult problem of modeling the dynamic relationship between prices and money in hyperinflation.

Moshin Khan [1975] has recently calculated the Durbin-Watson statistics for Cagan's [1956] regressions. They are reported in Table 5. These Durbin-Watson statistics provide evidence of residual correlation in all of Cagan's regressions except, possibly, Austria.<sup>17</sup> The autocorrelated residuals indicate some misspecification of either the money demand function or the anticipation formation mechanism in Cagan's

TABLE 5  
DURBIN-WATSON STATISTICS  
FOR CAGAN'S REGRESSIONS

	<u>D-W</u>
Austria	1.60
Germany	.33
Greece	.77
Hungary (1)	.37
Poland	.68
Russia	.76

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SOURCE: Khan's [1975], Table 1,  
p. 358.

model. My estimation results offer some evidence that the misspecification lies in the anticipation mechanism rather than in the money demand function.

My estimation procedure continues to employ the Cagan money demand function as a maintained hypothesis. But it replaces the adaptive anticipations hypothesis with the assumption that anticipations are formed rationally in the sense of Muth [1961]. My restricted regressions yield residuals which exhibit virtually no evidence of serial correlation in all except the Austrian and Russian cases. Furthermore, a restriction implied by the Cagan money demand specification can be rejected only in the Greek case. This suggests that at least for the German, Hungarian, and Polish cases residual autocorrelation in Cagan's regressions is due to his misspecified anticipation formation hypothesis and not to his money demand function specification.

If one believes that anticipations are formed rationally, then this evidence further implies that adaptive anticipations were not in fact rational in at least three of the hyperinflations.<sup>18</sup> In these cases at least, the proper way to go about investigating the hyperinflations seems not to restrict the money supply rule a priori so that adaptive anticipations are "rational," but rather to attempt to identify the money supply process directly from the data, and then to model anticipations rationally, based on the estimated money supply rule.

## 5. Summary

This paper has implemented a method of estimating the Cagan money demand function under rational expectations. The technique utilizes the side assumptions that (1) accurate contemporaneous information on the

price level and money stock is available to individuals and (2) unobservable noise in the portfolio balance schedule is negligible. Under these assumptions the estimation strategy delivers unbiased and consistent estimates of  $\alpha$ , the slope of the log of the demand for real balances with respect to anticipated inflation.

Application of this technique yields estimates of  $\alpha$  that are very reasonable by comparison with those obtained by other writers. In four of the six hyperinflations, the residuals from the theoretically restricted regression show no evidence of serial correlation. A theoretical restriction implied by the Cagan money demand specification cannot be rejected for five of the six hyperinflations. The restriction is clearly rejected in the Greek case, but this is potentially explained by the poor coverage of Cagan's Greek money supply data compared with data for the other hyperinflations.

A major attribute of my estimation procedure for  $\alpha$  is that it gets along with no restrictions on the money supply process. In particular the estimation strategy need not be embedded in a model of the entire inflationary process. My technique obtains a separation of the problem of estimating  $\alpha$  in the money demand function from the problem of modeling the dynamics of money and prices as a whole.

#### FOOTNOTES

<sup>1</sup>By subtracting  $\ln P_t$  from both sides of (4), it can be written:

$$\ln \frac{P_{t+1}}{P_t} = -\frac{\lambda}{\alpha} - \frac{1}{\alpha} \ln \frac{M_t}{P_t} + u_{t+1}$$

In other words, it relates period t real balances to period t+1 inflation.

<sup>2</sup>Errors in (4) are due to ex post money growth prediction errors. The forecast error could be due for example to noise in the money multiplier or to updated information on future money growth. If the government has to finance a fixed current level of real expenditures with current money creation, then the latter disturbance would cause the current price level to rise and thereby raise current money growth. In other words, zero velocity shocks do not rule out the possibility of feedback from inflation to money growth.

<sup>3</sup>At least, the econometrician must be prepared to believe that if there is noise in the portfolio balance function, it must be of minor importance compared to prediction errors on period t+1 money growth and information updates on future money growth.

<sup>4</sup>The regression was run on the MIT TROLL system.

<sup>5</sup>Barro's [1970] estimates of  $\alpha$  using different data are:

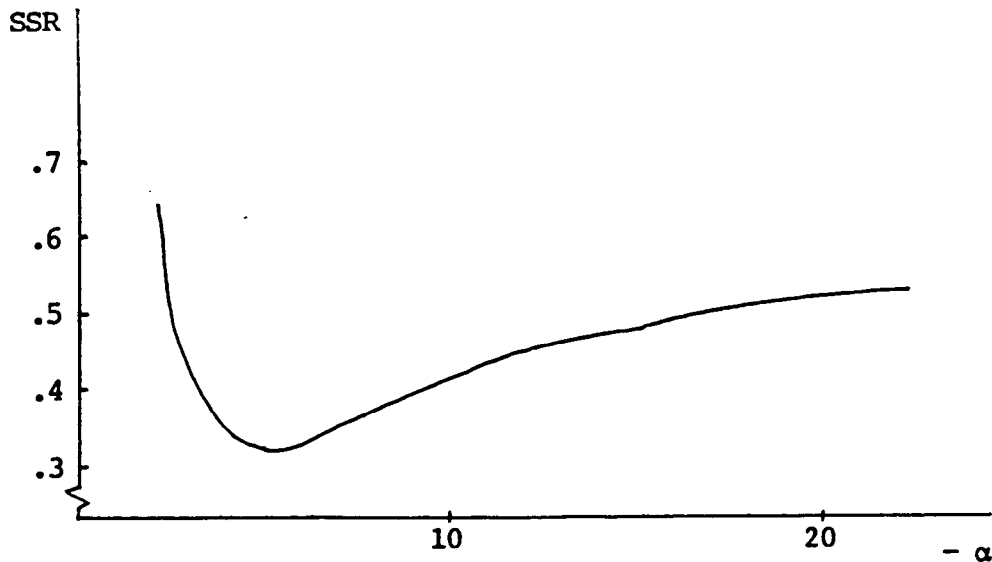
Austria	-4.09 (-3.6, -4.5) <sup>a</sup>
Germany	-3.79 (-3.3, -4.3)
Hungary	-5.53 (-4.6, -6.9)
Poland	-2.56 (-2.1, -3.3)

<sup>a</sup>95% confidence intervals.

<sup>6</sup>My sample periods are also similar to theirs.

<sup>7</sup>Barro's estimate of  $\alpha$  for the German case is much lower than these.

<sup>8</sup>The calculated SSR surface for  $\alpha$  in the restricted regression in the German case looks like:



<sup>9</sup>See Nerlove-Wallis [1966].

<sup>10</sup>The only country for which  $-\frac{\hat{1}}{\alpha}$  is positive and significant is Greece.

This is seriously at variance with the hypothesized sign restriction. A possible explanation for the poor performance of the model on Greek data is offered below.

<sup>11</sup>See Zellner and Palm, p. 34.

<sup>12</sup>Rodney Jacobs [1977, p. 124] has said that "Cagan's [estimation] procedure appears to work for a price series that is unrelated to the money stock only because  $\ln P_t$  cancels from both sides of the equation for real money balances." In other words, Jacobs argues that the appearance of  $\ln P_t$  on both sides of the equation Cagan estimated would guarantee a good "fit" even if the model were wrong. For what it's worth, the estimation strategy and restriction test employed here are not subject to Jacobs' criticism.

<sup>13</sup>Cagan [1956], p. 106.

<sup>14</sup>See Sargent [1977], p. 67.

<sup>15</sup>No such tendency is apparent in my estimates either.

<sup>16</sup>See Evans [1978] and Friedman [1978].

<sup>17</sup>The Durbin-Watson statistics from Barro's [1970] estimates of Cagan's model also indicate residual autocorrelation. Barro's D-W statistics are:

Austria	.53
Germany	.25
Hungary	.31
Poland	.32

<sup>18</sup>Evans' [1978] findings indicate that adaptive anticipations were not rational in the German hyperinflation.



## APPENDIX

### The Data

The data used here is taken from Cagan's study of hyperinflations in Austria, Germany, Greece, Hungary (1), Poland, and Russia.

The Cagan data consists of monthly time series on real balances and the rate of inflation. It is necessary for the present study to construct a money supply and price level time series from Cagan's series.

### Construction of a Price Level Series

Let  $t_0$  be the first month of the series. Assume  $\log P_{t_0} = c$  where  $c$  is a positive unknown constant. Then

$$\begin{aligned} \log P_{t_0} &= c \\ \text{constructed } \log P_{t_0+1} &= \log \frac{P_{t_0+1}}{P_{t_0}} \\ \text{" } \log P_{t_0+2} &= \log \frac{P_{t_0+2}}{P_{t_0+1}} + \log P_{t_0+1} \\ \text{" } \log P_{t_0+3} &= \log \frac{P_{t_0+3}}{P_{t_0+2}} + \log P_{t_0+2} \\ \text{" } \log P_{t_0+n} &= \log \frac{P_{t_0+n}}{P_{t_0+n-1}} + \log P_{t_0+n-1} \end{aligned} \left. \vphantom{\begin{aligned} \log P_{t_0} &= c \\ \text{constructed } \log P_{t_0+1} &= \log \frac{P_{t_0+1}}{P_{t_0}} \\ \text{" } \log P_{t_0+2} &= \log \frac{P_{t_0+2}}{P_{t_0+1}} + \log P_{t_0+1} \\ \text{" } \log P_{t_0+3} &= \log \frac{P_{t_0+3}}{P_{t_0+2}} + \log P_{t_0+2} \\ \text{" } \log P_{t_0+n} &= \log \frac{P_{t_0+n}}{P_{t_0+n-1}} + \log P_{t_0+n-1} \end{aligned}} \right\} \begin{array}{l} \text{construction} \\ \text{of price level} \\ \text{series} \end{array}$$

where constructed  $\log P_{t_0+n} = \log P_{t_0+n} - c$

$$\log P_{t_0+n} = \text{constructed } \log P_{t_0+n} + c$$

Construction of a Money Supply Series

$$\begin{aligned}\log M_{t_0+n} &= \log \left( \frac{M}{P} \right)_{t_0+n} + \text{actual } \log P_{t_0+n} \\ &= \log \left( \frac{M}{P} \right)_{t_0+n} + \text{constructed } \log P_{t_0+n} + c \\ &= -\log \left( \frac{P}{M} \right)_{t_0+n} + \text{constructed } \log P_{t_0+n} + c\end{aligned}$$

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