Working Paper 83-3

MEASUREMENT ERROR AND A REINTERPRETATION OF THE CONVENTIONAL MONEY DEMAND REGRESSION

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March 1983

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I) Introduction

It has been sixteen years since a partial adjustment model was first applied in empirical money demand studies by Chow [1966]. Since then the partial adjustment specification has become widely used, particularly in quarterly money demand studies. However, in spite of its widespread use, the theoretical rationalization for the partial adjustment specification has never been entirely satisfactory. Furthermore, a number of empirical characteristics of the conventional money demand regression based on a partial adjustment specification have proven difficult to interpret along partial adjustment lines.

This paper offers an interpretation of the conventional money demand regression that does not rely on a partial adjustment rationalization. Instead, money demand is assumed to adjust completely each period to appropriate current interest rate and transaction variables. An alternative interpretation of the conventional money demand regression is developed by taking into account the consequences of regressor measurement error. This interpretation is able to explain many characteristics of the conventional money demand regression that have proven difficult to interpret purely from a partial adjustment point of view.

A conventional money demand regression with a partial adjustment specification is described in Section II. Following this description, a number of specific difficulties in interpreting the conventional money demand regression are detailed. Theoretical weaknesses with the partial adjustment rationalization itself and difficulties with the partial adjustment interpretation of specific empirical characteristics of the conventional money demand regression are discussed. In Section III, coefficients in a conventional money demand regression are derived for the case where "true" money demand adjusts completely within each period to current appropriate interest rate and transaction variables, but the interest rate and transaction variable regressors include measurement error. In Section IV, empirical characteristics of the conventional money demand regression outlined in Section II are interpreted in terms of the measurement error model of Section III. The residual generating process implied by this measurement error view of the conventional money demand regression is described in Section V. Finally, some implications of the measurement error view for forecasting with the conventional money demand regression are discussed in Section VI. The analysis is summarized in the conclusion.

II) Difficulties with the Partial Adjustment Interpretation of the Conventional Money Demand Regression

A well-known discussion of the specification and interpretation of the conventional money demand regression is found in Goldfeld [1973]. To paraphrase Goldfeld, the specification usually proceeds by postulating the "desired" or long-run demand for real money balances as a function of a transaction variable and an interest rate opportunity cost variable such that

$$\tilde{m}(t) = a_0 + a_1 y(t) + a_2 r(t) \tag{1}$$

where $\tilde{m}(t) \equiv$ "desired" or long-run real period t money balances demanded

$y(t) \equiv$ real period t income

$r(t) \equiv$ the period t nominal rate of interest

Without an adjustment lag, real period t money balances would depend exclusively on real period t income and the period t nominal interest rate as in (1).

But with partial adjustment, real money balances adjust to the gap between the desired or long-run demand for real money balances and the previous period's holdings such that

(2) $$m(t) - m(t-1) = \lambda(\tilde{m}(t) - m(t-1))$$

where $m(t) \equiv$ short-run real period t money balances demanded

$\lambda \equiv$ coefficient of adjustment, $0 < \lambda < 1$

Using (1) to substitute for $\tilde{m}(t)$ in (2) and rearranging yields the conventional money demand regression specification

(3) $$m(t) = \lambda a_0 + \lambda a_1 y(t) + \lambda a_2 r(t) + (1-\lambda)m(t-1)$$

The form of the money demand specification with partial adjustment is the same as (1) except that a lagged real money balance variable is included in the specification.

Utilization of the partial adjustment mechanism has generally been motived by appeal to portfolio adjustment costs. The appeal has usually been vague. For example, Goldfeld [1973] motivates the partial adjustment mechanism by simply stating that "portfolio adjustment costs, both pecuniary and nonpecuniary, are assumed to prevent full, immediate, adjustment of actual money holdings to desired levels."[1/] Darby [1972] and Carr and Darby [1981] have argued that money balances serve as a shock absorber or buffer stock which temporarily absorbs unexpected variations in income (transitory income) until a portfolio of securities and consumer durable goods can be conveniently adjusted.[2/] Santomero and Seater [1981] derive an optimal Darby type partial adjustment specification based on the notion that it is costly to search for appropriate assets to buy with transitory money balances.

Appeal to portfolio adjustment costs may seem sufficient to generate partial adjustment in money demand. But such an appeal is not satisfactory, especially with respect to the demand for currency or checkable deposits in M1. Darby type rationalization for partial adjustment really describes costs involved in reallocating transitory income from temporary, safe, and easily accessible stores of value to a more desirable portfolio of securities and consumer durables. However, the argument does not explain partial adjustment for currency or checkable deposit holdings, since other equally safe and readily accessible stores of value such as savings deposits and recently money market mutual fund shares and repurchase agreements are available with higher rates of return. This suggests that the latter vehicles would be used as temporary stores of value and that currency and checkable deposit holdings would be adjusted on a period by period basis, in close relation to each period's planned net expenditure, taking into account each period's interest opportunity cost.[3/] Furthermore, portfolio adjustment costs are largely unrelated to the volume of adjustment that may be decided upon. Since the costs associated with such adjustment are largely fixed, it would be more costly, not less, to make a given desired adjustment gradually over time. Consequently, appeal to adjustment cost in this context can actually provide a justification for why such adjustment, once decided upon, would be carried out all at once.[4/]

The failure of the adjustment cost argument to rationalize a partial adjustment in the demand for M1 creates a serious problem for the standard interpretation of the conventional M1 money demand regression. Without a partial adjustment specification, the standard money demand regression would include only current income and interest rate variables as regressors.

In other words, without partial adjustment, standard theoretical models of money demand would predict lagged real money balances to be irrelevant to money demand. Yet, in general, the coefficient on lagged real money balances in the conventional money demand regression is positive and highly significant, and accounts for much of the explanatory power of the regression.

A second problem for the standard interpretation of the coefficient on lagged money along partial adjustment lines is that, to quote Goldfeld [1973] "the lags that result statistically for money adjustment appear too long to be explained on grounds of adjustment cost."[5] In other words, the estimated coefficient on lagged money is typically too high to be interpreted as representing a desired "speed of adjustment."

Finally, a third problem with the standard partial adjustment interpretation of the conventional money demand regression concerns residual autocorrelation. The standard theoretical specification of money demand coupled with partial adjustment does not predict residual autocorrelation. Yet, residuals from conventional money demand regressions generally exhibit highly significant positive serial correlation.

III) Measurement Error and a Derivation of Conventional Money Demand Regression Coefficients Without Partial Adjustment

An alternative interpretation of conventional money demand regression coefficients is developed in this section that does not rely on a partial adjustment specification. Instead, this alternative interpretation is based on the assumption that money demand adjusts completely within each period to current interest rate and transaction variables. It is shown that simply allowing for measurement error in the regressor variables is sufficient to account for characteristic features of the conventional money demand regression such as the significant positive coefficient on lagged real money balances and positive residual autocorrelation.6/

Transaction variable measurement error is likely to be significant for two reasons. First, even if GNP is the appropriate transaction variable, the reliability of statistically measured GNP is questionable.7/ Second, even if measured accurately, national income may only be imperfectly correlated with the theoretically appropriate transaction scale variable in money demand.8/

Interest rate variable measurement error may also be a problem for reasons roughly analogous to those above. First, interest rates are measured either as period averages or as end-of-period rates, both of which are only approximate measures of effective market interest rates. Second, even if measured accurately, an interest rate variable may only be imperfectly correlated with the theoretically appropriate interest opportunity cost in money demand.

The analysis proceeds by postulating the "true" demand for real money balances as a function solely of current appropriate transactions and interest opportunity cost variables

$$m(t) = a_0 + a_1 y^*(t) + a_2 r^*(t) + e(t) \tag{4}$$

where $m(t) \equiv$ real period t money balances demanded

$y^*(t) \equiv$ the appropriate period t transactions variable

$r^*(t) \equiv$ the appropriate period t interest rate variable

$e(t) \equiv$ the period t disturbance term

$a_1 > 0$

$a_2 < 0$

The appropriate transaction variable and interest rate generating processes are both assumed to be first-order autoregressive

(5) $$y^*(t) = \phi_0 + u(t) + \phi_1 y^*(t-1)$$

(6) $$r^*(t) = \theta_0 + q(t) + \theta_1 r^*(t-1)$$

where $u(t)$ and $q(t) \equiv$ period t transaction variable and interest rate generating process innovations respectively.

$\phi_0, \theta_0 > 0$

$0 < \phi_1 < 1$

$0 < \theta_1 < 1$

Measured period t income, $y(t)$, is assumed to move with the appropriate period t transaction variable, $y^*(t)$, plus a serially uncorrelated disturbance, $\zeta(t)$,

(7) $$y(t) = y^*(t) + \zeta(t)$$

The $\zeta(t)$ disturbance represents the error involved in taking measured period t income to represent the appropriate period t transaction variable.

Likewise, the measured period t interest rate, $r(t)$, is assumed to move with the appropriate period t interest rate variable, $r^*(t)$, plus a serially uncorrelated disturbance, $\eta(t)$,

(8) $$r(t) = r^*(t) + \eta(t)$$

The $\eta(t)$ disturbance represents the error involved in taking the measured period t interest rate to represent the appropriate period t interest rate variable. The $\eta(t)$ and $\zeta(t)$ disturbances are assumed to be uncorrelated with each other and with $u(t)$ and $q(t)$.

The conventional money demand regression is written

$$(9) \qquad m(t) = b_0 + b_1 y(t) + b_2 r(t) + b_3 m(t-1) + v(t)$$

where $\quad v(t) \equiv$ the period t residual

The conventional money demand regression differs from the assumed true money demand specification (4) both because the transaction and interest rate variables include measurement error and because a lagged real money balance variable is included in the regression.

The problem is to derive the b coefficients in the conventional money demand regression (9) in the context of the assumed true specification of money demand behavior described by (4) in an environment described by (5), (6), (7), and (8). In general, the disturbance term in the true money demand specification (4) could be correlated with the income or interest rate variables. But since the point of this paper is unrelated to such potential "simultaneous equation" problems, that potential correlation is assumed away. In other words, e(t) is assumed to be uncorrelated with $\zeta(t)$, $\eta(t)$, u(t), and q(t). In this case, the standard "normal equations" yield consistent estimators of the b coefficients. Specifically, solutions for b_1, b_2, and b_3 may be obtained from the following set of normal equations

$$(10) \qquad \begin{bmatrix} \sigma_{m(t)y(t)} \\ \sigma_{m(t)r(t)} \\ \sigma_{m(t)m(t-1)} \end{bmatrix} = \begin{bmatrix} \sigma^2_{y(t)} & \sigma_{y(t)r(t)} & \sigma_{y(t)m(t-1)} \\ \sigma_{y(t)r(t)} & \sigma^2_{r(t)} & \sigma_{r(t)m(t-1)} \\ \sigma_{y(t)m(t-1)} & \sigma_{r(t)m(t-1)} & \sigma^2_{m(t)} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \underline{9/}$$

Equations (4) through (8) may be used to derive the variances and covariances appearing in the normal equations in terms of measurement error parameters, appropriate interest rate and transaction variable generating process parameters, and the parameters of true money demand. These variances and covariances are reported in Appendix A, where they are used to derive the implied restrictions on the b coefficients that are relevant to the discussion of the conventional money demand regression to be carried out in the remainder of the paper. The relevant implied restrictions on the b coefficients are:

$$(11) \qquad b_0 = a_0(1-b_3) + [(1-b_3)a_1-b_1]\frac{\phi_0}{1-\phi_1} + [(1-b_3)a_2-b_2]\frac{\theta_0}{1-\theta_1}$$

$$0 < b_1 < a_1$$

$$a_2 < b_2 < 0$$

$$0 < b_3 < 1$$

IV) The Measurement Error View of Coefficients in the Conventional Money Demand Regression

1) The Coefficient on Lagged Money

The most interesting feature of the derived coefficients in the conventional money demand regression is that the coefficient on lagged money, b_3, is positive even though true money demand does not depend on lagged money. This is because the income and interest rate regressors in the conventional money demand regression differ from the appropriate transaction and interest rate variables by a random measurement error, so the estimated coefficients on these variables, b_1 and b_2, are biased toward zero as estimates of the true transaction and interest rate sensitivities a_1 and a_2.

In this circumstance, the sum of squared residuals in conventionally estimated money demand is reduced by including lagged money. To see why, consider the income variable. Money demand is positively contemporaneously correlated with the true transaction variable component of income.

But because b_1 is biased downward as an estimate of a_1, when the true transaction variable is above (below) its mean the regression tends to underpredict (overpredict) money demand. However, because the true transaction variable is positively autocorrelated, lagged money tends to be above (below) its mean when the current true transaction variable is above (below) its mean.[10/] In this case, lagged money enters the money demand regression with a positive coefficient because it helps to offset the underprediction of money demand when the true transaction variable is above its mean, and it helps to offset the overprediction of money demand when the true transaction variable is below its mean. An analogous argument follows for the interest rate variable.

Measurement error in either the interest rate or the transaction variable would be sufficient to generate a positive coefficient on lagged money. Furthermore, measurement error in both regressors has a reinforcing effect, producing a more positive coefficient on lagged money than for measurement error in either variable alone. However, as seen in Appendix A, measurement error cannot produce a coefficient on lagged money that exceeds one.

In fact, the coefficient on lagged money is invariably estimated to be positive and less than one, as predicted by the measurement error interpretation of that coefficient advanced here. In addition, as mentioned above the estimated coefficient on lagged money is generally too close to one to be interpreted as representing a desired "speed of adjustment." But since the measurement error view does not interpret the coefficient on lagged money as a "speed of adjustment," the proximity of the estimated coefficient on lagged money to unity presents no difficulty for the measurement error view.

2) The Income and Interest Rate Coefficients

As mentioned above, the measurement error interpretation views the income and interest rate coefficients, b_1 and b_2 respectively, to be biased toward zero as estimates of the true transaction and interest rate sensitivities of money demand. The so called "long-run" income and interest rate sensitivites, $\frac{b_1}{1-b_3}$ and $\frac{b_2}{1-b_3}$ respectively, are generally biased estimates of the true sensitivities as well. The direction of the bias depends on the magnitude of the bias in b_1 and b_2 toward zero relative to the size of the multiplicative term $\frac{1}{1-b_3}$. For the symmetric case where the variances of the transaction and interest rate measurement errors are the same order of magnitude, i.e., $\sigma_\zeta^2 \approx \sigma_\eta^2$, and the variances of the appropriate transaction and interest rate variable innovations are also the same order of magnitude, i.e., $\sigma_u^2 \approx \sigma_q^2$, the "long-run" income and interest rate sensitivities of money demand are also biased toward zero as measures of the true transaction and interest rate sensitivities, i.e., $a_1 > \frac{b_1}{1-b_3}$ and $a_2 < \frac{b_2}{1-b_3}$. However, this need not be the case.

3) The Constant Term Coefficient

The constant term coefficient, b_0, is generally a biased estimate of the true money demand constant term, a_0. However, the direction of bias is not even determinate in the symmetric case described above, where $(1-b_3)a_1-b_1 > 0$ and $(1-b_3)a_2-b_2 < 0$. As seen in equation (11), even in this case the direction of bias depends on all the parameters of the model.

V) The Residual Generating Process Implied by the Measurement Error View of the Conventional Money Demand Regression

Using equation (9) together with equations (4) through (8), the v(t) residual generating process implied by the measurement error view of the conventional money demand regression can be written

$$(12) \qquad v(t) = -b_1\zeta(t) - b_2\eta(t) + (a_1-b_1)u(t) + (a_2-b_2)q(t)$$

$$+ (\phi_1(a_1-b_1) - b_3a_1)\overline{y}^*(t-1)$$

$$+ (\theta_1(a_2-b_2) - b_3a_2)\overline{r}^*(t-1)$$

$$+ e(t) - b_3e(t-1)$$

where $\overline{y}^*(t) \equiv y^*(t) - \dfrac{\phi_0}{1-\phi_1}$

$$\overline{r}^*(t) \equiv r^*(t) - \frac{\theta_0}{1-\theta_1}$$

Even if true money demand is an exact function of appropriate transaction and interest rate variables so that σ_e^2 is zero, the residuals in the conventional money demand regression will be autocorrelated if the transaction and/or interest rate regressors are autocorrelated. For the case assumed above where both y* and r* are generated by AR1 processes, the v(t) residual generating process is ARMA(2,2) even when $\sigma_e^2 = 0$.11/ If, in addition, the transaction and interest rate variables are positively autocorrelated, then v(t) will be positively autocorrelated as well.

Since both the transaction variable and interest rate regressors are in fact positively autocorrelated, the measurement error interpretation of the conventional money demand regression provides an explanation for the positive residual autocorrelation typically found in conventional money demand regressions. It is worth stressing that the residuals in conventionally estimated money demand are autocorrelated in this view even if true money demand is an exact function of current appropriate transaction and interest rate variables.

VI) Measurement Error and Forecasting with a Conventional Money Demand Regression

The measurement error interpretation of the conventional money demand regression explains that the coefficient on lagged money can be significant even though lagged money plays no role in true money demand. Lagged money enters significantly because, with regressor measurement error and serially correlated appropriate interest rate or transaction variables, lagged money helps predict money. This section contains a discussion of the implications of this measurement error interpretation of lagged money for forecasting with a conventional money demand regression.

1) Forecasting and the Residual Generating Process

In order to more easily illustrate some key points relating to forecasting and the residual generating process, the interest sensitivity of true money demand is assumed zero and a version of the conventional money demand regression without an interest rate regressor is employed in this discussion. The version of the conventional money demand regression employed here is

$$(13) \quad m(t) = d_0 + d_1 y(t) + d_2 m(t-1) - d_1 \zeta(t) + (a_1 - d_1) u(t) + (\phi_1(a_1 - d_1) - d_2 a_1)\bar{y}^*(t-1) + e(t) - d_2 e(t-1)$$

where $0 < d_1 < a_1$

$0 < d_2 < 1$

$\bar{y}^*(t) \equiv y^*(t) - \frac{\phi_0}{1-\phi_1}$ [12/]

Prior to discussing the forecasting performance of conventional money demand regression (13), an operational model of the residual generating process must be derived. As presently written, none of the components of the residual generating process is observable. The simplest residual generating process occurs in the case where true money demand is an exact function of the appropriate transaction variable, i.e., $\sigma_e^2 = 0$. In this case, the term $\phi_1(a_1-d_1) - d_2a_1$ is zero so both the transaction variable and the true money demand disturbance components disappear.13/ The remaining components are white noise, so the residuals follow a serially uncorrelated mean zero process.

This result is interesting because it demonstrates that with a single input variable and an exact true money demand specification, lagged money in the conventional money demand regression can completely neutralize the effect of regressor measurement error on the serial correlation in the residuals. In this case, unbiased forecasts of the income regressor variable can be used together with the conventional money demand regression to yield unbiased forecasts of money demand, without the need to correct for residual serial correlation.

In general, however, the residuals in conventional money demand regressions are serially correlated. Specifically, when true money demand is not exact or when there are multiple serially correlated regressor variables, unbiased forecasts of money demand cannot be made with conventionally estimated money demand without correcting for residual serial correlation. For example, in the inexact one regressor case described in equation (13), the residual generating process is the sum of white noise, AR1, and MA1 components so that the residuals are generated by an ARMA(1,2) process.14/

In this case, the ARMA(1,2) model generating the residuals from the conventional money demand regression must be utilized together with the money demand regression to make unbiased forecasts of money demand.

Suppose the ARMA(1,2) residual generating process is

$$(14) \qquad (1-\alpha B)w(t) = (1-\beta B-\gamma B^2)\varepsilon(t)$$

where α, β, and $\gamma \equiv$ residual generating process parameters

$B \equiv$ a backshift operator

$w(t) \equiv -d_1\zeta(t) + (a_1-d_1)u(t)$

$+ (\phi_1(a_1-d_1)-d_2a_1)\overline{y}^*(t-1) + e(t) - d_2e(t-1)$

$\varepsilon(t) \equiv$ the period t residual innovation

Substituting the model of the residual generating process from (14) for the residual, $w(t)$, in (13) yields

$$(15) \qquad m(t) = d_0 + d_1y(t) + d_2m(t-1) + \alpha w(t-1) + (1-\beta B-\gamma B^2)\varepsilon(t)$$

Equation (15) is the conventional money demand regression with an appropriate correction for residual serial correlation. Because α, β, and γ are nonzero in this case, money demand forecasts not taking into account $w(t-1)$, $\varepsilon(t-1)$, and $\varepsilon(t-2)$ according to the residual generating process would in general be biased.

The analysis is relevant for the conventional method of correcting for residual serial correlation. The most common correction in money demand regressions is to simply fit an AR1 residual model. This amounts to fitting a model such as (14) with β and γ restricted to zero. Now, the case described above with a single regressor and inexact true money demand is the simplest for which the residuals in the conventional money demand regression are serially correlated. And even in this case, the AR1 model is too restrictive a correction. Not only does the AR1 model restrict β and γ to be zero when they should be free, but inappropriate zero restrictions on β and γ bias the estimate of the autoregressive parameter, α, in the AR1 model itself.

2) Regressor Generating Process Parameter Shifts and Forecast Performance

In the measurement error interpretation, each of the coefficients in the conventional money demand regression is a function of the appropriate interest rate and transaction variable generating process parameters, the parameters of true money demand, and the measurement error parameters. Specifically, as seen from the normal equations (10) and the variances and covariances reported in Appendix A, for the conventional money demand regression including both income and interest rate regressors each regression coefficient is a function of $\{\phi_0, \phi_1, \sigma_u^2; \theta_0, \theta_1, \sigma_q^2; a_0, a_1, a_2, \sigma_e^2; \sigma_\zeta^2 \text{ and } \sigma_\eta^2\}$. Furthermore, as seen in equation (12), the residual generating process in the conventional money demand regression depends on the regression coefficients; so coefficients in the appropriate correction for residual serial correlation are also functions of all the bracketed parameters above.

Consider using a conventional money demand regression to forecast money demand outside the sample period over which the regression was estimated. Because the conventional money demand regression coefficients depend on regressor generating process parameters as well as on parameters of true money demand, unbiased post-sample forecast performance depends not only on the parameters of true money demand remaining unchanged but also on parameters of the regressor generating processes remaining unchanged.

For example, an increase in regressor measurement error in the post-sample period relative to the sample period could cause post-sample forecasts to be biased. This is easily illustrated for the one regressor case described in Appendix B. In this case, the partial derivative of the lagged money coefficient, d_2, with respect to the measurement error variance, σ_ζ^2, is positive, which means that the size of the lagged money coefficient is positively related to the magnitude of the measurement error variance in the estimation period. Greater measurement error causes more downward bias in the transaction variable coefficient and causes the lagged money coefficient to increase, partially picking up some explanatory power lost due to the greater downward bias in the transaction variable coefficient. It follows, in this case, that a regression used to forecast over a post-sample period in which σ_ζ^2 has risen relative to the period over which the equation was estimated will appear to have too large a transaction variable coefficient and too low a coefficient on lagged money to adequately capture post-sample movements in money demand.

As another example, the mean level of either the interest rate or the transaction variable regressor could change in the forecast period relative to what it had been during the estimation period, and this could cause post-sample forecasts to be biased. Take the case where the post-sample transaction variable mean falls relative to its sample period level, e.g., ϕ_0 falls in the post-sample period. At the point of means, the conventional money demand regression imparts two potential sources of bias to the money demand prediction. Measurement error biases the transaction variable coefficient, d_1, below the true transaction variable sensitivity of money demand, a_1, tending to bias the money demand prediction downward. But the lagged money regressor with a positive coefficient tends to impart an upward bias to the money demand prediction. The transaction variable enters the regression directly with the d_1 coefficient and indirectly through lagged money with a d_2a_1 coefficient. In the one regressor case described in Appendix B, the net effect is to bias the prediction downward, i.e., $d_1 + d_2a_1 < a_1$. But since the regression constant term is determined to make the regression prediction unbiased at the point of means, d_0 includes a term $[a_1-d_1-d_2a_1]\frac{\phi_0}{1-\phi_1}$ completely offsetting the potential bias. As a result, the magnitude of the constant term, d_0, is positively related to the mean of the transaction variable in the estimation period. It follows, in this case, that a regression used to forecast over a post-sample period in which ϕ_0 has fallen relative to the period over which the regression was estimated will systematically underpredict money demand.

The fact that, with regressor measurement error, money demand regression coefficients are functions of the regressor generating process parameters has two particularly important implications related to forecasting. First, poor dynamic or static post-sample forecast performance does not necessarily indicate that parameters in true money demand have shifted. This possibility is interesting in light of the widely documented upward bias in the forecast performance of the conventional money demand regression that has emerged since 1974.[15/] This forecast bias has generally been interpreted as evidence of a downward shift in the true demand for money. But the measurement error interpretation of the conventional money demand regression suggests that the upward forecast bias could be due to shifts in the income or interest rate generating processes instead of a shift in true money demand. Since the implications for monetary policy of these alternative interpretations are very different, the measurement error interpretation merits serious analysis.

Second, the interest rate generating process is highly influenced by monetary policy. For example, monetary policy can affect the level of the interest rate, interest rate autocorrelation, and the variance of interest rate innovations. In terms of the parameters used here, monetary policy can affect θ_0, θ_1, and σ_ζ^2. Since these parameters, in turn, affect money demand regression coefficients, these regression coefficients can be expected to depend on the monetary policy being followed during the sample period over which the regression is estimated. It follows that post-sample predictive performance of a money demand regression could be adversely affected if monetary policy alters the post-sample interest rate generating process relative to the sample period.

Another way of putting this is to say that the money demand regression can be used to make unbiased <u>unconditional</u> forecasts of money demand as long as the policy induced interest rate generating process remains the same in the forecast period as in the estimation period. But, with regressor measurement error, the money demand regression cannot deliver unbiased <u>conditional</u> money demand forecasts for alternative policy induced interest rate generating processes because money demand regression coefficients are not invariant to shifts in the interest rate generating process.[16]

In order to make unbiased conditional forecasts of money demand for evaluation of alternative policies, it is necessary to compute changes in money demand regression coefficients as functions of proposed policy changes. Even in the case where the transaction variable generating process parameters, the measurement error parameters, and the parameters of true money demand are invariant with respect to monetary policy changes, this computation is not trivial. The computation requires that the above parameters be econometrically identified and that the policy induced shift in the interest rate generating process be deduced. Identification in the presence of measurement error is particularly delicate and the policy induced shift in the interest rate generating process can only be deduced in the context of a reasonably general model of interest rate determination.17/ Nevertheless, if unbiased conditional money demand forecasts are valuable in policy analysis, and if regressor measurement error is significant, then these difficulties need to be addressed.

VII) Conclusion

This paper has investigated the implications of regressor measurement error for interpreting the conventional money demand regression. Coefficients in the conventional money demand regression were derived for the case where true money demand adjusts completely within each period to current appropriate interest rate and transaction variables, but the interest rate and transaction variable regressors include measurement error. Notably, the coefficient on lagged money is positive, even though lagged money plays no role in true money demand. Lagged money enters significantly because, with regressor measurement error and serially correlated appropriate interest rate and transaction variables, lagged money helps predict money.

The measurement error interpretation has advantages over the typical partial adjustment interpretation of the lagged money coefficient for a number of reasons. At the theoretical level, partial adjustment for M1 transactions-type money balances is difficult to rationalize since equally safe and readily accessible stores of value are available with higher rates of return. At the empirical level, the measurement error view places no behavioral interpretation on the lagged money coefficient, but the estimated coefficient on lagged money is typically too high to be interpreted as representing a desired "speed of adjustment." In addition, residuals from conventional money demand regressions generally exhibit highly significant positive serial correlation. Yet the standard money demand specification coupled with partial adjustment does not predict the residuals from the conventional money demand regression to be serially correlated. By contrast, the residuals are expected to be positively serially correlated in the measurement error view.

In the measurement error interpretation, each of the coefficients in the conventional money demand regression is a function of all the parameters in true money demand and all the regressor generating process parameters. This has two important implications. First, poor post-sample forecast performance does not necessarily indicate that parameters in true money demand have shifted. In this view, the widely documented upward bias in the forecast performance of the conventional money demand regression since 1974 could be due to shifts in the income or interest rate generating processes instead of a shift in true money demand.

Second, the money demand regression cannot deliver unbiased conditional money demand forecasts for alternative policy induced interest rate generating processes because money demand regression coefficients are not invariant to shifts in the interest rate generating process. With regressor measurement error, in order to make unbiased conditional forecasts of money demand for evaluating alternative policies, it is necessary to compute changes in money demand regression coefficients as functions of proposed policy changes.

Appendix A

Equations (4) through (8) in the text are used to derive the following expressions for the variances and covariances appearing in the normal equations in terms of measurement error parameters, appropriate interest rate and transaction variable generating process parameters, and the parameters of true money demand.[18/],[19/]

(1) $$\sigma_{m(t)y(t)} = \frac{a_1\sigma_u^2}{1-\phi_1^2}$$

(2) $$\sigma_{m(t)r(t)} = \frac{a_2\sigma_q^2}{1-\theta_1^2}$$

(3) $$\sigma_{m(t)m(t-1)} = \frac{a_1^2\phi_1\sigma_u^2}{1-\phi_1^2} + \frac{a_2^2\theta_1\sigma_q^2}{1-\theta_1^2}$$

(4) $$\sigma^2_{y(t)} = \frac{\sigma_u^2}{1-\phi_1^2} + \sigma_\zeta^2$$

(5) $$\sigma_{y(t)r(t)} = \text{(unrestricted)}$$

(6) $$\sigma_{y(t)m(t-1)} = \frac{a_1\phi_1\sigma_u^2}{1-\phi_1^2}$$

(7) $$\sigma^2_{r(t)} = \frac{\sigma_q^2}{1-\theta_1^2} + \sigma_\eta^2$$

(8) $$\sigma_{r(t)m(t-1)} = \frac{a_2\theta_1\sigma_q^2}{1-\theta_1^2}$$

(9) $$\sigma^2_{m(t)} = \frac{a_1^2\sigma_u^2}{1-\phi_1^2} + \frac{a_2^2\sigma_q^2}{1-\theta_1^2} + \sigma_e^2$$

The determinant, Δ, of the right-hand-side matrix in equation (10) of the text may be written

$$(10) \qquad \Delta \equiv \sigma_\zeta^2\left[\frac{a_2^2(\sigma_q^2)^2}{1-\theta_1^2} + \frac{a_1^2\sigma_q^2\sigma_u^2}{(1-\theta_1^2)(1-\phi_1^2)} + \frac{\sigma_q^2\sigma_e^2}{1-\theta_1^2}\right] + \sigma_\eta^2\left[\frac{a_1^2(\sigma_u^2)^2}{1-\phi_1^2} + \frac{a_2^2\sigma_q^2\sigma_u^2}{(1-\phi_1^2)(1-\theta_1^2)} + \frac{\sigma_u^2\sigma_e^2}{1-\phi_1^2}\right] + \sigma_\zeta^2\sigma_\eta^2\sigma_{m(t)}^2 + Z$$

Since Δ would be positive if the measurement error variances, σ_ζ^2 and σ_η^2, were zero, $Z > 0$; so $\Delta > 0$.

For b_1:

Since $b_1^{\text{numerator}}$ contains no σ_ζ^2 term and $b_1 = a_1$ when $\sigma_\zeta^2 = \sigma_\eta^2 = 0$, b_1 has the form $b_1 = a_1\dfrac{x_1\sigma_\eta^2 + Z}{\Delta}$, where

$$x_1 \equiv \frac{a_1^2(\sigma_u^2)^2}{1-\phi_1^2} + \frac{a_2^2\sigma_u^2\sigma_q^2}{(1-\phi_1^2)(1-\theta_1^2)}(1-\phi_1\theta_1) + \frac{\sigma_u^2\sigma_e^2}{1-\phi_1^2};$$

therefore $0 < b_1 < a_1$.

For b_2:

Since b_2^{num} contains no σ_η^2 term and $b_2 = a_2$ when $\sigma_\zeta^2 = \sigma_\eta^2 = 0$, b_2 has the form $b_2 = a_2\dfrac{x_2\sigma_\zeta^2 + Z}{\Delta}$, where

$$x_2 \equiv \frac{a_2^2(\sigma_q^2)^2}{1-\theta_1^2} + \frac{a_1^2\sigma_q^2\sigma_u^2}{(1-\theta_1^2)(1-\phi_1^2)}(1-\theta_1\phi_1) + \frac{\sigma_q^2\sigma_e^2}{1-\theta_1^2};$$

therefore $a_2 < b_2 < 0$.

For b_3:

Since b_3 would be zero if the measurement error variances, σ_ζ^2 and σ_η^2, were zero, b_3 has the form

$$b_3 = \frac{x_3\sigma_\zeta^2 + x_4\sigma_\eta^2 + x_5\sigma_\zeta^2\sigma_\eta^2}{\Delta}\text{ , where}$$

$$x_3 \equiv \frac{a_1^2\phi_1\sigma_q^2\sigma_u^2}{(1-\theta_1^2)(1-\phi_1^2)},\quad x_4 \equiv \frac{a_2^2\theta_1\sigma_u^2\sigma_q^2}{(1-\phi_1^2)(1-\theta_1^2)}\quad \text{and } x_5 \equiv \sigma_{m(t)m(t-1)};$$

therefore $0 < b_3 < 1$.

For b_0:

The regression plane goes through the point of means, so that

$Em = b_0 + b_1Ey + b_2Er + b_3Em$

where
$$Em = a_0 + a_1\frac{\phi_0}{1-\phi_1} + a_2\frac{\theta_0}{1-\theta_1}$$

$$Eq = \frac{\phi_0}{1-\phi_1}$$

$$Er = \frac{\theta_0}{1-\theta_1}$$ 20/

Substituting for the expectations and solving for b_0 yields

$$b_0 = \phi_0(1-b_3) + [(1-b_3)a_1-b_1]\frac{\phi_0}{1-\phi_1} + [(1-b_3)a_2-b_2]\frac{\theta_0}{1-\theta_1}$$

For $[(1-b_3)a_1-b_1]$ and $[(1-b_3)a_2-b_2]$: The bracketed terms have the same sign as their numerators

$$[(1-b_3)a_1-b_1]^{num} = a_1[\sigma_\zeta^2(\frac{a_2^2(\sigma_q^2)^2}{1-\theta_1^2} + \frac{a_1^2\sigma_q^2\sigma_u^2}{(1-\theta_1^2)(1-\phi_1^2)}(1-\phi_1) + \frac{\sigma_q^2\sigma_e^2}{1-\theta_1^2})$$

$$+ \sigma_\zeta^2\sigma_\eta^2(\sigma_{m(t)}^2 - \sigma_{m(t)m(t-1)})$$

$$- \sigma_\eta^2\frac{\theta_1a_2^2\sigma_q^2\sigma_u^2}{(1-\phi_1^2)(1-\theta_1^2)}(1-\phi_1)]$$

$$[(1-b_3)a_2-b_2]^{num} = a_2[\sigma_\eta^2(\frac{a_1^2(\sigma_u^2)^2}{1-\phi_1^2} + \frac{a_2^2\sigma_q^2\sigma_u^2}{(1-\phi_1^2)(1-\theta_1^2)}(1-\theta_1) + \frac{\sigma_u^2\sigma_e^2}{1-\phi_1^2})$$

$$+ \sigma_\zeta^2\sigma_\eta^2(\sigma_{m(t)}^2 - \sigma_{m(t)m(t-1)})$$

$$- \sigma_\zeta^2 \frac{\phi_1 a_1^2\sigma_q^2\sigma_u^2}{(1-\theta_1^2)(1-\phi_1^2)} (1-\theta_1)]$$

Appendix B

This appendix describes the system in which true money demand depends only on an appropriate transaction variable $y^*(t)$. This single input version of true money demand is

$$(1) \qquad m(t) = a_0 + a_1 y^*(t) + e(t) \qquad\qquad a_1 > 0$$

The appropriate transaction variable, $y^*(t)$, is assumed to be generated as in equation (5) of the text. In this case the conventional money demand regression is written

$$(2) \qquad m(t) = d_0 + d_1 y(t) + d_2 m(t-1) + w(t)$$

where $\quad w(t) \equiv$ the period t residual

Measured period t income, $y(t)$, is assumed to be generated as in equation (7) of the text.

The coefficient solutions for the conventional money demand regression (2) are

$$d_0 \equiv a_0(1-d_2) + [(1-d_2)a_1-d_1]\frac{\phi_0}{1-\phi_1}$$

$$(3) \qquad d_1 \equiv \frac{\dfrac{a_1\sigma_u^2\sigma_e^2}{1-\phi_1^2} + \dfrac{a_1^3(\sigma_u^2)^2}{1-\phi_1^2}}{\Delta}$$

$$d_2 \equiv \frac{\dfrac{a_1^2\phi_1\sigma_u^2\sigma_\zeta^2}{1-\phi_1^2}}{\Delta}$$

where $$\Delta \equiv \sigma_\zeta^2\left[\frac{a_1^2\sigma_u^2}{1-\phi_1^2} + \sigma_e^2\right] + \frac{(a_1^2\sigma_u^2 + \sigma_e^2)\sigma_u^2}{1-\phi_1^2} > 0$$

The coefficient on the transaction variable mean, $\frac{\phi_0}{1-\phi_1}$, in the regression constant term, d_0, is

$$(1-d_2)a_1-d_1 = \frac{\dfrac{a_1^3(1-\phi_1)\sigma_u^2\sigma_\zeta^2}{1-\phi_1^2} + a_1\sigma_\zeta^2\sigma_e^2}{\Delta}$$

The coefficient on the $\overline{y}^*(t-1)$ term in the residual in equation (13) of the text is

$$\phi_1(a_1-d_1) - d_2a_1 = \phi_1a_1\sigma_\zeta^2\sigma_e^2$$

FOOTNOTES

1/ Goldfeld [1973], p. 582.

2/ See Carr and Darby [1981], pp. 184-87 and Laidler [1980], pp. 236-39 for criticism, from authors accepting some form of partial adjustment specification as appropriate, of specific models of partial adjustment that have been employed in money demand studies.

3/ See Baumol [1952] and Tobin [1956] for early analyses of the transaction demand for money. Barro and Fischer [1976], pp. 134-40 survey this literature.

4/ It is commonly assumed in the investment literature "that costs are associated with adjusting the capital stock at a rapid rate per unit time and that these costs rise rapidly with the absolute rate of investment, so that the firm never attempts to achieve a jump in its capital stock at any moment," (Sargent [1979], p. 127). Partial stock adjustment is optimal for this type of adjustment cost.

5/ Goldfeld [1973], p. 599.

6/ Johnston [1963], Chapter 6, contains a relatively extensive discussion of measurement error. Perhaps the most famous application of the measurement error model in econometrics is in Friedman [1956], where it is used to explain some empirical paradoxes in the consumption function literature. In recent years, many econometrics texts have offered only a very casual treatment of measurement error. See Goldberger [1972], pp. 993-99. A good example of the lack of attention currently given to measurement error is the recent article by Cooley and LeRoy [1981]. This article, which is an extensive critique of identification and estimation of money demand, does not mention measurement error at all.

7/ See Morgenstern [1963], pp. 242-82, Young [1974], "A Primer on Gross National Product Concepts and Issues" [1981], and "Gross National Product Data Improvement Project Report" [1977].

8/ See Miller and Orr [1966], pp. 425-27.

9/ See Sargent [1979], pp. 203-206.

10/ If neither the appropriate transaction nor interest rate variable were autocorrelated, i.e., $\phi_1 = \theta_1 = 0$, then the coefficient on lagged money would be zero.

11/ See Granger and Morris [1976], pp. 248-51, and Granger and Newbold [1977], pp. 28-31.

12/ These restrictions are derived in Appendix B.

13/ See Appendix B.

14/ See Granger and Morris [1976], p. 251.

15/ See Enzler, *et al*. [1976], Goldfeld [1976], and Porter and Offenbacher [1982].

16/ See Marschak [1953] and Lucas [1976] for discussions of conditional forecasting and policy evaluation.

17/ See Goldberger [1972], pp. 993-99.

18/ See Freund [1971], pp. 195-97 and Box and Jenkins [1976], pp. 56-8.

19/ Note that only measurement error variances enter the b regression coefficients. The b regression coefficients are affected by the presence of autocorrelation in the measurement errors only insofar as it affects the magnitude of the measurement error variances σ_ζ^2 and σ_η^2.

20/ If a measurement error has a non-zero mean, this affects the b_0 constant term in the conventional money demand regression. For example, suppose the transaction variable measurement error, $\zeta(t)$, had generating process $\zeta(t) = \Omega_0 + \Omega_1\zeta(t-1) + \delta(t)$. Then

$$Ey = \frac{\phi_0}{1-\phi_1} + \frac{\Omega_0}{1-\Omega_1}.$$

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