

**Working Paper 84-8**

**INFORMATIONAL IMPLICATIONS OF INTEREST RATE RULES**

**by**

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**November 1983  
revised September 1984**

We have benefited from the comments of Marvin Goodfriend and presentation of this paper at the Econometric Society meetings in December 1983. The National Science Foundation has supported the second author's participation in this research. The views expressed in this paper are not necessarily those of the NSF, of the NBER, or of the Federal Reserve Bank of Richmond.

### Abstract

Returning to a topic first systematically treated by Poole (1970) in a textbook Keynesian model, this paper compares interest rate and money supply rules. Our analysis, by contrast, is conducted within a rational expectations macro model that incorporates flexible prices and informational frictions. With differential information, interest rate targets can affect the information content of market prices and real activity, but these real consequences can always be replicated by an appropriately chosen money stock rule with feedback to economic activity. However, when the policy authority has incomplete information about the state of the economic system, it faces a discrete choice between an interest rate peg and strict money stock control. Depending on the parameters of the model, either of these policies may be optimal, given the informational constraints faced by the monetary authority.

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## I. Introduction

Policy discussions in central banks have long centered on the selection of an appropriate level of the interest rate. Recently, this focus has been translated into a search for an optimal interest rate rule. But this adherence to an interest rate policy has been challenged by monetarist critics.

William Poole's (1970) analysis of optimal monetary policy instruments in the context of a textbook Keynesian model yielded two major results relevant to this controversy. First, when the state of the economy is known by the monetary authority, money stock and interest rate policies are equivalent, so that optimal demand management can be achieved by either means. Second, when the central bank cannot fully observe the contemporaneous state of the economy, policymakers should employ the new information contained in the nominal interest rate to counteract the output effects of unobservable real and nominal shocks. Following this line, a policy of "leaning against" interest rate movements--i.e., a policy of positive contemporaneous money supply response to interest rate shocks--is typically desirable. In addition to providing the standard framework for analysis of monetary policy, Poole's (1970) work also hinted at a positive analysis of monetary authorities' observed concern with interest rate smoothing.<sup>1</sup>

In rational expectations models with flexible prices and informational frictions,<sup>2</sup> the implications of Poole's policy alternatives are dramatically altered.

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<sup>1</sup>See, for example, the discussion of interest rate smoothing in the context of a descriptive analysis of monetary policy provided by Poole (1975). Goodfriend (1984) offers a positive theory of monetary policy that incorporates an interest rate smoothing objective.

<sup>2</sup>Lucas (1972, 1973) provided initial models that stressed the importance of informational frictions for aggregate supply theory. More recent treatments incorporate economy-wide bond markets--Barro (1980), Grossman and Weiss (1982) and King (1983)--so that discussion of monetary policy choice becomes feasible.

In these models, an interest rate rule cannot be arbitrarily postulated but rather requires the specification of an underlying money supply rule, which serves as a nominal anchor to the system.<sup>3</sup> Thus, an even more fundamental equivalence obtains than in Poole's analysis. Further, the distribution of real activity is invariant to the sort of contemporaneous policy response discussed by Poole, because private agents efficiently utilize the information contained in the nominal interest rate, which is not affected by a known policy of leaning against surprise movements in the interest rate.<sup>4</sup>

This paper is concerned with the informational implications of interest rate rules in rational expectations models with flexible prices and informational frictions. Specifically, we consider a policy of actively targeting the nominal interest rate, which we define as adjusting its expected level to economic conditions. Because this policy of interest rate targetting is equivalent to a money supply rule with feedback to economic conditions, it alters the magnitude of fluctuations in real activity through the expectational channels described by King (1982), rather than through the standard feedback mechanisms analyzed in pre-rational expectations literature such as Poole (1970, Section V). That is, our class of interest rate rules alters the information content of market prices.<sup>5</sup>

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<sup>3</sup> Sargent and Wallace (1975) introduced the indeterminacy of the price level that obtains with an arbitrary interest rate rule under rational expectations. McCallum (1981b, 1984) discusses some alternative ways of resolving this indeterminacy, which all amount to specification of a nominal anchor for the system by a determinate path for the money supply.

<sup>4</sup> See King (1983), Dotsey and King (1983), and Canzoneri, Henderson and Rogoff (1983) for alternative discussions of this irrelevance result, which requires that agents observe nominal interest rates and that unanticipated but accurately perceived money growth has no real effects.

<sup>5</sup> King (1982) stresses that differential information on the part of economic agents is a necessary condition for monetary policy to affect the information content of prices.

Thus, we conclude that interest rate targeting can have an impact on the variability of real activity. But, in contrast to Poole, our analysis provides no reason to prefer an active interest rate policy to a money stock rule with feedback to economic conditions.

When the monetary authority must operate with incomplete information, so that such an optimal money supply rule or interest rate targeting scheme no longer is feasible, then one must compare two alternative non-activist policies, a strict money stock rule and an unconditional interest rate peg. Although such an interest rate peg destroys information, it also absorbs money demand disturbances and eliminates money supply shocks. Thus, in a conclusion reminiscent of Poole (1970, Section V), either a strict money stock rule or interest rate peg may be optimal when there are information constraints on the monetary authority.

The organization of the remainder of the paper is as follows. In Section II, we lay out the simple rational expectations model with flexible prices and informational frictions that we employ in our analysis of policy. In Section III, we discuss the solution of the model, with the details presented in a mathematical appendix. In Section IV, we consider how monetary policy potentially influences expectation formation and, hence, real activity in our model, with particular attention paid to the informational implications of alternative interest rate rules. Section VI is a brief summary and presents our conclusions based on this paper and related efforts.

## II. The Model

In this paper, we employ a simple aggregative model to demonstrate a set of results concerning interest rates and informational efficiency. But many of these results also hold in other more complicated models that have flexible

prices and informational frictions (such as King (1983) and Dotsey and King (1983)).

There are two elements in the model economy that are particularly important for our subsequent policy analysis. First, current commodity supply and demand depend on agents' rational expectations about the real rate of return as in Lucas (1972) and Barro (1980). Second, the economy is populated by two types of agents, who are differentiated by their endowment of information.<sup>6</sup> Specifically, a fraction of agents is accurately informed about the contemporaneous state of the economy. The remaining fraction  $(1-\lambda)$  is assumed to know only the current values of prices, but not the underlying shocks that determine these prices.<sup>7</sup> Taken together, these two elements dictate limitations on the role for monetary policy previously discussed by Barro (1976) and King (1982). That is, unless the monetary authority has superior information, current feedback to the state of the economy has no real effects, as perceived money growth is neutral. With differential information, prospective feedback can alter the information content of market prices and, hence, the distribution of real activity.

### Commodity Demand and Supply

Supply and demand at a given date  $t$  are aggregates of the actions of informed and uninformed agents. In common with other intertemporal substitution models

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<sup>6</sup>By viewing the information structure as exogenous, we abstract from equilibrium in the information market as considered by Edwards (1981). The endogenously determined fraction of informed traders would plausibly respond to policy, an effect which is not considered here.

<sup>7</sup>Our basic results do not require that one group is fully informed or, even, that some agents are better informed than others. The key assumption is that agents are differentially informed (see King (1982) and Dotsey and King (1983)). The assumption of fully informed agents yields, however, the simplest analytical solutions.

of business fluctuations, commodity supply and demand depend on the real rate of return expected by market participants. In our log-linear model the real rate of return between  $t$  and  $t+1$  is  $r_t = P_t + R_t - P_{t+1}$ , where  $P_t$  is the logarithm of the price level at date  $t$  and  $R_t$  is the level of the nominal interest rate. Informed agents form their rational expectation using a complete information set containing all shocks to the system in date  $t$  and earlier periods, which we denote  $I_t$ . Uninformed agents are limited to current information about the price level and the interest rate, an information set which we denote  $U_t = \{P_t, R_t, I_{t-1}\}$ .

Commodity supply and demand are specified as

$$(1) \quad y_t^s = (1-\lambda)\alpha^s \text{Er}_t|U_t + \lambda\alpha^s \text{Er}_t|I_t - \lambda\beta^s \text{Eg}_t|I_t - (1-\lambda)\beta^s \text{Eg}_t|U_t + \theta^s g_t$$

$$(2) \quad y_t^d = -(1-\lambda)\alpha^d \text{Er}_t|U_t - \lambda\alpha^d \text{Er}_t|I_t + \lambda\beta^d \text{Eg}_t|I_t + (1-\lambda)\beta^d \text{Eg}_t|U_t \\ + \theta^d g_t + \epsilon_t.$$

In addition to the intertemporal substitution influences of the rate of return, commodity supply and demand also depend on some current disturbances  $g_t$  and  $\epsilon_t$ . We think of  $g_t$  as being an unobservable level of government spending, which has direct supply effects ( $\theta^s g_t$ ) via productivity and demand effects ( $\theta^d g_t$ ), i.e., government's purchase of goods less substitution influences on private commodity demand. The coefficients  $\beta^s, \beta^d$  reflect wealth effects on commodity supply and demand. For a more detailed description of the effect of government purchases on supply and demand decisions see Barro (1981) or (1984)). The term  $\epsilon_t$  is a disturbance to private commodity demand.

Commodity market clearing requires that the real rate of return expected by uninformed agents satisfy

$$(3) \quad E r_t | U_t = \lambda (E p_{t+1} | I_t - E p_{t+1} | U_t) + \frac{\theta + \beta}{\alpha} g_t + \frac{(1-\lambda)\beta}{\alpha} (g_t - E g_t | U_t) + \frac{1}{\alpha} \varepsilon_t,$$

where we have defined the composite parameters  $\theta = \theta^d - \theta^s$ ,  $\alpha = \alpha^s + \alpha^d$ , and  $\beta = \beta^s + \beta^d$ . (The derivation also employs the fact that  $E g_t | I_t = g_t$ ). Substituting this expression into the supply schedule, we obtain the commodity market-clearing value of output

$$(4) \quad y_t = y_t^* + (1-\lambda) \frac{H}{\alpha} (g_t - E g_t | U_t),$$

where the full information level of output ( $y_t^*$ ) is

$$(5) \quad y_t^* = \frac{G}{\alpha} g_t + \frac{\alpha^s}{\alpha} \varepsilon_t.$$

In these expressions, we have used the composite parameters  $G$  and  $H$ , defined as  $G = \alpha^s(\theta - \beta) + \alpha(\theta^s + \beta^s)$  and  $H = \alpha^s \beta^d - \beta^s \alpha^d$ , which are treated as positive in our analysis.<sup>8</sup>

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<sup>8</sup> Given the results of Barro and King (1984), a few words concerning these assumptions are in order. Barro and King show that in models where agents' preferences are time separable and where commodities are nonstorable the parameter  $G$  is positive under standard assumptions but that  $H$  is zero. Therefore, output will never deviate from its full information value regardless of the degree of confusion about the actual values of  $m_t$  and  $g_t$ . In order for misperceptions of money and real disturbances to have an effect on output, one must do away with either the time separability or perishable commodity assumptions. However, the resulting models would be extremely complicated. We therefore view the assumption of  $H$  greater than zero as a convenient device for analyzing the consequences of misperceptions on output. In the context of the subsequent analysis, all we really desire is a reduced form solution in which misperceptions of nominal quantities have real effects. Since the underlying structural model plays only a limited role in the results obtained, the above assumptions have no qualitative effect on our results and significantly simplify the analysis.



### Money Demand and Supply

The demand for money is taken to have the semi-logarithmic form used by Sargent-Wallace (1975) and Barro (1980),

$$(6) \quad M_t^d = P_t + \delta y_t - \gamma R_t - kv_t - (1-k)v_{t-1},$$

where  $M_t^d$  is the logarithm of money demand and  $v_t$  is an aggregate velocity shock with persisting effects on the demand for money.<sup>9</sup>

Following our discussion above, we specify that the money supply rule involves both responses to interest rate surprises and feedback to the state of the economy.

$$(7) \quad M_t^s = \tilde{M}_t + \psi(R_t - ER_t | I_{t-1}) + f_t + m_t.$$

In this expression,  $\tilde{M}_t = M_0 + nt$  is the long-run growth path of money and  $m_t$  is a random shock to the money supply. Responses to interest rate shocks are captured by the term  $\psi(R_t - ER_t | I_{t-1})$ , with an interest rate peg obtaining when  $\psi$  is driven to infinity. We restrict attention in specifying feedback ( $f_t$ ) to responses to velocity shocks or past errors in monetary control, i.e.,

$$(8) \quad f_t = f_m m_{t-1} + f_v v_{t-1}.$$

Based on prior work of McCallum (1981, 1983) and Dotsey and King (1983), we know that one might alternatively view the authority as selecting an interest rate rule. We discuss this possibility in greater detail later in the paper.

### III. Rational Expectations Solution

Commodity market and monetary equilibrium yields two equations that link the price level and the nominal interest rate.

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<sup>9</sup>The first-order moving average parameterization of money demand disturbances was chosen for analytical tractability rather than empirical realism.

$$(9) \quad P_t = -R_t + E_{P_{t+1}}|U_t + \lambda(E_{P_{t+1}}|I_t - E_{P_{t+1}}|U_t) + \frac{\theta-\beta}{\alpha} g_t \\ + \frac{(1-\lambda)\beta}{\alpha} (g_t - E_{g_t}|U_t) + \frac{1}{\alpha} \epsilon_t$$

$$(10) \quad R_t = \frac{1}{\gamma+\psi} (P_t + \delta[\frac{G}{\alpha} g_t + (1-\lambda)\frac{H}{\alpha} (g_t - E_{g_t}|U_t) + \frac{\alpha}{\alpha} \epsilon_t] \\ - kv_t - (1-k)v_{t-1} - \tilde{M}_t + \psi ER_t|I_{t-1} - f_m m_{t-1} - f_v v_{t-1} - m_t)$$

Given the structure of the economy, the following undetermined coefficients solutions can be postulated:

$$(11) \quad R_t = \phi_0 + \phi_1 \tilde{M}_t + \phi_2 m_{t-1} + \phi_3 v_{t-1} + \phi_4 m_t + \phi_5 v_t + \phi_6 g_t + \phi_7 \epsilon_t,$$

$$(12) \quad P_t = \pi_0 + \pi_1 \tilde{M}_t + \pi_2 m_{t-1} + \pi_3 v_{t-1} + \pi_4 m_t + \pi_5 v_t + \pi_6 g_t + \pi_7 \epsilon_t,$$

The details of the solution method are spelled out in the appendix. As is frequently the case in this class of rational expectations models, one can first and most simply solve for the part of the equilibrium solution that involves the dependences of prices and interest rates on elements of  $I_{t-1}$ . These solutions have the following intuitive form

$$(13) \quad ER_t|I_{t-1} = \phi_0 + \phi_1 \tilde{M}_t + \phi_2 m_{t-1} + \phi_3 v_{t-1} = n - \frac{f_m}{1+\gamma} m_{t-1} - \frac{f_v + (1-k)}{1+\gamma} v_{t-1}$$

$$(14) \quad EP_t|I_{t-1} = \pi_0 + \pi_1 \tilde{M}_t + \pi_2 m_{t-1} + \pi_3 v_{t-1} \\ = \gamma n + \tilde{M}_t + \frac{f_m}{1+\gamma} m_{t-1} + \frac{f_v + (1-k)}{1+\gamma} v_{t-1}.$$

That is, the nominal interest rate has an unconditional mean  $n$  equal to the trend rate of monetary expansion (the real rate of interest is zero due to the absence of constant terms in (1) and (2)). The price level depends one-to-one

on the trend money stock ( $\tilde{M}_t$ ) and depends positively on the rate of monetary expansion (via the inverse effect on cash balances of expected inflation, the intensity of which is governed by  $\gamma$ ). Temporarily high values of the money stock ( $f_{m,t-1}$ ) raise the price level and lower the nominal interest rate via an expected deflation effect. Similarly, the net influence of  $v_{t-1}$  involves its own serial correlation (governed by  $k$ ) and policy influence (governed by  $f_v$ ) but otherwise works like a temporary money supply disturbances.

### Expectations of Uninformed Agents

Following Lucas (1973) and Barro (1976, 1980), we view uninformed agents as extracting information from available signals contained in prices and interest rates. Given that departures of output from its full information value are induced solely by  $g_t - E g_t | U_t$ , we focus on this expectation, which takes the form

$$(15) E g_t | U_t = b_P S_{Pt} + b_R S_{Rt}$$

where  $S_{Pt}$  and  $S_{Rt}$  are signals contained in the price level and interest rate.<sup>10</sup>

By observing the price level as expressed in (9) and the nominal interest rate given by (10) agents receive the following effective signals.

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<sup>10</sup>The conventional way to derive these signals, as in Lucas and Barro, would be to use the undetermined coefficients representation (11), so that the signal provided by the nominal interest rate would be  $\phi_4 m_t + \phi_5 v_t + \phi_6 g_t + \phi_7 \epsilon_t$ . Here, we employ an alternative solution strategy developed by Hercowitz (1980) which culls "effective signals" from prices and interest rates by using the fact that in equilibrium, agents know the influence of their own expectations on prices. This strategy frequently leads to sharper intuition and more readily obtainable solutions.

$$(16) S_{Pt} = \lambda \pi_2 m_t + \lambda \pi_3 v_t + \frac{\theta - \lambda \beta}{\alpha} g_t + \frac{1}{\alpha} \varepsilon_t$$

$$(17) S_{Rt} = \frac{1}{\gamma + \psi} \left\{ -m_t - kv_t + \left( \frac{G + (1 - \lambda)H}{\alpha} \right) g_t + \frac{\alpha^s}{\alpha} \varepsilon_t \right\}.$$

There are two important facts to notice about these signals. First, the price level signal is influenced by the expectations of informed agents,  $EP_{t+1}|I_t = \pi_2 m_t + \pi_3 v_t$ , so long as  $\lambda$  is not zero. Second, the information provided by the interest rate is not altered by any finite value of the policy parameter  $\psi$ , as agents can simply "rescale" and learn the same linear combination of fundamental disturbances. But, when  $\psi$  is infinite the interest rate is lost.

In interpreting our subsequent analysis, it will be useful to discuss expectation information in the case where there are no nominal shocks. Then, with the two signals  $S_{Pt}^*$  and  $S_{Rt}^*$  (the asterisk indicating the absence of nominal disturbances), depending only on the two underlying shocks  $g_t$  and  $\varepsilon_t$ , agents can accurately infer the value of  $g_t$ . Thus, in this case,

$$(18) Eg_t|U_t = b_P^* S_{Pt}^* + b_R^* S_{Rt}^* = g_t,$$

where  $b_P^*$  and  $b_R^*$  are population regression coefficients.

#### IV. Monetary Policies and Expectations

In this section, we explore the effects of some alternative monetary policies on expectations and, hence, on output.

A Strict Money Stock Rule. Under this policy, there is neither contemporaneous response to interest rates ( $\psi = 0$ ) nor feedback to unpredictable changes in money demand ( $f_v = 0$ ). Further, all policy errors ( $m_t$ ) are eliminated ( $f_m = 0$ ).

Contemporaneous Response to Interest Rates. As discussed previously, Poole (1970) puts forward the hypothesis that contemporaneous money supply response to interest rates can stabilize economic fluctuations. This is depicted by a nonzero, finite value of the parameter  $\psi$  in equation (7).

An Interest Rate Peg. An interest rate peg is the limiting case of a contemporaneous response to interest rates (i.e.,  $\psi = \infty$ ). Specifically, the monetary authority supplies any quantity of nominal balances demanded at the pegged rate. Under a peg, money supply disturbances are eliminated from the system and the signal  $S_{Rt}$  destroyed.<sup>11</sup>

An Interest Rate Target. We define a policy of interest rate targeting as adjusting its expected level ( $ER_t | I_{t-1}$ ) to economic conditions, but permitting surprise movements in the interest rate to occur in response to shocks. McCallum (1981, 1983) has taught us that such a policy of interest rate targeting is feasible under rational expectations, as long as (i) the monetary authority provides a nominal anchor to the system (such as  $\tilde{M}_t$  in our analysis) and (ii) the authority selects among a class of feasible interest rate rules. In our context, feasible interest rate targets with responses to  $m_{t-1}$  and  $v_{t-1}$  take the form

$$(19) \quad R_t^T = n + \tau_m m_{t-1} + \tau_v v_{t-1}.$$

Clearly, there is an equivalence between the specification of money supply feedback parameters ( $f_m, f_v$ ) and specification of the interest rate target

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<sup>11</sup>For a more detailed discussion of the determinacy properties of various pegs see McCallum (1981) and (1983) and Dotsey and King (1983). In general the resolution of indeterminacy involves the specification of an underlying money stock rule.

parameters  $(\tau_m, \tau_v)$ .<sup>12</sup> That is, altering  $f_m$  or  $f_v$  implies a change in  $ER_t | I_{t-1}$  and is therefore equivalent to moving the targeted level of the interest rate in the money supply rule (7).

Given the various monetary policies, which are specific cases of the money supply rule (4), we wish to compare the characteristics of each and decide which rule is optimal. Before doing so, we will present a general discussion of the effects of policy on the information content of prices and formulate a means of comparing policies. Placing our specific problem in such a general setting will suggest other contexts in which our results are likely to arise.

### Information and Policy.

In this section, we want to distinguish between two different ways in which policy can alter the informational state of the economy, building a foundation for our comparison of the various monetary policies discussed above.

To make our general analysis comparable to the specific problem addressed in the paper, we focus on a case in which economic agents are forming rational perceptions about a single variable  $x_t$ , which is itself not directly observable. For this purpose, agents have a vector of information variables or signals  $S_t = \langle S_{1t}, \dots, S_{qt} \rangle$ . If  $x_t, S_t$  are jointly normally distributed--conditionally on the information set  $A_{t-1}$ --then it follows that

$$(20) \quad E x_t | S_t, A_{t-1} = \mu_x + b_{xs} (S_t - \mu_s),$$

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<sup>12</sup>One can also view a more restricted form of an interest rate target as  $f(ER_t | I_{t-1} - ER_t)$ . In the present case where only the past history of velocity shocks is important this type of response would be equivalent to feedback on a velocity shock.

where  $\mu_x = E(x_t | A_{t-1})$ ,  $\mu_s = E(S_t | A_{t-1})$  and  $b_{xs} = \sigma_{xs} \Sigma_{ss}^{-1}$  for

$\sigma_{xs} = E x_t S_t' | A_{t-1}$  and  $\sigma_{ss} = E S_t S_t' | A_{t-1}$ . The conditional variance of  $x_t$  given  $S_t$  is

$$(21) \quad \sigma_{xx} - \sigma_{xs}' \Sigma_{ss}^{-1} \sigma_{xs}$$

where  $\sigma_{xx} = E x_t^2 | A_{t-1}$ .

Throughout our discussion, we use the magnitude of the variance of  $x_t$ , conditional on a specified information set, as our measure of the "informational state" of the agent or economy under study. That is, when there is a lower value of

$$(22) \quad E(x_t - E x_t | A_t)^2 | A_t,$$

where  $A_t$  is the current information set, we say that there is a better informational state.

As a result of our econometrics training and practice, much of our intuition about the effects of information is obtained from the sort of "regression" model outlined above. In the subsequent discussion, we use that intuition to discuss the two basic ways that policy can alter the information state of the economy.

The first and simplest way is to alter the list of signals while holding the covariance structure fixed. Then it is easy to determine the effect on the informational state. That is, let the information set available to agents include the covariance structure and a proper subset of the signal vector  $S_t$ ; then, from elementary statistical theory, we know that the conditional variance of  $x_t$  increases, lowering the informational state. That is, with the covariance structure fixed, a reduction in the number of signals worsens the informational

state. Viewing (18) as a population regression, this accords with our basic intuition, in that fewer independent variables lead to a larger population variance.

The second way that policy can effect the informational state is by altering the covariance structure  $x_t, S_{1t}, \dots, S_{qt}$ . In this case there is an ambiguous effect on the informational state. One needs to specify the precise nature of the alteration in covariance structure to determine the effect on the conditional variance of a prediction with fixed number of signals,

$$E(x_t - E x_t | A_t)^2 | A_t.$$

In models where agents form rational perceptions about state variables that are not directly observable, almost all policy interventions affect the covariance structure, while some policy interventions affect the number of signals.

## V. Alternative Monetary Policies

In this section, we consider some alternative feasible monetary policies. We start by considering an optimal feedback policy or, equivalently an optimal interest rate target. Then, we compare a money stock rule to an unconditional interest rate peg.

### Optimal Monetary Policy

Following the preceding discussion, we define the optimal interest rate or monetary policy as the one that produces the highest informational state of the economy. Specifically, the optimal policy is that which produces the lowest conditional variance of  $g_t$  given the information set of uninformed agents. Thus, our objective will be to find the policy that minimizes

$$E(g_t - E g_t | U_t)^2 | U_t.$$



From our discussion in Section III, we saw that agents could correctly infer  $g_t$  from  $S_{pt}^*$  and  $S_{Rt}^*$ . It is now easy to show that a feedback policy can effectively alter the information content of prices allowing agents to infer  $g_t$  even in the presence of nominal shocks. The conditional expectation of  $g_t$  is now given by

$$(23) \quad E g_t | U_t = b_p S_{pt} + b_R S_{RT} = b_p (S_{pt}^* + \lambda \pi_2 m_t + \lambda \pi_3 v_t) \\ + b_R (S_{Rt}^* - m_t - k_v)$$

If feedback parameters  $f_m$  and  $f_v$  are set so that  $b_p \lambda \pi_2 - b_R = 0$  and  $b_p \lambda \pi_3 - b_R k = 0$  then equation (23) will be identical to equation (18) with  $b_p = b_p^*$  and  $b_R = b_R^*$ . That is, agents will be able to infer  $g_t$  accurately with optimal feedback. In essence, optimal feedback is able to negate the contaminating influence of nominal shocks in the price and interest rate signals, allowing for a full information solution. We stress that this can only occur in the presence of differential information ( $\lambda \neq 0$ ), as in the analysis of King (1982) and Weiss (1980).

The optimal values of  $f_m$  and  $f_v$  are  $f_m^* = \frac{1+\gamma}{\gamma \delta \alpha^s}$  and  $f_v^* = \frac{k(1+\gamma)}{\gamma \delta \alpha^s} - (1-k)$  (see appendix for derivations). Further, an optimal targeting scheme with  $\tau_m^* = \phi_m^*$  and  $\tau_v^* = \phi_v^*$  results in a full information solution, where  $\phi_m^*$  and  $\phi_v^*$  the values of  $\phi_m$  and  $\phi_v$  when  $f_m$  and  $f_v$  are at their optimal levels. This reflects fundamental equivalence of these two policies, as in Poole's (1970) analysis. However, in contrast to Poole, the optimal policy does not involve responses to unpredictable movements in the current interest rate, nor does it involve any consideration of relative variances.

### An Interest Rate Peg Versus a Money Stock Rule

Suppose, however, that the monetary authority is unable to follow a feedback rule of the type discussed, perhaps because information on lagged shocks is unavailable. Since the information embodied in  $P_t$  and  $R_t$  is unaffected by finite values of  $\psi$ , we find a policy that contemporaneously responds to interest rates produces the same solution as a pure money stock rule. As Dotsey and King (1983) stress, this result occurs because agents are both rational and observe the interest rate. Therefore, movements in money caused by responses to interest rates are perfectly perceived and have no consequence for output or the information content of prices.

This leaves the monetary authority with a choice between a strict money stock rule and a policy of pegging the interest rate at its unconditional expected value  $n$ . These alternative policies are the only feasible ones given the limited information possessed by the monetary authority and correspond to the passive policies of Poole (1970). The comparison of these two policies in terms of the informational state produced by each is nontrivial, since the peg alters (i) the number of signals and (ii) the covariance structure of the model when compared with the money stock rule. That is, under a peg, the interest rate is no longer a signal. Furthermore, under a peg, money supply disturbances no longer arise and velocity shocks are completely absorbed by changes in the money stock.

To compare these two policies, we find it useful to decompose the conditional variance of  $g_t$  in a way that highlights the signalling role of the interest rate. We start by calculating the expectation of  $g_t$  conditional on observation of the price level. Then we revise this expectation using the information contained

in the interest rate. (One can view this procedure as arising because agents receive information sequentially.) Formally,

$$(24) \quad E g_t | S_{Pt}, S_{Rt} = E g_t | S_{Pt} + a(S_{Pt} - E S_{Pt} | S_{Rt})$$

where  $E g_t | S_{Pt} = [\sigma_{gp} / \sigma_{pp}] S_{Pt}$ ,  $E R_t | S_{Pt} = [\sigma_{RP} / \sigma_{PP}] S_{Pt}$

$$a = \{\sigma_{gR} - \sigma_{gP} \sigma_{RP} / \sigma_{PP}\} / \{\sigma_{RR} - \sigma_{RP}^2 / \sigma_{PP}\}, \quad \sigma_{PP} = \text{var}(S_{Pt}),$$

$\sigma_{RR} = \text{var}(S_{Rt})$ ,  $\sigma_{gP} = \text{cov}(g_t, S_{Pt})$ , and  $\sigma_{RP} = \text{cov}(S_{Rt}, S_{Pt})$ . The second term

on the right-hand side reflects the extent to which agents revise their expectations based on information contained in the interest rate. The analogous formula for the variance is

$$(25) \quad E[(g_t - E g_t | U_t)^2] = (\sigma_{gg} - \frac{\sigma_{gP}^2}{\sigma_{PP}}) - a(\sigma_{gR} - \frac{\sigma_{gP} \sigma_{RP}}{\sigma_{PP}}).$$

The second term on the right-hand side of (25) is nonnegative. Thus, the information contained in  $S_{Pt}$  cannot worsen the variance of the prediction error.

By examining (24) and (25) it is easy to see that the peg destroys a signal, wiping out the second right-hand term in (24) and (25). From this standpoint the peg reduces the informational state of the economy. However, it also reduces the variance of the price signal, which no longer contains any nominal disturbances, thereby lowering the first right-hand side term in (25) and improving the informational state of the economy.

In general, neither the peg nor the money stock rule is dominant. Instead the comparison depends on relative magnitudes of the variance of the disturbances and on the parameters of the model, a conclusion which is reminiscent of Poole (1970, Section V). For example, as the variance of real shocks is smaller

( $\sigma_\epsilon^2 \rightarrow 0$ ) pegging the interest rate would allow the price level to accurately communicate  $g_t$  and full information solution would obtain. However, as  $\sigma_\epsilon^2 \rightarrow \infty$ , the added information contained in the interest rate would imply a dominance of the money stock rule.

## V. Summary and Conclusions

This paper explores the informational implications of interest rate policies in rational expectations models with flexible prices and informational frictions. Overall, our analysis suggests that there is little to be gained by discussing monetary policy in terms of interest rate rules. Yet, it does not rule out real effects of systematic actions framed in those terms.

More specifically, we have three major findings of our analysis. First, as in some earlier analyses, we find that contemporaneous response of the variety discussed by Poole (1970) is not an important determinant of real activity because it simply represents a perceived monetary action and hence, is neutral. Second, and more importantly, we find that interest rate targets can affect the distribution of real activity, via the information content of prices, but only in a manner identical to the effects of money supply feedback rules. Finally, we find that either a strict money stock rule or interest rate peg may be a dominant policy, when the monetary authority operates in a situation of incomplete information.

This third finding suggests a potentially rewarding avenue of further research. Suppose that only a subset of the aggregate state of the economy is observable by the monetary authority and the private sector. Then, it appears that there would be a nontrivial choice between a money supply feedback rule, based on observable state elements, and a policy of pegging the interest rate at a level conditional on the observable state elements. This latter policy appears to characterize the actual behavior of the monetary authority in the U.S. and could potentially be a desirable response in a situation of incomplete information.

As shown in the text, equilibrium in the goods and money market gives the following two conditions:

$$(A1) \quad P_t = -R_t + EP_{t+1}|U_t + \lambda(EP_{t+1}|I_t - EP_{t+1}|U_t) + \frac{\theta-\beta}{\alpha} g_t \\ + \frac{(1-\lambda)\beta}{\alpha} (g_t - Eg_t - Eg_t|U_t) + \frac{1}{\alpha} \varepsilon_t$$

$$(A2) \quad R_t = \frac{1}{\lambda+\psi} (P_t + \delta[\frac{G}{\alpha} g_t + (1-\lambda)\frac{H}{\alpha} (g_t - Eg_t|U_t) + \frac{\alpha^s}{\alpha} \varepsilon_t]) \\ - kv_t - (1-k)v_{t-1} - \tilde{M}_t + \psi ER_t|I_{t-1} - f_m m_{t-1} - f_v v_{t-1} - m_t$$

Using (A1) and (A2) and the undetermined coefficients solutions postulated in (11) and (12) we can solve for the undetermined coefficients attached to the elements of  $I_{t-1}$ , namely  $\tilde{M}_t$ ,  $m_{t-1}$  and  $v_{t-1}$ . These solutions are

$$(A3) \quad \begin{array}{ll} \pi_0 = \gamma & \phi_0 = 0 \\ \pi_1 = 1 & \phi_1 = 0 \\ \pi_2 = \frac{f_m}{1+\gamma} & \phi_2 = -\frac{f_m}{1+\gamma} \\ \pi_3 = \frac{f_v + (1-k)}{1+\gamma} & \phi_3 = -\frac{f_v + (1-k)}{1+\gamma} \end{array}$$

and are independent of the policy parameter  $\psi$ , which simply controls policy responses to shocks.

To study incomplete information, we note that the price level and interest rates are equivalent to observing signals

$$(A4) \quad S_{Pt} = \lambda\pi_2 m_t + \lambda\pi_3 v_t + \frac{\theta-\lambda\beta}{\alpha} g_t + \frac{1}{\alpha} \varepsilon_t$$

and

$$(A5) \quad S_{Rt} = -m_t - kv_t + \delta \frac{G+(1-\lambda)H}{\alpha} g_t + \delta \frac{\alpha^s}{\alpha} \varepsilon_t$$

where the term  $\frac{1}{\gamma+\psi}$  has been omitted since it is merely a scaling factor.

The solutions for  $b_p^*$  and  $b_R^*$  in the regression  $Eg_t | U_t = b_p^* S'_{Pt} + b_R^* S'_{Rt}$  are straight forward to obtain, by employing  $g_t = Eg_t | U_t$  and  $S'_{Pt} = \frac{\theta - \lambda \beta}{\alpha} g_t + \frac{1}{\alpha} \epsilon_t$  and  $S'_{Rt} = \delta \frac{G + (1 - \lambda)H}{\alpha} g_t + \frac{\alpha^S}{\alpha} \epsilon_t$ . We find that  $b_p^* = \alpha \alpha^S / [(\theta - \lambda \beta) \alpha^S - (G + (1 - \lambda)H)]$  and  $b_R^* = -\alpha / \delta [(\theta - \lambda \beta) \alpha^S - (G + (1 - \lambda)H)]$ . The solutions for  $f_m^*$  and  $f_v^*$  are found by using the expressions for  $\pi_2$  and  $\pi_3$  and solving the restrictions  $b_p^* \lambda \pi_2 - b_R^* = 0$  and  $b_p^* \lambda \pi_3 - kb_R^* = 0$ .

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