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THE DEMAND FOR CURRENCY IN THE UNITED STATES

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ABSTRACT:

The idea of totally deregulating the financial system and implementing monetary policy through currency control has received renewed attention. An important aspect concerning the desirability of using currency as the instrument of policy is the behavior of the demand for currency. If currency demand is not well behaved, then a policy of controlling the nominal supply of currency could produce drastic swings in the price level and interest rates. The adverse consequence of such effects could outweigh the benefits of deregulation. It is therefore crucial that the behavior of currency demand be well understood before unequivocally advocating total financial deregulation and currency control. This paper takes a step in that direction by analyzing the demand for currency over the period 1921-1980.

I. Introduction

The idea of totally deregulating the financial system and implementing monetary policy entirely through the control of currency has recently received attention. Fama (1980, 1983) has resurrected the idea suggested by Patinkin (1961) that a well determined nominal policy can be achieved through the control of currency, and that other forms of financial regulation with their associated welfare losses need not be tools of monetary control. However, an important aspect concerning the desirability of using currency as the sole instrument of policy is the behavior of the demand for currency. If currency demand is not well behaved, then a policy of controlling the nominal supply of currency could produce drastic swings in the price level and interest rates. The adverse effects of the additional noise in these variables could very well outweigh the benefits of deregulation.

It is therefore crucial that the behavior of currency be well understood before one can unequivocally advocate total financial deregulation. This paper examines the behavior of real currency balances over the period 1921-1980 and finds that currency behaves in a systematic and explainable manner over this period. This is somewhat encouraging with regard to the policy advocated by Fama, since this fairly long sample is characterized by a variety of monetary regimes. These include a gold standard with sterilization in the 1920's followed by a gold standard without sterilization in the 1930's. The 1940's and early 1950's may best be described as a policy of a pure interest rate peg, while monetary policy over the remainder of the sample followed an adjustable peg. However, in view of the Lucas critique, one still needs to be cautious since direct control of currency was not attempted during the sample and explicit interest rates on demand deposits were regulated over most of the sample.

The point of departure for analyzing the demand for currency is an inventory model of the transactions demand for money. The model developed in this paper is based on a number of extensions to the original Baumol-Tobin model. Specifically, the work of Feige and Parkin (1971) and Barro and Santomero (1972) are jointly incorporated to yield a framework that includes two media of exchange as well as goods. Therefore, the model is analytically similar to Santomero (1979) and Dotsey (1983), and is rich enough to explain movements in the relative importance of currency and demand deposits as transaction media. That is, both the proportion of expenditures made with currency and the average amount of currency held are investigated. Indeed the two are closely related.

The behavior of currency is empirically examined by employing Laurent's (1970) methodology for constructing estimates of the value of currency transfers. These estimates, coupled with data on debits to demand deposits, are used to construct a time series on the percentage of expenditures made with currency, which for simplicity will also be referred to as the expenditure ratio. The behavior of the expenditure ratio and the demand for currency are then jointly investigated. The behavior of both variables show remarkable stability and the empirical work can be viewed as generally supportive of the inventory theory of money demand. Further, the use of annual data in the empirical work removes any need for considering lagged dependent variables, and thus avoids a growing dispute in monetary economics over the interpretation of these variables.

A very important implication of the inventory model is that the proportion of transactions income spent using currency is the relevant transactions variable for determining the demand for currency, and that GNP, which is typically used in most studies of currency demand, is likely to be a poor proxy for this variable. Therefore, a comparison between regressions using these two transactions variables is analyzed. The poor performance of GNP casts doubt on the empirical procedures used in a number of other studies on currency.

The framework of the paper is as follows. Section II develops the theoretical model. Section III discusses the empirical implementation of the model, while in

Section IV the empirical results are discussed. Section V critically examines some past empirical work on currency, and Section VI contains a brief summary.

IIa. The Model

The framework used to determine the optimal method of purchasing goods is an extension of the basic inventory models of money demand developed by Baumol (1952) and Tobin (1956). In order to investigate the way that individuals choose to make expenditures, at least two media of exchange are needed. Therefore, both currency and demand deposits are included.[2/] In this regard, the model is similar to Barro and Santomero (1972). In order to motivate the use of one media of exchange over another, goods must also be explicitly introduced into the model. Different characteristics of goods with respect to transactions costs are employed to help generate a preference for using one media of exchange over another. Also, the model includes a short-term store of value, called a savings account, that allows an individual to economize on transactions balances.

The individual's problem is to maximize the profits (π) of managing his savings balance, his transactions balances, and his goods inventories given the transactions costs associated with going to the bank, cashing a check, and purchasing goods, as well as the different rates of return on each asset. The choice variables for the individual are the number of trips he makes to the bank (n_s), the number of times he cashes a check <u>in between</u> trips to the bank (n_d), the number of trips to the store made <u>in between</u> check cashings (n), and the medium of exchange used in purchasing a particular good. (To facilitate the exposition, Table 1 provides a summary of notation.) Because the average holdings of each asset are determined by the choice variables, the results of the profit maximization also yield expressions for the various asset demands. Since, the paper is concerned with currency, its average balance is given by equation (1),

$$(1)\quad \bar{C} = \frac{YT}{2n_s(n_d+1)}\left[\bar{e} - \frac{1}{n}\int_c p(i)di\right]$$

where $\frac{YT\bar{e}}{n_s(n_d + 1)}$ is the amount of currency withdrawn at the start of each goods purchasing cycle and $\frac{YT\bar{e}}{2n_s(n_d + 1)n}\int_c p(i)di$ are the average goods inventories held when the goods are purchased with currency. Here p(i) is the percentage of income, YT, spent on a particular good and $\int_c p(i)di$ integrates this percentage over all goods purchased with currency. Therefore, $YT\int_c p(i)di$ represents the value of goods purchased with currency. From (1), it is clear that the maximization of transactions profits in terms of n_s, n_d, n, and $\bar{e}$ will yield a demand for average currency holdings in terms of the exogenous variables of the model.

Formally, an individual maximizes

$$(2)\quad \pi = r_sT\bar{S} + r_dT\bar{D} + r_cT\bar{C} + r_gT\bar{G} - a_sn_s - a_dn_sn_d$$
$$- (\tau + A(\bar{h}) + B(\bar{h}))n_s(n_d + 1)n$$

where r_s, r_d, r_c, and r_g are the nominal yields on average savings balances, $\bar{S}$, average demand deposit balances, $\bar{D}$, average currency holdings, $\bar{C}$, and average goods inventories, $\bar{G}$. The transactions cost of going to the savings deposit is a_s while the transactions cost of cashing a check is a_d.

The term $(\tau + A(\bar{h}) + B(\bar{h}))$ represents the transactions cost associated with the purchase of goods. Each good is associated with two different proportional transactions costs. One is associated with the use of currency and the other with demand deposits. $A(\bar{h})$ and $B(\bar{h})$ represent the total proportional transactions costs incurred using currency and demand deposits, respectively. Further, each shopping trip incurs a fixed cost, τ, which includes the value of an individual's time as well as transportation costs. One can therefore think of the model in terms of going to a fixed location (e.g., a shopping mall), and making purchases at a number of different

TABLE 1

TABLE OF NOTATION

T = an exogenously determined payments interval

X = the individual's total expenditure on goods over a period of length T

Y = X/T, the rate at which an individual spends

$z(i)$ = the amount spent on good i on each shopping trip

$p(i)$ = the proportion of total expenditure made on good i

$\bar{e}$ = the proportion of total expenditure made with currency, also referred to as the expenditure ratio

S = the average savings account balance

D = the average demand deposit balance

C = the average amount of currency held

G = the average goods inventory held

n_s = the number of trips made to the saving account

n_d = the number of check cashings made in between trips to the bank

n = the number of trips to the store made in between check cashings

a_s = the cost of going to the bank

a_d = the cost of cashing a check

$\alpha(i)$ = the proportional transactions cost associated with purchasing good i with cash

$a(i)$ = $o(i)z(i)$, the transactions cost associated with good i on each shopping trip

$\beta(i)$ = the proportional transactions cost associated with purchasing good i with a check

$b(i)$ = $\beta(i)z(i)$

τ = the lump sum transactions costs associated with a trip to the store

r_s = the interest paid on savings

r_d = the interest paid on demand deposits

r_c = the interest paid on currency

r_g = the interest earned on goods including depreciation shops, where for simplicity each shop sells a particular type of good.

In order to obtain a function that implicitly defines the optimal proportion of goods purchased with currency, it is assumed that there exists an infinite number of goods lying along the continuum from 0 to 1.[3/] Further, it is assumed for simplicity that goods whose indices are elements of $(0, \bar{h})$ are purchased with currency and those whose indices are elements of $(\bar{h}, 1)$ are purchased by check.[4/]

The proportional cost of purchasing the i^{th} good on any particular shopping trip with currency can be expressed as $\alpha(i)z(i)$, where $\alpha(i)$ is the proportional transactions cost associated with the i^{th} good and $z(i)$ is the amount spent on the i^{th} good on each shopping trip. Similarly, let the proportional cost of purchasing the i^{th} good with a check be denoted as $\beta(i)z(i)$. That is, different goods may have different transactions costs for both currency and demand deposits. For instance, a small purchase such as a pack of chewing gum or a newspaper would be much cheaper to perform with cash than with a check, while a large purchase such as a refrigerator may be more efficiently done with a check. Carrying large amounts of cash might be associated with a psychic discomfort of possible loss, while sellers may not accept checks for small items. Also, one may wish to have a written record for large purchases, a service which accompanies checks. For notational ease let
$a(i) = \alpha(i)z(i)$ and $b(i) = \beta(i)z(i)$. Then the cost of any particular shopping trip can be written as $\tau + \int_0^{\bar{h}} a(i)di + \int_{\bar{h}}^1 b(i)di$ where $\int_0^{\bar{h}}(\int_{\bar{h}})$ is the integral over the set of all goods purchased with cash (demand deposits). Defining $A(\bar{h}) = \int_0^{\bar{h}} a(i)di$ and and $B(\bar{h}) = \int_{\bar{h}}^1 b(i)di$ gives the proportional transactions costs incurred using currency and checks, respectively.

Given the structure of the model, the various inventories of goods and assets can be expressed in terms of the choice variables n_s, n_d, n and the proportion of goods bought with currency, $\bar{e}$. (For a detailed description, see the appendix.) The optimal values of n_s, n_d, n, and $\bar{h}$ (and therefore $\bar{C}$) are solved for in two steps. First, for a given $\bar{h}$, the profits from managing the frequency of transactions is maximized. This yields n_s, n_d, and n as functions of $\bar{h}$ and the exogenous variables of the model. It also gives expressions for the average holdings of the various assets in terms of $\bar{h}$. In particular the average value of currency $\bar{C}(\bar{h})$ is depicted by

$$3. \quad \bar{C}\ (\bar{h}) = 1/2Y\bar{e} \left\{ \left[\frac{2a_d}{\bar{e}Y(r_d-r_c)} \right]^{\frac{1}{2}} - \left[\frac{2(\tau + A(\bar{h}) + B(\bar{h}))}{Y((1-\bar{e})(r_d-r_g) + \bar{e}(r_c-r_g))} \right]^{\frac{1}{2}} \right\}$$

One therefore observes that for any given $\bar{h}$, average currency balances increase when transactions income, the cost of cashing a check, and the opportunity costs of holding goods (r_d-r_g and r_c-r_g) rise, and that average currency balances fall when the opportunity cost of holding currency r_d-r_c and the transactions costs of making a shopping trip increase.

A complete solution to the model is obtained by substituting in the solutions for $n_s(\bar{h})$, $n_d(\bar{h})$, and $n(\bar{h})$ into the expression for profits yielding an equation in which profits are a function of $\bar{h}$ and the exogenous variables of the model. This expression is then maximized with respect to $\bar{h}$, and the resulting first order necessary conditions yield an implicit function of $\bar{h}$. Since $\bar{e}$ is directly related to $\bar{h}$ by $\bar{e} = \int_0^{\bar{h}} p(i)di$, solving for the optimal value $\bar{h}$ yields the optimal proportion of goods bought with currency.

The optimal $\bar{h}$, $\bar{h}*$, occurs at a value that exactly offsets changes in interest earnings and transactions costs. Further, the first order necessary conditions imply that $a(\bar{h}*) < b(\bar{h}*)$. Since demand deposits pay a higher yield than currency, one would only use currency if it had a lower transactions cost. However, a check might still be used for the i^{th} good if $a(i) < b(i)$ since there is an opportunity cost of holding currency instead of demand deposits. Also, the second order conditions for profit maximization imply that the derivative of $b(\bar{h}*)$, $b'(\bar{h}*)$ must be less than $a'(\bar{h}*)$.

Therefore, if currency is to be used at all it must provide a relative advantage in transacting for some goods, and the transactions costs of using currency must increase faster with respect to the ordering of goods. These conditions seem reasonable since the cost of using a check does not seem to vary much over goods, while the transactions costs of currency increases sharply with the amount spent on the good (especially if one includes the cost of expected currency loss).

IIb. Comparative Statics.

The effects of changes in the exogenous variables on $\bar{h}^*$ (and hence $\bar{e}^*$) and the demand for currency will now be examined. Since the demand for currency will experience both a direct effect as well as an indirect effect due to changes in transactions patterns when exogenous variables change, the comparative static results for the expenditure ratio will be examined first. (For detailed results see the Appendix).

The Effects on the Expenditure Ratio

Regarding the expenditure ratio, one first observes that changes in a_s, r_s, and Y do not affect the optimal method of purchase as long as they do not influence the consumption bundle. This is because a_s and r_s are only involved in determining the optimal number of trips to the saving account and therefore are not related to expenditure patterns, while transactions income serves solely as a variable that scales the level of expenditure. However, to the extent that movements in these variables change expenditure patterns (i.e., the distribution of p(i)), they would affect the expenditure ratio.

The effects of changes in the various transactions costs a_d, $b(\bar{h}^*)$, $a(\bar{h}^*)$ are straightforward. A decrease in the cost of cashing a check implies that more goods will be bought with currency. As check cashing becomes less expensive, a smaller opportunity cost is incurred when goods are bought with cash since an individual can economize on currency balances by using demand deposits as a store of value. In the limit as a_d approaches zero, the model would be equivalent to having two means of payment both stored as demand deposits. In this case an individual's currency balances would be zero, and he would purchase a good using cash or check depending on whether $a(i) \lessgtr b(i)$. An increase in $b(\bar{h}^*)$ relative to $a(\bar{h}^*)$ causes more goods to be purchased with currency since the proportional transactions costs of using currency becomes relatively cheaper.

The effect of a change in the fixed cost of going to the store cannot be unambiguously signed, nor can the effects of changes in r_d, r_c and r_g. However, there is an exact relationship between the comparative static effects of these variables that hinges on the sign of the following inequality:

$$(4)\quad (1/T)(Y/2)^{\frac{1}{2}}p(\bar{h}^*)(r_d - r_c) \gtrless (b(\bar{h}^*) - a(\bar{h}^*))n_s(n_d + 1)n,$$

where $n_s(n_d + 1)n$ is the number of trips made to the store over each payments periods. The left-hand side of this expression represents the gain of using demand deposits relative to currency, since the level of currency and deposit balances are affected by the amount spent on the marginal good. The right-hand side represents the additional cost of using a check multiplied by the number of trips made to the store.

Increases in the fixed cost of going to the store, τ, r_g and r_d will cause the the expenditure ratio to rise, while an increase in r_c will cause it to fall if the left-hand side of (3) is greater than the right-hand side. With respect to increases in τ and r_g, both cause fewer trips to the store as well as a decline in the additional amount of demand deposts and currency that would have to be held if an additional good was purchased with either medium. Therefore, the transactions benefit of using currency declines, as does the opportunity cost of holding currency. Whichever declines by more will determine the net effect on the expenditure ratio.

An increase in the own rate on currency causes the number of trips to the store to increase making currency's use relatively more attractive. However, it also increases the average holdings of currency reducing the attractiveness of using currency. If the gain due to transactions use outweighs the increased opportunity cost due to higher balances, then more goods will be bought with cash.

An increase in the rate paid on demand deposits has two conflicting effects on the expenditure ratio. That is, an increase in r_d should produce opposite results to those encountered with r_g and r_c. It can be shown that the substitution effect between demand deposits and currency dominates, if the left-hand side of (1) is smaller than the right side implying an increase in the expenditure ratio.

Therefore, although an exact sign can not be given to the comparative static results of a number of the exogenous variables, there does exist a well-defined relationship between their various effects. For instance, a rise in the fixed cost of going to the store will cause the expenditure ratio to move in the same direction as an increase in the yield on goods inventories or demand deposits, while an increase in the yield on currency should cause the expenditure ratio to move in the opposite direction. Therefore, although one can not predict the way in which the expenditure ratio will shift when interest rates or τ move, there is a well-defined relationship between the effects generated by movements in these variables. This is summarized in table 2.

The Effect on Average Currency Balances

The effect on average currency balances of a change in an exogenous variable, x, can be separated into two components. One is a direct effect for any given expenditure pattern, while the other involves the effect of a change in expenditure patterns. Therefore, it is important to establish the shape of $\bar{C}(\bar{h})$ with respect to $\bar{h}$. Initially, as one begins to use currency, currency balances rise. In the neighborhood of $\bar{h}^*$, $C(\bar{h})$ will be increasing as the number of goods purchased with currency increase if $(1/T)(Y/2)^{\frac{1}{2}}\, p(\bar{h}^*)(r_d-r_c) < (b(\bar{h}^*) - a(\bar{h}^*))(n_s(n_d(\bar{h}^*)+1)n(\bar{h}^*)$, which turns out to be the case empirically (for more detail see Appendix A). Satisfying this inequality implies that the amount of currency held in between check cashings rises by more than the average inventories of goods, implying that average currency balances rise. Notationally,

$$(5) \quad \frac{d\bar{C}(\bar{h}*)}{dx} = \frac{\partial C(\bar{h}*)}{\partial x} + \frac{\partial C(\bar{h})}{\partial h*} \frac{\partial \bar{h}*}{\partial x}$$

The inequality that determines the sign of $\partial\bar{C}(\bar{h}*)/\partial\bar{h}*$ is the same inequality that determines the sign of $\partial\bar{h}*/\partial x$. Therefore, it will turn out that the expenditure ratio effects embodied in the second term on the right-hand side of (4) will reinforce the direct effects of the exogenous variables if column 2 of table 2 is relevant. Since this turns out to be the case empirically, the comparative static effects of changes in exogenous variables can be summarized by

$$(6) \quad \bar{C} = C(\underset{o}{a_s}, \underset{+}{d_d}, \underset{+}{\bar{e}Y}, \underset{+}{\bar{e}}, \underset{o}{r_s}, \underset{-}{r_d}, \underset{+}{r_c}, \underset{-}{b(\bar{h}*)/a(\bar{h}*)}).$$

where the signs under the variable indicate the sign of the partial derivate of currency with respect to that variable.

III.a <u>Developing the Empirical Tests</u>.

The preceding section suggests a simultaneous equation systems approach for estimating the model. The two step maximization procedures yields equations determining both the expenditure ratio and average currency balances which can be expressed as

$$(7) \quad \bar{e} = f(Y, a_s, a_d, \tau, a(\bar{h}), b(\bar{h}), r_s, r_d, r_c, r_g, u_t)$$

$$(8) \quad \bar{C} = g(\bar{e}Y, \bar{e}, a_s, a_d, \tau, a(\bar{h}), b(\bar{h}), r_s, r_d, r_c, r_g, v_t)$$

where u_t and v_t are stochastic disturbance terms. Alternatively, one could substitute (7) into (8) and represent $\bar{C}$ as a function of exogenous variables. However, the coefficients from such an equation would be a complicated combination of direct and indirect effects on average currency holdings and therefore be far less revealing than

TABLE 2

SUMMARY OF COMPARATIVE STATICS[†]

Exogenous Variable	If $(1/T)(Y/2)^{1/2}p(\bar{h}^*)(r_d-r_c) > (b(\bar{h}^*) - a(\bar{h}^*))n_s(n_d+1)n$	If $(1/T)(Y/2)^{1/2}p(\bar{h}^*)(r_d-r_c) < (b(\bar{h}^*) - a(\bar{h}^*))n_s(n_d+1)n$
a_s[‡]	0	0
r_s[‡]	0	0
Y[‡]	0	0
a_d	+	+
r_g	+	-
r_c	-	+
r_d	+	-
τ	+	-
$b(\bar{h}^*)/a(\bar{h}^*)$	+	+

[†]The signs in columns 2 and 3 indicate the sign of $\frac{\partial \bar{h}^*}{\partial x}$ where x is an exogenous variable. A zero indicates no change.

[‡]As mentioned in the text, changes in a_s, r_s, and Y do not affect the expenditure ratio only if they do not affect the consumption bundle.

the procedure outlined above. Performing the estimation on the system (7) and (8) allows one to discriminate between the effects a variable has on expenditure patterns and any additional effects the variable has on the demand for currency.

Another important consideration in implementing the empirical tests, concerns the derivation of measures which approximate the variables of the model. A description of these follows.

III.b The Dependent Variables

The real per capita value of currency balances was used as the dependent variable in the currency demand regressions. Its calculation is straightforward. However, the empirical measure of the expenditure ratio represents a fairly novel procedure. Ideally, the expenditure ratio, $\bar{e}$, is given by the ratio of goods and services bought with currency to the total amount spent on goods and services using both currency and demand deposits. The volume of purchases made with currency is calculated by a method devised by Laurent (1970), while debits to demand deposits (excluding New York City debits) are used to measure expenditures made by check.

Using debits to demand deposits is appropriate only if purchases of goods and services made by check is a fairly constant fraction of total debits. Examining the behavior of the ratio of debits to demand deposits to GNP in figure 2 indicates that this is unlikely to be the case. Beginning in the early sixties, debits to demand deposits began to grow at a much faster rate than GNP. The conjectured reason for this behavior is that transactions of a purely financial nature have become an increasingly important part of total demand deposit debits. Because the volume of demand deposit debits involving financial transactions are not a constant fraction of total demand deposit debits, this series does not represent a consistent indicator of the volume of goods and services purchased by check. Therefore, the sample period emphasized in the empirical work on the expenditure ratio covers the years 1920-1965, with the belief that nonfinancial debits are a relatively constant fraction of total debits over this period.

The measure of expenditures made with currency is obtained by using Laurent's procedure. Laurent's methodology uses data on issuance and redemptions of currency by denomination in order to calculate the value of transactions made with currency, given that each bill performs a certain number of transfers before being retired.[5] The actual number of transfers that the average bill makes is calculated by maximizing the correlation between the sum of currency transfers and debits to demand deposits to GNP. Over the period 1920-1965 the number of transfers that an average bill made before being retired was 85. Using evidence from Laurent this would imply that a one-dollar bill was involved in a transaction approximately every six days.[6]

The real value of currency transfers, CT, obtained from this procedure is depicted in figure 2. This series is quite volatile and reaches a peak during World War II. The large wartime use of currency may have been associated to some degree with black market activity arising from the rationing of goods. It could also have been partially due to an increased proportion of the population in military service and to tax evasion (see Cagan (1958)).

The expenditure ratio is given by

$$(9)\quad \bar{e} = CT/(CT + DEBITS)$$

where DEBITS are the real value of debits to demand deposits. As indicated by the graph of the expenditure ratio (figure 3), currency has been an important media of exchange accounting for approximately 60 percent of transfers in 1933 and 1934 and falling to roughly 20 percent of transfers in 1965. It should be noted that the figures obtained for CT and $\bar{e}$ represent an upper limit for the value and percentage of the value of goods and services purchased with currency. This is because most transactions involving demand deposits are written for their exact amount, while most transactions undertaken with currency involve total transfers which are greater than the value of the purchase. For example, purchasing an \$8.00 item with a ten-dollar bill involves a currency transfer of \$12.00.

Figure 1

RATIO OF DEBITS TO DEMAND DEPOSITS TO GNP

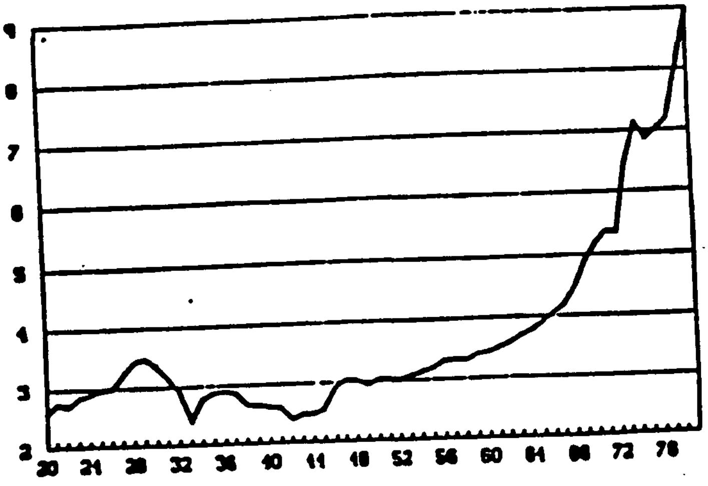


Figure 2

REAL VOLUME OF CURRENCY TRANSFERS

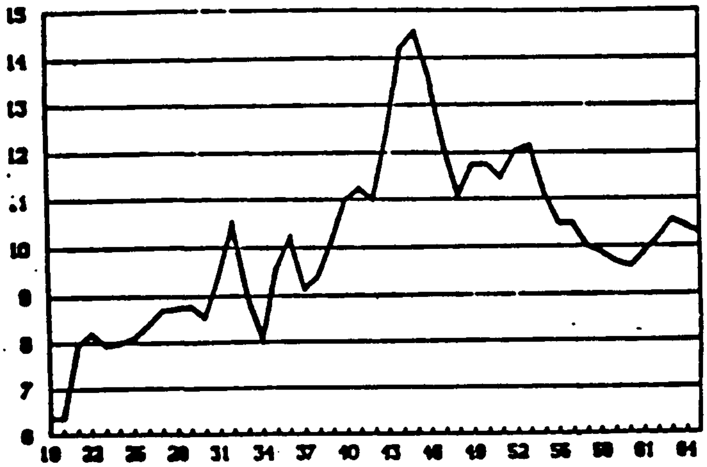
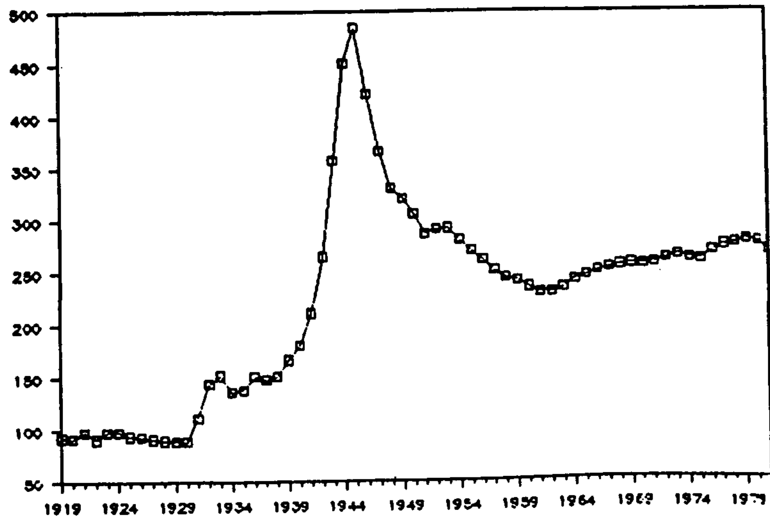


Figure 3

PERCENT OF EXPENDITURES MADE WITH CURRENCY

Figure 4

Real per capita currency



IIIc. The Independent Variables

The next step is to examine the independent variables used as determinants of the expenditure ratio and the demand for currency. In Section II, it was shown that both dependent variables should depend upon the transactions costs of cashing a check and purchasing goods, on the rate yielded by demand deposits, currency, and goods, and on the distribution of expenditure over goods. In order to proxy for the fixed cost component of transactions costs, the real wage rate, W, on average hourly earnings of production workers in manufacturing was used. Two implicit rates on demand deposits are tried. One, RD, is equal to one minus the maximum reserve requirement ratio on demand deposit times the commercial paper rate. This measure essentially assumes that banks totally avoid Regulation Q, and remit services at a competitive rate. The other, RDI, is the average implicit rate derived by Startz (1979). The inflation rate, INF, is used to approximate the positive component of the return to goods inventories. Depreciation is therefore implicitly assumed to be constant. Real per capita consumption, CONS, is included to pick up changes in the types of goods purchased caused by changes in wealth. The percentage of consumption comprised by durables, PDUR, is also used as a proxy for changes in expenditure patterns. Since durables are likely to intensively involve demand deposit use, a shift toward durable goods purchases should lower the expenditure ratio. However, once the effect of PDUR and CONS are accounted for in currency transfers, they should have no additional effect on average currency balances.[7/]

Three other variables, the percentage of bank failures, PFAIL, the marginal tax on labor income, MTAX, derived by Barro and Sahasakul (1983),[8/] and the rate paid on savings, RS, are included. The first is intended to capture the loss of liquidity (as well as actual losses) suffered by deposit holders when banks are suspended. This loss of liquidity on demand deposits should make currency a relatively more attractive medium of exchange. Also, currency is a useful medium for avoiding income taxes since no written records need to be kept. Because the advantage of hiding income increases with marginal tax rates, it is likely that currency will be used more intensively when marginal tax rates are high. Indeed Cagan (1958) argues that much of the increase in currency holding during World War II was due to its increased use in order to avoid taxes. It is also possible that an increase in MTAX could cause individuals to hoard some of their wealth in the form of currency. Therefore, MTAX could also affect the demand for currency above and beyond its affect on expenditure patterns.

The rate paid on savings accounts are included to capture the effect of credit usage. This rate will also be a determinant of average currency holdings for individuals that do not use a checking account. The purchase of goods on credit essentially allows an individual to use his savings account as a transaction medium. That is, money can be left in the savings account until the bill becomes due. Since the model reasonably requires a trip to the bank each payments period,[9/] no additional costs are borne in transferring funds. An increase in the rate paid on savings would intensify the purchase of goods with credit. To the extent that credit purchases substitute for currency (since a credit purchase eventually causes a debit to demand deposits), the expenditure ratio should be negatively related to this variable. However, under the assumption that goods are purchased more frequently than the payments period, credit use should have no direct effect on average currency balances beyond the effects that are captured by the amount of currency transfers. If, this assumption is violated, the use of credit could have a substantial effect on average currency holdings.

A direct measure of the intensity of credit is also used. This measure (PCR) is constructed as the ratio of installment retail credit, check credit, noninstallment retail credit and service credit to consumption. Optimally, one would like to construct a variable that measured the amount of credit extended in relation to purchases over a particular period. However, no such measure is available over the sample period and PCR uses the average amount of credit outstanding.

Before presenting the results, some potential difficulties should be discussed.[10/] One difficulty is capturing the effects of illegal activities, besides those of tax evasion. No measure was derived for this type of behavior. Also, hoarding of currency by foreign residents could affect the data on issuance and redemptions thus biasing the derivation of currency transfers. Again no measure was found to account for this. A dummy variable taking on values of 1 in 1942-1945 was tried in order to pick up the effects of rationing and black market activity during World War II, but it was insignificant.

IVa <u>Empirical Results: The Expenditure Ratio 1920-1965</u>

The basic regression (run over the period 1920-1965) is depicted in equation (10), where the notation "ln" signifies the natural logarithm of a variable.

$$(10)\quad \ln(\bar{e}_t) = a_0 + a_1\ln(CONS_t) + a_2\ln(RD_t) + a_3\ln(RSAV_t) + a_4 INF_t + a_5\ln(W_t) + a_6\ln(PDUR_t) + a_7(MTAX_t) + a_8 PFAIL_t + u_t$$

The results of this regression are reported in column 1 of table 3. The regression is also run with a correction for serial correlation of order two.[11/] The errors of the corrected regression appear to be white noise.

Column 1 of table 3 represents the results of estimating equation (9) and will be emphasized in the following discussion. Column 2 substitutes RDI for RD. The interpretation adopted here is that RD is an appropriate proxy for the implicit rate earned on demand deposits, and that column 1 contains the most meaningful regression.

It is also important to report that the results of table 3 are not significantly affected by altering the value of G, the number of times a bill is assumed to circulate over its lifetime. Similar results were found for values of G equal to 50 and 110. Regressions were also run using the logarithm of the ratio of currency transfers to demand debits with much the same qualitative results.

Examining the results of column 1, the coefficient on real per capita consumption indicates that as wealth increases there is a significant change in the types of goods purchased. Specifically, an increase in the level of consumption alters the bundle of goods purchased and implies that individuals purchase more of the types of goods associated with checking account transactions. The coefficient on the percentage of consumption comprised by durables reaffirms this conclusion. That is, as individual's buying habits change, so does their method of purchase.

The negative coefficient on inflation establishes the condition that $(1/T)(Y/2)^{\frac{1}{2}}p(\bar{h}^*)(r_d-r_c) < (b(\bar{h}^*) - a(\bar{h}^*))n_s(n_d+1)n$. Therefore, one would expect the coefficients on the value of time as proxied by W, and on the implicit rate on demand deposits to be negative, while variables that capture an implicit yield on currency, say MTAX, to have a positive sign. This is indeed the case. However, the coefficient on the real wage rate does seem rather large. This could reflect the fact that as time becomes more valuable individuals lump the purchases of goods together, and find it more convenient to carry out this procedure with checks. The assumption of the model developed in section II is that agents purchase all goods during the same trip, which is not a totally accurate assumption. As the value of time increases one would expect a growing preference for stores that sold a large variety of merchandise, allowing individuals to lump together purchases. In this case it may be that using checks becomes relatively more convenient, and that the wage rate variable is picking up this effect as well. The variable PFAIL also has the correct sign. (The coefficient is significant at the 11 percent level.)

The yield on the savings account is also significant and negative. As mentioned before, it may be proxying for the return on credit purchases, since credit in a sense allows one to use the store of value as a means of exchange. To the extent that credit purchases substitute for cash, they would tend to lower the expenditure ratio. A substitution of credit for a check would have little effect on the ratio since most credit purchases are eventually paid off with a check. (In some instances using credit issued through a third party could result in two demand debits per purchase. For example, American Express would pay the merchant and you would pay American

TABLE 3

REGRESSION RESULTS FOR THE EXPENDITURE RATIO OVER THE PERIOD 1920-1965

Independent Variables	Column 1	Column 2
CONSTANT	-2.95(-6.76)**	-2.57(-4.82)**
ln(CONS)	-.48(-5.56)**	-.61(-6.16)**
ln(RD)	-.057(-3.86)**	
ln(RSAV)	-.18(-9.56)**	-.21(10.08)**
INF	-.50(-4.59)**	-.32(-2.56)*
ln(W)	-.76(-12.45)**	-.61(-10.43)**
ln(PDUR)	-.067(-2.38)*	-.10(- 3.25)**
ln(MTAX)	.046(2.60)*	.013(.62)
PFAIL	.120(1.64)	.14(1.49)
ln(RD1)		-.055(-1.21)
RHO(1)	.62(4.97)**	.54(4.04)**
RHO(2)	-.56(-4.84)**	-.41(-3.55)**
$\bar{R}^2$	.99	.99
S.E.R.	.0239	.0279
D.W.	2.16	1.80

Column 1 represents the results for equation (6). RHO(1) and RHO(2) are the coefficients for second order serial correlation. $\bar{R}^2$ is the corrected correlation coefficient, S.E.R. is the standard error of the regression, and D.W. is the Durbin-Watson statistic. Numbers in parentheses are t-statistics. **, *, and † indicate significance at the 1%, 5%, and 10% significance levels, respectively.

Column 2 is equation (6) with RD1 substituted for RD.

Express.) The variable PCR, which measures the intensity of credit use was tried and its coefficient was insignificantly different from zero, perhaps implying that the interpretation given to RSAV is incorrect or that the credit variable was not a good measure of credit usage.

The regression results of equation (4) were also checked for stability using the cusum of squares test developed by Brown, Durbin, and Evans (1975). Since the sample period contains a number of very different economic environments, such tests are certainly called for. The maximum cusum of square statistics were .1131 for the forward recursion and .1686 for the backward recursion. Both values were well below the 10 percent critical value of .2093, therefore rejecting instability.[12/] Also, a specification test suggested by Plosser, Schwert, and White (1982) which involves overdifferencing of the equation failed to reject the level specification used in estimation. The test statistic for the regression given in column I of table 3 is distributed χ^2 with 8 degrees of freedom and has a value of 11.48, which is below the 5 percent critical value of 15.51.

It is interesting to examine the behavior of the expenditure ratio that is implied by the results of the regressions. During the Depression the percent of expenditures made with currency rose from .47 in 1930 to a high of approximately .63 in 1933, a 33 percent increase. The largest part of this increase is explained by the 19 percent drop in real per capita consumption, while significant portions are explained by the 25 percent fall in PDUR, the 21 percent fall in RSAV, the 53 percent fall in RD, and the 39 percent increase in the marginal tax rate. However, the sum of these effects only accounts for a 20 percent rise in $\bar{e}$. The percentage of bank failures rose by 400 percent between 1930 and 1933, which would account for an additional rise in $\bar{e}$ of 8 percent, thereby explaining most of the increase in the expenditure ratio during the Depression.

During World War II, the continued high value of $\bar{e}$ can, to a large extent, be attributed to a rapid increase in the marginal tax rate from 6.5 percent in 1940 to

26.2 percent in 1945, an increase of over 300 percent. The decline in the percentage of consumption composed by durables from 10.8 percent in 1940 to 6.7 percent in 1945, a decline of 38 percent, was also significant in maintaining the relatively high values of $\bar{e}$. The changes in these two variables largely offset the effects of a 23 percent rise in real per capita consumption and an 18 percent increase in real wages. These results therefore support Cagan's contention that the rapid rise in tax rates during World War II caused currency to be used more intensively than it otherwise would have been.

After the war the expenditure ratio experienced a rather consistent decline. This is largely attributable to the growth in per capita consumption, the increased rate paid on both savings and demand deposits, and the rising real wage rate.

IVb The Demand for Currency

The basic regression examining the demand for real currency balances is given by

$$(11)\quad d\ln\bar{C}_t = b_0 + b_1 d\ln(CT) + b_2 d\ln(\bar{e}_t) + b_3 d\ln(RD_t) + b_4 d\ln(RSAV_t) + b_5 d(INF_t) + b_6 d\ln(W_t) + b_7 d\ln(PDUR_t) + b_8 d\ln(PCR_t) + b_9 d\ln(MTAX_t) + b_9 d(PFAIL_t) + v_t$$

where "d" indicates that all variables are first differenced,[13/] and estimation is carried out using 2SLS. Column 1 of table 4, reports the results of this regression. The coefficient on the real per capita currency transfers, CT, is highly significant as are the coefficients on PFAIL, and W. The signs and size of these coefficients do not violate any of the predictions of the inventory model. The insignificance of PDUR implies that currency is not intensively used in the purchase of durables, while the insignificance of MTAX implies that currency was not hoarded in response to changes in tax rates.

The insignificance of RSAV is consistent with the adjaceny property of the inventory model if most individuals use a checking account and therefore do not use a savings account as a temporary store of value for currency. The effect of the commercial paper rate was also investigated and its coefficient was insignificant further confirming the adjacency property. The insignificance of RD and INF are a little puzzling. Regarding RD, it may only measure the marginal rate on corporate balances while the marginal rate on individual's demand deposits is probably zero. Since the behavior of corporations has a large effect on debits to demand deposits this would account for the significance of this variable in the expenditure ratio regressions. However, corporations only hold a small fraction of cash balances, which could account for the lack of significance of this variable in explaining the behavior of the stock of currency. Therefore, with the exception of the results on INF and PCR, the simple inventory model examined in this paper performs quite well.

The results concerning credit are indeed puzzling since these are not consistent with the results reported for the expenditure ratio in which PCR was insignificant, implying that credit usage did not directly substitute for currency. The significance of PCR in the currency regressions can only be accounted for if credit is a substitute for currency and if the goods purchased are those that are bought less frequently than the payments period.[14/] It is only in this case that credit can have an effect on average currency balance beyond any affects already accounted for by changes in expenditure patterns. To see this, consider the following example. Initially, suppose currency is used to purchase two types of goods, one, good a, is purchased more frequently than the payments period and the other, good b, is purchased every other payments period, and that the individual does not meet the interior conditions of the model regarding the use of demand deposits or a savings account. In that case, cash balances would accumulate in week one for expenditure on good b in week 2. In other words the one week's worth of interest is insufficient for this individual to use a savings account. But, if the individual could purchase good b with credit, he could keep money in his savings account for a much longer period, since credit payments usually involve a minimum of a one month lag. This would increase the likelihood that the individual would employ a savings account, with an accompanying decline in his average cash balances. While this example can account for the effect of credit in equation (11) it can not jointly explain the results on the expenditure ratio and the currency equation. As mentioned earlier, the variable PCR is a somewhat crude measure and it is clear that more work needs to be done regarding the effect of credit.

Finally, the expenditure ratio does not appear to be significant beyond its effect on CT (recall CT = $\bar{e}$Y). This is somewhat of a blessing in disguise since $\bar{e}$ is the only variable that can not be accurately measured over the entire sample 1921-80. Indeed as indicated in column 2 of table 2, omitting $\bar{e}$ does not substantially alter the regression results. Therefore, a regression omitting $\bar{e}$ is examined over the entire sample period in column 3. The only major change is that the marginal tax rate is now significant at the 10 percent level, implying that increases in tax rates lead to hoarding of currency.

The stability of the regressions in columns 2 and 3 of table 4 were checked using two procedures developed for simultaneous equation systems. One developed by Erlat (1982) is essentially an extension of the usual Chow test and the other developed by Giles (1981) extends the use of the Brown-Durbin-Evans cusum of squares test to simultaneous equations. Over the period 1921-1965, with the sample broken at 1944 which is the date that the Quandt log likelihood ratio takes on its maximum value, the test statistic which is distributed F^{21}_{12} takes on a value of .31. This is well below the 5 percent critical value. The maximum cusum of squares statistic takes on a value of .2637 which is below the 5 percent critical value of .2857.[15/] Regarding the 1921-1980 period, the sample was broken at 1968 with the test statistic being distributed F^{15}_{34} = .268 which is again well below the 5 percent critical value. The maximum cusum of squares statistic is .2932 which is above the 5 percent critical value of .244 but below the 1 percent critical value of .2972. Therefore, the tests broadly suggest that the demand for currency is stable over the period.

As depicted in figure 4, the major changes in real per capita currency balances occurs during the Depression and during World War II. After WWII, currency declines

TABLE 4

REGRESSION RESULTS FOR CURRENCY

Independent Variables	Column 1 1920-1965	Column 2 1921-1965	Column 3 1921-1980
CONSTANT	.020(1.31)	.016(1.15)	.012(1.17)
dlnCT	.63(3.46)**	.71(4.14)**	.60(4.06)**
dln$\bar{e}$	.43(1.21)	--	--
dlnRD	.042(1.11)	.017(.52)	-.003(-.14)
dlnRSAV	-.062(-.54)	-.075(-.68)	-.090(-.94)
dINF	.29(1.13)	.14(.67)	.013(.07)
dlnW	.87(2.31)*	.62(1.99)†	.57(2.14)*
dlnRDUR	-.004(-.04)	-.052(-.59)	-.02(-.29)
dlnPCR	-.45(-2.73)*	-.44(-2.80)**	-.44(-3.57)**
dlnMTAX	.042(.93)	.05(1.14)	.067(1.74)†
dPFAIL	.38(2.62)*	.46(3.60)**	.45(3.84)**
RHO	.44(3.15)**	.39(2.94)**	.37(3.17)**
D.W.	2.39	1.94	1.88
S.E.R.	.0514	.0500	.0452
$\bar{R}^2$	.741	.757	.733

Numbers in parenthesis are t- statistics. **, *, † indicate significance at the 1 percent, 5 percent, and 10 percent significance levels, respectively.

TABLE 5

REDUCED FORM REGRESSION RESULTS

Independent Variables	Column 1 1921-1965	Column 2 1921-1965
dln(gnp)	---	-.005(-.03)
dln(C-BAR+DEBITS)	.76(2.27)*	---
dln(CONS)	-.85(1.67)†	---
dlnRD	-.023(-.56)	-.004(-.10)
dlnRSAV	-.22(-1.96)†	-.27(-2.22)*
dlnINF	-.12(-.67)	-.31(-1.47)
dlnW	.092(.24)	-.17(-.42)
dlnPDUR	-.031(-.24)	-.12(-1.00)
dlnPCR	-.61(-3.54)**	-.60(-3.42)**
dlnMTAX	.064(1.24)	.11(1.99)†
dPFAIL	.70(3.34)**	.46(2.25)*
P.W.	1.78	1.74
S.E.R.	.0590	.0629
$\bar{R}^2$	.73	.60

sharply and then grows at a relatively steady rate. During 1931 real per capital currency balances grew by 22 percent and in 1932 they grew by 25 percent. This growth is largely but not totally explained by the increase in the use of currency and the large number of bank failures. The increased use of currency occurs, even though income was falling during this period. In fact, CT grew at a rate of 8.8 percent in 1931 and 10.4 percent in 1932.

The largest growth in currency occurs during the war. Over that period real per capita balances grew by 15.5 percent in 1941, 22.9 percent in 1942, 29.8 percent in 1943 and 23.2 percent in 1944. This rapid growth was also sharply reversed immediately after the war. The rapid rise was partially accounted for by the increased use of currency during the war, especially in 1943 and 1944 when CT grew by at an annual rate of roughly 13 percent. A look at figure 2 shows that CT reached a peak during the war and thereafter experienced a somewhat uneven pattern of decline. The behavior of credit also helps explain movements in currency balances. Credit fell substantially during the war, but began to grow quite strongly after the war. The growth in wage rates during the war also helps explain the increase in currency. Wage rates also fell immediately after the war, and then grew fairly consistently through the rest of the sample. As mentioned earlier, marginal tax rates also grew substantially during the war. However, the coefficient on the marginal tax rate is only significant when the sample is extended through 1980. This provides some evidence that currency hoarding is associated with high tax rates, although the evidence is far from conclusive.

An examination of the residuals of the currency regressions indicate that the large swings in currency holdings that occurred over the sample are captured quite well by the regressions. Only the residuals for the years 1931, 1935, and 1949 are somewhat large. The above results therefore indicate that currency behaves in a systematic fashion and that this behavior is consistent with the predictions of an inventory model of money demand. Indeed the inventory model is especially useful in indicating that the proper transactions scale variable involves a measure of the actual expenditures made with currency. As will be shown in the next section, simply employing real per capita GNP in a reduced form regression yields poor results and casts doubt on previous studies of the demand for currency.

V. A Critical Examination of the Literature

The empirical methodology of section IV was used to estimate the structural parameters of equations (10) and (11). Based on the parameter estimates obtained for these structural equations, the reduced form regression for currency consistent with the structural equations would be

$$(12)\quad C = .71d\ln(CT + DEBITS) - .34d\ln(CONS) - .23\, d\ln(RD) - .17d\ln(RSAV) + .08d\ln(MTAX) - .047\, d\ln(PDUR) - .45d\ln(PCR) + .54d\ln PFAIL - .36dINF + .08d\ln(W)$$

where CT + DEBITS are in real per capita terms. The estimated reduced form over the period 1921-1965 is given in column 1 of table 5. The estimated coefficients, with the exception of the coefficient on CONS, are all extremely close to the values given in (12), and all the coefficients are within one standard error of the coefficients in (12).

If one, instead runs a regression using real per capita GNP, gnp, (or real per capita consumption) as the transactions scale variable the results are markedly different. As seen in column 2 table 5, the coefficient on dlngnp is insignificantly different from zero. A similar result is found for the entire sample period, which make spurious correlation a strong possibility with regard to a number of results reported in the literature.

For example, Becker (1975) uses the level of retail trade as a scale variable on quarterly regressions over the period 1953-1971. Although he obtains a significant positive coefficient on this variable, the coefficient on lagged currency is .98, and may indicate that this coefficient is picking up a good deal of serial correlation.

Garcia and Pak (1976) find that the log of real GNP is significant and positive in quarterly regressions over the period 1952:2-1967:4 and 1952:2-1977:2. The coefficients on lagged currency are between .80 and .86 while the first order serial correlation coefficients are .70 and .53 respectively. Ochs and Rush (1983) also find that the log of real GNP is significantly positive, but obtain a very high coefficient on lagged currency. In all these studies using first differencing without lagged dependent variables appears to be a needed specification check.

VI. Conclusions

Building on an inventory model of currency demand, the empirical work in this paper indicates that the demand for currency has behaved systematically over a sample period that includes very diverse conditions. The inventory model indicates the importance of correctly specifying the transactions variable, and this importance is confirmed in the empirical work. A number of other properties of the inventory model are consistent with the data, among which are the insignificance of the commercial paper rate in the currency demand regressions and the sign patterns on the coefficients in the expenditure ratio equations. The systematic and largely explainable behavior of currency over the sample lends support to the view that currency control may be a viable alternative to current monetary policy.

FOOTNOTES

1/ Implicit in this argument is the notion that the demand for currency would not radically change under a regime of targeting currency. Given the wide disparity of the economic environments examined in the empirical work, my guess is that the demand for currency would, if anything, exhibit more systematic behavior under a regime of constant growth.

2/ Introducing the intertemporal problems associated with credit would severely complicate the model.

3/ The qualitative results of this model are not sensitive to a variety of alternative specifications. Similar results were found for a model containing a continuum of goods with no fixed transactions cost associated with the purchase of goods and for a model that contained a discrete number of goods, the purchase of each being associated with a different fixed cost. For a detailed discussion of the latter model, see Dotsey (1983).

4/ If goods are equally weighted in the budget, then goods can be ordered in such a way that purchasing behavior would accord with this assumption. If goods were ordered so that $(b(i) - a(i)) \geq (b(i+\varepsilon) - a(i+\varepsilon))$, then buying the first goods with cash will be optimal. For unequally weighted goods I have not been able to derive a similar ordering.

5/ The volume of transactions negotiated with coins is not considered.

6/ The correlation coefficient series generated by this procedure is extremely flat making it difficult to state a maximal value with a high degree of statistical certainty. However, the results of the regressions presented in section IV are not very sensitive to changes in G. Similar results were obtained for values of G = 50 and G = 110.

7/ The method of expenditure could have a direct effect on average currency balances if goods purchased with currency are bought less frequently than the payments period (see Grossman and Policano (1975)). An increase in the proportion of goods that are bought less frequently than the payments period ((behavior which is likely to be associated with durables) would cause average cash balances to rise. However, if durables are not generally purchased with currency, an increase in the proportion of expenditure made on durables would only affect currency through its indirect effect on the expenditure ratio.

8/Barro and Sahasakul's derivation is an improvement over earlier work done on marginal tax rates. In particular, they calculate a rate derived from the actual tax schedule and therefore avoid downward biases caused by deductions to income. They also include the effects of social security taxes.

9/A 1979 survey by the Federal Reserve Bank of Atlanta indicates that individuals make on average 3 deposits per month to their demand deposit accounts.

10/These difficulties are potentially severe since the latest Federal Reserve Study could account for less than 15 percent of the U.S. currency stock.

11/The best fit for the errors from an uncorrected regression was ARIMA (2,0,0) with the AR parameters equalling .51 with a standard error of (.15) and -.49 with a standard error of (-.15).

12/A maintained hypothesis of the Brown-Durbin-Evans test is white noise errors. The test was therefore also run after the variables were filtered using an AR2 filter. The maximum cusum of squares statistics were .1780 for the forward recursion and .2009 for the backward recursion, which are below the 10 percent critical value of .2140.

13/The results of the specification test suggested by Plosser, Schwert, and White can be extended to simultaneous systems and indicate that the level regression is an inapproriate specification.

14/For a detailed discussion of the effects of the relative frequency of goods purchases on money balances, see Grossman and Policano (1975).

15/In the stability tests a maintained hypothesis is that the errors of the regression are white noise. Therefore, the variables are filtered before running the tests so that the resulting errors are indeed white noise.

LITERATURE CITED

1. Barger, Harold. Outlay and Income in the United States: 1921-1938. Nat. Bur. Econ. Res., New York, 1942.
2. Barro, Robert J. "Integral Constraints and Aggregation in an Inventory Theory of Money Demand." J. Finance 31 (March 1976), 77-87.
3. Barro, Robert J., and Sahasakul, Chaipat. "Average Marginal-Tax Rates from Social Security and the Individual Income Tax." NBER Working Paper 1214, October 1983.
4. Barro, Robert J., and Santomero, Anthony M. "Household Money Holdings and the Demand Deposit Rate." J.M.C.B. 4 (May 1972), 397-413.
5. Baumol, W. J. "The Transactions Demand for Cash--An Inventory-Theoretic Approach." Q.J.E. 66 (November 1952), 545-56.
6. Becker, William E. "Determinants of the United States Currency-Demand Deposit Ratio." J. Finance 30 No. 1 (March 1975), 57-74.
7. Brown, R. L.; Durbin, J.; and Evans, J. M. "Techniques for Testing the Constancy of Regression Relationships Over Time." J. of the Royal Statistical Society, Series B (1975), 149-63.
8. Cagan, Phillip. "Demand for Currency Relative to the Total Money Supply." J.P.E. 66 (August 1958): 303-29. Reprinted in Nat. Bur. Econ. Res. Occasional Paper 62.
9. Dotsey, Michael. "A Study of the Relative Use of Currency and Demand Deposits." Ph.D. dissertation (unpublished), Department of Economics, University of Rochester (1983).
10. Dutton, Dean S., and Graham, William P. "Transactions Costs, the Wage Rate and the Demand for Money." A.E.R. 63 (September 1973), 652-65.
11. Erlat, Haluk. "A Note on Testing for Structural Change in a Single Equation Belonging to a Simultaneous System." Ec. Letters 13 (1983), 185-189.

12. Fama, Eugene F. "Banking in the Theory of Finance," JME 5 (April 1980): 39-57.

_______. "Financial Intermediation and Price Level Control." JME Vol. 12 No. 1 (July 1983): 7-28.

13. Feige, Edgar, and Parkin, M. "The Optimal Quantity of Money, Bonds, Commodity Inventories and Capital." A.E.R. 61 (June 1971), 335-49.

14. Garcia, G. and Pak S. "The Ratio of Currency to Demand Deposits in the United States." J. Finance 34 (June 1986), 703-715.

15. Giles, David E.A. "Testing for Parameter Stability in Structural Econometric Relationships." Ec. Letters 7 (1981), 323-326.

16. Grossman, Herschel I., and Policano, Andrew. "Money Balances, Commodity Inventories, and Inflationary Expectations. J.P.E. 83, no. 6 (1975), 1093-1112.

17. Karni, E. "The Value of Time and the Demand for Money." J.M.C.B. 6 (February 1974) 45-64.

18. Kuznets, S. National Income and Its Composition. Nat. Bur. Econ. Res., New York, 1941.

19. Laidler, David E. W. The Demand for Money Theories and Evidence. 2nd ed. New York: Dun-Donnelley, 1977.

20. Laurent, Robert D. "Currency Transfer by Denomination." Ph.D. dissertation (unpublished), University of Chicago (1970).

21. Ochs, Jack and Rush, Mark. "The Persistence of Interest Rate Effects on the demand for Money." JMCB 15 (November 1983), 499-505.

22. Patinkin, Don. "Financial Intermediaries and the Logical Structure of Monetary Theory," AER 51 (March 1960), 95-115.

23. Plosser, Charles I.; Schwert, G. William; and White, Halbert. "Differencing as a Test of Specification." I.E.R. 23 (October 1982): 535-52.

24. Santomero, Anthony J. "Optimal Transactions Behavior and the Demand for Money." Ph.D. dissertation, Brown University (1971).

________. "The Role of Transactions Costs and Rates of Return on the Demand Deposit Decision." J. Monetary Economics 5 (1979), 343-64.

25. Startz, Richard. "Implicit Interest on Demand Deposits." J. Monetary Economics 5 (1979), 515-34.

26. Tobin, James. "The Interest-Elasticity of Transactions Demand for Cash." R.E.S. 38 (August 1956), 241-47.

APPENDIX A

Given the structure of the model, the various inventories of goods and assets can be expressed in terms of n_s, n_d, n, and the proportion of goods bought with currency, $\bar{e}$. Recall, that in the expressions below it is assumed for simplicity that all goods lie along the continuum from 0 to 1 and that the goods whose indices are an element of $(0, \bar{h})$ are purchased with currency, while those whose indices are an element of $(\bar{h}, 1)$ are purchased by check. The average holdings of savings, demand deposits, currency, and goods are given by equation (A1a)-(A1d), where p(i) is the proportion of the budget spent on good i, T is the length of the payments period and Y is the flow of transactions income over the period. [Hence z(i) is related to p(i) and YT, by z(i) = p(i)YT/(Number of shopping trips in each period).]

(A1a) $\bar{S} = 1/2YT[1 - 1/n_s]$

(A1b) $\bar{D} = \frac{YT}{2n_s}\left[1 - \frac{\bar{e}}{n_d+1} - \frac{1}{n(n_d+1)} \int_{\bar{h}}^{1} p(i)di\right]$

(A1c) $\bar{C} = \frac{YT}{2n_s(n_d+1)}\left[\bar{e} - \frac{1}{n} \int_{0}^{\bar{h}} p(i)di\right]$

(A1d) $\bar{G} = \frac{YT}{2n_s(n_d+1)n} \int_{0}^{1} p(i)di = \frac{YT}{2n_s(n_d+1)n}$

where $\bar{e} = \int_0^{\bar{h}} p(i)di$ represents the proportion of the budget spent using currency and is therefore called the expenditure ratio.

An individual maximizes the profits, π, of managing his transactions with respect to his choice variables. Specifically he maximizes

$$\text{(A2)} \quad \pi = r_s\overline{TS} + r_d\overline{TD} + r_c\overline{TC} + r_g\overline{TG} - a_s n_s - a_d n_s n_d$$

$$- (\tau + A(\overline{h}) + B(\overline{h}))n_s(n_d+1)n$$

where $A(\overline{h}) = \int_0^{\overline{h}} a(i)di$ and $B(\overline{h}) = \int_{\overline{h}}^{1} b(i)di$. Using expressions (A1a)-(A1d), equation (2) may be rewritten in the following form.

$$\text{(A2')} \quad \pi = 1/2r_s YT^2 - \frac{1/2YT^2}{n_s}(r_s-r_d) - \frac{1/2YT^2}{n_s(n_d+1)}\overline{e}(r_d-r_c)$$

$$- \frac{1/2YT^2}{n_s(n_d+1)n}\left(\int_{\overline{h}}^{1} r_d p(i)di + \int_0^{\overline{h}} r_c p(i)di - r_g\right)$$

$$- (a_s-a_d)n_s - a_{dc}n_s(n_d+1) - (\tau + A(\overline{h}) + B(\overline{h}))n_s(n_d+1)n$$

The optimal values of the choice variables are derived in two steps. First, for any given $\overline{h}$, the optimal values of n_s, n_d+1, and n are expressed as functions of $\overline{h}$ and the parameters of the model. These solutions are then substituted back into (A2'), resulting in an expression which depends on $\overline{h}$. The optimal value of $\overline{h}$ is then implied by the first order condition for profit maximization. The solutions for n_s, n_d+1, and n are given by equations (A3a)-(A3c).

$$\text{(A3a)} \quad n_s = T\left[\frac{Y(r_s-r_d)}{2(a_s-a_d)}\right]^{\frac{1}{2}}$$

$$\text{(A3b)} \quad n_d+1 = \left[\frac{\overline{e}(r_d-r_c)(a_s-a_d)}{a_d(r_s-r_d)}\right]^{\frac{1}{2}}$$

$$\text{(A3c)} \quad n = \left[\frac{((1-\overline{e})(r_d-r_g) + \overline{e}(r_c-r_g))a_d}{\overline{e}(\tau + A(\overline{h}) + B(\overline{h}))(r_d-r_c)}\right]^{\frac{1}{2}}$$

As can be seen from the above solutions, the number of trips to the bank is independent of $\bar{h}$ and therefore $\bar{e}$. This independence occurs because the optimal average savings balance only depends on the relative yields between savings and demand deposits and on the incremental cost of going to the bank over the cost of cashing a check. These considerations are removed from the optimal manner in which goods are purchased. The number of check cashings is sensitive to the manner in which goods are purchased, since the checking account is used to control average currency holdings. If a large portion of income is spent using cash, then it will be more worthwhile to economize on currency holdings by cashing more checks. Similarly the number of trips to the store given by $n_s(n_d+1)n = T\left[\frac{(1-\bar{e})Y(r_d-r_g) + \bar{e}Y(r_c-r_g)}{2(\tau + A(\bar{h}) + B(\bar{h}))}\right]^{\frac{1}{2}}$ depends on $\bar{h}$, but it is not immediately clear in which direction the dependence lies. It can be shown that both the numerator and denominator are decreasing functions of $\bar{h}$. Intuitively, since demand deposits earn a greater return than currency one would only buy a particular good, say the i^{th}, with currency if $a(i) < b(i)$. Therefore in the neighborhood of the optimal $\bar{h}$, as $\bar{h}$ increases $A(\bar{h})$ increases but by less than $B(\bar{h})$ decreases, so that the denominator falls. It is clear that the numerator is declining in $\bar{e}$ and therefore $\bar{h}$.

For a given $\bar{h}$, the transactions behavior of an individual with respect to interest rates and transactions costs is straightforward. The greater the relative yield on an asset, the higher will be the optimal holdings of that asset. This result is accomplished by making more trips to the asset. Conversely, a higher transactions cost of using an asset will imply fewer trips. For example, a rise in r_d (or a fall in a_d) implies more check cashings. Also, to further increase the holding of demand deposits more trips to the bank and the store will be made. With respect to goods, a rise in r_g or an increase in the transactions costs of purchasing goods will result in a rise in desired goods inventories. This is accomplished by purchasing goods in larger amounts and, therefore, by making fewer trips to the store.

The necessary conditions for an individual to adhere to the above inventory model are that $n_s \geq 2$, $n_d+1 \geq 1$, and $n \geq 1$. These imply the following relationships.

(A4a) $1/2YT^2(r_s-r_d) \geq 4(a_s-a_d)$

(A4b) $\overline{e}(r_d-r_c)(a_s-a_d) \geq a_d(r_s-r_d)$

(A4c) $[(1-\overline{e})(r_d-r_g) + \overline{e}(r_c-r_g)]a_d \geq \overline{e}(\tau + A(\overline{h}) + B(\overline{h}))(r_d-r_c)$

Equation (A4a) states that in order for one to use a savings account the interest rate earnings on savings relative to demand deposits must outweigh the increased transactions costs of making a trip to the bank relative to cashing a check. (A4b) and (A4c) have similar interpretations regarding the use of the checking account as a store of value and the holding of positive quantities of C and D respectively.

To examine the meaning of equations (A4a)-(A4b) more fully, a numerical example is presented. For a payments period of 2 weeks, a salary of \$1000 every 2 weeks, and $r_s = .05$ per year and $r_d = .02$ per year, (A4a) will only hold if $a_s - a_d \leq 14.5$ cents. It may therefore be true that many individuals use demand deposits as a store of value for transactions income. The model, however, would not be substantially changed. In this case $n_s = 1$ in (A1a)-(A1d), and the solution for $n_d+1 = T\left[\frac{\overline{e}Y(r_d-r_c)}{2a_d}\right]^{\frac{1}{2}}$, which is merely $n_s(n_d+1)$ in the model in the text. The solution for n is unchanged. Finally, if there doesn't exist a store of value, the optimal n equals the expressions for $n_s(n_d+1)n$ in the text. The only consequence for the comparative statics would be that more parameters would have no direct effect on the optimal value of $\overline{h}$.

Substituting (A3a)-(A3c) into (A1a)-(A1d) yields expressions for average asset holdings in terms of $\bar{h}$. These are given in (A1a')-(A1d').

$$\text{(A1a')}\quad \bar{S}(\bar{h}) = 1/2\,YT\left\{1 - \frac{1}{T}\left[\frac{2(a_s-a_d)}{Y(r_s-r_d)}\right]^{\frac{1}{2}}\right\}$$

$$\text{(A1b')}\quad \bar{D}(\bar{h}) = 1/2\,Y\left\{\left[\frac{2(a_s-a_d)}{Y(r_s-r_d)}\right]^{\frac{1}{2}} - \bar{e}\left[\frac{2a_d}{\bar{e}Y(r_d-r_c)}\right]^{\frac{1}{2}} - (1-\bar{e})\left[\frac{2(\tau + A(\bar{h}) + B(\bar{h}))}{Y((1-\bar{e})(r_d-r_g) + \bar{e}(r_c-r_g))}\right]^{\frac{1}{2}}\right\}$$

$$\text{(A1c')}\quad \bar{C}(\bar{h}) = 1/2\,Y\left\{\bar{e}\left[\frac{2a_d}{\bar{e}Y(r_d-r_c)}\right]^{\frac{1}{2}} - \bar{e}\left[\frac{2(\tau + A(\bar{h}) + B(\bar{h}))}{Y((1-\bar{e})(r_d-r_g) + \bar{e}(r_c-r_g))}\right]^{\frac{1}{2}}\right\}$$

$$\text{(A1d')}\quad \bar{G}(\bar{h}) = 1/2\,Y\left[\frac{2(\tau + A(\bar{h}) + B(\bar{h}))}{Y((1-\bar{e})(r_d-r_g) + \bar{e}(r_c-r_g))}\right]^{\frac{1}{2}}$$

Also, substituting (A3a)-(A3c) into (A2') gives an expression for profits which depends on $\bar{h}$ and the exogenous variables of the model.

$$\text{(A5)}\quad \pi(\bar{h}) = 1/2YT^2r_s - T[2Y(r_s-r_d)(a_s-a_d)]^{\frac{1}{2}} - T[2Y\bar{e}(r_d-r_c)a_d]^{\frac{1}{2}}$$
$$- T[(2Y(1-\bar{e})(r_d-r_g) + 2Y\bar{e}(r_c-r_g))(\tau + A(\bar{h}) + B(\bar{h}))]^{\frac{1}{2}}$$

This expression is then maximized with respect to $\bar{h}$, and the resulting first order necessary conditions yield an implicit function $\bar{h}$. This is given by equation (A6)

$$\text{(A6)}\quad -1/2T[2Y\bar{e}(r_d-r_c)a_d]^{\frac{1}{2}}\,\frac{p(\bar{h})}{\bar{e}} + 1/2T\left[\frac{2Y(\tau + A(\bar{h}) + B(\bar{h}))}{(1-\bar{e})(r_d-r_g) + \bar{e}(r_c-r_g)}\right]^{\frac{1}{2}}(r_d-r_c)p(\bar{h})$$

$$- 1/2T\left[\frac{2(1-\bar{e})Y(r_d-r_g) + 2\bar{e}Y(r_c-r_g)}{\tau + A(\bar{h}) + B(\bar{h})}\right]^{\frac{1}{2}}(a(\bar{h}) - b(\bar{h})) = 0$$

More intuitively (A6) can be expressed as,

$$\text{(A7)}\quad T\left(\frac{\partial\overline{D}}{\partial\overline{h}}\right)r_d + T\left(\frac{\partial\overline{C}}{\partial\overline{h}}\right)r_c + T\left(\frac{\partial\overline{G}}{\partial\overline{h}}\right)r_g - \frac{\partial(n_s(n_d+1))}{\partial\overline{h}}a_d$$

$$- \frac{\partial(n_s(n_d+1)n)}{\partial(\overline{h})}(\tau + A(\overline{h}) + B(\overline{h})) - n_s(n_d+1)n(a(\overline{h}) - b(\overline{h})) = 0$$

Equation (A7) states that the change in the interest rate earnings and transactions cost is exactly balanced at the optimal $\overline{h}$.

Using condition (A4c) it can be shown that the sum of the first two terms in (A6) is negative. This implies that $a(\overline{h}^*) < b(\overline{h}^*)$ for profit maximization, where $\overline{h}^*$ denotes the optimal value of $\overline{h}$. Also, for the second order conditions to hold $\left(\frac{\partial^2\pi}{\partial\overline{h}^2} \leq 0\right)$, it is necessary for the derivative of $b(\overline{h}^*)$, $b'(\overline{h}^*)$ to be sufficiently less than $a'(\overline{h}^*)$ (see Appendix B).

Comparative Statics

Equation (A6) will now be used to examine how changes in transactions costs, income, and interest rates affect $\overline{h}^*$ and the expenditure ratio, $\int_0^{\overline{h}^*} p(i)di$. Since the relationship between $\overline{h}$ and the expenditure ratio is monotonic, the signs of the effects will be the same. Therefore, only changes in $\overline{h}^*$ are analyzed. It is assumed that the conditions of the implicit function theorem are met so that a function $\overline{h} = f(x)$, where x is a vector of parameters, exists in the neighborhood of $\overline{h}^*$. The derivative of $\overline{h}$ with respect to any particular parameter, $\frac{\partial\overline{h}}{\partial x_i}$, will be of the same sign as $\frac{\partial}{\partial x_i}\left(\frac{\partial\pi}{\partial\overline{h}}\right)$ evaluated at $\overline{h}^*$. Therefore, the latter expression is all that is derived.

First it is easy to see that changes in a_s, r_s, and Y do not affect the optimal method of purchase. However, this is only true to the extent that the consumption bundle remains unchanged, or more explicitly that the distribution of p(i) is invariant to movements in Y, a_s, and r_s. While this may be a reasonable assumption regarding r_s, it is unlikely to be true for changes in transactions income.

The qualitative effect of a change in the cost of cashing a check is given by equation (A8).

$$\text{(A8)} \quad \frac{\partial}{\partial a_d}\left(\frac{\partial\pi}{\partial\bar{h}}\right) = -(T/2)(Y/2)^{\frac{1}{2}}\left[\frac{r_d-r_c}{a_d\bar{e}^*}\right]^{\frac{1}{2}} p(\bar{h}^*) \leq 0$$

Therefore, a decrease in the cost of cashing a check implies that more goods will be bought with currency.

The comparative static effects for changes in the various interest rates cannot unambiguously be signed, since they depend on the relative values of interest rates and transactions costs. For instance, equation (A9) gives the results for a change in r_g (inflation).

$$\text{(A9)} \quad \frac{\partial}{\partial r_g}\ \frac{\partial\pi}{\partial\bar{h}^*} = \frac{T}{2}\left(\frac{Y}{2}\right)^{\frac{1}{2}}\left\{\frac{TC^{\frac{1}{2}}}{I^{\frac{3}{2}}}(r_d-r_c)p(\bar{h}^*) - \frac{1}{TC^{\frac{1}{2}}I^{\frac{1}{2}}}(b(\bar{h}^*) - a(\bar{h}^*))\right\}$$

where for notational simplicity $TC = \tau + A(\bar{h}^*) + B(\bar{h}^*)$ and $I = (1-\bar{e}^*)(r_d-r_g) + \bar{e}^*(r_c-r_g)$. This expression is positive if $p(\bar{h}^*)(r_d-r_c) > (b(\bar{h}^*) - a(\bar{h}^*))\ \frac{I}{TC}$. Using the expression for $n_s(n_d+1)n$ yields equation (3) in the text.

Regarding a change in the own rate on currency, there are two effects which support each other. This can be observed by examining (A10).

$$\text{(A10)} \quad \frac{\partial}{\partial r_c}\left(\frac{\partial\pi}{\partial\bar{h}^*}\right) = T\left(\frac{Y}{2}\right)^{\frac{1}{2}}\left\{\frac{1}{2}\left[\frac{a_d}{\bar{e}^*(r_d-r_c)}\right]^{\frac{1}{2}} p(\bar{h}^*) - \frac{TC^{\frac{1}{2}}}{I^{\frac{1}{2}}}p(\bar{h}^*)\right.$$

$$\left. - \frac{1}{2}\frac{TC^{\frac{1}{2}}}{I^{\frac{3}{2}}}\bar{e}^*p(\bar{h}^*)(r_d-r_c) + \frac{1}{2}\frac{1}{TC^{\frac{1}{2}}I^{\frac{1}{2}}}\bar{e}^*(b(\bar{h}^*) - a(\bar{h}^*))\right\}$$

Using the first order condition (A6), to substitute for the first term in brackets, the sign of equation (A10) can be reduced to the sign of the following expression.

$$\left[\frac{I}{TC}(b(\overline{h}^*) - a(\overline{h}^*)) - p(\overline{h}^*)(r_d - r_c)\right]\left[1 + \frac{\overline{e}(r_d - r_c)}{I}\right]$$

An increase in r_c causes more trips for goods to be made. If currency is relatively cheap at the margin to transact with when compared to its weighted opportunity cost, individuals will use more currency.

The response of $\overline{h}^*$ to a change in r_d also reflects two avenues. On the one hand a change in r_d should have an opposite effect to that of r_g, while on the other it should have an opposite effect with respect to r_c, since D and C are substitutes. It can be shown that the substitution effect between D and C dominates and that $\overline{h}^*$ rises with r_d when

$$p(\overline{h}^*)(r_d - r_c) > (b(\overline{h}^*) - a(\overline{h}^*))\ \frac{I}{TC}\ .$$

The outcome of a change in the fixed cost of going to the store, τ, is similar to the results encountered for changes in various interest rates. Specifically an increase in τ will cause $\overline{h}^*$ to rise if $p(\overline{h}^*)(r_d - r_c) > (b(\overline{h}^*) - a(\overline{h}^*))\frac{I}{TC}$. This is the same condition encountered for a change in r_g.

Regarding $\overline{C}(\overline{h})$, comparative static effects will involve a number of considerations. First, there will be direct effects on currency independent of changes in transactions patterns. Secondly, since transactions patterns change when exogenous variables change, there will be an additional effect on currency balances. The change in currency holdings due to a change in transactions patterns will depend on whether more or less goods are bought with currency and whether currency balances are increasing or decreasing around the optimal transactions pattern. The shape of the currency function $C(\bar{h})$ can be ascertained by examining $\partial C(\bar{h})/\partial\bar{h}$. This is given by

$$\text{(A11)}\quad \frac{\partial C(\bar{h})}{\partial\bar{h}} = \frac{p(\bar{h})}{2\bar{e}}\,C(\bar{h}) - 1/2\left(\frac{Y\cdot I}{2\cdot TC}\right)^{\frac{1}{2}}\left[\frac{TC\;p(\bar{h}) + \bar{e}(a(\bar{h}) - b(\bar{h}))}{I} - \frac{\bar{e}\;TC\;p(\bar{h})(r_d - r_c)}{I^2}\right]$$

It is easy to see that at $\bar{h} = 0$ this derivative is positive. Also, using the F.O.C. from (A6), the derivative evaluated at $\bar{h}*$ can be expressed as

$$\text{(A12)}\quad \frac{\partial C(\bar{h}*)}{\partial\bar{h}} = 1/2\,\frac{r_d - r_g}{I}\left(\frac{Y\cdot I}{2\cdot TC}\right)^{\frac{1}{2}}\left[\frac{b(\bar{h}*) - a(\bar{h}*)}{r_d - r_c} - \frac{TC\;p(\bar{h}*)}{I}\right]$$

which is positive if $p(\bar{h}*)(r_d - r_c) < \frac{I}{TC}(b(\bar{h}*) - a(\bar{h}*))$ or if

$(1/T)(Y/2)^{\frac{1}{2}}p(\bar{h}*)(r_d - r_c) < (b(\bar{h}*) - a(\bar{h}*))n_s(n_d+1)n.$

This turns out to be the empirically relevant case in the text and therefore in the neighborhood of the optimal expenditure ratio an increase in the proportion of goods purchased with currency will cause average currency balances to rise.

The comparative static effects caused by changes in the exogenous variables on $C(\bar{h}*)$ are $dC(\bar{h}*)/dx = \frac{\partial C(\bar{h}*)}{\partial x} + \frac{\partial\bar{C}(\bar{h}*)}{\partial\bar{h}*}\frac{\partial\bar{h}*}{\partial x}$. It is easy to show that $\partial\bar{C}(\bar{h}*)/\partial r_c > 0$, $\partial C(\bar{h}*)/\partial r_d < 0$, $\partial C(\bar{h}*)/\partial r_g < 0$, $\partial\bar{C}(\bar{h}*)/\partial a_d > 0$, and $\partial C(\bar{h}*)/\partial TC > 0$.

The empirical results on $\bar{e}$, which imply that column 2 of table 2 is relevant, also imply that all indirect effects reinforce the direct effects that changes in exogenous variables have on currency holdings.

APPENDIX B

Derivation of 2nd order condition.

Then 2nd order condition is expressed in equation (B1), where a prime over a variable indicates its derivative with respect to $\bar{h}$. For notational simplicity, the symbol $TC = t + A(\bar{h}) + B(\bar{h})$, and the symbol $I = (1-\bar{p})(r_d - r_g) + \bar{p}(r_c - r_g)$.

$$\text{(B1)} \quad \frac{\partial^2 \pi}{\partial \bar{h}^2} = - T\left(\frac{Y}{2}\right)^{\frac{1}{2}} \Bigg\{ p'(\bar{h}) \left[\frac{(r_d - r_c)a_d}{\bar{p}}\right]^{\frac{1}{2}} - 1/2\bar{e}^{-\frac{3}{2}} \; p(\bar{h})^2 [a_d(r_d - r_c)]^{\frac{1}{2}}$$

$$+ 1/2 \frac{p(\bar{h})(r_d - r_c)(b(\bar{h}) - a(\bar{h}))}{(I^{\frac{1}{2}})(TC)^{\frac{1}{2}}} - 1/2 \frac{(TC)^{\frac{1}{2}} p(\bar{h})(r_d - r_c)}{(I)^{\frac{3}{2}}}$$

$$- p'(\bar{h})(r_d - r_c)\left[\frac{TC}{I}\right]^{\frac{1}{2}} + 1/2 \frac{p(\bar{h})(r_d - r_c)(b(\bar{h}) - a(\bar{h}))}{(I)^{\frac{1}{2}}(TC)^{\frac{1}{2}}}$$

$$- 1/2 \frac{(I)^{\frac{1}{2}}}{(TC)^{\frac{3}{2}}} (b(\bar{h}) - a(\bar{h}))^2 - \left[\frac{I}{TC}\right]^{\frac{1}{2}} (b'(\bar{h}) - a'(\bar{h})) \Bigg\}$$

(B1) will be less than zero if the expression in brackets is greater than zero. Using the first order condition to eliminate the first term in brackets, and performing the necessary algebra reduces the bracketed term to the following expression.

$$-1/2\bar{e}^{-\frac{3}{2}}p(\bar{h})^2[a_d(r_d-r_c)]^{\frac{1}{2}} + \frac{p'(\bar{h})}{p(\bar{h})}\left[\frac{I}{TC}\right]^{\frac{1}{2}}\left[(b(\bar{h}) - a(\bar{h})) - \frac{p(\bar{h})}{p'(\bar{h})}(b'(\bar{h}) - a'(\bar{h}))\right]$$

$$+ 1/2\left[\frac{b(\bar{h}) - a(\bar{h})I^{\frac{1}{2}}}{TC} - \frac{p(\bar{h})(r_d-r_c)}{I^{\frac{1}{2}}}\right]\left[\frac{p(\bar{h})(r_d-r_c)TC^{\frac{1}{2}}}{I} - \frac{b(\bar{h}) - a(\bar{h})}{TC^{\frac{1}{2}}}\right]$$

The last term is composed of two terms of different sign and is therefore negative as is the first term. For profit maximization the second term must be sufficiently positive, or $b(\bar{h}) - a(\bar{h}) - \frac{p(\bar{h})}{p'(\bar{h})}(b'(\bar{h}) - a'(\bar{h}))$ must be sufficiently positive. This reduces to $(b(\bar{h}) - a(\bar{h}))\frac{p'(\bar{h})}{p(\bar{h})} > b'(\bar{h}) - a'(\bar{h})$, which is certainly true if $b'(\bar{h}) < a'(\bar{h})$. If goods are of the same weight around the optimal $\bar{h}^*$, then $b'(\bar{h}^*) < a'(\bar{h}^*)$ is required.