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Aggregate Fluctuations and Economic Growth: A Case of Random–Walk Hypothesis

Ching–Sheng Mao *

Federal Reserve Bank of Richmond

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ABSTRACT

This paper presents a model economy in which the "balanced" growth is determined endogenously. The growth process in this economy does not depend on exogenous specifications such as human capital accumulation or technological progress. Rather, it is determined within the model and governed by two economic forces: (1) the intertemporal substitution of consumption and labor and (2) the intertemporal production opportunities. In equilibrium, the real quantities (i.e., consumption, capital, employment and output) will all evolve as logarithm random walks with drift. Therefore, the time series generated by this model is not trend stationary and the propagation of technological disturbances is permanent. This result is consistent with the empirical findings of Nelson and Plosser (1982).

AGGREGATE FLUCTUATIONS AND ECONOMIC GROWTH: A CASE OF RANDOM–WALK HYPOTHESIS

1. INTRODUCTION

That the study of aggregate fluctuations can be conceptually separated from the theory of economic growth seems to be the implicit view in the modern theory of business cycles. Based on this view, models of economic fluctuations have been examined primarily in a context where growth is either assumed away or triggered by exogenous considerations such as population growth and technological progress. Consistent with this view is the empirical strategy in which economic time series were often subject to a variety of detrending schemes in order to remove the growth component. To a large extent, this view may be attributed to the empirical observation that many economic aggregates like consumption and real GNP tend to wander regularly along a steadily growing path over a long span of time. This empirical regularity has often been taken to suggest that the economic cycles can be explained without reference to the growth of the economy which, presumably, has little to do with short–run and transitory perturbations. Because of its conceptual appeal and empirical convenience, this view has received wide acceptance in the literature.

By contrast, the approach taken in this paper is that the nature of aggregate fluctuations cannot be abstracted from the process of economic growth, a point emphasized in the recent works of Romer (1985) and King and Robelo (1986). It is my contention that business cycles can be better understood when growth dynamics are taken into account. In a sense, these contrasting views reflect the longstanding debate among economists about the proper ways to describe economic processes. For instance, can aggregate economic data, say real output, be best described by a stationary stochastic process? The fact that many economic aggregates appear to be "explained" fairly well using high–order stationary autoregressions does not imply that they are indeed generated by such processes. Nelson and Plosser (1982), for example, could not reject the hypothesis that many U.S. aggregate time series data are characterized by logarithmic random walks (more generally, nonstationary integrated processes). Certainly, this issue cannot be settled on a purely statistical and empirical basis.

A somewhat confusing element that has contributed to the above debate is the stochastic theory of economic growth. In a well–known result, existing growth models assert that per capita quantities, including output, capital and consumption, possess stationary distributions under reasonable assumptions (see Brock and Mirman (1972) for an example). This result implies that technological shocks are transitory in nature and real quantities can therefore be practically described by stationary processes. However, the stationarity of per capita quantities does not necessarily carry over to the aggregates. To see this, one must recognize that most existing growth models assume, implicitly or explicitly, an exogenous process for fixed factors of production such as labor. Depending upon how these driving processes are specified, stationary as well as nonstationary time series can be generated.

The difficulty in endogenously determining economic growth is related to the well–known dilemma of indeterminacy of firm size when a constant returns to scale technology is specified. That is, any factor allocation fulfilling the firm's zero profit and marginal conditions will be equally acceptable from the firm's point of view. Therefore, aggregate dynamics are usually generated by artificially specifying a stochastic process for some set of scale factors. As shown in this paper, however, constant returns to scale need not present this problem when in a general equilibrium framework consumers' labor supply decisions can be used by firms to pin down their sizes. Based on this observation, a model economy can be constructed in which labor supply (or population) is endogenously determined rather than exogenously specified. Firms then use this information in formulating their optimal production plan. The process of sustained growth determined through this mechanism does not originate from the increasing returns of technology as in Romer (1985), nor is it related to human capital technology as in King and Robelo (1986). Instead, it is a self–induced process governed by two economic forces: (1) the intertemporal substitution of consumption and labor and (2) the intertemporal production opportunities.

The novelty of this model is that diminishing returns to factors of production do not constrain the growth of the economy as they do in conventional models. The reason for this is the absence of fixed factors in the technology. As a result, economic agents are fully capable of transferring resources across time. This transfer permits greater propagation in technological disturbances. In fact, with identically and independently distributed (i.i.d.) productivity shocks and homogeneous preferences, the model predicts that the aggregate capital stock, output, consumption, and labor employment will all evolve as logarithmic random walks with drift. Temporary real shocks that occur at a particular point in time will be propagated permanently into the future and shift the entire equilibrium path. Such processes are not trend stationary and clearly do not possess stationary distributions.

This result is quite similar to that obtained by King and Robelo (1986), but the growth mechanisms in our model do not depend on an explicit exogenous technology for producing human capital. Rather, the mechanisms are endogenously governed by the consumers' willingness and ability to smooth out their consumption over time. The model has important implications for macroeconometrics. In particular, because of the nonstationarity in levels, we have to search for transformation schemes in order to apply stationary time series methods. Our results suggest the use of first differencing as a way of achieving stationarity, a longstanding practice that has rarely been justified on the basis of coherent economic theory. It is noteworthy that the random walk process in our model identifies the specific form of nonstationarity used in the test of Nelson and Plosser (1982).

The rest of the paper is organized as follows. In section 2, I construct a model economy where population growth is endogenously determined. The model incorporates important elements of both real business cycles and stochastic growth models. Section 3 discusses the determination of the equilibrium. A solution for the optimal decision rules and value function is presented in section 4. The quantity dynamics are analyzed in section 5. The paper is concluded in section 6.

2. THE MODEL

Consider an economy populated by a large number of identical households, whose preferences depend positively on the amount of goods consumed and negatively on the labor participation of the family. The preferences of the stand–in representative household are characterized by the utility function $u(c_t, n_t)$, where c_t and n_t denote the household's consumption and labor participation at time t, respectively. The labor participation n_t in this model measures the population or the labor force of the representative family. One may imagine that at the beginning of each period, the head of the household decides how many children he is willing to bear which will then engage in the economy's production process. Under this interpretation, the production of children generates a negative service to the household's utility because it represents the amount of labor supply.[1] For simplicity, we abstract from the problem of family formation and assume the child bearing cost is negligible.

For analytical tractability, we confine ourselves to the case where the utility function $u(c_t, n_t)$ is homogeneous of degree γ, where γ is a constant between 0 and 1. Note that the parameter γ can also be used to measure the intertemporal substitution elasticity of consumption and labor, which is equal to $1/(1-\gamma)$. In addition, we assume

(a) $u_1 > 0$, $u_2 < 0$, $u_{11} < 0$, $u_{22} < 0$;
(b) The "preferred set" $\{(c_t,n_t) | \ u(c_t,n_t) \geq \bar{u}\}$ is strictly convex;
(c) For a given utility level, $\lim_{n_t \to 0} [-u_2/u_1] = 0$ and $\lim_{c_t \to \infty} [-u_2/u_1] = \infty$.

These assumptions are standard in economic analysis. Specifically, (a) requires the diminishing marginal utility of consumption and increasing penalty of labor supply. Assumption (b) implies that the indifference curve is upward sloping and strictly convex toward the axis of n_t. A necessary and sufficient condition for this to hold is

$$2u_1u_2u_{12} - u_1^2u_{22} - u_2^2u_{11} > 0$$

Assumption (c) requires that the slope of the indifference curve be 0 as $n_t \to 0$ and infinite as $c_t \to \infty$. This assumption is sufficient for ruling out the corner solutions.

Now, we turn to the the production side of the economy. At the beginning of time t, the representative household is equipped with pre–determined capital stock k_{t-1} and labor supply n_{t-1}. He observes a technological shock λ_t and produces y_t units of goods according to the following constant returns to scale technology:

$$y_t = \lambda_t F(k_{t-1}, n_{t-1})$$

The technological shock λ_t is assumed to be a positive i.i.d. random variable drawn from a known probability distribution. The production technology specified above is identical to that in Long and Plosser (1983) and Mao (1986). Because of the assumed lag structure in the technology, the economy behaves like a stochastic endowment economy.

The economic problem of the representative household is to achieve maximal lifetime utility by allocating the stochastic output y_t, which is given at the beginning of each period, between consumption and savings (capital) over time, and by choosing whatever labor supply (population) that will bring about this optimal plan. For simplicity, we assume the capital stock is fully depreciated so that k_t can be regarded as investment at time t. This assumption also serves to insure that any persistence of the equilibrium quantities does not derive from the storage technology of the capital stock. The commodity feasibility constraint is therefore

$$c_t + k_t \leq \lambda_t F(k_{t-1}, n_{t-1}) \equiv y_t$$

With an immortal family, the optimization problem can be formally stated as follows:

$$\max_{\{c_t,\ k_t,\ n_t\}} E_0\left[\sum_{t=0}^{\infty} \beta^t\, u(c_t, n_t)\right] \qquad 0 < \beta < 1$$

subject to

$$c_t + k_t \leq \lambda_t F(k_{t-1}, n_{t-1}) \equiv y_t, \quad \text{for all t}$$

$$\text{(1)} \qquad c_t \geq 0,\ k_t \geq 0,\ n_t \geq 0, \quad \text{for all t}$$

$$c_0 + k_0 \leq y_0$$

The initial stock y_0 is given

where E_0 is the expectations operator conditional on information available at time 0, and β is the time preference discount factor. The model is closed by requiring that the household's expectations be rational.

The model as it is stated is a typical real business cycle model in that the intertemporal production technology plays an important role in generating quantities dynamics. Instead of treating labor as hours worked as in previous models (e.g., Long and Plosser (1983) and Kydland and Prescott (1982)), however, labor supply in this model is a measure of "bodies" that corresponds to the usual employment statistics. Since the household can fully adjust its labor supply at any point in time, production opportunities will not be constrained by a fixed supply of factors, such as time endowment, which will limit the degree of intertemporal substitution in production. Because of this, the economic agent has greater flexibility to smooth his consumption over time in response to unexpected random shocks. This flexibility permits a greater propagation of technological disturbances as opposed to previous business cycles models.

The model can also be regarded as one example of a stochastic growth model. However, conventional growth theory is mainly concerned with the process of capital accumulation and its relation to consumption smoothing. As mentioned before, the decision of labor supply is either indeterminate or implicitly assumed to be exogenous. From the viewpoint of business cycles, this is not a desirable feature because the variation of labor employment constitutes an indispensable part of economic fluctuations. By allowing endogenous determination of individual's labor supply, the model can display self–sustained growth even when technological progress does not persist in the economy. The way we approach this problem is different from King and Robelo (1986), in which endogenous growth depends on a process of human capital accumulation which is a specific form of technological progress.

In the tradition of the business–cycle literature, our major concern is with the stochastic properties of the time series generated by this model. Without solving the model explicitly, the next section proceeds to define an equilibrium for this economy and to discuss some of its properties.

3. EQUILIBRIUM

The central planning problem specified in the last section defines a sequence of Pareto allocations over time. Associated with this sequence is a set of supportive "shadow prices" that can be used to map the Pareto optima into competitive equilibria. The basic approach here is to interpret the optimal shadow prices as the competitive prices which consumers and producers used to solve their own optimization problems in decentralized markets. These prices will consist of not only spot prices, such as the wage rate, but also the interest rate that prevails in the contingent market for future consumption and output. Given these shadow prices, agents (consumers and firms) can be induced to behave in a way that conforms to the rules specified by Pareto optima. In other words, the optimal decision rules of the central planning model will duplicate a competitive equilibrium determined in markets (see Sargent (1980) and Prescott and Mehra (1983) for a formal argument). We will not spell out details of the market arrangements here.

In view of the above remarks, we study the equilibrium using the dynamic programming algorithm. Let $V(y_t, \lambda_t)$ be the value function associated with the problem (1), defined recursively by

$$(2) \qquad V(y_t, \lambda_t) = \max_{\{k_t, n_t\}} u(y_t - k_t, n_t) + \beta E_t\Big[V[\lambda_{t+1}F(k_t, n_t), \lambda_{t+1}]\Big]$$

Note that V(.) is the maximal obtainable expected return at time t given the current realization of y_t and λ_t. An equilibrium is characterized by the decision rules, k_t^* and n_t^* (presumably, a function of y_t and λ_t), together with a bounded value function that solves the above optimality equation. Associated with this solution is a transient probability distribution for outputs that are generated by the optimal decision rules and the random shocks. One of our goals is to characterize such a probability distribution by extracting information from the above optimization problem.

The equilibrium prices for this economy are determined simultaneously with real quantities Specifically, assuming that the value function exists and is differentiable with respect to y_t, then the process $\{V_1(y_t,\lambda_t)\}$ is a sequence of spot output prices that will support the Pareto allocations. In our model, this supporting price is equal to the marginal utility of consumption evaluated at the optimum.[2] Given these prices, the expected real interest rate at time t, denoted r_t^e, is defined to be

$$r_t^e = \frac{V_1(y_t,\ \lambda_t)}{\beta\ E_t\,[V_1(y_{t+1},\lambda_{t+1})]} - 1$$

That is, $(1 + r_t^e)$ is the ratio of the present value prices for outputs at time t and t+1. It is the one period ahead ex ante (prior to the occurrence of λ_{t+1}) interest rate that will clear the futures market for output.

Now, assuming that the value function $V(y_t,\ \lambda_t)$ exists, then the solutions of the optimality equation (2) must satisfy the following first order conditions:

(3) $$u_1(y_t-k_t^*,\ n_t^*) = \beta\, E_t[V_1(y_{t+1}^*,\ \lambda_{t+1})\ \lambda_{t+1}]F_1(k_t^*,\ n_t^*)$$

(4) $$-u_2(y_t-k_t^*,\ n_t^*) = \beta\, E_t[V_1(y_{t+1}^*,\ \lambda_{t+1})\ \lambda_{t+1}]F_2(k_t^*,\ n_t^*)$$

where y_{t+1}^* is the optimal output at time t+1 contingent on the realization of λ_{t+1}. Equations (3) and (4) require that the marginal cost and benefit arising from the trade–off of consumption and labor over time be equal in equilibrium. In other words, a small deviation from the optimal plan should leave the life time utility intact. As usual, this optimal plan should also fulfill the following transversality condition:

$$\lim_{t \to \infty} \beta^t \, E_0\left[V_1(y_t^*, \lambda_t)\, y_t^*\right] = 0$$

The transversality condition says that the expected present value of output cannot be valued at infinity, which implies, as in growth models, that the capital stock and labor supply will not grow "too fast" in equilibrium. We assume this condition is fulfilled for the rest of the paper.

The difference equations (3) and (4) impose certain restrictions on the equilibrium paths of the capital stock and the labor supply. These restrictions, although not explicitly specified, reflect two basic choices facing individuals: (i) the substitution between goods and (the negative of) labor at a given point of time and (ii) the allocation of output and consumption over time. To see how the equilibrium is determined by these forces, divide (4) by (3) to get

$$(5) \qquad -u_2(y_t-k_t^*, n_t^*)/u_1(y_t-k_t^*, n_t^*) = F_2(k_t^*, n_t^*)/F_1(k_t^*, n_t^*)$$

Equation (5) is the efficiency condition that governs the allocation of goods and labor at a given point of time. It requires that the rates of contemporaneous substitution in preferences and production be equal in equilibrium. That is, the isoquant must be tangent to the indifference curve at each point in time. As shown in figure 1, this condition alone does not determine the equilibrium because there are a large number of candidate allocations on the tangent loci. To locate the equilibrium, one must take into account the allocation of goods over time.

The individual's intertemporal decision is illustrated in figure 2, in which the horizontal and vertical axes measure the output at time t and t+1, respectively. For the sake of exposition, let us assume for the moment that the labor supply is fixed at some given level, say, n_t^1. Then, according to the production technology, there is a transformation possibility between current and future consumption, conditional on n_t^1 and the realization of the random shock λ_{t+1}. Since λ_{t+1} is unknown at time t, the expected frontier $E(\lambda_{t+1})F(k_t, n_t^1)$ is employed as the relevant transformation scheme. The slope of this transformation curve (measured in absolute value) is the expected marginal product of capital and measures the feasible substitution of consumption over time. Given this production opportunity, the individual allocates a portion of current output into saving to maximize utility. Clearly, the optimum will be achieved at the point where the expected rate of return on saving (i.e., the marginal product of capital) equals the rate of intertemporal substitution in consumption. That is, the indifference curve will be tangent to the production frontier in equilibrium. This is the standard efficient condition that governs the allocation of goods over time. With labor supply being fixed at n_t^1, the point E_1 is the best outcome that can be obtained.

Now, suppose the individual is allowed to adjust his labor supply. Then, for each level of labor supply we have a different efficient allocation, which is represented by a point on the tangent loci AB. Moving along this tangent loci reflects the fact that the individual is now capable of altering the transformation possibilities through the choice of labor supply. To pin down the equilibrium, recall that the capital stock and labor supply must also satisfy equation (5), the contemporaneous efficiency condition. The allocations fulfilling this condition are represented by a curve CD which corresponds to the tangent loci shown in the figure 1. Clearly, the equilibrium will occur at the point E where AB and CD intersect. This is the point where both intertemporal and contemporaneous efficiency conditions are fulfilled at the same time.

The foregoing analysis illustrates the importance of the labor supply as an endogenous device in coordinating the use of resources. This feature is either absent or limited in existing models. For example, consider the classical growth model where the labor supply is inelastic and exogenously given by the size of population. Intuitively, the equilibrium of this economy corresponds to a particular point on the tangent loci AB, say E_1, figure 2. Since the economic agent can only move along a given production frontier, his ability to transfer resources over time is limited by the productivity of capital, which is diminishing by assumption. It is clear that, unless there is some kind of persistent technological progress or the population is growing to such an extent that it will offset the decreasing marginal product of capital, the economy cannot experience sustained growth.[3] This is the underlying reason why stationarity will prevail in the classical model. Now, with endogenous labor supply, the individual is no longer constrained to a fixed production schedule. In fact, by changing his labor supply decision, not only can he alter the rate of return on investment, but he can also manipulate intertemporal production opportunities. Notice that his ability to do so is not limited by feasible constraints such as a fixed time endowment. The extra degree of freedom leaves the household additional room for smoothing consumption and income over time. Therefore, one would expect that the equilibrium in this economy would display a stronger tendency for persistence. In fact, as will be shown later, the equilibrium quantities of this model possess an extreme form of smoothing property — logarithmic random walks with drift.

The next section explores the homogeneous structure of the underlying economy and provides a solution to the optimization problem. The method we use is motivated by Boyd (1985) and Mao (1986).

4. SOLUTION

Solving the optimization problem (1) with a homogeneous structure relies on a simple observation. As a specific example, consider the standard textbook problem where consumers maximize utility over a linear feasible set. If the utility function is homogeneous (or, more generally, homothetic), then the equilibrium will be invariant up to a scalar shift of the budget line. In other words, the income path is a straight line starting from the origin. The underlying reason for this, of course, is that the scalar transformation of the budget set does not alter the preference ordering, and the equilibrium can therefore be mapped to another under the same transformation.[4] This simple fact allows us to derive some useful properties of the value function.

To begin with, let us rewrite the optimality equation (2) as follows:

$$
\begin{aligned}
V(y_t, \lambda_t) = \ & \max \ u(c_t, n_t) + E_t\Big[\sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s, n_s)\Big] \\
\text{(2')} \qquad & \text{s.t.} \quad c_s + k_s = \lambda_s F(k_{s-1}, n_{s-1}) \equiv y_s, \quad s \geq t+1 \\
& \qquad\ \ c_t + k_t = y_t
\end{aligned}
$$

Note that the optimization problem above amounts to the problem (1) from the vantage point of time t. Let us assume that the sequence $\{c_s^*, k_s^*, n_s^*\}_t^{\infty}$ solves (2') with y_t as the initial stock. Then the maximum utility can be evaluated along this optimal time path as follows:

$$
V(y_t, \lambda_t) = u(y_t - k_t^*, n_t^*) + E_t\Big[\sum_{s=t+1}^{\infty} \beta^{s-t} u(y_s^* - k_s^*, n_s^*)\Big]
$$

Now, suppose the individual is endowed with a different initial income given by

τy_t, $\tau > 0$. For our purposes, it does not matter whether the variation of y_t is due to an unexpected shock at time t or a change in the production decision made at time t–1. Given this new initial stock, what is the new equilibrium? Consider the sequence $\{\tau k_s^*, \tau n_s^*\}_t^\infty$ which is a scalar transformation of the old equilibrium. We claim that this sequence is optimal with τy_t as the initial income. First, the path $\{\tau k_s^*, \tau n_s^*\}_t^\infty$ lies within (in fact, on the boundary of) the feasible set because the production technology exhibits constant returns to scale. Secondly, since the period's utility function is homogeneous of degree γ, we have

$$\begin{aligned} & u(\tau y_t^* - \tau k_t^*, \tau n_t^*) + E_t\Big[\sum_{s=t+1}^{\infty} \beta^{s-t} u(\tau y_s^* - \tau k_s^*, \tau n_s^*)\Big] \\ &= \tau^\gamma u(y_t - k_t^*, n_t^*) + \tau^\gamma E_t\Big[\sum_{s=t+1}^{\infty} \beta^{s-t} u(y_s^* - k_s^*, n_s^*)\Big] \\ &= \tau^\gamma V(y_t, \lambda_t) \end{aligned}$$

Clearly, if the sequence $\{k_s^*, n_s^*\}_t^\infty$ solves the problem for y_t and λ_t, then the transformation $\{\tau k_s^*, \tau n_s^*\}_t^\infty$ also solves the problem for τy_t and λ_t.[5] This implies that the value function obeys the following relationship:

$$\begin{aligned} V(\tau y_t, \lambda_t) &= u(\tau y_t^* - \tau k_t^*, \tau n_t^*) + E_t\Big[\sum_{s=t+1}^{\infty} \beta^{s-t} u(\tau y_s^* - \tau k_s^*, \tau n_s^*)\Big] \\ &= \tau^\gamma V(y_t, \lambda_t) \end{aligned}$$

Thus the value function is homogeneous of degree γ in y_t for a given realization of λ_t. Hypothetically, the value function takes the following form:

(6) $$V(y_t, \lambda_t) = A(\lambda_t) y_t^\gamma, \quad 0 < \gamma < 1$$

where $A(\lambda_t)$ is an unknown function, presumably nondecreasing in λ_t. Notice that the value function is strictly concave in the state variable y_t. Economically, this means that the househould is risk averse toward output fluctuations. The relative risk aversion coefficient of Arrow and Pratt is $1-\gamma$, the reciprocal of which is just the intertemporal substitution elasticity of consumption. The slope of the value function evaluated at y_t is the equilibrium spot price of the current output.

The homogeneity of the value function is a building block in our solution procedure. Specifically, it allows simplification of the Euler equations. Using (6), the Euler equation (3) can be rewritten as follows:

$$u_1(y_t-k_t^*, n_t^*) = \gamma\,\beta\,E\left[\lambda_{t+1}^{\gamma}A(\lambda_{t+1})\right]F(k_t^*, n_t^*)^{\gamma-1}F_1(k_t^*, n_t^*)$$

where use has been made of $y_{t+1}^* = \lambda_{t+1}F(k_t^*, n_t^*)$. Also, we have replaced the conditional expectation operator by an unconditional one because the process $\{\lambda_t\}$ is serially uncorrelated. By assumption, $u_1(.,.)$ and $F(.,.)$ are homogeneous of degree $\gamma-1$ and 1, respectively. We can divide both sides by $(n_t^*)^{\gamma-1}$ to obtain

$$u_1[(y_t-k_t^*)/n_t^*, 1] = \gamma\,\beta\,E\left[\lambda_{t+1}^{\gamma}A(\lambda_{t+1})\right]F(k_t^*/n_t^*, 1)^{\gamma-1}F_1(k_t^*/n_t^*, 1) \tag{7}$$

This equation involves only the consumption–labor ratio and capital–labor ratio. Now, because the rate of contemporaneous substitution in preferences and production can also be expressed in terms of a ratio, the tangency condition (5) can be written as

$$g[(y_t-k_t^*)/n_t^*] = f(k_t^*/n_t^*)$$

where $f = F_2/F_1$ and $g = -u_2/u_1$. This equation defines a one–to–one correspondence between the equilibrium capital–labor ratio and the consumption–labor ratio. The determination of either one will automatically determine the other. Assume that the function g is invertible, then the equilibrium consumption–labor ratio can be expressed in terms of the capital–labor ratio:

(8) $$(y_t - k_t^*)/n_t^* = g^{-1}f(k_t^*/n_t^*)$$

Substituting (8) into the left side of (7), we have

(9) $$u_1[g^{-1}f(k_t^*/n_t^*),1] = \gamma\,\beta\,E\left[\lambda_{t+1}^{\gamma}A(\lambda_{t+1})\right]F(k_t^*/n_t^*,1)^{\gamma-1}F_1(k_t^*/n_t^*,1)$$

Equation (9) summarizes the theoretical restrictions that must be placed on the equilibrium capital–labor ratio.[6] Given the expectations of the random shocks and the unknown function $A(\lambda_{t+1})$, the only possible solution for the equilibrium capital–labor ratio is a constant.[7] Thus, let

(10) $$k_t^*/n_t^* = a$$

where a is a constant dictated by the parameters of preferences, technology, and most importantly, the stochastic properties of the random shocks. From (8), the equilibrium consumption–labor ratio is also a constant:

$$(y_t - k_t^*)/n_t^* = g^{-1}f(a)$$

Rearranging and using (10), we obtain

$$(11) \qquad k_t^* = \left[\frac{a}{a + g^{-1}f(a)}\right] y_t \equiv \phi\, y_t, \quad 0 < \phi < 1$$

By definition,

$$(12) \qquad c_t^* = y_t - k_t^* = \left[\frac{g^{-1}f(a)}{a + g^{-1}f(a)}\right] y_t \equiv (1-\phi)\, y_t$$

Also,

$$(13) \qquad n_t^* = k_t^*/a = \left[\frac{1}{a + g^{-1}f(a)}\right] y_t = \frac{\phi}{a}\, y_t$$

Substituting (11)–(13) into the optimality equation (2'), one can determine the unknown function $A(\lambda_{t+1})$, which is a constant and equals [8]

$$A = \frac{u[g^{-1}f(a),1]/[a+g^{-1}f(a)]^{\gamma}}{1 - \beta\, E(\lambda_{t+1}^{\gamma})\ \mu^{\gamma}}, \quad \text{where} \quad \mu \equiv \frac{F(a,\ 1)}{a + g^{-1}f(a)}$$

The constant A is a meaningful quantity if and only if $[\beta\, E(\lambda_{t+1}^{\gamma})\mu^{\gamma}] < 1$. This is also a necessary and sufficient condition for the transversality condition to hold. Later we will show that the quantity μ dictates the growth rate of this economy.

Equations (11)–(13) are the equilibrium decision rules for investment, consumption and labor supply. The linearity of the decision rules implies that unexpected productivity shocks at each point of time will be distributed proportionately between consumption and savings (i.e., capital stock). The labor supply will also change proportionally in order to satisfy the efficiency conditions. Therefore, the equilibrium quantities will move together in response to random disturbances. In principle, such changes are governed by the substitution and wealth effects, but because of the general equilibrium nature of the model prices and wealth (or permanent income) cannot be held exogenous to uncover these effects. Some general observations, however, can be made.

The relationship between employment and output fluctuations is particularly interesting. Consider the situation of higher than expected current income. First, the individual tends to reduce his labor supply because he is wealthier. Secondly, because both current and future consumption are normal goods, the individual has an incentive to spread some portion of current income into the future. This is done by increasing the accumulation of capital stock. Given the initial labor supply, the future marginal products of capital and labor will then decrease and increase, respectively. Of course, this cannot be an equilibrium because it creates potential profits that can be exploited by increasing the labor supply. As long as the exploitable gains outweigh the wealth effect, the labor supply will continue to rise. In other words, the individual is always willing to substitute labor into periods where the net expected returns are higher. This mechanism differs from traditional growth models, in which the smoothing of consumption relies entirely on the adjustment of capital stock.

It is not surprising that the incentive for smoothing consumption over time also tends to smooth the equilibrium price ratios, such as the expected real interest rate. In fact, in the case of random walk consumption, the equilibrium rate of interest (i.e., r_t^e) remains constant over time. It should be mentioned, however, that the ex post interest rate, which has nothing to do with the savings decision, will fluctuate over time in response to random shocks.

5. DYNAMICS

The dynamic properties of the system can be studied in terms of the reduced forms of the decision rules. For convenience, we will omit the superscript "*" hereafter. Using the production technology and the decision rule for labor supply (13), equilibrium output can be derived as follows:

$$y_t = \lambda_t F(a, 1)\, n_{t-1} = \lambda_t \left[\frac{F(a,\ 1)}{a\ +\ g^{-1} f(a)}\right] y_{t-1} = \lambda_t\, \mu\, y_{t-1}$$

Similarly, from (11)–(13), the equilibrium capital stock, consumption and employment are

$$k_t = \phi\, y_t = \lambda_t \left[\frac{F(a,\ 1)}{a\ +\ g^{-1} f(a)}\right] a\, n_{t-1} = \lambda_t\, \mu\, k_{t-1}$$
$$c_t = y_t - k_t = \lambda_t\, \mu\, (y_{t-1} - k_{t-1}) = \lambda_t\, \mu\, c_{t-1}$$
$$n_t = k_t/a = \lambda_t \left[\frac{F(a,\ 1)}{a\ +\ g^{-1} f(a)}\right] n_{t-1} = \lambda_t\, \mu\, n_{t-1}$$

Taking natural logarithms, we have

(14) $\log y_t = \log \mu + \log y_{t-1} + \log \lambda_t$

(15) $\log k_t = \log \mu + \log k_{t-1} + \log \lambda_t$

(16) $\log c_t = \log \mu + \log c_{t-1} + \log \lambda_t$

(17) $\log n_t = \log \mu + \log n_{t-1} + \log \lambda_t$

Equations (14)–(17) are the reduced forms of the dynamic system. Since the random shock $\log \lambda_t$ is i.i.d., the equilibrium quantities, in logs, will all evolve as random walks with a constant drift $\log \mu$.[9]

One of the most important implications of random walks is that temporary real disturbances will be propagated permanently into the future and shift the equilibrium time path once and for all. The time series generated by such processes will be unpredictably fluctuating along a constant mean path, a feature that appears to agree with a casual observation of the real economy and is consistent with the findings of Nelson and Plosser (1982) that many U.S. economic aggregates, including real GNP, consumption, capital, and employment may be generated by autoregressive processes with an unit root.[10]

The drifted random walks (14)–(17) imply that the economy is characterized by balanced growth in which all endogenous variables grow at the same rate. In fact, with an i.i.d. random shock, the equilibrium growth rate is a white noise. Rewrite (14) as follows:

$$\Delta \log y_t = \log \mu + \log \lambda_t$$

where Δ is the first difference operator. Clearly, the growth rate of this economy, $\Delta \log y_t$, consists of two components: (1) the expected or "natural" growth rate, $\log \mu$, governed by the deep parameters of preferences and technology and the probability distribution of the random shock, and (2) the stochastic part, $\log \lambda_t$, driven by exogenous disturbances. The equilibrium growth rate, therefore, is determined endogenously by economic agents, which is different from conventional growth models where balanced growth is exogenously given by population growth. Further, the endogenous growth is not related to an exogenous technology for producing human capital, which is essential for the models of King and Robelo (1986) and Hercowitz and Sampson (1986).[11]

The random walk as a data generating process contrasts with other real business cycles models. In Long and Plosser (1983), for example, the output process can be characterized as a stationary autoregression of first order, for which the propagation of shocks decays with the share of physical capital in output (see King and Plosser (1986)). Since the share is empirically small (about one third in U.S.), the internal mechanisms of the Long and Plosser model — consumption smoothing and capital accumulation — do not produce quantitatively important serial correlation in economic time series. In this regard, the random walk with a constant drift may be interpreted as an extreme form of serial correlation in the levels of economic aggregates. However, it is questionable to measure serial correlation for time series generated by nonstationary processes because they do not possess a stable autocorrelation function. In our case, however, stationarity can be achieved simply by first differencing the time series and considering the growth rate, which is a white noise.

For the last century, capitalistic economies have been growing at a somewhat constant pace. In the case of the U.S. economy, for example, the annual growth rates of real GNP have been 3 percent on average, which is consistent with the prediction of our model. However, the growth of many aggregate time series (notably, output, employment, and consumption) also displays a strong cyclical pattern over the course of economic fluctuations, which our model fails to capture. The reason for this is due to the nature of technological shocks assumed in the model. It is not difficult to see that, if random shocks follow a Markov process, then equilibrium quantities may be characterized as an integrated process, in which case the variation in growth rates will be serially correlated. The economic reason for this is because a particular realization of technological shocks that are serially correlated will alter the curvature of the future production schedules, and hence the intertemporal efficiency condition. Despite its ad hoc nature, this device have been widely used in real business cycles models to enhance persistence in economic time series (e.g., Kydland and Prescott (1982)). The above discussion suggests that the presence of utility impulses may also introduce additional persistence into the system through its effect on the intertemporal substitution of consumption. This factor is often ignored in the literature. Whether or not it is quantitatively important is left for further research.

6. CONCLUSION

The main objective of this paper has been to provide a theoretical ground on which economic time series should be interpreted. We start by asking whether the observed macroeconomic data can be modeled as a nonstationary process, a hypothesis that has important implications for the study of business cycles. To empirical economists, this view has been accepted implicitly for a long time, but rarely justified on a theoretical basis. Part of the problems may be attributed to the difficulty of identifying different sources of the nonstationarity embedded in the economic data. This paper provides an explicit example in which the economic system itself is capable of generating nonstationary aggregate data even in the absence of nonstationary shocks. In this economy, the nonstationarity cannot be removed by detrending the time series as it is usually done in empirical studies. Instead, first differencing is a proper way of achieving stationarity.

The model analyzed in this paper may be used as a benchmark to study the implications of fiscal policy. For example, the model may incorporate the government as the third agent who claims a certain amount of resources (i.e., lump–sum taxes) to meet its expenditure needs. In this case, the variation of production shocks may be interpreted as a change in the expenditure or tax policy. Since shocks have permanent effects in our model, we expect that a change in the tax or expenditure policy will shift the time path of the equilibrium quantities. In the long run, however, this should not affect the "natural" growth rate at the steady state unless the government also changes the stochastic properties of its policy such as the mean and variance of its expenditure.

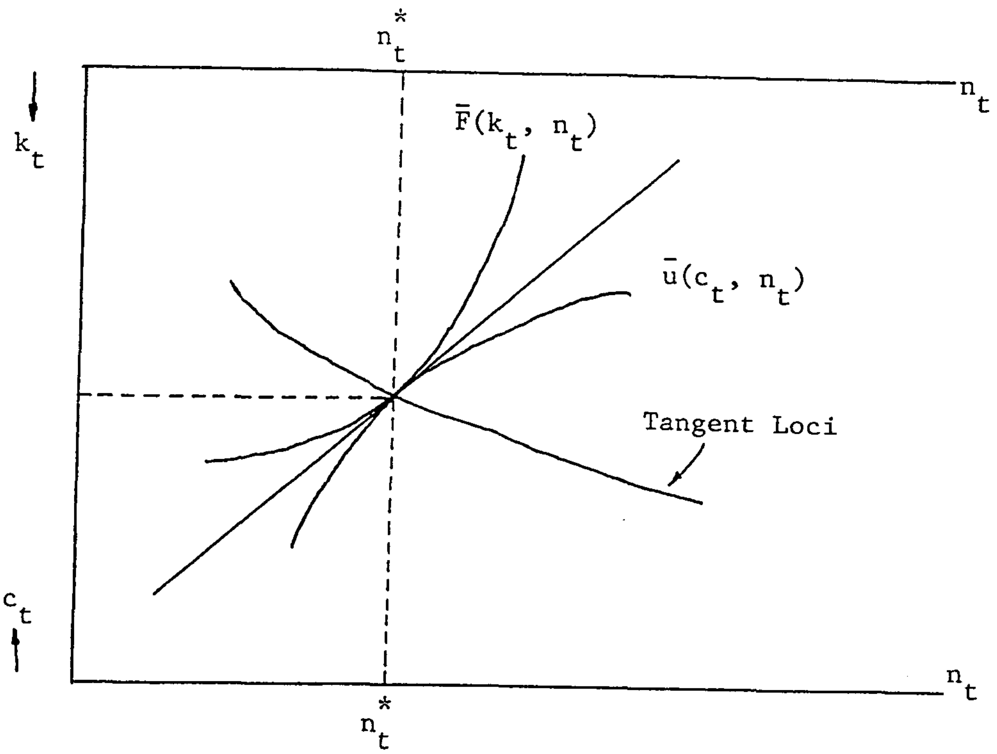

Figure 1 — Contemporaneous Efficiency Condition

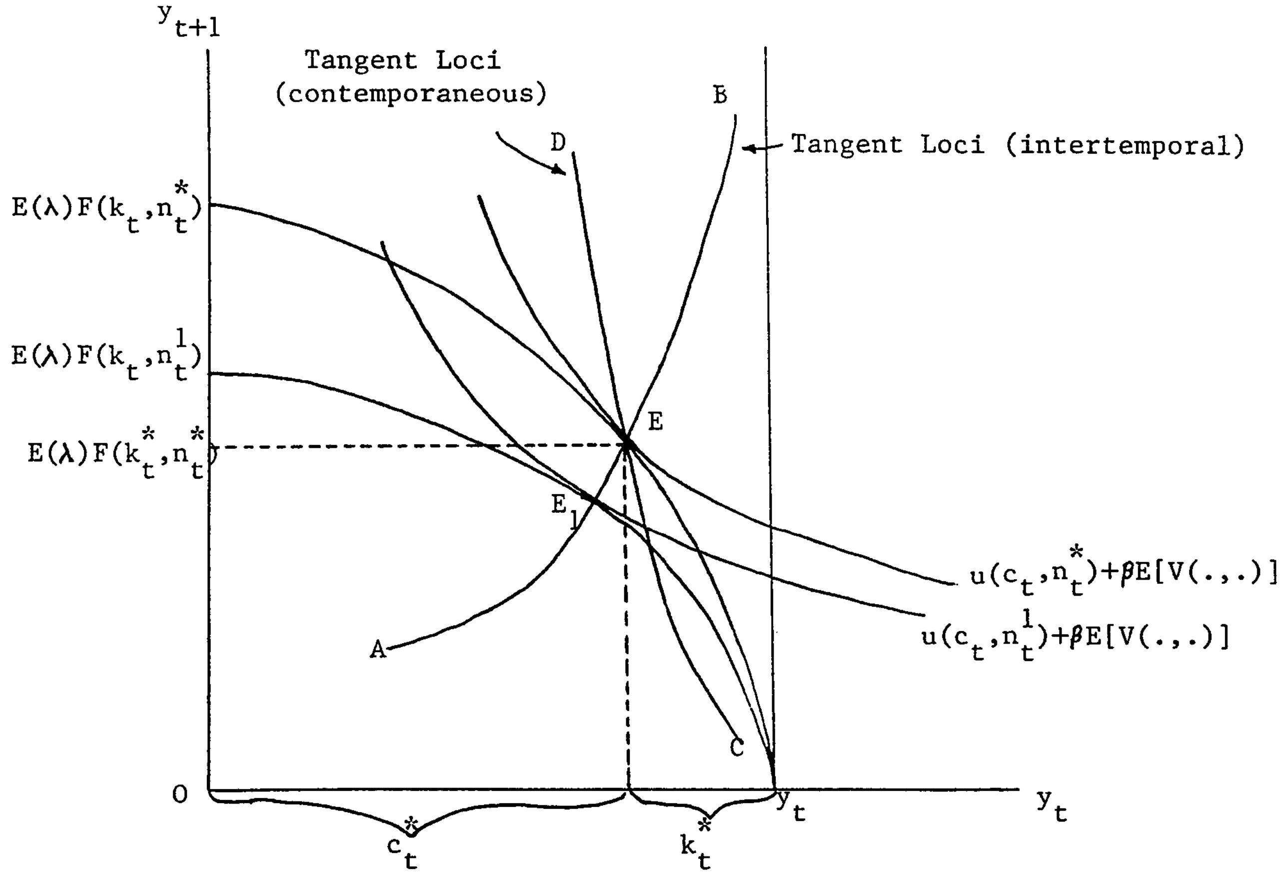

Figure 2 — Intertemporal Efficiency Condition

FOOTNOTES

1. This formulation is different from the dynastic utility function used by Barro and Becker (1986), in which the production of children generates a positive service to the household's utility because parents care about the welfare of their offsprings.

2. This relationship can be argued intuitively. Let us suppose that the output increases marginally at time t due to unexpected random shocks. This extra income can be either consumed at time t or used as production input for future consumption. But either way should leave the total returns intact at optimum, and poses no differences to the economic agent. Therefore, the value of such extra income, which is the price of the current output, should be equal to the marginal utility of consumption. Note that this quantity should be adjusted accordingly if capital is not fully depreciated.

3. In this sense, the classical model does not really explain the phenomenon of growth because it relies on exogenous mechanisms.

4. Recently, Boyd (1986) has shown that this principle can be applied to dynamic and stochastic settings as well. In fact, according to Boyd, the transformation need not be linear as long as it preserves the preference ordering. Technically speaking, any such transformation is "isomorphic" and preserves the geometrical structure of the equilibrium. Using this property, a complicated stochastic model might be converted to a easy–to–solve form. See Boyd (1986) for details.

5. Another way of looking at this is to check the Euler conditions. Since $V_1(.,.)$ equals marginal utility of consumption in equilibrium, equation (3) can be written as follows:

$$u_1(y_t - k_t^*, n_t^*) = \beta\, E_t\left[u_1[\lambda_{t+1} F(k_t^*, n_t^*) - k_{t+1}^*, n_{t+1}^*]\ \lambda_{t+1}\right] F_1(k_t^*, n_t^*)$$

Since both u and F are homogeneous, this equation still holds under the scale transformation $\{\tau y_s^*, \tau k_s^*\}_t^\infty$. The same argument applies to equation (4) as well.

6. Note that equation (9) can be viewed as an algebraic equation rather than a functional equation. In the latter case, the solution is in general more difficult to obtain because expectations of the future endogenous variable have to be taken into consideration. In the present case, however, only the current endogenous variables are involved.

7. Like Mao (1986), one can imagine that the equilibrium capital–labor ratio as a function of y_t and λ_t, or more generally, a function of the current and past random shocks $\{\lambda_t, \lambda_{t-1}, \ldots\}$. Then, equation (9) can be rearranged as follows:

$$H(\lambda_t, \lambda_{t-1}, \ldots) \equiv \gamma\beta E\left[\lambda_{t+1}^{\gamma} A(\lambda_{t+1})\right]$$

where H(.) is the implicit equilibrium function implied by the equilibrium capital labor ratio. Since the right hand side is constant over time, H(.) must be a constant function. That is, the implicit coefficients of the random shocks must be identically equal to zero, which implies a constant capital–labor ratio.

8. The quantity $A(\lambda_t)$ can be derived in a different way. Using the homogeneity of the utility and production function, the optimality equation (2) becomes

$$\begin{aligned} A(\lambda_t)y_t^{\gamma} &\equiv u(y_t - k_t^*, n_t^*) + \beta E[\lambda_{t+1}^{\gamma} A(\lambda_{t+1})]F(k_t^*, n_t^*) \\ &\equiv \frac{1}{\gamma}[u_1(y_t - k_t^*, n_t^*)(y_t - k_t^*) + u_2(y_t - k_t^*, n_t^*)n_t^*] + \\ &\quad \frac{1}{\gamma}\beta E[\lambda_{t+1}^{\gamma} A(\lambda_{t+1})][\gamma F_1(k_t^*, n_t^*)k_t^* + \gamma F_2(k_t^*, n_t^*)n_t^*] \end{aligned}$$

From the Euler equations (3) and (4), the above reduces to

$$\begin{aligned} A(\lambda_t)y_t^{\gamma} &\equiv \frac{1}{\gamma}[u_1(y_t - k_t^*, n_t^*)y_t] \\ &\equiv \frac{1}{\gamma}\left[u_1[g^{-1}f(a), 1](n_t^*)^{\gamma-1}y_t\right] \equiv \left[\frac{u_1[g^{-1}f(a), 1]}{\gamma\,[a + g^{-1}f(a)]^{\gamma-1}}\right]y_t^{\gamma} \end{aligned}$$

Therefore,

$$A(\lambda_t) \equiv \left[\frac{u_1[g^{-1}f(a),\, 1]}{\gamma\ [a + g^{-1}f(a)]^{\gamma-1}}\right]$$

It can be shown that this expression is equivalent to that in the text.

9. The wealth (or permanent income), measured in log, will also evolve as a drifted random walk. Define the wealth w_t as the expected present value of current and future income, discounted by the equilibrium real rate of interest, i.e.,

$$w_t = E_t\left[\sum_{s=t}^{\infty}\left[\frac{1}{1 + r_s^e}\right]^{s-t} y_s\right]$$

where r_t^e is constant in equilibrium. Use (14) and complete the summation, we obtain

$$w_t = \left[\frac{1}{1 - \beta\ E(\lambda_{t+1}^{\gamma})\ \mu^{\gamma}}\right] y_t$$

Therefore, the wealth is also linear in y_t, which implies that it will also evolve as a drifted random walk. Note that, in this case, the current income and permanent income amount to the same thing.

10. It should be mentioned that the test conducted by Nelson and Plosser (1982) has a low power against the alternative processes.

11. A common feature of the models of King and Robelo (1986) and Hercowitz and Sampson (1986) is that both impose some kind of fixed factor dynamics. Although the growth in these models is endogenously generated, it can be shown that it may not be sustainable in the absence of technical progress. In fact, the data generating process in this case will be stationary. This is most apparent in the model of Hercowitz and Sampson.

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