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Working Paper 90-10

Liquidity Constraints in Commercial Loan Markets
with Imperfect Information and Imperfect Competition

by

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November 1990

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This research was begun while the first author was on the faculty at Michigan State University. We gratefully acknowledge the financial support of Michigan State, the Investors in Business Education Fund at the University of Illinois and the National Science Foundation (SES 89-09242). We wish to thank Dan Bechter, Kathryn Combs, Michael Dotsey, Donald Hodgman, Charles Holt, David Humphrey, Ayse Imrohoroglu, Jeffrey Lacker, David Mengle, Loretta Mester, Neil Wallace and seminar participants at the Federal Reserve Bank of Richmond for their suggestions. The views expressed in this paper do not necessarily reflect the view of the Federal Reserve System or the Federal Reserve Bank of Richmond.

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Abstract

This paper presents a simple general equilibrium model of the commercial loan market in which liquidity constraints arise endogenously because of imperfect information and imperfect competition. The information and market structure generate a discriminatory interest rate schedule and loan size restrictions, which we interpret as liquidity constraint phenomena. The model's predictions are consistent with actual lending policies observed in the commercial loan industry. Further, the lender and all borrowers are at least as well off under this solution as they would be if faced with any single interest rate policy other than the competitive rate.

I. INTRODUCTION

Most researchers who have assumed the existence of liquidity constraints when developing models to match macroeconomic data have carefully qualified this practice by acknowledging the need for a theoretical basis. We provide one such basis in this paper by constructing an environment within which liquidity constraints arise endogenously because of imperfect information and imperfect competition. Specifically, we develop static and dynamic versions of a simple general equilibrium model with a commercial loan market in which there is a single lender and a large number of borrowers who differ only in terms of their net worth, and hence, ability to repay. The imperfect information assumption is the same as that used by Stiglitz and Weiss (1981): the lender knows the distribution of borrowers' types, but not the identity of any particular borrower.

Our assumptions on market and information structure generate a discriminatory interest rate schedule that matches the pricing policies observed in the commercial loan industry. The price discrimination arises despite the absence of differences across borrowers in risk, credit histories, or loan administration costs--factors commonly believed to explain any observed differences in loan interest rates across borrowers. Further, the endogeneity of the liquidity constraints, along with the model's microfoundation structure, allows us to conduct a normative analysis of such pricing policies. We find that the discriminatory interest rate policy can be Pareto superior to most uniform pricing schedules. In addition, smaller borrowers are

shown to be more liquidity constrained than larger borrowers, and thus bear a larger share of the distortion induced by the market imperfections.

The literature on liquidity constraints and the distortions they introduce into market allocations and interest rates is proliferating. Among the most recent publications in this area are Cox (1990), Fissel and Jappelli (1990), Hayford (1989), Hayashi (1987), Jappelli (1990), and Zeldes (1989), to name just a few.¹ One factor differentiating these many papers is the way the term "liquidity constraint" is defined. Some authors maintain that an individual is liquidity constrained if completely denied access to capital markets; others contend that liquidity constraints exist when individuals face different interest rates for borrowing and lending, or when interest rates are relatively high. In this paper, we say that borrowers are liquidity constrained if they face restrictions on the amount they can borrow at a given rate or if the interest rate at which at least some of them can borrow is higher than the rate they would face in a competitive market without distortions.

II. PREVIOUS RESEARCH ON LIQUIDITY CONSTRAINTS

The study of liquidity constraints has a very long history. Hodgman (1960) is generally credited with the inception of the "modern" literature on liquidity constraints.² He presents a theory of credit rationing by a rational, profit-maximizing lender in a competitive loan market in which the risk of loss from default is a function of loan size. Jaffee and Modigliani (1969) extend this approach by

introducing demand factors and make two noteworthy modifications. First, they assume that the lending market is imperfectly competitive, although, like Hodgman, they do not empirically justify their assumed market structure. Second, they exogenously assume that perfect price discrimination is impossible. This assumption is crucial for their analysis because a lender able to discriminate perfectly among individuals charges borrowers their reservation interest rates but does not ration credit.³

Baltensperger and Devinney (1985, p. 480) discuss Jaffee and Modigliani's assumption of imperfect price discrimination and note that

the question of the origin and importance of these [liquidity] constraints [on price setting] . . . was not given much attention by Jaffee-Modigliani. The fact that perfect price differentiation is impossible is introduced as an entirely exogenous element into their theory. They mention interest rate ceilings . . . and appeal to a vague concept of 'moral costs' and 'consideration of good will,' which make it inadvisable to charge widely different rates to different customers. In a way, their discussion returns to the point where Hodgman started out, in that the proof of the consistency of credit rationing with rational behavior relies on precisely those legal, 'moral,' or 'psychological' constraints and rigidities that Hodgman tried to avoid. . . .

One purpose of this paper is to show that imperfect price discrimination, and hence credit rationing, emerge naturally in an economic environment with imperfect competition and imperfect information.

More recent theoretical research on liquidity constraints has focused on the role of private information. Stiglitz and Weiss (1981) produced the seminal work of this type. Their model differs from ours

in the assumed structure of the commercial loan market and the features distinguishing borrowers. Specifically, the Stiglitz and Weiss model assumes perfect competition and that borrowers differ in their levels of wealth, which are private information, and perhaps the riskiness of their investment projects. As a consequence, the Stiglitz and Weiss model generates loan quantity rationing, where some borrowers receive the loan they request while other observationally identical borrowers are denied loans completely. In contrast, we obtain loan size rationing, where identical borrowers receive identical loans but the amounts received are smaller than those they would receive in a loan market without the imperfections assumed in this paper.

Even more recently, Gale and Hellwig (1985), Green (1987), Townsend (1988), and Williamson (1986) have developed contracting models of private information economies in which liquidity constraints arise endogenously. In these models, optimal allocations are derived from various types of Pareto problems (i.e. maximization of utility subject to private information and/or resource constraints). Williamson's model contains the most fully developed credit rationing analysis; hence we will compare our results with his in Section V. However, as Green (1987, p. 21) notes, whether the allocations that result from these Pareto problems with private information can be supported by a system of Walrasian prices is often difficult to establish. Consequently, we pursue an alternative approach. We specify a simple general equilibrium model of a private information economy and solve for the constrained optimal prices and quantities directly.

III. EMPIRICAL EVIDENCE FROM COMMERCIAL LOAN MARKETS

Our model is constructed to generate results that are consistent with the available evidence on business practices common in commercial loan markets and the nature of the credit rationing that occurs. In particular, using interest rate data reported by the Board of Governors of the Federal Reserve System in the Federal Reserve Bulletin, Goldberg (1982, 1984) finds substantial evidence suggesting that banks use "base rate pricing" practices. That is, they quote a prime rate and then offer selective price concessions to certain customers. He concludes (1984, p. 280) that

this activity appears to reflect an attempt by bankers to be competitive with the direct credit markets without lowering their prime rate quotes. The effect of this credit innovation has been to allow banks to segment customers on the basis of the elasticity of their demands for bank credit, and to allow the banks to be price-takers for those customers with access to direct credit markets and to be price-setters for their remaining customers.

Goldberg also notes that banks often make commercial loans with a variable interest rate; the loan rate on outstanding business loans is tied to the bank's prime rate. Such loan terms (known as most-favored-customer clauses) deter cuts in the prime by requiring lenders to provide lower interest rates to current borrowers if they later reduce the rates to subsequent borrowers.⁴

The prime rate's use as a "base rate," its use as an anchor in variable-rate loans, and its public disclosure may facilitate coordination in commercial loan markets. Leander (1990) illustrates this point with a fascinating description of a recent prime rate cut.⁵ He reports that on the morning of January 8, 1990, First National Bank

of Chicago instructed its public relations department to inform the news services (e.g. Dow Jones and Reuters) of its cut in the prime rate from 10.5 to 10 percent. According to Leander, "As the news that First Chicago had dropped the prime flashed across computer screens in bank trading rooms and executive offices, hasty meetings were called among credit committee officers, resulting in the cascade of announcements [from other banks]." In fact, within hours of First Chicago's announcement, 30 of the nation's largest banks, as well as many smaller banks, had jumped into what Leander calls "the biggest game of follow-the-leader in American business: changing the prime rate." Citibank was the first to follow the rate cut, acting just 24 minutes after First Chicago's action. Interestingly, a president of a medium-sized bank claimed, "we were just waiting for the money center banks to make their moves."

Further empirical support for our results comes from evidence on the nature of credit rationing in commercial loan markets. Data on bank lending terms reported by the Board of Governors of the Federal Reserve System in the Federal Reserve Bulletin (1990) indicates consistently that the average effective interest rate charged on a loan varies inversely with loan size. In addition, Evans and Jovanovic (1989) report empirical results consistent with loan size, rather than loan quantity, rationing. Specifically, they develop and estimate a model of entrepreneurial choice using data from the National Longitudinal Survey of Young Men (1976-78); the tightness of a liquidity constraint is a parameter in the model. They find that a relatively small percentage of applicants are denied loans completely and that

credit rationing is prevalent among all but the wealthiest of the remaining applicants. Furthermore, when applicants are rationed, the liquidity constraint usually takes the form of a limitation on the size of the loan made to each borrower.⁶

Of course, the evidence just described does not validate our assumption that commercial loan markets are characterized by imperfect competition. There is no doubt, however, that lenders can identify their competitors and are well aware of their strategic interdependence regarding market outcomes. Moreover, the pricing policies observed in the commercial lending industry match the pricing policies that our model predicts, regardless of the market's actual structure. That is, there is evidence consistent with price discrimination based on loan size, with smaller loans bearing higher interest rates. The data do not explain why such pricing might occur, but our model suggests that imperfect information and imperfect competition are sufficient to generate price discrimination among borrowers based solely on firm net worth or resources available for repayment.⁷ Risk and loan administration cost differences are unnecessary in our model.

IV. STRUCTURE OF THE MODEL

In this section we present two models with similar characteristics that can accommodate the empirical phenomena discussed in the previous section. The first is a static (two-period) general equilibrium nonuniform pricing model, and the second is a dynamic (overlapping generations) version of the first model. Clearly, the two-period model is a special case of the stationary overlapping generations

model. We consider the models separately for three reasons. First, this presentation makes clear that the overlapping generations structure is not essential for our results. Equivalent results can be obtained from the two-period model. Second, because most recent credit rationing results have been obtained from two-period models, this presentation facilitates comparison of our model with these contract models. Finally, use of the overlapping generations framework makes our model immediately applicable to the study of a wide range of dynamic macroeconomic problems (e.g. dynamic fiscal policy, the Ricardian equivalence Proposition, the cost of business cycles) for which an endogenous theory of liquidity constraints is important.⁸

A. The Two-period Model

Consider an economy with n types of two-period lived borrowers and a single lender; n is a positive and finite number. There are N_i borrowers of each type i , with $i = 1, 2, \dots, n$, who may be thought of as privately-owned firms that operate for two periods.⁹ Firms are assumed to differ only with respect to their deterministic endowments of physical good, or net worth. Specifically, each firm of type i has a net worth of $w_1^i > 0$ in its first period of operation and $w_2^i > 0$ in its second period. All firms have the same first period net worth: $w_1^i = w_1$ for all i ; however, higher index firms have larger second period net worth: $w_2^{i+1} > w_2^i$.¹⁰ Because firms are privately held, we assume that each type i firm has preferences that are representable by a twice differentiable, strictly increasing, and strictly concave utility function, $u[x_1^i, x_2^i]$, where x_t^i is the amount of time

$t = 1, 2$ good consumed by the owner of the firm. We complete our description of borrowers by assuming that x_t^i is a normal good. Given these assumptions, the net worth pattern results in higher index firms being larger borrowers.

The single lender in this economy wishes to maximize the profit obtained from revenues generated by loan repayments, less the cost of making new loans. Assume that the lender's endowment of physical good at time 1 is sufficient to support its lending policy, and suppose that the following information restriction exists:¹¹ the lender and all borrowers know the utility function u , the net worth pattern, and N_i for all i , but cannot identify the type of any individual borrower. Thus, a borrower's type is private information. The implications of this information restriction are two-fold. First, it prevents perfect price discrimination by the lender but allows for the possibility of imperfect discrimination via self-selection policies (i.e. policies that result in borrowers correctly sorting themselves into groups by choosing the loan package designed for their type). Second, it precludes borrowers from sharing loans because they are unable to identify each other prior to receiving a loan.

The lender's monopoly problem is to choose a total repayment (i.e. principal plus interest) schedule for period 2, denoted by $P(q)$, such that any firm that borrows amount q in period 1 must repay amount P in period 2. Let $R_i(q)$ denote the reservation outlay for loans of size q by a type i borrower (i.e. the maximum amount a type i borrower is willing to pay at time 2 for a time 1 loan of size q), and let $R_i'(q)$ denote the derivative of $R_i(q)$ (i.e. the inverse demand for loans of

size q). Further, let $q_0 = 0$ and $R_i(0) = 0$, which indicates that the lowest index group borrows nothing and that the reservation value from borrowing zero is zero for all groups. The lender's two-period nonuniform profit maximization problem can now be stated as follows:

$$\max_{(q_1, P(q_1)), \dots, (q_n, P(q_n))} \sum_{i=1}^n N_i [P(q_i) - q_i] \quad (1)$$

$$\text{subject to: } R_i(q_i) - P(q_i) \geq R_i(q_j) - P(q_j) \text{ for all } i. \quad (2)$$

Equation (1) is the lender's profit function at time 2. Clearly the lender's profit is the aggregate amount repaid at time 2 by all borrowers (i.e. the lender's total revenue) minus the aggregate amount lent at time 1 to these agents (i.e. the lender's total cost).

Equation (2) summarizes the self-selection constraints for all i classes of borrowers. These constraints indicate that borrower i 's consumer surplus from choosing a loan of size q_i must be at least as great as the consumer surplus received from choosing a loan of some other size q_j . These self-selection constraints are designed to induce borrowers to correctly reveal their type. Thus, the lender's two-period problem is to choose an amount to lend at time 1, q_i , and a total repayment schedule for time 2, $P(q_i)$, for every type $i = 1, \dots, n$.

B. The Overlapping Generations Model

Consider now a stationary, discrete time overlapping generations model with the same structure as the two-period model. Each generation $t \geq 0$ has n types of two-period lived borrowers, with N_i borrowers of each type and $i = 1, \dots, n$. These borrowers may again be

thought of as privately held firms who operate for two periods. Each firm i of generation t commences operation in its first period (time t) with net worth $w_t^i(t) > 0$ and has net worth $w_{t+1}^i(t)$ in its second period (time $t+1$). As before, all firms have identical first period net worth and higher index firms have larger second period net worth; that is, $w_t^i(t) = w_1$ and $w_{t+1}^i(t) = w_2^i$ with $w_2^{i+1} > w_2^i$ for all i and t . We assume that each privately held firm's preferences are representable by a twice differentiable, strictly increasing, and strictly concave utility function, $u[x_1^i(t), x_2^i(t)]$ where $x_t^i(t)$ is the amount of time $t = 1, 2$ good consumed by the owner of the firm. Finally, $x_t^i(t)$ is a normal good. Clearly, higher index firms are again larger borrowers.

There is a single lender in the economy who, without loss of generality, is assumed to operate for all periods. The lender and all borrowers are subject to the same information restriction specified in the two-period model, and the lender's endowment at the initial date is assumed to be sufficient to support its stationary lending policy. Observe that because the economy is stationary, time notation can be suppressed. Thus, the lender's stationary nonuniform profit maximization problem is formally the same as the two-period problem stated in Section IV.A: maximize (1), the lender's stationary profit function at time t , subject to (2), the self-selection constraints for the i classes of borrowers.

However, the interpretation of equation (1) is somewhat different in the overlapping generations model. In particular, the lender's stationary profit at time t is now the difference between the total

revenue it obtains from loans repaid at time t by all borrowers of all types from generation $t-1$ and the total cost of new loans granted to new borrowers from generation t . Thus, in the stationary overlapping generations model the lender chooses an amount to lend, q_t , and a total repayment schedule, $P(q_t)$, for every period $t \geq 0$ and every type $i = 1, \dots, n$, but its borrowers and lenders are members of different generations. All other aspects of the two models are identical.

V. THE NATURE OF THE OPTIMAL SOLUTION

The models developed in Section IV are general equilibrium versions of the Spence (1980) nonuniform pricing model. We prove this in the appendix by showing that the models' assumptions on preferences and net worth generate reservation outlay and total repayment functions with the properties assumed by Spence. Thus, following Spence (1980, pp. 822-823), the lender's profit maximization problem can be solved as follows. First, observe that at an optimum, equation (2) is satisfied with equality. Using this fact and the assumptions that $q_0 = 0$ and $R_i(0) = 0$, and making successive substitutions into (2), one can show that

$$P(q_t) = \sum_{j=1}^i [R_j(q_j) - R_j(q_{j-1})]. \quad (3)$$

Equation (3) gives the lender's profit-maximizing repayment schedule, given the loan quantities q_1, \dots, q_n . Second, the profit-maximizing

loan quantities can be determined as follows. Define $M_i = \sum_{j=1}^n N_j$,

$i = 1, \dots, n$, where M_i measures the finite cumulative distribution of consumer types from i to n , with $M_{n+1} = 0$ because n is the highest group. Substituting (3) into (1), differentiating with respect to q_i , and using the definition of M_i yields

$$R'_i(q) = \left[\frac{M_{i+1}}{N_i + M_{i+1}} \right] R'_{i+1}(q) + \left[\frac{N_i}{N_i + M_{i+1}} \right] \text{ for } i = 1, \dots, n. \quad (4)$$

Equation (4) summarizes the optimal loan size formulae for each group.

A. Economic Interpretation of the Solution

We can now interpret the results (i.e. equations (3) and (4)) for an economy with a commercial lending industry. Equation (4) indicates that the loan size, q_i , offered to group $i = 1, \dots, n-1$ is strictly less than the size available in a competitive market for all groups except the largest. In particular, equation (4) indicates that the profit-maximizing loan size for each group should be chosen so that the implicit marginal (reservation) value of a loan of size q to type i borrowers, $R'_i(q)$, equals a weighted average of the implicit marginal value of the loan to the next highest group, $R'_{i+1}(q)$, and the marginal cost of lending, which is one. This lending policy clearly violates the competitive prescription that requires the lender to equate only the marginal value of a loan to group i with the marginal cost of a loan to group i (which is unity for all $i = 1, \dots, n$).

Equation (4) is also essential for showing that the highest group suffers no quantity or price distortion, but the degree of credit rationing experienced by borrowers from all other groups $i = 1, \dots, n-1$

is inversely related to their index. In particular, because $M_{n+1} = 0$ by construction, (4) reduces to $R'_n(q) = 1$ for group n . This is the standard competitive prescription to equate marginal benefit to marginal cost. Hence, the competitive quantity is supplied at the competitive rate for this group. To establish that the pattern of distortion is regressive, consider the following. In the appendix we prove that our assumptions on preferences and net worth imply that $R'_{i+1}(q) > R'_i(q)$. Using this fact, along with appropriate restrictions on the distribution of borrower types (i.e. N_i), equation (4) implies that low index borrowers are relatively more constrained than high index borrowers.¹²

The final result pertains to the welfare properties of the nonuniform price and quantity scheme given by equations (3) and (4). It is well known in the nonuniform pricing literature (e.g. Spence (1980, p. 823)) that for any uniform price different from marginal cost, there is a nonuniform outlay schedule that weakly benefits all borrowers and the lender without side payments. In other words, if the borrowers and lender were given a choice between (i) any single interest rate policy that differs from the competitive interest rate, and (ii) a quantity-dependent array of interest rates, with one rate appropriate for each group, they would all prefer or at least be indifferent to the latter policy without coercion. This result indicates that there exists some quantity-dependent interest rate policy that is Pareto superior to any single interest rate policy except for the single rate that prevails in the competitive market.

Two other interesting features of the solution merit discussion. Because imperfect information prevents perfect price discrimination, the lender must ensure that the loan size/interest rate package designed for each group satisfies the self-selection constraint. The ordering of loan sizes so that $q_i \geq q_{i-1}$ for all i , is necessary for this constraint to be satisfied. This condition states that the lender must offer loans to high index (i.e. large net worth) borrowers that are at least as large as those offered to low index borrowers. Further, $P(q)/q$ is weakly decreasing in q . This condition indicates that large borrowers pay lower average interest rates than small borrowers.

Both of these features of the solution stem from the lender's need to ensure that each group selects the "correct" loan size-interest rate package. In particular, the lender must make the selection of a small loan undesirable for high index borrowers. This is done by allowing the average interest rate to fall with loan size, thus letting larger borrowers keep some of their consumer surplus. The lender must also ensure that small borrowers do not select loans designed for large borrowers. The information restriction guarantees that such selections are not made. Specifically, the information restriction prevents borrowers from identifying each other. Consequently, they are unable to pool their net worths to share a loan designed for larger borrowers.

B. Comparison to the Related Literature

We interpret the preceding results on loan size and interest rate distortions as liquidity constraint phenomena. In particular, in our

model all but the largest borrowers are prohibited from obtaining loans as large as they would choose under perfect competition and perfect information, and the lower a borrower's net worth, the more troublesome (i.e. distorting) the constraints are for the group. Thus, the liquidity constraints bind and are distorting for much of the population. These theoretical predictions appear to be consistent with the empirical results on loan size rationing reported by Evans and Jovanovic (1989) and the Board of Governors of the Federal Reserve System in the Federal Reserve Bulletin (1990).

The intuition behind these credit rationing results is as follows. The model consists of numerous borrowers who differ along a single dimension (second period net worth). The price-leading lender has market power and wishes to maximize profit. The lender knows the distribution of borrower types in the economy, but does not know the identity of any particular borrower. This information restriction prohibits policies, such as perfect price discrimination, that lead to a Pareto optimal allocation of resources. However, the lender can exploit the correlation of borrowers' market choices with their net worth; this is done by offering a nonuniform interest rate schedule that rations loan sizes to all but the largest group. The information implicitly revealed by self-selection allows the lender to partially offset its inability, because of imperfect information about borrower characteristics, to design borrower-specific interest-rate schedules. Thus, the quantity constraints, which we interpret as liquidity constraints on the size of loans that borrowers can obtain, arise

endogenously as an optimal response to the information restriction in an imperfectly competitive market.

It is interesting to compare our results with recent contracting models with private information. We regard Williamson (1986) as an especially good example of this type of model. In particular, Williamson considers a two-period model with three key features: asymmetrically informed borrowers and lenders, costly monitoring, and project divisibilities. These features generate increasing returns to scale from delegated monitoring, and hence a single intermediary emerges endogenously. The emergence of a single lender who pools loans and monitors borrowers is a well known feature of delegated monitoring models of banking (e.g. in addition to Williamson, see also Diamond (1984) or Krasa and Villamil (1990)). Hence our assumed market structure is identical with the structure that emerges endogenously in these recent theoretical models. However, unlike the loan size rationing that emerges in our model, Williamson obtains "all or nothing" loan quantity rationing. This result stems from an asymmetry in the payoff functions of lenders and borrowers in his model, which in turn arises from the costly monitoring of borrowers. In contrast, loan size rationing is a manifestation of price discrimination in our model.

VI. CONCLUSION

This paper has presented a theoretical model of the commercial loan market that formalizes the traditional folklore that lenders can maximize profit by using third-degree price discrimination. This

price discrimination is a form of loan size rationing that occurs despite the absence of differences across borrowers in terms of default risk or costs of loan administration. It is also consistent with the empirical evidence regarding commercial loan rates. Moreover, our analysis shows that all loan market participants--the lender and all borrowers--are at least as well-off with the discriminatory interest rate schedule as they would be if faced with any uniform interest rate other than the competitive rate. Finally, the paper fills a gap in the macroeconomics literature by presenting a framework that yields endogenous liquidity constraints while being tractable for the study of various dynamic macroeconomic problems.

APPENDIX

Villamil (1988) establishes that an analogue of the overlapping generations model specified in this paper is a special case of the widely-used Spence nonuniform pricing model. A straightforward adaptation of this argument will be used to show that the assumptions on preferences and net worth made in Section IV imply reservation outlay functions (i.e. $R_i(q)$ and $R'_i(q)$) that satisfy the assumptions of the Spence (1980, p. 822) nonuniform pricing model:

S.1: Borrower types can be ordered so that for all q , $R_{i+1}(q) > R_i(q)$ and $R'_{i+1}(q) > R'_i(q)$.

S.2: Firms can refrain from borrowing, and if they do, $P(0) = 0$ and $R_i(0) = 0$.

Property S.1 implies that borrower types can be ordered so that for all q , $R_{i+1}(q) > R_i(q)$ and $R'_{i+1}(q) > R'_i(q)$. As a consequence, a schedule representing $R_{i+1}(q)$ as a function of q lies above a schedule representing $R_i(q)$ and has a steeper slope. From S.2, firms may borrow nothing, and if they do, $P(0) = 0$ and $R_i(0) = 0$. This implies that the consumer surplus of a borrower of type i from a loan of size $q \geq 0$, $R_i(q) - P(q)$, is at least as great as the reservation price for purchasing nothing, which is zero.

Spence assumes, as do we, that the monopolist knows $R_i(q)$ and N_i for all i , but does not know the identity of any particular borrower. Thus, it remains to show that our model satisfies Spence's assumptions S.1 and S.2. This is accomplished in the following proposition.

Proposition: The assumptions on preferences and net worth made in Section IV imply reservation outlay functions for consumption in excess of net worth in the first period that satisfy S.1 and S.2.

Proof: Let p denote the price of date $t+1$ good in terms of date t good. Let q denote the amount borrowed, i.e. the amount of first period consumption in excess of w_1 , and let $h_i(p)$ denote the excess demand for first period consumption by a type i borrower. From the assumptions that $u(\cdot)$ is concave and that x_1 is a normal good, $h_i(p)$ is single-valued and decreasing in p where $h_i(p) > 0$. Thus, for all $q \geq 0$, $h_i(p)$ has an inverse that we shall denote by $R'_i(q)$. From the assumptions on preferences and net worth, $h_{i+1}(p) > h_i(p)$, and consequently $R'_{i+1}(q) > R'_i(q)$ for all $q \geq 0$. Further, letting

$$R_i(q) = \int_0^q R'_i(z) dz, \text{ we have that } R_{i+1}(q) > R_i(q) \text{ for all } q \geq 0.$$

Clearly, S.1 is satisfied. Property S.2 is also satisfied because any borrower can refuse to apply for a loan, in which case his/her repayment obligation and reservation outlay are zero (i.e. $P(0) = R_i(0) = 0$).

The proposition establishes that the lender's problem that we study in Section IV is a special case of the Spence (1980) nonuniform pricing model; hence, Spence's results apply directly. The nature and interpretation of these results in a loan market context are discussed in Section V.

FOOTNOTES

1. These studies all focus on consumer, not commercial, loan markets. They find consistently that approximately twenty percent of consumers face some form of borrowing constraint. In contrast, our paper focuses on commercial loans.
2. See Baltensperger and Devinney (1985) for an excellent survey of this literature.
3. Jaffee and Modigliani (1969, p. 851) define credit rationing as a situation in which there is "an excess demand for commercial loans at the ruling commercial loan rate."
4. Further support for our theoretical and Goldberg's empirical findings comes from at least two sources. Grether and Plott (1984) report evidence of imperfectly competitive outcomes in laboratory experiments with markets in which most-favored-customer (MFC) clauses are used. Holt and Scheffman (1987) present a theoretical model in which MFC clauses facilitate tacit collusion in the setting of base (list) prices, such as the prime rate, and make base prices immune to discounts that must be given to all current and potential buyers.
5. The prevailing prime rate is said to change when sixteen of the thirty largest banks change their prime rates.
6. In contrast, Berger and Udell (1989, p. 3) find that credit "rationing is not likely to be an important macroeconomic phenomenon." A close reading of their paper indicates, however, that they use the phrase "credit rationing" as a synonym for "credit allocation" or "credit control," expressions commonly used (see, e.g. Merris (1975)) to refer to the impact of monetary policy on credit availability through its effect on market interest rates. They make clear that their data set cannot be used to identify the type of rationing that we study: the rationing of credit to certain groups of borrowers under all credit market conditions.
7. Our model assumes a single, profit-maximizing lender and hence is not subject to the Rothschild and Stiglitz (1976) non-existence problem. In particular, Rothschild and Stiglitz note that competition among suppliers of a homogeneous good may lead to non-existence of a price discriminating equilibrium. However, using a spatial model of monopolistic competition, Borenstein (1985) shows that third-degree price discrimination almost always occurs, even though equilibrium profit is zero, as long as firms can sort buyers based on their willingness-to-pay. His work suggests that product differentiation (e.g. different collateral requirements, maturities, etc.) may be important for ensuring

existence of an equilibrium in multi-lender price discrimination models. We believe that price discriminating equilibria are important (either in price leadership or monopolistically competitive settings) because recent work by Milde and Riley (1988) suggests that imperfect information by itself may not be sufficient to generate credit rationing, at least when signalling opportunities exist. In particular, on page 120 they note that the (asymmetric) private information in their competitive loan market gives rise to a separating equilibrium, but not rationing.

8. See, for example, Auerbach and Kotlikoff (1987), Barro (1974), Bernheim (1987), Hayford (1989), Imrohorglu (1989), and Scheinkman and Weiss (1986).
9. The notion of a firm that we employ merits discussion. Classical general equilibrium theory treats the firm as a production technology. In contrast, a recent paper by Prescott and Boyd (1987) models the firm as a coalition of agents in an overlapping generations model. The agents in their model are identically endowed and two-period lived, with identical utility functions defined on consumption. The firm is an on-going coalition of these agents who produce the consumption good each period. The primary emphasis in our paper is on the relationship between the lender and borrowers in the commercial loan market, not the nature of commercial borrowers (firms) per se. However, our interpretation of the firm is consistent with a highly simplified version of the Prescott and Boyd model. Specifically, each privately held firm in our model corresponds to a singleton (i.e. coalition of one) in their model, and production is replaced by exogenous net worth, measured in terms of the consumption good. The implication of liquidity constraints in their richer setting for classic industrial organization questions, such as the distribution of firm size and firm growth, remains an interesting open research problem.
10. Because endowment patterns are deterministic, there is no default risk in this model if the lender induces each type of borrower to self-select the "correct" loan size-interest rate package. In what follows, we specify self-selection constraints that ensure that agents prefer the "correct" package. Consequently, our third-degree price discrimination obtains despite the absence of differences across borrowers in default risk. See Azariadis and Smith (1989) for a model with default risk.
11. In this model, all loans are loans of physical good. This follows in the tradition of Samuelson (1958).
12. See Spence (1980, p. 824) for a discussion of constraints on the distribution of consumer types.

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