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Welfare-Improving Credit Controls

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Working Paper 91-1

WELFARE-IMPROVING CREDIT CONTROLS*

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Credit controls are generally believed to result in an inefficient allocation of resources. This paper presents a counterexample. It displays a general equilibrium, multi-good model with spatial separation for which steady state equilibria exist in which both cash (i.e. fiat currency) and trade credit are used in exchange. Transaction costs, restrictions on the timing of trade, and a positive nominal interest rate cause the laissez-faire equilibrium to be non-optimal. A quantitative restriction on the use of trade credit can yield a Pareto superior allocation.

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1. Introduction

Credit controls have been used widely in many countries. In the United States, controls were used most recently in 1980 as part of President Jimmy Carter's anti-inflation program. Most quantitative credit controls--including the 1980 U.S. program--have been designed to decrease aggregate demand without raising interest rates during periods of high inflation.[1] Economists, however, generally believe that credit controls misallocate resources and reduce welfare. This paper presents a counterexample by focusing on the role of credit as a substitute for money in exchange.

Section 2 displays a model economy in which the government finances its deficit with money creation and yet, contrary to general belief, credit controls can improve welfare. The model allows for multiple means of payment--cash and trade credit--and focuses explicitly on the representative individual's decision of which to use in exchange. Cash transactions are assumed to involve an opportunity cost in terms of foregone interest when the nominal interest rate is positive. Trade credit allows individuals to buy goods without incurring the opportunity cost of holding cash, but its use involves a real resource cost. When making purchases, the individual chooses between these means of payment by balancing the resource cost of using trade credit against the interest opportunity cost of cash. The interest opportunity cost is not, however, a social cost, whereas the resource cost is. Consequently, market failure occurs when the nominal interest rate, and thus the opportunity cost, is positive. The positive interest rate drives a wedge between the private and social costs of using cash and trade credit. As a result, individuals make excessive use of trade credit. This market failure is analogous to that empirically quantified in Humphrey and Berger (1990).

Section 3 analyzes a government credit control policy that prevents individuals from using "too much" trade credit. It shows that credit controls can, under some circumstances, produce an allocation Pareto superior to the laissez-faire competitive equilibrium. This result obtains for two reasons. First, the controls directly reduce transaction costs incurred, allowing any level of production to support more consumption. Second, when labor supply, and thus output, is relatively insensitive to the quantity of trade credit allowed, the controls induce individuals to substitute toward leisure and away from consumption. This alteration in the consumption-leisure allocation partially offsets the distortion caused by institutional restrictions on the timing of economic activity. The robustness of this policy result is discussed in Section 4 along with concluding remarks.

2. The Model

Preferences, Endowments, and Technologies

This paper analyzes a multi-good, discrete time economy at dates $t \geq 0$. At each date, a continuum of identical infinitely-lived agents resides at each location on a circle with a circumference of one. Each location Z agent, $Z \epsilon [0,1]$, is endowed in every period with $e>0$ units of time and a technology for producing a location-specific good. The technology available to agents at location Z turns n units of labor into n units of non-storable good z. At date $t=0$, the agents hold equal shares of the M_0 units of fiat currency outstanding.

The period utility function $U(W(c_{tz}), e-n_t)$ represents an agent's preferences at each date t. The argument c_{tz} is a vector with typical element being consumption of good z $(0 \leq z \leq 1)$ at date t, and n_t is labor supply at that date. Thus, $e-n_t$ is leisure. Assume that U is twice differentiable, strictly increasing, and strictly concave. Also assume that consumption goods available at the same date are perfect complements; this assumption is made for tractability. Consequently, $W(c_{tz}) = \inf_z \{c_{tz}\}$. Without loss of generality, the utility maximizing bundle is taken to be $c_{tz}=c_t$ for all z.[2] The period utility function may now be written as $U(c_t,e-n_t)$, and the representative agent's objective is to maximize

$$\sum_{t=0}^{\infty} \beta^t U(c_t,e-n_t), \qquad 0<\beta<1.$$

Note that the assumptions on population, preferences, endowments, and the spatial set-up are consistent with the operation of a perfectly competitive goods market and securities market at each point on the circle.

There is also an infinitely-lived government in the economy. The government runs a real deficit of $g\geq0$ per capita at each date, which it finances with revenues from seignorage. It adjusts the money supply according to $M_t=xM_{t-1}$, where $x\geq1$ is chosen to satisfy the government's budget constraint at each date $t\geq0$:

$$g= \frac{M_t-M_{t-1}}{p_t} = \frac{(x-1)M_{t-1}}{xp_{t-1}}.$$

As in standard cash-in-advance models, economic activity proceeds sequentially each period, as illustrated in Figure 1. Immediately after the period begins, agents repay outstanding debts. Next, the securities markets operate; at this time, agents borrow and lend cash (i.e. exchange currency for claims to currency in the securities market at the next date). No other transactions may be made in these markets. After the securities markets close, the goods markets operate. Each agent may be thought of as a two-person household consisting of a worker and a shopper. The worker stays home, producing and selling the consumption good, while the shopper travels completely around the circle once, buying goods at each point. Cash purchases of goods are financed with fiat currency held at the close of the securities trading session. The shopper finances credit purchases (i.e. uses trade credit) by issuing the seller a claim to currency at the opening of the following period. The government also makes goods purchases at this time with newly-created currency; it is assumed without loss of generality to make no trade credit purchases.[3] Finally, consumption occurs. Notice that an agent does not have the opportunity to trade securities again after the close of the goods markets but before the next date. This is of critical importance in the agent's expenditure-financing decision.

A desirable feature of this model is its allowance for two types of credit: cash loans available in the securities markets and trade credit issued in the goods markets. Assume that obtaining either type of credit involves a real transaction cost of kDy, where k is a non-negative constant, D denotes the distance between a borrower's residence and lender's location, and y is the total real value of the loan (units of the good bought in the case of a trade credit purchase, and real balances lent in the case of a cash loan). Here D is the shortest distance around the circle, taking values between zero and one-half because the circle has a circumference of one. The transaction cost is measured in terms of goods used in extending credit. Under perfect competition, lenders pass such costs onto borrowers. Sellers cannot lend currency received from cash sales before the beginning of the next date, so there is no opportunity cost in terms of foregone interest associated with trade credit. Moreover, because the price level is the same when trade credit is issued as when it is repaid (because the money supply is adjusted when the goods markets open), kd, the real transaction cost, is the rate of interest charged on a credit purchase.

The transaction cost may be thought of as a cost of verifying the value of claims on borrowers. This verification is necessary because the spatial separation limits the information that agents have about each other. Once this cost is incurred, an accurate evaluation of identity is obtained; no fraudulent securities are issued. The proportionality of the transaction cost might stem from borrowers' incentives to misrepresent their identities. If the incentive to misrepresent oneself increases with the value of the security, the total cost of verification might be increasing in the loan size. This is the case if a seller must use the verification technology for a time period proportional to a borrower's desired expenditure to get an accurate appraisal.[4]

Note that the spatial set-up and cost structure imply that transaction costs are never incurred in making cash loans. Because a large and equal number of agents resides at each point on the circle, the securities markets at different locations are identical. Only cash loans trade in these markets, and a cash loan's value in exchange is independent of where it was issued. However, because the transaction cost increases with distance, cash loans are more costly for agents the farther the distance between the agent's and the lender's home locations. Thus, agents only obtain loans in the market at their home location, incurring no transaction costs.

To determine the agent's use of cash and trade credit, let p_t denote the cash price of a unit of the consumption good bought at date t. This paper's focus on equilibria in which each good sells for the same price is common to models with Clower and cash-in-advance constraints (e.g. Lucas (1980)) and is motivated by the symmetry in the environment. Perfect competition and inelastic goods demands result in buyers bearing the full burden of the transaction costs. Thus, on a trade credit purchase from a seller located a distance D_t from his residence, an agent pays $p_t(1+kD_t)$ for the good. The cost of making the purchase with cash arises because the agent either borrows the cash needed for the purchase or foregoes lending by making the purchase. Let r_t denote the net nominal one-period interest rate on securities at date t. The cost of using cash is then $p_t(1+r_t)$. An optimizing agent chooses the least costly method of financing his purchases; that is, he uses trade credit if the distance D_t satisfies

$$p_t(1+kD_t) \leq p_t(1+r_t). \tag{1}$$

Let D^* denote the distance satisfying (1) with equality; that is, $D^*=r/k$. Equation (1) implies then that an agent wishing to finance his expenditures in the least costly way uses *trade credit to buy all goods sold at a distance* D from his home for which $D \leq D^*$. In other words, he uses trade credit when shopping at stores sufficiently close to home and cash otherwise.[5]

The Representative Agent's Problem

The representative agent solves a two-stage competitive lifetime choice problem. First, he chooses at each date an amount of each good to consume and the least costly way to finance his purchases (i.e. a value of D^* from equation (1)). Next, he chooses sequences for consumption, c_t, labor supply, n_t, currency holdings at the close of the securities markets, m_t, currency holdings at the close of the goods markets, m_t', and net lending of one-period securities, b_t, treating prices, p_t and r_t, and sales of his goods, s_t, parametrically. These choices maximize

$$\sum_{t=0}^{\infty} \beta^t U(c_t, e-n_t), \qquad 0<\beta<1, \tag{2}$$

subject to *(after integrating over distance)*[6]

$$b_t + m_t \leq m'_{t-1} + (1+r_{t-1})b_{t-1} - p_{t-1}c_{t-1}(2D^*_{t-1}+kD^{*\,2}_{t-1}) + (2D^*_{t-1}+kD^{*\,2}_{t-1})p_{t-1}s_{t-1} \tag{3}$$

$$m_t' = [n_t-(2D^*_t+kD^{*2}_t)s_t]p_t + m_t - (1-2D^*_t)p_tc_t \tag{4}$$

$$(1-2D^*_t)p_tc_t \leq m_t \tag{5}$$

$$c_t\geq 0;\ 0\leq n_t\leq e \tag{6}$$

$$\lim_{t\to\infty}\beta^t b_t=0;\ \lim_{t\to\infty}\beta^t m_t=0 \tag{7}$$

Equation (3) states that uses of wealth during the securities trading session cannot exceed beginning of period wealth. Equation (4) is an accounting identity that defines m_t', making use of the linear production technology, while (5) is an endogenous cash-in-advance constraint, applying only to purchases that the agent chooses to make with cash. Finally, (6) specifies the non-negativity constraints, and (7) states the transversality conditions.

The first order conditions for this problem are

$$\beta^t U_1(c_t,e-n_t) = p_t\{\ \mu_{2,t}(1-2D^*_t) + \mu_{1,t+1}(2D^*_t+kD^{*2}_t) + \mu_{3,t}(1-2D^*_t)\ \} \tag{8}$$

$$\beta^t U_2(c_t,e-n_t) = p_t\mu_{2,t} \tag{9}$$

$$\mu_{1,t+1}(1+r_t)-\mu_{1t} = 0 \tag{10}$$

$$\mu_{2,t}-\mu_{1,t}+\mu_{3,t} \leq 0,\quad = \text{if } m_t > 0 \tag{11}$$

$$-\mu_{2,t}+\mu_{1,t+1} = 0 \tag{12}$$

where U_i is the partial derivative of $U(\cdot,\cdot)$ with respect to its ith argument, and $\mu_{1,t}$, $\mu_{2,t}$, $\mu_{3,t}$ are the Lagrange multipliers on constraints (3), (4), and (5), respectively, at date t.

Define $V(c_t,e-n_t) \equiv U_1(c_t,e-n_t)/U_2(c_t,e-n_t)$. Assume that V is decreasing in

c_t and increasing in $e\text{-}n_t$, and that $\lim_{c_t \to 0} V(c_t, e\text{-}n_t) = \infty$ and $\lim_{n_t \to e} V(c_t, e\text{-}n_t) = 0$. From (8)-(12), an agent chooses consumption and labor supply to satisfy

$$V(c_t, e\text{-}n_t) = 1 + 2D_t^*(kD_t^*/2) + r_t(1-2D_t^*)$$
$$= 1 + kD_t^{*2} + r_t(1-2D_t^*). \qquad (13)$$

The left side of (13) is the marginal rate of substitution of leisure for consumption, while the right side is the cost to the agent of turning leisure into consumption goods using the production process. From the constant returns to scale production technology, the marginal rate of transformation is one. Thus, (13) reveals that there is a wedge between the marginal rates of substitution and transformation. This wedge measures the effective cost of a unit of consumption purchased optimally. An optimizing agent buys a fraction $2D^*$ of his purchases on trade credit at an average interest rate (transaction cost) of $kD_t^*/2$ and a fraction $1\text{-}2D_t^*$ on cash at an average cost (opportunity cost) of r_t.

Equilibrium

A steady state equilibrium is a set of constants $\{c, n, m', m, b, D^*, r\}$ and a value of p_t for each $t \geq 0$ for which, given M_0 and $g \geq 0$,

i. agents minimize transaction costs by choosing D^* to satisfy (1) at equality and maximize (2) subject to (3)-(7)

ii. the government satisfies its budget constraint:

$$g = \frac{M_t - M_{t-1}}{p_t} = \frac{(x-1)M_{t-1}}{xp_{t-1}}$$

iii. each securities market clears: $b_t = 0$ for each location Z.

iv. the money market clears: $M_t = m_t + p_t g$.

v. all goods markets clear: $n = (1+kD^{*2})c + g$.

By Walras' law, if the securities and money markets clear, so will the goods markets.

The following proposition states conditions guaranteeing the existence of a steady state equilibrium and characterizes such equilibria.

PROPOSITION ONE: EXISTENCE OF AN EQUILIBRIUM WITH CASH AND TRADE CREDIT

If $k>0$ and $[2x/(2+k)]<\beta<1$, given $x\geq 1$, then there exists a stationary equilibrium with $r>0$, $0<p<+\infty$ and $D^{*}\epsilon(0,1/2)$.

The proof of this proposition, like that for all subsequent results, is presented in the appendix.[7]

3. Optimality and Credit Controls

Optimal steady state allocations for this economy maximize the representative agent's utility subject to the condition that at each date government and private consumption plus transaction costs not exceed total resources. The solution to this maximization problem indicates that a Pareto optimal allocation must satisfy two conditions: that no transaction costs are incurred and that the marginal rate of substitution between current consumption and current leisure (MRS(c,e-n)) equals the marginal rate of transformation (MRT) of one.[8] Equations (1) and (13) show that a steady state equilibrium will satisfy these conditions only if the nominal interest rate, r, is zero. From proposition one, then, the equilibria of economies with $k>0$ and $\beta<1$ are not Pareto optimal because they involve positive values of r and thus D^{*}. The distortion in the consumption-leisure allocation in such equilibria also arises in the cash-in-advance literature (e.g. Lucas (1982)) and in both cases stems from the assumed institutional restriction that securities and goods markets operate sequentially. This restriction forces the representative agent to trade current leisure for future consumption, creating a wedge between the MRS(c,e-n) and the MRT when the nominal interest rate is positive, even if $g=0$.[9]

One obvious way of eliminating the resource waste from transaction costs is with a credit control policy--a legal restriction on the fraction of purchases made with trade credit. Although a credit control policy cannot eliminate the inefficiency in the consumption-leisure allocation, credit controls can be welfare-improving.[10]

Consider the impact of a credit control policy in an economy with $k>0$ and $[2x/(2+k)]<\beta<1$ and thus a laissez-faire steady state equilibrium that has $r\epsilon(0,k/2)$ and $D^*\epsilon(0,1/2)$ and so is not Pareto optimal. In designing a credit control poicy, the government chooses the fraction of purchases that may be financed with trade credit.[11] Let 2δ denote this fraction, with $\delta\epsilon[0,D^*)$; thus, 2δ is less than $2D^*$, the fraction of purchases financed with trade credit in the absence of credit controls.[12] The government selects δ to maximize the representative agent's indirect utility subject to equations (3)-(5) and the market clearing conditions, all with D^* set equal to δ. Although the solution to this maximization problem cannot be derived analytically, the agent's utility under some binding credit control policies (i.e. some value of $\delta\epsilon[0,D^*)$) can be shown to be higher than under laissez-faire. This is the approach taken here.

A steady state equilibrium under the *quantitative credit control policy* described above has $1+r=(x/\beta)$ and $V(c,e-n)=1+k\delta^2+r(1-2\delta)$, both from the first-order conditions, and $n=[(1-2\delta)x+2\delta+k\delta^2]c$ from constraints (3)-(5) combined with the market clearing conditions. Thus, equilibrium labor supply satisfies

$$V\left(\frac{n}{[(1-2\delta)x+2\delta+k\delta^2]},e-n\right) = 1+k\delta^2+r(1-2\delta) \qquad (14)$$

given δ, where $r = \frac{x-\beta}{\beta}$. The following results summarize the effects of credit controls:

<u>LEMMA</u>: CONDITIONS FOR LABOR SUPPLY AND OUTPUT TO BE INCREASING IN δ

$\delta \geq \frac{x-1}{k}$ is sufficient for $\frac{dn}{d\delta}>0$

Note that, because $1+r=x/\beta$ in a steady state equilibrium, the condition $\delta \geq \frac{x-1}{k}$ may be written as $\delta \geq [\beta(1+r)-1]/k$, which never exceeds $[\beta(1+k/2)-1]/k$; the latter expression is less than one-half. Thus, the condition can be satisfied for meaningful values of δ (i.e. values between zero and one-half). In addition, if $g=0$ so that the government sets $x=1$, this lemma states that for any $\delta \geq 0$, $\frac{dn}{d\delta}>0$.

<u>PROPOSITION TWO</u>: CREDIT CONTROLS CAN BE WELFARE-IMPROVING

$\frac{dn}{d\delta}>0$ is a sufficient condition for steady state utility to be higher under a partial credit control policy (a setting of $\delta \epsilon (0,D^*)$) than if trade credit use is completely banned ($\delta=0$). If, in addition, labor supply is not too sensitive to changes in δ, steady state utility is higher with a partial credit control than none at all (a setting of $\delta=D^*$).

Utility functions exist for which a partial credit control policy yields an allocation that is Pareto superior to the competitive equilibrium. One example is $U(c,e-n)= \ell n(c)+\ell n(e-n)$.

Intuitively, the credit control scheme's desirability stems from the second-best nature of the competitive equilibrium allocation. There are two sources of inefficiency in the economy. First, trade credit use requires that resources be exhausted on transaction costs. At the aggregate level, these transaction costs drive a wedge between output and feasible consumption (see equilibrium condition (v), which is also the economy's feasibility constraint). This wedge reflects the fact that the social cost of using cash is zero, while the cost of trade credit is positive. It is minimized when only cash is used in exchange ($D^*=0$).

Second, the restriction requiring sequential trade prevents an individual from financing goods purchases with labor income earned in the period the purchases are made. This restriction distorts decision-making by driving a wedge between the MRS(c,e-n) and the MRT of leisure for consumption goods when the nominal interest rate is positive. With a positive nominal rate, an individual perceives cash as having a positive opportunity cost, despite the fact that cash purchases pose no social cost. Given this perception, he chooses to use trade credit when it is relatively less costly. That is, equation (1), which results in D^* being equated to r/k, determines his use of cash and trade credit. By setting D^* in this manner, he minimizes the distortion (i.e. wedge) in his consumption-leisure allocation. He does, nevertheless, choose less consumption and more leisure than is Pareto optimal.

Only government policies that drive the nominal interest rate to zero can eliminate both wedges completely. The credit control policy studied here does not affect the equilibrium interest rate. Rather, it affects equilibrium utility in two ways: directly, through the effect on transaction costs incurred, and indirectly, by altering the consumption-leisure allocation. Specifically, by reducing trade credit sales, the policy decreases the resources used on transaction costs, thereby directly increasing the level of consumption supported by any level of labor supply. Thus, the direct effect is to increase utility. A credit control also may affect utility indirectly by reducing labor supply (if the condition in the lemma is satisfied), shifting the equilibrium allocation toward leisure and away from consumption, and so lowering utility from its level in a laissez-faire equilibrium.

The superiority of the partial ban on trade credit stems from the dominance of the direct effect for sufficiently loose credit controls (values of δ close to r/k) and of the indirect effect for relatively stringent controls (values of δ close to zero). That is, in a laissez-faire economy, marginally tightening the credit control policy by reducing δ slightly below r/k decreases the wedge between output and feasible consumption without affecting the wedge between the MRS(c,e-n) and the MRT. This is welfare-improving. Analogously, in an economy with a complete ban on trade credit, marginally loosening the credit control policy by increasing δ has no effect on the wedge in the feasibility constraint, but decreases slightly the wedge between the MRS(c,e-n) and the MRT. This, too, is welfare-improving. More generally, as δ approaches zero from above, the wedge between output and feasible consumption decreases, and the wedge between the MRS(c,e-n) and the MRT rises. The value of δ that maximizes utility represents a trade-off between the two distortions in the economy, balancing the costs incurred from increasing opportunity costs against the gains from reducing the exhaustion of resources on transaction costs.

4. Conclusion

With a positive nominal interest rate, an individual incurs a positive opportunity cost for using cash. In the model presented here, he responds by economizing on his cash holdings, substituting credit for cash up to the point where the resource cost of using credit just equals the opportunity cost of cash at the margin. However, the private opportunity cost of cash is not a social cost, although the private resource cost of credit is. Hence, the use of credit is excessive from the social point of view. Credit controls can be socially beneficial, in this second best situation, because they mitigate somewhat the distortions of a positive nominal interest rate.

The case for credit controls would be present in any model of money in which the private and social costs of various media of exchange diverge. The result also holds in an overlapping generations version of the model (see Schreft (1987)). Similarly, in a money-in-the-utility function version of the model, a wedge between the marginal rate of substitution and the marginal rate of transformation would still exist, although its functional form would change because the opportunity cost associated with the use of cash would be offset somewhat by the utility gain from holding real balances. Thus, credit controls would still improve welfare. Finally, and perhaps most important, the model's policy conclusion does not require the assumed transaction cost structure. An inventory-theoretic model of money in which cash and checks are means of payment, with checks bearing a real verification or processing cost and cash bearing a real "shoeleather" cost of trips to the bank, would also generate the same conclusion.

This paper does not, however, show or claim to show that all types of credit controls can improve welfare in all environments. In this representative agent model, for instance, a ban of any magnitude on credit extended in the securities markets would have no real effects because no trade occurs in these markets in equilibrium. With heterogeneity, such a credit restriction would <u>reduce</u> welfare. In addition, more general preferences would not affect the result that agents use both cash and trade credit in exchange. They would, however, affect which goods are purchased in which manner, the size of the distortion from trade credit use, and the extent to which transaction costs are borne by buyers.

In practice, governments have monopolized the provision of currency, not paid interest on currency, and been unwilling or unable to maintain low nominal interest rates. This paper highlights the possibility that, in this second best world, some control of credit as a means of payment may be socially beneficial. Of course, in practice, the costs of implementation and enforcement of credit controls would have to be considered. The model provides a simple, tractable framework for identifying the social loss caused by the private sector's substitution of credit means of payment (i.e. trade credit, checks, credit cards) for cash when nominal interest rates are positive. As such, it can serve as a useful point of departure for thinking about the imposition of credit controls in inflationary environments.

Appendix

Proof of Proposition One:

Agent optimization requires that (8)-(12) hold. As already indicated, these can be combined to yield equation (13). Further, (8) for dates t and t+1, (9) and (10) combined imply that $1+r=x/\beta$ in a stationary equilbrium, and thus that $D^*=r/k=(x-\beta)/\beta k$. For $k>0$ and $2x/(2+k)<\beta<1$, $0<D^*<1/2$, meaning that both cash and trade credit are used in exchange. From the government budget constraint combined with the money market clearing condition, $g=(x-1)m/p$. Equation (5) yields equilibrium real balances: $m/p=(1-2D^*)c$. The equations for g and m/p may be combined with the goods market clearing condition to get $n=\gamma c$, where $\gamma=2D^*+kD^{*2}+x(1-2D^*)$. Finally, this equation may be combined with (13) and the solution for D^* to obtain one equation in n that combines optimization, satisfaction of the government budget constraint, and market clearing: $V(n/\gamma,e-n) = 1 + (x-\beta)/\beta - (x-\beta)^2/\beta^2k$. The monotonicity assumptions on $V(\cdot,\cdot)$ and the boundary conditions, $\lim_{c_t\to 0}V(c,e-n)=\infty$ and $\lim_{n_t\to e}V(c,e-n)=0$, guarantee that the left side of this equation is decreasing in n, and thus equals the constant on the right side at some $n>0$. This solution for n can be used with the market clearing conditions, the government budget constraint, and equations (3)-(5) to determine c, m, m′, p and g, all positive and finite, and $b=0$.■

Proof of Lemma:

Totally differentiating (14) yields

$$\frac{dn}{d\delta} = \frac{(-2x+2+2k\delta)V_1n+2k\delta-2r}{V_1[(1-2\delta)x+2\delta+k\delta^2]-V_2}$$

where V_i is the derivative of V(c,e-n) with respect to its ith argument. By assumption, $V_1<0$ and $V_2>0$. Also, any binding credit control has $\delta<D^*\leq 1/2$. This implies that $(1-2\delta)x+2\delta+k\delta^2 > 0$. Thus, the denominator of this derivative is negative. Looking at the numerator, $2k\delta-2r<0$ because $\delta<D^*=\frac{r}{k}$. In any equilibrium, $n\geq 0$ (see equation (6)). The numerator is negative if $-2x+2+2k\delta\geq 0$; that is, $\delta\geq\frac{x-1}{k}$ implies $\frac{dn}{d\delta}>0$.■

Proof of Proposition Two:

Steady state utility under the policy is

$$U\left(\frac{n}{(1-2\delta)x+2\delta+k\delta^2}, e-n\right), \tag{A.1}$$

where n is the labor supply function. Differentiating (A.1) with respect to δ yields

$$\frac{\partial U}{\partial \delta} = \frac{-2U_1 n(-x+1+k\delta)}{[(1-2\delta)x+2\delta+k\delta^2]^2} + \frac{dn}{d\delta}\left[\frac{U_1}{(1-2\delta)x+2\delta+k\delta^2} - U_2\right]. \tag{A.2}$$

At $\delta=0$, this derivative equals $\frac{2U_1 n(x-1)}{x} + \frac{dn}{d\delta}\left[\frac{U_1}{x} - U_2\right]$. The first term in the sum is nonnegative because $x\geq 1$ and $U_1>0$. From (14) evaluated at $\delta=0$, $V(\cdot,\cdot) = U_1/U_2=1+r=x/\beta>x$ because $0<\beta<1$. Thus, the bracketed term is positive and $\frac{dn}{d\delta}>0$ is sufficient for $\frac{\partial U}{\partial \delta}>0$ when $\delta=0$.

To prove the second part of the proposition, evaluate (A.2) at $\delta=r/k=D^*$. This derivative is negative iff

$$\frac{dn}{d\delta}\left[\frac{U_1 k}{(k-2r)x+2r+r^2} - U_2\right] < \frac{2U_1 n(-x+1+r)k^2}{[(k-2r)x+2r+r^2]^2}. \tag{A.3}$$

The right side of this inequality is positive. Using (14), the bracketed term on the left side is positive. Thus, $\frac{dn}{d\delta}$ must be sufficiently small for welfare to be improved by the imposition of a partial ban on trade credit.[13]■

Figure 1

date t begins	debts are repaid	securities markets operate	goods markets operate	consumption occurs	date t+1 begins

Footnotes

1. See Hodgman (1973) for a survey of credit control use in Western Europe.

2. The assumption that the various goods available at a given date are perfect complements is made for tractability and is often used in models with both cash and credit (e.g. see Englund and Svensson (1988)). Note that utility maximizing bundles consistent with this assumption may involve consumption of larger quantities of some goods than of others, provided that the set of goods consumed in larger quantity is of measure zero. Here, c_t may be thought of as consumption of a composite good at date t.

3. If trade credit purchases are costly for the government, then the assumption that no such purchases are made is the same as the assumption that the government raises only the minimum amount of revenue needed for its expenditures. Of course, if trade credit purchases are not costly for the government, perhaps because the government is located at every point on the circle and thus known to everyone, the government would be indifferent between using cash and trade credit; either type of purchase would require the government to print the same amount of currency before debts are repaid to finance its expenditures.

4. Kroncke, Nemmers and Grunewald (1978, pp. 126-127) describe the information gathering activities firms undertake before approving trade credit sales. Goodfriend (1988) discusses the role that verification and monitoring costs have played in the development of payments systems.

5. Note that preferences are such that the representative agent travels around the circle once during each shopping period, buying an equal amount of goods at each point and financing purchases in the least costly manner. Consequently, the model does not allow the agent to shop more frequently at some stores than at others and thus to be better known and able to use trade credit at lower cost from the stores he visits more often.

6. Note that the agent's total cost of using trade credit to buy goods sold at distances $D \leq D^*$ from his home is $2\int_0^{D_t^*}(1+kD)p_t c_t dD$ which, after integrating, is simply $(2D_t^*+kD_t^{*2})p_t c_t$. This reflects that a fraction $2D_t^*$ of total date t purchases is financed with trade credit and that kD_t^{*2} is spent on transaction costs. (Recall that D measures the closest distance in each direction and that the circle has a circumference of one, so $D^* \epsilon [0,1/2]$.) The term kD_t^{*2} may be written $(2D_t^*)(kD_t^{*2}/2)$, where $kD_t^{*2}/2$ is the average interest rate (or transaction cost) incurred on trade credit purchases made. Analogously, the total expenditure on cash purchases is $2\int_{D_t^*}^{1/2} p_t c_t dD$ or, after integrating over distance, $(1-2D_t^*)p_t c_t$. This indicates that a fraction $1-2D_t^*$ of date t purchases is financed with cash. Equations (3)-(5) are written for simplicity after integrating over distance.

7. In contrast, if $k=0$ or $[2x/(2+k)] \geq \beta$, then a nonmonetary steady state equilibrium exists. See Schreft (1987).

8. Balasko and Shell (1980) show that a stationary equilibrium exists in which these necessary conditions are satisfied and the real interest rate is nonnegative is Pareto optimal.

9. A government deficit (i.e. a value of $g>0$) financed with an inflation tax as modelled here only increases the wedge, worsening the distortion in the consumption/leisure allocation already present from the environment's trading arrangements.

10. Schreft (1989) shows for a version of this economy with $g=0$ that the government can achieve a Pareto optimal allocation by deflating at rate $1/\beta$. Optimal deflation is not a feasible policy for the economy studied here because $g>0$ is possible, and all government expenditures are assumed to be financed with money creation.

11. A quantitative restriction on all credit extensions, including those that would be considered analogues of the trade credit sales in this model, was imposed in the United States in the spring of 1980 by the Federal Reserve Board at the request of President Jimmy Carter. The program's effects were pervasive. To monitor credit extensions, the Federal Reserve System required member and non-member banks, consumer finance companies, credit unions, thrifts, retails stores, and airlines to submit regular reports on their credit extensions. The Fed's Fifth District, for example, received reports from Bailey Lumber Company (Bluefield, WV), Hofheimer's Inc. (a Norfolk, VA retail shoe store chain), Allegheny Airlines (Washington, DC), The Melart Jewelers, Inc. (Silver Springs, MD), and Air Conditioning Corporation (a Greensboro, NC mechanical contractor). For a detailed analysis of the 1980 experience with credit controls, see Schreft (1990).

12. This credit control policy could be implemented by requiring that all sellers file with the government periodic reports revealing their extensions of trade credit, and imposing a fine for not reporting. The reports could be used to monitor compliance with a direct quantitative restriction on the amount of trade credit issued or to impose an ad valorem sales tax on all trade credit purchases. In the model of section 2, an ad valorem sales tax on trade credit purchases would be completely passed onto consumers because the goods markets are perfectly competitive and demand is inelastic. Thus, an agent buying a unit of consumption goods from a seller at a distance D from home must pay $p(1+kD)(1+\tau)$ if $\tau \geq 0$ is the ad valorem sales tax rate. Using this cost in the left side of equation (1) yields $D^* = (r-\tau)/k(1+\tau)$ as the distance at which the agent is indifferent between the use of cash and credit. Clearly, any direct quantitative restriction on trade credit to a fraction 2δ of total purchases, for $\delta \epsilon [0,r/k)$, is equivalent to an ad valorem trade credit sales tax with the tax rate set to satisfy $\delta = (r-\tau)/k(1+\tau)$. In practice, the cost of implementing such a sales tax will be low if other types of sales taxes are already in place.

13. For the utility function $u(c,e-n) = \ell n(c)+\ell n(e-n)$, the bracketed term on the left side of (A.3) is positive and, if $\delta>(x-1)/k$, $\frac{dn}{d\delta}$ is positive but sufficiently small.

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