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External Increasing Returns, Short-Lived Agents and Long-Lived Waste

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EXTERNAL INCREASING RETURNS,

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ABSTRACT

External Increasing Returns, Short-lived Agents and Long-lived Waste

Actions that affect environmental quality both influence and respond to macroeconomic variables. Further, many environmental and macroeconomic consequences of current actions will have uncompensated effects that outlive the actors. This paper presents an overlapping-generations model of environmental externalities and capital accumulation: consumption of the old generates long-lived garbage as a by-product, while young agents invest in both capital and destruction of the existing garbage stock. The model also assumes external increasing returns: increases in the capital stock increase the future productivity of capital. In the model, increases in the natural rate of degradation do, and improvements in society's ability to dispose of garbage may, encourage capital accumulation. Multiple Pareto-ranked equilibria can arise as a consequence of the interaction between garbage and capital accumulation. Underaccumulation of garbage, analogous to dynamically inefficient overaccumulation of capital, can arise in the model.

The family which takes its mauve and cerise, air-conditioned, power-steered, and power-braked automobile out for a tour passes through cities that are badly paved, made hideous by litter, blighted buildings, billboards, and posts for wires that should long since have been put underground. They pass on into a countryside that has been rendered largely invisible by commercial art... They picnic on exquisitely packaged food from a portable icebox by a polluted stream and go on to spend the night at a park which is a menace to public health and morals. Just before dozing off on an air mattress, beneath a nylon tent, amid the stench of decaying refuse, they may reflect vaguely on the curious unevenness of their blessings. Is this, indeed, the American genius?

J. K. Galbraith, *The Affluent Society* (1958)

What stands between us and a decent environment is not the curse of industrialization, not an unbearable burden of cost, but just the need to organize ourselves consciously to do some simple and knowable things. Compared with the possibility of an active abatement policy, the policy of stopping economic growth in order to stop pollution would be incredibly inefficient. It would not actually accomplish much, because one really wants to reduce the amount of, say, hydrocarbon emission to a third or a half of what it is now. *And what no-growth would accomplish, it would do by cutting off your face to spite your nose.*

R. M. Solow, "Is the End of the World at Hand?" (1973)

I. INTRODUCTION

There is currently a great deal of concern about the state of the environment. From the perspective of standard economic analysis, the problem might seem evident, and the solution straightforward: production and consumption decisions generate external effects, implying a need for Pigovian taxes or the creation of appropriate markets. Such analysis ignores two important aspects of environmental problems. First, the macroeconomic perspective is missing. Actions that affect the environment both influence and respond to macroeconomic variables. Environmental policy decisions frequently concern the intertemporal allocation of resources, and so have implications for economic growth, capital accumulation, saving and interest rates. Second, the externalities are inter- as well as intragenerational: actions taken today affect the welfare of generations as yet unborn. Such external effects are intrinsically hard to internalize; their existence almost surely alters the set of desirable policies.

This paper considers how agents make waste production and disposal decisions in a setting where these have long-lasting effects. Waste in this model represents the long-lived, noxious by-products of consumption, be they groundwater pollution from solid-waste landfills, toxic waste dumps, spent nuclear fuel rods, medical debris on New Jersey beaches, or whatever. The model is formulated in terms of the accumulation of a stock of waste, but this stock could equally represent some more general scalar index of environmental quality. Consumption activities generating waste as a by-product are utility-enhancing, but the waste produced imposes a negative externality on future generations that must either pay the costs of waste disposal or accept a lower standard of living.

The analysis draws on a number of important and diverse economic literatures. The model utilizes the overlapping-generations framework of Allais (1947) and Samuelson (1958), as extended by Diamond (1965) to include capital accumulation, since this demographic structure lends itself to analysis of situations where agents' actions have consequences beyond their own lifetimes. As Diamond (1965) embeds the features of Solow's (1956) neoclassical growth model in an overlapping-generations framework, it provides the natural starting point for analysis of the interaction between capital accumulation and intergenerational environmental diseconomies.

A number of significant recent contributions to growth theory emphasize the role of external increasing returns in the growth process; see Romer (1990). Romer (1986), Stokey (1988), Lucas (1988), and Schmitz (1989) all study external increasing returns in infinitely-lived agent models, while Freeman and Polasky (1989) and Mourmouras (1989) do so in an overlapping-generations framework. Such "new growth theory" models with overlapping generations exhibit positive external effects across generations: agents alive today cannot capture all the benefits of their actions. External increasing returns are a feature of this model also, permitting analysis of both positive and negative intergenerational spillovers.[1]

This work on growth theory is related in turn to the literature on coordination failure in macroeconomics (see, for example, Cooper and John (1988)). That literature considers circumstances in which rational agents, acting in their own self-interest, may achieve an inefficient equilibrium; coordinated behavior may be required to achieve a Pareto-superior outcome. Such coordination failures can arise in the model here. Like Azariadis and Drazen (1990), Murphy, Shleifer and Vishny (1989), and Weil (1989), among others, this paper provides a link between these two strands of the literature.

Environmental questions have been analyzed extensively in the environmental and natural resource literatures (see, for example, Baumol and

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[1]Chatterjee and Cooper (1989), Pagano (1989) and John (1988) also examine overlapping-generations models with externalities, but intergenerational externalities are not their principal focus.

Oates (1988), Conrad and Clark (1987), Dasgupta and Heal (1979) and Neher (1990)). The externalities considered in this literature are often divided into three categories: common-property problems (Dasgupta (1982) and Weitzman (1974a)); the upstream-firm problem (Loehman and Whinston (1970)) and pollution problems (Kneese and Mäler (1973), Plourde (1972) and Weitzman (1974b)). Researchers have investigated mechanisms under which a decentralized economy might successfully internalize environmental externalities. Such mechanisms include markets for effluents, pollution licenses and Pigovian taxes. On the basis of such analysis, policy prescriptions have been offered and in some cases successfully implemented (Hahn (1989)). But by assuming that the life span of individuals and the life span of the economy are the same (possibly infinite), researchers in environmental economics have for the most part restricted themselves formally to the analysis of intragenerational conflict. While our analysis also investigates policies that internalize the externalities present in the model, the design of such policies is complicated by our explicitly intergenerational focus.

Questions of intergenerational equity have been extensively discussed in the exhaustible-resource literature (see, for example, Solow (1974, 1986)), but not, for the most part, in models of pollution. It is particularly striking that there is almost no use of an overlapping-generations framework in either the exhaustible-resource or pollution literatures, despite the appeal of this demographic structure for explicit analysis of intergenerational issues. Kemp and Long (1980), Mourmouras (1990) and Sandler (1982) are important exceptions: Kemp and Long and Mourmouras construct overlapping-generations models of natural resources;

Sandler analyzes the optimal provision and maintenance of club goods in a finite-horizon economy.[2] Intergenerational externalities are also a feature of John, Pecchenino and Schreft's (1989) analysis of the stockpiling of nuclear weapons.

In a sense, this paper revisits the "limits to growth" debate of the 1960s and 1970s, as perhaps exemplified by the Galbraith and Solow quotes at the start of the paper. The questions addressed in that literature are again timely, since modern theory is providing new arguments for subsidizing growth at a time when environmental concerns are gaining prominence in political debate. Environmental models with an explicit intergenerational and macroeconomic focus are needed.

[2] Schimmelpfennig (1990) studies the greenhouse effect in an economy with renewable resources and an overlapping-generations structure. Howarth and Norgaard (1990) use a three-period model with distinct generations to consider the impact of property rights on intergenerational equity.

II THE ENVIRONMENT

Preliminaries

Consider an infinite-horizon economy comprised of finitely-lived individuals and perfectly competitive firms. A new generation (called generation t) is born at each date $t = 1, 2, \ldots,$ and lives for two periods. Assume no population growth and normalize the size of each generation to unity. Young agents have preferences defined over consumption in old age, c_{t+1}, and the stock of garbage when they consume, g_{t+1}. These preferences are represented by the utility function

$$U(c_{t+1}) - \phi(g_{t+1}).$$

Assume that $U(\cdot)$ is strictly concave, $\phi(\cdot)$ is strictly convex, and both are increasing and twice continuously differentiable. Assume also that $\lim_{c \to 0} U'(\cdot) = \infty$.[3]

Young agents are each endowed with one unit of labor which they supply to firms inelastically. They divide their wage, w_t, between saving for old age consumption, s_t, and investment in the destruction of garbage, d_t. When old, agents supply their saving inelastically to firms and earn the gross return $(1 + \bar{r}_{t+1})$.

The garbage stock evolves according to

$$g_{t+1} = (1 - b)g_t + \beta c_t - \gamma d_t$$

where b is the natural biodegradation of garbage, $b \in [0, 1]$, βc_t is the augmentation of the garbage stock by the consumption of the old at t, $\beta > 0$,

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[3]Note that we exclude intergenerational altruism.

and γd_t is the diminution of the stock of garbage by the destruction efforts of the young at t, $\gamma > 0$.

To enjoy the fruits of their labor fully, agents may choose to clean up their environment prior to consuming. While its efforts reduce the existing stock of waste for itself and for future generations, the present generation is concerned only with its own welfare and ignores any benefits bestowed on its progeny. Consumption of those now alive, however, produces garbage that impinges on future generations' utility; this effect, too, is ignored by those currently alive.

The firms are perfectly competitive, profit maximizers who produce output using the production function $Y_t = a\psi(K_{t-1})F(K_t,N_t)$. The function F() is a standard neoclassical constant-returns production function, implying that output per worker can be written as $y_t = a\psi(K_{t-1})f(k_t)$, where k_t is the capital-labor ratio, and where $f'(\cdot) > 0$, $f''(\cdot) \le 0$ and $\lim_{k\to\infty} f'(\cdot) = 0$. The function $\psi(K_{t-1})$ represents enhancements to productivity from last period's capital ($\psi'(\cdot) > 0$); it is thus a technological externality. Because K_{t-1} is predetermined at time t, $\psi(K_{t-1})$ enters the production technology as a constant from the perspective of current producers. Since the population is normalized to one, $\psi(K_{t-1})$ can be written as $\psi(k_{t-1})$. It is assumed that the capital stock depreciates at rate δ, with depreciation occurring during the production process. Finally, a is a technology parameter.

The inclusion of last period's capital stock in the current production technology is motivated by the recent literature on external increasing returns in growth models, emphasized in particular by Romer (1986); see also Weil (1989). Romer's insight, drawing on Arrow's (1962) analysis of learning-by-doing and Young's (1928) analysis of increasing returns, is that production generates knowledge as a by-product. Although production at any time period is a constant-returns activity, the model exhibits increasing returns from an intertemporal social perspective.

Since our focus is on external effects across generations, we wish to abstract from the well-understood free-rider problems within a generation. We assume that each generation elects a government at the beginning of each date for a one-period term. This government behaves myopically, carrying out policies made solely for the welfare of agents alive during its term in office. The government has the power to levy lump-sum and distorting taxes and transfers. Specifically, it levies lump-sum taxes on the young to achieve the desired destruction of garbage. That is, an agent's choice of destruction can be interpreted as arising from the collective provision of waste depletion, a public good. This allocation could be achieved as a Lindahl equilibrium.

In our analysis we also consider the effects of a distorting tax (transfer) on the return to capital and a proportional (lump-sum) tax (transfer) on wages accompanied by a lump-sum transfer of the proceeds to the old agents. These policies may be used to alter the rate of capital accumulation and thus the augmentation of the stock of garbage. A myopic government, concerned only about the short-run effects of garbage stock diminution, would never have an incentive to impose such taxes. By contrast, an infinitely-lived institution, set up with the goal of implementing policies to improve the welfare of the current and all future generations, may choose a policy that internalizes the intergenerational externalities. The problem of such an institution is considered in Section VI. In the remainder of this section and in Sections III - V, only a short-lived government is assumed to exist.

The representative agent's optimization problem

The representative agent takes as given the wage, w_t, the return on saving, $\bar{r}_{t+1}$, and the stock of garbage at the beginning of period t, g_t. Thus, the competitive lifetime choice problem of a representative agent is to choose c_{t+1}, d_t and s_t to maximize

$$U(c_{t+1}) - \phi(g_{t+1}) \tag{1}$$

subject to

$$w_t = s_t + d_t \tag{2}$$

$$c_{t+1} = (1 + \bar{r}_{t+1})s_t \tag{3}$$

$$g_{t+1} = (1 - b)g_t + \beta c_t - \gamma d_t \tag{4}$$

$$c_{t+1},\ d_t,\ s_t \geq 0.$$

After substituting equations (2) - (4) into (1), utility maximization yields the following first-order condition:

$$U'(\cdot)(1 + \bar{r}_{t+1}) - \gamma\phi'(\cdot) = 0. \tag{5}$$

The individual chooses s_t to equate the marginal utility of consumption, multiplied by the return to saving, to the marginal disutility of garbage, multiplied by the return from destruction. Equation (5) assumes an interior solution with a positive level of destruction, but nothing in the model precludes the possibility that agents might choose not to engage in any destruction, and hence set $d_t = 0$. We discuss this case further below.[4]

The representative firm's problem

The individual firm takes the wage and the rental rate on capital as given. It hires labor until the marginal product of labor equals the wage,

$$a\psi(k_{t-1})[f(k_t) - k_t f'(k_t)] = w_t, \tag{6}$$

and hires capital until the marginal product of capital equals the rental rate $[r_t]$,

$$a\psi(k_{t-1})f'(k_t) = r_t. \tag{7}$$

Goods market clearing

For the goods market to clear the demand for goods must equal the supply of goods. Thus,

$$s_t + d_t + c_t = w_t + r_t k_t + (1 - \delta)k_t. \tag{8}$$

Combining equations (2), (3) and (8) yields

$$k_t = s_{t-1}, \tag{9}$$

given that $\bar{r}_{t+1} = r_{t+1} - \delta$: the return on saving equals the net return on capital. Thus, the capital stock at t is fully determined by saving at t - 1.

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[4]Note that the boundary conditions on the utility function ensure that agents will always wish to choose a strictly positive level of consumption. In the absence of transfers to the old, this is sufficient to guarantee that saving will also be strictly positive.

III THE STEADY STATE

Definition

A competitive steady-state equilibrium for this economy is given by (c, d, g, w, r, s, k) such that:

(a) agents maximize (1) subject to (2) - (4), given w, r, δ, and g;

(b) the factor markets clear:

$$w = a\psi(k)[f(k) - kf'(k)] \tag{10}$$

$$r = a\psi(k)f'(k); \tag{11}$$

(c) the goods market clears:

$$w + (1 + r - \delta)k = s + d + c; \tag{12}$$

(d) the stock of garbage is constant:[5]

$$g = \frac{\beta}{b} c - \frac{\gamma}{b} d. \tag{13}$$

A steady-state equilibrium can be characterized by the first-order condition (5) evaluated at the steady state, the market clearing conditions, and the law of motion for the stock of garbage. Equations (10) - (12) and the agent's budget constraint can be combined to yield the following equilibrium relationships:

$$s = k, \tag{14}$$

$$r(k) = a\psi(k)f'(k), \tag{15}$$

$$y(k) = a\psi(k)f(k), \tag{16}$$

$$w(k) = y(k) - r(k)k \tag{17}$$

$$c(k) = [1 + r(k) - \delta]k, \tag{18}$$

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[5]This is the steady-state condition for $b > 0$. We consider the case where $b = 0$ below.

$$d(k) = w(k) - k. \tag{19}$$

Substituting (17) - (19) into (13), we obtain

$$g(k) = (\beta/b)(c(k)) - (\gamma/b)[w(k) - k].$$

It is convenient to write this as

$$g(k) = \left(\frac{1}{b}\right)\left([\beta(1-\delta) + \gamma]k + \rho(k)\right) \tag{20}$$

where

$$\begin{aligned} \rho(k) &= \beta r(k)k - \gamma w(k) \\ &= \left((\beta + \gamma)\left(\frac{r(k)k}{y(k)}\right) - \gamma\right)y(k). \end{aligned} \tag{21}$$

Equation (20) gives the steady-state level of waste as a function of the capital stock. Suppose, counterfactually, that young agents devoted their entire wage to destruction. Then labor's share of output would go towards destruction and capital's share of output would go to consumption. The net effect on garbage would be given by $\rho(k) = \beta r(k)k - \gamma w(k)$. Now the young actually devote some of their wages to saving. Each unit of saving (capital) implicitly represents one less unit of destruction and $(1-\delta)$ more units of consumption when old; thus each unit of capital also contributes $(\beta(1-\delta) + \gamma)$ to the garbage stock. The net addition to the garbage stock each period therefore equals $[\beta(1-\delta) + \gamma]k + \rho(k)$, which must equal the natural depletion of the garbage stock (bg) in steady-state equilibrium.

We can thus identify a number of ways in which changes in the capital stock will affect the steady-state stock of garbage. First, as just noted, additional capital directly implies less destruction and more consumption, so that if $\rho(k) = 0$, garbage increases linearly with the capital stock. Second, increases in the capital stock increase output, and thus imply higher payments to capital and labor. The effect of this change on the garbage stock depends on whether or not the young's destruction exceeds the old's addition to garbage at the margin; that is, it depends upon whether $\rho(k)$ is positive or negative. The sign of $\rho(k)$ depends on the relative shares of capital and labor, the size of the consumption externality, and the productivity of the destruction technology. Third, changes in the capital stock may change the stock of garbage by altering relative factor shares. If, for example, a higher capital stock is associated with higher payments to capital (so $r(k)k/y(k)$ is increasing in k), then increases in the capital stock will, through this channel, increase garbage.

A steady-state equilibrium for this economy is represented by the steady-state garbage equation (20) and the first-order condition

$$U'(c(k))(1 + r(k) - \delta) - \gamma\phi'(g) = 0, \tag{22}$$

where $c(k)$ and $r(k)$ are as just defined. An equilibrium for this model is illustrated in Figure 1.

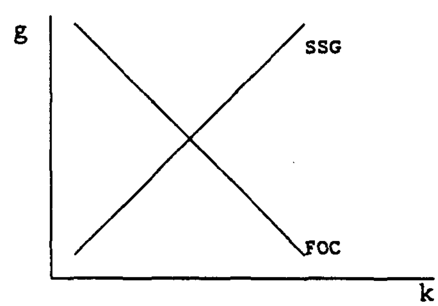

Figure 1

This figure is drawn under the assumptions that steady-state garbage (SSG) is increasing in k and that the first-order condition (FOC) defines an inverse relationship between k and g. We discuss these assumptions further below. Before considering the comparative static properties of the model and the possibility of multiple equilibria, we briefly turn to some special cases, to which we will then refer throughout the paper.

Special Case I: zero destruction ($d = 0$)

As noted in the discussion of the agent's problem, the equilibrium may entail a corner solution where agents engage in zero destruction. If young agents are not engaging in destruction, then they are saving their entire wage income. This implies in turn that, in steady state,

$$k = s = w(k) = a\psi(k)[f(k) - kf'(k)]. \tag{23}$$

This equation implicitly defines the capital stock, k_{zd}. Assuming zero destruction, the garbage stock is given by

$$g = g_{zd}(k) = \left(\frac{\beta}{b}\right)(c(k)). \tag{24}$$

Recalling that $g(k) = (\beta/b)c(k) - (\gamma/b)d(k)$, it follows that $g(k_{zd}) = g_{zd}(k_{zd})$. Finally, note that zero destruction will be an equilibrium if and only if

$$U'(c(k_{zd}))(1 + r(k_{zd}) - \delta) - \gamma\phi'(g_{zd}) \geq 0. \tag{25}$$

This equilibrium is illustrated in Figure 2, where equation (24) is graphed as SSG_{zd}.

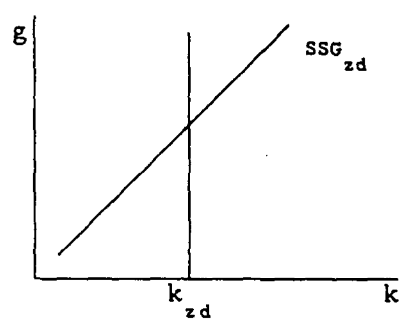

Figure 2

We can divide k-g space into two regions depending upon whether or not agents engage in positive destruction. A member of generation t takes as given the garbage stock at time t and the capital stock at t and t-1. We obtain the zero-destruction locus (ZDL) dividing the regions by setting $k_{t-1} = k_t$ and finding (k_t, g_t) pairs such that the first-order condition (5) is satisfied with equality when $d_t = 0$. We illustrate positive- and zero-destruction equilibria in Figures 3a and 3b.[6]

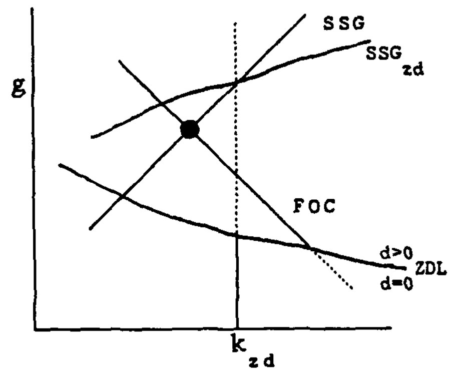

Figure 3a

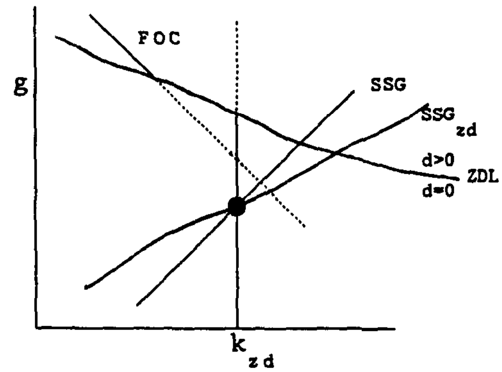

Figure 3b

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[6]Under the assumptions that $d_t = 0$ and $k_t = k_{t-1}$, it follows that

$$k_{t+1} = w(k_t),$$

where w() is the steady-state function defined in equation (17). Therefore,

$$r_{t+1} = \tilde{r}(k_t) = a\psi(k_t)f'(w(k_t));$$

$$c_{t+1} = \tilde{c}(k_t) = (1 + \tilde{r}(k_t) - \delta)w(k_t).$$

Also,

$$g_{t+1} = (1 - b)g_t + \beta c(k_t).$$

Using these definitions, the first-order condition will be satisfied with equality at time t, assuming $d_t = 0$, when

$$U'(c_{t+1})(1 + r_{t+1} - \delta) - \gamma\phi'(g_{t+1}) = 0.$$

The ZDL is thus defined by

$$\Rightarrow \quad U'(\tilde{c}(k))[\tilde{c}(k)/w(k)] - \gamma\phi'((1 - b)g_{ZDL} + \beta c(k)) = 0.$$

At k_{zd}, $\tilde{r}(k) = r(k)$, $\tilde{c}(k) = c(k)$ and $w(k) = k$. Comparing this equation with equation (22), it follows that the FOC will lie above the ZDL at k_{zd} if and only if $(\beta/b)c(k_{zd}) > g_{ZDL}$. Since the left-hand side of this inequality is simply $g_{zd}(k_{zd})$, we can observe either Figure 3a, where the SSG_{zd} line lies above the FOC, which in turn lies above the ZDL at k_{zd}; or we can observe Figure 3b, where the ordering is reversed. Note, finally, that the ZDL can be shown to be downward sloping if $(1 - \sigma) > 0$, if $c'(k_t) > 0$, and if there are no external increasing returns.

For a positive destruction equilibrium, it must be the case that the SSG line intersects the FOC line above the ZDL line, as in Figure 3a.[7] In a zero-destruction equilibrium, the SSG_{zd} line must lie below the ZDL at k_{zd}, as illustrated in Figure 3b.

Special Case II: zero garbage ($\beta = 0$)

Since an aim of this paper is to understand how an environmental externality affects capital accumulation, it is useful to consider the no-garbage economy as a reference point. The economy in this case can be understood as a special case of the no-destruction economy, since agents will obviously not engage in destruction if there is nothing to destroy. Agents thus save all their wage income, and the equilibrium capital stock will, as above, be implicitly defined by equation (23). Note that this example is also the special case of the Diamond (1965) model where agents do not consume in youth.

Special Case III: zero degradation ($b = 0$)

In practice, the natural rate of biodegradation depends in part on how garbage is stored. When biodegradable garbage is placed in the anaerobic conditions of a landfill, the biodegradation rate has been found to be extremely slow. Hanson (1989) reports that, after 25 years in a landfill,

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[7] In addition, it must be the case that $w(k) > k$. Unlike the zero-destruction locus, which summarizes agents' behavior, this is essentially a technological restriction on k for destruction to be positive. In terms of the figures, it implies that SSG must lie below SSG_{zd} at a positive-destruction equilibrium. Figures 3a and 3b are drawn such that $w(k) > k$ for $k < k_{zd}$.

waste retains its original weight, volume and shape.[8] Given this, it is worth examining the limiting case where garbage does not degrade at all.[9]

A steady-state equilibrium in the absence of degradation will be characterized by garbage destruction by the young that exactly offsets the addition to the garbage stock as a result of the consumption of the old. That is, at a steady state with $b = 0$, it must be the case that

$$\beta c(k_{b=0}) - \gamma d(k_{b=0}) = 0$$

$$\Rightarrow \quad \rho(k) = - [\beta(1-\delta) + \gamma]k \tag{26}$$

In this case, the garbage equation serves to pin down the equilibrium capital stock. This equilibrium is illustrated in Figure 4.

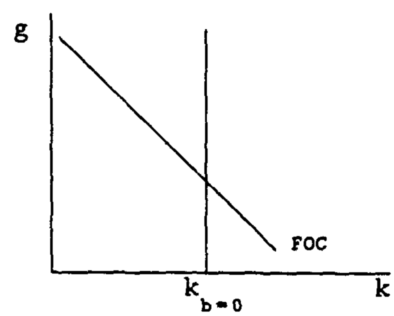

Figure 4

[8] William Rathje, an archeologist and organizer of a study of the waste production of households (*Le Projet du Garbàge*), has excavated a number of landfills, finding, among other things, forty-year-old readable newspapers and hot dogs that are still recognizable after decades. See Rathje (1984) for a discussion of *Le Projet du Garbàge*, and Luoma (1990) for a more general discussion of Rathje's work.

[9] Recall also that the variable we call garbage might represent some more general index of environmental quality. Under such an interpretation, it is not obvious that a positive rate of degradation is an appropriate assumption.

Special Case IV: Absence of External Increasing Returns ($\psi(k) = 1, \forall k$)

At various points throughout the paper, we consider the role of external increasing returns for the analysis. For the present, we note simply that we can eliminate the external increasing returns from the analysis by setting $\psi(k)$ everywhere equal to 1.

IV COMPARATIVE STATICS

The comparative static results that follow are based on the equilibrium with positive destruction, as illustrated in Figure 1. Where applicable, we note how our results are altered for the special cases considered above. Total differentiation of equations (20) and (22) yields the following system:

$$\begin{bmatrix} 1 & -\left(\frac{1}{b}\right)\left(\beta(1-\delta) + \gamma + \rho'(k)\right) \\ -\gamma\phi'' & \varsigma \end{bmatrix} \begin{bmatrix} dg \\ dk \end{bmatrix} =$$

$$\begin{bmatrix} -\frac{d}{b} & \frac{c}{b} & -\frac{g}{b} & \frac{\rho}{ab} & -\frac{\beta}{b}k \\ \phi' & 0 & 0 & -\frac{1}{a}U'(1-\sigma)r & U'(1-\sigma) \end{bmatrix} \begin{bmatrix} d\gamma \\ d\beta \\ db \\ da \\ d\delta \end{bmatrix} \qquad (27)$$

where $\varsigma = U'()\left[(1-\sigma)r'(k) - \sigma\frac{c}{k^2}\right]$ and $\sigma = -U''()c/U'()$.[10]

The determinant of the left-hand side matrix is

$$\Delta = U'(\cdot)\left[(1-\sigma)r'(k) - \sigma\frac{c}{k^2}\right] - \gamma\phi''\left(\frac{1}{b}\right)\left(\beta(1-\delta) + \gamma + \rho'(k)\right).$$

The following conditions are together sufficient for $\Delta < 0$:

(i) $(1-\sigma)r'(k) \leq 0$,

(ii) $\rho'(k) \geq 0$.

The $(1-\sigma)$ term in condition (i) indicates the response of saving to changes in the interest rate. If $(1-\sigma) > 0$, then the substitution effect dominates and saving is increasing in the interest rate, whereas if $(1-\sigma) < 0$, the income effect dominates and saving is decreasing in the

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[10]The elasticity of substitution, σ, is a function of k unless the utility function exhibits constant relative risk aversion. For compactness of notation, we suppress this dependence.

interest rate. In the absence of external increasing returns, higher values of the capital stock are associated with a lower rate of interest, so $r'(k) < 0$. If external increasing returns are present, $r'(k)$ may be positive.

To understand condition (ii), recall that

$$\rho(k) = \left[(\beta + \gamma)\frac{r(k)k}{y(k)} - \gamma\right]y(k).$$

If factor shares are constant, then $\rho(k)$ will be (perhaps inversely) proportional to output $(y(k))$.[11] In this case, $\rho(k) > 0$ implies and is implied by $\rho'(k) > 0$. This is true a fortiori if capital's share of output increases as the capital stock increases.

Referring back to Figure 1, condition (i) implies that the first-order condition is downward-sloping (since it implies that $\varsigma < 0$). and condition (ii) ensures that steady-state garbage is increasing in k. The following propositions set out the comparative static behavior of the model when conditions (i) and (ii) hold.[12]

Proposition 1: Economies with better destruction technologies accumulate less garbage in steady state, but may have more or less capital than economies with worse technologies.

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[11]For example, if $f(k) = k^{\nu}$, then $\rho(k) = [(\beta + \gamma)\nu - \gamma]a\psi(k)k^{\nu}$.

[12]Conditions (i) and (ii) are considered in greater detail in Section V, where we discuss the possibility of multiple equilibria in the model. The comparative static results derived below are easily reinterpreted if (i) or (ii) does not hold but Δ is still negative. If $\Delta > 0$, the equilibrium is not locally stable.

Proof: An economy's destruction technology is better if γ is higher. From the system of equations characterizing the steady state,

$$\frac{dg}{d\gamma} = \frac{1}{\Delta b}\left[-\ d(k)\varsigma + \phi'\left(\beta(1-\delta) + \gamma + \rho'(k)\right)\right] < 0;$$

$$\frac{dk}{d\gamma} = \frac{\phi'}{\Delta}\left[1 - \left(\frac{\gamma d(k)}{bg}\right)\left(\frac{\phi'' g}{\phi'}\right)\right] \gtrless 0. \circ$$

Agents allocate their wages so that at the margin they are indifferent between destroying garbage and consuming. Thus, an economy with a more productive destruction technology can devote fewer resources to destruction to achieve a given garbage stock. Agents in such economies thus have an incentive to substitute consumption and saving for destruction, but saving is relatively less effective and destruction relatively more effective for increasing utility. In terms of Figure 1, an increase in γ shifts both curves downward.

Changes in the productivity of the destruction technology are naturally irrelevant in any zero-destruction equilibrium. In the economy with no degradation, improvements in the destruction technology unambiguously increase the capital stock.

Proposition 2: More wasteful economies accumulate less capital and more garbage.

Proof: A more wasteful economy has a higher β. By the system of equations characterizing the steady state,

$$\frac{dk}{d\beta} = \frac{\gamma\phi'' c}{b\Delta} < 0;$$

$$\frac{dg}{d\beta} = \frac{c}{b\Delta}\varsigma > 0. \circ$$

In wasteful economies each unit of consumption generates a lot of garbage. Thus, to equate the marginal utility of consumption with the marginal disutility of garbage accumulation, individuals must dedicate more resources to destruction, leaving less to save and consume. In terms of Figure 1, the SSG curve lies further to the left. In a zero-destruction equilibrium, increases in β have no effect on the capital stock but still result in increased garbage in steady state. When b = 0, the comparative static results are unchanged.

Proposition 3: Economies with high biodegradation rates accumulate more capital and less garbage.

Proof: The parameter b is higher in an economy with a higher natural rate of biodegradation. Differentiating, we obtain

$$\frac{dk}{db} = - \frac{\gamma\phi'' g}{b\Delta} > 0;$$

$$\frac{dg}{db} = - \frac{g}{b\Delta} \zeta < 0. \quad \circ$$

The finding that the capital stock is increasing in the rate of degradation is intuitive: the higher is the natural rate of degradation, the smaller is the amount of destruction that need be undertaken by the agents in the economy. In Figure 1, the SSG line lies further to the right. Once again, changes in b have no effect on the capital stock in a zero-destruction equilibrium.

As noted earlier, the biodegradation rate may be very small in reality. Such slow rates may lead economies to find alternative forms of waste destruction, such as incineration; to reduce waste generation, perhaps through recycling; or to increase the rate of biodegradation, perhaps by switching from foam to paper products. These decisions may in turn have implications for the rate of capital accumulation.[13]

Proposition 4: More productive economies have higher or lower equilibrium stocks of capital and garbage than less productive economies.

Proof: A more productive economy has a higher a. Differentiating,

$$\frac{dk}{da} = -\frac{1}{a\Delta}\left[U'r(1-\sigma) - \frac{\gamma\phi''\rho}{b}\right] \lesseqgtr 0$$

where $(1 - \sigma)$ is of either sign, and $\rho \gtrless 0$. Further,

$$\frac{dg}{da} = \frac{1}{ab\Delta}\left[\varsigma\rho - [\beta(1-\delta) + \gamma + \rho']U'(1-\sigma)r\right] \gtrless 0$$

since both terms are either positive or negative. o

Increases in productivity will increase the interest rate, increasing the incentive to accumulate capital if $(1 - \sigma) > 0$; thus the first term in the first equation will be positive. Increases in productivity also increase total output, implying an increase in garbage if $\rho > 0$. This gives agents an incentive to destroy more and save less, implying that the second term is then negative. The response of garbage to changes in productivity is ambiguous for similar reasons.

In a no-destruction equilibrium, improvements in productivity increase the capital stock if w(k) is concave, but still have an ambiguous

[13]We plan to examine recycling and landfill space in more detail in future work.

effect on steady-state garbage.[14] Similarly, when $b = 0$, improvements in the technology unambiguously increase the steady-state capital stock. (In this case, the first term in the expression for dk/da equals zero, and the second term can be signed since no degradation implies that $\rho() < 0$.) The effect on garbage remains ambiguous.

Proposition 5: Economies with higher rates of capital depreciation have higher or lower equilibrium stocks of garbage and capital than economies with lower rates of capital depreciation.

Proof: The higher the rate of capital depreciation the higher is δ. Then,

$$\frac{dk}{d\delta} = \frac{1}{\Delta}\left[U'(1-\sigma) - \frac{\gamma\phi''\beta k}{b}\right] \gtrless 0,$$

since the first bracketed term is of either sign and $\left[-\frac{\gamma\phi''\beta k}{b}\right] < 0$. Further,

$$\frac{dg}{d\delta} = \frac{1}{\Delta b}\left[-\beta k\zeta + U'(1-\sigma)[\beta(1-\delta) + \gamma + \rho']\right] \lessgtr 0,$$

since $(1-\sigma) \gtrless 0$. $\circ$

The ambiguity of these comparative static results arises from the response of saving to the interest rate. If $(1-\sigma) \leq 0$ (i.e., if the interest elasticity of saving is non-positive), then $dk/d\delta$ is unambiguously positive. We obtain the unusual result that increases in the depreciation rate will increase the steady-state capital stock; further, this result can arise even without a backward-bending savings function. The intuition is that an increase in the depreciation rate reduces the consumption externality, thus

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[14] If $c'(k) > 0$, however, the garbage stock increases when a increases. In a zero-destruction equilibrium, $c(k) = y(k) - \delta k$, so that $c'(k) > 0$ if the economy is dynamically efficient.

discouraging destruction and encouraging saving. If, by contrast, the interest elasticity of saving is non-negative ($(1 - \sigma) \geq 0$), then increases in the depreciation rate unambiguously decrease the stock of garbage.

In the no-destruction economy, increases in the depreciation rate have no effect on the equilibrium capital stock and cause a decrease in the steady-state level of garbage. In the no-degradation economy, increases in the depreciation rate unambiguously increase the capital stock, while the effect on garbage remains ambiguous.

V MULTIPLE EQUILIBRIA

It is well-known that multiple equilibria can be obtained in overlapping-generations models by assuming strong income effects (see, for example, Azariadis and Guesnerie (1986)). It has also been shown (Weil (1989)) that external increasing returns can generate multiple equilibria (see also Chatterjee, Cooper and Ravikumar (1990)). These results carry over to this model, as illustrated in Figure 5.

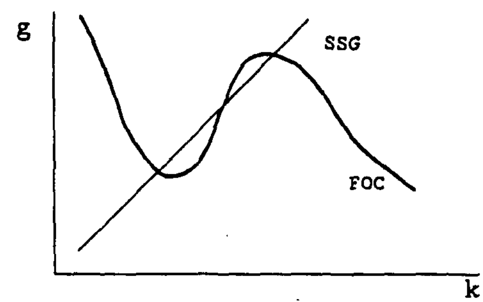

Figure 5

In contrast to Figure 1, the first-order condition does not have negative slope everywhere. Assuming that steady-state garbage is still monotonically increasing in k, it is evident from this figure that a necessary condition for multiplicity is that the first-order condition somewhere have positive slope. The first-order condition implicitly defines g as a function of k.[15] Differentiation of the first-order condition yields

$$\left.\frac{\partial g}{\partial k}\right|_{FOC} = \frac{\zeta}{\gamma\phi''}.$$

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[15]In terms of the economics, it is natural to think of the first-order condition's determining the level of saving, given the stock of garbage. As Figure 5 shows, however, this relationship may be a correspondence, rather than a function. By the assumptions on $\phi()$, we can write g as an implicit function of k.

Recalling that $\varsigma = U'()\left[(1 - \sigma)r'(k) - \sigma c/k^2\right]$, a necessary condition for multiplicity is $(1-\sigma)r'(k) > 0$.[16] This in turn requires either strong income effects ($(1 - \sigma) > 0$) or $r'() > 0$. The latter is possible only in the presence of external increasing returns, since

$$r'(k) = \psi(k)f''(k) + \psi'(k)f'(k),$$

where the first term is negative by the concavity of f().

In Weil's model, equilibria with higher k are Pareto-preferred to low-k equilibria. Such a result need not hold in this model. Increases in the capital stock are generally associated with greater utility,[17] but high-k equilibria also exhibit a larger steady-state garbage stock, which reduces utility. The disutility from higher garbage may more than offset the increased utility from the higher capital stock. Recalling the quote at the start of this paper, this might be termed a Galbraithian view.

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[16]In Weil's model, multiplicity is not possible when the interest elasticity of savings is negative (see his Proposition 1). This is consistent with the result here, since Weil's model imposes increasing returns.

[17]The condition for utility to be increasing in k, given g, is

$$(1 + r - \delta)a\psi'()f() - (r - \delta)r'(k)k > 0.$$

(In the absence of external increasing returns, $\psi'() = 0$, $r'() < 0$, and the condition simply reduces to $r > \delta$ (dynamic efficiency). In the presence of external increasing returns the first term is positive but $r'(k)$ may be positive.) To see this, note that we can write the indirect utility of a young agent as

$$V(k, g) = \max_d U((w(k) - d)(1 + r(k) - \delta)) - \phi(g - \gamma d)$$

$$\Rightarrow \quad \frac{\partial V}{\partial k} = U'()[r'(k)k + (1 + r - \delta)w'(k)],$$

by the envelope theorem. The result follows from the fact that

$$w'(k) = y'(k) - r'(k)k - r = a\psi'(k)f(k) - r'(k)k.$$

Multiplicity can arise in this model in the absence of both strong income effects and external increasing returns if, at high values of the capital stock, economies engage in sufficient destruction to reduce the steady-state stock of garbage. This is illustrated in Figure 6.

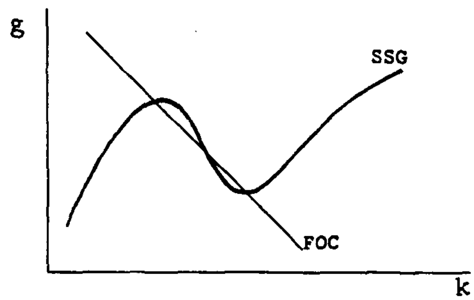

Figure 6

At the low-k, high-g equilibrium, agents have relatively more of an incentive to destroy. Since agents are putting resources into destruction, they do not save much and the capital stock is low; this implies that agents have low income and so cannot engage in much destruction, validating the high garbage stock. By contrast, at the high-k, low-g equilibrium, agents have less of an incentive to destroy, leading to a higher capital stock and a greater ability to engage in destruction. These complementarities lie behind the multiple equilibria.

A necessary condition for multiplicity when the first-order condition is downward sloping is that steady-state garbage is decreasing in k over some range. Recall that

$$g(k) = \left(\frac{1}{b}\right)\left([\beta(1-\delta) + \gamma]k + \rho(k)\right)$$

$$\Rightarrow \quad \frac{\partial g}{\partial k}\Big|_{SSG} = \left(\frac{1}{b}\right)\left(\beta(1-\delta) + \gamma + \rho'(k)\right).$$

A necessary condition for multiplicity is thus that $\rho'(k) < 0$. It is easily confirmed that

$$\rho'(k) = (\beta + \gamma)r(k)[1 - \eta_{rk}] - \gamma y'(k),$$

where $\eta_{rk} = - r'(k)k/r(k)$. In the absence of external increasing returns, $y'(k) = r(k)$, and the necessary condition for multiplicity becomes

$$r(k)[\beta(1 - \eta_{rk}) - \gamma\eta_{rk}] < 0.$$

External increasing returns are thus not required for this type of multiplicity.[18] External increasing returns imply increases in y'() and decreases in η_{rk}, and so have an ambiguous effect on $\rho'()$.

This model is consistent with observations of relatively poor economies with serious pollution problems; many countries in Eastern Europe currently fit this description. Contrary to the popular perception that pollution is associated with high GNP, it may rather be the case that it is only rich countries that are able to spare the resources to combat environmental problems -- a conclusion more Solovian than Galbraithian.

Multiplicity: Examples

In order to demonstrate the possibility of multiplicity more formally, we now consider two simple examples.

Proposition 6: Suppose that $U(c) = \ln(c)$, $\phi(g) = (\theta/2)g^2$, $f(k) = k^\nu$ and $\psi(k) = k^\epsilon$. Let $\epsilon = 3/2$, $\nu = 1/2$, $\beta = 5$, $\gamma = 7$, $\delta = 1$, $\theta = 1/56$, $b = 1/2$, and $a = 3$. Then there are two equilibria.

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[18] This is explicitly proved (by example) in Proposition 7 below.

Proof: The steady-state first-order condition from this model is $g = 1/\gamma\theta k$. Further, it is easily shown that $\rho(k) = [\beta\nu - \gamma(1-\nu)]ak^{\epsilon+\nu}$. The steady-state solution for k is defined by

$$Z(k) = (\theta\gamma/b)[(\beta(1-\delta) + \gamma)k + \rho(k)] - 1/k = 0,$$

which, for the parameter values specified, implies

$$Z(k) = (k/4)(7 - 3k) - 1/k = 0.$$

This equation has two positive roots: $k = \{1, 2\}$. It is easily confirmed that destruction is positive at these values of k. o

Proposition 7: Suppose that $U(c) = \ln(c)$, $\phi(g) = (\theta/2)g^2$, $f(k) = k/(1 + k)$, $\psi(k) = 1 \ \forall\ k$. Let $\gamma = \delta = 1$, $b/\theta = 18/25$, $\beta = 257/313$, and $a = 313/50$. Then there are three equilibria.

Proof: As in Proposition 6, the first-order condition is $g = 1/\theta\gamma k$. For the CES production function specified, it can be shown that

$$g(k) = \frac{1}{b}\left([\beta(1-\delta) + \gamma]k + (\beta - \gamma k)\left[\frac{ak}{(1+k)^2}\right]\right).$$

Given the parameter values specified, the equilibrium values of k are defined by

$$Z(k) = k + \left(\frac{257}{313} - k\right)\left(\frac{313}{50}\right)\left(\frac{k}{(1+k)^2}\right) - \frac{18}{25k} = 0.$$

This equation has three positive roots: $k = \{1, 3/2, 2\}$. Destruction is positive at these values of k. o

Note that the functional forms assumed for these examples are completely standard (log utility, quadratic costs, CES production functions). Note also that, since the first-order condition slopes downward in k-g space, multiplicity must arise through the garbage effect just discussed. In the first example, multiplicity arises because of external increasing returns combined with a negative value of $\rho(k)$. In the second example, multiplicity arises because labor's share of output increases as k increases; this effect is sufficiently strong to cause garbage to be a decreasing function of k over some range.

Multiple equilibria can arise under the special cases discussed earlier. Considering first zero degradation, there will be multiple equilibria if equation (26) (reproduced here) has multiple roots:

$$\rho(k) = - [\beta(1-\delta) + \gamma]k \qquad (26)$$

It is easy to construct examples where this is true. Multiple equilibria with zero destruction will arise if equation (23) ($w(k) = k$) has multiple roots, and if the steady-state garbage line lies below the zero-destruction locus at these values of k. Again, examples are easily constructed.

IV WELFARE ANALYSIS

Since environmental damage may outlive its perpetrators, overlapping-generations models provide the most appropriate demographic structure for analysis of environmental externalities. Adoption of this framework has direct implications for welfare analysis. First, intergenerational externalities are intrinsically hard to internalize: those imposing the externalities are not alive at the same time as those who enjoy or suffer the consequences. Second, an overlapping-generations structure complicates the analysis of Pareto-improving policies, since the welfare of multiple generations must be considered.

We first consider the golden-rule allocation, solving the problem of a social planner who treats all generations symmetrically. The social planner solves

$$\text{Maximize } U(c) - \phi(g) \tag{28}$$

subject to

$$y + (1 - \delta)k = c + d + k \tag{29}$$

$$bg = \beta c - \gamma d \tag{30}$$

$$y = a\psi(k)f(k), \tag{31}$$

where (29) and (31) combined represent economic feasibility and (30) is the steady-state stock of garbage. Substitute (30) into (28) and form the Lagrangian

$$\underset{c,\ d,\ k}{\text{maximize}}\ \mathcal{L} = U(c) - \phi\left(\frac{\beta}{b} c - \frac{\gamma}{b} d\right) + \lambda(a\psi(k)f(k) - \delta k - c - d). \tag{32}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial c} = U' - \phi' \frac{\beta}{b} - \lambda = 0 \tag{33}$$

$$\frac{\partial \mathcal{L}}{\partial d} = \phi' \frac{\gamma}{b} - \lambda = 0 \tag{34}$$

$$\frac{\partial \mathcal{L}}{\partial k} = \lambda(a[\psi f' + \psi' f] - \delta) = 0 \tag{35}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = a\psi(k)f(k) - \delta k - c - d = 0. \tag{36}$$

From (35), the planner sets capital at the level at which net output is maximized:[19]

$$a[\psi f' + \psi' f] = r + a\psi' f = \delta. \tag{37}$$

Equation (37) is essentially the familiar condition for the golden-rule level of capital. Note that the planner's choice internalizes the externality from the external increasing returns. From (33) and (34), the planner sets the marginal utility from consumption equal to the marginal utility from destruction:

$$U' - \phi'\frac{\beta}{b} = \phi'\frac{\gamma}{b}. \tag{38}$$

Comparing (38) with (5) establishes that the planner internalizes the consumption externality (the $\phi'\beta/b$ term) and the destruction externality (the $1/b$ in the $\phi'\gamma/b$ term).

Intuitively, the planner's problem decomposes in this way because, for any given value of the capital stock, the planner can divide output between consumption and destruction to achieve any constant garbage stock. Thus the planner can translate an increase in output into increased consumption while keeping garbage constant, unambiguously increasing utility. The planner therefore chooses k to maximize output, and then divides it optimally. As Proposition 8 shows, the social planner can achieve this social optimum by means of taxes and/or subsidies on wage and rental income.

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[19] Note that if there are sufficiently strong external increasing returns net output might be unbounded.

Proposition 8: The decentralized economy with taxes on wages and capital income can achieve the first-best (golden-rule) steady-state allocation.

Proof: Suppose the planner taxes the net return on capital and wages at the rates τ_k and τ_w, respectively. In the presence of taxes, a representative agent's choice problem is to choose c_{t+1}, d_t, and s_t to solve

$$U(c_{t+1}) - \phi(g_{t+1})$$

subject to

$$w_t(1 - \tau_w) = s_t + d_t$$

$$c_{t+1} = (1 + (r_{t+1} - \delta)(1 - \tau_k))s_t + T_{t+1}$$

$$g_{t+1} = (1 - b)g_t + \beta c_t - \gamma d_t$$

$$c_{t+1},\ d_t,\ s_t \geq 0$$

where T_{t+1} is a transfer (lump-sum tax).

A steady-state equilibrium for the economy with taxes is now a vector $\{c, d, g, w, r, s, k, T\}$ such that agents optimize, factor and goods markets clear, the stock of garbage is constant, and the government budget constraint is satisfied: $\tau_w w + \tau_k(r - \delta)k = T$. At a steady state the representative agent's indirect utility can be defined as a function of the tax parameters:

$$V(\tau_k, \tau_w) = U\{[1 + (r(\tau_k,\tau_w) - \delta)]k(\tau_k,\tau_w) + \tau_w w(\tau_k,\tau_w)\} -$$
$$\phi\left\{\frac{\beta}{b}\{[1+(r(\tau_k,\tau_w)-\delta)]k(\tau_k,\tau_w) + \tau_w w(\tau_k,\tau_w)\} - \frac{\gamma}{b}\{w(\tau_k,\tau_w)(1 - \tau_w) - k(\tau_k,\tau_w)\}\right\}$$

where $r(\tau_k,\tau_w)$, $k(\tau_k,\tau_w)$, and $w(\tau_k,\tau_w)$ are the implicit functions defining the interest rate, capital, and the wage rate, respectively, in steady-state equilibrium.

The planner chooses τ_w and τ_k to maximize the representative agent's indirect utility function. The first-order conditions of the planner's constrained problem are

$$\frac{\partial V()}{\partial \tau_w} = U'()w + U'()\left[(1 + r - \delta) + kr' + \tau_w[y' - r - kr']\right]\frac{\partial k}{\partial \tau_w} -$$

$$\phi'()\left(\frac{\beta + \gamma}{b}\right)w - \phi'()\left[\frac{\beta}{b}\left((1 + r - \delta) + kr' + \tau_w[y' - r - kr']\right) -\right.$$

$$\left.\frac{\gamma}{b}\left([y' - r - r'k](1 - \tau_w) - 1\right)\right]\frac{\partial k}{\partial \tau_w} = 0$$

$$\frac{\partial V()}{\partial \tau_k} = U'()\left[(1 + r - \delta) + kr' + \tau_w[y' - r - kr']\right]\frac{\partial k}{\partial \tau_k} -$$

$$\phi'()\left[\frac{\beta}{b}\left((1 + r - \delta) + kr' + \tau_w[y' - r - kr']\right) -\right.$$

$$\left.\frac{\gamma}{b}\left([y' - r - r'k](1 - \tau_w) - 1\right)\right]\frac{\partial k}{\partial \tau_k} = 0,$$

which simplify to

$$U'() = \left(\frac{\beta + \gamma}{b}\right)\phi'()$$

$$\partial y(\tau_k, \tau_w)/\partial k = \delta.$$

These equations define τ_k and τ_w. Note that $\partial y/\partial k = a[\psi f' + \psi' f]$. Thus, the first-order conditions for the constrained and unconstrained planner's problems are identical: the planner can choose τ_k and τ_w to achieve the social optimum.∘

The planner requires two tax parameters to direct the economy to the social optimum because there are two market distortions: one in the goods market, the other in the capital market.

As is well-known, the competitive equilibrium of a Diamond (1965) type, overlapping-generations model can be dynamically inefficient (see, for example, Blanchard and Fischer (1989, p. 103)). If so, all generations would be better off if they saved less (accumulated less capital) and consumed more. Not surprisingly, such a result carries over to this model. There is an analogous possibility of inefficiency in terms of the garbage stock. That is, agents may underaccumulate garbage, implying that all generations could be made better off by destroying less and consuming more. To see this, consider Figure 7.

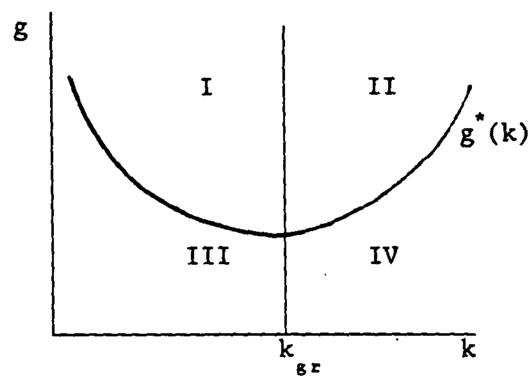

Figure 7

Equation (37) defines the golden-rule level of the capital stock (k_{gr}). Equation (38) can be interpreted as defining the optimal level of garbage for a given value of the capital stock ($g^*(k)$), in steady state.[20] Together, these divide k-g space into four regions. In regions II and IV, the capital stock exceeds its golden-rule level, the economy is dynamically inefficient, and it is possible to increase all generations' utility by increasing consumption and decreasing saving. In regions III and IV, below $g^*(k)$, agents are underaccumulating garbage, and it is possible to increase all

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[20] Using the facts that, in steady state, $bg = \beta c - \gamma d$ and $y(k) - \delta k = c + d$, we can show that $c = [\gamma(y(k) - \delta k) + bg]/(\beta + \gamma)$. Substituting this into equation (38), it is easily shown that $g^*(k)$ attains a minimum at the golden-rule capital stock.

generations' utility by increasing consumption and decreasing destruction.[21] In the absence of external increasing returns, the economy cannot be in equilibrium in region III; that is, there are no dynamically efficient steady-state equilibria where agents underaccumulate garbage. In the presence of external increasing returns, however, equilibria in region III are possible, as established by the following proposition.

Proposition 9: Underaccumulation of garbage in a dynamically efficient equilibrium is possible only in the presence of external increasing returns.

Proof: The first-order condition from the agent's maximization problem is given by equation (22):

$$U'(c)(1 + r - \delta) - \gamma\phi'(g) = 0. \tag{22}$$

In any steady state,

$$c = \frac{\gamma(y(k) - \delta k) + bg}{\beta + \gamma},$$

so equation (22) can be interpreted as defining the steady-state value of g for a given k. Now $g^*(k)$ is implicitly defined by (38):

$$U'(c) - \phi'(g^*)\frac{\beta}{b} = \phi'(g^*)\frac{\gamma}{b} \tag{38}$$

$$\Rightarrow \quad U'(c) = \left(\frac{\beta + \gamma}{b}\right)\phi'(g^*),$$

with consumption given by the same expression as before. Since U() is concave and $\phi()$ is convex, comparison of (22) and (38) reveals that $g < g^*$ if and only if

$$\frac{b}{\beta + \gamma} > \frac{1 + r - \delta}{\gamma}$$

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[21] If the economy is in region II, III or IV, Pareto-improving policies thus entail transfers from the young to the old. Such Pareto improvements could be supported by social contracts of the type discussed by Kotlikoff, Persson and Svensson (1988).

$$\Rightarrow \quad 1 + r - \delta < \frac{b\gamma}{\beta + \gamma} < 1,$$

since $b \leq 1$ and $\beta > 0$. It follows that a necessary condition for $g < g^*$ is $r < \delta$. In the absence of external increasing returns, this condition implies dynamic inefficiency. But in the presence of external increasing returns, the economy is dynamically inefficient iff $r + a\psi' f < \delta$, so this condition does not contradict dynamic efficiency. ∘

In region I, simple Pareto-improving policies are not so easily found. Pareto improvements are possible in general if agents are at an equilibrium with positive destruction or if there are no external increasing returns. This is proved in Proposition 10.

Proposition 10: Pareto-improving policies are generically possible in any equilibrium with positive destruction or with no external increasing returns.

Proof: For the sake of economy of notation, we suppose that the economy is initially in steady-state equilibrium (k, g), although the method of proof is applicable to any equilibrium. Consider the following perturbation to this steady state. At time τ, increase saving by one unit and decrease destruction by one unit. At time $\tau+1$, set $s_{\tau+1} = k$ and let $d_{\tau+1} = d + \alpha$. At time $\tau+2$, set $s_{\tau+2} = k$ and let $d_{\tau+2} = d + \epsilon$. Choose ϵ such that $g_{\tau+3} = g$ and choose α such that the utility of generation $\tau+1$ is $\tilde{U}$ (steady-state utility).

Since $k_{\tau+2} = k_{\tau+3} = k$, and $g_{\tau+3} = g$, the utility of all generations born at $\tau+2$ and after can be maintained at $\tilde{U}$. The utility of all generations up to and including $\tau-1$ is unchanged, and the utility of generation $\tau+1$ is unchanged by construction. It thus remains to consider the effect of this perturbation on generation τ. Note first that $g_{\tau+1} = g + \gamma$ and $y_{\tau+1} = y + r$. Since $d_{\tau+1} = d + \alpha$, feasibility at τ+1 implies

$$c_{\tau+1} = c + (1 + r - \delta) - \alpha.$$

The change in utility of generation τ is thus given by

$$U'(c)(c_{\tau+2} - c) - \phi'(g)(g_{\tau+2} - g) = U'(c)\left[(1 + r - \delta) - \alpha + \left(\frac{1+r-\delta}{\gamma}\right)\gamma\right]$$
$$= - \alpha U'(c),$$

using the first-order condition from the agent's problem. Using the expressions for $c_{\tau+1}$ and $d_{\tau+1}$, we obtain

$$g_{\tau+2} = g + \beta(1 + r - \delta) + \gamma(1\text{-}b) - \alpha(\beta + \gamma).$$

Since $y_{\tau+2} = y + a\psi'f$ and $d_{\tau+2} = d + \epsilon$, feasibility at τ+2 implies

$$c_{\tau+2} = c + a\psi'f - \epsilon.$$

It then follows that

$$g_{\tau+3} = g + (1\text{-}b)[\beta(1 + r - \delta) + \gamma(1\text{-}b) - \alpha(\beta + \gamma)] + \beta a\psi'f - (\beta + \gamma)\epsilon.$$

Choosing ϵ such that $g_{\tau+3} = g$ implies

$$\epsilon = \frac{\beta a\psi'f}{\beta + \gamma} + \left(\frac{1 - b}{\beta + \gamma}\right)\left(\beta(1 + r - \delta) - \gamma(1\text{-}b)\right) - \alpha(1\text{-}b).$$

The change in utility of generation τ+1 is given by

$$U'(c)(c_{\tau+2} - c) - \phi'(g)(g_{\tau+2} - g)$$
$$= U'(c)\left(a\psi'f - \epsilon + \left(\frac{1 + r - \delta}{\gamma}\right)\left(\beta(1 + r - \delta) + \gamma(1\text{-}b) - \alpha(\beta + \gamma)\right)\right).$$

Choosing α to set this equal to zero and substituting in for ϵ yields finally

$$\alpha = \frac{\beta(1 + r - \delta) + \gamma(1\text{-}b)}{\beta + \gamma} - \frac{a\psi'f\gamma^2}{(\beta + \gamma)[(\beta + \gamma)(1 + r - \delta) - \gamma(1\text{-}b)]}.$$

If the external increasing returns are sufficiently strong, so that the second term in this expression dominates and $\alpha < 0$, then this perturbation will be Pareto-improving. Since the proposed perturbation entails decreasing destruction, it is only feasible in an equilibrium with positive destruction. In the absence of external increasing returns, the second term in this expression equals zero and α is positive. In this case the proposed perturbation would unambiguously lower the utility of generation τ; increasing destruction by one unit and lowering saving by one unit would be Pareto-improving. o

Proposition 10 shows that, even when agents do not overaccumulate capital or underaccumulate garbage, Pareto-improving policies may be possible because of the externalities present in the model. With sufficiently strong external increasing returns, the reallocation suggested in the proposition entails a reduction in destruction, which is not feasible in a zero-destruction equilibrium. Increasing saving in period τ while leaving destruction unchanged is not a Pareto-improving policy, since, by feasibility, this reduces the consumption, and hence the utility, of the old alive at time τ. This possibility is a direct consequence of the overlapping-generations structure. Note, finally, that Proposition 10 does not prove that no Pareto-improving reallocations are possible when the suggested perturbation fails, although we conjecture this to be the case.

SECTION VI CONCLUSIONS

Actions that affect environmental quality both influence and respond to macroeconomic variables, and many environmental and macroeconomic consequences of agents' actions have uncompensated effects that outlive the actors. This paper has examined an overlapping-generations model of environmental externalities and capital accumulation in which young agents either invest in capital or in destruction of garbage, and where the consumption of the old augments the stock of garbage.

The model in this paper demonstrates that it may be misleading to address environmental and macroeconomic concerns in isolation. Changes in parameters describing the evolution of the stock of waste also have effects on the accumulation of capital. In particular, we find that increases in the natural rate of degradation of waste encourage the accumulation of capital, and that improvements in society's ability to dispose of waste may reduce capital accumulation. Also, a higher depreciation rate of capital may be associated with a higher equilibrium capital stock.

Multiple Pareto-ranked equilibria -- coordination failures -- are possible in our model. Such multiplicities can arise not only from the well-understood sources of strong income effects or external increasing returns, but also from the interaction of garbage and capital accumulation. An economy may get stuck in a low-output, high-garbage equilibrium, where the high level of garbage reduces agents' incentives to invest, and the resulting low level of income prevents them from destroying the stock of garbage, even though a better (low-garbage high-output) equilibrium exists.

We plan to extend the model of this paper in a number of directions. First, we are investigating the dynamic behavior of the model in order to understand the effects of the environmental externality on growth paths. Second, we intend to generalize the model to allow for consumption in youth and old age, so that there are distinct saving and destruction decisions. Third, we wish to explore the implications of introducing interest-bearing government debt, in order to gain insight into the effects of macroeconomic policies on decisions that affect the environment. Finally, we will apply our model to specific environmental concerns, such as recycling and the shortage of landfill space.

References

Allais, M., 1947, *Economie et Interêt*, Imprimerie Nationale, Paris.

Arrow, K., 1962, "The economic implications of learning by doing," *Review of Economic Studies 29*, 155-73

Azariadis, C. and A. Drazen, 1990, "Threshold externalities in economic development, *Quarterly Journal of Economics 105*, 501-26.

Azariadis, C. and R. Guesnerie, 1986, "Sunspots and cycles," *Review of Economic Studies 58*, 725-37.

Baumol, W., and W. Oates, 1988, *The Theory of Environmental Policy*, Cambridge University Press, Cambridge.

Blanchard, O., and S. Fischer, 1989, *Lectures on Macroeconomics*, MIT Press, Cambridge.

Chatterjee, S. and R. Cooper, 1989, "Multiplicity of equilibria and fluctuations in dynamic imperfectly competitive economies," *American Economic Review*, 353-57.

Chatterjee, S., R. Cooper, and B. Ravikumar, 1990, "Participation dynamics: sunspots and cycles," mimeo, University of Iowa.

Conrad, J., and C. Clark, 1987, *Natural Resource Economics*, Cambridge University Press, Cambridge.

Cooper, R., and A. John, 1988, "Coordinating coordination failures in Keynesian models," *Quarterly Journal of Economics 103*, 441-64.

Dasgupta, P., 1982, *The Control of Resources*, Harvard University Press, Cambridge, Mass.

Dasgupta, P., and G. Heal, 1979, *Economic Theory and Exhaustible Resources*, Cambridge University Press, Cambridge.

Diamond, P., 1965, "National debt in a neoclassical growth model," *American Economic Review 55*, 1126-50.

Freeman, S. and S. Polasky, 1989, "Knowledge-based growth," University of California, Santa Barbara, Working Paper in Economics #4-89.

Galbraith, J., 1958, *The Affluent Society*, Houghton Mifflin, New York.

Hahn, R., 1989, "Economic prescriptions for environmental problems: how the patient followed the doctor's orders," *Journal of Economic Perspectives* 3, 95-114.

Hanson, D., 1989, "Solid waste problem receiving more attention from Congress," *Chemical and Engineering News*, 23-4.

Howarth, R. and R. Norgaard, 1980, "Intergenerational resource rights, efficiency, and social optimality," *Land Economics 66*, 1-11.

John. A., 1988, "Imperfect competition, Keynesian dynamics and the paradox of thrift," mimeo, Michigan State University.

John, A., R. Pecchenino and S. Schreft, 1989, "The macroeconomics of Dr Strangelove: an overlapping-generations model of the apocalypse," Econometrics and Economic Theory Working Paper #8813, Michigan State University.

Kemp, M. and N. Long, 1980, "The underexploitation of natural resources: a model with overlapping generations," in *Exhaustible Resources, Optimality and Trade*, M. Kemp and N. Long (eds.), North Holland, Amsterdam.

Kneese. A. and K. Mäler, 1973, "Bribes and charges in pollution control: an aspect of the Coase controversy," *Natural Resources Journal 13*, 703-16.

Kotlikoff, L., T. Persson and L. Svensson, 1988, "Social contracts as assets: a possible solution to the time-consistency problem," *American Economic Review 78*, 662-77.

Loehman, E., and A. Whinston, 1970, "The welfare economics of water resource allocation over time," *Applied Economics 2*, 75-99.

Lucas, R., 1988, "On the mechanics of economic development," *Journal of Monetary Economics 21*, 3-42.

Luoma, J., 1990, "Trash can realities," *Audubon 92*, 86-97.

Mourmouras, A., 1989, "An overlapping generations model of endogenous growth with human and physical capital accumulation and endogenous labor supply," mimeo, University of Cincinnati.

Mourmouras, A., 1990, "Renewable resources, the environment and intergenerational equity in overlapping generations models," mimeo, University of Cincinnati.

Murphy, K., A. Shleifer, and R. Vishny, 1989, "Industrialization and the big push," *Journal of Political Economy 97*, 1003-26.

Neher, P., 1990, *Natural Resource Economics: Conservation and Exploitation*, Cambridge University Press.

Pagano, M., 1990, "Imperfect competition, underemployment equilibrium, and fiscal policy," *Economic Journal 100*, 440-63.

Plourde, C., 1972, "A model of waste accumulation and disposal," *Canadian Journal of Economics 5*, 117-25.

Rathje, W., 1984, "The garbàge decade," *American Behavioral Scientist 28*, 9-29.

Romer, P., 1986, "Increasing returns and long-run growth," *Journal of Political Economy 94*, 1002-37.

Romer, P., 1990, "Are nonconvexities important for understanding growth?" *American Economic Review 80*, 97-103.

Samuelson, P., 1958, "An exact consumption-loan model of interest with or without the social contrivance of money," *Journal of Political Economy 66*, 467-82.

Sandler, T., 1982, "A theory of intergenerational clubs," *Economic Inquiry 20*, 191-208.

Schimmelpfennig, D., 1989, "Equilibrium responses of overlapping generations to the greenhouse effect," mimeo, Michigan State University.

Schmitz, J., Jr., 1989, "Imitation, entrepreneurship, and long-run growth," *Journal of Political Economy 97*, 721-39.

Solow, R., 1956, "A contribution to the theory of economic growth, *Quarterly Journal of Economics 71*.

Solow, R., 1973, "Is the end of the world at hand?" *Challenge 16*, 39-51.

Solow, R., 1974, "The economics of resources of the resources of economics," *American Economic Review 64*, 1-14.

Solow, R., 1986, "On the intergenerational allocation of natural resources," *Scandinavian Journal of Economics 88*, 141-9.

Stokey, N., 1988, "Learning by doing and the introduction of new goods," *Journal of Political Economy 96*, 701-17.

Weil, P., 1989, "Increasing returns and animal spirits," *American Economic Review 79*, 889-94.

Weitzman, M., 1974a, "Free access vs. private ownership as alternative systems for managing common property," *Journal of Economic Theory 8*, 225-34.

Weitzman, M., 1974b, "Prices vs. quantities," *Review of Economic Studies 41*, 477-91.

Young, A., 1928, "Increasing returns and economic progress," *Economic Journal 38*, 527-42.