Working Paper Series

Scope Economies: Fixed Costs, Complementarity, and Functional Form

WP 91-03

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Working Paper 91-3

Scope Economies: Fixed Costs, Complementarity, and Functional Form*

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February, 1991

This is a preprint of an article published in the Journal of Business ©, July 1993, v. 66, iss.3, pp. 437-62

*The opinions expressed are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Richmond or the Federal Reserve System. The authors thank John Boschen for his comments on an earlier version of this paper. Support for Lawrence Pulley was received from the Summer Research Grant Program at the College of William and Mary.

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Abstract. Bank scope economies have been derived from either the standard or generalized (Box-Cox) multiproduct translog (or other logarithmic) functional form. Reported results have ranged from strong economies to diseconomies and are far from conclusive. The problem is functional form. An alternative composite form is shown to yield stable SCOPE results both at the usual point of evaluation and for points associated with quasi-specialized production (QSCOPE). Unstable results are obtained for the other forms. Scope economies are shown to exist for large U.S. banks in 1988 and to depend on the number of banking outputs specified. The scope estimates are also separated into their two sources--fixed-cost and cost-complementarity effects.

Scope Economies: Fixed Costs, Complementarity, and Functional Form

I. Introduction.

Results from previous studies of economies of scope in banking have been inconclusive. Some researchers have found large scope economies (Gilligan, Smirlock, and Marshall (1984), Kim (1986), Buono and Eakin (1990)): others have found only small scope economies--or even diseconomies (Lawrence and Shay (1986), Mester (1987b), Berger, Hanweck, and Humphrey (1987)). One reason no consensus exists on this important issue is the use of the popular multiproduct translog cost function. Even with outputs generalized through the use of Box-Cox transformations, the translog is inherently nonrobust when used to evaluate scope economies.[1]

There are at least three sources of misinterpretation or error in existing studies of scope economies in banking. First, a number of researchers have chosen to measure only cost complementarities and use these results to infer the existence of scope economies. While cost complementarities are more easily obtained from the translog than are direct estimates of economies of scope, the sufficient conditions for scope economies require that weak complementarities exist for all pairs of outputs and at all output levels below the initial point of evaluation (usually the means). As pointed out by Mester (1987b), these conditions are rarely tested properly because of the effort involved. Unfortunately, researchers pursuing simpler alternatives to this strict test have often tested inappropriate hypotheses such as jointness rather than economies of scope (Gilligan, Smirlock, and Marshall (1984)).

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[1] In contrast, scale economy estimates using these forms are reasonably robust. See the surveys of Mester (1987a), Clark (1988), and Humphrey (1990).

Second, even when scope economies are computed directly, they cannot be measured using the standard translog specification since the logarithmic function approaches $-\infty$ as its argument approaches zero. This difficulty has prompted a number of ad hoc modifications. One modification involves substituting a small positive value for zero output levels both for estimation and scope economy evaluation. Another is the use of a Box-Cox transformation on output (and other variables) to derive a generalized translog form which can admit zero values (Lawrence (1989); Buono and Eakin (1990)). As we will show below, these modifications do not eliminate the problem of bias in using the translog form and are unsatisfactory.

Third, measuring scope economies for banks requires evaluating the estimated cost function for production levels that generally lie well outside the sample observations. Unlike other industries, few banks produce zero levels of certain outputs and fewer still produce only one output. Thus, bank scope estimates are typically based on extrapolated cost measurements having little empirical support.

We address these problems in the following way. While cost complementarities are an interesting concept, we do not compute them in lieu of measuring scope economies directly, as is frequently done in the literature. Rather, the functional form we use allows us not only to compute scope economies but also to assess the actual contribution of cost complementarities to the scope value obtained. Indeed, the two sources of scope economies--spreading fixed cost over a broader product mix and cost complementarity--can be formally separated. This has not been done before and is important in understanding why scope economies vary with the degree of output aggregation specified. Functions that are multiplicative (or log additive), such as the translog, make it impossible to identify and measure these two effects.

The serious problem of bias from using either the standard or the generalized (Box-Cox) translog form in obtaining scope economies is overcome by using an alternative form--a composite function. This form exhibits the quadratic output structure recommended by Baumol, Panzar, and Willig (1982) for examining economies of scope and is related to the specification used recently by Röller (1990) to address problems with the translog form.

Lastly, although we can not alleviate the need to extrapolate beyond the confines of our bank data set to derive scope estimates, we can evaluate the relative stability and sensitivity of scope estimates to this extrapolation problem. This is done through a suitable modification to the standard formula for measuring economies of scope that enables us to examine joint cost effects relative to differing degrees of output specialization. The resulting "quasi-scope" values should not vary significantly for small departures from completely specialized production. Stability over this region is demonstrated for our composite function and shown not to exist for either the standard or the generalized translog forms.

Economies of scope, and measures of their two sources--fixed costs and cost complementarity, are formally defined and derived in the next section. In Section III, the problems and biases in using the standard and generalized translog forms are discussed and our composite model, which overcomes these difficulties, is introduced. Scope and quasi-scope estimates for large U.S. banks in 1988 are presented in Section IV. These are obtained from estimated composite, translog, and generalized translog cost functions. The stability of these results is assessed and illustrated for a number of different output aggregations. In addition, the contributions of fixed cost and cost complementarities to scope economies are shown for the composite form. This permits us to explain why scope economies vary with the level of output aggregation. Summary and concluding comments are contained in Section V.

II. Measuring Economies of Scope: Fixed Costs and Complementarity.

Definition And Measurement. Economies of scope reflect cost savings resulting from simultaneously producing several outputs in the same firm, rather than producing each one separately in a specialized firm. These savings arise from two sources: reducing excess capacity by producing a broader product mix, thereby lowering the fixed costs allocated to existing products, and reducing the costs of joint production through production complementarities.

In banking, daily deposit accounting for different types of accounts jointly or the processing of different types of loans jointly often requires much the same type and level of overhead as each would if produced separately. Thus, excess computer, branch-network, and loan-production office capacity can be reduced as more banking products are offered, lowering the cost of each product compared to a smaller product line. Cost complementarities arise when account and credit information developed for deposit products can be used to reduce the information and monitoring requirements for installment, mortgage, and other specialized loan products for the same customer base. Thus, expansion of a deposit base, on either the corporate or retail side, can lower the cost of providing loan products.

Overall economies of scope are measured as the percentage cost savings from producing all outputs jointly as opposed to producing each output separately:

$$(1) \qquad SCOPE = [C(q_1,0,.,0;\underline{r}) + C(0,q_2,0,.,0;\underline{r}) +\ldots+ C(0,.,0,q_m;\underline{r}) - C(q_1,q_2,.,q_m;\underline{r})]/C(q_1,q_2.,.,q_m;\underline{r})$$

where $C(\cdot)$ refers to the cost function, q_i (i=1,.,m) refers to outputs, and $\underline{r}$ is a vector of n input prices. Since SCOPE is derived from an estimated multiproduct cost function, choice of functional form is important. In addition to being a "proper" cost function (nondecreasing in outputs and nonegative, nondecreasing, concave, and linearly homogeneous in input prices), the function must be capable of representing cost relationships adequately over a region which includes zero levels of production in some outputs.[2] While the translog form can be altered to meet this latter requirement, the fix-ups involved still yield scope economy estimates that are nonrobust (as is demonstrated below).

<u>Appeal Of A Quadratic Specification For Outputs.</u> It is interesting to note that when Baumol, Panzar, and Willig (1982) discussed the SCOPE measure, they recommended estimating it with a cost function having a quadratic specification for outputs. This is a direct and proper method to account for zero outputs in estimation and evaluation. In addition, such a form allows us to identify separately the fixed-cost and complementarity effects contained in the scope measure.

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[2] For the other outputs, SCOPE estimates are generally evaluated at the sample means or medians of the produced output and input-price variables.

The appeal of such a form is evident from the following example of a two-output cost function that is quadratic in outputs and (for simplicity) strongly separable in input prices:

$$C(\underline{q},\underline{r}) = [\alpha_0 + \alpha_1 q_1 + \alpha_2 q_2 + \tfrac{1}{2}\alpha_{11} q_1^2 + \alpha_{12} q_1 q_2 + \tfrac{1}{2}\alpha_{22} q_2^2] \cdot f(\underline{r})$$
$$= h(\underline{q}) \cdot f(\underline{r}).$$

Although illustrative, this example is precisely the form of the multiproduct cost function used recently by Röller (1990) to reexamine the multiproduct structure of the Bell System. And, it leads to very nearly the same expression for measuring economies of scope as the composite cost function introduced in the next section.

The estimate of economies of scope from the cost function above is given by:

$$\text{SCOPE} = \{[(\alpha_0 + \alpha_1 q_1 + \tfrac{1}{2}\alpha_{11} q_1^2) \cdot f(\underline{r})] + [(\alpha_0 + \alpha_2 q_2 + \tfrac{1}{2}\alpha_{22} q_2^2) \cdot f(\underline{r})]$$
$$- [(\alpha_0 + \alpha_1 q_1 + \alpha_2 q_2 + \tfrac{1}{2}\alpha_{11} q_1^2 + \alpha_{12} q_1 q_2 + \tfrac{1}{2}\alpha_{22} q_2^2) \cdot f(\underline{r})]\}/C(\underline{q},\underline{r}).$$

Simplifying,

(2) $$\text{SCOPE} = (\alpha_0 - \alpha_{12} q_1 q_2)/h(\underline{q}).$$

The calculation generalizes in a straightforward manner to the case of m outputs:

(3) $$\text{SCOPE} = [(m-1)\alpha_0 - \Sigma_{i=1}\Sigma_{j>i}\alpha_{ij} q_i q_j]/h(\underline{q}).$$

<u>The Separation Of Fixed-Cost And Complementarity Effects.</u> The separate contributions of the two factors determining economies of scope--fixed costs and complementarity--are evident in (2) and (3). The first term in the formulas measures the savings resulting from reduced excess capacity, in which fixed costs are spread across a broader product line. In calculating economies of scope the formula implies that specialized production of each

output would require the same level of fixed costs as joint production of all m outputs. In comparing joint production with specialized production for an m-output firm, the level of fixed costs will therefore be "saved" m-1 times. The second term in the SCOPE formulas shows how cost complementarities between outputs contribute to economies of scope. If variable inputs are shared by different product lines, as when deposit and other information serves both deposit and loan products, the α_{ij} coefficients would be expected to be negative and economies of scope would increase.

As expression (3) makes clear, the contribution to economies of scope resulting from saved fixed costs will in all likelihood increase with increases in the number of outputs specified in the cost function. This could produce differences in measured economies of scope across banking studies that result solely from differences in output specification or level of aggregation. Since there remains a debate in the literature regarding the appropriate number and specification of banking outputs, we derive SCOPE estimates using three different levels of output aggregation.

The portion of economies of scope attributable to cost complementarities only is expressed as:

(3′) $$SCOPE_{CC} = - \Sigma_{i=1}\Sigma_{j>i}\alpha_{ij}q_i q_j/h(\underline{q})$$

while that attributable to fixed cost is:

(3′′) $$SCOPE_{FC} = SCOPE - SCOPE_{CC} = (m-1)\alpha_0/h(\underline{q}).$$

Multiplicative or log-additive cost functions such as the popular translog specification do not allow the fixed-cost and complementarity components of economies of scope to be separated. Consider the translog cost function for the case of m outputs and n input prices:

(4) $$\ln C = \alpha_0 + \Sigma\alpha_i \ln q_i + \tfrac{1}{2}\Sigma\Sigma\alpha_{ij}\ln q_i \ln q_j + \Sigma\Sigma\delta_{ik}\ln q_i \ln r_k + \Sigma\beta_k \ln r_k + \tfrac{1}{2}\Sigma\Sigma\beta_{kl}\ln r_k \ln r_l$$

where i,j refer to the m outputs and k,l refer to the n input prices. To compute economies of scope from (1) using the translog cost function, we must first exponentiate both sides of the cost function in (4). This yields a cost specification of the following form:

(5) $$C = e^{a0} \cdot e^{\alpha 1 \ln q1} \cdot e^{\alpha 2 \ln q2} \cdot \ldots$$

The problem of handling zero output values aside, we see that "fixed costs" (e^{a0}) enter as a scaling factor and will therefore cancel out of the SCOPE formula in (1) so we are unable to isolate the two.[3] In our application below, we provide separate estimates of the fixed-cost and cost complementary components of economies of scope for large U.S. banks in 1988.

III. Choice of an Appropriate Cost Function Specification.

Problems In Using The Translog Form. It is well known that there are problems with using the translog cost function to examine scope economies; nonetheless, the translog remains the most popular model in banking studies. Three alternatives are available to researchers who prefer the multiproduct translog model. The first alternative is to avoid computing scope economies directly but instead to infer their existence using sufficient conditions based on cost complementarities. Baumol, Panzar, and Willig (1982) have shown

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[3] A similar result holds for the Box-Cox variant of the translog form, as well as for the (logarithmic) Minflex Laurent form recently applied by LeCompte and Smith (1990) and Hunter, Timme, and Yang (1990). Thus for neither of these two forms, even when used in a frontier estimation framework (Ferrier and Lovell (1990)), is it possible to separate fixed-cost from complementarity effects.

that a twice-differentiable cost function will display economies of scope at $\underline{q}^*$ if it exhibits weak cost complementarities, i.e., if $\delta^2 C(\underline{q}',\underline{r})/\delta q_i \delta q_j \le 0$ $i \neq j$, for all output pairs at all output vectors $\underline{q}'$ with $\underline{0} \le \underline{q}' \le \underline{q}^*$ (with the inequality holding strictly for some subset). As discussed in Mester (1987b), particular care must be exercised when using this approach. To conduct the test properly the condition must be examined for a very broad range of output vectors, $\underline{q}'$, and this makes testing so cumbersome that it rarely is done correctly. Furthermore, some authors (Gilligan and Smirlock (1984) and Gilligan, Smirlock, and Marshall (1984)) have tested the estimated translog cost function for jointness and mistakenly interpreted their results as implying economies of scope.[4]

However, if we view the translog cost function as a log-quadratic approximation to the true unknown cost function, it is possible to develop a test of the complementarity condition that is based on cost function coefficients only and does not involve variable values (Denny and Pinto (1978)). In this case, the condition $\delta^2 C(\underline{q}',\underline{r})/\delta q_i \delta q_j \le 0$ reduces to the requirement that output coefficients satisfy $\alpha_i \alpha_j < \alpha_{ij}$ for all i and j, $j \neq i$.[5]

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[4] By restricting the cost function parameters, they impose the condition $\delta^2 C(\underline{q},\underline{r})/\delta q_i \delta q_j = 0$, $i \neq j$ (which for the translog specification is given by $\delta^2 \ln C(\underline{q},\underline{r})/\delta \ln q_i \delta \ln q_j + (\delta \ln C(\underline{q},\underline{r})/\delta \ln q_i)\ (\delta \ln C(\underline{q},\underline{r})/\delta \ln q_j) = 0$ $i \neq j$). Likelihood-ratio tests show that the restrictions are rejected by the data and this is taken by the authors as evidence of economies of scope. However, the test performed is a test of jointness only: rejection of the condition is consistent with either $\delta^2 C(\underline{q},\underline{r})/\delta q_i \delta q_j < 0$ (implying economies of scope) or $\delta^2 C(\underline{q},\underline{r})/\delta q_i \delta q_j > 0$ (implying increased costs for joint production). Without further testing it is impossible to determine which alternative has empirical support.

[5] Lawrence (1989) employs a variant of this procedure to test for economies of scope in banking. It is not clear from his discussion whether he conducted his likelihood ratio tests against the null hypothesis that there are no scope economies ($\alpha_i \alpha_j \ge \alpha_{ij}$) or the null that there are no economies or diseconomies

The Denny and Pinto condition ($\alpha_i\alpha_j<\alpha_{ij}$) is based on the relationship between the parameters of the true cost function and those of the log-quadratic (translog) approximation at the point of expansion. However, White (1980) has shown that the translog coefficients estimated using OLS need not coincide with the parameters of the approximation, even at the point of expansion. From a practical standpoint, therefore, this test is not very useful.

The second alternative is to estimate the translog cost function and substitute some small positive value in place of the zero outputs necessary to evaluate the SCOPE measure (1). This practice should not continue. Since the logarithmic function becomes arbitrarily steep and its value approaches $-\infty$ as its argument approaches zero, the translog cost function is badly behaved not only precisely at zero, but in a region around zero. Furthermore, the size of this region and the extent of the problem depend on parameter estimates and thus it is impossible to determine <u>ex ante</u> at which point cost behavior improves. The procedure of substituting small positive values for zero in the translog is apt to produce large estimates of scope economies (if most of the α_{ii} coefficients on squared outputs are positive) or large estimates of scope diseconomies (if most of the α_{ii} coefficients are negative). This characteristic of the translog is referred to as the "flip-flop" property by Röller (1991) and has been evident in a number of banking studies (e.g., Berger, Hanweck, and Humphrey (1987)).

The third alternative for measuring economies of scope using the

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($\alpha_i\alpha_j=\alpha_{ij}$). At any rate, it would have been preferable if he had conducted a joint test over all output pairs rather than test individual output pairs separately.

translog methodology is to estimate a generalized version of the translog model. Caves, Christensen, and Tretheway (1980) obtained a generalized translog cost function by applying a Box-Cox transformation to the output variables in the translog model. The Box-Cox transformation is defined as:

$$q^{(\phi)} = (q^{\phi}-1)/\phi \qquad \text{if } \phi\neq 0$$
$$= \ln q \qquad \text{if } \phi=0.$$

The generalized multiproduct translog cost function of Caves, Christensen, and Tretheway is therefore given by:

$$\text{(6)} \qquad \ln C = \alpha_0 + \Sigma\alpha_i q_i^{(\phi)} + \tfrac{1}{2}\Sigma\Sigma\alpha_{ij}q_i^{(\phi)}q_j^{(\phi)} + \Sigma\Sigma\delta_{ik}q_i^{(\phi)}\ln r_k + \Sigma\beta_k \ln r_k + \tfrac{1}{2}\Sigma\Sigma\beta_{kl}\ln r_k \ln r_l$$

The standard translog cost function results if $\phi = 0$. The generalized translog specification in (6) does admit zero output values (except in the case of $\phi = 0$) and can be examined for economies of scope using the SCOPE formula (1).[6]

From an estimation standpoint, the generalized translog often turns out to be less of an improvement than anticipated. In their study of the structure of the Bell System, Evans and Heckman (1984) estimate a generalized translog cost function and find the optimal value of ϕ to be fairly close to zero, the value corresponding to the translog. Similarly, when Lawrence (1989) estimates ϕ using banking data, it is found to be close to zero.

Upon further reflection, this result is not unexpected. The translog

[6] Some studies, rather than estimating the Box-Cox transformation parameter on output, have instead assumed that $\phi = 1$ (Berger, Hanweck, and Humphrey (1987); Buono and Eakin (1990)). While this alleviates the problem of zero output for scope estimation, the reported results suggested scope diseconomies and economies respectively.

cost function is a second-order Taylor expansion of the logarithm of costs (lnC) in terms of logarithms of outputs and input prices (lnq and lnr). We should therefore expect that the "optimal" Box-Cox transformation of q would turn out to be very close to lnq, i.e. a ϕ very close to zero, since the log transformation is still applied to the cost and input-price variables. If this is the case, then the estimated generalized translog model will be very close to the estimated translog model. Although we may then compute cost estimates for zero outputs (as long as the optimal ϕ is not exactly zero), the difficulties with the translog cost behavior in the neighborhood of zero will remain. The behavior of scope economies for the generalized translog at output points at or close to zero, illustrated below, confirms this observation. Thus, there appear to be no straightforward procedures for obtaining reliable estimates of economies of scope from the translog model.[7]

A More Appropriate Specification: The Composite Form. A more satisfying way to address the problems associated with scope measurement is to develop a multiproduct cost function by adding a suitable input price structure to the quadratic output structure recommended for examining multiproduct technologies by Baumol, Panzar, and Willig (1982) [hereafter, BPW]. One such family of

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[7] Yet another possibility is to generalize completely the log-quadratic (translog) cost specification by applying Box-Cox transformations not only to outputs, but also to input prices and the dependent variable, costs, as well. Such a cost function specification was examined in Lawrence (1989). However, since the transformation parameter estimates obtained by Lawrence are not statistically different from those of the standard translog form, little advantage is gained in using this approach to estimate economies of scope. In general, the procedure can produce quite complicated expressions for the SCOPE formula, particularly because of the transformation applied to the dependent variable. Another significant drawback is that it is difficult to restrict the cost function to be linearly homogeneous in input prices, as required by theory.

cost functions discussed by BPW (Chapter 15) is that suggested by Pulley and Braunstein (1982, 1984):

(7) $$C(\underline{q},\underline{r},) = (\alpha_0 + \Sigma\alpha_i q_i + \tfrac{1}{2}\Sigma\Sigma\alpha_{ij}q_i q_j) \cdot f(\underline{r})$$

where f(•) must be easily restricted to be linearly homogeneous in $\underline{r}$: Pulley and Braunstein use a simple Cobb-Douglas form for f(•). A member of the same class of cost functions has been used recently by Röller (1990) who shows that such cost functions satisfy the condition of "output flexibility".[8] The problem with cost functions of the form of expression (7), however, is that they are strongly separable in outputs and input prices. As BPW point out, separable cost functions are not well suited to model the effects of input prices on industry structure because of the restrictions imposed on potentially important input price-output quantity interactions. At a minimum, separability should be framed as a testable hypothesis rather than imposed *a priori*.

The cost function model we use in this study is a variant of the composite cost function described in Pulley and Braunstein (1990). It retains the output flexibility of the specification in expression (7) but adds output and input-price interaction terms so that separability is no longer imposed. Specifically, the composite cost function combines the quadratic output specification recommended for examining multiproduct technologies by BPW with the log-quadratic input price structure exhibited by the translog cost function--an input price structure that is easily restricted to be linearly

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[8] Röller estimates a CES-quadratic form where f($\underline{r}$) is a constant-elasticity-of-substitution (CES) specification while outputs follow a quadratic specification.

homogeneous. The composite cost function will be estimated in logarithmic form:[9]

$$\text{(8)} \qquad \ln C = \ln(\alpha_0 + \Sigma\alpha_i q_i + \tfrac{1}{2}\Sigma\Sigma\alpha_{ij} q_i q_j + \Sigma\Sigma\delta_{ik} q_i \ln r_k) + \Sigma\beta_k \ln r_k + \tfrac{1}{2}\Sigma\Sigma\beta_{kl} \ln r_k \ln r_l = \ln[h(\underline{q};\underline{\ln r})] + f(\underline{\ln r}).$$

The specification (8) cannot be obtained through generalizations of the sort examined by Lawrence (1989).[10] In what follows, we demonstrate that the composite form (8) is robust to points of evaluation at or close to zero while the standard or generalized translog forms are not.

IV. Estimates of Scope and Quasi-Scope for Large U.S. Banks.

Outputs, Inputs, And Estimation. Banks produce a variety of payment, safekeeping, intermediation, and accounting services for deposit and loan customers (Benston and Smith (1976); Mamalakis (1987)). However, some have argued that banks primarily produce loans. With this (asset) approach, the production of deposit services is viewed as merely payment in kind for the use of funds from which to make loans (Sealey and Lindley (1977)). In effect, this is a reduced-form model of the banking firm: the production of deposit services is treated as an intermediate output to depositors who provide loanable funds inputs for loans, so deposit services are netted out.

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[9] See Pulley and Braunstein (1990) for a more general specification of the composite model.

[10] The formula for economies of scope, derived by substituting the composite cost function into the SCOPE formula in (1), simplifies to yield the formulas in (2) for two outputs and (3), (3′), and (3″) for m outputs, the only change being $h(\underline{q};\underline{\ln r})$ replaces $h(\underline{q})$ in the denominator.

But there is no need to focus on only a single type of banking output like loans, especially since the production of deposit services accounts for half of all physical capital and labor input expenditures. Because deposit services are such a large component of bank value added, explicit modeling of their productive structure, along with that for loans, will yield a more accurate structural description of the bank as a whole. This objective can be achieved using a structural model of a multiproduct banking firm. In such a model, the production of deposit services is included among the set of bank outputs.

For our purposes, banks are considered to produce payment and safekeeping outputs (associated with the value of demand deposits (DD) and savings and small denomination time deposits (TS)) as well as intermediation and loan outputs (associated with the value of real estate (RE), installment and credit card (IN), and commercial and industrial loans (CI)). Over the last decade, these 5 output categories accounted for 75 to 80 percent of value added in banking (Berger and Humphrey (1991), Table 1). Such a categorization of bank output, with one exception, is consistent with that identified in the user cost approach to determine bank inputs from outputs (Hancock (1986); Fixler and Zieschang (1991)). Three inputs are specified: physical capital, labor, and funds (composed of core deposits plus purchased funds) and their prices enter the model directly.[11]

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[11] There is no inconsistency in specifying the value of core deposits as an output category and, at the same time, including interest expenses on these funds as contributing to total cost. The value of the stock of these deposits is taken as an indicator of the underlying payment and deposit and withdrawal transactions that comprise the actual flow of bank demand deposit and time and savings output, which is not available for large banks. And, using the same logic, the value of

Bank Call Report data on 205 large U.S. banks, all with assets greater than $1 billion in 1988, are used to illustrate the effect of employing different functional forms to obtain scope estimates. These banks account for almost 45 percent of all bank assets. Our preferred specification incorporates the 5-output, 3-input grouping noted above.[12] To illustrate the effect of aggregation on these results, scope estimates are also presented where first deposits and then deposits and loans are separately summed so that the 5 outputs become 4 and then 2, respectively.[13]

Standard procedures were used to estimate the composite cost function (8), the standard translog cost function (4), and the generalized translog (6). Sheppard's Lemma was used to develop the three input cost shares, and the cost functions were estimated jointly with the labor and interest cost share equations. The form of the share equation for input k from the

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the stock of loans is used as an indicator of the flow of new loan outputs. Since interest expenses, including those of purchased funds, comprise some 70% of bank total costs, it is clear that the interest cost of loanable funds is an important component of total cost and needs to be included. The controversy in the literature over whether deposits are an input or an output is not an issue here as both aspects are incorporated in our model. In any event, recent studies treating deposits first as an input then as an output have found that scale economy estimates and subadditivity are little affected either way (Hunter, Timme, and Yang (1990)).

[12] The input prices were: (1) new-contract replacement cost per square foot of bank and office building space for 9 U.S. regions; (2) expenditures on labor divided by the number of full-time-equivalent workers; and (3) the average nominal interest rate paid on core deposits.

[13] Specifically, the 4-output model is (DD+TS),RE,IN,CI while the 2-output case is (DD+TS),(RE+IN+CI). A model containing only bank assets was specified, but we experienced convergence problems and dropped it as an illustration. The problems with convergence are not too surprising since loan assets directly account for less than half of all bank costs (so a reduced-form specification is more difficult to identify locally than a structural model).

composite model is:

$$S_k = [\alpha_0 + \Sigma\alpha_i q_i + \tfrac{1}{2}\Sigma\Sigma\alpha_{ij}q_iq_j + \Sigma\Sigma\delta_{ik}q_i\ln r_k]^{-1} \cdot \Sigma\delta_{ik}q_i + \beta_k + \Sigma\beta_{kl}\ln r_l.$$

The form of the share equation for the translog model is:[14]

$$S_k = \Sigma\delta_{ik}\ln q_i + \beta_k + \Sigma\beta_{kl}\ln r_l.$$

All models were estimated using the nonlinear-least-squares procedure LSQ of the Time Series Processor (TSP) package.[15] If the errors are normally distributed, the estimates will be maximum likelihood estimates.

Quasi-Scope: A Measure Of Extrapolation Sensitivity. Although the composite model--unlike the translog--imposes no arbitrary restrictions on estimated cost behavior for zero outputs, the points examined in computing economies of scope for banks almost always lie outside the sample. We can have only limited confidence in estimated costs using these points, no matter what model is used for the cost function.

More confidence could be obtained if the SCOPE formula in (1) were generalized to examine a broader range of output values and shown to be stable at many points of evaluation, including values circumscribed by the sample observations. Therefore, instead of defining the costs of specialized production as the costs associated with producing a given amount of one output and none of the others, i.e. $C(0,.,q_i,.,0;\underline{r})$, the effects of quasi-specialized production are considered in which small amounts of all the other outputs are

[14] The share equation for the generalized translog is identical except the Box-Cox transformation $q_i^{(\phi)}$ replaces $\ln q_i$.

[15] Parameter restrictions for symmetry and linearly homogeneity in input prices were imposed. The restrictions are identical for the composite, translog, and generalized translog cost functions and are given by: Symmetry: $\alpha_{ij} = \alpha_{ji}$; $\beta_{kl} = \beta_{lk}$; and Linear Homogeneity: $\Sigma_k\delta_{ik}=0$, for all i; $\Sigma\beta_k=1$; $\Sigma_l\beta_{kl}=0$, for all k.

produced as well.[16] Production of the specialized output is adjusted so that the total amount produced via quasi-specialized production is equal to the quantity produced in joint production. Defining the parameter ϵ to be the proportion of the other outputs produced, the SCOPE formula in (1) may be modified to measure what can be referred to as "quasi" economies of scope:[17]

$$(9) \quad QSCOPE = [C(\{1-(m-1)\epsilon\}q_1,\epsilon q_2,..,\epsilon q_m;\underline{r}) + C(\epsilon q_1,\{1-(m-1)\epsilon\}q_2,\epsilon q_3,..,\epsilon q_m;\underline{r}) + ... + C(\epsilon q_1,..,\epsilon q_{m-1},\{1-(m-1)\epsilon\}q_m;\underline{r}) - C(q_1,q_2,..,q_m;\underline{r})]/C(q_1,q_2,..,q_m;\underline{r}).$$

As ϵ increases, the production points examined become less and less extreme relative to the sample observations. Using QSCOPE we can demonstrate the relative stability and sensitivity of scope estimates from the composite, standard translog, and generalized translog forms to the common problem of extrapolation beyond the confines of the banking data set.

When $\epsilon = 0$, QSCOPE becomes the traditional SCOPE measure in (1), capturing the fixed-cost savings from single-firm, as opposed to m-firm, production and the cost-complementarity effects from joint as opposed to specialized production. Strictly speaking, for $\epsilon > 0$ QSCOPE is an empirical subadditivity measure examining both scope and scale effects for a particular m-firm division of total output. The maximum value for ϵ in the QSCOPE calculations is 1/m. When $\epsilon = 1/m$, the quantity $\{1-(m-1)\epsilon\}$ equals 1/m and the

[16] If positive amounts of only some of the other outputs are produced, our concept is similar to the diversification measure developed by Grosskopf, Hayes, and Yaisawarng (1990), an approach discovered subsequent to our analysis here.

[17] This concept is closely related to expansion path subadditivity (Berger, Hanweck, and Humphrey (1987)), where there are only two firms producing quasi-specialized production. One of them produces the output level and mix of the average firm observed at a point on the industry expansion path while the other (synthetic) firm produces output equal to the difference between this firm's production and that of an average firm further out on the expansion path.

costs of quasi-specialized production, $C(\epsilon q_1,.,\{1-(m-1)\epsilon\}q_i,.,\epsilon q_m;\underline{r})$, equal the costs of producing the proportion 1/m of all outputs, $C(q_1/m,.,q_i/m,.,q_m/m;\underline{r})$. At this point the distinction of specialized production is lost and QSCOPE is a measure of the fixed costs effects just described and scale economies. Since banks have been shown to exhibit approximately constant returns to scale,[18] QSCOPE should reflect only fixed-cost savings for ϵ approaching 1/m.

SCOPE And QSCOPE Estimates. Table 1 contains the results of evaluating QSCOPE for the 5-output specifications of the composite, translog, and generalized translog cost models. The evaluation takes place at the sample medians.[19] QSCOPE values are obtained for eight values of ϵ (0--corresponding to SCOPE, .0001, .001, .01, .05, .10, .15, and .20).[20] Asymptotic standard errors are computed for each of the QSCOPE estimates using the procedure described in the Appendix. In addition to the QSCOPE estimates and standard errors, Table 1 also contains summary statistics from the estimation of the cost models. The optimal value of the Box-Cox transformation parameter, ϕ, is also reported for the generalized translog model. Tables 2 and 3 contain the results from the 4-output (DD+TS,RE,CI,IN) and 2-output (DD+TS,RE+CI+IN) models.

The results for the composite model in Tables 1, 2, and 3 show

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[18] See the surveys referenced in footnote 1.

[19] The sample medians of the five outputs (DD,TS,RE,CI,IN) are 681170, 1588700, 810860, 833870, and 444460, respectively--measured in $000's.

[20] Since $\epsilon = 0$ cannot be used with the translog cost function, we use $\epsilon = .000001$ in place of $\epsilon = 0.0$ in this case only.

significant economies of scope for all output specifications. The estimates of traditional economies of scope (corresponding to $\epsilon = 0.0$) indicate that the costs of specialized production of the five (four, two) banking outputs would be 50 (36, 30) percent greater than the costs of joint production. Importantly, for small ϵ equal to .0001, .001, or .01 the QSCOPE estimates are virtually identical to the traditional SCOPE estimate. This consistency of the estimated values for such modifications in the evaluation procedure mitigates our concern over the extrapolation problem discussed above. As ϵ increases, the composite function QSCOPE estimates--though still significantly different from zero--decline as specialized production becomes less and less concentrated in the given output; therefore, the advantages to specialization diminish. For example, when $\epsilon = .20$, the QSCOPE values are roughly half as large as the values when $\epsilon = 0$.[21]

As expected, the QSCOPE estimates for the translog models for ϵ values close to zero cannot be taken seriously. Large estimates of economies of scope are obtained because the coefficients on squared outputs (the α_{ii}'s) are positive in sign. The procedure of computing economies of scope by inserting small positive values in place of zero in the estimated translog function is unsatisfactory. Although the estimated asymptotic standard errors reflect the imprecision of the procedure, it does not seem prudent to attempt to estimate scope economies using a methodology that is expected _a priori_ to produce meaningless estimates. Furthermore, the magnitudes of the estimates for the

[21] When $\epsilon = .20$, the QSCOPE estimate in the 5-output model measures the cost savings from producing the median values of the five outputs jointly in one firm as opposed to producing one-fifth of the medians of each of the five outputs in five identical firms.

translog models do not result from taking the logarithm of arbitrarily small numbers. Even for small ϵ values, the arguments that actually enter the logarithmic functions are not trivial.[22]

The optimal value of the Box-Cox parameter in the generalized translog model is estimated to be .20 in the 5-output and 4-output models, and .10 in the 2-output model. Our expectation that the optimal value would be fairly close to zero--the value that results in the log-quadratic translog specification--is confirmed. Similarly, the QSCOPE estimates derived from the generalized translog cost function for small ϵ values exhibit the same unstable patterns as those derived from the translog.

For larger ϵ values the translog and generalized translog specifications yield QSCOPE estimates that are both significantly different from zero and similar to the estimates obtained from the composite model. Therefore, for $\epsilon \geq .10$ the QSCOPE estimates obtained from the composite cost function are robust across the three types of cost function models examined here. This finding buttresses the arguments made earlier regarding the appeal of the composite model in examining economies of scope. Furthermore, the differences between the estimates from the composite and the two translog models and the dramatic fluctuations in scope estimates for the latter models with small changes in ϵ (when ϵ is itself small) buttress earlier arguments against the use of the translog and generalized translog models in such examinations.

While the composite form is clearly the most reliable and stable one for investigating scope economies, all three forms yield very similar estimates of

[22] For example, given the medians of the banking outputs reported above, the value of ϵq_i ranges from 44 to 158 when $\epsilon = .0001$.

banking scale economies. This agreement is anticipated since scale estimates are based on curvature conditions at points were the sample information is complete: nonetheless, it is important since greater accuracy in identifying scope economies should not be offset by inaccuracies or biases in measuring scale economies. All three forms gave point estimates that indicate slight scale economies for the set of large U.S. banks examined here, when evaluated at the medians of their 5 outputs in 1988.[23] Other studies of large banks, both cross-section (Noulas, Ray, and Miller (1990)) and over time (Hunter, Timme, and Yang (1990); Humphrey (1991)), find average costs to have a slight U-shape, which is consistent with weak economies or diseconomies of scale, depending on the exact point of evaluation.

Marginal Costs And Statistical Fit. Well-behaved cost functions exhibit nonnegative marginal costs at all output vectors. As Berger, Hanweck, and Humphrey (1987) point out, a number of previous studies of scope economies obtained negative marginal costs for some banking outputs using the translog cost function. Since nonnegative marginal costs are part of the maintained hypothesis for using the estimated cost function to measure economies of scope, the validity of findings from such studies can be questioned. For all output specifications of the composite, translog, and generalized translog models, we evaluated the marginal costs of each output for each of the 205 banks in our sample. The marginal costs for the first output (demand deposits--DD) in the 5-output models were negative for the majority of the

[23] The scale economy values for the composite, standard translog, and generalized translog forms are, respectively, .941, .952, and .938 (indicating slight economies).

banks for all three cost function models. Except in a handful of cases, however, the marginal costs for all outputs under all three cost function specifications are positive in the 4-output and 2-output models. Thus, while the 5-output model is our preferred one, the theoretical conditions are more fully met in the more aggregative models.

The summary statistics for the models in Tables 1, 2, and 3 indicate that all three cost functions and associated share equations provide fairly comparable descriptions of the banking data. Since the translog is a constrained version of the generalized translog, log-likelihood ratios can be used to test whether it provides a statistically indistinguishable fit. The translog model is rejected in favor of the generalized translog model for the 5-output and 4-output specifications, while the two models are not statistically different in the 2-output case. Judged on the basis of sum-of-squared-error and adjusted-R^2 criteria, the composite specification provides a slightly better overall fit for the cost function and share equations for the 5-output and 4-output models in Tables 1 and 2, while the translog and generalized translog provide slightly better descriptions when only two outputs are specified. The generalized translog cost function provides the better fit for the cost function considered separately, although the differences in adjusted R^2 values are trivial.[24]

<u>Fixed-Cost And Complementarity Effects.</u> Estimates of economies of scope from the composite model increase in magnitude from 30 to 50 percent as the number of outputs increases. As discussed in Section II, this is to be

[24] To examine this issue correctly, single-equation estimation must be performed.

expected given the treatment of fixed costs in the SCOPE and QSCOPE formulas. Table 4 presents estimates of the fixed-cost ($SCOPE_{FC}$) and cost-complementarity ($QSCOPE_{CC}$) components of the scope and quasi-scope economies estimated for the composite function.[25]

The fixed-cost component increases from 6 to 15 to 26 percent as the number of outputs rises from 2 to 4 to 5. Thus the proportion of scope economies accounted for by fixed costs rises from 20 to 44 to 52 percent, respectively, illustrating that scope economies rise with reductions in the level of output aggregation. Since the fixed cost component of economies of scope is independent of the level of output, $SCOPE_{FC}$ is not a function of ϵ.

The cost complementarity component of scope economies behaves differently with respect to the level of output aggregation. Unlike the effect of spreading fixed costs across a broader product mix, the cost complementarity effects should be relatively stable regardless of the number of outputs specified. When $\epsilon = 0$, cost complementarities are between 20 to 24 percent for all output specifications,[26] indicating that complementarities are responsible for making the costs of joint production some 20 to 24 percent lower than the costs of specialized production. For the 5-output case, this is around the same percentage cost reduction that is attributable to reducing fixed costs. However, as the point of evaluation moves away from the zero

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[25] It is not possible to separate fixed-cost from complementarity effects in either the standard or the generalized translog forms.

[26] This consistency is all the more impressive when it is recognized that the number of output interaction terms (the α_{ij}, where $j>i$) on which the calculation is based ranges from only one in the 2-output model to ten in the 5-output model.

output levels associated with scope economies, the cost-complementarity effect becomes smaller and smaller. When $\epsilon = .20$, the cost-complementarity effect on QSCOPE is zero or close to it, and the only remaining effect is from spreading fixed costs (i.e. QSCOPE = $SCOPE_{FC}$ at large ϵ values). Recall from the earlier discussion of the QSCOPE measure that as ϵ approaches 1/m, the cost-complementarity effect converges to a pure scale effect. Since the estimated cost function shows almost no scale economies, the only influence left to affect QSCOPE derives from fixed cost. To sum up, increases in SCOPE associated with reductions in the level of output aggregation are due to the effects of spreading fixed costs over a larger product mix while reductions in QSCOPE associated with moving the point of evaluation away from zero outputs are due to the diminishing effects of cost complementarity and the absence of scale economies.[27]

V. Summary and Conclusions.

To date, scope economies in banking have been derived from logarithmic functional forms. This includes the standard multiproduct translog form (with its ad hoc fix-ups to accommodate zero outputs), the generalized translog (with a Box-Cox transformation on outputs), and the (logarithmic) Minflex Laurent form. Banking studies have reported scope results that have ranged from strong economies to diseconomies and are far from conclusive. The

[27] This decomposition can be generalized to the expansion path subadditivity measure (EPSUB) of Berger, Hanweck, and Humphrey (1987) and the diversification measure (DIVERS) of Grosskopf, Hayes, and Yaisawarng (1990). Like QSCOPE, the levels of EPSUB and DIVERS are determined from the net effect of both expanding the number of outputs produced at all specializing firms and by the level of quasi-specialized production.

problem is largely attributable to the choice of functional form, as noted in another context by Röller (1990). The instability of the logarithmic functional form is demonstrated using data on 205 large banks for 1988, accounting from almost 45 percent of total U.S. banking assets. An alternative functional form--the composite form--is introduced, which addresses these problems and is shown to yield stable scope results. This holds both at the usual point of evaluation (where non-specialized outputs are zero--SCOPE) and for points associated with quasi-specialized production (QSCOPE). When measurement is restricted to output levels much greater than zero, all the functional forms estimated here yield similar estimates of quasi-scope. This result, along with our other findings, suggests that the translog form (and its variants) should not be used to determine scope economies in banking, although they can be used to determine other efficiency measures such as QSCOPE and scale economies.

Using our composite functional form, significant scope economies on the order of 30 to 50 percent are shown to exist for large U.S. banks. The size of these economies depends on the number of banking outputs specified, becoming larger with increased disaggregation. Because the composite form permits us to decompose scope economies into their two sources--spreading fixed cost over a broader product mix and cost complementarity, this result can be investigated further. The component of economies of scope attributable to spreading fixed costs across product lines is a function of the number of banking outputs, and this is the reason why larger scope economies are measured when more, less-aggregated outputs are included in the cost function. In contrast, the component of scope economies attributable to cost complementarities is relatively stable with respect to the number of banking outputs specified. This effect accounts for between 20 and 24 percentage points of the 30 to 50 percent values obtained for overall scope economies. However, as the point of evaluation moves away from specialized production (the point of evaluation for SCOPE) and toward proportional production, the cost complementarity effect becomes smaller and approaches a measure of scale economies. Since banking scale economies are very slight, the net effect is to reduce our quasi-scope (QSCOPE) measure down to the point where it is almost solely comprised of cost reductions associated with spreading fixed costs over a larger product mix. Such insights into the reasons why the efficiency of joint production varies positively with output disaggregation but negatively with less-specialized production are not possible using the functional forms applied in current banking studies.

APPENDIX

Procedure for Computing Asymptotic Standard Errors for Estimates of Economies of Scope Derived from Multiproduct Cost Functions

Asymptotic variances for a (twice-differentiable) nonlinear function $f(b_1,\ldots,b_n)$ of n random coefficients may be approximated by $Var(f(b_1,\ldots,b_n)) = (\delta f/\delta \underline{b})'\Sigma_b(\delta f/\delta \underline{b})$, where $\delta f/\delta \underline{b}$ is the n-dimensional column vector representing the gradient of $f(\cdot)$ with respect to the random coefficients and Σ_b is the variance-covariance matrix of the coefficients. If the random coefficients are themselves maximum likelihood estimates, $f(\cdot)$ will have a limiting normal distribution. Here $f(\cdot)$ is the SCOPE or QSCOPE formula and the random coefficients are the estimated parameters of the cost function. The procedure is described in Thiel (1971) Chapter 8, especially problems 3.1-3.3, 3.6, and 3.7. Note that the medians of the output and price data in the SCOPE and QSCOPE formulas are treated as constants. Although this practice is standard and conforms to the procedure used in Mester (1987b), it does ignore the sample variability associated with computing medians.

Table 1
QSCOPE Estimates Based on Composite, Translog, And Generalized Translog Cost Functions
(5-Output Specification: DD,TS,RE,CI,IN)

QSCOPE VALUES
(Asymptotic Standard Errors in Parentheses)

Value of ϵ	Composite Model	Translog Model	Generalized Translog (Optimal ϕ: .20)
0.0[a] (=SCOPE)	.50 (.16)	6.41E+09 (1.32E+11)	4.14 (4.67)
.0001	.50 (.16)	3239.5 (27960.)	1.46 (1.44)
.001	.50 (.16)	33.6 (144.9)	.85 (.80)
.01	.48 (.15)	1.64 (3.44)	.43 (.47)
.05	.40 (.11)	.33 (.49)	.31 (.17)
.10	.32 (.09)	.20 (.18)	.30 (.09)
.15	.27 (.08)	.16 (.08)	.28 (.05)
.20	.25 (.07)	.14 (.06)	.27 (.04)

MODEL SUMMARY STATISTICS

	Composite Model	Translog Model	Generalized Translog
Cost Function:			
SSE[b]	4.78	5.00	4.71
Adj. R^2	.975	.974	.975
Labor Share Equation:			
SSE[b]	.295	.341	.323
Adj. R^2	.416	.325	.360
Interest Share Equation:			
SSE[b]	.722	.872	.817
Adj. R^2	.531	.434	.470
Log Likelihood	792.68	764.72	777.07

a ϵ=.000001 was used in place of ϵ=0.0 in the translog model.

b SSE refers to sum of squared errors.

Table 2
QSCOPE Estimates Based on Composite, Translog, And Generalized Translog Cost Functions
(4-Output Specification: DD+TS,RE,CI,IN)

QSCOPE VALUES
(Asymptotic Standard Errors in Parentheses)

Value of ϵ	Composite Model	Translog Model	Generalized Translog (Optimal ϕ: .20)
0.0[a] (=SCOPE)	.36 (.11)	4.51E+09 (3.90E+10)	754. (1867)
.0001	.36 (.11)	3460.5 (13557.)	75.0 (130.)
.001	.35 (.11)	38.9 (93.0)	24.2 (33.0)
.01	.34 (.10)	1.87 (3.42)	5.22 (4.61)
.05	.27 (.08)	.32 (.65)	1.27 (.65)
.10	.22 (.06)	.18 (.28)	.59 (.20)
.15	.18 (.05)	.15 (.13)	.35 (.08)
.20	.15 (.04)	.15 (.07)	.23 (.04)

MODEL SUMMARY STATISTICS

	Composite Model	Translog Model	Generalized Translog
Cost Function:			
SSE[b]	5.42	5.60	4.84
Adj. R^2	.972	.971	.975
Labor Share Equation:			
SSE[b]	.322	.359	.347
Adj. R^2	.363	.285	.310
Interest Share Equation:			
SSE[b]	1.03	1.09	1.08
Adj. R^2	.330	.293	.300
Log Likelihood	748.54	724.86	743.51

a ϵ=.000001 was used in place of ϵ=0.0 in the translog model.

b SSE refers to sum of squared errors.

Table 3
QSCOPE Estimates Based on Composite, Translog, And Generalized Translog Cost Functions
(2-Output Specification: DD+TS,RE+CI+IN)

QSCOPE VALUES
(Asymptotic Standard Errors in Parentheses)

Value of ϵ	Composite Model	Translog Model	Generalized Translog (Optimal ϕ: .10)
0.0[a] (=SCOPE)	.30 (.10)	8.34E+14 (3.88E+15)	4.86E+09 (1.93E+10)
.0001	.30 (.10)	3.03E+06 (6.17E+06)	1327 (1682)
.001	.29 (.10)	3626 (4109)	111 (94.2)
.01	.29 (.10)	30.2 (15.4)	10.1 (4.81)
.05	.25 (.08)	2.41 (.70)	1.92 (.58)
.10	.21 (.07)	.69 (.20)	.86 (.21)
.15	.17 (.05)	.19 (.09)	.47 (.11)
.20	.14 (.04)	-.05 (.05)	.30 (.06)

MODEL SUMMARY STATISTICS

	Composite Model	Translog Model	Generalized Translog
Cost Function:			
SSE[b]	6.02	5.61	5.59
Adj. R^2	.969	.971	.971
Labor Share Equation:			
SSE[b]	.327	.313	.311
Adj. R^2	.352	.380	.384
Interest Share Equation:			
SSE[b]	1.11	1.01	1.01
Adj. R^2	.281	.346	.346
Log Likelihood	726.49	742.54	743.83

a ϵ=.000001 was used in place of ϵ=0.0 in the translog model.

b SSE refers to sum of squared errors.

Table 4
Estimates of Fixed Costs and Cost Complementarity Effects

$SCOPE_{FC}$: Economies of Scope Resulting from Fixed Cost Only

5-Output Model	4-Output Model	2-Output Model
.26 (.08)	.15 (.04)	.06 (.01)

$QSCOPE_{CC}$: Economies of Scope Resulting from Complementarities Only
(Asymptotic Standard Errors in Parentheses)

Value of e	5-Output Model	4-Output Model	2-Output Model
0.0 (= $SCOPE_{CC}$)	.24 (.12)	.20 (.08)	.24 (.10)
.0001	.24 (.12)	.20 (.08)	.24 (.10)
.001	.24 (.12)	.20 (.08)	.24 (.10)
.01	.22 (.11)	.19 (.08)	.23 (.09)
.05	.13 (.07)	.13 (.05)	.19 (.08)
.10	.06 (.03)	.07 (.03)	.15 (.06)
.15	.01 (.01)	.02 (.01)	.12 (.05)
.20	-.00 (.01)	-.00 (.01)	.09 (.04)

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