Variance Properties of Solow's Productivity Residual and Their Cyclical Implications

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Abstract

For the United States economy (1960-1989), the correlation between the growth rates of the Solow residual and the real price of energy (government spending) is -0.55 (0.09). The Solow residual confounds movements in energy prices and government spending with those in true technology. Why? To address this question, this study develops a model to see if it quantitatively captures the endogenous transmission mechanism underlying the observed Solow residual correlations. It does. The transmission mechanism depends on endogenous capital utilization. With this transmission mechanism in place, and with the occurrence of shocks to `true' technology, energy prices, and government spending, the model economy accounts for 76 or 89 percent of U.S. output volatility, well matches the U.S. empirical regularities involving capital utilization and the Solow residual, and is generally consistent with other features of U.S. business cycles.

I. Introduction

Popular discussion often refers to energy price movements as shocks, shocks that are equivalent, in some sense, to technology shocks and important sources of fluctuations in economic activity. In fact, Hall (1988, 1990) rejects the invariance properties of Solow's productivity residual, a measure of technology shocks, primarily because it reflects oil price changes.[1] More exactly, using United States data (1953-1984), he finds that the most striking evidence against the invariance of the Solow residual to movements in exogenous variables (other than true technology) is that of significantly negative correlations between the growth rates of sectoral Solow residuals and the nominal price of oil, for most sectors of the economy. Another fact, documented by the present study for the United States (1960-1989), is that the correlation between the growth rate of the aggregate Solow residual and the real price of energy is -0.55. Finally, evidence for the postwar United States on the significance of the relationship wherein oil price increases preceed most recessions, is in Hamilton (1983) and Dotsey and Reid (1992).

Shocks to government spending are possibly important sources of economic fluctuations also, although they appear to be quite different from technology shocks. Consider that Hall (1988, 1990) finds significantly positive correlations between the growth rates of sectoral Solow residuals and military purchases for only a few sectors of the economy. Furthermore, the present study shows that the correlation between the growth rates of the aggregate Solow residual and total government spending is only 0.09.

These facts prompt the questions: How do energy price shocks transmit so strongly to the Solow residual? Why is it, simultaneously, that government spending shocks impact only slightly on the Solow residual? Does the explicit accounting for this transmission mechanism mean that the variance of the isolated, `true' technology component of the Solow residual is negligible? What is the quantitative importance of `true' technology, energy price, and government spending shocks, occurring in the presence of the Solow residual's transmission mechanism, in generating business cycle phenomena? In short, what do the variance properties of Solow's productivity residual imply for cyclical fluctuations? These questions are the focus here.

To address these questions, this study develops a model with perfect competition and constant returns to scale, that is quantitatively capable of capturing the endogenous production channels underlying the observed Solow residual correlations. The channels are: capital services, for a given stock of capital, and capital depreciation, which influences the stock of capital over time. Each depends on endogenous capital utilization.

Since energy enters the production function only because it is essential to the utilization of capital, the endogenous fluctuations in utilization and energy use are tightly linked. Solow residual growth can significantly differ from `true' technology growth by incorporating the effects of fluctuations in capital utilization, operating through the two indicated channels. Given the endogeneity of utilization and its linkage to energy use, all shocks, but especially energy price shocks, will impact on the Solow residual. Lucas (1987) also points out, in principle, that movements in the Solow residual possibly cloud those in true technology because of fluctuations in capital utilization.

The model's production structure is novel. As suggested above, there are two costs to the capital utilization decision: an energy and a depreciation cost. The production structure extends that of Taubman and Wilkinson (1970) and Greenwood, Hercowitz and Huffman (1988) by admitting the energy cost to utilization. Also, the production structure differs from that of a traditional energy model (see e.g. Rasche and Tatom (1981)), where capital utilization is fixed and the elasticity of substitution between the capital stock and energy is unity. The difference stems from the existence of the depreciation cost of utilization and the linkage between it and energy use. This creates an indirect channel, working through the stock of capital, in addition to the direct production function channel, by which fluctuations in energy infiltrate the economy.

The imposition of the model's first-order condition for utilization, capital accumulation equation and production function on published U.S. time series data (on output, labor hours, investment and energy prices) generates U.S. time series on utilization, `true' capital and `true' technology. A crucial finding is that this `true' technology measure is impervious to movements in U.S. energy prices and government spending. The upshot is that the fluctuations in U.S. capital utilization, responding to changes in U.S. energy prices and government spending, actually do offer a quantitative explanation of the Solow residual correlations observed in the U.S. data.
The explanation is consistent with perfect competition and constant returns to scale. Therefore, it sharply differs from Hall's (1988, 1990) explanation, which relies on imperfect competition and increasing returns to scale. It also turns out that the variance of `true' technology is not substantially smaller than that of the Solow residual itself.

Both the energy and depreciation cost margins of the capital utilization decision play a crucial role in making the U.S. measure of `true' technology impervious to changes in U.S. energy prices. Absent the energy cost margin, then the U.S. capital utilization series does not respond to energy price changes. This implies that the associated U.S. `true' technology measure is not free from influence of energy price changes. Absent the depreciation cost margin, then the elasticity of output with respect to energy use must be reduced to essentially equal the energy share of output. This number is too small to generate a U.S capital utilization series that is sufficiently responsive to energy price movements to render the associated U.S. `true' technology measure pure from the effects of energy price changes.

Calibrating the model economy to the U.S. data and examining its cyclical implications allows evaluation of the model and assessment of the quantitative importance of `true' technology, energy price and government spending shocks in generating cyclical phenomena. The evaluation gauges the model's ability to account for fluctuations in the U.S. time series on capital utilization, `true' capital, the Solow residual and standard macroeconomic variables. The empirical regularities obtaining for the former series constitute new dimensions for the evaluation of business cycle models.

The model, with all three shocks operating, accounts for 76 or 89 percent of U.S. output volatility, well matches the U.S. regularities involving capital utilization and the Solow residual, and is generally consistent with other facts characterizing U.S. business cycles. Energy price shocks promote the model's ability to match the U.S. data along many dimensions. Shocks to government spending exert quite a mixed influence on the model's explanation of the U.S. data.

Section II outlines the model and solution technique. Section III describes the empirical data and measures of technology growth. Section IV notes the calibration and evaluation procedures. Section V presents and discusses the findings. Section VI concludes the paper.

II. The Model and Solution Technique

(i) The Economic Environment

Consider an environment with a representative firm and household and a government. The representative firm is a price taker on all markets, solving the following problem:

$$(1) \qquad \max_{(l_t, k_t h_t)} \quad \Pi_t = y_t - w_t l_t - r_t k_t h_t$$

subject to the production function:

$$(2) \qquad y_t = F(z_t l_t,\ k_t h_t) = (z_t l_t)^{\theta}\ (k_t h_t)^{(1-\theta)} \quad , \quad 0 < \theta < 1$$

where: Π is per-capita profit, y is per-capita output, w is the wage rate for labor, l is per-capita labor hours, r is the rental rate for capital services, k is the per-capita stock of capital in place at the beginning of the period, h is the utilization rate of k, z is the exogenous technology variable, θ is the labor share of output and subscript t denotes time t. The production function, F, satisfies standard properties, constant returns to scale and a unitary elasticity of substitution between l_t and k_t. Given constant returns to scale, permanent technological change must be of labor-augmenting form to ensure that the model is consistent with balanced growth (see King, Plosser and Rebelo (1988)). This rationalizes the way in which z_t enters (2). The production function differs from the standard neoclassical one solely by the inclusion of h_t, representing the intensity of capital utilization (i.e., the number of hours per period and/or the speed per hour at which the capital stock is operated). For a given k_t, h_t determines the flow of capital services, $k_t h_t$. The manner in which h_t enters (2) follows Taubman and Wilkinson (1970) and Greenwood, Hercowitz and Huffman (1988), admitting flexible proportions between l_t and h_t and a direct relationship between labor's productivity and h_t.

The representative household is infinitely-lived with preferences over consumption and leisure defined by:

$$E_0 \sum_{t=o}^{\infty} \beta^t u(c_t,l_t) \quad , \quad u(c_t,l_t) = \log c_t + \gamma \log(1-l_t) \quad , \quad 0 < \beta < 1 \ ,$$

where: c is per-capita consumption, β is the discount factor, γ is a preference parameter and the time endowment is normalized at unity. The momentary utility function, u, satisfies standard properties and a unitary elasticity of substitution between consumption and leisure. The latter restriction ensures that the model is consistent with balanced growth and a stationary allocation of time to market work (see Kydland (1984)).

The household's capital stock evolves according to:

$$k_{t+1} = [1 - \delta(h_t)]k_t + i_t \quad , \quad \delta(h_t) = h_t^{\omega}/\omega \quad , \quad 0 < \delta(\cdot) < 1 \quad , \quad \omega$$

where: i is per-capita gross investment and ω is a parameter. Equation (4) differs from the standard capital accumulation equation by allowing variable depreciation; δ is an increasing convex function of h_t. This specification also follows that in Taubman and Wilkinson (1970) and Greenwood, Hercowitz and Huffman (1988). It captures Keynes's notion of the user cost to capital, with higher utilization causing faster depreciation, at an increasing rate, because of wear and tear. In the present environment, utilization also involves an energy cost. Specifically:

$$(5) \quad e_t/k_t = a(h_t) \quad , \quad a(h_t) = h_t^{\nu}/\nu \quad , \quad \nu \geq 1$$

where: e is per-capita energy usage and ν is a parameter. Equation (5) is a technical relationship capturing the idea that energy is essential to the utilization of capital, with an increase in utilization increasing energy usage, per unit of capital, at an increasing rate. Jorgenson and Griliches (1967) espoused a similar idea: electricity and utilized capital are complementary in production. The convexity of the function, a, is motivated by considerations of diminishing marginal energy efficiency.

Allowing an energy cost dimension to the capital utilization decision marks an important difference between the production structure here and that in Taubman and Wilkinson (1970) and Greenwood, Hercowitz and Huffman (1988). Also, consider the following. Use (5) to substitute for h_t in (2), obtaining:

$$(2') \quad y_t = (z_t l_t)^{\theta} \, [k_t^{(1-1/\nu)} \, e_t^{(1/\nu)} \, \nu^{(1/\nu)}]^{(1-\theta)}$$

This production function is identical in form to one that holds capital utilization fixed and maintains a unitary elasticity of substitution between the capital stock and energy, as in some earlier energy models (e.g. Rasche and Tatom (1981)). But, the production structure here differs crucially from that of those earlier energy models by allowing depreciation to depend on utilization and, through it, on energy use. This creates an indirect channel, working through the stock of capital, in addition to the direct production function channel, by which fluctuations in energy impact on the economy.

The household's budget constraint is:

(6) $w_t l_t + (1-\tau) r_t k_t h_t = c_t + i_t + p_t e_t + x_t$

where: τ is the tax rate on capital income, p is the exogenous relative price of energy and x is the lump-sum tax. Equation (6) sets total income equal to total expenditure. The household is a price taker on all markets, views transfers and taxes as given, and maximizes expected lifetime utility in (3) by choosing c_t, l_t, k_{t+1}, h_t, and therefore i_t and e_t, subject to the technical and budget constraints in (4) - (6).

Government enters the economy by purchasing goods and taxing income according to:

(7) $g_t = x_t + \tau r_t k_t h_t$

where: g is per-capita exogenous government purchases. This is a simple specification of fiscal policy; government's budget balances each period, government spending is exogenous and there is only one type of distortional income taxation. (See Greenwood and Huffman (1991) and Cooley and Hansen (1992) for analyses of various types of distortional taxes.) Shocks to government spending impact on the economy only through wealth effects. Section III indicates the reason for explicitly including capital taxation.

The stochastic exogenous shock structure is:

(8) $\log(z_{t+1}) = \log(z_t) + \log(\bar{z}) + u_{zt+1}$

(9) $\log(\bar{g}_{t+1}) = \rho_g \log(\bar{g}_t) + (1-\rho_g)\log(\bar{g}) + u_{gt+1}, \quad \bar{g}_t \equiv g_t/z_t \quad , \quad 0 < \rho_g < 1$

(10) $\log(p_{t+1}) = \rho_p \log(p_t) + (1-\rho_p)\log(p) + u_{pt+1}, \quad 0 < \rho_p < 1$

where: $\log(\bar{z})$ is the mean growth of z_t, $\log(\bar{g})$ is the mean of $\log(\bar{g}_t)$, $\log(p)$ is the mean of $\log(p_t)$ and ρ_g, ρ_p are parameters. The innovations u_{zt+1}, u_{gt+1} and u_{pt+1} have zero means, and are the realizations from the stationary Markov distribution function $\Phi(u_{t+1} \mid u_t)$ at the beginning of time (t+1); where u_{t+1} is a vector comprising of the three innovations. The specification of the g_t process implies that movements in z_t generate permanent movements in g_t, while changes in $\bar{g}_t$ cause temporary fluctuations in g_t.

(ii) The Competitive Equilibrium

The economy's competitive equilibrium obtains when the firm and household solve their problems and the government budget constraint holds. It is implicitly defined by:

(11) $w_t = F_1(z_t l_t, k_t h_t) z_t$

(12) $r_t = F_2(z_t l_t, k_t h_t)$

(13) $-u_2(c_t, l_t) = u_1(c_t, l_t) w_t$

(14) $\delta'(h_t)k_t + a'(h_t)p_t k_t = (1-\tau) r_t k_t$

(15)

$$u_1(c_t, l_t) = \beta \underset{t}{E}\left[u_1(c_{t+1}, l_{t+1})\left((1-\tau)\, r_{t+1}\, h_{t+1} + 1 - \delta(h_{t+1}) - a(h_{t+1})p_{t+1}\right)\right]$$

(16) $y_t - p_t e_t = c_t + i_t + g_t$

(2), (4) - (5) and (7) - (10).

Equations (11) and (12) equate factor prices to the respective marginal productivities. The efficiency condition governing l_t is equation (13). The sum of the marginal depreciation and energy costs is set equal to the after-tax marginal return to an increase in h_t, in equation (14), determining the efficient value of h_t. Equation (15) is the efficiency condition governing capital accumulation. It differs from the standard one not only by including h_{t+1} but also by subtracting the marginal energy cost from the after-tax marginal return to an increase in k_{t+1}. The resource constraint for the economy is equation (16), obtained by substituting (7) into (6) while noting (2), (11) and (12). It sets income, $y_t - p_t e_t$, equal to expenditure, $c_t + i_t + g_t$, for the representative agent. One interpretation of the term $p_t e_t$ is that it is value added to the production of final goods, y_t, by a foreign economy at price, p_t. In this interpretation, the domestic economy exports (imports) final (intermediate) goods to (from) this foreign economy in the amount $p_t e_t$; it is the only international trade that occurs and trade balances each period.

In this economy a positive shock to p_t will directly cause a negative income effect (see (16)) that works to decrease c_t and increase l_t. From (14), the efficient value of h_t falls, which in turn reduces labor's marginal productivity and promotes an intratemporal substitution effect to decrease c_t and l_t (see (11) and (13)). In addition, the fall in h_t directly impacts on the production function, working to reduce y_t and to enhance the negative income effect of the shock to p_t. This is the sense, then, in which a positive energy price shock is tantamount to a negative technology shock in the present environment. If the increase in p_t is somewhat persistent, intertemporal substitution margins are affected as follows (see (15)): capital accumulation declines as agents smooth consumption and anticipate lower returns to investment.

A positive shock to g_t will also cause a negative income effect (see (16)) that tends to reduce c_t and raise l_t. The increase in l_t increases the marginal productivity of capital services and thus also the efficient value of h_t (see (12) and (14)). Labor's marginal productivity falls as l_t rises, but it does so by a smaller amount than it would in the absence of the increase in h_t. The fall in labor's marginal productivity prompts an intratemporal substitution effect that enhances the decrease in c_t and mitigates the increase in l_t. The increases in l_t and h_t cause y_t to increase, dampening the negative income effect of the shock to g_t. To the extent that the shock is temporary, it is likely that investment falls as agents smooth consumption.

(iii) The Solution Technique

An exact solution for the competitive equilibrium is not possible. An approximate solution was obtained using the technique advanced by King, Plosser and Rebelo (1988). Appendix 1 indicates the key steps.

III. The Empirical Data and Measures of Technology Growth

(i) The Empirical Data

The empirical data are annual, real, per-capita data for the United States over the period 1960-1989. The calibration and evaluation use this data. The choice of periodicity and time period stems from the desire to use the longest and most relevant data series on energy usage available. Appendix 2 presents full details and sources of the published data.

Energy usage is the sum of electricity, coal, natural gas and petroleum usage by the private non-energy production sector of the economy. The four components of this energy good serve as weights in the construction of the energy price deflator. The real price of energy is the ratio of the energy price deflator to the gross domestic product price deflator. Output is gross domestic product plus energy usage less the sum of gross housing, government and energy-sector products. Consumption is personal consumer expenditure on nondurables and

services minus that on housing services and energy goods. Investment is gross private domestic fixed investment in nonresidential capital, excluding that component for the energy sector. Government spending is government purchases of goods and services. Labor hours are the product of employment and average hours per worker per year,
where employment is private non-energy sector employment.

One measure of the capital stock, denoted by k_t^*, is the net stock of private domestic fixed nonresidential capital, excluding that component for the energy sector.[2] The Perpetual Inventory Method underlies the construction of k_t^*; it assumes a constant depreciation rate. Accordingly, k_t^* is not the empirical counterpart to the model's capital stock, k_t. Published data also do not provide a satisfactory empirical counterpart to h_t. Existing utilization measures consist of the detrended component of manufacturing output and a survey measure for only part of the economy that includes the utility sector (defined as mining, manufacturing and utilities).

However, the model's structure, combined with other published data, implies empirical series for h_t and k_t. Specifically:

(14') $$h_t^{(\omega-1)} + h_t^{(\nu-1)}p_t = (1-\tau)(1-\theta)y_t/(k_t h_t),$$ and

(4) $$k_{t+1} = [1 - \delta(h_t)]k_t + i_t \quad , \quad \delta(h_t) = h_t^{\omega}/\omega,$$

together with published data on p_t, y_t and i_t, imply empirical series for h_t and k_t.[3] Equation (14') derives from (14), by noting (2), (12) and functional forms for $\delta(h_t)$ and $a(h_t)$. Values for parameters in (14') and (4) come from the model calibration, which uses growth observations, estimated parameters of the exogenous processes, and other studies.

This data generation process revealed the necessity of including realistic distortional capital income taxation in the model to bring the model's balanced-growth-path value for (y_t/k_t) into line with the average U.S. data value of (y_t/k_t) (which is 0.95). The initial value of k_t was next chosen to achieve equality between those two values (it is 0.0148). Figures 1 and 2 show the resultant h_t and k_t.

(ii) The Empirical Measures of Technology Growth

The imposition of (2) on the empirical data gives rise to the measure of `true' technology growth (i.e., true according to the model):

$$(17) \quad \Delta\log z_t = [\ \Delta\log y_t - \theta\ \Delta\log l_t - (1-\theta)(\Delta\log k_t + \Delta\log h_t)\ ]/\theta$$

The standard measure of technology growth, Solow residual growth, is:

$$(18) \quad \Delta\log sr_t = [\ \Delta\log y_t - \theta\ \Delta\log l_t - (1-\theta)\ \Delta\log k_t^*\ ]/\theta$$

where: sr denotes the Solow residual. The value of θ comes from the model calibration. Figures 3 and 4 display z_t and sr_t. The two technology measures differ in their treatment of utilization and measurement of capital. The distinction is an important one. Consider the time series properties in Table 1.

Table 1

VARIABLE	% SD	CORRS	CORRZ
$\Delta \log sr_t$	2.60	1.00	0.77 (0.000)
$\Delta \log z_t$	2.25	0.77 (0.000)	1.00
$\Delta \log g_t$	2.95	0.09 (0.632)	0.02 (0.921)
$\Delta \log p_t$	10.39	-0.55 (0.002)	-0.001 (0.995)

Key: % SD denotes the percentage standard deviation.
CORRS denotes the correlation with $\Delta \log sr_t$.
CORRZ denotes the correlation with $\Delta \log z_t$.
Parentheses contain two-tailed marginal significance levels for a t-test.

The correlations between $\Delta \log sr_t$ and $\Delta \log g_t$ and between $\Delta \log z_t$ and $\Delta \log g_t$ are both mildly positive and insignificant.

A sharp difference emerges across the correlations between $\Delta\log sr_t$ and $\Delta\log p_t$ and between $\Delta\log z_t$ and $\Delta\log p_t$. The former is strongly negative and significant while the latter is negligible and insignificant. These findings are consistent with those in Hall (1988,1990) for the United States (1953-84). Hall finds significantly negative (positive) correlations between the growth rates of sectoral Solow residuals and nominal oil prices (real military purchases), for many (a few) sectors of the economy.

The significant Solow residual correlations make nonsense of viewing it as a measure of true technology. Hall (1988, 1990) argues that such correlations stem from the existence of market imperfection combined with increasing returns to scale. Given the insignificant correlations involving $\Delta\log z_t$ in Table 1, an alternative explanation is possible. The fluctuations in capital utilization, responding to movements in energy prices and government spending, explain the Solow residual correlations. They do so in a manner consistent with perfect competition and constant returns to scale. In addition, with view to the standard deviations of $\Delta\log z_t$ and $\Delta\log sr_t$ in Table 1, this explanation obtains while the variance of $\Delta\log z_t$ is not substantially smaller than that of $\Delta\log sr_t$.

Both the energy and depreciation cost margins of the capital utilization decision play a critical role in achieving the insignificant correlation between $\Delta \log z_t$ and $\Delta \log p_t$. Consider (14') once again:

$$(14')\qquad h_t^{(\omega-1)} + h_t^{(\nu-1)}p_t = (1-\tau)(1-\theta)y_t/(k_t h_t)$$

Absent the energy cost margin, then the term including p_t disappears from (14'), implying that h_t is unresponsive to p_t. The upshot is that the associated U.S. `true' technology measure is not purged of the influence of energy price changes. Next, rearrange (14'), noting (5) and the functional forms for $\delta(h_t)$ and $a(h_t)$, to get:

$$(14'')\qquad \frac{p_t e_t}{y_t} = (1-\tau)\frac{(1-\theta)}{\nu} - \frac{\omega}{\nu}\,\frac{k_t}{y_t}\,\delta(h_t)$$

Absent the depreciation cost margin, then the term involving $\delta(h_t)$ disappears from (14"), implying that the elasticity of output with respect to energy use, $(1-\theta)/\nu$, (multiplied by $(1-\tau)$), equals the energy share of output.

Calibrating the model to match average values of the U.S. energy and capital shares of output, and capital income taxes, then requires a value for ν that is "too high." It is too high to generate a h_t series sufficiently responsive to p_t (see (14')) to render the associated U.S. `true' technology measure pure from the effects of energy price movements.

Maintaining the more restrictive assumptions of a fixed proportionate relationship between the hours worked by capital and labor, and constant depreciation, as in Kydland and Prescott (1988, 1991), does not give rise to a satisfactory $\Delta \log z_t$ series. Specifically, that $\Delta \log z_t$ exhibits very similar dynamics to those of $\Delta \log sr_t$. The correlation between the two series is 0.996 and $\Delta \log z_t$ shows a correlation with $\Delta \log p_t$ ($\Delta \log g_t$) equal to -0.54 (0.07). This result, essentially, obtains not only because of the small capital share but also because the correlation between the rate of change of hours per worker and $\Delta \log p_t$ is not sufficiently negative. (Contrast the latter correlation, equalling -0.41, to the correlation between $\Delta \log h_t$ and $\Delta \log p_t$, equalling -0.86.)

Hall (1988, 1990) rules out fluctuations in capital utilization as being quantitatively capable of explaining the Solow residual correlations. The reason for the apparent inconsistency between that argument and the one advanced here concerns the modelling of the utilization rate. Hall maintains a fixed proportionate relationship between capital utilization and total labor hours per unit of the capital stock, as well as constant depreciation. This is very similar to the Kydland and Prescott (1988, 1991) model. The present model does not impose such restrictions.

IV. Calibration and Evaluation

This section outlines the calibration and evaluation procedures, first advanced by Kydland and Prescott (1982). Define the model's time period to be one year. To denote the steady state values of model variables, use the notation introduced earlier except omit time subscripts and use a bar to signify the stationary counterpart of a variable (except for z).

(i) Calibration

First consider the exogenously-set values for parameters and variables, based on U.S. data averages or other studies. Imposing balanced growth and equation (2) on U.S. data gives $\bar{z}$ = 1.0162, which equals the average gross growth rate of U.S. y_t. The gross, after-tax real return to capital along the model's balanced growth path, $\bar{z}/\beta$, is set equal to 1.0650, the value in King, Plosser and Rebelo (1988). θ equals 0.70, the value in Greenwood, Hercowitz and Krusell (1992). l is set equal to 0.3529, the U.S. average value for the ratio of hours worked to total nonsleeping hours (per worker). $\delta(h)$ equals 0.0796, the average depreciation rate of k_t^*; computed as the average value of the U.S. series: $(i_t - k_{t+1}^* + k_t^*)/k_t^*$. The government share of output, $\bar{g}/\bar{y}$, is 0.2695, equalling the average value of U.S. g_t/y_t. The energy share of output, $(p\bar{e})/\bar{y}$, is 0.0430, equal to the average value of U.S. $(p_t e_t)/y_t$. p is set equal to 0.9386, which is the average value of U.S. p_t. τ equals 0.35, the value in Greenwood and Huffman (1991).

Next consider the endogenously-derived parameter values. No outside guide is available for the values of ω and ν. However, using the foregoing exogenously-set values together with: (a) the steady-state conditions determining h and the energy-capital ratio along the balanced growth path, $\bar{e}/(\overline{kz}^{-1})$ and (b) the definitions of $\delta(h)$ and a(h), allows simultaneous solution for h, ω and ν. The resultant value of $\omega(\nu)$ is 1.4435 (1.7260). There is no direct evidence on the value of γ. The foregoing exogenously-set values combined with the steady state conditions of the model imply $\gamma = 2.1874$.

Least-squares regressions give consistent estimates of the parameters of the stochastic exogenous processes. The most parsimonious and adequate specifications are:

$$(8') \qquad \log(z_{t+1}) = \log(z_t) + \log(\bar{z}) + u_{zt+1}, \qquad u_{zt+1} = \epsilon_{zt+1} + \eta_z\epsilon_{zt}$$

$$(9') \qquad \log(\bar{g}_{t+1}) = \rho_g\log(\bar{g}_t) + (1-\rho_g)\log(\bar{g}) + u_{gt+1}, \qquad u_{gt+1} = \epsilon_{gt+1} + \eta_g\epsilon_{gt}$$

$$(10') \qquad \log(p_{t+1}) = \rho_p\log(p_t) + (1-\rho_p)\log(p) + u_{pt+1}, \qquad u_{pt+1} = \epsilon_{pt+1} + \eta_p\epsilon_{pt}$$

where: ϵ_{it} is a stationary, zero-mean, serially-uncorrelated innovation and η_i is a parameter (i= z, g, p). Table 2 presents the findings (the key indicates new notation). The coefficient estimates are significantly greater than zero (at approximately the 5% significance level). Therefore, they provide values for ρ_g, ρ_p, η_z, η_g and η_p. The point estimates: $\hat{\sigma}_z$, $\hat{\sigma}_g$ and $\hat{\sigma}_p$ give values for σ_z, σ_g and σ_p, respectively. Of the covariance estimates, only $\hat{\sigma}_{zg}$ is significantly different from zero (based on t-tests on the coefficients of least-squares regressions of $\hat{e}_{it}$ on $\hat{e}_{jt}$ ($i = z, g, p$)). Therefore, set $\sigma_{zg} = \hat{\sigma}_{zg}$, $\sigma_{zp} = 0$ and $\sigma_{gp} = 0$. Analysis of residual autocorrelations suggests that the residuals are serially uncorrelated. Table 3 lists parameter and steady-state variable values.

There is no inconsistency across the findings of a significant correlation between innovations to $\Delta\log z_t$ and $\log \bar{g}_t$ and an insignificant correlation between $\Delta\log z_t$ and $\Delta\log g_t$. Consider an application of the Granger Representation Theorem. Begin by supposing that $\log g_t$ and $\log z_t$ are I(1), cointegrated processes with independent innovations and cointegrating vector, [1 -1]. An example of the error-correction-form for the vector stochastic process, $[\Delta\log g_t \ \Delta\log z_t]'$, is:

$$(19) \quad \begin{bmatrix} \Delta\log g_t \\ \Delta\log z_t \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}) \begin{bmatrix} \alpha_3 \\ 0 \end{bmatrix} [1 \ -1] \begin{bmatrix} \log g_{t-1} \\ \log z_{t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

where ϵ_{1t} and ϵ_{2t} are stationary, zero-mean, independent innovations and α_1, α_2 and α_3 are positive scalars. From (19) it follows that:

$$\log \bar{g}_t = (1-\alpha_3)\log \bar{g}_{t-1} + (\alpha_1-\alpha_2) + (\epsilon_{1t}-\epsilon_{2t}) \tag{20}$$

Under balanced growth, $E(\Delta\log z_t) = E(\Delta\log g_t)$. Using this hypothesis and

taking expectations in (19) gives:

$$\alpha_1-\alpha_2 = \alpha_3\log \bar{g} \tag{21}$$

Substituting (21) into (20) implies:

$$\log \bar{g}_t = (1-\alpha_3)\log \bar{g}_{t-1} + \alpha_3\log \bar{g} + (\epsilon_{1t}-\epsilon_{2t}) \tag{22}$$

The second equation in (19) and equation (22) have exactly the same structures as equations (8') and (9'), respectively. From (19) and (22) it is clear that the innovations to $\Delta\log z_t$ (ϵ_{2t}) and to $\log \bar{g}_t$ $(\epsilon_{1t} - \epsilon_{2t})$ will exhibit negative covariation, while the independence of ϵ_{1t} from ϵ_{2t} may be sufficient to cause an insignificant correlation between $\Delta\log z_t$ and $\Delta\log g_t$.

Table 2

Coefficient Estimates

$\hat{\eta}_z = 0.3598\ (0.1895)$

$\hat{\rho}_g = 0.9098\ (0.0354)$ $\hat{\eta}_g = 0.7862\ (0.1961)$

$\hat{\rho}_p = 0.9039\ (0.0652)$ $\hat{\eta}_p = 0.3376\ (0.2041)$

Residual Properties

$\hat{\sigma}_z = 0.0210$ $\hat{\sigma}_{zg} = -0.0003$ $\hat{c}_{zg} = -0.5417$

$\hat{\sigma}_g = 0.0284$ $\hat{\sigma}_{zp} = -0.0002$ $\hat{c}_{zp} = -0.0982$

$\hat{\sigma}_p = 0.0966$ $\hat{\sigma}_{gp} = -0.0002$ $\hat{c}_{gp} = -0.0610$

Autocorrelations

(S.E. = 0.185)

	$\hat{e}_{zt}$	$\hat{e}_{gt}$	$\hat{e}_{pt}$
Lag 1	0.01	-0.06	0.03
Lag 2	0.25	0.25	0.16
Lag 3	-0.02	-0.06	0.13
Lag 4	0.16	0.06	-0.06
Lag 5	0.04	-0.03	0.04
	Q(5) = 2.61	Q(5) = 2.11	Q(5) = 1.41
	$\chi^2_4 = 9.49$	$\chi^2_3 = 7.81$	$\chi^2_3 = 7.81$

Key: (i) ^ denotes an estimated quantity.
(ii) Standard errors are in parentheses.
(iii) σ_i is the standard deviation of ϵ_{it} (i = z, g, p).
σ_{ij} (c_{ij}) is the covariance (correlation) between ϵ_{it} and ϵ_{jt} (i,j = z, g, p).
(iv) S.E. denotes standard error.
Q is the Box-Pierce statistic.
χ^2_i is the critical value of the chi-square, at the 5% significance level and i degrees of freedom.
(v) Sample period: 1961-1989.

Note: The calibrated values of $\bar{z}$ and p were imposed on (8') and (10') during the estimation. The mean of U.S. $\bar{g}_t$ was imposed on (9') during its estimation.

Table 3

Preferences

$\beta = 0.9542$
$y/(\overline{kz^{-1}}) = 0.9513$
$\gamma = 2.1874$

Production

$\theta = 0.70$
$e/(\overline{kz^{-1}}) = 0.0436$
$\omega = 1.4435$
$\nu = 1.7260$

Tax rate

$\tau = 0.35$

Steady State Variables

$\bar{y} = 0.1896$
$\bar{c} = 0.1113$
$\bar{i} = 0.0191$
$\bar{g} = 0.0511$ $\qquad \bar{g}/\bar{y} = 0.2695$
$\bar{p} = 0.9386$
$e = 0.0087$ $\qquad p\bar{e}/\bar{y} = 0.0430$
$\bar{l} = 0.3529$
$k = 0.2026$ $\qquad \delta(h) = 0.0796$
$h = 0.2234$

Stochastic Structure

$\bar{z} = 1.0162$ $\qquad \eta_z = 0.3598$ $\qquad \sigma_z = 0.0210$ $\qquad \sigma_{zg} = -0.0003$
$\sigma_{zp} = 0$

$\bar{g} = 0.0511$ $\qquad \rho_g = 0.9098$ $\qquad \sigma_g = 0.0284$ $\qquad \sigma_{gz} = \sigma_{zg}$
$\eta_g = 0.7862$ $\qquad \sigma_{gp} = 0$

$p = 0.9386$ $\qquad \rho_p = 0.9039$ $\qquad \sigma_p = 0.0966$ $\qquad \sigma_{pz} = 0$
$\eta_p = 0.3376$ $\qquad \sigma_{pg} = 0$

(ii) Evaluation

(a) Simulate time paths for the logarithmic levels of variables of interest, using the Markovian decision rules and laws of motion of the exogenous variables for the nonstationary economy.

The time paths have 30 observations, the size of the U.S. data sample. Any one simulation corresponds to one sample of 30 realizations of the vector $\epsilon_t = [\epsilon_{zt}\ \epsilon_{gt}\ \epsilon_{pt}]$. Two alternative approaches are taken to obtain this sample: [1] uses a normal random number generator; [2] uses the actual sequence of residuals from the estimation exercise. The approach in [1] is generally the one taken in the existing literature. Its advantages include the possibility of reducing dependency on initial conditions as well as on sampling uncertainty. Its disadvantage is that it imposes the assumption of normally distributed innovations. The approach in [2] reverses this scenario. Its disadvantages lie in its dependency on initial conditions and exposure to the idiosyncracies of a sample realization. Its advantage is that it does not impose a strong distributional assumption on ϵ_t. This may be an important advantage in the present context, where ϵ_{pt} especially is unlikely to have a normal distribution. In order to reap the advantages for the approach in [1], 500 independent samples, each initially consisting of 200 observations, are simulated; then, the first 170 observations are discarded from each sample. For each simulation, in each approach, the steady state values of state variables and $z_0 = 1$ provide initial conditions.

(b) For each model sample, filter the data. Then compute summary statistics for the filtered data. For the approach in [1], the statistics are averages across the 500 samples.

(c) Compare the statistics for the model data to the corresponding statistics for the U.S. filtered, logarithmic-level data.

The Hodrick-Prescott filtering method underlies most of the statistics because of its prominence in quantitative macroeconomic studies (see Kydland and Prescott (1990)). The smoothing parameter for the Hodrick-Prescott filter is set at 400, the value commonly used for annual data. The first-difference filter underlies the statistics relating to analysis of the Solow residual since the interest in these stems from the documented regularities at the first-difference frequency (in Section III).

The foregoing evaluation is undertaken for the model described earlier, henceforth referred to as the basic model. It is also undertaken for two special variants: one that abstracts from energy price shocks (by setting $\sigma_p = 0$) and one that abstracts from shocks to the stationary component of government spending (by setting $\sigma_g = 0$). The latter two experiments permit isolation of the contribution of energy-price and temporary government spending shocks to the basic model. To keep this isolation pure, the experiments use the same sets of innovations.

V. The Findings

(i) Basic Model

Consider the findings for the basic model, starting with Table 4.[4]

In the U.S. data, the salient features of the standard deviations are: the well-known facts that consumption, labor hours and capital are less volatile, while investment is more volatile than output; energy usage, utilization and depreciation are quite volatile. The model accounts for 76 or 89 percent of the volatility of U.S. output. It captures the aforementioned relative volatilities, except that of consumption for the normal innovations case, and generally captures the absolute volatilities. The model significantly exaggerates the volatility of investment and, for the normal innovations approach, somewhat understates the volatility of depreciation. The predicted energy-usage volatility is intermediate to that of the two alternative U.S. energy-usage measures.

Each series in the U.S. data exhibits high persistency. The model mimics this well. Only the persistency of consumption and, for the normal innovations case, of labor hours is somewhat understated.

The U.S. data show that all series are strongly procyclical, except for capital and the average productivity of capital services, which are countercyclical. The model predicts this dimension closely. Exceptions are that the model does not predict the countercyclicality of U.S. k_{t+1} and, for normal innovations approach, it underestimates the countercyclicality of U.S. k_t. Also, when using normal (actual) innovations, labor hours (consumption) are not procyclical enough.

The U.S. correlation between labor hours and its average productivity is positive and the U.S. correlation between capital services (energy usage) and its average productivity (energy prices) is negative. The model generally performs well in this regard. One significant discrepancy is that the model, when using normal innovations, fails to predict the positive correlation between labor and its average productivity. The predicted correlation between energy usage and energy prices is intermediate to that for the two alternative U.S. energy-usage measures.

In the U.S. data, output exhibits a positive (negative) correlation with technology and government spending (energy prices). The model closely captures this dimension for the actual innovations case, somewhat less closely for normal innovations case. In particular, regarding the correlation between output and energy prices, the model predictions of -0.41 or -0.62 are greater than or come close to, respectively, the U.S. data value of -0.68.

Table 5 shows that the model fits the U.S. Solow residual facts. Notice especially, for the correlation between the growth rates of the Solow residual and energy prices, the model predictions of -0.43 or -0.47 are close to the U.S. data value of -0.55.

In short, Tables 4 and 5 suggest that the model explains a high fraction of U.S. output volatility, quite well matches the U.S. regularities involving energy prices, energy usage, capital utilization and the Solow residual, and is generally consistent with other features of U.S. business cycles. Discrepancies between the model and U.S. data, for both simulation approaches, that seem significant are the overstatement of investment volatility and the understatement of both the persistency of consumption and the countercyclicality of capital. [5]

It is possible that these discrepancies partly stem from lack of support for the assumption of a unitary elasticity of intertemporal substitution in consumption (see Finn, Hoffman and Schlagenhauf (1990)). Lower values of this elasticity imply less willingness to substitute consumption intertemporally, making investment less volatile and consumption more procyclical and persistent. With regard to the discrepancy involving k_{t+1}, consider the underlying behavior of $\delta(h_t)$, k_t and i_t. For these variables, the most noticeable differences across the model and U.S. data are the excessively high correlation between k_t and y_t and the standard deviation of i_t. Both of these differences form the prime reasons for why k_{t+1} is too procyclical. The excessively high correlation between k_t and y_t, in turn, seems to reflect that the intertemporal substitution effect, encouraging capital accumulation, is too strong relative to the wealth effect, discouraging capital accumulation, when anticipated increases in next period's output occur.

(ii) Contribution of Energy Price Shocks to Basic Model

Tables 6 and 7 present the findings for the model with $\sigma_p = 0$. Compare these tables with Tables 4 and 5, respectively. Energy-price shocks contribute 7.47 or 18.75 percent to the percentage of U.S. output volatility accounted for by the basic model. The quantitatively significant effects arising from the inclusion of these shocks are:

(a) The increase in the volatilities of investment, energy usage, utilization, depreciation and, for the actual innovations case, of the average productivity of capital services.

(b) The persistency of investment increases when using actual innovations.

(c) A switch from strongly procyclical to countercyclical average productivity of capital services and, for the normal innovations approach, a fall in the procyclicality of energy usage.

(d) The change from a strong positive to a strong negative correlation between capital services and its average productivity and, when using actual innovations, from a negative to a positive correlation between labor and its average productivity.

The effects along these dimensions constitute improvements in the basic model's ability to match the U.S. data, with the one exception of the effect on investment volatility. In addition, it is only by including energy-price shocks that the basic model can predict the strong negative correlations between energy prices and each of energy usage, output and the Solow residual manifested in the U.S. data. Some intuition about these effects follows.

A positive energy-price shock strongly decreases utilization and capital services, prompting a fall in output and a rise in the average productivity of capital services. The shock is a major source of negative covariation between the average productivity of capital services and each of output and capital services. A positive energy-price shock, by reducing utilization, also reduces energy usage, implying that it is a source of positive covariation between energy usage and output. But, it must be a weaker source of this positive covariation than technology or government spending shocks since including energy-price shocks causes the procyclicality of energy usage to fall. As Section II indicates, a positive energy-price shock, by decreasing utilization, also decreases the marginal productivity of labor hours. This creates an intratemporal substitution effect to reduce labor hours and to enhance positive comovement between labor hours and output.

(iii) Contribution of Temporary Government Spending Shocks to Basic Model

Consider the findings for the model with $\sigma_g = 0$ in Tables 8 and 9. In particular, compare these tables to Tables 4 and 5, respectively. Temporary government spending shocks change the percentage of U.S. output volatility accounted for by the basic model by -7.82 or 9.90 percent. The quantitatively significant effects arising from the inclusion of these shocks are:

(a) Consumption volatility switches from being smaller to greater than output volatility, for the normal innovations case.

(b) The decrease (increase) in the persistency of investment (labor), when using actual innovations.

(c) The procyclicality of consumption (labor) falls for the actual (normal) innovations case.

(d) The correlation between labor and its average productivity decreases, especially when using normal innovations.

(e) The decrease in the procyclicality of government spending, for normal innovations case.

(f) The correlation between the Solow residual and government spending decreases.

The effects along these dimensions constitute improvements in the basic model's ability to match the U.S. data, except for the effects on the relative volatility of consumption, the procylicality of consumption and labor and the correlation between labor and its average productivity. Some intuition for these effects follows.

The reduced volatility of output stems from the negative covariance between innovations to the temporary component of government spending and to technology, and the fact that both types of innovations cause output movements in the same direction. To highlight this, consider that for a model economy (with normal innovations) identical in all respects to the basic model economy except for setting $\sigma_{zg} = 0$, the standard deviation of output is 3.02. This exceeds the standard deviation of output in the model economy with

$\sigma_g = 0$ (and normal innovations), 2.85, and in the basic model economy (with normal innovations), 2.58.

As indicated in Section II, a positive shock to government spending causes a negative income effect (and sets in motion intratemporal substitution effects) that decreases consumption and increases labor, utilization and output. The shock affects consumption and labor more strongly than it does output, and is a source of negative (positive) covariation between consumption (labor) and output. Temporary government spending shocks must be a weaker source of positive covariation between labor and output than are technology and energy-price shocks, since inclusion of the former reduces the procyclicality of labor. Given the property of diminishing labor productivity, a positive government spending shock, by increasing labor, decreases its average productivity. The shock is a major source of negative covariation between these two variables.

Following the simulation approach using normal innovations, it is interesting to elucidate the strong impact of maintaining $\sigma_{zg} < 0$ on the correlations between labor and each of output and labor's average productivity (the tables explain new notation). For the model economy with $\sigma_g = 0$: $c(l_t, y_t) = 0.69$ and $c(l_t, APl_t) = 0.53$. A model economy that is identical in all respects to the basic model economy except for setting $\sigma_{zg} = 0$, displays: $c(l_t, y_t) = 0.61$ and $c(l_t, APl_t) = 0.24$. Finally, for the basic model economy, where $\sigma_{zg} < 0$: $c(l_t, y_t) = 0.30$ and $c(l_t, APl_t) = -0.13$.

Both output and the Solow residual are highly correlated with technology. The perfect linkage between technology and government spending, obtaining by construction, breaks by allowing temporary shocks to government spending. Also, government spending impacts positively but less strongly on output and the Solow residual than does technology, since the former can only work through the endogenous responses of labor and/or utilization. Including temporary government spending shocks, therefore, reduces the correlations between government spending and each of output and the Solow residual.

VI. Conclusion

For the United States economy (1960-1989), the correlation between the growth rates of the Solow residual and the real price of energy (government spending) is -0.55 (0.09). These correlations suggest that the Solow residual confounds movements in energy prices and government spending with those in true technology. The question arises as to how energy price and government spending shocks transmit to the Solow residual. Furthermore, with this transmission mechanism in place, what is the quantitative importance of energy price, government spending and true technology shocks in generating business cycle phenomena?

To address these questions, this study develops a model featuring perfect competition and constant returns to scale, that is quantitatively capable of capturing the endogenous production channels underlying the observed Solow residual correlations. These channels depend on endogenous capital utilization. Solow residual growth can, then, significantly differ from `true' technology growth because it absorbs the effects of fluctuations in utilization. Given the endogeneity of capital utilization and its close linkage to energy use, all shocks, but especially energy price shocks, will impact on the Solow residual.

The model, together with published U.S. time series data, generates U.S. time series on utilization. An important finding is that fluctuations in this utilization series, responding to movements in U.S. energy prices and government spending, actually do provide a quantitative explanation of the Solow residual correlations observed in the U.S. data. Since the explanation is consistent with perfect competition and constant returns to scale, it sharply differs from Hall's (1988, 1990) explanation that relies on imperfect competition and increasing returns to scale.

Incorporating shocks to `true' technology, energy prices, and government spending, the model economy accounts for 76 or 89 percent of U.S. output volatility, well matches the U.S. empirical regularities involving capital utilization and the Solow residual, and is generally consistent with other features of U.S. business cycles. Energy price shocks promote the match between the model and U.S. data along many dimensions. Government spending shocks exert mixed effects on the coherence between the model and U.S. data.

Extending the model to address questions concerning the dynamics of small open economies (see Finn (1990) and Mendoza (1991)), particularly their real exchange rate dynamics, and international business cycle behavior (see Stockman and Tesar (1990)) seems an exciting avenue for future research.

Table 4: Basic Model and U.S. Data (H-P Filtered Data)

Variable	I			II			III		
	Model, Normal Innovations			Model, Actual Innovations			U.S. Data, 1960-1989		
	% SD	AUTO1	CORRY	% SD	AUTO1	CORRY	% SD	AUTO1	CORRY
y_t	2.58	0.69	1.00	3.02	0.82	1.00	3.40	0.68	1.00
c_t	2.63	0.56	0.77	2.33	0.53	0.61	1.95	0.80	0.86
i_t	12.86	0.38	0.75	14.03	0.46	0.88	7.40	0.55	0.81
e_t	6.84	0.65	0.73	9.18	0.85	0.82	3.55 (10.02)	0.68 (0.81)	0.81 (0.76)
l_t	1.10	0.31	0.30	1.59	0.59	0.65	2.26	0.63	0.87
k_t	1.54	0.76	-0.09	1.65	0.77	-0.18	1.68	0.86	-0.36
k_{t+1}	1.54	0.76	0.31	1.65	0.77	0.35	1.68	0.86	-0.22
h_t	4.13	0.64	0.71	5.51	0.83	0.82	6.03	0.80	0.79
$\delta(h_t)$	5.96	0.64	0.71	7.95	0.83	0.82	8.70	0.80	0.79
APl_t	2.48	0.63	0.90	2.32	0.68	0.86	1.83	0.72	0.79
$APks_t$	2.73	0.62	-0.10	3.38	0.78	-0.35	4.03	0.78	-0.19

$c(l_t, APl_t)$ $c(ks_t, APks_t)$ $c(e_t, p_t)$ $c(l_t, APl_t)$ $c(ks_t, APks_t)$ $c(e_t, p_t)$ $c(l_t, APl_t)$ $c(ks_t,$

APks$_t$)	c(e_t, p_t)
-0.81	-0.68 (-0.97)

-0.13	-0.74	-0.92	0.17	-0.84	-0.96	0.38	

c(y_t, z_t)	c(y_t, g_t)	c(y_t, p_t)	c(y_t, z_t)	c(y_t, g_t)	c(y_t, p_t)	c(y_t, z_t)	c(y_t, g_t)
0.80	0.34	-0.41	0.63	0.72	-0.62	0.48	

c(y_t, p_t)	
0.57	-0.68

Key: (1) APl$_t$ is the average product of l_t.
APks$_t$ is the average product of ks_t.
$ks_t \equiv k_t h_t$.

(2) % SD denotes the percentage standard deviation.
AUTO1 denotes the first-order autocorrelation coefficient.
CORRY denotes the correlation with y_t.
c(.,.) denotes the correlation between the indicated variables.

(3) In panel III two values are reported for each statistic involving e_t. The first value pertains to the case when e_t is measured using the published data described in Appendix 2. The second value, in parentheses, pertains to the case when e_t is measured by using equation (5) and the empirical measures of h_t and k_t described in Section III. The second measure of e_t was constructed and its properties were summarized due to the reservations about the first measure, which are discussed in Appendix 2.

Table 5: Basic Model and U.S. Data (First-Differenced Data)

	I			II			III		
Variable	Model, Normal Innovations			Model, Actual Innovations			U.S. Data, 1960-1989		
	% SD	CORRS	CORRZ	% SD	CORRS	CORRZ	% SD	CORRS	CORRZ
sr_t	3.08	1.00	0.87	2.92	1.00	0.85	2.60	1.00	0.77
z_t	2.20	0.87	1.00	2.24	0.85	1.00	2.25	0.77	1.00
g_t	3.17	0.13	0.14	3.08	0.24	0.12	2.95	0.09	0.02
p_t	10.40	-0.43	0.01	10.44	-0.47	-0.02	10.39	-0.55	-0.001

Key: % SD denotes the percentage standard deviation.
CORRS denotes the correlation with sr_t.
CORRZ denotes the correlation with z_t.

Table 6: Model with $\sigma_p = 0$ (H-P Filtered Data)

	I			II		
Variable	Model, Normal Innovations			Model, Actual Innovations		
	% SD	AUTO1	CORRY	% SD	AUTO1	CORRY
y_t	2.33	0.69	1.00	2.38	0.76	1.00
c_t	2.40	0.56	0.73	2.23	0.58	0.48
i_t	10.26	0.23	0.70	10.66	0.18	0.77
e_t	2.61	0.68	0.998	2.67	0.75	0.998
l_t	1.08	0.33	0.31	1.52	0.58	0.56
k_t	1.36	0.74	-0.10	1.47	0.71	-0.18
k_{t+1}	1.36	0.74	0.30	1.47	0.71	0.39
h_t	1.80	0.64	0.87	1.88	0.68	0.88
$\delta(h_t)$	2.59	0.64	0.87	2.71	0.68	0.88
APl_t	2.23	0.63	0.88	1.99	0.67	0.77
$APks_t$	0.99	0.64	0.87	1.15	0.66	0.86

	$c(l_t, APl_t)$	$c(ks_t, APks_t)$	$c(l_t, APl_t)$	$c(ks_t, APks_t)$
	-0.15	0.68	- 0.10	0.59
	$c(y_t, z_t)$	$c(y_t, g_t)$	$c(y_t, z_t)$	$c(y_t, g_t)$
	0.90	0.38	0.83	0.56

Key: see key to Table 4.

Table 7: Model with $\sigma_p = 0$ (First-Differenced Data)

	I			II		
Variable	Model, Normal Innovations			Model, Actual Innovations		
	% SD	CORRS	CORRZ	% SD	CORRS	CORRZ
sr_t	2.73	1.00	0.98	2.66	1.00	0.98
z_t	2.20	0.98	1.00	2.24	0.98	1.00
g_t	3.17	0.15	0.14	3.08	0.13	0.12

Key: see key to Table 5.

Table 8: Model with σ_g = 0 (H-P Filtered Data)

	I			II		
Variable	Model, Normal Innovations % SD	AUTO1	CORRY	Model, Actual Innovations % SD	AUTO1	CORRY
y_t	2.85	0.69	1.00	2.68	0.76	1.00
c_t	2.26	0.57	0.94	2.19	0.64	0.94
i_t	11.36	0.55	0.81	12.26	0.70	0.82
e_t	6.97	0.65	0.72	8.55	0.82	0.72
l_t	0.67	0.30	0.69	0.64	0.29	0.64
k_t	1.40	0.81	-0.11	1.66	0.85	-0.12
k_{t+1}	1.40	0.81	0.30	1.66	0.85	0.33
h_t	4.19	0.64	0.71	5.17	0.81	0.71
$\delta(h_t)$	6.04	0.64	0.71	7.47	0.81	0.71
APl_t	2.44	0.64	0.98	2.33	0.71	0.98
$APks_t$	2.76	0.63	-0.02	3.53	0.81	-0.22

$c(e_t, p_t)$	$c(l_t, APl_t)$	$c(ks_t, APks_t)$	$c(e_t, p_t)$	$c(l_t, APl_t)$	$c(ks_t, APks_t)$
-0.95	0.53	-0.68	-0.90	0.46	-0.85
$c(y_t, p_t)$	$c(y_t, z_t)$	$c(y_t, g_t)$	$c(y_t, p_t)$	$c(y_t, z_t)$	$c(y_t, g_t)$
-0.48	0.90	0.90	-0.37	0.83	0.83

Key: see key to Table 4.

Table 9: Model with σ_g = 0 (First-Differenced Data)

	I			II		
Variable	Model,	Normal	Innovations	Model,	Actual	Innovations
	% SD	CORRS	CORRZ	% SD	CORRS	CORRZ
sr_t	3.16	1.00	0.88	2.98	1.00	0.86
z_t	2.20	0.88	1.00	2.24	0.86	1.00
g_t	2.20	0.88	1.00	2.24	0.86	1.00
p_t	10.40	-0.42	0.01	10.44	-0.46	-0.02

Key: see key to Table 5.

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Endnotes

1. Solow residual growth is output growth less the share weighted growth rates of labor hours and the capital stock. The shares are factor shares in a Cobb-Douglas production function. This approach to measuring technology growth is due to Solow (1957).

2. Energy usage was added to obtain the output measure because gross domestic product of the private non-energy production sector is value added by that sector. Gross housing product and consumer spending on housing services were subtracted in obtaining the output and consumption measures, respectively, because they are activities associated with household production (see Greenwood and Hercowitz (1991)). The output, consumption, investment, employment and capital measures are net of energy sector activities since the model does not explain them.

3. This procedure uses equation (14') rather than (5) to minimize dependency on the published e_t series. As pointed out in Appendix 2, the latter measure is not an accurate one.

4. The coefficient values of the four fundamental linear Markovian decision rules for the stationary basic economy are:

$$\begin{bmatrix} \hat{k}_{t+1} \\ \hat{c}_t \\ \hat{l}_t \\ \hat{h}_t \end{bmatrix} = \begin{bmatrix} 0.81 & -0.81 & -0.01 & -0.03 & -0.25 & 0.14 & 0.03 \\ 0.27 & -0.27 & -0.15 & -0.09 & 0.16 & -0.09 & -0.02 \\ -0.20 & 0.20 & 0.23 & -0.01 & -0.23 & 0.13 & 0.03 \\ -0.67 & 0.67 & 0.13 & -0.31 & -0.13 & 0.07 & 0.02 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ u_{zt} \\ \hat{g}_t \\ \hat{p}_t \\ \epsilon_{zt} \\ \epsilon_{gt} \\ \epsilon_{pt} \end{bmatrix}$$

The innovation, u_{zt}, enters the stationary economy as a negative, serially-correlated technology shock. The white-noise innovations: ϵ_{zt}, ϵ_{gt} and ϵ_{pt} impact on the economy through their positive influence on expectations of future technology, government spending and energy price shocks. The adjustment coefficient, 0.81, is smaller than that reported in other studies which assume a fixed utilization rate (e.g. 0.95 for the divisible-labor economy model in King, Plosser and Rebelo (1988)). This suggests that endogenous utilization results in faster adjustment to disturbances since it provides an additional margin along which agents can respond. The signs of the above coefficients can be rationalized by considering the interaction between wealth, intertemporal and intratemporal substitution effects.

5. An earlier version of this paper, evaluates an indivisible-labor model. The indivisible-labor model modifies the basic model by specifying utility as a linear function of leisure (see e.g. Hansen (1985)) and by changing the calibrated value of γ to 3.38. In general, the volatility (persistency) of fluctuations is higher (lower) in the indivisible-labor economy than in the basic economy, implying a better match with the U.S. data along some dimensions but a worse match along others. In particular, the enhanced volatility of labor (investment) implies a better (worse) fit with the U.S. data. These findings suggest that, in the presence of technology, government spending, and energy price shocks, the increase in the substitutability of leisure inherent in the indivisible labor specification may be too strong.

Appendix 1: The Solution Technique

Step [1]: A stationarity-inducing transformation of model variables is undertaken since exogenous growth occurs, stemming from the growth of z_t. Denote the new stationary variables by:

$\bar{s}_t \equiv s_t/z_t$, for $s_t = w_t, c_t, y_t, e_t, i_t, x_t, n_t$. $\bar{k}_{t+1} \equiv k_{t+1}/z_t$.

The variables l_t, h_t, r_t, p_t, $\bar{g}_t$ and innovation vector u_t are also stationary. Competitive equilibrium for the stationary economy is implicitly defined by:

$$(11')\quad \bar{w}_t = \theta\, \bar{y}_t/l_t$$

$$(12')\quad r_t = (1-\theta)\,\bar{y}_t/[\bar{k}_t h_t \exp(-u_{zt})\bar{z}^{-1}]$$

$$(13')\quad \gamma/(1-l_t) = \bar{w}_t/\bar{c}_t$$

$$(14')\quad h_t^{(\omega-1)} + h_t^{(\nu-1)}\, p_t = (1-\tau)r_t$$

(15')

$$\bar{c}_t^{-1} = \beta\, \underset{t}{E}\left[\bar{c}_{t+1}^{-1} \exp(-u_{zt+1})\bar{z}^{-1}\left((1-\tau)r_{t+1}\, h_{t+1} + 1 - \frac{h_{t+1}^{\omega}}{\omega} - \frac{h_{t+1}^{\nu}}{\nu}\, p_{t+1} \right) \right]$$

$$(16')\quad \bar{y}_t - p_t\bar{e}_t = \bar{c}_t + \bar{i}_t + \bar{g}_t$$

$$(2')\quad \bar{y}_t = l_t^{\theta}\,[\,\bar{k}_t h_t \exp(-u_{zt})\bar{z}^{-1}\,]^{(1-\theta)}$$

$$(4')\quad \bar{k}_{t+1} = [1 - h_t^{\omega}/\omega]\bar{k}_t \exp(-u_{zt})\bar{z}^{-1} + \bar{i}_t$$

(5') $\bar{e}_t/[\overline{k_t}\exp(-u_{zt})\bar{z}^{-1}] = h_t^{\nu}/\nu$

(8') $\bar{g}_t = \bar{x}_t + \tau r_t \bar{k}_t h_t \exp(-u_{zt})\, \bar{z}^{-1}$

and (9) - (10). This system derives from the competitive equilibrium for the nonstationary economy, by noting the stationarity transformation, equation (8) and the functional forms for F, u, δ, and a.

Step [2]: Find the deterministic steady state of the stationary economy.

Step [3]: Obtain a linear approximation of the stationary system around the deterministic steady state and invoke certainty equivalence. This involves expressing each equation in terms of the innovations and variables that are percentage deviations from their steady state values:

$\hat{s}_t \equiv \log(s_t/s)$, for $s_t = \bar{w}_t, \bar{c}_t, \bar{y}_t, \bar{e}_t, \bar{i}_t, \bar{x}_t, \bar{n}_t, \bar{k}_t, l_t, h_t, r_t, p_t$ and $\bar{g}_t$, $s \equiv$ steady state value of s_t.

Step [4]: Specify the MA(1) structures of the innovation processes:

$$u_{zt} = \epsilon_{zt} + \eta_z \epsilon_{zt-1}$$

$$u_{gt} = \epsilon_{gt} + \eta_g \epsilon_{gt-1}$$

$$u_{pt} = \epsilon_{pt} + \eta_p \epsilon_{pt-1}$$

where: ϵ_{it} is a zero-mean, white noise innovation process and η_i

is a parameter (i = z, g, p). Section IV discusses these specifications.

Step [5]: Obtain the particular solution to the approximate system that is consistent with transversality condition: $\lim_{t\to\infty} \beta^t \lambda_t \bar{k}_{t+1} = 0$, where: λ_t is the lagrange multiplier associated with the resource constraint (16').

Step [6]: The solution, in general form, is:

$$X_{t+1} = A_1 X_t + A_2 \epsilon_{t+1}$$

$\hat{s}_t = B X_t$, for $\hat{s}_t = \hat{w}_t, \hat{c}_t, \hat{y}_t, \hat{e}_t, \hat{\imath}_t, \hat{x}_t, \hat{n}_t, \hat{l}_t, \hat{h}_t, \hat{r}_t$

where: $X'_t \equiv [\hat{k}_t \ u_{zt} \ \hat{g}_t \ \hat{p}_t \ \epsilon_{zt} \ \epsilon_{gt} \ \epsilon_{pt}]$ is the state vector at time t.

$\epsilon'_{t+1} \equiv [\epsilon_{zt+1} \ \epsilon_{gt+1} \ \epsilon_{pt+1}]$ is the white noise innovation vector at time t+1.

A_1, A_2 and B are matrices, of appropriate size, whose elements are scalar functions of the parameters of the approximate system.

Step [7]: Use the solution in [6], the definition of $\hat{s}_t$ in [3], the definitions of $\bar{s}_t$ and $\bar{k}_{t+1}$ in [1] and equation (8) to find the approximate competitive equilibrum process for the nonstationary economy.

Appendix 2: The Data

The data are annual, real, per-capita data for the United States (1960-1989).

(i) Energy Usage, Prices and Product

The sources for this data are: (1) State Energy Data Report: Consumption Estimates 1960-1989, Energy Information Administration (SEDR); (2) Annual Energy Review 1990, Energy Information Administration (AER). The conversion factors in the Appendices of the AER are used to establish BTU measures. Some important reservations about the accuracy of the energy usage series include:

(a) Commercial sector energy usage is inaccurate. Its usage is sometimes part of residential sector usage and vice-versa. Its coal usage, particularly, only roughly separates from that of the residential sector. This coal usage series also includes government usage. The present study attempts to isolate commercial sector natural gas and petroleum usage from that of the government by using employment share data.

(b) Transportation sector motor gasoline usage is approximated by taking a constant fraction (0.25) of published motor gasoline usage. The latter also includes government and private non-business usage.

(c) Energy-production sector energy usage is not entirely excluded.

(d) The conversion factors used in obtaining BTU measures are approximate.

Energy Usage (trillions of BTUs): the sum of electricity (ELEC), coal (COAL), natural gas (NATG) and petroleum (PETR) usage by the private non-energy production sector of the economy.

ELEC = CSE + ISE + TSE

CSE = commercial sector electricity usage. Series is in Table 94 AER.

ISE = industrial sector electricity usage. Series is in Table 12 SEDR.

TSE = transportation sector electricity usage. Series is in Table 13 SEDR.

COAL = CSC + ISC + TSC

CSC = commercial sector coal usage. Series is in Table 11 SEDR.

ISC = industrial sector coal usage. Series in Table 12 SEDR (includes net imports of coke) less the coke plant coal usage from Table 83 AER.

TSC = transportation sector coal usage. Series is in Table 13 SEDR.

<u>NATG</u> = CSG + ISG

CSG = commercial sector natural gas usage. Series in Table 11 SEDR multiplied by the commercial sector employment share series (subsection (ii) defines and documents this series, (a)).

ISG = industrial sector natural gas usage. Series in Table 12 SEDR less the lease and plant fuel series in Table 77 AER.

<u>PETR</u> = CSP + ISP + TSP

CSP = commercial sector petroleum usage. Series in Table 11 SEDR less the strategic petroleum reserve acquisition series in Table 66 AER, multiplied by the commercial sector employment share series (subsection (ii) defines and documents this series, (b)).

ISP = industrial sector petroleum usage. Series is in Table 12 SEDR (excluding those components listed in the asphalt and road oil, lubricants and `other' categories).

TSP = transportation sector petroleum usage. Series is in Table 13 SEDR (excluding that component listed in the lubricants category and 0.75 of that component listed in the motor gasoline category).

Energy Prices (dollar prices per trillion BTUs).

pelec = price of electricity. Series is in Table 100 AER.

pcoal = price of coal. Series is in Table 88 AER.

pnatg = price of natural gas. Series is in Table 79 AER.

ppetr = price of petroleum = $(x_1 + x_2)/x_3$.

x_1 = dollar value of total production plus net imports of oil and petroleum products. Series are in Tables 32-34 AER.

x_2 = dollar value of natural gas plant liquids production evaluated at domestic crude oil prices (series from Tables 29, 51 AER).

x_3 = economy-wide consumption of petroleum, measured in trillion BTUs. Series is in Table 9 SEDR.

Energy Usage (billions of current dollars):

(pelec. ELEC + pcoal. COAL + pnatg. NATG + ppetr. PETR), scaled appropriately. **Energy Usage** (billions of 1987 dollars): the constant 1987 price counterpart to the foregoing energy usage series.

Energy Price Deflator (1987=100): the ratio of energy usage in current dollars to energy usage in 1987 dollars.

Energy Product (billions of current dollars): the sum of the value of fossil fuel production (series is in Table 32 AER) and value added by the electricity-producing sector. The latter's definition is sales less the values of oil, coal and natural gas inputs (series are in Table 92 AER; also, the price series described above are used).

(ii) All Other Data

Sources for remaining data are Citibase and: (1) National Income and Wealth Division, BEA, U.S. Department of Commerce (DC); (2) "Fixed Reproducible Tangible Wealth in the United States, Revised Estimates" by John C. Musgrave, Survey of Current Business, BEA, U.S. Department of Commerce, January 1992, pp. 106-137 (SCB). Unless otherwise stated, the source is Citibase.

Population (thousands of persons): civilian non-institutional population aged sixteen and over.

Aggregate Price Deflator (1987=100): gross domestic product price deflator.

Output (billions of 1987 dollars): gross domestic product plus energy usage minus the sum of gross housing, government and energy-sector products (subsection (i) indicates definitions and sources for the energy items).

Consumption (billions of 1987 dollars): personal consumer expenditure on nondurable goods and services minus the sum of that on housing services, gasoline and oil, fuel oil and coal, electricity and gas.

Investment (billions of 1987 dollars): gross private domestic fixed investment in nonresidential capital excluding that component for the coal mining, oil and gas extraction, petroleum- and coal-product manufacturing, electricity and gas-service sectors. Source: DC.

Government Spending (billions of 1987 dollars): government purchases of goods and services.

Labor hours: the product of employment and hours per worker per year. Employment (thousands of persons) is total employment (civilian plus resident armed forces) minus government employment plus armed forces overseas minus the sum of employment in the coal mining, oil and gas extraction, petroleum-and coal-product manufacturing, electricity, gas-and sanitary-service sectors. Hours per worker per year is an average across all workers in all industries.

Capital Stock (billions of 1987 dollars): net stock of private domestic fixed nonresidential capital excluding that component for the coal mining, oil and gas extraction, petroleum-and coal-product manufacturing, electricity and gas-service sectors. Source: SCB.

Commercial Sector Employment Share Series.

Series (a): equals $1 - x_1/(x_1 + x_2)$, where x_1 = employment (thousands of persons) in government, and x_2 = employment (thousands of persons) in services, finance, insurance and real estate, wholesale and retail trade, communications and agriculture, forestry and fishing.

Series (b): equals $1 - x_1/(x_1 + x_3)$, where $x_3 = x_2$ less employment (thousands of persons) in agriculture, forestry and fishing.

Relative Price of Energy (1987=1): ratio of energy price deflator to aggregate price deflator.

Table L1 [Low energy share economy]

VARIABLE	% SD	CORRS	CORRZ
$\Delta \log sr_t$	2.60	1.00	0.66 (0.00)
$\Delta \log z_t$	2.47	0.66 (0.000)	1.00
$\Delta \log g_t$	2.95	0.09 (0.632)	0.16 (0.412)
$\Delta \log p_t$	10.39	-0.55 (0.002)	-0.10 (0.623)

Table L2 [Low energy share economy]

Coefficient Estimates

$\hat{\eta}_z = 0.24\ (0.19)$

$\hat{\rho}_g = 0.91\ (0.04)$ $\hat{\eta}_g = 0.48\ (0.20)$

$\hat{\rho}_p = 0.90\ (0.07)$ $\hat{\eta}_p = 0.34\ (0.20)$

Residual Properties

$\hat{\sigma}_z = 0.0240$ $\hat{\sigma}_{zg} = -0.0004$ $\hat{c}_{zg} = -0.5427$

$\hat{\sigma}_g = 0.0301$ $\hat{\sigma}_{zp} = -0.0004$ $\hat{c}_{zp} = -0.1794$

$\hat{\sigma}_p = 0.0966$ $\hat{\sigma}_{gp} = 0.0000$ $\hat{c}_{gp} = -0.0095$

Autocorrelations
(S.E. = 0.185)

	$\hat{e}_{zt}$	$\hat{e}_{gt}$	$\hat{e}_{pt}$
Lag 1	0.00	0.02	0.03
Lag 2	0.29	0.24	0.16
Lag 3	-0.15	-0.22	0.13
Lag 4	0.12	0.04	-0.06
Lag 5	-0.08	-0.17	0.04
	Q(5) = 3.58	Q(5) = 3.94	Q(5) = 1.41
	$\chi^2_4 = 9.49$	$\chi^2_3 = 7.81$	$\chi^2_3 = 7.81$

Table L3: Low Energy Share Economy Model and U.S. Data (H-P filtered data)

	I			II			III		
Variable	Model, Normal Innovations			Model, Actual Innovations			U.S. Data, 1960-1989		
	% SD	AUTO1	CORRY	% SD	AUTO1	CORRY	% SD	AUTO1	CORRY
y_t	2.62	0.67	1.00	3.36	0.85	1.00	3.40	0.68	1.00
c_t	2.61	0.55	0.84	2.53	0.57	0.75	1.95	0.80	0.86
i_t	9.92	0.53	0.80	12.27	0.69	0.92	7.40	0.55	0.81
e_t	6.42	0.65	0.71	9.36	0.87	0.88	3.55 (9.28)	0.68 (0.55)	0.81 (0.70)
l_t	0.93	0.37	0.27	1.43	0.67	0.66	2.26	0.63	0.87
k_t	1.32	0.81	-0.03	1.61	0.84	-0.13	1.34	0.76	-0.32
k_{t+1}	1.32	0.81	0.35	1.61	0.84	0.34	1.34	0.76	-0.18
h_t	3.45	0.64	0.69	4.97	0.86	0.88	4.88	0.55	0.73
$\delta(h_t)$	5.09	0.64	0.69	7.33	0.86	0.88	7.19	0.55	0.73
APl_t	2.52	0.61	0.93	2.65	0.71	0.91	1.83	0.72	0.79
$APks_t$	2.32	0.62	0.09	2.61	0.75	-0.30	3.69	0.47	0.07

	I			II			III		
	$c(l_t, APl_t)$	$c(ks_t, APks_t)$	$c(e_t, p_t)$	$c(l_t, APl_t)$	$c(ks_t, APks_t)$	$c(e_t, p_t)$	$c(l_t, APl_t)$	$c(ks_t, APks_t)$	$c(e_t, p_t)$
	-0.08	-0.60	-0.88	0.29	-0.75	-0.95	0.38	-0.71	-0.68 (-0.71)
	$c(y_t, z_t)$	$c(y_t, g_t)$	$c(y_t, p_t)$	$c(y_t, z_t)$	$c(y_t, g_t)$	$c(y_t, p_t)$	$c(y_t, z_t)$	$c(y_t, g_t)$	

$c(y_t, p_t)$	0.88	0.43	-0.32	0.82	0.77	-0.69	0.62	0.57	-0.68

Table L4: Low Energy Share Economy Model and U.S. Data (first-differenced data)

	I			II			III		
Variable	Model,	Normal	Innovations	Model,	Actual	Innovations	U.S. Data, 1960-1989		
	% SD	CORRS	CORRZ	% SD	CORRS	CORRZ	% SD	CORRS	CORRZ
sr_t	3.19	1.00	0.92	3.11	1.00	0.91	2.60	1.00	0.66
z_t	2.44	0.92	1.00	2.46	0.91	1.00	2.47	0.66	1.00
g_t	2.97	0.26	0.25	3.10	0.34	0.25	2.95	0.09	0.16
p_t	10.40	-0.34	0.01	10.44	-0.45	-0.12	10.39	-0.55	-0.10

Table L5: Constant Depreciation Economy Model (H-P filtered data)

	I			II		
Variable	Model, % SD	Normal AUTO1	Innovations CORRY	Model, % SD	Actual AUTO1	Innovations CORRY
y_t	2.22	0.69	1.00	2.55	0.83	1.00
c_t	2.37	0.60	0.75	2.11	0.60	0.56
i_t	8.20	0.36	0.74	8.77	0.37	0.79
e_t	13.01	0.63	0.51	17.45	0.84	0.69
l_t	1.01	0.47	0.24	1.44	0.70	0.62
k_t	1.49	0.81	0.18	1.64	0.82	0.17
k_{t+1}	1.49	0.81	0.56	1.64	0.82	0.65
h_t	2.86	0.62	0.49	3.80	0.83	0.68
APl_t	2.19	0.65	0.88	2.01	0.70	0.83
$APks_t$	2.70	0.65	0.17	3.18	0.76	-0.10

$c(e_t, p_t)$	$c(l_t, APl_t)$	$c(ks_t, APks_t)$	$c(e_t, p_t)$	$c(l_t, APl_t)$	$c(ks_t, APks_t)$
-0.99	-0.20	-0.71	-0.99	0.07	-0.80

$c(y_t, p_t)$	$c(y_t, z_t)$	$c(y_t, g_t)$	$c(y_t, p_t)$	$c(y_t, z_t)$	$c(y_t, g_t)$
-0.60	0.82	0.39	-0.37	0.65	0.72

Table L6: Constant Depreciation Economy Model (first-differenced data)

	I			II		
Variable	Model,	Normal	Innovations	Model,	Actual	Innovations

	% SD	CORRS	CORRZ	% SD	CORRS	CORRZ
sr_t	2.57	1.00	0.91	2.63	1.00	0.91
z_t	2.20	0.91	1.00	2.24	0.91	1.00
g_t	3.17	0.13	0.14	3.08	0.24	0.12
p_t	10.40	-0.40	0.01	10.44	-0.42	-0.02