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Customer Flows, Countercyclical Markups, and the Persistent Effects of Monetary Shocks

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Peter N. Ireland
Federal Reserve Bank of Richmond

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CUSTOMER FLOWS, COUNTERCYCLICAL MARKUPS, AND THE PERSISTENT EFFECTS OF MONETARY SHOCKS

Peter N. Ireland*

Research Department
Federal Reserve Bank of Richmond

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*The opinions expressed herein are my own and do not necessarily represent those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

Abstract

This paper develops a general equilibrium model in which households face fixed costs associated with searching for a new supplier of consumption goods. These search costs provide firms with some monopoly power over their existing customers and generate the kind of customer flow dynamics first considered by Phelps and Winter. Customer flows, in turn, cause markups of price over marginal cost to vary countercyclically, both amplify and propagate the effects of technology shocks on output, and allow the effects of monetary shocks on output to persist.

I. Introduction

Following Lucas' (1972) pathbreaking work, economists have sought to explain business cycle fluctuations in output and employment using general equilibrium models featuring optimizing agents with rational expectations. Hall (1975) criticizes this approach, noting that while models such as Lucas' can account for the existence of a Phillips curve relationship between innovations in the nominal money supply or the nominal price level on the one hand and innovations in real output or employment on the other, they fail to explain why movements in output and employment appear to be serially correlated over the business cycle. Modigliani (1977), Tobin (1977), and Gordon (1981) all echo Hall's criticism.[1]

In response to this repeated criticism, much subsequent work in business cycle theory has augmented Lucas' original model with additional mechanisms that allow monetary shocks to have persistent effects on output. Lucas (1975) himself introduces physical capital accumulation as a propagation mechanism, while Taylor (1980) considers staggered nominal wage contracts. Blanchard (1983) considers staggered nominal price setting; Blinder and Fischer (1981) consider inventory accumulation; Sargent (1987, Ch.18) considers adjustment costs in the labor market; and both Wright (1986) and Howitt (1988) consider labor market search. This paper explores yet another channel through which monetary shocks can have persistent effects on output and employment: the customer flow dynamics originally modeled by Phelps and Winter (1970).

Bils (1989) presents a partial equilibrium model of customer flow dynamics, in which a single monopolist supplies output to a set of customers who arrive in an infinite sequence of overlapping generations. Prior to actually buying the firm's output, each customer is unsure of how well the good will suit his tastes. Thus, a customer who does purchase the good in his first period of life and finds it to his liking develops an attachment to the firm. In this case, he becomes willing to pay a higher price for the good in his second period of life. Thus, in Bils' model as in Phelps and Winter's, the firm faces a trade-off: it can raise its price and thereby extract more surplus from its existing customers, or it can lower its price and thereby attract more new customers.

Bils demonstrates that these customer flow dynamics induce the monopolist to charge a markup of price over marginal cost that varies countercyclically.[2] In some periods, high demand results from an especially large inflow of newly-born customers. It is precisely during these periods that the firm finds it most rewarding to lower its price to expand the size of its customer base. Hence, periods of high demand coincide with periods of low prices and, since marginal costs are constant, low markups as well. These countercyclical markups imply that market clearing prices work in Bils' model to amplify, rather than stabilize, movements in output relative to demand.

This paper begins by modifying and extending Bils' analysis so that it takes place in a general equilibrium setting where all agents are infinitely-lived. The model developed here pairs each household with one of a large number of firms as its initial supplier of consumption goods and confronts the representative household with a fixed cost of searching for a new supplier. This search cost gives the representative firm some monopoly power, enabling it to raise its price above those of all other firms while still retaining some of its customers.

In the simplest version of the model, considered in sections II and III, disturbances to the marginal product of labor drive fluctuations in aggregate output and employment. When output increases after a positive technology shock, so does the size of each household's purchase of consumption goods. This increase in the size of each purchase strengthens the representative household's incentive to search for a new supplier. Consequently, the representative firm's monopoly power erodes during periods of high output, decreasing the equilibrium markup of price over marginal cost. Again, countercyclical markups work to amplify the effects of real shocks on aggregate output.

Section IV then augments the model to account for persistence in the effects of technology shocks. There, households acquire new information through the process of search today that lowers their costs of search tomorrow. As before, an increase in output yields an increase in the number of searching households. In this case, however, the increase in the number of searching households today also increases the number of households with low search costs tomorrow, leading to an increase in the number of searching households tomorrow, and so on. Thus, even following a purely transitory technology shock, the representative firm's monopoly power erodes for a number of periods. Movements in price relative to marginal cost not only amplify movements in output during the period of the shock, but also serve to generate persistent movements in output during the periods following the shock.

Section V introduces a nominal sector and shows how the customer flow dynamics that both amplify and propagate the effects of technology shocks also allow the effects of monetary shocks to persist. Section VI concludes.

II. Customer Flow Dynamics

A. The Economic Environment

The economy consists of a continuum of firms indexed by $i \in [0,1]$ and a continuum of households indexed by $j \in [0,1] \times [0,1]$. Thus, there is both a large number of firms and a large number of households per firm. Time is discrete and indexed by $t=0,1,2,\ldots$.

At the beginning of period t, each firm i is matched with a continuum of households of measure x_{it}. These households are the firm's initial customers, and the firm is the households' initial supplier during period t. Each household observes its initial supplier's price, but knows only the distribution of the other firms' prices. Firms begin period t=0 with equal numbers of customers, so that $x_{i1}=1$ for all $i \in [0,1]$. In addition, each household is matched with an initial supplier during each period t, so that

$$\int_0^1 x_{it} di = 1.$$

Household j has preferences described by the expected utility function

$$E\left\{ \sum_{t=0}^{\infty} \beta^t [2(\theta_{jt}^{1-s_{jt}} c_{jt})^{1/2} - n_{jt} - \kappa s_{jt}] \right\},$$

where $\beta \in (0,1)$ is a constant discount factor, $\kappa > 0$ is a constant, c_{jt} is the household's consumption, and n_{jt} is its labor supply during period t. The idiosyncratic shock $\theta_{jt}>0$ takes on values in the interval $[\theta_l,\theta_h]$ and is iid over time for each household with distribution function F and density function f. It is assumed that the distribution of shocks across households is such that during each period t, a fraction $F(\theta)$ of each firm's initial customers have $\theta_{jt}\leq\theta$.[3] It is also assumed that $f'(\theta)\geq 0$ for all $\theta\leq 1$; the significance of this assumption is discussed below. The θ_{jt} shocks are realized before agents have made any of their decisions for period t.

Household j's choice variable s_{jt} equals zero if the household purchases output from its initial supplier during period t; s_{jt} equals one if the household purchases output from any other firm. Thus, as in Ball and Romer (1990), θ_{jt} measures household j's preference for remaining with its initial supplier during period t: in terms of utility, it values one unit of output from its initial supplier the same as θ_{jt} units of output from any other firm. The θ_{jt} shocks, therefore, provide households with a motive for search; households with lower values of θ_{jt} have a stronger incentive to look for a new supplier during period t.

If household j decides to leave its initial supplier during period t, it must pay a fixed search cost κ, measured in terms of utility; since the household's utility function is linear in labor supply, κ may be interpreted as a time cost. Through the process of search, the household is matched randomly with one of the other firms, which becomes its second supplier during period t. The probability that a searching household gets firm i as a second supplier is proportional to x_{it}, the size of the firm's initial customer base. More precisely, the probability that a household will have a second supplier with index $i \le I$ during period t is given by

$$G_t(I) = \int_0^I x_{it} di$$

for all $I \in [0,1]$. During period t, the household's decision to search is final and can be made only once: if household j does leave its initial supplier, it must purchase output from its second supplier. The household's second supplier during period t then becomes its initial supplier during period t+1.

Labor, meanwhile, is perfectly mobile across firms and is paid the competitive wage w_t during period t. Firm i hires l_{it} units of labor to produce y_{it} units of output according to the technology

$$y_{it} = z_t l_{it},$$

where z_t is an aggregate productivity disturbance during period t. For simplicity, it is assumed that all agents know the entire sequence $\{z_t\}_{t=0}^{\infty}$ at the beginning of period t=0.

During period t, therefore, firm i earns profits

$$\pi_{it} = p_{it}y_{it} - w_t l_{it} = (p_{it} - w_t/z_t)y_{it},$$

where p_{it} is its output price. It distributes these profits in equal shares to all households.

The analysis that follows focuses on symmetric equilibria, in which all firms charge the same price during period t, so that $p_{it}=p_t$ for all $i \in [0,1]$. Let $p^0=\{p_t\}_{t=0}^{\infty}$ denote the entire sequence of these prices; let $p^t=\{p_{t+k}\}_{k=0}^{\infty}$ denote the subsequence of prices from period t forward. In such an equilibrium, no firm can have an incentive to charge a different sequence of prices $q^0=\{q_t\}_{t=0}^{\infty}$, with $q_t \ne p_t$ for some $t \ge 0$. Hence, the analysis begins by considering the optimizing behavior of a representative household whose initial supplier plans to charge a set of prices $q^t=\{q_{t+k}\}_{k=0}^{\infty}$ from period t forward while all other firms charge prices p^t. This behavior--and, in particular, its effects on the firm's profits--will then be used to characterize the conditions under which the firm has no incentive to deviate from a candidate equilibrium price sequence p^0.

B. Household Optimization

Consider a representative household whose initial supplier during period t plans to charge prices given by the sequence q^t. If all other firms charge prices p^t, the household's knowledge of the distribution of the other firms' prices allows it to know with certainty that if it leaves its initial supplier, it will be matched with a second supplier charging p^t.

Let $u(\theta_{jt};q^t)$ denote the maximized value of the household's expected utility from period t forward, given that it experiences shock θ_{jt} during period t and its initial supplier plans to charge prices q^t from period t forward. Then $u(\theta_{jt};q^t)$ must satisfy the Bellman equation

$$(1)\quad u(\theta_{jt};q^t) = \max 2[(1-s)(\theta_{jt}c^0)^{1/2}+s(c^1)^{1/2}] - (1-s)n^0 - s(n^1+\kappa) + \beta E[(1-s)u(\theta_{jt+1};q^{t+1})+su(\theta_{jt+1};p^{t+1})],$$

where the expectation is taken over realizations of θ_{jt+1}. The maximization in (1) is by choice of positive constants c^0, n^0, c^1, and n^1 and $s\in\{0,1\}$ that satisfy the budget constraints

$$(2)\quad \pi_t + w_t n^0 \geq q_t c^0$$

and

$$(3)\quad \pi_t + w_t n^1 \geq p_t c^1,$$

where π_t denotes the household's share of firm profits in period t, c^0 and

n^0 are the household's consumption and labor supply if it decides to remain with its initial supplier during period t, and c^1 and n^1 are its consumption and labor supply if it decides to search for a second supplier.

The first-order conditions for (1) imply that optimal choices for c^0, n^0, c^1, and n^1 are

$$(4) \qquad c^0 = \theta_{jt}(w_t/q_t)^2 \qquad \text{and} \qquad n^0 = \theta_{jt}w_t/q_t - \pi_t/w_t$$

and

$$(5) \qquad c^1 = (w_t/p_t)^2 \qquad \text{and} \qquad n^1 = w_t/p_t - \pi_t/w_t.$$

Substituting (4) and (5) back into (1) reveals that the household will remain with its initial supplier, choosing s=0, if and only if

$$(6) \qquad \theta_{jt} \geq \frac{q_t}{p_t}\left\{1 - \frac{\kappa + \beta E[u(\theta_{jt+1};q^{t+1})-u(\theta_{jt+1};p^{t+1})]}{w_t/p_t}\right\}.$$

C. Firm Optimization

Next, consider the behavior of a representative firm that begins period t with a set of initial customers of measure x_{it}. If this firm charges prices q^t from period t forward while all other firms charge prices p^t, it retains all of its initial customers with θ_{jt} satisfying equation (6).

Equation (6) reveals that here, as in Bils (1989), a time consistency problem arises. Specifically, the representative firm can prevent some of its initial customers from leaving during period t by promising to charge a low price q_{t+1} during period t+1, since the representative household's gain in expected future utility from remaining with the firm instead of searching for a new supplier, given by $E[u(\theta_{jt+1};q^{t+1})-u(\theta_{jt+1};p^{t+1})]$ in equation (6), is decreasing as a function of q_{t+1}. Once period t+1 arrives, however, the firm has an incentive to charge a higher price q_{t+1} than originally promised.

This problem is resolved here, as in Bils (1989), by focusing on time-consistent equilibria in which firms choose their prices sequentially, period-by-period. Thus, when choosing its optimal price q_t during period t, the representative firm takes q^{t+1} as given, to be determined by decisions made in the future. Hence, the firm also takes

$$\theta_t^0 = 1 - \frac{\kappa + \beta E[u(\theta_{jt+1};q^{t+1})-u(\theta_{jt+1};p^{t+1})]}{w_t/p_t} \tag{7}$$

as given when choosing q_t.

Accordingly, if the representative firm decides to charge price q_t during period t, it retains all of its initial customers with $\theta_{jt} \geq (q_t/p_t)\theta_t^0$. This set of customers has measure $x_{it}\{1-F[(q_t/p_t)\theta_t^0]\}$, and by equation (4), each of these customers purchases $\theta_{jt}(w_t/q_t)^2$ units of output. Since all other firms charge price p_t, and since searching households are matched with the representative firm in proportion to x_{it}, the firm also gains a set of new customers of measure $x_{it}F(\theta_t^0)$; equation (5) implies that each of these customers purchases $(w_t/q_t)^2$ units of output. Hence, the firm faces total demand

$$x_{it}(w_t/q_t)^2 h(q_t/p_t;\theta_t^0),$$

where

$$h(q_t/p_t;\theta_t^0) = F(\theta_t^0) + \int_{(q_t/p_t)\theta_t^0}^{\theta_h} \theta f(\theta)d\theta,$$

and earns profits of

$$\pi_{it} = (q_t - w_t/z_t)x_{it}(w_t/q_t)^2 h(q_t/p_t;\theta_t^0)$$

during period t.

The firm then begins period t+1 with a new set of initial customers of measure

$$x_{it+1} = x_{it}\delta(q_t/p_t;\theta_t^0),$$

where

$$\delta(q_t/p_t;\theta_t^0) = 1 - F[(q_t/p_t)\theta_t^0] + F(\theta_t^0).$$

Note, in particular, that $\delta(q_t/p_t;\theta_t^0)\geq 1$ when $q_t \leq p_t$ and $\delta(q_t/p_t;\theta_t^0)\leq 1$ when $q_t \geq p_t$. Thus, as in Phelps and Winter (1970), the representative firm gains customers if it charges a price below those of all other firms and loses customers if it charges a price above those of all other firms. As in Gottfries (1986), each household's θ_{jt} must be random over time to make δ a fixed function of q_t/p_t and θ_t^0.

Firms face no uncertainty in this environment; they operate with perfect foresight. The representative firm seeks to maximize its current market value, equal to the present value of its current and future profits, discounted over time by the representative household's marginal utility of income, which in this case is simply

$$\sum_{k=0}^{\infty} \beta^k (\pi_{it+k}/w_{t+k}) \tag{8}$$

during period t.

Let $v(x_{it};z_t)$ denote the maximized value of (8) when the representative firm begins period t with a set of initial customers of measure x_{it} and the aggregate productivity disturbance is z_t. Then $v(x_{it};z_t)$ must satisfy the Bellman equation

$$(9) \quad v(x_{it};z_t) = \max_{q>0} (q/w_t-1/z_t)x_{it}(w_t/q)^2h(q/p_t;\theta_t^0) + \beta v[x_{it}\delta(q/p_t;\theta_t^0);z_{t+1}].$$

Equation (9) indicates that here, as in Phelps and Winter (1970) and Bils (1989), the firm faces a trade-off. By raising its current price q, the firm extracts more surplus from some of its customers. At the same time, however, this price increase drives away other customers; this effect works not only to reduce current profits through the term $h(q/p_t;\theta_t^0)$, but also to reduce future profits through the term $\delta(q/p_t;\theta_t^0)$ in (9).

The first order condition for (9) determines the outcome of this trade-off:

$$\begin{aligned}(10) \quad 0 = {} & x_{it}(1/w_t)(w_t/q)^2h(q/p_t;\theta_t^0) \\ & - 2(q/w_t-1/z_t)x_{it}(w_t/q)^2(1/q)h(q/p_t;\theta_t^0) \\ & + (q/w_t-1/z_t)x_{it}(w_t/q)^2(1/p_t)h'(q/p_t;\theta_t^0) \\ & + \beta x_{it}(1/p_t)\delta'(q/p_t;\theta_t^0)v'(x_{it+1};z_{t+1}),\end{aligned}$$

where

$$h'(q/p_t;\theta_t^0) = -(q/p_t)(\theta_t^0)^2f[(q/p_t)\theta_t^0],$$

$$\delta'(q/p_t;\theta_t^0) = -\theta_t^0f[(q/p_t)\theta_t^0],$$

and $v'(x_{it+1};z_{t+1})=\partial v(x_{it+1};z_{t+1})/\partial x_{it+1}$. The envelope condition for (9) states that

$$(11) \quad v'(x_{it};z_t) = (q/w_t-1/z_t)(w_t/q)^2h(q/p_t;\theta_t^0) + \beta\delta(q/p_t;\theta_t^0)v'(x_{it+1};z_{t+1}).$$

D. *Equilibrium*

In a symmetric equilibrium, $q=p_t$, $q^{t+1}=p^{t+1}$, and $x_{it}=x_{it+1}=1$ for all $t\geq 0$. Imposing these conditions, and noting that they imply $\delta(1;\theta_t^0)=1$, equation (10) becomes

$$0 = (w_t/p_t)h(1;\theta_t^0) - 2(w_t/p_t)[1-(w_t/p_t)(1/z_t)]h(1;\theta_t^0)$$
$$+ (w_t/p_t)[1-(w_t/p_t)(1/z_t)]h'(1;\theta_t^0) + \beta\delta'(1;\theta_t^0)v'(1;z_{t+1}),$$

while (11) becomes

$$v'(1;z_t) = (w_t/p_t)[1-(w_t/p_t)(1/z_t)]h(1;\theta_t^0) + \beta v'(1;z_{t+1}).$$

Equation (7), meanwhile, reduces to

$$\theta_t^0 = 1 - \frac{\kappa}{w_t/p_t}.$$

Note that these conditions only pin down the relative price $\omega_t=w_t/p_t$; determining absolute prices requires a choice of numeraire. Hence, they can be written

$$(12)\quad 0 = \omega_t h(1;\theta_t^0) - 2\omega_t(1-\omega_t/z_t)h(1;\theta_t^0)$$
$$+ \omega_t(1-\omega_t/z_t)h'(1;\theta_t^0) + \beta\delta'(1;\theta_t^0)v'(1;z_{t+1}),$$

$$(13)\quad v'(1;z_t) = \omega_t(1-\omega_t/z_t)h(1;\theta_t^0) + \beta v'(1;z_{t+1}),$$

and

$$(14)\quad \theta_t^0 = 1 - \kappa/\omega_t.$$

Given the sequence $\{z_t\}_{t=0}^{\infty}$, equations (12)-(14) determine equilibrium values for ω_t, $v'(1;z_t)$, and θ_t^0 for all $t\geq0$. Given these solutions, all other equilibrium quantities can be easily constructed. Hence, (12)-(14) completely summarize an equilibrium for this economy.

III. Countercyclical Markups

A. Steady State Markups

When $z_t=z$ for all $t\geq0$, equations (12)-(14) describe steady state equilibria in which $\omega_t=\omega$ and $\theta_t^0=\theta^0$ for all $t\geq0$. In particular, (14) implies that $\theta^0=1-\kappa/\omega$, while (12) and (13) imply that ω must satisfy

$$(15)\quad 0 = \omega h(1;\theta^0) - 2\omega(1-\omega/z)h(1;\theta^0) + \omega(1-\omega/z)h'(1;\theta^0) + [\beta/(1-\beta)]\omega(1-\omega/z)h(1;\theta^0)\delta'(1;\theta^0).$$

Equation (15) is highly nonlinear; in general, analytic solutions fail to exist. However, two special cases for which explicit solutions can be found serve to highlight one of the model's key implications.

First, as κ becomes arbitrarily large, (15) implies that the steady state markup of price over marginal cost, z/ω, approaches two. In this case, infinite search costs give firms full monopoly power. Equation (4) shows that each household's elasticity of demand is $\varepsilon=2$. Hence, the usual formula for the monopolistic markup applies: $\varepsilon/(\varepsilon-1)=2$.

Second, as κ becomes arbitrarily small and as both θ_l and θ_h approach unity, (15) implies that z/ω also approaches unity. In this case, households regard all goods as perfect substitutes and can move freely between firms. Hence, the competitive outcome obtains: price equals marginal cost.

More generally, the steady state markup lies somewhere between the monopolistic and competitive solutions; as in Phelps and Winter (1970), the representative firm enjoys only limited monopoly power over its customers. And while analytic solutions to (15) are no longer available, numerical solutions can be found by choosing a specific form for the distribution function F and assigning values to the model's parameters.

Thus, let θ have a normal distribution with mean $\bar{\theta}$ and standard deviation σ, truncated at $\theta_l>0$ and $\theta_h<\infty$. With $\bar{\theta}=1.01$, households prefer to remain with their initial suppliers, on average. With $\theta_l=0.001$, the distribution is truncated close to zero; with $\theta_h=2.019$, the distribution is symmetric about $\bar{\theta}$. In this case, with $\beta=0.95$, $\kappa=0.01$, $\sigma=0.0075$, and $z=z^0=1$, equation (15) has the solution $\omega/z=1.0366$, implying a steady state markup of 3.66 percent.

When z increases to $z^1=(1.01)^{1/2}$, the steady state markup declines to 3.58 percent. Aggregate output, which by (4)-(7) is given by

$$y_t = \omega_t^2 h(1;\theta_t^0)$$

in a symmetric equilibrium, increases by 1.15 percent. Thus, the steady state markup varies inversely with output. Under perfect competition, the increase from $z^0=1$ to $z^1=(1.01)^{1/2}$ generates an increase in output of only one percent. Here, as in Bils (1989), the countercyclical markup works to amplify the effects of aggregate real disturbances on output.

B. The Response of Markups to Transitory Shocks

Comparing steady states, as above, illustrates the effects of a permanent change in the marginal product of labor. Next, consider the effects of a purely transitory shock, as captured by the sequence $\{z_t\}_{t=0}^{\infty}$ with $z_0=z^1=(1.01)^{1/2}$ and $z_t=z^0=1$ for all $t\geq1$.

Using the same parameter values as before, equations (12)-(14) yield the solutions $z_0/\omega_0=1.0169$ and $z_t/\omega_t=1.0366$ for all $t\geq1$. Thus, the transitory shock causes the equilibrium markup to fall from 3.66 percent to 1.69 percent. Output, meanwhile, increases by 4.94 percent during the period of the shock. Once again, the markup is countercyclical, serving to amplify the effects of the technology shock.

C. The Sources of Countercyclical Markups

To understand how customer flow dynamics give rise to countercyclical markups in this environment, note first that equations (6), (7), and (14) imply that in a symmetric equilibrium, household j remains with its initial supplier during period t if and only if

(16) $$\kappa \geq \omega_t(1-\theta_{jt}).$$

For a household with $\theta_{jt}<1$, the right-hand side of (16) represents the utility cost of remaining with its initial supplier. The size of this cost increases with ω_t since, as shown by equations (4) and (5), the size of the household's purchase also increases with ω_t. The left-hand side of (16) measures the utility cost of searching for a new supplier; this cost is fixed relative to the size of the purchase. Hence, (16) implies that as the size its purchase increases, the household's incentive to search becomes stronger.

Next, consider equations (6) and (7) from the perspective of the representative firm. If the firm charges price q_t while all other firms charge price p_t, it retains a fraction $1-F[(q_t/p_t)\theta_t^0]$ of its initial customers. Thus, as the firm raises its price infinitesimally above p_t, it loses a fraction $\theta_t^0 f(\theta_t^0)$ of its customers. Differentiating this expression with respect to θ_t^0 yields

(17) $$f(\theta_t^0) + \theta_t^0 f'(\theta_t^0).$$

Note from equation (14) that θ_t^0 is increasing in ω_t. Since (14) also implies that $\theta_t^0 \leq 1$, the assumption $f'(\theta) \geq 0$ for all $\theta \leq 1$ guarantees that the expression in (17) is positive.

Thus, when the assumption $f'(\theta) \geq 0$ for all $\theta \leq 1$ holds, equation (17) implies that the strengthened incentive for any individual household to search translates into an increase in the number of households that actually search when the firm raises its price above p_t. In this case, the increase in ω_t that increases both aggregate output and the size of each household's purchase also makes the demand facing the representative firm more elastic, inducing the firm to lower its markup. Hence, when output rises, the markup falls.

The numerical results presented above also show that a temporary increase in the productivity parameter z_t yields a larger drop in the markup, and hence a larger increase in output, than a permanent increase in z_t. In the case of a transitory shock, the representative firm knows that it will be able to charge a higher markup in the future, when output will be lower. Higher markups in the future provide the firm with a stronger incentive to cut its price today in an effort to expand its future customer base. Hence, in equilibrium, the markup falls by more than when the shock is permanent.

IV. Introducing Persistence

While results from the previous section illustrate that customer flow dynamics can generate countercyclical markups that amplify the effects of technology shocks on output, these effects do not persist. In particular, both the markup and output return to their steady state levels in the period immediately following a transitory shock to productivity. Thus, accounting for persistence in the effects of technology shocks requires some modification to the model of sections II and III.

Accordingly, suppose that if household j searches for a new supplier in period t-1, it acquires valuable information about market opportunities that reduce its search costs during period t; for simplicity, consider the limiting case where the search cost is reduced to zero. In this case, the household's search cost during period t is given by $\kappa(1-s_{jt-1})$, which equals κ if $s_{jt-1}=0$ and zero if $s_{jt-1}=1$. Household j's utility function becomes

$$E\left\{\sum_{t=0}^{\infty}\beta^t[2(\theta_{jt}^{1-s_{jt}}c_{jt})^{1/2}-n_{jt}-\kappa s_{jt}(1-s_{jt-1})]\right\},$$

where, for period t=0, the initial condition s_{j-1} is taken as given.

Having made this change to the search technology, let $u(\theta_{jt},s_{jt-1};q^t)$ denote the maximized value of the household's expected utility from period t forward, given that it experiences shock θ_{jt} during period t, given that it made search decision s_{jt-1} during period t-1, and given that its initial supplier plans to charge prices q^t from period t forward. Then $u(\theta_{jt},s_{jt-1};q^t)$ must satisfy the Bellman equation

(18) $$u(\theta_{jt},s_{jt-1};q^t)$$
$$= \max 2[(1-s)(\theta_{jt}c^0)^{1/2}+s(c^1)^{1/2}] - (1-s)n^0 - s[n^1+\kappa(1-s_{jt-1})] + \beta E[(1-s)u(\theta_{jt+1},0;q^{t+1})+su(\theta_{jt+1},1;p^{t+1})],$$

where the maximization is again by choice of c^0, n^0, c^1, and n^1 and $s\in\{0,1\}$ satisfying (2) and (3).

The first order conditions for (18) indicate that optimal choices for c^0, n^0, c^1, and n^1 continue to be given by (4) and (5). Thus, if household j remained with its initial supplier during period t-1, so that $s_{jt-1}=0$, (18) implies that the household continues to remain with its initial supplier during period t, choosing s=0, if and only if

(19) $$\theta_{jt} \geq (q_t/p_t)\theta_t^0,$$

where now

(20) $$\theta_t^0 = 1 - \frac{\kappa + \beta E[u(\theta_{jt+1},0;q^{t+1})-u(\theta_{jt+1},1;p^{t+1})]}{w_t/p_t} .$$

If, on the other hand, the household searched for a new supplier during period t-1, so that $s_{jt-1}=1$, (18) implies that the household remains with its initial supplier during period t if and only if

(21) $$\theta_{jt} \geq (q_t/p_t)\theta_t^1,$$

where

(22) $$\theta_t^1 = 1 - \frac{\beta E[u(\theta_{jt+1},0;q^{t+1})-u(\theta_{jt+1},1;p^{t+1})]}{w_t/p_t} .$$

Since $\kappa>0$, equations (19)-(22) show that the reduction in search costs makes the representative household more likely to search for a new supplier during period t if searched during period t-1.

As before, the representative firm takes θ_t^0 and θ_t^1 as given when choosing its price q_t for period t. If all other firms charge price p_t, the representative firm retains a subset of its initial customers of measure $x_{it}(1-\sigma_t)\{1-F[(q_t/p_t)\theta_t^0]\}+x_{it}\sigma_t\{1-F[(q_t/p_t)\theta_t^1]\}$, where σ_t denotes the fraction of all households who searched for a new supplier during period t-1 and hence have low search costs during period t.[4] Each of these customers demands $\theta_{jt}(w_t/q_t)^2$ units of output. The firm also gains a set of new customers of measure $x_{it}(1-\sigma_t)F(\theta_t^0)+x_{it}\sigma_tF(\theta_t^1)$; each of these customers demands $(w_t/q_t)^2$ units of output. Hence, the firm faces total demand

$$x_{it}(w_t/q_t)^2h(q_t/p_t;\sigma_t,\theta_t^0,\theta_t^1),$$

where

$$h(q_t/p_t;\sigma_t,\theta_t^0,\theta_t^1) = (1-\sigma_t)[\ F(\theta_t^0) + \int_{(q_t/p_t)\theta_t^0}^{\theta_h} \theta f(\theta)d\theta\ ]$$

$$+ \sigma_t[\ F(\theta_t^1) + \int_{(q_t/p_t)\theta_t^1}^{\theta_h} \theta f(\theta)d\theta\ ],$$

and earns profits of

$$\pi_{it} = (q_t-w_t/z_t)x_{it}(w_t/q_t)^2 h(q_t/p_t;\sigma_t,\theta_t^0,\theta_t^1)$$

during period t.

The firm then begins period t+1 with a new set of initial customers of measure

$$x_{it+1} = x_{it}\delta(q_t/p_t;\sigma_t,\theta_t^0,\theta_t^1),$$

where

$$\delta(q_t/p_t;\sigma_t,\theta_t^0,\theta_t^1) = (1-\sigma_t)\{\ 1 - F[(q_t/p_t)\theta_t^0] + F(\theta_t^0)\ \}$$

$$+ \sigma_t\{\ 1 - F[(q_t/p_t)\theta_t^1] + F(\theta_t^1)\ \}.$$

As before, the firm chooses q_t to maximize its current market value, given by the discounted sum in equation (8). The firm's Bellman equation becomes

$$v(x_{it};z_t) = \max_{q>0}\ (q/w_t-1/z_t)x_{it}(w_t/q)^2 h(q/p_t;\sigma_t,\theta_t^0,\theta_t^1) \tag{23}$$

$$+ \beta v[x_{it}\delta(q/p_t;\sigma_t,\theta_t^0,\theta_t^1);z_{t+1}].$$

To simplify the notation, let $v_t=v'(1;z_t)$, $u_t^0=E[u(\theta_{jt},0;p^t)]$, and $u_t^1=E[u(\theta_{jt},1;p^t)]$. After the equilibrium conditions $q=p_t$, $q^{t+1}=p^{t+1}$, and $x_{it}=x_{it+1}=1$ are imposed, the first order and envelope conditions for (23) can be written

$$(24)\quad 0 = \omega_t h(1;\sigma_t,\theta_t^0,\theta_t^1) - 2\omega_t(1-\omega_t/z_t)h(1;\sigma_t,\theta_t^0,\theta_t^1) + \omega_t(1-\omega_t/z_t)h'(1;\sigma_t,\theta_t^0,\theta_t^1) + \beta\delta'(1;\sigma_t,\theta_t^0,\theta_t^1)v_{t+1}$$

and

$$(25)\quad v_t = \omega_t(1-\omega_t/z_t)h(1;\sigma_t,\theta_t^0,\theta_t^1) + \beta v_{t+1},$$

where

$$h'(q/p_t;\sigma_t,\theta_t^0,\theta_t^1) = -(1-\sigma_t)(q/p_t)(\theta_t^0)^2 f[(q/p_t)\theta_t^0] - \sigma_t(q/p_t)(\theta_t^1)^2 f[(q/p_t)\theta_t^1]$$

and

$$\delta'(q/p_t;\sigma_t,\theta_t^0,\theta_t^1) = -(1-\sigma_t)\theta_t^0 f[(q/p_t)\theta_t^0] - \sigma_t\theta_t^1 f[(q/p_t)\theta_t^1].$$

Equations (20) and (22) simplify to

$$(26)\quad \theta_t^0 = 1 - [\kappa+\beta(u_{t+1}^0-u_{t+1}^1)]/\omega_t$$

and

$$(27)\quad \theta_t^1 = 1 - \beta(u_{t+1}^0-u_{t+1}^1)/\omega_t.$$

In a symmetric equilibrium, each firm earns the same level of profits, so that $\pi_{it}=\pi_t$ for all $i\in[0,1]$ where

$$(28)\quad \pi_t/w_t = \omega_t(1-\omega_t/z_t)h(1;\sigma_t,\theta_t^0,\theta_t^1).$$

Using (4), (5), and (19), (21), and (28), equation (18) implies

$$(29)\quad u_t^0 = \omega_t[\, F(\theta_t^0) + \int_{\theta_t^0}^{\theta_h} \theta f(\theta)d\theta \,] + \omega_t(1-\omega_t/z_t)h(1;\sigma_t,\theta_t^0,\theta_t^1) - F(\theta_t^0)\kappa + \beta[1-F(\theta_t^0)]u_{t+1}^0 + \beta F(\theta_t^0)u_{t+1}^1$$

and

$$(30)\quad u_t^1 = \omega_t[\, F(\theta_t^1) + \int_{\theta_t^1}^{\theta_h} \theta f(\theta)d\theta \,] + \omega_t(1-\omega_t/z_t)h(1;\sigma_t,\theta_t^0,\theta_t^1) + \beta[1-F(\theta_t^1)]u_{t+1}^0 + \beta F(\theta_t^1)u_{t+1}^1.$$

the search cost parameter is increased slightly to $\kappa=0.011$. This increase in κ increases the steady state markup as well as the reduction in search costs during period t obtained from searching during period t-1. With all other settings the same as in figure 1, and with the same transitory increase in z_t, figure 2 shows that output increases by 4.68 percent during the period of the shock and remains 1.39 percent above steady state during period t=1. Output is 0.57 percent above steady state for t=2, 0.23 percent above steady state for t=3, and 0.10 percent above steady state for t=4.

V. The Persistent Effects of Monetary Shocks

The models described in sections II-IV are purely real; they cannot be used to study the effects of monetary shocks. In principal, the nominal sector could be accounted for here by adding a cash-in-advance constraint to the representative household's optimization problem; this, however, is not a straightforward exercise. Since households face idiosyncratic shocks, they will accumulate real balances not only for the purpose of buying consumption goods, as required by the cash-in-advance constraint, but also in an attempt to insure against adverse θ_{jt} shocks. Hence, as in Lucas (1980), the monetary equilibrium will feature a nondegenerate distribution of real balances across households; constructing such an equilibrium will involve the difficult task of characterizing this distribution. Moreover, the cash-in-advance constraint will introduce a second intertemporal consideration, in addition to the search decision, into the household's already complicated problem.

Thus, in order to preserve tractability, let aggregate real money demand be described by the simple quantity equation

$$m_t/p_t = y_t = \omega_t^2 h(1;\sigma_t,\theta_t^0,\theta_t^1), \tag{32}$$

where the second equality defines aggregate output in the model of section IV, m_t is the aggregate nominal money supply during period t, and p_t and $w_t=\omega_t p_t$ are now interpreted as the nominal price level and the nominal wage rate during period t. Ball and Romer (1990) also take this approach to introduce money in their model.

When combined with the other seven equilibrium conditions (24)-(27) and (29)-(31), equation (32) indicates that so long as prices and wages are fully flexible, any change in the nominal money supply can be met by proportional changes in nominal prices and wages that leave the real wage, ω_t, and all other real variables unchanged. In this case, the setting for m_t amounts only to a choice of numeraire. As noted by Gordon (1981) and Ball and Romer (1990), customer flow dynamics are the outcome of real, rather than nominal, rigidities; considered in isolation, they cannot explain how changes in the nominal money supply affect real output.

When coupled with some form of nominal rigidity, however, customer flow dynamics can allow monetary shocks to have persistent effects on real output. As an example, consider the effects of a once-and-for-all change in the nominal money supply, made after firms have posted their common nominal price p_0 for period t=0. Then for t=0, the representative firm's first order and envelope conditions (24) and (25) no longer apply; with m_0 given by policy and p_0 sticky for one period, equations (26), (27), and (29)-(32) determine ω_0, θ_0^0, θ_0^1, u_0^0, u_0^1, and σ_1. In particular, (32) implies that y_0 varies proportionately with the post-shock nominal money supply m_0. For $t \geq 1$, the price level is free to adjust; the seven original equations (24)-(27) and (29)-(31) determine the real variables ω_t, θ_t^0, θ_t^1, v_t, u_t^0, u_t^1, and σ_{t+1}, and (32) once again serves only to determine the nominal variables p_t and w_t.

Figure 3 traces out the effects of a one percent increase in the nominal money supply when, as described above, nominal prices are sticky for period t=0. The marginal product of labor is constant, with $z_t=1$ for all $t \geq 0$, but all other parameter settings are as in figure 2.

The one percent increase in money generates a one percent increase in output when nominal prices are sticky for one period. This increase in output also increases the number of searching households during period t=0, which increases the number of households with low search costs during period t=1. Hence, customer flow dynamics allow the effects of the monetary shock to persist. Output remains 0.31 percent above its steady state level during period t=1, 0.13 percent above steady state during period t=2, and 0.05 percent above steady state during period t=3.

VI. Conclusion

The results of sections IV and V illustrate that customer flow dynamics of the kind first considered by Phelps and Winter (1970) not only can serve to amplify the effects of shocks on aggregate output, as they do in Bils (1989), but can also work to propagate those effects over time.

The model of customer flows developed here, while extending those of

Phelps and Winter and Bils, remains highly stylized along some dimensions: preferences and technologies are described by simple functional forms, while capital accumulation is abstracted from entirely. These features of the model keep the analysis tractable and serve to isolate the effects of customer flows from other sources of propagation. On the other hand, they also preclude any assessment of the model's ability to match the US data. Accordingly, this paper must be read as one that simply illustrates the theoretical possibility that customer flow dynamics can work as a mechanism for generating persistent effects of real and monetary shocks. Whether this mechanism can produce effects that are quantitatively important remains a task for future research.

Notes

[1]Cogley and Nason (1995) suggest that this criticism applies to real business cycle models as well; they argue that those models have only weak internal mechanisms for propagating the effects of technology shocks and therefore cannot account for the observed serial correlation in aggregate output.

[2]Bils (1987) and Rotemberg and Woodford (1991) find evidence of countercyclical markups in the US data.

[3]One might guess that the more basic assumption that θ_{jt} is iid across households, together with the law of large numbers, would guarantee that the individual household's probability $F(\theta)$ of drawing $\theta_{jt} \leq \theta$ coincides with the fraction of the firm's total customers having $\theta_{jt} \leq \theta$. As shown by Judd (1985) and Feldman and Gilles (1985), however, the law of large numbers will not generally apply when there is a continuum of households. Feldman and Gilles indicate that probabilities for the individual household and fractions for the firm can be made to coincide by allowing θ_{jt} to be correlated across households; in fact, the analysis that follows is not inconsistent with any required cross-sectional dependence in θ_{jt}.

[4]Again, the cross-sectional distribution of θ_{jt} is assumed to be such that the fraction of all households who remain with their initial suppliers during period t-1 and experience shock $\theta_{jt} \leq \theta$ during period t is equal to $(1-\sigma_t)F(\theta)$. Similarly, the fraction of all households who search during

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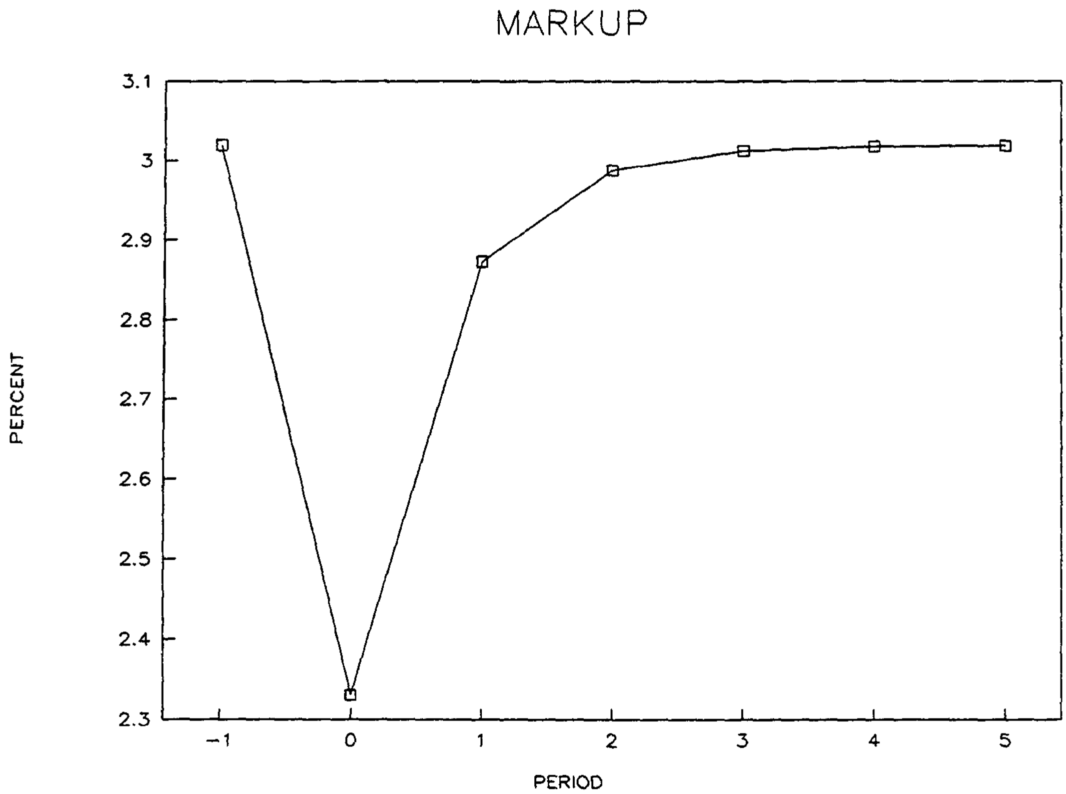
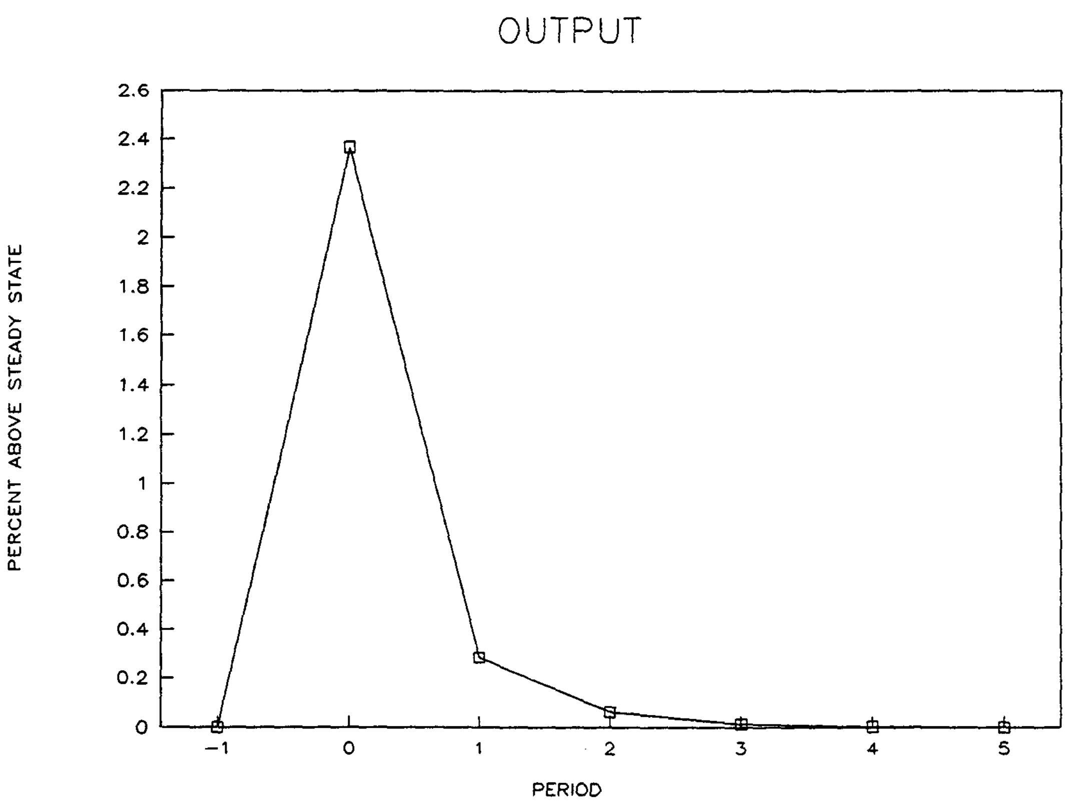
Figure 1

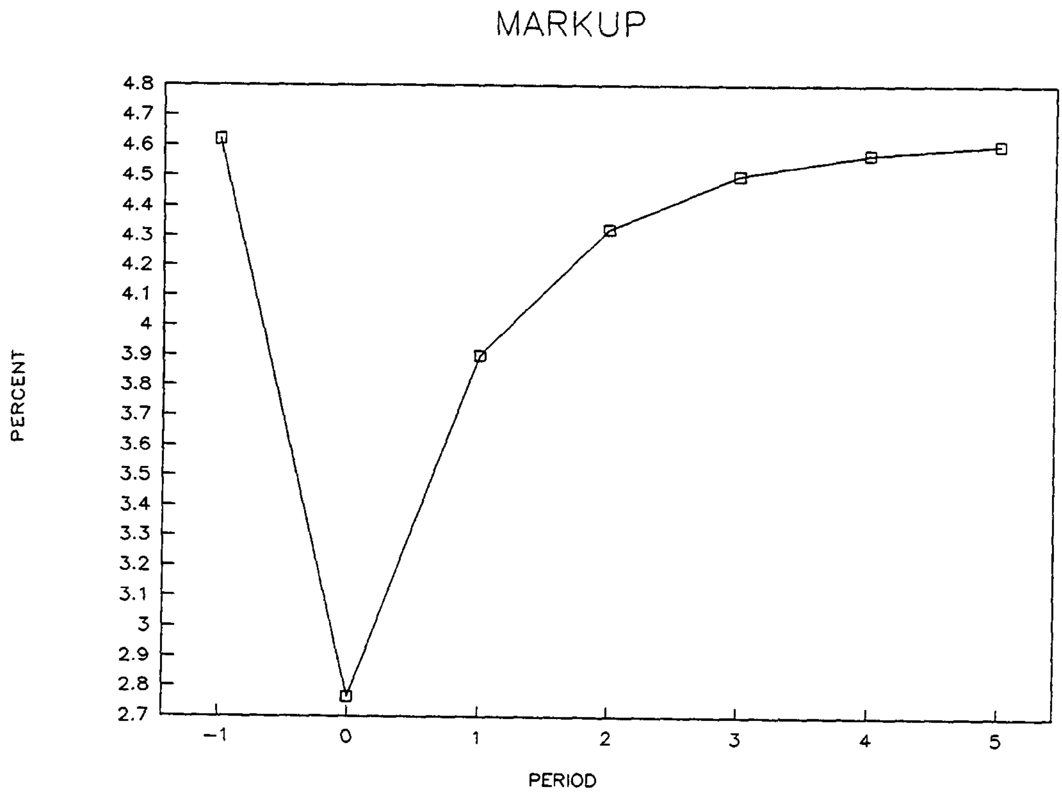


MARKUP
PERCENT
3.1
3
2.9
2.8
2.7
2.6
2.5
2.4
2.3
-1
0
1
2
3
4
5
PERIOD

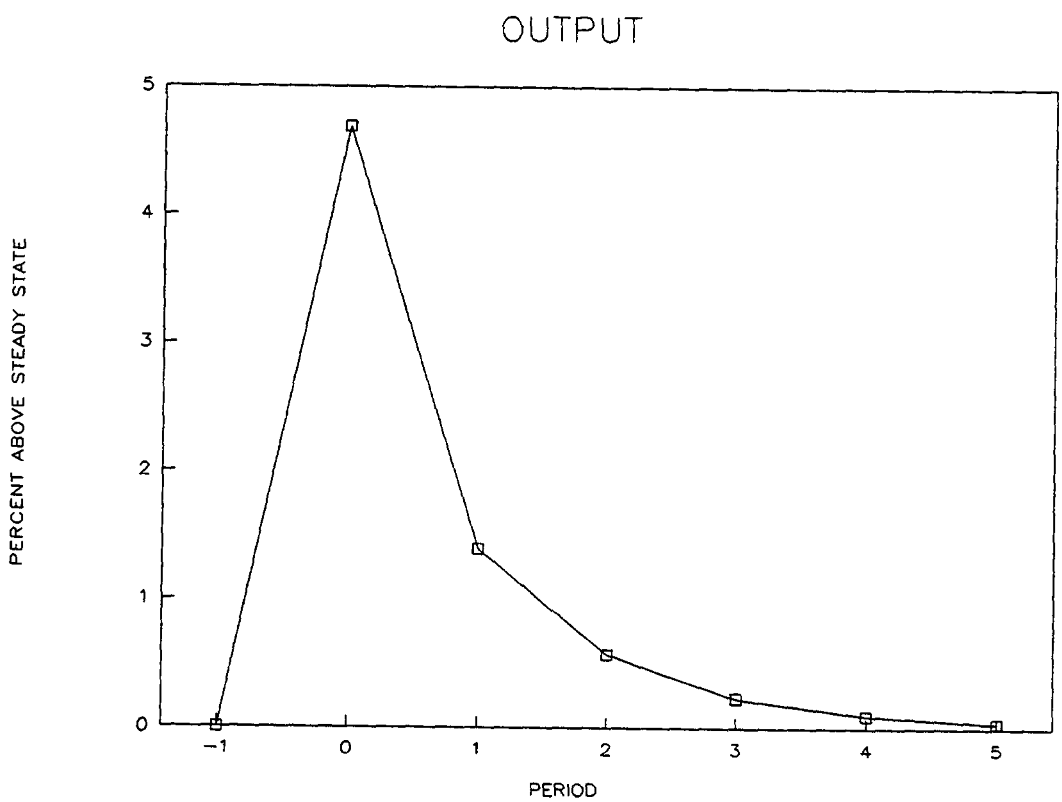


OUTPUT
PERCENT ABOVE STEADY STATE
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-1
0
1
2
3
4
5
PERIOD

Figure 2

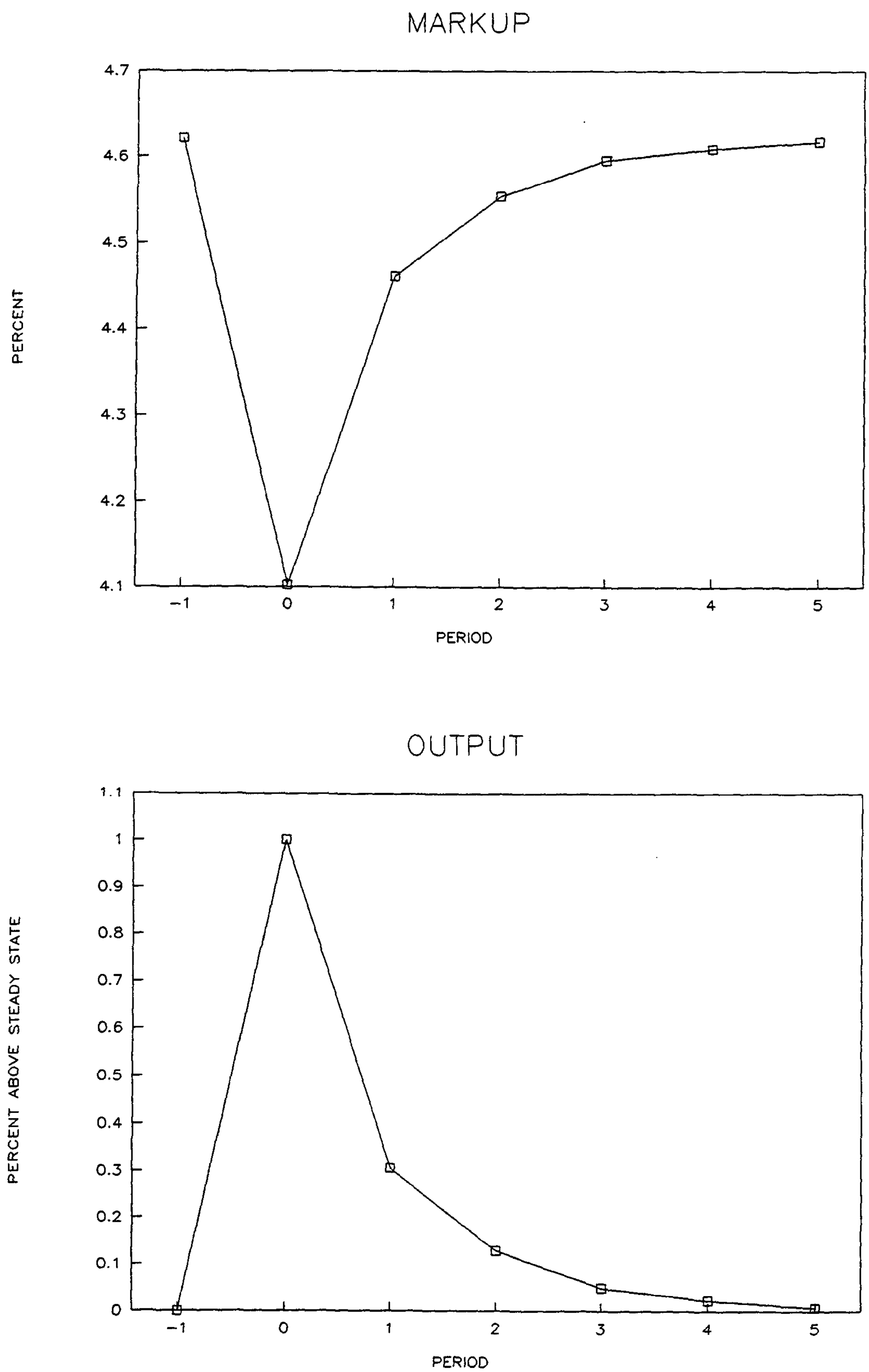


MARKUP
PERCENT
4.8
4.7
4.6
4.5
4.4
4.3
4.2
4.1
4
3.9
3.8
3.7
3.6
3.5
3.4
3.3
3.2
3.1
3
2.9
2.8
2.7
-1
0
1
2
3
4
5
PERIOD



OUTPUT
PERCENT ABOVE STEADY STATE
5
4
3
2
1
0
-1
0
1
2
3
4
5
PERIOD

Figure 3


MARKUP
4.7
4.6
4.5
4.4
4.3
4.2
4.1
PERCENT
-1
0
1
2
3
4
5
PERIOD
OUTPUT
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
PERCENT ABOVE STEADY STATE
-1
0
1
2
3
4
5
PERIOD