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State-Dependent Pricing and the Dynamics of Business Cycles *

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Abstract

The nature of price dynamics has long been thought important for the origin and duration of business cycles. To investigate this topic, we construct a dynamic stochastic general equilibrium macroeconomic model in which monopolistically competitive firms face fixed costs of changing the nominal prices of final goods. These prices are thus changed infrequently and discretely. The framework captures major features of the price dynamics stressed by the New Keynesian research program, particularly work on (s,S) pricing rules. However, by treating firms as heterogeneous with respect to the size of fixed costs of price adjustment, we are able to study a wider range of issues than in the prior literature. For example, we explore how the nature of optimal price-setting depends on (i) the extent of persistence of variations in the money stock and (ii) the interest elasticity of money demand. Further, our model can be used to study a wide range of aspects of the positive and normative economics of monetary policy. We illustrate these topics by considering the consequences of changing the rate of inflation and by evaluating alternative policy rules.

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The nature of price dynamics has been long thought important for the origin and duration of business cycles. Recent work in New Keynesian macroeconomics has reemphasized the empirical observation that the prices of individual firms frequently remain fixed for substantial periods of time. This literature has attributed price stickiness to costs of changing prices at the level of the firm—sometimes called menu costs—which lead individual firms to adjust prices only when there are sufficiently large variations in costs or demand.[1]

The standard development of dynamically optimal pricing policy with menu costs follows an inventory theoretic approach, yielding price adjustment rules that are (s,S) . Early work by Barro [1972], and Sheshinski and Weiss [1983] on (s,S) policies has been followed by a large number of recent studies, most notably those of Caplin and Spulber [1987] and Caplin and Leahy [1991]. Yet, there is as yet no work that incorporates optimal (s,S) policies into a fully articulated dynamic macroeconomic model. This shortcoming occurs for two reasons. First, price adjustments in a menu cost setting are *state-dependent.* Second, the individual firm's price is adjusted in a *discrete* manner in response to the state of the economy. This discreteness makes it difficult to characterize optimal aggregate price dynamics in a way that permits integration into a complete macroeconomic model. For example, in response to a shock of a given size, the simplest (s,S) model would predict that no firm (or all firms) would adjust their prices. Richer (s,S) models—such as those in Caplin and Spulber [1987], Caplin and Leahy [1991] or Caballero and Engel [1991]—moderate this prediction, by introducing heterogeneity in the circumstances of individual firms. Yet, these richer models typically involve a "curse of dimensionality", in the sense of Bellman [1957], so that they can be solved under very restrictive conditions (random walk driving processes and interest inelastic money demand). These limitations mean that it is very difficult to perform dynamic studies similar to those undertaken by real business cycle analysts. One cannot, for example, explore the consequences of alternative driving processes for the money stock or alternative rules for monetary policy, which are necessary ingredients to modern business cycle analysis.

[1]Recent surveys that discuss the empirical evidence for infrequent price adjustment and its macroeconomic consequences are Rotemberg [1987] and Weiss [1993]. The twin volumes edited by Mankiw and Romer [1991] contain core references in New Keynesian macroeconomics, including some of the work on price adjustment that is most closely related to the topic of this paper.

Thus, most recent work on the role of price dynamics in business cycles has used an alternative approach, which is called *time-dependent* pricing. The most attractive version is due to Calvo [1983], who develops a model in which the timing of price adjustment by an individual firm is governed by an exogenous mechanism which specifies that the probability of adjustment is a constant, independent of calendar time and also of the length of time elapsed since the last adjustment. An attractive feature of Calvo's model is that it leads to a very simple representation of aggregate price dynamics.[2] While the circumstances of individual firms are random and price adjustments discrete, there is a sufficient number of firms that aggregate price dynamics are described by a simple, low order expectational difference equation. Thus, in contrast to the complexity of the state-dependent pricing rules that arise from the (s,S) literature, it is easy to incorporate time-dependent price dynamics into standard macroeconomic models.[3]

In this paper, we derive a similarly tractable representation of state-dependent pricing and imbed it in a small-scale macroeconomic model. We then contrast the nature of business cycles with state-dependent pricing to those arising with an identical steady-state pattern of time-dependent price adjustment. We thus undertake two extensions of an earlier literature on forward-looking pricing that uses the Calvo setup, including work of Buiter and Miller [1985], Ball [1994] and others. On the basis of that time-dependent pricing model, one can reach three major conclusions about the influence of changes in monetary policy on economic activity. First, if changes in the money stock are perceived to be temporary by firms, then there is little response of prices. Second, if changes in the money stock are perceived to be permanent, then there is a much larger change by firms adjusting prices: they would choose to adjust most of the way toward the proportionately higher price level that would prevail in the long run. However, the overall price level would still behave sluggishly because many firms would not adjust. Consequently, there would be important aggregate effects of the monetary change on real economic activity when the shift was temporary or permanent, but there is some presumption that the effect is larger when the shock is temporary because of more sluggish price adjustment. Third, if the monetary authority permanently

[2]However, there is an important limitation of Calvo's specification. As shown by King and Wolman [1996], if the marginal probability of nonadjustment in a quarterly model is greater than .9, then firms will choose not to operate in the Calvo setup when there are inflation rates of 10 percent or more. These difficulties are not shared by the extension of the Calvo model that we develop below.

[3]Recent examples include Yun[1994], King and Wolman[1996], and King and Watson [1996].

increases the inflation rate in a credible manner, then there is little real effect or, at least, little departure from the real effects that would prevail in a flexible price model.

We explore two modifications of the Calvo framework below. In the first, we allow the probability of price adjustment to be lower for firms that have recently adjusted their price and higher for those that have not adjusted their price for many periods. That is, we study the effects of a richer pattern of time-dependent price adjustment. Looking at the dynamic response of an economy to some standard monetary changes, we find that there are some similarities with the basic setup, but also some important differences. Notably, with a richer probability structure, there is no longer a short-run superneutrality: a permanent increase in the inflation rate temporarily increases real activity.

In the second of these modifications, we permit a response of the pattern of adjustment to the state of the economy: more firms undertake costly adjustments when there is a larger present value of benefits to adjustment. Relative to a time-dependent model with the same steady state patterns of adjustment, there are many important differences. Some of these are relatively simple and intuitive. For example, when there is a permanent increase in the quantity of money, there is a faster pattern of adjustment in the price level because a larger portion of firms adjust. Other implications are more complicated and less intuitive: we find that there is a tendency for the dynamic responses of prices and output to display oscillatory paths. Overall, though, we find that state-dependent pricing is important for the analysis of inflation/disinflation policy: for permanent and credible changes in the inflation rate, it restores approximate superneutrality.

The organization of the paper is as follows. Section 1 reports on some empirical aspects of price dynamics at the micro and macro levels that we want our theory to capture. To undertake a comparison between the two models of pricing, we must begin by extending the prior literature in two directions. In section 2, we extend the Calvo [1983] framework to allow for a richer pattern of time dependence in price adjustment. In the original setup, the conditional probability of an individual firm's being able to adjust its price in any period is independent of how long it had been since its last adjustment. The extension that we consider permits this conditional probability to vary in an arbitrary manner. In section 3, we develop an analogous representation of the economy when there is costly price adjustment. To do so, we begin by studying the price setting problem of a firm which faces a fixed cost of price adjustment that is random, rather than certain as in much of the (s,S) literature. We derive a discrete individual choice rule for the firm: it will choose to adjust its price only if the gains from doing so are sufficiently large to warrant payment of the fixed cost. Thus, our model captures price inflexibility at the level of the individual firm. However, we also assume that there are many firms in the economy (technically a continuum) and that each faces a different level of fixed costs. Thus, there is always a marginal firm that is indifferent between adjustment and nonadjustment even when there are no differences in demand conditions across firms. We also impose the restrictions that the randomness in adjustment costs is independent over time implying that all firms that adjust price choose the same price. This common price outcome means that there is a low dimension of the state of the economy. With such smoothness and dimensionality conditions satisfied, it is direct to use conventional linear approximation model solution procedures.[4] In section 4, we discuss the structure of the rest of the small-scale macroeconomic model used in this investigation.[5] In section 5, we develop the steady state of this model and discuss issues of calibration, including the structure of adjustment costs.

Section 6 reports on how the state-dependent model of price adjustment works with respect to basic changes in the quantity of money. To provide a reference point for this discussion, we also consider a time-dependent model with an identical steady-state pattern of adjustment probabilities. Our discussion considers the effects of three basic monetary policy shocks, assuming that the monetary authority makes the money stock its instrument. We contrast the effects of a temporary increase in the quantity of money, a permanent increase in the quantity of money, and a persistent increase in the money growth rate. We find that the nature of the monetary policy shock determines whether there are quantitatively important differences between state-dependent and time-dependent model responses: for a purely temporary change in money, there is little difference but there are very important differences in other cases, which generally increase the responsiveness of the price level to monetary shocks.

Section 6 also reports on the interaction between optimal price adjustment and the structure of the rest of the macroeconomic model, specifically on the specifi-

[4]That is, we use linear approximation methods as in the real business cycle analyses of Kydland and Prescott [1982] or King, Plosser and Rebelo [1988]. Our specific implementation of this approach draws on the model solution theory and algorithms of King and Watson [1995a,b].

[5]This model is developed from those employed in King and Watson [1995c] and King and Wolman [1996].

cation of money demand which is typically assumed to be a quantity equation in most standard (s,S) price adjustment analyses. We demonstrate that the impact effects of money on output depend in a quantitatively important manner on the specification of money demand. When we employ a money demand specification that captures long-run interest sensitivity of real balances (based on the shopping time specification estimated in Wolman [1996]), we find that there are only minor effects of temporary monetary changes on output and prices: a one percent increase in the money stock has about a .2 percentage point effect on output and a negligible effect on the price level. When the persistence of monetary variations is increased, there are larger effects on both output and on the price level. By contrast, with a constant velocity specification imposed, the effect of money on output is roughly one-for-one when the shock is temporary, but is increasingly dissipated by price adjustment as the monetary changes are assumed to be more persistent.

The discussion then turns to the implications of the state-dependent pricing model for various alternative representations of monetary policy. In section 7, we show that our pricing structure can be used with both interest rate rules and with price level rules. In terms of the former, we trace out how an interest rate shock would affect macroeconomic activity under state and time-dependent pricing policies. In terms of the latter, we consider the effects of a permanent productivity disturbance on the path of output under a policy of stabilizing the price level, paralleling the prior investigation of King and Wolman [1996]. We find that the time-dependent and state-dependent pricing models give responses that are essentially identical to each other and also to the responses produced in a frictionless price adjustment (real business cycle) model. In section 8, we consider the transition between alternative rates of inflation in various sticky price models. Our reference point for this discussion is the prior work by Buiter and Miller [1985], Ball [1994] and King and Wolman [1996], which show two major results for the Calvo model. The first of these is that there is no stimulative effect of an increase in the inflation rate if it is simply increased in an unexpected manner. This finding arises from a combination of the price adjustment mechanism and the fact that there is an accommodation of the changing level of real demand for money by the monetary authority.[6] The second of these is that a substantial expansion

[6]Indeed, in the King and Wolman [1996] version of this experiment, there is a modest decline in output that occurs as individuals substitute out of market activity into nonmarket substitutes for monetized exchange.

occurs for several quarters if there is a permanent increase in the growth rate of money. Reconsidering these inflation experiments, we find three major results in this section. First, our more general time-dependent model does not share the implication of the Calvo model for unexpected changes in the inflation rate: an expansion arises when the inflation rate increases. This result is traced to the fact that our time-dependent model assigns a small marginal probability of adjustment to firms that have recently adjusted price. Second, this expansion is virtually eliminated by state-dependent pricing . Third, in both state-dependent and time-dependent setups, there continues to be a quantitatively important difference between increases in the money growth rate and increases in the inflation rate. Section 9 provides a brief summary, reports our main conclusions, and discusses directions for future research.

1. Price Dynamics

In this section, we begin by describing six facts about the dynamics of prices that we think any macroeconomic model should be able to capture. We use these facts as a basis for evaluating some existing models of price dynamics and as a rationale for our current work.

1.1. Stylized Facts

The first four of the stylized facts stressed by the New Keynesian macroeconomics apply to the behavior of individual prices and the price level; the latter two involve the behavior of individual prices and the inflation rate.

Four facts concerning individual prices and the price level: Figure 1 displays some hypothetical examples of paths of price adjustment that display four sets of facts that motivate our investigation. These sorts of paths could be selected, for example, from the Stigler and Kindahl [1970] data studied by Carlton [1986] or the Israeli data that is discussed in Weiss's [1993] recent summary of the case for sticky prices. More specifically, these paths capture the range of price adjustment facts produced in the recent set of studies by Lach and Tsiddon [1992, 1996].

First, as is shown in panel A of Figure 1, the paths of individual prices are adjusted in ways that include **infrequent adjustments, irregularly timed adjustments** and **changes of differing sizes** (including many very small changes). Second, as is shown in panel B of Figure 1, the adjustments of individual firms are sufficiently **imperfectly correlated** that the path of adjustment for the price level is relatively smooth.

Two more facts about inflationary situations: In situations of inflation, it is necessary to add two additional facts to this list, namely that the changes in individual firm prices become **larger** and they also occur with **greater frequency.**

1.2. Implications for model building

Macroeconomists have developed models of price dynamics that seek to explain these stylized facts. One approach is to simply assume that a representative firm faces quadratic costs of adjusting its price (as in Rotemberg [1982]): this approach captures the dynamics of the price level well, but is inconsistent with the dynamics of individual prices. Another approach is to assume that an individual firm has an exogenously timed, random pattern of opportunities to adjust its price (as in Calvo [1983]): this approach can capture five of the six stylized facts, but it cannot explain the greater frequency of price change in inflationary settings. Each of these approaches has been criticized, for example by Blanchard and Fischer [1989], for being too mechanical. However, each is very tractable and can be used to evaluate the macroeconomic consequences of alternative monetary policies in line with the general methodological recommendations of Lucas ([1976], [1980]).

Most recent work has followed Barro [1972] and Sheshinski and Weiss [1983] in building models in which firms face constant real costs of changing nominal prices. These models can also capture five of the six five stylized facts, but a different subset than is produced by the Calvo setup. The (s,S) models can readily explain the response of the frequency of price adjustment to inflation, but they cannot easily rationalize the existence of many small price changes.

Further, despite a great deal of hard work—by Caballero and Engel [1991] among others—it has not proved possible to mold the (s,S) model into a useful tool. There are two main reasons. First, to avoid the implication that all firms adjust simultaneously, it is necessary to introduce heterogeneity in the initial conditions (prices) or in the demand or cost conditions that firms face. This heterogeneity has been introduced in a manner that requires development of extensive aggregation technology, which is itself unwieldy. Second, the optimality of the simple (s,S) policy depends on assumptions about the driving processes of the economy which are highly restrictive, specifically that micro and macro shocks are continuous time random walks, as well as the absence of any effects of inflation on the demand for money. The aggregate results in the (s,S) literature thus typically require strong restrictions on forcing processes and behavior. These methods also preclude building complete macroeconomic models. For these reasons, business cycle researchers have mainly turned to the time-dependent pricing approach that we will describe in the next section.[7]

The state-dependent pricing approach that we will develop in this paper can capture all six of the facts discussed above, as well as being sufficiently simple that it can be incorporated into a dynamic stochastic general equilibrium model that can be solve in a rapid manner using linear systems methods. After we develop the approach in the next two sections of the paper, we will use it to learn about the consequences of discrete and occasional price setting for various aspects of business cycles.

2. Time-dependent Pricing

We begin by exploring the rational pricing practices of a monopolistically competitive firm that is required to hold its nominal price fixed for an interval of random length that is exogenously determined. This firm is also assumed to satisfy all demand at the posted price, which is the conventional assumption in sticky price models. New Keynesian macroeconomists have stressed that as long as price exceeds marginal cost, a positive response of quantity to demand will be an optimal policy for a monopolistic competitor with a fixed price. Initially, we will focus on expositing the adjustment structure and we then describe the nature of dynamically optimal pricing policies.

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[7]Two natural questions that arise when one looks at Figure 1 are as follows. What type of economic activity would not be subject to the discrete and infrequent adjustment at the microeconomic level? In what sense are sticky prices special, relative to these other categories of economic activity? King and Thomas [1996] provide a detailed discussion of the continuum economy strategy that we use in this paper, together with a set of examples from labor economics, suggesting that it can be applied in many other contexts. It is not clear to us that prices are indeed special in terms of microeconomic stickiness; Hamermesh [1989] argues that employment is discretely and infrequently adjusted at the firm level but that even relatively coarse aggregation mechanisms (less than 10 firms) produce series that one might plausibly model as smooth.

2.1. Adjustment structure

Shortly after a firm enters a period of our discrete time setup, there is a realization of a random "signal" that determines whether it will be able to adjust its price. We will let α_i be the conditional probability that a firm whose price has been fixed for $i-1$ periods will be allowed to adjust its price in the ith period. Correspondingly, we will define η_i as the probability of nonadjustment, $\eta_i = 1-\alpha_i$. We assume that there is a maximum time period, J, at which adjustment takes place for certain.

The adjustment process is displayed in Figure 2: it shows the pattern of flows of firms of each type within each period of the discrete time structure and the manner in which these flows alter the stock of firms of each type within the next period. At the beginning of a given period of time t, a fraction of firms θ_{jt} has not adjusted its price for j periods, for j=1,2,...J. Subsequently, a fraction α_j of each type of firms receives the adjustment signal and a fraction of firms η_j receives the nonadjustment signal. There is thus a fraction of firms, equal to $\sum_{i=1}^{J} \alpha_i \theta_{it}$, which adjusts its price within period t. Comparably, there is a fraction of firms $\eta_j \theta_{jt}$, in each category $j = 1, 2, ...J$, which continues to charge the nominal price set j periods ago.

Thus, the fractions of firms are governed by a system of linear difference equations:

$$\theta_{j+1,t+1} = \eta_j \theta_{jt} \text{ for } j = 1, 2, ...J-1 \tag{2.1}$$

$$\theta_{1,t+1} = \sum_{j=1}^{J} \eta_j \theta_{jt} \tag{2.2}$$

It is easy to calculate stationary values of the θ (the stationary distribution of price-setters in terms of duration of price fixity),

$$\theta_j = \frac{\varphi_j}{\sum_{h=1}^{J} \varphi_h} \text{ with } \varphi_j = [\eta_{j-1} * \eta_{j-2} * .... * \eta_1 * 1]. \tag{2.3}$$

The Calvo [1983] version of this time-dependent price adjustment specification is that $\alpha_j = \alpha$ and that $J \to \infty$. Under this assumption, it follows that $\varphi_j = \eta^{j-1}$ and that $\theta_j = (1-\eta)\eta^{j-1}$. This case will form a benchmark for some of our later discussion.

With stationary probabilities, the time-dependent pricing structure makes it easy to describe the evolution of firms through time and easy to create aggregates.

For example, if we let P^*_{t-h} be the nominal price that was set by all adjusting firms h periods ago, then a fixed weight price index could be calculated as

$$[\sum_{h=1}^{J-1}(\alpha_h\theta_h)]P^*_t + \sum_{h=1}^{J-1}(\eta_h\theta_h)\ P^*_{t-h}$$

since fraction $\eta_h\theta_h$ of firms is stuck with prices that they set prices h periods ago.

2.2. Optimal price setting with exogenous adjustment

The optimal pricing policy of a firm that is a monopolistic competitor can be developed as follows. We define $V_h(P^*_{t-h}, S_t)$ as the nominal market value of a firm that set its price h periods ago and $\Delta(S_{t+h}, S_t)$ as the nominal discount factor for contingent cash flows in state S at date t+h. S is the state of the aggregate economy, which includes all the factors governing the general price level P. We define $\Pi_h(P^*_{t-h}, S_t)$ as the flow profits accruing to a firm that last adjusted its price h periods ago and set the price P^*_{t-h}.

2.2.1. Firms that are not adjusting

For firms that have not received the price adjustment signal, there is a dynamic programming recursion of the form,

$$\begin{aligned} V_h(P^*_{t-h}, S_t) &= \{\Pi_h(P^*_{t-h}, S_t) \\ &+\alpha_{h+1}E[\Delta(S_{t+1}, S_t)V_0(S_{t+1})]|S_t \\ &+\eta_{h+1}E[\Delta(S_{t+1}, S_t)V_{h+1}(P^*_{t-h}, S_{t+1})]|S_t\} \end{aligned} \tag{2.4}$$

where the customary max operation is omitted because we are specializing our discussion to the case in which there are no decisions that the firm must make other than price setting. This specification reflects the fact that the firm will adjust next period with probability α_{h+1}, in which case it will have nominal value $V_0(S_{t+1})|S_t$, and will be unable to adjust with probability η_{h+1}, in which case it will have nominal value $V_{h+1}(P^*_{t+1-h}, S_{t+1})|S_t$. Notice that we are assuming that there is no effect of the length of the interval of price fixity on the value of the firm if it adjust. The value recursions imply an "envelope theorem" condition for each vintage,

$$\frac{\partial V_h(P^*_{t-h}, S_t)}{\partial P^*_{t-h}} = \{\frac{\partial \Pi_h(P^*_{t-h}, S_t)}{\partial P^*_{t-h}} \tag{2.5}$$

$$+\eta_{h+1} E[\Delta(S_{t+1}, S_t)\frac{\partial V_{h+1}(P^*_{t+1-h}, S_{t+1})}{\partial P^*_{t-h}}]|S_t,$$

which will play an important role below.

2.2.2. Firms that are adjusting

For a firm that is capable of adjusting its price, we have the dynamic programming recursion,

$$V_0(S_t) = \max_{P_t^*}\{\Pi_0(P_t^*, S_t)$$

$$+\alpha_1 E[\Delta(S_{t+1}, S_t)V_0(S_{t+1})]|S_t + \eta_1 E[\Delta(S_{t+1}, S_t)V_1(P_t^*, S_{t+1})]|S_t\}. \tag{2.6}$$

Assuming differentiability of the value functions V_h for $h = 1, ...J-1$, it follows that efficient price setting satisfies the first-order condition

$$0 = \frac{\partial \Pi_0(P_t^*, S_t)}{\partial P_t^*} + \eta_1 E[\Delta(S_{t+1}, S_t)\frac{\partial V_1(P_t^*, S_{t+1})}{\partial P_t^*}]|S_t \tag{2.7}$$

Using updated versions of the envelope theorem conditions together with this first-order condition, it follows that we can write

$$0 = \sum_{h=0}^{J-1} E\{\varphi_h[\Delta(S_{t+h}, S_t)\frac{\partial \Pi_h(P_t^*, S_{t+h})}{\partial P_t^*}]|S_t \tag{2.8}$$

This condition indicates that the firm must be equating probability weighted discounted marginal costs and revenues from a change in price.

2.2.3. A dynamic markup equation

We can combine the expressions of the model to generate an equation that is a "dynamic markup equation", which indicates how firms adjust their prices in response to interest rates and to their expectations about future costs and demand.

Assuming that there is a constant elasticity demand for its product, with $-\varepsilon$ being the elasticity of demand, it follows that the optimal price satisfies

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{h=0}^{J-1} E\{\varphi_h[\Delta(S_{t+h}, S_t)\Psi(S_{t+h})d(P_t^*, S_{t+h})]|S_t}{\sum_{h=0}^{J-1} E\{\varphi_h[\Delta(S_{t+h}, S_t)d(P_t^*, S_{t+h})]|S_t}. \quad (2.9)$$

In this expression, $\Psi(S_{t+h})$ is marginal cost at date t+h and $d(P_t^*, S_{t+h}) = (P_t^*/P_{t+h})^{-\varepsilon} y_{t+h}$ is the level of real demand for the firm's product at date $t+h$, given its choice of price today and the future values of the price level and aggregate demand. (Notice that the terms involving P_t^* can be dropped from the right hand side of the expression, since they enter in the numerator and denominator in the same fashion). If adjustment were immediate or if costs were constant over time, the price would be set as a simple markup over cost, $P_t^* = \mu\Psi_t$ with $\mu = \frac{\varepsilon}{\varepsilon-1}$.

Accordingly, within the time-dependent pricing setup, there is a distribution of firms in terms of prices and markups. King and Wolman [1996] follow Calvo [1983] in studying the special case of this price adjustment specification in which $\varphi_h = \eta^h$. They show that a higher rate of inflation increases the *marginal* markup, i.e. that which an adjusting firm chooses. For small inflations, they show that there is little effect on the *average* markup that is charged by all firms in the economy (including those that are adjusting and those that are not). At higher rates of inflation (those in excess of 5%), by contrast, further increases in expected inflation raise the marginal markup sufficiently so that the average markup actually rises. However, the time-dependent price adjustment structure makes it impossible to examine the effects of inflation on the frequency of price adjustment, since that is specified exogenously.

3. State-dependent pricing

For the purpose of studying the effects of steady state inflation and the dynamics of business cycles, we now develop a closely related model in which the frequency of price adjustment is endogenous. This model will imply that the fractions of firms in the various "bins" in Figure 2 evolves through time and becomes part of the state of the economy. Within this setting, for example, an increase in the average inflation rate will mean there will be higher values of α_j for every j and potentially a smaller value of J. Inflation will make it more likely that an individual firm will find it worthwhile to pay the (fixed) costs of adjusting its price. However, our state-dependent pricing model will also have implications for the dynamics of business cycles, as similar trade-offs emerge in response to business cycle developments.

We want to stay as close as possible to the model of the previous section. To produce a similarly tractable pattern of state-dependent adjustment, we need three alterations in the framework above. First, we need to specify the nature of the (fixed) costs of price adjustment that are present in our model. Second, we need to determine the nature of the optimal adjustment decisions of firms which govern the evolution of the fractions α_{jt}. Third, we need to explore the nature of optimal price setting on the part of firms.

3.1. Adjustment costs and adjustment rates

The key alteration of the previous model is that we explicitly allow for heterogeneity among firms in terms of discrete costs of adjustment, although the previous model may be reinterpreted as one for which each firm learns the realization of a random variable that implies either zero or infinite costs of adjustment. In contrast to this reinterpretation, we model the size of these fixed costs as a continuous function of the fraction of firms that are adjusting.

To see how this structure works, let's focus momentarily on a specific firm in bin j. We assume that there is a fixed labor cost of ξ_{jt} hours that must be paid if this firm chooses to adjust its price. Since the nominal commodity cost is $W(S_t)\xi_{jt}$, there will be variations due to changes in real wages and in the general price level. These fixed costs will mean that our individual firm will choose either to adjust or not to adjust, so that individual actions will be discrete.

While the individual decisions are discrete, we assume that—at the level of the jth bin—there is a continuum of firms differentiated by the level of ξ_{jt}: there is a continuous function on the unit interval, $0 \leq \alpha_{jt} \leq 1$, such that the real labor cost of the marginal firm is $\xi_j(\alpha_{jt})$ if the fraction of firms α_{jt} is adjusting. This function is graphed in Figure 3: the key properties of this function are (i) that it originates at the origin (so that there is always some firm with no cost of adjustment) and (ii) that it is continuous and everywhere increasing.

We continue to use the same notation as in the previous section of the paper to denote the value of a firm that has not adjusted price for j periods, $V_j(P^*_{t-j}, S_t)$. For consistency we view this as describing the value of a firm that currently does not incur any fixed cost of adjustment, but will face future adjustment costs. Firms of this type will face a continuum of adjustment costs and it follows that there will be a critical value of α_{jt} such that a firm will just be indifferent between adjusting and not,

$$W_t \xi_j(\alpha_{jt}) = V_0(S_t) - V_j(P^*_{t-j}, S_t), \tag{3.1}$$

or there will be full adjustment (as with the firms in the Jth bin) if

$$W_t \xi_J(1) < V_0(S_t) - V_J(P^*_{t-J}, S_t).$$

Thus (3.1) describes the endogenous determination of the fraction of firms that are adjusting price. Increases in wages reduce the fraction of firms adjusting; increases in the value difference $V_0(S_t) - V_j(P^*_{t-j}, S_t)$ raise the fraction of firms adjusting within the jth bin.

3.2. Values of firms and optimal pricing decisions

We next want to make this pattern of adjustment consistent with the rest of the model developed in the previous section. There are two main issues here.

Uniformity of action levels: We want all firms that adjust to take the same price setting action, i.e., to select the same P^*_t, so that it is necessary for us to carry along only a single price for each "bin" rather than a distribution of prices. For this reason, we assume that firms face a fixed cost that is a serially independent random variable.

Effects of expected future adjustment costs on firm value: We also need to modify the value function recursions above to introduce expected future costs of adjustment. We assume that these costs are entirely born by the firms that undertake the adjustment. Consequently, conditional on adjustment, the expected fixed cost is $\Xi_j(\alpha_{jt}) = [\int_0^{\alpha_{jt}} \xi_j(x)dx]/(\alpha_{jt})$.

The value function recursions are as follows. First, for firms in $h = 1, 2, ..J$,

$$V_h(P^*_{t-h}, S_t) = \{\Pi_h(P^*_{t-h}, S_t)$$

$$+E[\alpha_{h+1,t+1}\Delta(S_{t+1}, S_t)(V_0(S_{t+1}) - W_{t+1}\Xi_1(\alpha_{h+1,t+1}))]|S_t \tag{3.2}$$

$$+E[\eta_{h+1,t+1}[\Delta(S_{t+1}, S_t)V_{h+1}(P^*_{t-h}, S_{t+1})]|S_t\}$$

By the notation, $\alpha_{h+1,t+1}$ and $\eta_{h+1,t+1}$, we mean to indicate that these adjustment rates are functions of the firm's price and the general state of the economy, i.e., $\alpha_{h+1}(P^*_{t-h}, S_{t+1})$ and $\eta_{h+1}(P^*_{t-h}, S_{t+1})$. It is the essence of (3.1) that adjustment is state-dependent in precisely this manner.

For a firm that chooses to adjust its price, we have the dynamic programming recursion,

$$V_0(S_t) = \max_{P_t^*}\{\Pi_0(P_t^*, S_t)$$

$$+E[\alpha_{1,t+1}\Delta(S_{t+1}, S_t)(V_0(S_{t+1}) - W_{t+1}\Xi_1(\alpha_{1,t+1}))]|S_t \tag{3.3}$$

$$+E[\eta_{1,t+1}[\Delta(S_{t+1}, S_t)V_1(P_t^*, S_{t+1})]|S_t\}$$

A nice feature of this model is that the efficiency condition for price-setting is **very** close to that in the time-dependent setup, which is a special case when the adjustment rates are exogenous. The partial derivative of the right hand side with respect to price is:

$$0 = \frac{\partial\Pi_0(P_t^*, S_t)}{\partial P_t^*} + E[\eta_{1,t+1}\Delta(S_{t+1}, S_t)\frac{\partial V_1(P_t^*, S_{t+1})}{\partial P_t^*}]|S_t \tag{3.4}$$

It may appear that there should be additional terms in this expression, which take into account the effect that the price has on the probability of future adjustment. However, these additional terms are zero when we impose the requirement that there is zero value for the marginal adjusting firm in (3.1).[8] This expression can be iterated to reproduce a version of (2.9), with the only modification being that the conditional probabilities of maintaining price stickiness for h periods are now functions of the future state of the economy. In this sense, the time-dependent model is an approximation to the more general state-dependent setup in which there is small variation in the probabilities.

3.3. Dynamics and aggregation

There are now a new set of endogenous state variables for our aggregate model, the fractions of firms that enter each period in the J bins of the economy. These fractions of firms are governed by a system of linear difference equations:

[8] The additional terms are as follows,

$$E\{\Delta(S_{t+1}, S_t)[V_0(S_{t+1}) - W_{t+1}\frac{\partial(\alpha_{1,t+1}\Xi_1(\alpha_{1,t+1}))}{\partial\alpha_{1,t+1}} - V_1(P_t^*, S_{t+1})]\frac{\partial\alpha_1(P_t^*, S_{t+1})}{\partial P_t^*}\}|S_t\}$$

from straightforward differentiation. From the definition of Ξ, it follows that $\frac{\partial(\alpha_{1,t+1}\Xi_1(\alpha_{1,t+1}))}{\partial\alpha_{1,t+1}} = \xi(\alpha_{1,t+1})$. It follows that there is no contribution from this area, since (3.1) implies that the bracketed term is zero.

$$\theta_{j+1,t+1} = \eta_{jt}\theta_{jt} \text{ for } j = 1, 2, \ldots J-1 \tag{3.5}$$

$$\theta_{1,t+1} = \sum_{j=1}^{J} \alpha_{jt}\theta_{jt} \tag{3.6}$$

One major difficulty with state-dependent pricing models is that they are very difficult to aggregate, which Blanchard and Fischer [1989] stress as a key reason that there has not been more work on the business cycle implications of these models. Our setup makes aggregation almost as easy as in the time-dependent pricing model framework of the previous section, because our model also has the implication that there is a finite number of lags and there is a single price set by all adjusting firms. The fixed weight price index described above then is

$$[\sum_{h=1}^{J-1}(\alpha_{ht}\theta_{ht})]P_t^* + \sum_{h=1}^{J-1}(\eta_{ht}\theta_{ht})\ P_{t-h}^*,$$

since fraction $\eta_{ht}\theta_{ht}$ of firms chooses to maintain the fixed price set h periods ago. However, in line with the underlying monopolistic competition structure of the model, we employ an alternative price level aggregate in our analysis below,

$$P_t = \{[\sum_{h=1}^{J-1}(\alpha_{ht}\theta_{ht})]P_t^{*(1-\varepsilon)} + \sum_{h=1}^{J-1}(\eta_{ht}\theta_{ht})(P_{t-h}^*)^{(1-\varepsilon)}\}^{(\frac{1}{1-\varepsilon})}. \tag{3.7}$$

in which, as above, $-\varepsilon$ is the (constant) elasticity of demand for each firm's product. Like the simpler fixed weight price index, this price level is affected by both the prices set by adjusting firms and the fractions of firms in various "bins."[9]

4. The rest of the model

We comment only very briefly on the structure of the rest of the model, since it has been extensively discussed in King and Wolman [1996].[10].

[9]This price index is a natural outcome of the Dixit-Stiglitz preferences commonly used in this class of models. For more detail, see Blanchard and Kiyotaki [19xx].

[10]King and Watson [1995c] provide a very detailed discussion of the real side of the model, but they use a specification of time dependent pricing that is an *ad hoc* approximation to the rule used by King and Wolman [1996].

Households: The households in our model are infinitely lived representative agents, selecting contingency plans for consumption, labor supply and real balances. These plans are chosen to maximize the expected value of a discounted, time separable utility function subject to an intertemporal budget constraint, a time constraint, and a "shopping time" technology that specifies that money reduces the time that one would otherwise need to devote to transactions activity.

Firms: The firms in our households choose contingency plans for labor demand, investment, and prices so as to maximize their expected discounted value.

Markets: The labor market in our model is perfectly competitive and the commodity market is imperfectly competitive. A full set of markets in state contingent claims is presumed to exist.

Government: There is no fiscal policy in our model economy. The decisions of the monetary authority are described by a policy rule. We consider some alternative policy rules in the discussion below.

5. The steady state and calibration

The model economy that we are studying has a nonstochastic steady state that is relatively complex when compared to other models in the literature. We need to understand this steady state because we are going to study the near steady-state dynamics of the model using linear approximation methods. In this section, we briefly outline the nature of the steady state computations that we have undertaken. We then discuss other issues of calibration. Finally, we make some comparisons across inflationary steady states of our model.

5.1. Computing the stationary distribution

The steady state of our model economy involves a stationary distribution of firms in terms of the time since the date of last price adjustment. For given value of the wage rate, the nominal interest rate, and the adjustment cost function $\xi(\alpha)$, we can describe how the steady state will operate and how it must be computed.

There is a fixed point problem in this economy because it contains is what Bertsekas [1976] and Rust [1985] call a "regenerative optimal stopping problem." To begin, the problem is an optimal stopping problem because there is an unknown horizon J (the last bin in Figure 2) which must be determined according to the rule that it is always optimal to pay the costs of fully adjusting. As discussed above, this requires that

$$W\xi_J(1) < \widehat{V_0} - V_J(P^*). \tag{5.1}$$

In contrast to many optimal stopping problems, this stopping point is influenced by the value V_0 that is associated with restarting the process. It is in this sense that it is regenerative.

For arbitrary $\widehat{V_0}$ and P, it is easy to determine optimal stopping time and indeed to then construct (via backward induction) the remainder of the value functions. Taking the future adjustment policy as given (from the prior step in the dynamic programming recursions), it follows that:

$$V_h(P) = \{\Pi_h(P) + \alpha_{h+1}\Delta\gamma_P[\widehat{V_0} - W\Xi_{h+1}(\alpha_{h+1})] + \eta_{h+1}\Delta[V_{h+1}(P)] \tag{5.2}$$

In this latter expression, γ_P is the inflation rate in the steady-state that we are studying, which is introduced when we make the problem stationary, and Δ is the discount factor on one period nominal cash flows. Accordingly, the value function recursions involve a real discount factor $\Delta\gamma_P$. Implicitly, in these expressions, we are treating the price as set at an arbitrary at an earlier date 0.

Given the value functions, it is then direct to compute the optimal policy using

$$W\xi_h(\alpha_h) = \widehat{V_0} - V_h(P) \tag{5.3}$$

simply by "inverting" the ξ function.

Proceeding through the value recursions, we can then determine a maximum discounted profit at the initial date by optimizing over the various values of P to find the value function (V_0) and the policy function (P^*) for a firm which takes the choke value $\widehat{V_0}$ as exogenously specified. We then must find a fixed point, i.e., a value of V_0 that is optimal when $\widehat{V_0} = V_0$. The outcome of this process is a pricing rule (value of P^*) and a set of adjustment fractions that we can use as an ingredient to our study of business cycles. The pricing rule is in the form of a markup over marginal cost that depends on the likelihood of future adjustments. The steady state has basic homogeneity properties. The value functions, wage rate and price are all homogenous of degree one in the price level; the optimal adjustment policy is unaffected by the general level of prices.

5.2. Interaction with the rest of the steady state

We specify a parametric representation of the rest of our steady state as is commonly done in real business cycle models. We can solve for the stationary state of the rest of the model economy in a fairly straight-forward manner, conditional on the outcome of the previous section. As it happens, we must iterate between these two tasks. To see why, recognize that the real wage rate in our model is given by $w = W/P = \widehat{\mu}[a\frac{\partial f(k,n)}{\partial n}]$, where $\widehat{\mu}$ is an exogenously specified value of the steady state markup and $[a\frac{\partial f(k,n)}{\partial n}]$ is the marginal product of labor. Accordingly, the material of section 5.1 may be viewed as determining an optimal markup μ conditional on a hypothesized value of the markup ($\widehat{\mu}$, which enters in the wage rate). Again, we must seek a fixed point.

The computation of the steady state is thus somewhat involved, taking a few minutes on a Pentium PC. This contrasts with the seconds that are typically involved in either solving for the steady state of a typical RBC model or in computing the dynamic outcomes that we'll consider further below.

5.3. The effects of inflation on adjustment frequency

We now use the model economy to provide a sample discussion of the effects of inflation on the pattern of price adjustment. We specify an adjustment cost function of the form

$$\xi = B(\alpha)^b \tag{5.4}$$

with $B = .07$ and $b = 1$. This value of B means that if all firms adjusted fully within the period, then there would be labor costs of 35% of market time (market time is .20 of total time in the economy that we construct). However, because firms choose to adjust only infrequently, there will be much smaller costs in the calibrated steady state.[11]

Figure 4 shows the steady-state distribution of firms by duration of price adjustment in three models. First, there is the result of computing the steady state

[11] Since this specification governs the "marginal" fixed costs and is linear, our model has a form of "quadratic adjustment costs" that may explain why it produces price dynamics somewhat similar to those of Rotemberg [1982], in which the individual firm faces quadratic costs of adjusting prices. However, our model economy is not exactly the same as the quadratic cost-of-changes model, yielding additional state variables that describe the distribution of firms across the "bins" of Figure 2 that make for more complicated dynamics.

of an economy as described above. This economy involves 5% annual inflation: we call it our benchmark model. Second, there is a Calvo model with the same expected duration of price fixity (7.9 quarters).[12] In our benchmark case, the structure of adjustment shown in Figure 4 implies that only .54% of total market labor time is spent in price adjustment: it is thus consistent with the frequently expressed view that small "menu costs" can produce a relatively protracted average pattern of adjustment (as suggested, for example, by Mankiw [1985] and Rotemberg [1987]) . Third, we compute an alternative steady state with 10% inflation under our given price adjustment structure (5.4): we call this our high inflation model. Higher inflation results in more time allocated to price adjustment; 1.05% with 10% inflation. By way of reference, this increase in time cost is smaller than the time cost associated with economizing on transactions costs that result from the same increase in inflation, which King and Wolman [1996] estimate as about one percent of market time.

The main points to be made about this figure are as follows. First, the first panel of the figure shows that the steady state of the benchmark model involves virtually no chance that a firm will adjust its price within the first quarter ($\alpha_1 = .005$) and roughly 63% chance that price fixity will last for a year or more. However, the model also implies that these probabilities rise sharply through time after the first year and the maximum lag is nine quarters. At the same time, as shown in panel B, the steady state also implies that 12.7% of the firms are adjusting each quarter. Second, to have the same expected duration of price fixity, the Calvo model is characterized by very different overall patterns of adjustment. The Calvo model, therefore, appears to be a poor approximation to an economic environment characterized by menu-costs and state-dependent pricing. The more general time-dependent models that we developed above may be more useful approximations, especially if one is primarily concerned with issues involving steady states. Third, there are quantitatively important effects of changing the average rate of inflation on the average pattern of price adjustment. Increasing the inflation rate from 5% to 10% sharply increases the frequency of price adjustment for the parameters that we employ: the expected duration of price fixity drops from 7.9 to 5.6 quarters and the maximum lag drops from 20 to 12.

The choice of these two parameter values, $b = 1$ and $B = .07$, is meant to help us illustrate the nature of the modeling approach. We make no pretense that

[12]This was obtained by choosing a value of η to produce the same expected duration as in the benchmark model.

these parameters are calibrated to match any aspect of actual price dynamics. However, the exercise of this section makes it clear that one could use data on how the steady state pattern of price adjustment depends on the inflation rate to select these parameter values.

6. Implications of state-dependent adjustment

We now explore how our model economy with state-dependent pricing responds to a basic set of monetary policy shocks. We think that these experiments shed light on four important issues. The first issue concerns the size of the price changes in response to various types of innovations in monetary policy. The second issue pertains to the varying degrees of price level sluggishness and output responsiveness associated with different types of price adjustment mechanisms. The third issue involves the type of information that is useful for forecasting the behavior of prices. The fourth issues involves the extent to which optimal pricing is related to other aspects of the macroeconomic model, especially the money demand function.

6.1. Dynamic effects of monetary expansions

To study the dynamic effects of monetary expansions, we have conducted a set of experiments; Figures 5-8 report some summary some summary results on these. We use the convention that the state-dependent model results are given by (o's) and the time-dependent model results are given by (+'s), which we also use later in the paper.

In Figures 5 and 6, we study the effects of a one percent monetary expansion at date $t = 1$: the two panels of Figure 5 make the monetary expansion temporary in alternative ways and the two figures of Figure 6 make the monetary expansion permanent in alternative ways. By looking across these two figures, we can thus determine how the price and output effects of a monetary expansion depends on the nature of the monetary policy rule.

6.1.1. Temporary expansions

Figure 5 shows the effects of a one percent increase in the quantity of money under two alternative scenarios that make the increase temporary. In the "fully transitory" case of Figure 5A, the money stock is increased for one quarter of a year (at date 1) and then returns to its normal level at date 2 and all future periods. In Figure 5B, the money stock is increased by one percent in quarter one and then is slowly brought back to path (the money stock rule is $\log(M)_t = .8 * \log(M_{t-1}) + \varepsilon_t$ so that the effect of $\varepsilon_t = 1$ on $\log(M_t)$ is .8 in quarter 2, .64 in quarter 3 etc.)

In the fully transitory case in Figure 5A, there is essentially no effect of money on the price level. The one percent change in the quantity of money has less than a .01 percent effect on the price level in both the state-dependent and time-dependent models. There is also little difference between the two models in terms of effects on output, the nominal interest rate or on the markup. In Figure 5B, the disturbance is more persistent, but ultimately transitory.[13] The effect on the price level is larger than in Figure 5A, but continues to be small (less than .1 percent) in both the state-dependent and time-dependent cases.

It is noteworthy that when the monetary change is more persistent, there are *larger* output effects at date 1 in the time-dependent model: investment and consumption respond more to the sustained monetary change. Investment responds more dramatically because increased persistence implies future real demand will be higher (a "rational expectations accelerator" effect); consumption responds more dramatically because increased persistence implies that there are changes in income of larger present value (a "permanent income" effect).

However, with increased persistence, we also begin to see a difference between time-dependent and state-dependent pricing cases, which generally works to make output less responsive to demand and prices more responsive to demand.

6.1.2. Permanent increases in the quantity of money

Figure 6 shows the dynamic response of the two model economies to permanent increases in the quantity of money: panel A concerns a one-time change in the quantity of money and panel B concerns a sustained, but ultimately temporary increase in the money growth rate. In this Figure, the benchmark results for the time-dependent model are again given by the +'s and o's are the results for the state-dependent price adjustment.

A once-and-for-all increase in money: There are three basic findings in panel A of Figure 6. The first is a continuation of a finding in Figure 5: increasing the

[13]The total effect on the stock of money is $1 + .8 + .64 + ... = \frac{1}{1-.8} = 5$ so that "mean lag" reasoning would suggest that the effects are approximately equivalent to the money stock being one percent higher for 5 quarters.

persistence of the monetary disturbance all the way to a unit root continues to increase the effect of a monetary shock on date 1 output. By way of reference, the response of output to money is roughly one-for-one in the time-dependent model of Figure 6A, while it was about .2 in panel B of Figure 5. The second is a recurrent finding about how sticky price macroeconomic models with rational expectations respond to permanent monetary shocks: nominal interest rates rise as a result of a monetary expansion. The finding is recurrent because the price level is largely predetermined in the short-run, but will ultimately rise in the long-run. There is thus an increase in expected inflation and a necessary rise in the nominal interest rate. The third lesson concerns the difference between state-dependent and time-dependent models: a major effect of state-dependent pricing is to mitigate the real effects of the monetary expansion. For example, in panel A of Figure 6, we see that there is an impact effect on output of about one-half the size of the time-dependent model. Firms with low real price and consequent high demand to are willing to pay to adjust. Typically, it is the firms that have not adjusted for several quarters who find it most desirable to make the adjustment so that the monetary shock induces a change in the distribution away from its steady state levels.[14]

Transition to a higher path of money. Panel B of Figure 6 displays the effect of a shock to money when the driving process is $\log(M_t) - \log(M_{t-1}) = .67 * [\log(M_{t-1}) - \log(M_{t-2})] + \varepsilon_t$, which is a form suggested by estimates of univariate autoregressive models of M1 money growth over the post-war period. According to this model, a positive one percent shock ($\varepsilon_t = .01$) will raise the level of money by three percent in the long-run. Such a shock thus induces a gradual transition from an initial money supply path to a higher one. The three basic findings of panel A are continued in panel B: (i) given that future money increases by more than current money, there is a larger output effect at the impact date; (ii) nominal interest rates rise rather than fall in response to a monetary injection; and (iii) state-dependent pricing substantially mitigates the effects of money on output.

[14]With state dependent pricing, there is also a recurrent tendency for the price level and real quantities to display oscillatory responses to the new steady-state, i.e., for a period of overshooting to arise. More generally, oscillatory patterns for the adjustment fractions (the α's and the θ's) are a recurrent part of smooth macroeconomic models with discrete individual choices and, in principle, these oscillations can therefore be translated to other variables. We return later to consideration of the extent of variation in the α's.

6.2. Factors affecting the price level

In this section, we explore the factors affecting the general level of prices, when there is a once-and-for-all increase in the general level of prices: the influence of these factors is shown graphically in Figure 7. The central message of this figure is that changing patterns of adjustment are quantitatively very important for the price level, particularly so a short horizons. Our analysis proceeds in two stages.

First, we recall that the price level is given as

$$P_t = \{[\sum_{h=1}^{J-1}(\alpha_{ht}\theta_{ht})]P_t^{*(1-\varepsilon)} + \sum_{h=1}^{J-1}(\eta_{ht}\theta_{ht})(P_{t-h}^*)^{(1-\varepsilon)}\}^{(\frac{1}{1-\varepsilon})}, \tag{6.1}$$

in (3.7) above. Thus, in response to a permanent increase in the quantity of money, there will be an important influence of lagged prices (P_{t-h}^*). However, the extent of this influence depends on the fraction of firms that choose to hold prices fixed: with P_t^* higher than P_{t-h}^*, the price level will increase if more firms choose to adjust. On impact, in panel A of Figure 7, this is precisely what happens: the price level increases by about one-half of its long run increase, with most of it stemming from increases in $\alpha_{h,t=1}$ relative to their steady state levels. More specifically, panel A of Figure 7 shows the decomposition of P_t into components attributable to changing probabilities (the dashed line, representing the effects of variations in θ, η and α) and changes in these probabilities and in the prices chosen by firms (the solid line). In the short-run, shifting weights are dominant. In the long-run, the increases in prices are entirely due to shifts in chosen prices, with the α's, η's and θ's returning to their steady state values. One implication of this decomposition is that empirical distributed lags "price equations" would be subject to important econometric misspecification if they omitted the determinants of the α's and θ's.

Second, we turn to the prices chosen by adjusting firms. In the state-dependent pricing model, this takes the form,

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1}\frac{\sum_{h=0}^{J-1} E\{\varphi_h[\Delta(S_{t+h}, S_t)\psi(S_{t+h})(P(S_{t+h}))^{1+\varepsilon}y(S_{t+h})]|S_t}{\sum_{h=0}^{J-1} E\{\varphi_h[\Delta(S_{t+h}, S_t)(P(S_{t+h}))^{\varepsilon}y(S_{t+h})]|S_t}. \tag{6.2}$$

In this expression, φ_h is the probability of nonadjustment between t and $t+h$, $\varphi_h = \eta_h(S_{t+h})\eta_{h-1}(S_{t+h-1})...\eta_1(S_{t+1})$, $\psi(S_{t+h})$ is real marginal cost at date $t+h$, $P(S_{t+h})$ is the price level at date $t+h$ and $y(S_{t+h})$ is aggregate output at date

$t+h$. The form of (6.2) thus suggests a decomposition of movements in P^* into components attributable to the discount factors applied to future cash flows (including both market discounting (Δ) and probability discounting (φ)) and the factors affecting cash flows: real marginal cost; the price level; and real output.

Figure 7, panel B, produces this decomposition for the case of a once-and-for-all increase in the quantity of money. This diagram shows that in the "long run" of about twenty quarters P^* increases by one per cent: all of this "long run" variation is due to an increase in the general level of prices. However, on impact, matters are substantially different. Increases in real marginal cost raise P^* by about .2% since there is an expansion of real economic activity and a rising real wage is necessary to clear the labor market (this effect is presented with the solid line in the diagram). Changes in the adjustment probabilities push up P^* by an additional 0.1% on impact. This reflects the fact that since many firms adjust on impact, there is a slight decrease in the likelihood of adjustment in the near future, meaning firms set a higher price today. Finally, the rising general price level contributes a full1% upward impetus to P^*: in our calibrated model, this effect operates nearly entirely through nominal marginal cost, although there is also a "demand switching" effect (represented by $P(S_{t+h})^{\epsilon}$) that is present in theory. Similarly, there are effects of market discounting (Δ) and aggregate demand (y) that are present in theory but are not quantitatively important in our calibrated model.

6.3. Price and output dynamics with a constant velocity specification

A notable feature of Figures 5 and 6 is that increased persistence of the monetary driving process raises the responsiveness of both the price level and real output to a monetary shock. This finding is sensitive to the assumed form of the money demand function.

As a result of the "shopping time" model, there is a money demand function which takes the approximate form near our steady state:

$$\log(M_t) = \log(P_t) + \nu \log(c_t) + (1-\nu)\log(w_t) - \vartheta R_t.$$

That is, money demand depends positively on real consumption and the real wage rate, with coefficients summing to unity (this restriction arises from our requirement that there be constant velocity of money constant along a steady state growth path driven by technical progress). The nominal interest rate has a negative effect on the demand for money, as it induces substitution from money into alternative time-consuming activities. Our choices of the money demand parameters are based on a nonlinear shopping time model estimated on long U.S. time series data, as in King and Wolman [1996], and are $\nu = .29$ and $\vartheta = 24$. In terms of interpreting the semi-elasticity, we assume that interest rates are measured as decimals, not percentages, and at quarterly rates. The implied semi-elasticity for an annual, percentage rate, is thus .06, which is not too different from the .10 estimate that Stock and Watson [1993] derive similar long-term U.S. data. It is nevertheless large relative to many estimates of the interest semi-elasticity of the demand for money applicable over the horizons studied here.

An alternative money demand model—utilized in the studies of Caplin and Spulber [1987] and Caplin and Leahy [1991]—is the constant velocity specification:

$$\log(M_t) = \log(P_t) + \log(y_t).$$

Under that alternative specification, there must be trade-off between the response of prices to money and the real output effects of money.

In this subsection, we describe a variant of our model that replaces the shopping time demand for money with the constant velocity specification, while otherwise leaving the rest of the model unaltered. The results are presented in Figure 8: panel A shows the response to a purely temporary monetary increase and panel B shows the response to a once-and-for-all increase in the quantity of money. There are three features that are worth stressing. First, an increase in the quantity of money has a smaller impact effect under state-dependent pricing than under time-dependent pricing, in line with our earlier findings in Figures 5 and 6. Second, increased persistence of money raises the responsiveness of prices and thus cuts the responsiveness of output to the monetary shock. Third, the permanent monetary disturbance raises the nominal interest rate, in line with our earlier results in Figures 5 and 6.

7. Alternative monetary policy rules

In this brief section, we discuss how state-dependent pricing is important for the consequences of monetary and real disturbances under alternative policy rules. While we treat it only briefly in the current paper, we think that it is of substantial practical interest that the choice of the policy rule can have an important influence on the structure of price adjustment.

7.1. Interest rate shocks

Some monetary economists have argued that Federal Reserve policy is better modeled as an interest rate rule rather than a money stock rule (see, for example, Bernanke and Blinder [1992], Goodfriend [1987] and Sims [1989]). Our state-dependent pricing framework is capable of modeling macroeconomic activity under such interest rate rules; we assume that there is a mild, positive dependence of the interest rate on the price level so that there is a unique stable equilibrium.[15]

Figure 9 shows the dynamic response of two model economies to a persistent, but ultimately transitory, policy-induced variation in the nominal interest rate. In this Figure, the benchmark results are given by the +'s and o's are the results for the state-dependent price adjustment.

The major effect of state-dependent pricing is again to mitigate the real effects of the expansionary monetary disturbance (a decline in the nominal interest rate) in line with our earlier findings. Looking again at the output responses in panel A of Figure 9, we see that there is an impact effect on output of about one-half the size of that in the corresponding time-dependent model. We also again see some tendency for oscillatory dynamics in real and nominal variables.

7.2. Targeting the price level

In the Calvo version of the time-dependent model of price adjustment, King and Wolman [1996] study the effects of conducting monetary policy so as to target a path for the price level. In this subsection, we assume that there is a permanent productivity improvement under a policy rule that adjusts the money stock so that there is a fixed path for the price level. Figure 10 shows the consequences of a one percentage point permanent increase in productivity for macroeconomic activity when there is a P rule in place. There is an equivalence of three sets of real outcomes: those under flexible prices (a monopolistically competitive RBC model), those under time-dependent prices and those under state-dependent prices.[16] As

[15] The interest rate rule is $R_t - R = f(\log(P_t) - \log(P)) + x_t$ with $f = .1$. The analysis of Kerr and King [1995] suggests that any $f > 0$ will produce a unique equilibrium in many macroeconomic models. Our chosen value of f means that the monetary authority will raise the nominal interest rate by 40 basis points if the price level is one percent above its target level. We choose a relatively small, positive value of f to achieve determinacy while making a great deal of the response of interest rates exogenous at short horizons.

[16] This equivalence is not exact, but is accurate to the extent that one cannot distinguish visually between the outcomes when graphed as in Figure 10.

stressed by King and Wolman [1996], by targeting the path of the price level, the central bank makes the quantity of money move in ways that eliminate a potential demand-deficiency that would occur if an M rule was alternatively used.[17]The results of this section thus reinforce the previous conclusions concerning the desirability of targeting the path of the price level.

An novel element of the price level targeting rule is that it makes state and time-dependent pricing equivalent.. With no change in the inflation rate, there is little incentive for individual agents to change the time pattern of adjustment. This contrasts sharply with the differences documented for other policy rules above.

8. Changing the rate of inflation

We now use our framework to explore the implications of permanently increasing the rate of inflation. We take as background to our analysis the results of comparable experiments in King and Wolman [1996, section 6.2] that concern the effects of such permanent changes in inflation within the Calvo version of the time-dependent model, although we do not explicitly reproduce those results here. The findings of that earlier analysis and those of Buiter and Miller [1985] and Ball [1994] were that disinflations introduced by a decline in the money growth rate could cause large recessions, but that a disinflation undertaken with an initial monetary accommodation could be costless.

8.1. A permanent increase in money growth

Figure 11 shows the effect of increasing the money growth rate by two percentage points, so that inflation must also by two percent in the long run. In both the time-dependent model and the state-dependent model, there is a large initial increase in the level of real economic activity, although the initial "boom" is substantially smaller in the model with state-dependent pricing (the peak effect on output is about 12 percent relative to trend with time-dependent pricing, while it is about 9 percent with state-dependent pricing). Inflation must temporarily exceed the

[17]In our setting, an M rule would lead to an initial contraction of labor input in response to a productivity disturbance, although the magnitude of this decline would be somewhat smaller with state dependent rather than time dependent pricing.

long run level, as the economy transits to a new lower level of real balances in the new steady-state situation.

These findings are broadly in line with the results of similar experiments in King and Wolman [1996], with the Calvo form of time-dependent pricing, which in turn are similar to the earlier findings of Buiter and Miller [1985] and Ball [1994].

8.2. A permanent increase in inflation

Figure 12 shows that the effects of unexpected, permanent increase in the inflation rate from 5% to 7% per annum. As shown in panel B, this requires a complicated pattern of money stock adjustment as the central bank accommodates the time-varying demand for money that arises as a result of this shock. Notably, in contrast to earlier experiments with the Calvo form of time-dependent pricing, there is a temporary expansion of real economic activity (a peak effect of about 4% on output, which is significantly smaller than that which arises with the sustained monetary in Figure 11). This result can be traced to the fact that our time-dependent model assigns a small marginal probability of adjustment to firms that have recently adjusted price and large marginal probabilities to who have not recently adjusted price. Accordingly, to increase the inflation rate, the average markup of price over marginal cost must fall, resulting in an expansion of economic activity.

However, the initial expansion is eliminated by the introduction of state-dependent pricing. Essentially, firms with a lengthy interval since their last price adjustment choose to adjust in response to the change in inflation, so that the average markup and real activity respond minimally to the policy shift.

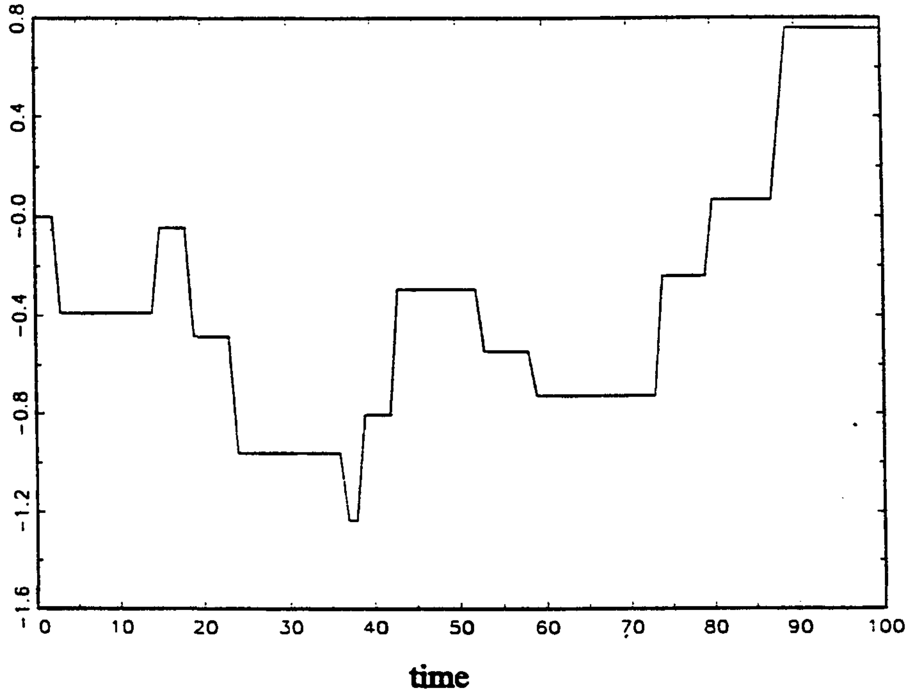
9. Summary and conclusions

In this paper, we derived a tractable representation of state-dependent pricing, which we used to explore the dynamics of business cycles. It is flexible enough that we were able to study the impact of real and nominal shocks under a wide variety of alternative monetary policies and general equilibrium specifications. More specifically, we illustrated the use of this approach with a range of "quantitative theory" experiments, contrasting it to a time-dependent price adjustment mechanism with the same steady state properties. Based on this comparison, we conclude that there is an important potential difference between time and state-dependent pricing: the average (steady state) pattern of price rigidity may be a poor description of the marginal pattern of price rigidity in response to specific shocks.

Our initial set of experiments with this framework also highlighted the role that expectations about the future play in the pace and pattern of price adjustment. Under some scenarios, we found that there can be little real effect of changes in inflation if there are state-dependent prices. Thus, in our future research, we plan an analysis of disinflations that are viewed as imperfectly credible by private agents, who must learn about the true state of the monetary authority's objectives by studying a path of money growth outcomes.

Figure 1. Representative price dynamics

A. Individual price dynamics



B. Individual and aggregate price dynamics

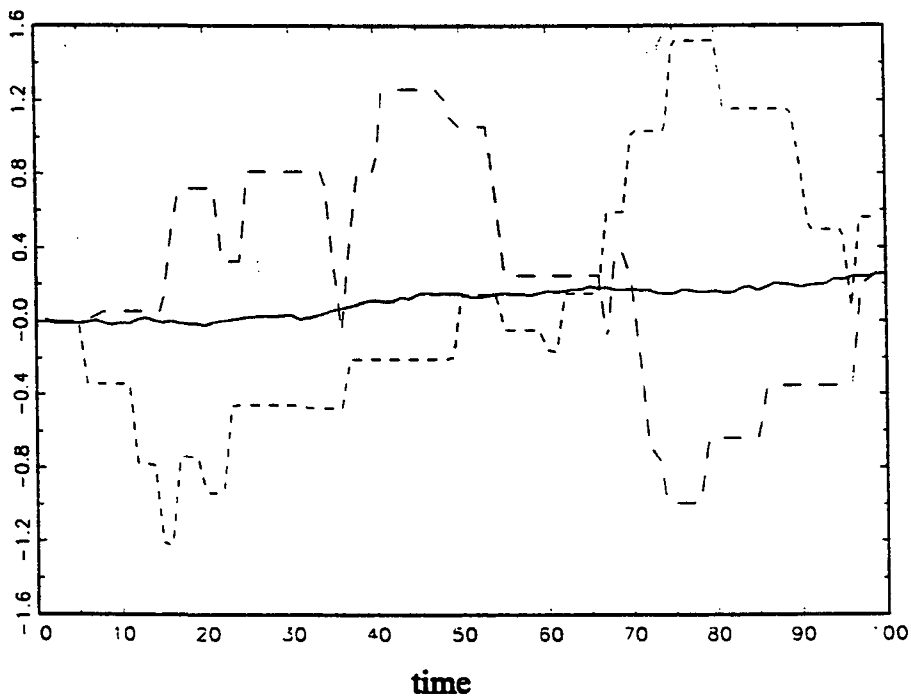


Figure 2. Evolution of "vintages" of price-setters

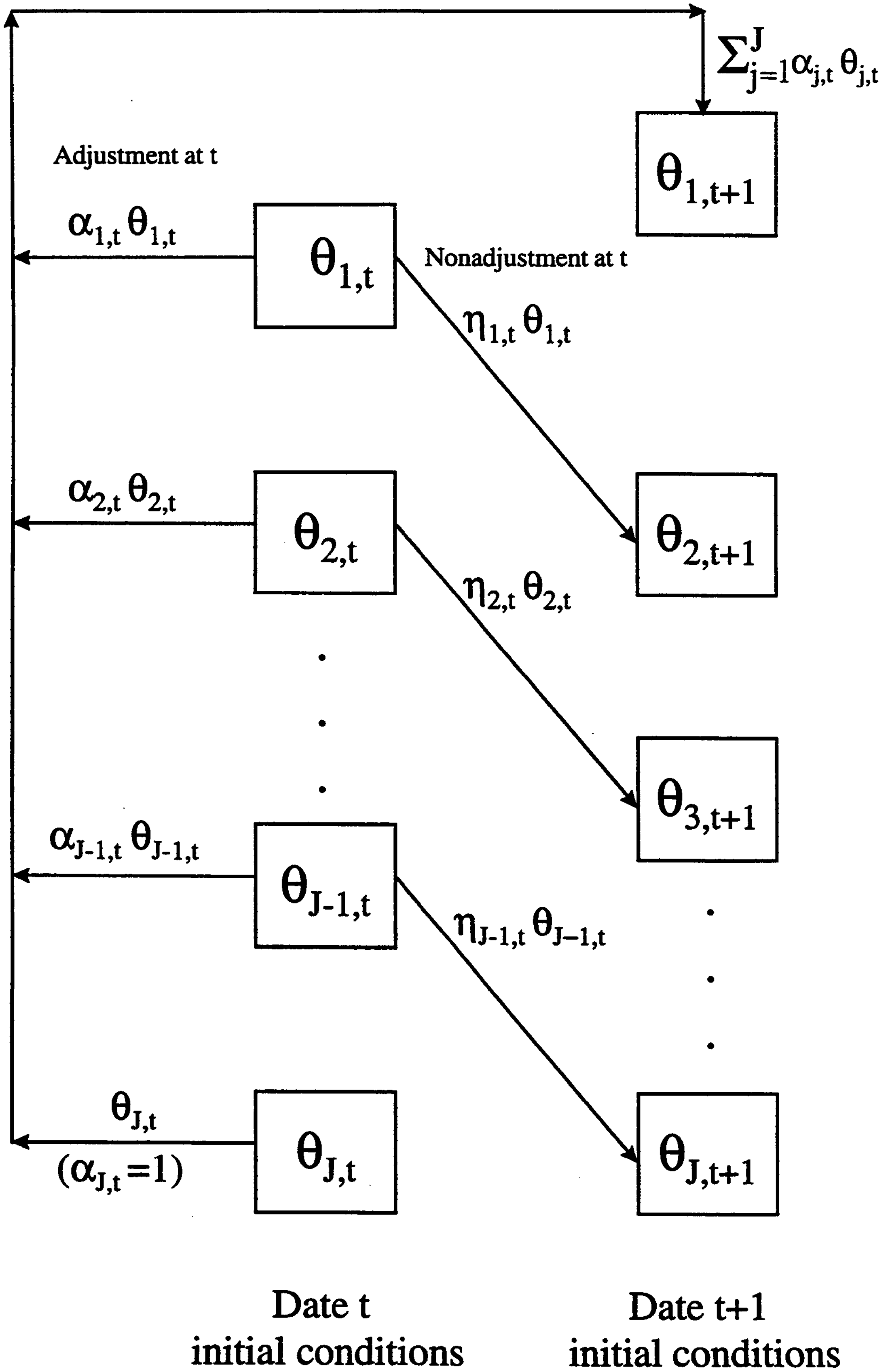


Figure 3. Determination of the marginal firm

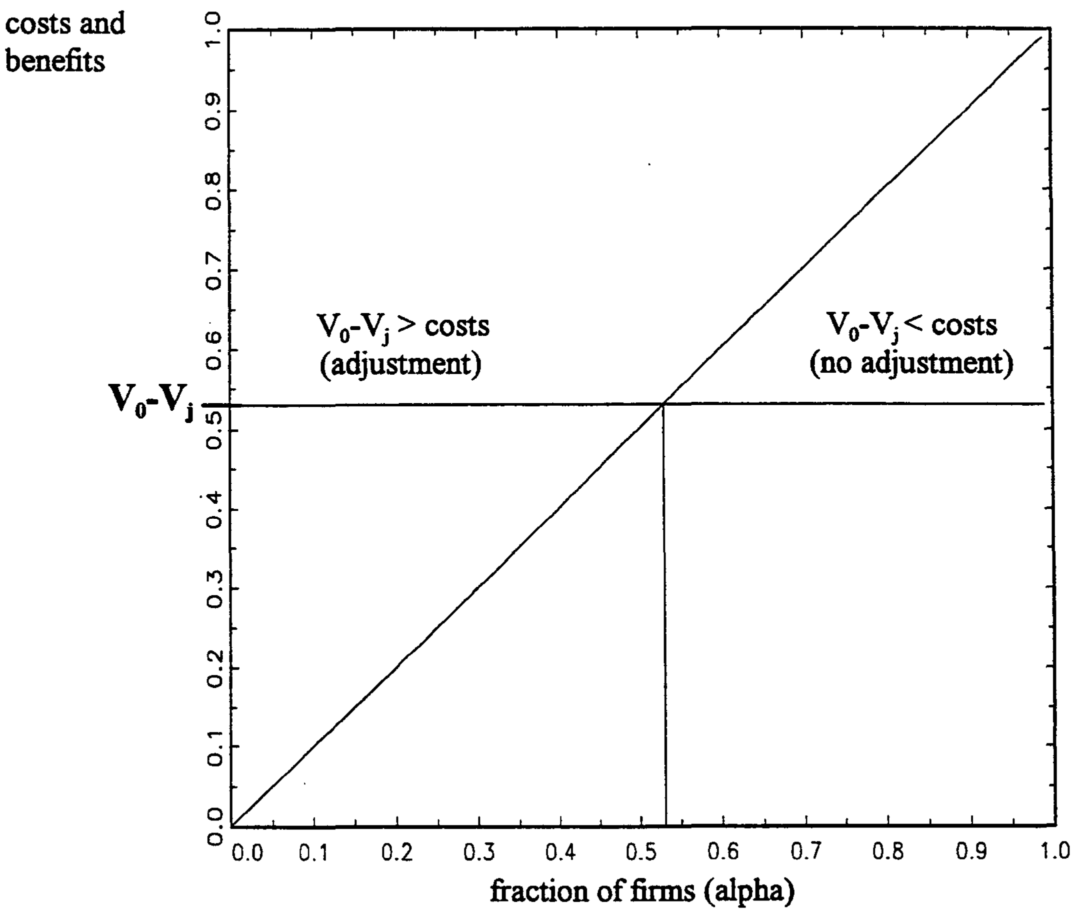
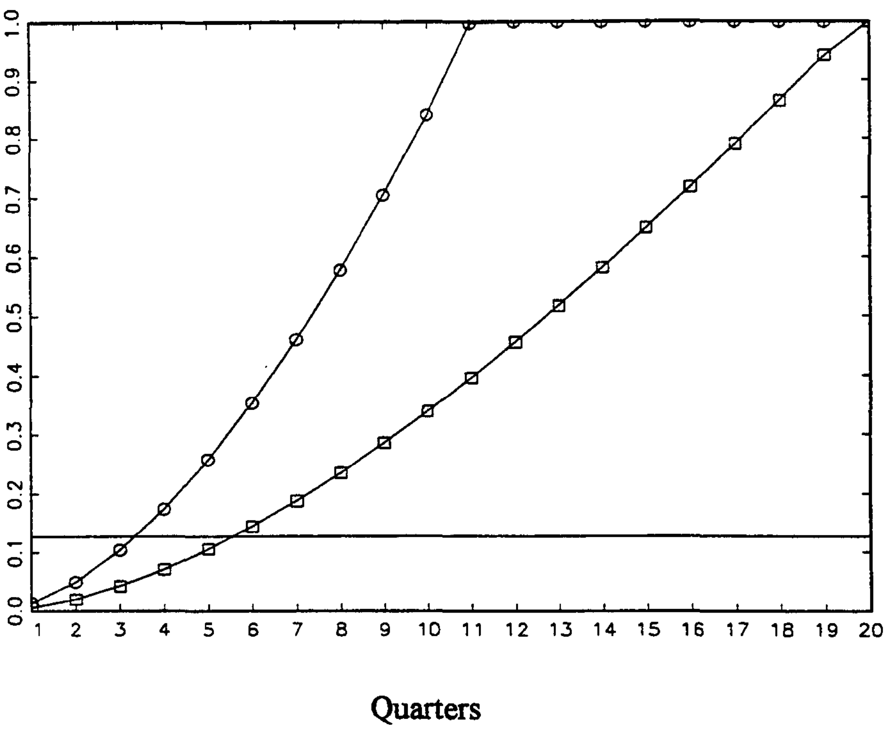


Figure 4. Steady State Distribution of Firms
Benchmark (□) 10% inflation (o) Calvo (-)

A. Conditional Adjustment Probabilities



B. Fraction of Firms at Each P^*_{t-j}

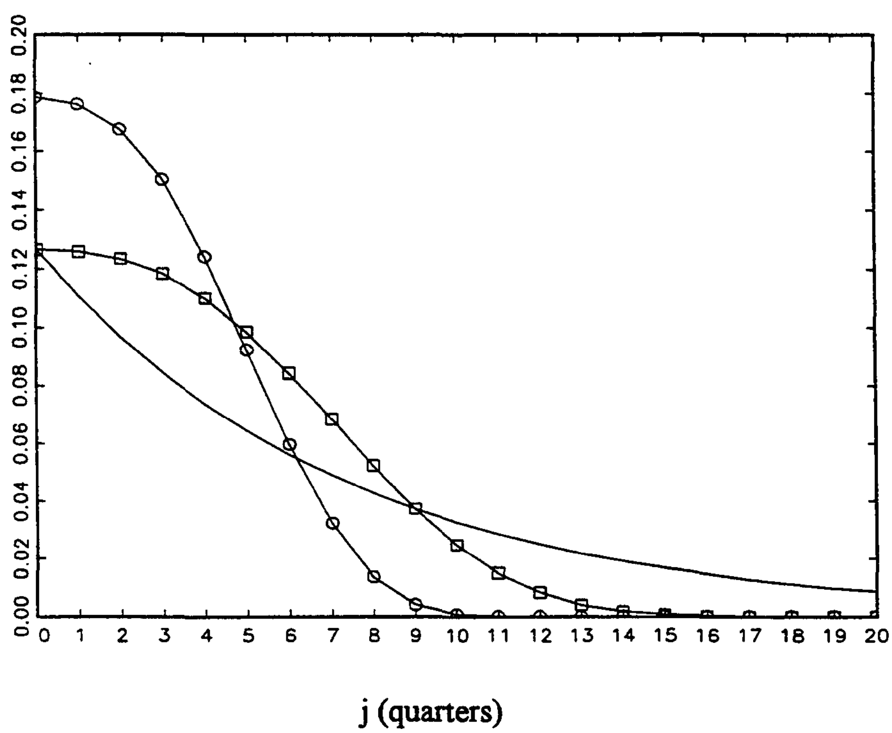


Figure 5. Temporary Increase in the Quantity of Money
Pricing: State Dependent (o) Time Dependent (+)

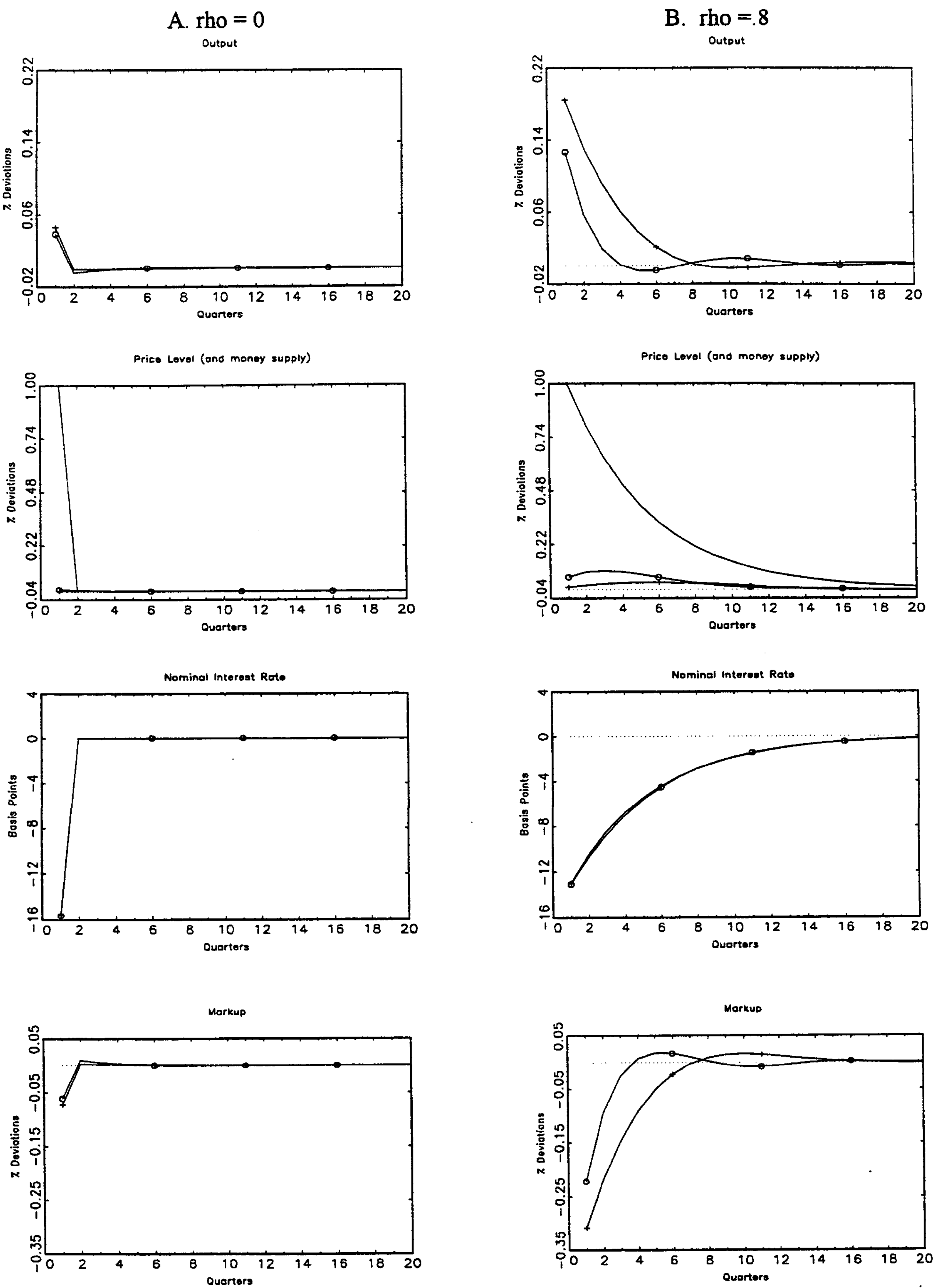


Figure 6. Integrated Driving Processes for M

Pricing: State Dependent (o) Time Dependent (+)

A. random walk M shock

B. serially correlated money growth shock

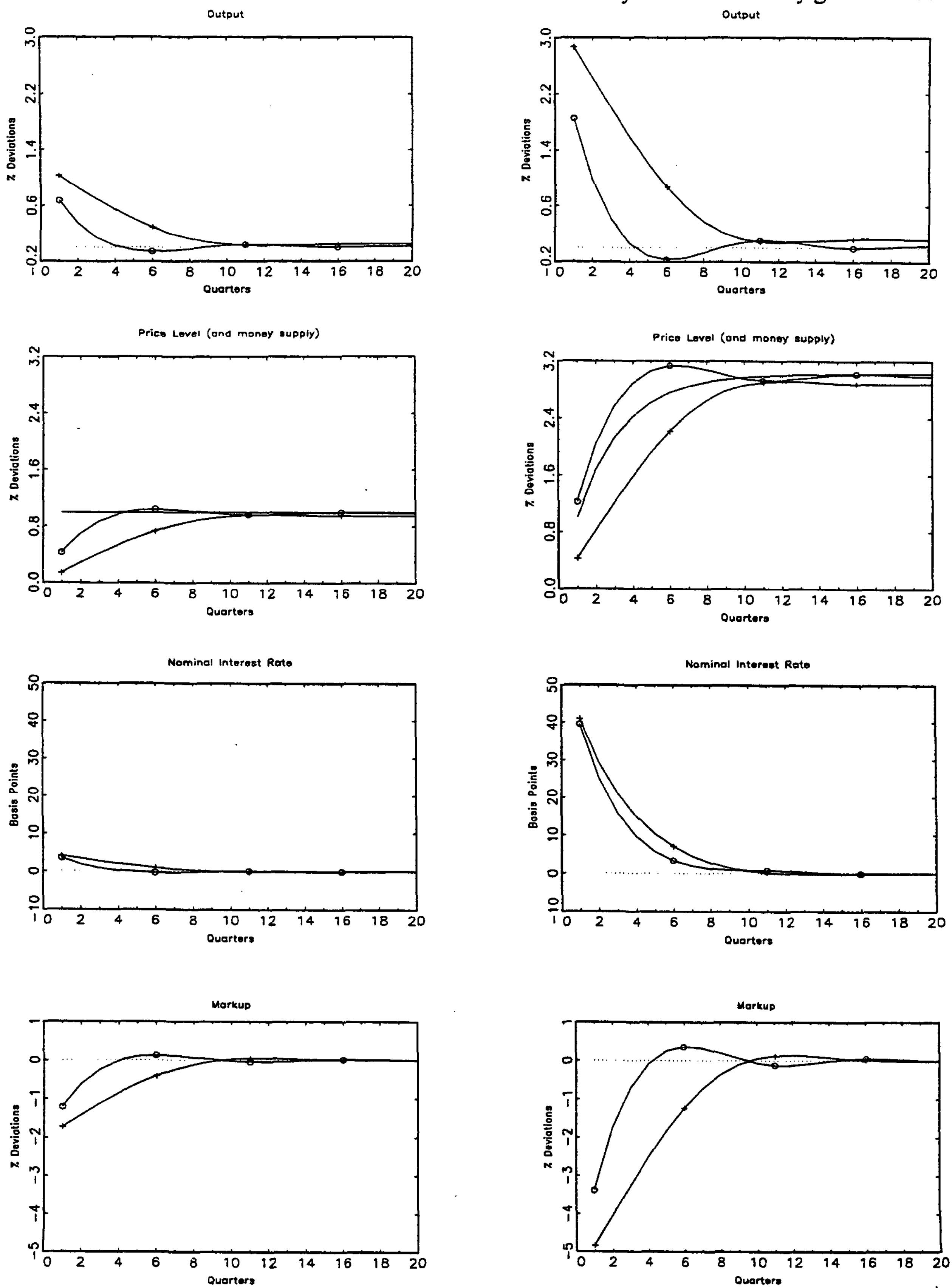
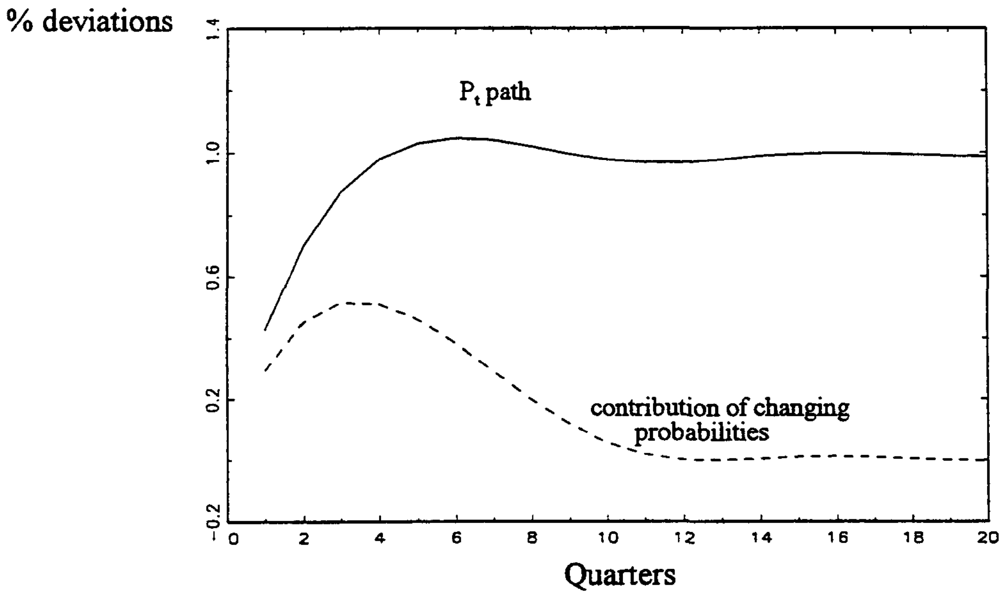


Figure 7. Price Level Decompositions

A. Decomposition of P_t



B. Decomposition of P^*_t

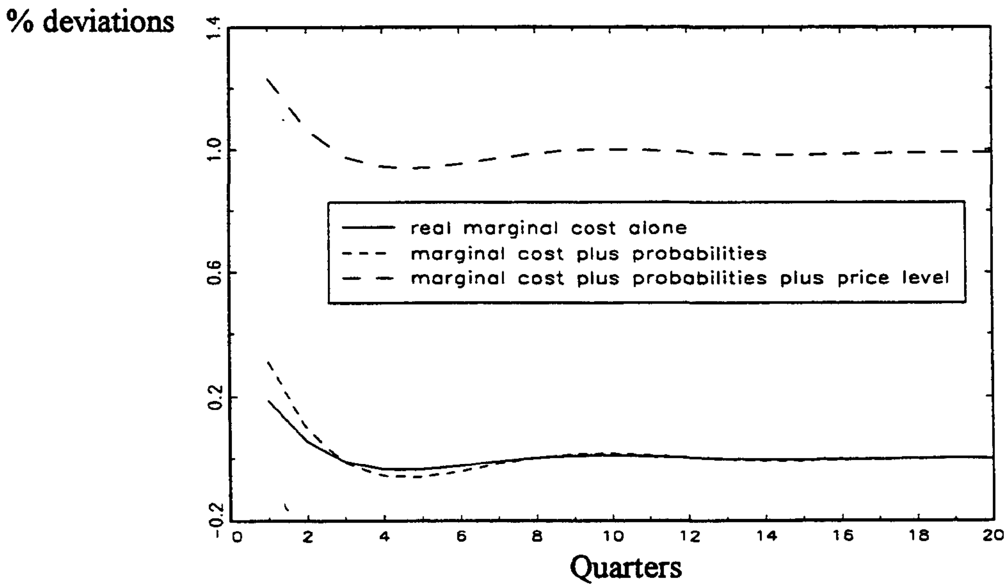


Figure 8. Interest Inelastic Money Demand (M/P=Y)
Pricing: State Dependent (o) Time Dependent (+)

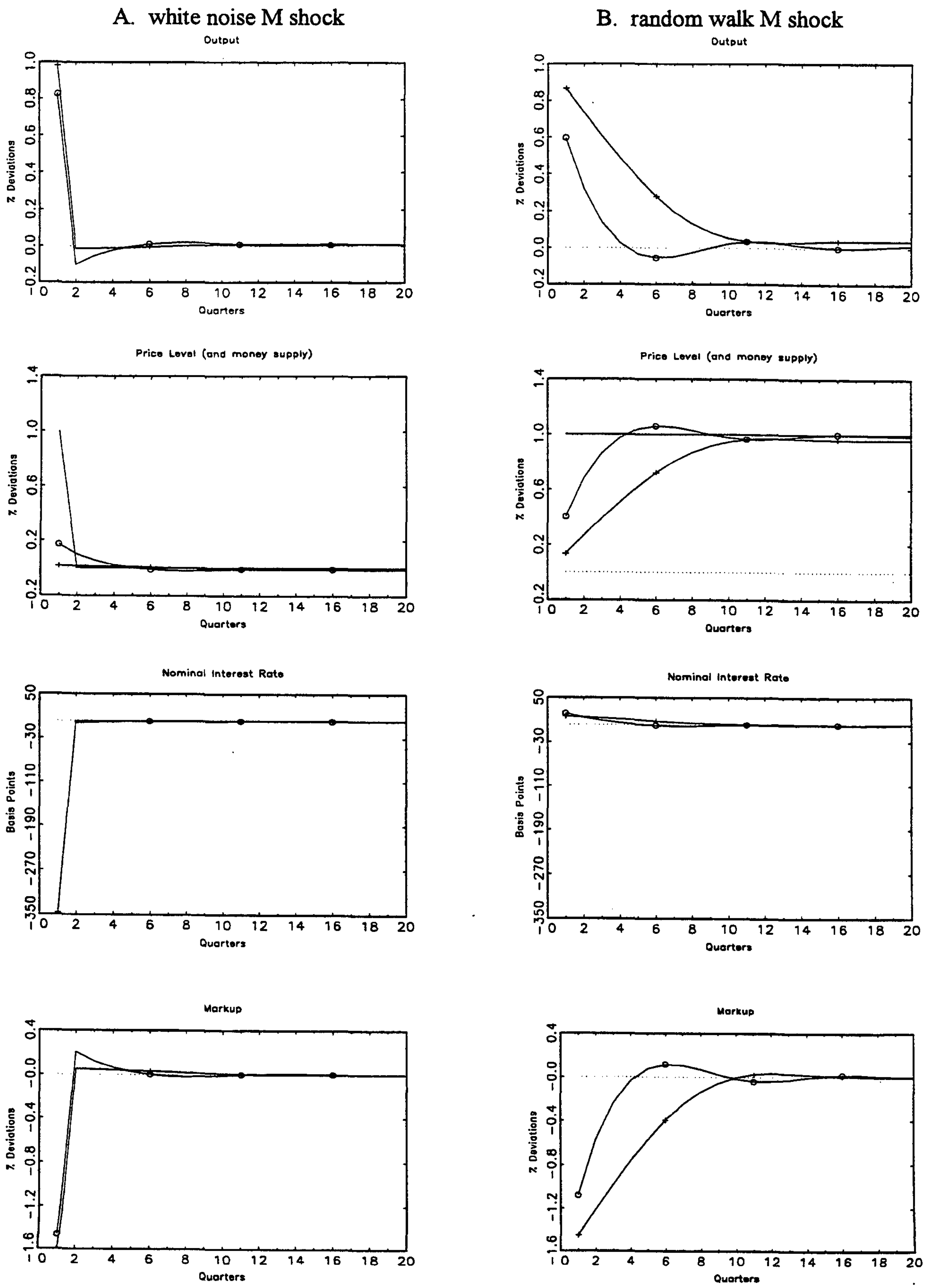


Figure 9. Persistent Interest Rate Shock (rho=.9)
(Weak Feedback Rule)
Pricing: State Dependent (o) Time Dependent (+)

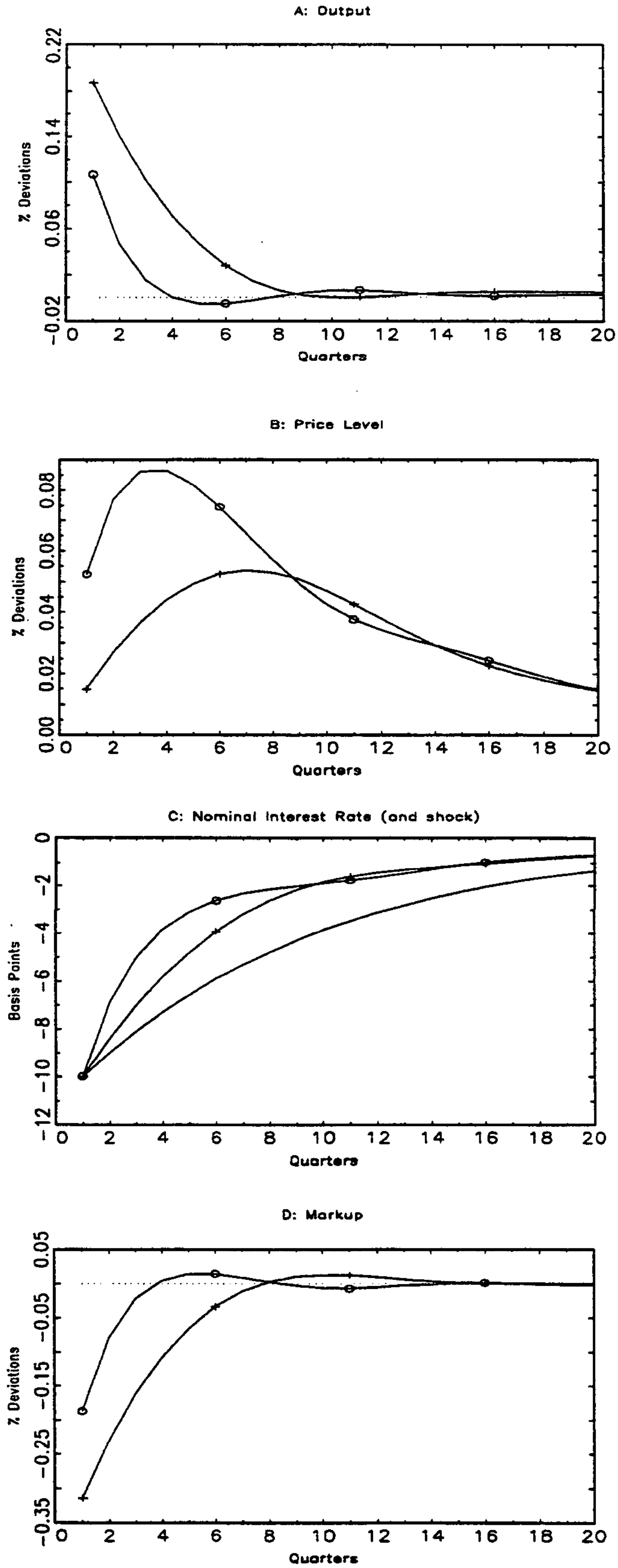


Figure 10. Permanent Productivity Shock
Price Level Targeting
Pricing: State Dependent (o) Time Dependent (+)

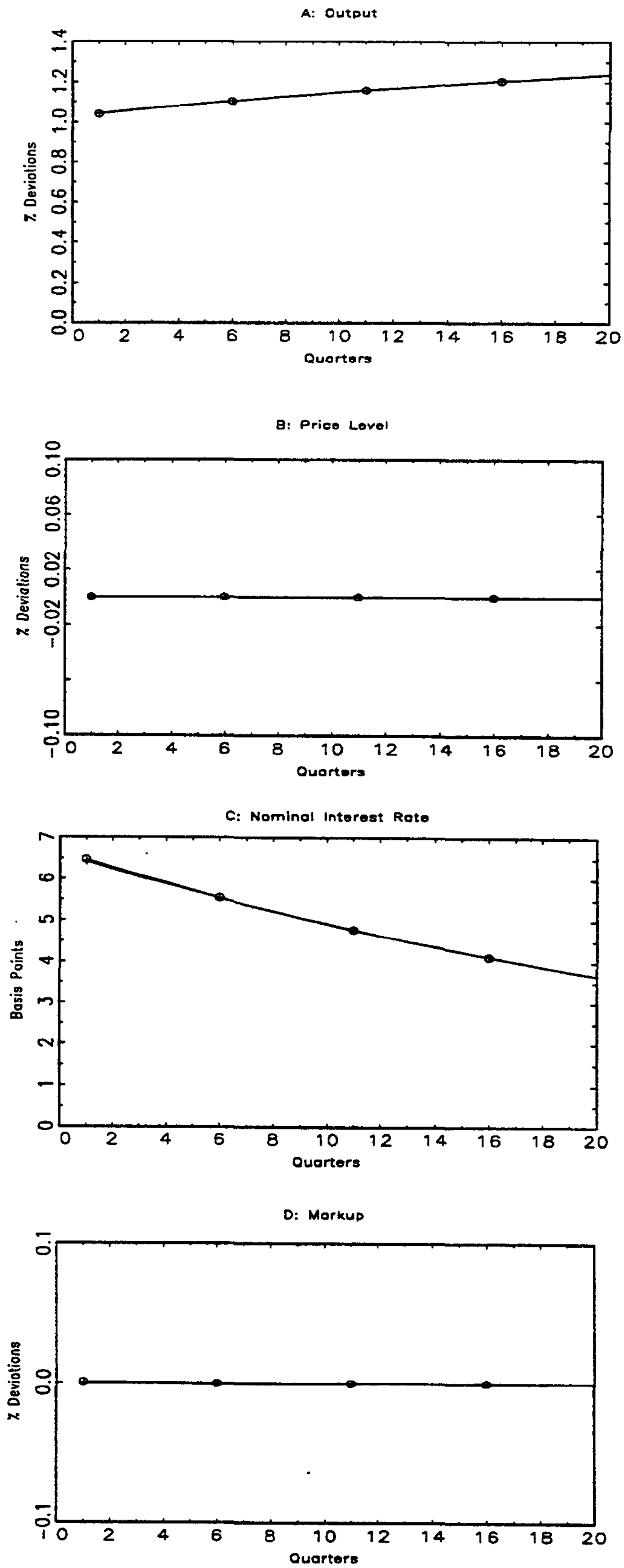


Figure 11. Permanent Increase in Money Growth

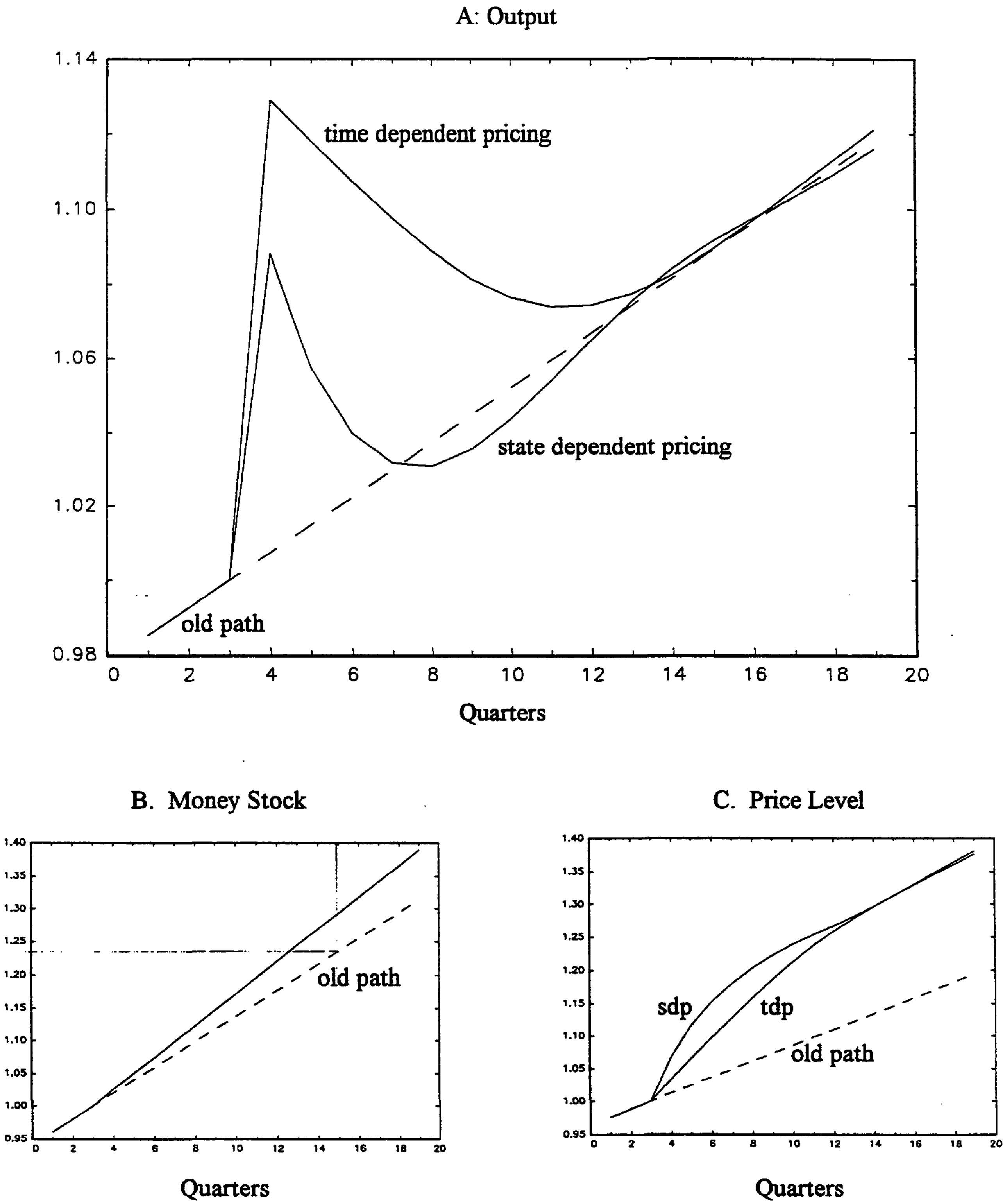
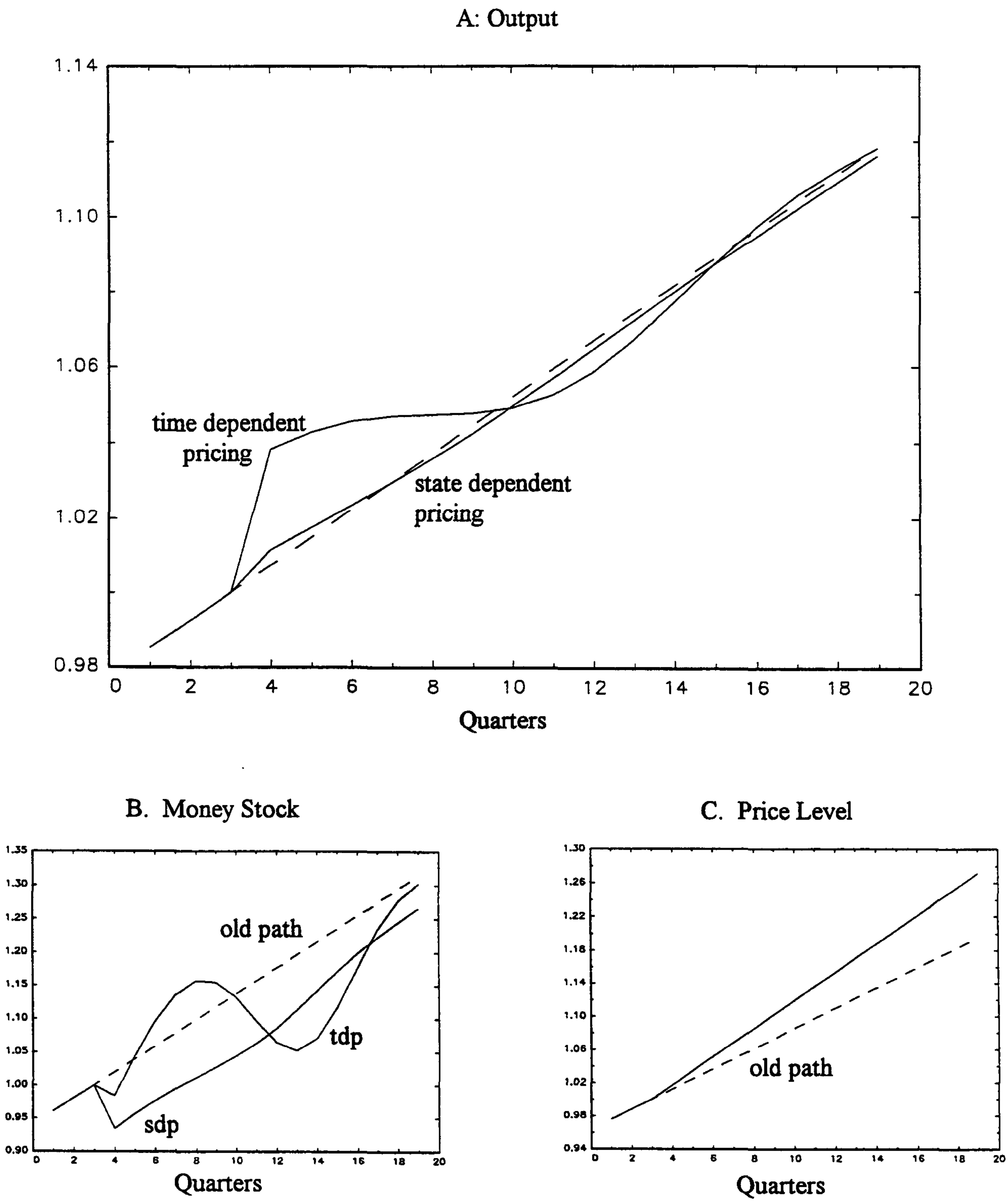


Figure 12. Permanent Increase in Inflation



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